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## A dynamical systems approach to modeling human development

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## *Department of Economics*

Working Paper 2014:9

### *A DYNAMICAL SYSTEMS APPROACH TO MODELING HUMAN DEVELOPMENT*

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## A DYNAMICAL SYSTEMS APPROACH TO MODELING HUMAN DEVELOPMENT

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# A DYNAMICAL SYSTEMS APPROACH TO MODELING HUMAN DEVELOPMENT

*By* SHYAM RANGANATHAN\*, RANJULA BALI SWAIN AND DAVID J.T.

SUMPTER

31 OCTOBER 2014

A key aim of economics is to set goals and investigate the relationship between various socio-economic indicators. By fitting time series data using a Bayesian dynamical systems approach we identify non-linear interactions between GDP, child mortality, fertility rate and female education. We show that reduction in child mortality is best predicted by the level of GDP in a country over the preceding 5 years. Fertility rate decreases when current or predicted child mortality is low, and is weakly dependent on female education and economic growth. As fertility drops, GDP increases producing a cycle that drives the demographic transition.

JEL: C51, C52, C53, C61, J13, O21

Keywords: Demographic transition, Human Development, dynamical systems, Bayesian, data-driven, GDP, child mortality, fertility rate

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## I. Introduction

The Industrial Revolution that brought unprecedented economic growth to Western Europe and North America also coincided with a new epoch in population dynamics (Galor, 2005). Countries started moving from a regime of high mortality and high fertility to a regime of low mortality and low fertility, a process that demographers call the demographic transition. Under the assumption that these twin processes of economic and demographic transition in the post-Industrial Revolution epoch are connected, economists have built theoretical models to study the demographic transition and its effects on economic growth. Of particular interest is the general question as to whether economic growth can spur the demographic transition or vice versa. In this paper, we build a dynamical systems model of the interactions between economic growth, child mortality and fertility and use this model to answer this general question.

Among the various microeconomic models proposed to understand the interactions between population and economic growth, Becker (1981) is an influential paper that looks at how income growth results in women making a quantity-quality tradeoff in their fertility choice. Fertility choice is analyzed as a problem of maximizing the mother's (or, in general, the family's) utility and child mortality is a constraint that creates the need for a high fertility rate. This approach has been extended and tested in a variety of settings (Barro and Becker, 1989; Tamura, 1996; Doepke, 2005).

Growth economists have primarily taken a statistical approach: focusing on GDP and treating child mortality and fertility as important covariates and analysing the problem from that perspective (Durlauf, Johnson and Temple, 2005). Grounded in the theory of the Solow growth model (Barro,

1991a), growth econometrics has employed regression models to relate per capita GDP growth rate to a whole range of factors suggested by the available data (Sala-i Martin, 1997).

There are limitations to both the microeconomic approach of Becker and the statistical approach of growth econometrics. The microeconomic models provide a lot of useful information about the mechanisms through which the demographic transition takes place. Their strength lies in the fact that the model predictions can be tested against data on individual behaviour. However, such empirical evidence cannot be conclusive as to the actual mechanisms, since it remains unclear if other alternative models may have the same or better predictive power at the macro-level. There remains uncertainty as to which theoretical model is the correct one.

In general, independent of whether they take a macro or micro approach, existing models tend to focus on one aspect of a multi-dimensional process of the demographic transition, be it GDP, average child per woman, or child mortality. For instance, the economic growth variable is the critical variable in the growth econometric models and the model studies the effect of the covariates on this key variable. This leaves out the interesting complex interactions between the different indicator variables.

We argue that a better approach is to characterise the complex system comprising the different variables as a whole and study how the mutual interactions affect each variable. In this paper, we model the demographic transition as a dynamical system that is described by the indicator variables GDP, child mortality, fertility rate and female education. Changes in each variable are described as a function of the levels of the other variables and other non-linear interaction terms. We use Bayesian statistics to estimate

the best fit models to the available data (Ranganathan et al., 2014; Ley and Steel, 2009).

The fitted model allows us to disentangle the interactions taking place during the demographic transition. The first-order terms in the fitted model allow us to estimate the effects of each variable on the changes in the variable of interest and suggest mechanisms by which variables influence each other. The higher order terms and the interaction terms capture the essential nonlinearities and the interactions between the variables in this complex system. This approach provides a view of the demographic transition as a complete process.

The key results we obtain in this paper are as follows. Firstly, we demonstrate that our fitted model captures how countries move from a high mortality, high fertility, low prosperity regime to a low mortality, low fertility and high prosperity regime. More detailed analysis of the equations shows that, unlike the suggestion of Barro, economic growth does not directly impact the fertility rate but influences it through the intermediate variable of child mortality. Similarly, and this time in support of Barro (1991*b*), the fertility rate affects child mortality only indirectly by increasing or decreasing the economic growth.

We also use our model to test the effect of female education on fertility rates. This is an important problem for policymakers and a number of initiatives have been undertaken to reduce poverty in sub-Saharan Africa, India etc. by reducing fertility rates through investments in female education on the basis of prior research (Cochrane, 1979). However, other research (Cleland, 2000) suggests reductions in child mortality might be more critical in reducing fertility rates. Our model shows that up to first-order effects,

female education is an important variable in reducing fertility rates. But when we account for the non-linearities in the system and higher order effects, reducing child mortality is more important than improving female education for reducing fertility rate.

The paper is organised as follows. Section II describes the existing theoretical and empirical literature on economic growth and the demographic transition. Section III presents the data used in the paper. Section IV provides a brief overview of the methodology. Section V presents the results of our modeling. Section VI analyses the results and provides some robustness checks. In Section VII we conclude our paper with some directions for future research.

## II. Literature

The demographic transition is an important phenomenon for policymakers to understand because of its implications for society (Kalemli-Ozcan, 2002). The key idea is that improved health and life expectancy (especially lower child mortality) leads to lower fertility which leads to economic growth and improved quality of life.

### *A. Theoretical Literature*

The basic models of economic growth and its relationship to technological growth have been well-studied for more than fifty years (Solow, 1956). To understand the relationship between child mortality, fertility and economic growth, these models have been extended to include more factors.

Some studies have attempted an ambitious unified model that combines features of demographic transition and economic growth (Galor, 2005; Galor and Weil, 1996, 1999; Galor and Moav, 2002; Lucas, 1988). These models



rely on the idea that technological progress leads to a quality-quantity trade-off between investing in children's well-being and number of children. This in turn results in a fertility transition from having many under-educated children to having a few well-educated children in a more developed society which then causes a sustained growth in income. For instance, Galor and Moav (2002) argue that technological progress led to an increase in returns to education in the post-industrial revolution period. Galor and Weil (1996) suggest that the declining gender wage gap has led to higher wages for women. This has led to a fertility decline because of the change in women's choices trying to maximise their overall utility.

Economic theories that attempt to integrate the demographic transition with theories of long-run economic growth mostly rely on the fertility aspect of demographic transition (Galor and Weil, 1999; Greenwood and Seshadri, 2002). Such models tend to neglect or minimise the effects of mortality.

To include the effects of mortality in a fertility transition setup, at the individual level, Becker's classic paper (Becker, 1981) has been extended in Barro and Becker (1989) where child mortality impacts the overall cost of a surviving child. Thus, declining child mortality reduces the cost of the surviving child, and may increase net fertility initially but a tradeoff between quantity and quality of children results in a fertility decline following a mortality decline. This suggests that richer countries which have lower child mortality levels will undergo a fertility transition first. But Doepke (2005) notes that fertility rates have declined in countries with significantly different levels of income. Further refinements to this approach have been made to allow for uncertainty and sequential fertility choice (Sah, 1991; Kalemli-Ozcan, 2002; Wolpin, 1997; Doepke, 2005).

The link between child mortality and fertility may also be explained in terms of replacement and hoarding responses. If fertility choice is sequential, the parent can choose a target number of births and *replace* a dead child with another. Thus the number of surviving children equals the target chosen by the parent. As child mortality declines, total fertility also declines. However, if instead of replacing children, parents decide to raise fertility (or *hoard* on children) in anticipation of future deaths, a decline in child mortality may lead to an increase in net fertility, provided this hoarding effect is sufficiently strong.

Another research strand emphasises human capital accumulation as the main driver of economic growth. Mortality and fertility rates also impact the human capital investment decisions of parents. Clearly, the consumption of goods of families would be affected by the costs of raising a child and providing for the child's healthcare. Extending this idea to the country level, Heckman et al. (2002) argues that the return to human capital is highest before the age of five years and hence child mortality and fertility rate have a critical impact on a country's future economic growth.

