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An Indicator for Ecosystem Externalities in Fishing
- Supporting Material

Lars Ravn-Jonsen, Ken H. Andersen, and Niels Vestergaard

AN INDICATOR FOR ECOSYSTEM EXTERNALITIES IN FISHING

SUPPORTING MATERIAL

Lars Ravn-Jonsen^{a,c}, Ken H. Andersen^b, Niels Vestergaard^a

ABSTRACT

This supporting material for An Indicator for Ecosystem Externalities in Fishing provides the estimations of the cost model parameters, establishes a price model, performs sensitivity analyses of the cost model parameters and analyzes sensitivity with respect to the control variable.

Keywords: Production function, Price model, Sensitivity analyses

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A. ESTIMATION OF COST MODEL PARAMETERS

A.1. DATA

Data from three sources were used to estimate a production function. The data sources were accounting statistics, landing statistics and the ICES (International Council for Exploration of The Sea) stock assessment summary (ICES 2010b). The Danish Accounting Statistics are based on a voluntary sample of yearly accounts and comprise a total of 2 691 observations for the years 2001-2009. The data are confidential and are provided by Statistics Denmark. The landing and catch statistics record each fishing vessel's data for each landing broken down into species, size class and quality. The landings statistics are provided by the Danish Ministry of Food, Agriculture and Fisheries and are merge with the accounting statistics on Statistics Denmark's server using a specific key for each vessel.

The data are analyzed using the following model:

$$\log y_i = \alpha_0 + \sum_j \alpha_j \log x_{j,i} + \frac{1}{2} \left(\sum_j \sum_{j'} \beta_{jj'} \log x_{j,i} \log x_{j',i} \right) + \gamma(t_i) + \epsilon_i$$

$$\beta_{jj'} \equiv \beta_{j'j} \quad \epsilon_i \sim NID$$

In economics, this model is known as a translog function; however, it can be used as a second-order approximation for any functionality. Due to its second-order properties, it smoothly approximates any systematic found in the dataset. In this model, $i = 1, 2, \dots, 2691$ are the observations equivalent to the observations in the accounting statistics. The dependent variable y is the total output in terms of whole fish weight from the Accounting Statistics, and