### *B. Empirical Literature*

The literature on economic growth has pointed out the importance of both child health and fertility as important variables affecting GDP of a country (Durlauf, Johnson and Temple, 2005). The Barro regressions, the research strand originating from Barro's seminal paper (Barro, 1991*b*), have repeatedly shown the importance of fertility rates and child mortality to economic growth across different panel data.

Since infant mortality and child mortality <sup>1</sup> are both important indicators of child health, it is necessary to look at both their effects on fertility decline. The evidence on whether decrease in infant mortality causes a decline in fertility is mixed (Van de Walle, 1986; Galloway, Lee and Hammel, 1998; Rosero-Bixby, 1998; Eckstein, Mira and Wolpin, 1999). This suggests that child mortality might have a more direct bearing on fertility decline and, hence, on the demographic transition.

But, even for child mortality, the relationship with fertility rates is not straightforward as Galor (2005) notes in his survey on the demographic transition literature. The paper also provides a number of other possible factors such as the rise in demand for human capital and the rise in level of income per capita that may have caused the transition in Western Europe and the United States. Based on data for the period 1960-1963 in Israel, Ben-Porath (1976) finds that a decline in child mortality would lead to an increase in net fertility<sup>2</sup>. Barro (1991*b*) finds that while child mortality is positively correlated with total fertility, there is no significant effect of child mortality on net fertility. Haines (1998), however, finds that for the United States census data from 1900 and 1910, mortality decline raises total fertility while lowering net fertility.

These observations can be explained using the replacement and hoarding effects, the mechanisms by which women make fertility choices in an uncertain mortality environment. For developing countries, factors other than replacement and hoarding behaviour could also play an important role. Rut-

<sup>1</sup>The number of children dying before age 1 per thousand live births is the infant mortality rate indicator variable while the number of children dying before age 5 per thousand live births is the child mortality rate.

<sup>2</sup>The total fertility rate is the average number of children that would be born to a woman in her reproductive lifetime based on prevailing age-specific rates and the net fertility is net of fertility and mortality.

stein and Medica (1978) shows that infant mortality may lead to a decrease in subsequent fertility due to secondary effects. The authors argue that when health problems like infectious disease lead to a child’s death, the mother’s health may also be affected and it may lead to a decline in her subsequent fertility. Other studies show that factors such as sex of the child or birth order also impact fertility due to socio-cultural norms in the country being studied. For instance, increase of subsequent fertility is significantly lower if there are surviving sons as opposed to surviving daughters (Heer and Wu, 1978).

In developed countries, economic factors may also play a crucial role in fertility decline as suggested by the Becker fertility choice model. For example, Eckstein, Mira and Wolpin (1999) study the Swedish fertility data and find that while more than two-thirds of the decline in fertility is explained by reduction in child mortality, the rest is explained by increases in real wages.

### **III. Bayesian Dynamical Systems Models**

Many theoretical models make assumptions about specific causative mechanisms, and then test these assumptions against data. The model comes first and the data is used to support the model. We adopt a different methodological approach to the demographic transition. Using the data available, we fit a general non-linear differential equation model that allows for polynomial terms representing the interactions between indicator variables. Instead of restricting the types of interactions allowed (as would be done in a theoretical model with specific assumptions on mechanisms), we use Bayesian statistics to choose the best possible differential equation model that fits the

available data. The polynomial terms in the model themselves suggest the most feasible mechanism that might drive the process forward. Below, we briefly describe the methodology adopted for a system with two variables. A full description of the methodology adopted here can be found in Ranganathan et al. (2014). We also provide a toolbox in R (Ranganathan et al., 2013) for performing the fitting procedure.

We illustrate our approach in modelling the yearly changes in the indicator variables, say  $x_1$  and  $x_2$ , as possibly non-linear functions of  $f_1(x_1, x_2)$  and  $f_2(x_1, x_2)$ . Thus, the underlying model for the system is given by

$$\begin{aligned} (1) \quad \frac{dx_1}{dt} &= f_1(x_1, x_2) + \epsilon_1 \\ (2) \quad \frac{dx_2}{dt} &= f_2(x_1, x_2) + \epsilon_2 \end{aligned}$$

$\epsilon_1$  and  $\epsilon_2$  are noise variables and we assume that they are independent Gaussian variables with mean of zero and variance equal to the sample variance in the data. We then use polynomial functions where each term is of power  $-1, 0, 1$  or is the product of such powers of the variables. The product terms capture non-linearities due to interactions between the variables. We also include quadratic terms in the variables and their reciprocals to capture non-linear effects due to the variables themselves. For instance, the model for change in  $x_1$  is

$$\begin{aligned} f_1(x_1, x_2) = & a_0 + \frac{a_1}{x_1} + \frac{a_2}{x_2} + a_3x_1 + a_4x_2 + \frac{a_5}{x_1x_2} + \frac{a_6x_2}{x_1} + \frac{a_7x_1}{x_2} + \\ & + a_8x_1x_2 + a_9x_1^2 + a_{10}x_2^2 + \frac{a_{11}}{x_1^2} + \frac{a_{12}}{x_2^2} \end{aligned}$$

Including all possible terms in a model would make it unwieldy and overfit the data. There are 13 models with one term and, in general  $\binom{13}{m}$ , models with  $m$  terms.

To do model selection efficiently, we use a two-step algorithm. First, we rank the models with a given number of terms  $m$  according to the log-likelihood values. Specifically, the log-likelihood of the best fit for  $dx$  models with  $m$  terms is

$$(3) \quad L(m) = \log P(dx_1|x_1, x_2, m, \phi_m^*)$$

where  $\phi_m^*$  is the set of unique parameter values obtained from the best fit regression out of all of the  $\binom{13}{m}$  models with  $m$  terms. This gives us a measure of how the models best fit the available data (Bishop, 2006).

The log-likelihood value increases with increasing complexity of the model as the data can be fit better with additional terms. However, there is a diminishing returns effect due to overfitting. More complex models fit the available data better but fare poorly when confronted with new data. So, in the second step of the algorithm, we choose the most robust of these models by computing the model with the highest Bayes factor (Robert, 1994),

$$(4) \quad B(m) = \int_{\phi_m} P(dx|x, y, m, \phi_m^*) \pi(\phi_m) d\phi_m$$

The model that most efficiently describes the available data is then the model with the highest Bayes factor that is chosen by the second step of our algorithm. In order to see how the number of terms affect model fit, we usually plot  $L(m)$  and  $B(m)$  as a function of  $m$  for different variable combinations. This allows us to quickly assess the relative advantage of a

particular model in predictive power.

In the above description, we have assumed that the modelling errors in Eq. 2 are uncorrelated. However, in most real systems, the errors are correlated due to the presence of omitted, latent variables that affect both  $x_1$  and  $x_2$ . Such correlated errors can be handled in a straightforward fashion using the seemingly unrelated regressions approach (Amemiya, 1985). If the noise variables are also correlated over time, which is a realistic assumption in social systems where there is systematic distortion, we need more complex, time-series methods to handle these. The presence of lagged effects (the current child mortality level may influence future fertility rate values and not the current value, for instance) is another commonly observed phenomenon in realistic, social systems. We model the lagged effects by allowing for polynomial terms that are the lagged variables and use the same approach as before to find the best model. Similarly, it is straightforward to extend approach to three or more variables, and details of this are given in Ranganathan et al. (2014).

#### IV. Data

The data used in the paper has primarily been taken from the World Bank ‘World Development Indicators’ dataset. This contains data for nearly 200 countries for a period of more than 50 years. For the economic indicator, we use the GDP per capita (in constant 2005 dollars) from the publicly available Gapminder dataset. Documentation for this is provided at [www.gapminder.org](http://www.gapminder.org). In the years of interest for us 1950-2009, the data is identical to the World Bank dataset. We use the log GDP value and call

the variable  $G$  in the analysis.

We use child mortality as the mortality indicator (denoted by  $C$ ). Child mortality refers to the number of children not surviving to age 5 per 1,000 live births and is a strong indicator of child health. The total fertility rate is the fertility measure (denoted by  $A$ ) and is defined as the average number of children a woman has in the course of her lifetime. The data on these indicator variables are available in both the World Bank and the Gapminder datasets.

## V. Results

Our demographic transition model has three indicator variables  $C$ ,  $G$  and  $A$ . However, we first illustrate the method by constructing a two variable model that explains the relationship between  $C$  and  $G$ .

### A. *The effect of economic growth on child mortality*

Fig. 1 shows the phase portrait of the  $C$  and  $G$  data. A phase portrait refers to a trajectory plot of a dynamical system. In Fig. 1, the yearly changes in the indicator variables  $C$  and  $G$  are plotted as vectors in the  $C$  and  $G$  plane. The yearly change is different for different countries and in different years. In general, the yearly changes are a function of current levels of  $C$  and  $G$  and Fig. 1 shows this relationship.

We then use the data shown in the phase portrait to fit a model to the interactions between  $C$  and  $G$ . The following model has the highest log-likelihood for two polynomial terms, i.e. maximum value for  $L(2)$ . The



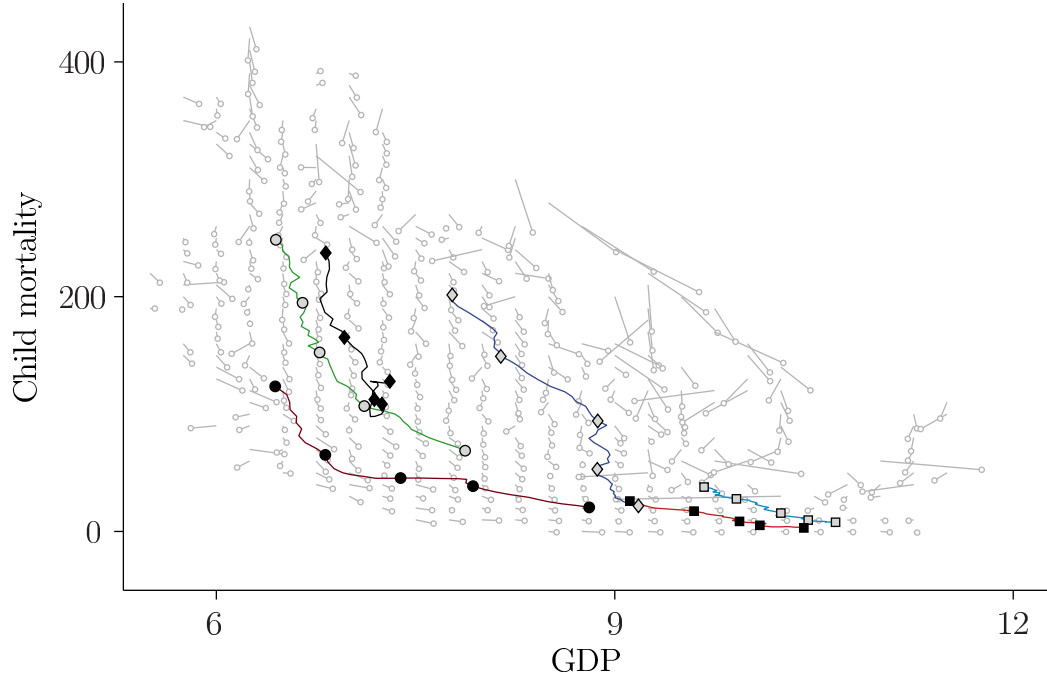


FIGURE 1. DATA PHASE PORTRAIT FOR CHILD MORTALITY AND GDP. DOTS REPRESENT DEVELOPMENT STATES AND LINES SHOW AVERAGE YEARLY CHANGE IN INDICATORS. DEVELOPMENT STATISTICS SHOW CHILD MORTALITY DECREASING AND GDP INCREASING ALMOST THROUGHOUT. THE CONTINUOUS LINES REPRESENT TRAJECTORIES FOR DIFFERENT COUNTRIES OVER THE LAST FIFTY YEARS. THE COUNTRY CODE IS: SOLID CIRCLE - CHINA, HOLLOW CIRCLE - INDIA, SOLID DIAMOND - KENYA, HOLLOW DIAMOND - BRAZIL, SOLID SQUARE - SWEDEN, HOLLOW SQUARE - USA

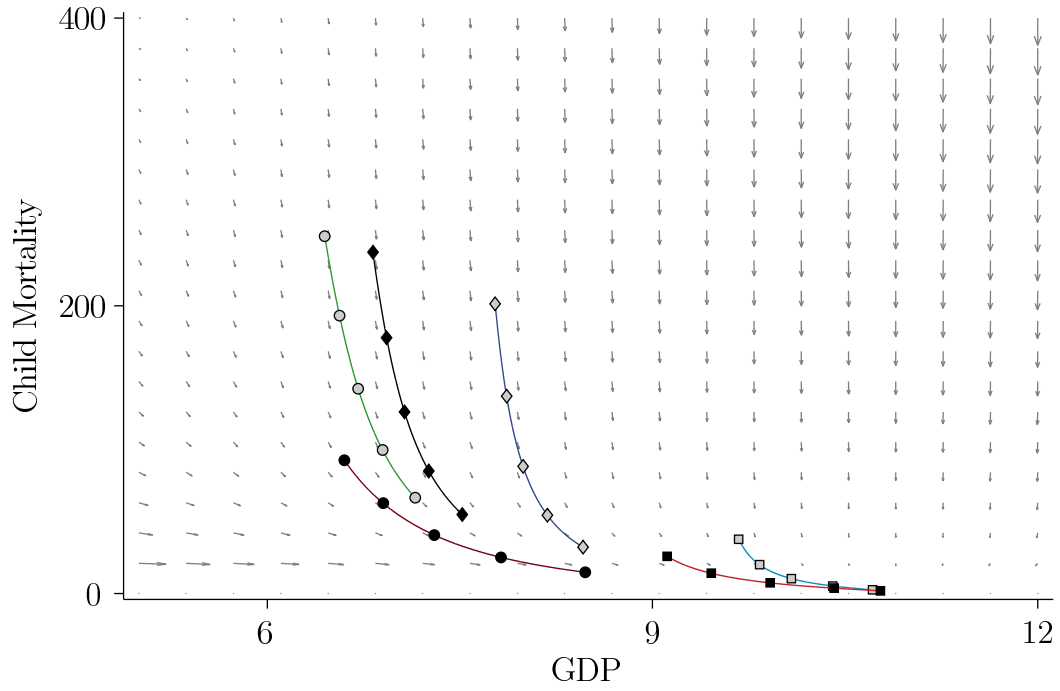


FIGURE 2. MODEL PHASE PORTRAIT FOR CHILD MORTALITY AND GDP. LINES AND DOTS REPRESENT THE DEVELOPMENT STATES AND PREDICTED CHANGES BASED ON THE MODELS. THE DARK, CONTINUOUS LINES REPRESENT TRAJECTORIES FOR DIFFERENT COUNTRIES OVER THE LAST FIFTY YEARS. THE COUNTRY CODE IS: SOLID CIRCLE - CHINA, HOLLOW CIRCLE - INDIA, SOLID DIAMOND - KENYA, HOLLOW DIAMOND - BRAZIL, SOLID SQUARE - SWEDEN, HOLLOW SQUARE - USA

phase portrait for this model is shown in Fig. 2.

$$\begin{aligned} (5) \quad \frac{dC}{dt} &= -0.0028C(1.6G - 0.02C) \\ (6) \quad \frac{dG}{dt} &= \frac{2.5}{CG}(10.9 - G) \end{aligned}$$

These equations summarize a number of important facts about how these indicators have changed over time. Firstly, we note that child mortality declines on average, with the mean fractional decrease per year equal to

$$0.0028(1.6G - 0.02C)$$

Percentage decrease in child mortality is therefore larger when GDP is high and when child mortality is low.  $G$  is log GDP, and as a result the right hand side of Eq. 6 gives the percentage change in GDP. Eq. 6 implies that GDP increases faster when child mortality and GDP are low, and decreases when  $G > 10.9$ . This value, incidentally, is close to Switzerland's GDP per capita in 2007.

The above discussion simply tells us the best 2-term model for fitting the data and tells us nothing about how reliable the model is compared to alternatives. Fig. 3 gives log-likelihood  $L(m)$  and Bayes factor  $B(m)$  as a function of the number of terms  $m$  in the model for child mortality. Here only the best possible models with  $m$  terms obtained from linear regression analysis are considered. While  $L(m)$  increases with number of terms,  $B(m)$  increases very little or decreases for  $m > 2$ . Looking at the plot, we see that the three term model for  $\frac{dC}{dt}$  involves adding a constant term and changing the constants related to the effect of  $G$  and  $C$ . Importantly, the additional

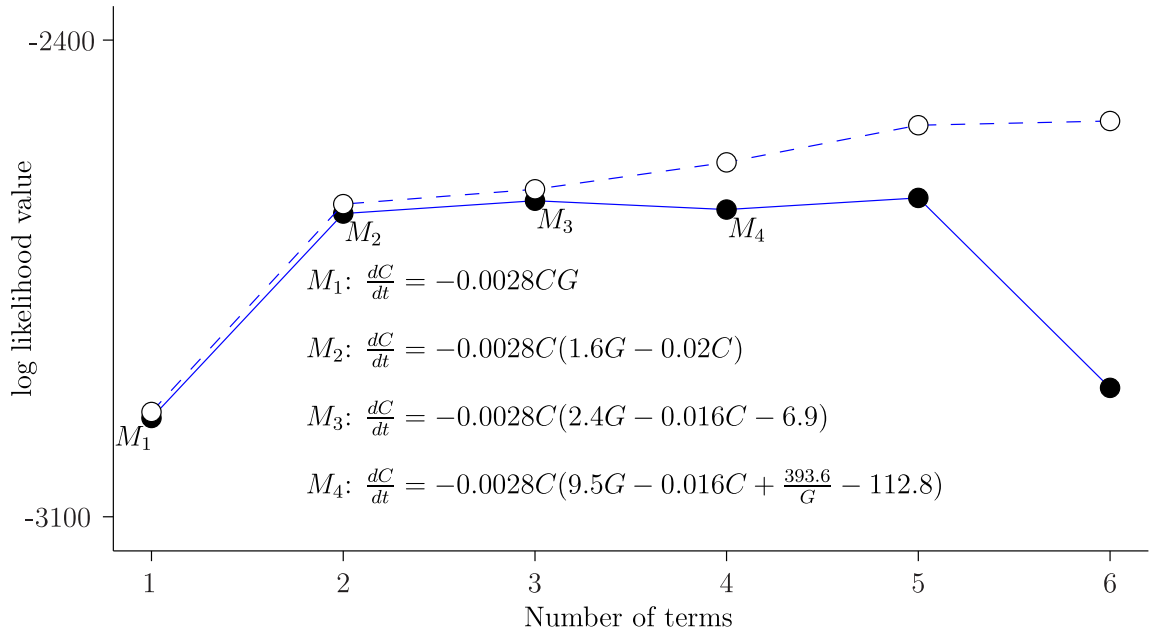


FIGURE 3. THE LOG-LIKELIHOOD (HOLLOW CIRCLES) AND LOG-BAYES FACTOR (SOLID CIRCLES) FOR  $dC$  MODELS. LOG-LIKELIHOOD VALUE INCREASES WITH NUMBER OF TERMS BUT BAYES FACTOR DECREASES AFTER  $m = 3$

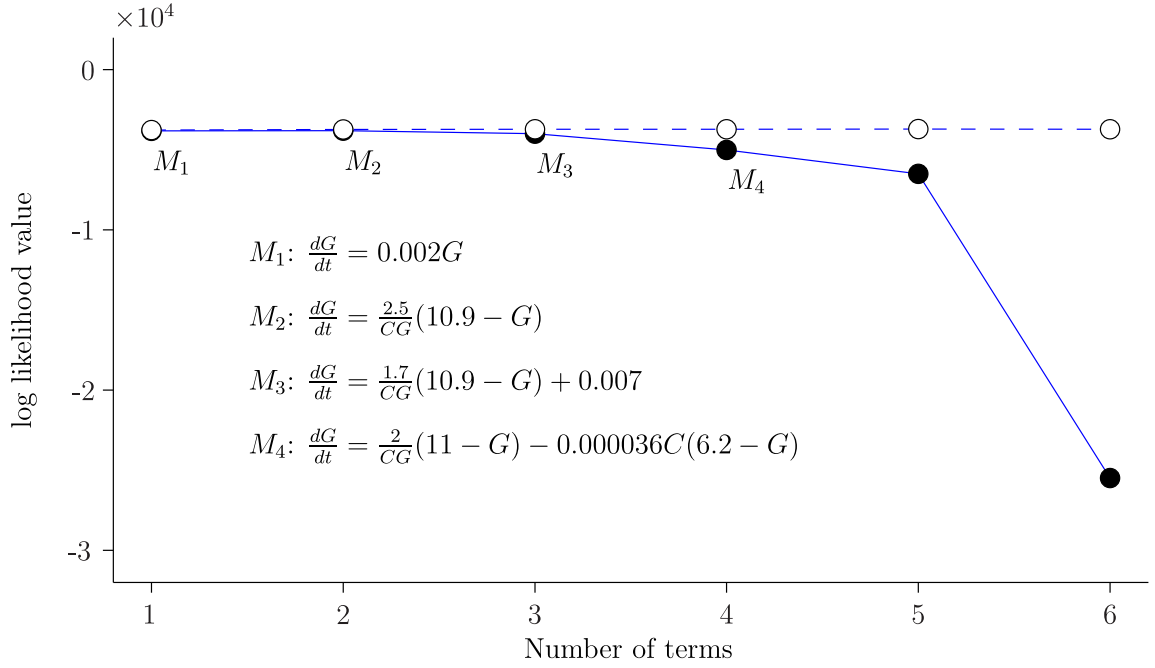


FIGURE 4. THE LOG-LIKELIHOOD (HOLLOW CIRCLES) AND LOG-BAYES FACTOR (SOLID CIRCLES) FOR  $dG$  MODELS. LOG-LIKELIHOOD VALUE INCREASES WITH NUMBER OF TERMS BUT BAYES FACTOR DECREASES CONTINUOUSLY

term does not change the role of  $G$  and  $C$  in determining  $\frac{dC}{dt}$ . As in the two term model, higher GDP and lower child mortality still cause greater decreases in child mortality. Similarly the four term model adds another term ( $1/G$ ) to the three term model and slightly alters the parameter values for the other terms. The overall effect of this addition remains the same, so that the percentage change in child mortality is high when GDP is high and child mortality is low. This conclusion reassures us about the robustness of Eq. 5 as a model. If by adding an extra term we had completely changed the interpretation of the model, then we would have less confidence that this model provides a reflection of an underlying reality.

The fit of the two term model for  $\frac{dG}{dt}$  improves only slightly relative to the one term model (Figure 4). This result reduces our confidence in Eq. 6 as a good model. The one term model with the highest log-likelihood is

$$\frac{dG}{dt} = 0.002G$$

implying that child mortality is not an important factor in economic growth. The exponential growth model, where  $\frac{dG}{dt} = 0.018$ , is also reasonably close to the best model in terms of Bayes factors. These results are consistent with earlier conclusions that child mortality does not directly impact economic growth (Fernandez, Ley and Steel, 2001; Kalemli-Ozcan, 2002).

Finally, we estimate the error covariance matrix for these models. We find that the off-diagonal terms in the covariance matrix (scaled such that the diagonal elements are 1) are -0.1261. This indicates that the error terms are largely uncorrelated. Applying the seemingly unrelated regression

approach described in detail in (Ranganathan et al., 2014), we see that the best parameters also change only slightly compared to our original estimate. This gives good support to Eqs. 5 and 7 as a model of the interactions between GDP and child mortality.

### *B. Economic growth, Child Mortality and Fertility*

The natural next step in modeling the demographic transition is to add fertility rate to our two variable model of child mortality and GDP. We first fit models for the yearly changes of  $\frac{dC}{dt}$ ,  $\frac{dG}{dt}$  and  $\frac{dA}{dt}$  using polynomial functions of the three variables,  $C$ ,  $G$  and  $A$ . We use the Bayes factor  $B(m)$  to choose the best fit models as a function of the number of terms,  $m$ . Fig. 5 gives Bayes factor values for the best model for  $\frac{dC}{dt}$  as a function of all three variables (solid circles), and as a function of just two variables  $C$  and  $G$  (hollow circles), and  $C$  and  $A$  (crosses). Here we see that the addition of a fifth term makes little improvement in the Bayes factor for the three variable model, and we conclude that a model with four terms is sufficiently good. We note also that of the models with only the  $C$  and  $G$  indicators has a Bayes factor much closer to the three variable model than the model with only  $C$  and  $A$ . This observation suggests that fertility rate is less important than GDP as a predictor of changes in child mortality. This is in line with the observation in Barro (1991b) that most of the correlation between fertility rate and child mortality can be accounted for by the number of children who do not survive past age 4. GDP is an important factor in changes in child mortality is because a high GDP correlates with larger investments in child health etc.

In the case of GDP (Fig. 6), the two variable  $A$  and  $G$  model has a higher

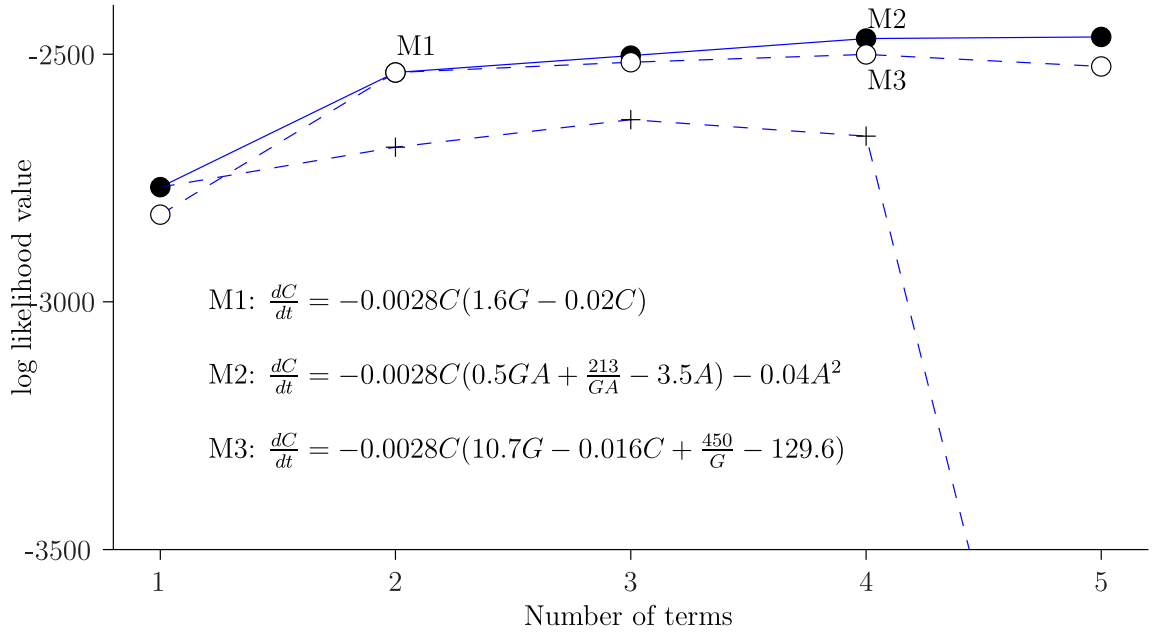


FIGURE 5. BAYES FACTOR PLOT FOR  $\frac{dC}{dt}$  MODELS. SOLID CIRCLES CORRESPOND TO MODELS WITH ALL THREE VARIABLES INCLUDED, HOLLOW CIRCLES TO MODELS WITH ONLY  $C$  AND  $G$  AND  $+$  TO MODELS WITH ONLY  $C$  AND  $A$



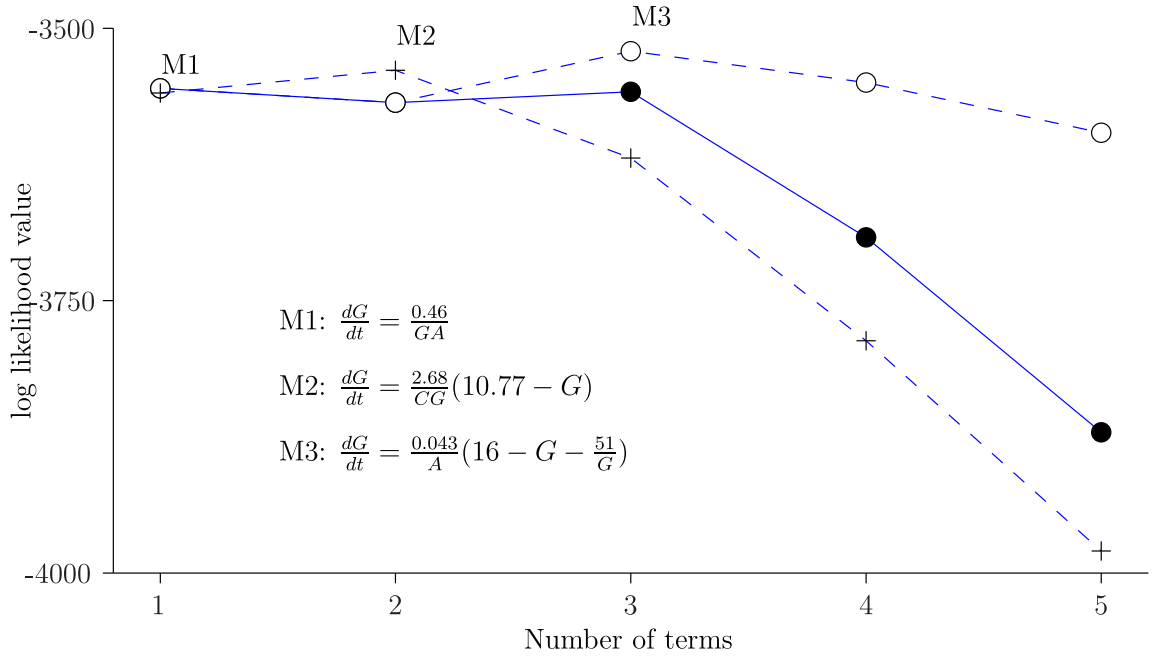


FIGURE 6. BAYES FACTOR PLOT FOR  $\frac{dG}{dt}$  MODELS. SOLID CIRCLES CORRESPOND TO MODELS WITH ALL THREE VARIABLES INCLUDED, HOLLOW CIRCLES TO MODELS WITH ONLY  $G$  AND  $A$  AND  $+$  TO MODELS WITH ONLY  $G$  AND  $C$

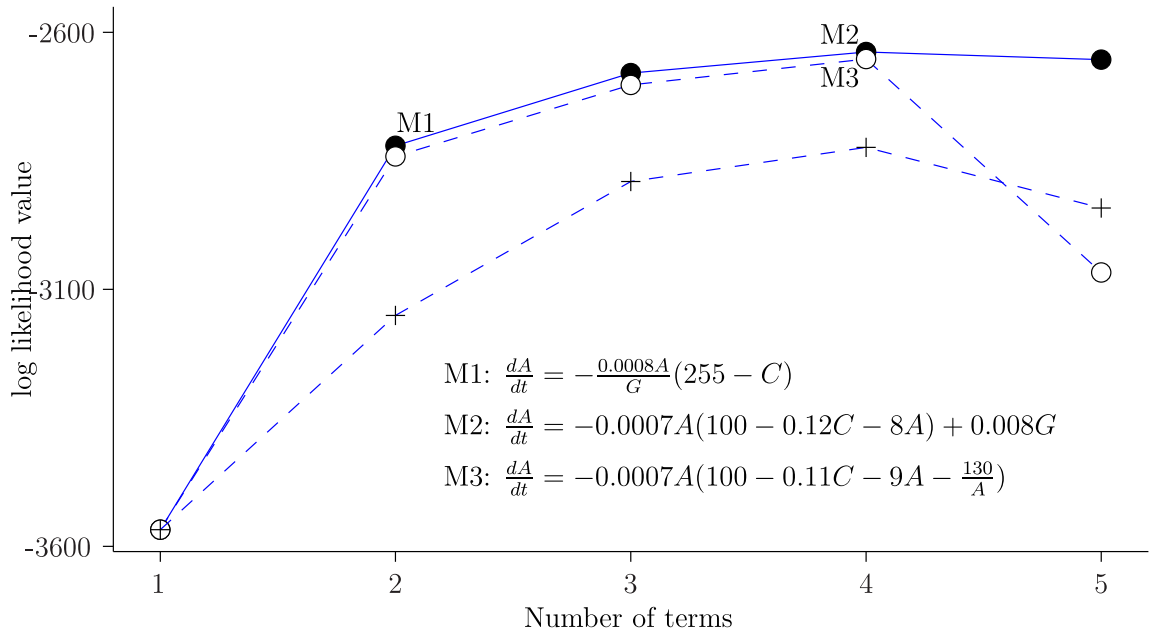


FIGURE 7. BAYES FACTOR PLOT FOR  $\frac{dA}{dt}$  MODELS. SOLID CIRCLES CORRESPOND TO MODELS WITH ALL THREE VARIABLES INCLUDED, HOLLOW CIRCLES TO MODELS WITH ONLY  $A$  AND  $C$  AND  $+$  TO MODELS WITH ONLY  $A$  AND  $G$

Bayes factor than the three variable model which includes  $C$ . This provides further support for the earlier conclusion that child mortality is not a good predictor of changes in GDP. For fertility (Fig. 7), the two variable  $A$  and  $C$  model has Bayes factor close to the three variable model, while the  $A$  and  $G$  model has a much lower Bayes factor. This suggests that  $C$  is a more important explanatory variable for fertility rate than  $G$ .

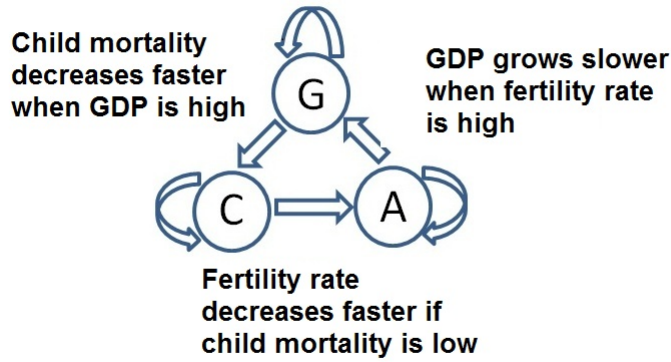


FIGURE 8. MOST SIGNIFICANT PREDICTOR FOR GDP, CHILD MORTALITY AND FERTILITY RATE - TWO VARIABLES CAPTURE MOST OF THE INFORMATION IN THE DATA FOR CHANGES IN THE INDICATORS

The Bayes factor approach provides a measure of likelihood of various models and Figs. 5-7 allow us to weigh up the relative value of particular models. For GDP we see that the model

$$(7) \quad \frac{dG}{dt} = \frac{0.043}{A} \left( 16 - G - \frac{51}{G} \right)$$

has the highest Bayes factor. In this model, a high fertility rate slows economic growth. Solving  $\frac{dG}{dt} = 0$  gives two equilibrium points,  $G^* = 4.4$  and  $G^* = 11.6$ , suggesting that there is a slowdown in growth at both low and high GDP. Note that neither of these values is seen in the data as they represent extremely low and extremely high GDP values. We may however interpret the steady state value of  $G^* = 4.4$  as evidence of a ‘poverty trap’ where countries are forced by certain self-reinforcing mechanisms to be trapped in a state of poverty without escape except through external means (Bowles, Durlauf and Hoff, 2006). The other steady state  $G^* = 11.6$  can be interpreted not so much as an upper limit on the economic growth, but more as a slowing of growth in rich countries.

For the child mortality and fertility rate models, the Bayes factor for models with only two variables ( $G$  and  $C$  in the case of child mortality, and  $C$  and  $A$  in the case of fertility rate) is relatively close to that of the models where all 3 variables are used. To provide an economical description of the available data, this suggests that two variable models are sufficient as the addition of one more variable does not provide much more information. Thus the following equations capture the essential nature of the interactions of these indicators.

$$(8) \quad \frac{dC}{dt} = -0.0028C(1.6G - 0.02C)$$

$$(9) \quad \frac{dA}{dt} = -0.0007A(100 - 0.11C - 9A - \frac{130}{A})$$

These equations, combined with Eq. 7, provide an overall structure for how the indicators interact. This is illustrated digramatically in Fig. 8.

The overall cycle illustrated here is that child mortality decreases faster with higher GDP, fertility rate decreases faster when child mortality is low and decreased fertility rate is associated with growth in GDP. In going through the cycle, we can see that there is a tendency of countries to go from the regime of high mortality, high fertility and low prosperity to a regime of low fertility, low fertility and high prosperity as seen repeatedly in the experience of developed countries. The cycle also allows us to understand why certain countries may fail to reach higher prosperity levels and suggests where interventions may be useful through policy initiatives.

In addition to pointing to the basic structure of interactions, the models above also show the non-linearities involved. For example, we see that the fertility rate decreases faster when it is itself high, but this decrease is slowed if child mortality is also high. There is also a secondary effect which slows the percentage decrease in the fertility rate when it is very low. The model shown above has two non-trivial equilibrium points (at roughly  $A^* = 10$  and  $A^* = 1.5$ ) obtained by solving the Eq.  $\frac{dA}{dt} = 0$ .

These equilibrium points are not observed in the data itself and are representative of extreme values. The steady state value  $A^* = 10$  would correspond to a country with relatively low  $G$  and high  $C$ , and corresponds to the high mortality, high fertility and low economic growth regime in the demographic transition. The steady state value  $A^* = 1.5$ , which is less than the replacement fertility rate of around 2, would correspond to a low mortality, low fertility and high economic growth regime. Thus the two steady states correspond to the two opposite ends of the spectrum described in the demographic transition literature.

Finally, as a robustness check we look if the error terms in the differential

equation model are uncorrelated. We find there is only limited correlation (the maximum off-diagonal term in the scaled covariance matrix is 12% of the diagonal term). Hence we are justified in assuming that the errors across variables are almost uncorrelated and we use the models obtained using this assumption.

### *C. The effect of Education on fertility*

As seen in Barro (1991*b*) or Galor (2005), there a number of other important covariates to be considered. Many policymakers work with female education as an important tool to reduce fertility rate and increase economic growth (UN, 2002; Cochrane, 1979). International organizations such as the United Nations Population Fund and the World Bank advocate better schooling for girls as a means of achieving lower child mortality and fertility rate. However, the evidence is not conclusive as significant fertility declines have occurred without noticeable changes in female education (Basu, 2002; Cleland, 2000). To test the hypothesis on whether female education is significant for fertility decline, we construct a model relating it to total fertility and test it against a model that relates child mortality to fertility.

We define the educational indicator  $E$  to be the average years of schooling for female population as collected in the Barro-Lee dataset (Barro and Lee, 2010). Since the data is available only on a five-yearly basis in that dataset, we use linear interpolation to obtain the yearly data points in order to apply our method to the data. We then ask if we can improve the fit of the model in Eqs. 7 to 9 by including educational attainment.

We find the best models for the fertility rate ( $\frac{dA}{dt}$  models) containing only

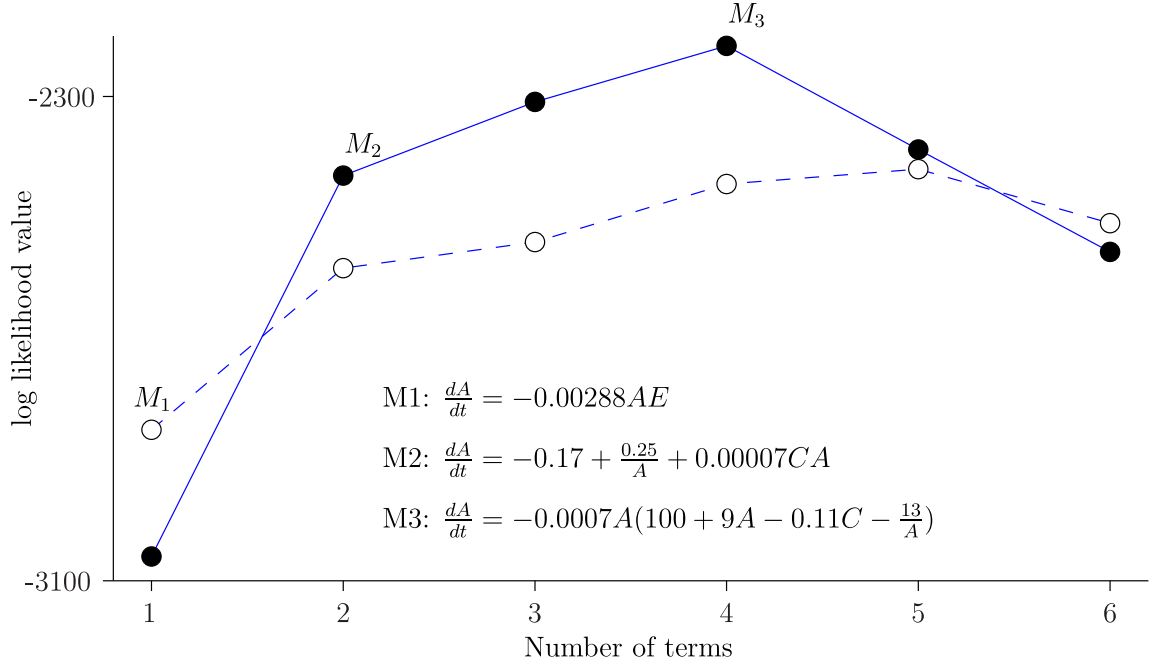


FIGURE 9. THE LOG-BAYES FACTOR PLOTS FOR A-C MODELS (SOLID CIRCLES) AND A-E MODELS (HOLLOW CIRCLES) SHOWING THAT CHILD MORTALITY IS MORE IMPORTANT THAN AVERAGE YEARS OF SCHOOLING AS AN EXPLANATORY VARIABLE FOR FERTILITY RATE. BUT FOR THE SIMPLEST ONE-TERM MODELS, THE EDUCATION INDICATOR SEEMS MORE CRUCIAL.

the two variables  $A$  and  $C$  and test them against  $\frac{dA}{dt}$  models obtained containing only  $A$  and  $E$ . If  $C$  is a more significant predictor of changes in  $A$  than  $E$ , then those models will have higher Bayes factors. We plot the Bayes factor values for both sets of models for different numbers of terms in Fig. 9.

We see that when a single term model is required to explain decrease in fertility rate, education is the best single explanatory variable. However, for models that contain 2, 3 and 4 polynomial terms, we see that the models with  $C$  are better than models with  $E$  as the explanatory variable. If we go on to calculate  $\frac{dA}{dt}$  models with three variables ( $A$ ,  $E$  and  $C$ ) we find that the

2, 3 and 4 term models with the highest Bayes factor involve only  $A$  and  $C$ . We conclude based on this that while higher female educational attainment does predict first order decreases in fertility rate well, child mortality is the more effective predictor overall.

This explains the findings of Basu (2002) and Cleland (2000): while investments in female education are valuable, it seems that improvement in child health and investments in healthcare systems in general might be more important.

## VI. Lags and leads

In socio-economic systems, the indicators may have lagged effects on one another. For instance, it is reasonable to assume that in the case of total fertility rate, women in a country may change their reproductive rates based on child mortality rates prevalent from a few years before the current year. Lead effects are also plausible, with women adjusting their fertility in anticipation of economic growth.

One way to check for lagged effects is to look at the cross-correlation between the change in one variable and the level of all other variables. The correlation coefficient between two variables  $X$  and  $Y$  defined as

$$\frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sqrt{E[(X - \mu_X)^2]E[(Y - \mu_Y)^2]}}$$

The *cross-correlation* is then the correlation co-efficient a function of the lag between the change variable and the level variables. The top left plot in Fig. 10 gives the cross-correlation coefficient between  $dC(t)$  and  $C(t - \tau)$ , where  $\tau$  is the lag in number of years, plotted as a function of  $\tau$ . In this case, the point of maximum correlation (negative in this case) is at a value



of  $\tau = 0$  implying that the current yearly change in  $C$  is best predicted by the current level of  $C$ .  $A$  also have the maximum correlation with  $dC$  (negative) at a lag of 0, suggesting that current levels of the variable is more important than past (or future) levels in predicting the current change  $dC$  but for  $dC$  with  $G$ , there is no corresponding sharp peak.

The relationship is more complicated when we consider between-variable effects. For example, there is an indication of a possible lag effect of  $C$  on  $dG$ , and of  $C$  on  $dA$ . In these cases, the correlation approach cannot provide information about the effect of lags and leads, because as we have already seen in the previous section, the interactions between the variables are non-linear.

Another issue with using this approach to identifying the best lag value is that each country is weighted equally when calculating the average correlation coefficient. Due to random effects and missing data for some countries, this might lead to spurious values when estimating the correlation coefficient between two variables.

A better approach is to study the lag effects within our Bayesian framework. We look at our best models and find the Bayes factor of these models with a lagged variable instead of the actual variable. For example, to further investigate the possible lag effect for  $G$  in the  $dC$  model, we calculate Bayes factor for

$$\frac{dC}{dt} = -0.0028C(1.6G(t - \tau) - 0.02C)$$

for values of  $\tau$  ranging between  $-15$  and  $15$  years. As a result we obtain a

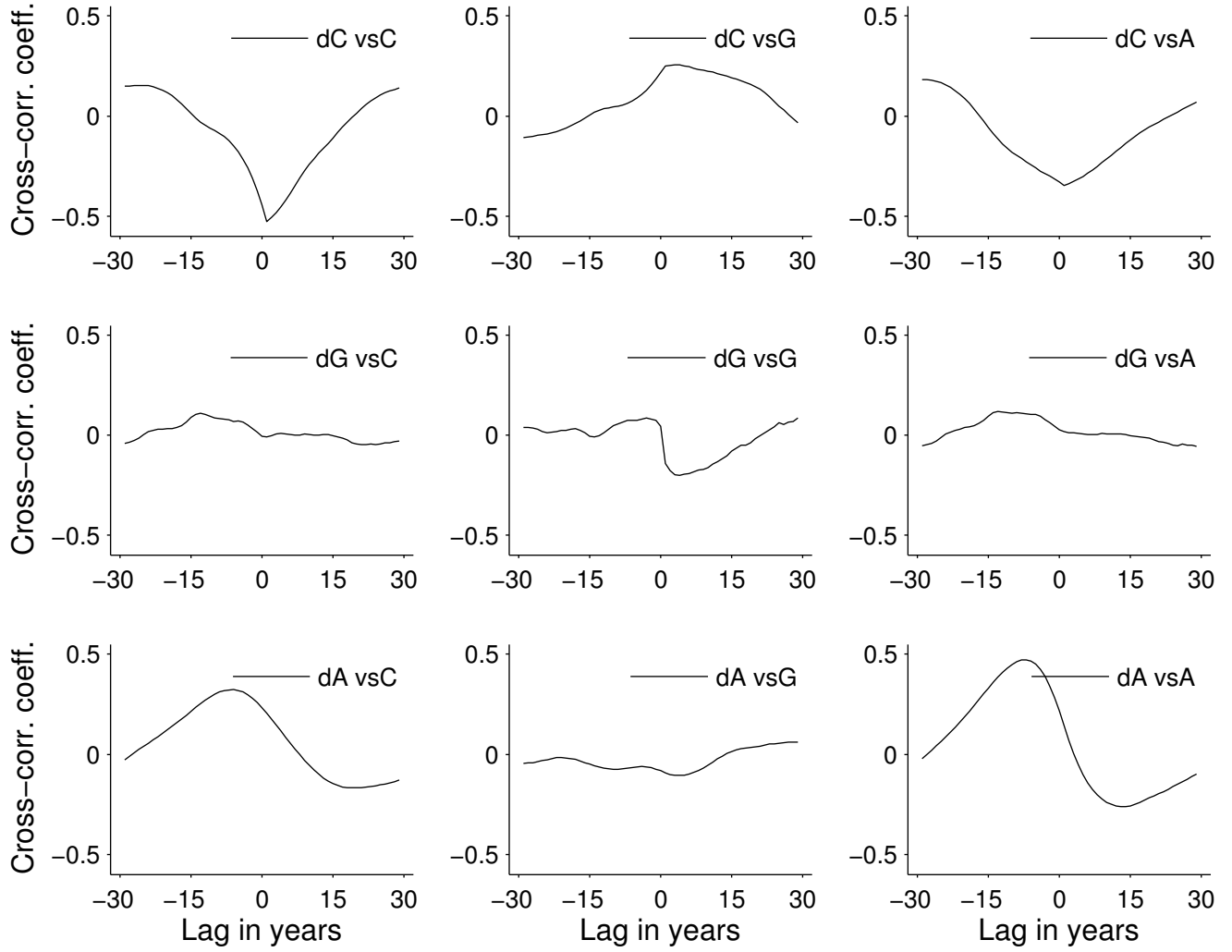


FIGURE 10. CROSS-CORRELATION COEFFICIENT BETWEEN CHANGES IN VARIABLES AND THE DIFFERENT VARIABLES. ALL THE PLOTS ASSUME A LAG IN THE LEVEL VARIABLE SO THAT A PEAK AT A LAG TIME OF 5 IN THE  $dC$  vs  $G$  PLOT CORRESPONDS TO THE OBSERVATION THAT  $dC(t)$  HAS PEAK CORRELATION WITH  $G(t - 5)$ .

plot of the Bayes factor for the model as a function of the various lags  $\tau$  and use this to identify the best lag factor for the model. Fig. 11 a shows that a lag of around 5 years gives the best model fit. Since child mortality in the

data is measured by combining various estimates over a 5 year period (detailed description in the documentation available in the data sources) this result makes sense. Thus the rate of decrease in child mortality depends on the level of GDP in the preceding 5 years.

We repeat the same procedure for the  $dA$  model

$$\frac{dA}{dt} = -0.0007A(100 - 0.11C(t - \tau) - 9A - \frac{130}{A})$$

now with  $C(t - \tau)$  as the lagged variable. The results shown in Fig. 11(b) suggest that the longer lead we use on child mortality the better prediction we get on average child per woman. This result suggests that women are using their future prediction of the probability of their child dying when making the decision how many children to have. The greater the probability of future survival, the fewer the number of children produced. However, some caution is required here. The data set for lagged variables is necessarily shorter than that used in the original non-lagged fitting (we have to use the same size data set for every lag in order to compute a consistent Bayes factor). Given the significant amount of missing data for poorer countries, the long lags might be a selection effect for richer countries where this data is available. As larger datasets become available for developing countries these results should become clearer.

Finally, the  $dG$  model does not show significant improvement when using a lagged variable (results not shown).

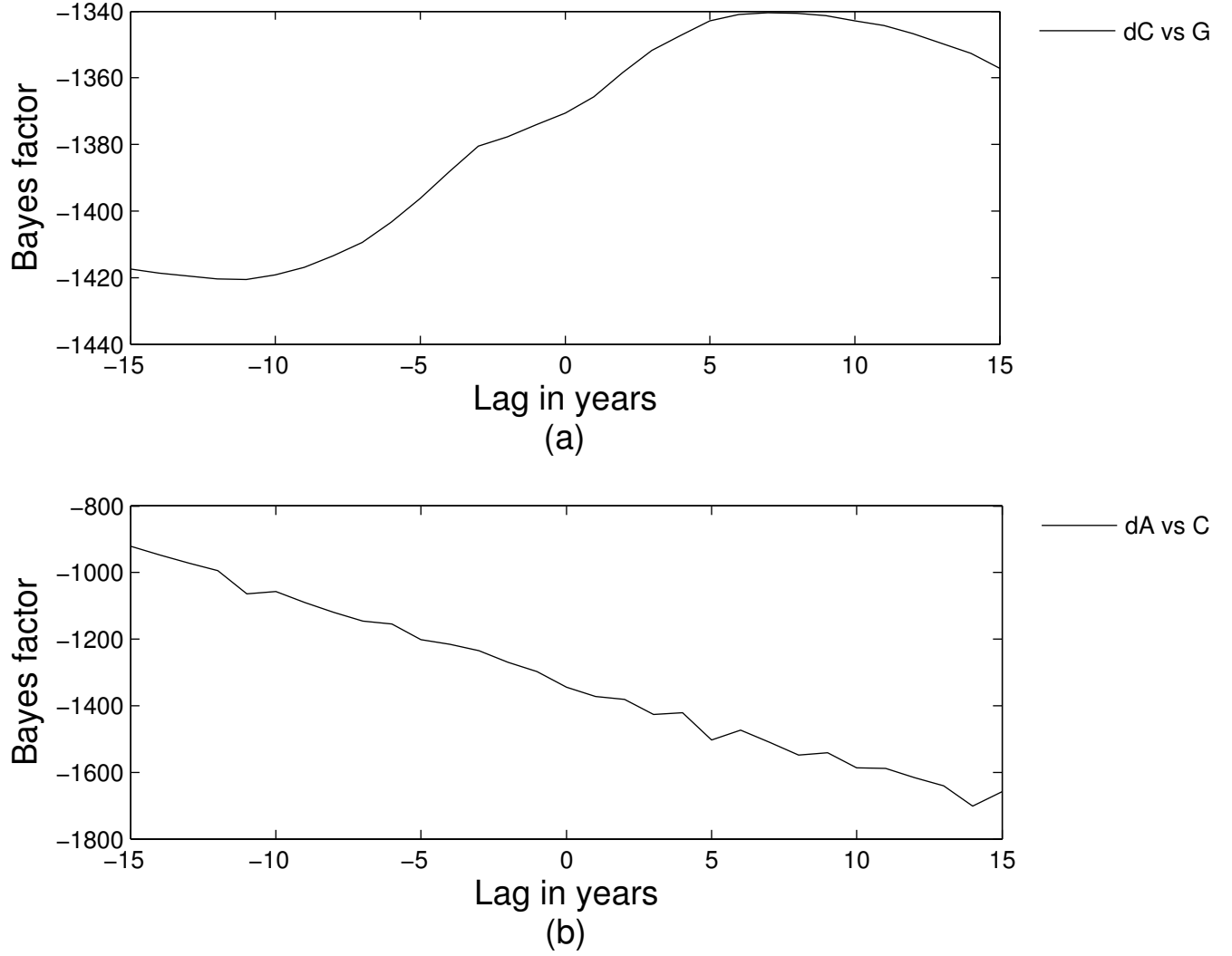


FIGURE 11. BAYES FACTOR PLOTS FOR THE  $dC$  AND  $dA$  MODELS AS A FUNCTION OF THE LAG IN  $G$  AND  $C$  RESPECTIVELY. THE  $dC$  MODEL SUGGESTS THAT A LEAD TIME OF AROUND 5 YEARS IN  $G$  IS THE BEST PARAMETER FOR THE  $dC$  MODELS. THE OPTIMAL LAG TIME FOR  $C$  IN THE  $dA$  MODEL IS NOT CLEAR FROM THE DATASET.

## VII. Causation implied by models

The innovation of the approach we have presented here lies in identifying the dynamic interactions that best explain the demographic transition.

Our approach provides (i) the phase portrait visualization of the data which is more natural to complex dynamical systems; (ii) an emphasis on yearly changes instead of long-run equilibria that may not be attained; (iii) the modelling of non-linearities in interaction terms which is the norm in most realistic complex systems; and (iv) a robust model that best explains empirical evidence on the demographic transition. A more controversial question is how we interpret our results in terms of causal mechanisms. Can we use the equations we have derived to understand the actions of people living in the countries from which data was collected? To address this question we now give an interpretation of the models we have obtained in the context of earlier theoretical literature.

Various causative mechanisms are proposed for the onset of the demographic transition. Becker (1981) and a large body of literature following his work explains the demographic transition as a consequence of increased investments in human capital due to technological change. Increased returns on education are also thought to initiate the demographic transition and therefore a decline in fertility (Galor and Weil, 1999).

Our analysis and the cycle presented in Fig. 8 emphasizes lowered child mortality over increased economic opportunities as the more immediate cause of drops in fertility rates. While higher GDP lowers child mortality, probably as a result of improvements in economic and social conditions, the best single predictor of decreases in fertility rate is child mortality. From an individual mother's point of view, if the probability of children surviving is lower, then having more children increases biological fitness. Similarly, we find that while female education does predict decreases in fertility, child mortality remains a better predictor of these decreases. The decision whether

or not to have a child may well involve a tradeoff against other economic and education opportunities (Becker, Murphy and Tamura, 1990), but it is changes in the costs of child bearing which have the greatest role in decreasing fertility.

Although the emphasis on child mortality in the relational cycle implied by Fig. 8 is different from that emphasized in earlier work, the change in focus is relatively small. Importantly, none of our findings shift us a long way from those hypotheses previously proposed about human development. Instead, our analysis sharpens the picture by finding those models that are closest to all the available aggregate data. By fitting rate of change of indicators to their current state we have looked explicitly at how the state of the world in one year leads to the state of the world the following year. However, if we are interested in establishing the causes of interactions we have to check whether our model is consistent with the already available literature. A model that doesn't make causal sense should not be accepted. In Bayesian language, such a model would have a low prior probability.

The approach we have taken in this paper can be contrasted with one that starts from the point of view of underlying micro-level interactions of economic agents. There are a number of limitations to such an approach, with respect to providing succinct and empirically accurate models of data. Firstly, although based on observations, such models do not necessarily provide the best fit to the existing data. Instead, correlational evidence is provided for particular assumptions or predictions of the model. For example, Doepke (2005) tests the Barro-Becker model against fertility and child mortality data for various European countries. In such examples, the data shows how the model fits the data within a certain error range, but this

does not rule out the existence of a large number of alternative models each of which has some degree of empirical support in the available data. The advantage of the Bayes factor-based analyses we have performed here is that they provide a likelihood measure over all plausible models.

A second limitation is that economic models usually involve specific mathematical forms that are less suited to capturing non-linear interactions in data. Equilibrium analysis plays a big role and hence the range of models which can be studied formally using the available tools is limited. While these restrictions help mathematical analysis, they are not necessarily feasible and restrict the degree to which non-linearities in the data can be captured by the model. A third limitation is that, despite their mathematical tractability, the statement of neoclassical economic models is often very complicated in comparison to a set of differential equations such as Eqs. 7 to 9.

### VIII. Conclusions

We have constructed a model of the demographic transition using the key indicator variables  $G$ ,  $C$  and  $A$ . We see that phase transitions in our dynamical system model correspond to features of the demographic transition as studied by demographers and economists. Our model captures the mechanisms by which demographic transition occurs and the specific terms show the specific quantitative impact of each state variable on the changes in the other variables. Since the system of three differential equations provides a succinct picture of the entire process, further analysis of the process is possible.

The important substantive findings in our results relate to the sequence of events in the demographic transition. It appears that the child mortality is reduced as a result of economic growth during the preceding five years. The reduction in child mortality, or possibly anticipation of this change, then drives fertility rates down. Although a predictor of fertility, female education plays a less important role than child mortality. Finally, economic growth is mostly independent of the other indicators, but is weakly driven by lowered fertility. The link back to child mortality completes the cycle through which the demographic transition takes place.

The approach we have outlined can match data well but with no underlying assumptions and thus no *a priori* causal basis. This criticism must be taken seriously because, without identifying underlying mechanisms, there are always a multitude of possible models. But it helps us discuss, in a post-analysis stage as we have done above, how the derived models relate to the micro-level motives of economic actors. Indeed, looking at the interactions we have identified and evaluating whether they agree with already known empirical evidence is an essential part of gaining a better model of a process such as the demographic transition. Models which do not correspond to reasonable mechanisms should be discarded in favour of those that do. A process of understanding interactions is not possible using statistical analysis alone, but it can be incorporated by our changing the prior probabilities of certain models. Our approach is also generally flexible in that we can incorporate specific terms to the model if their significance has been suggested by other independent research.



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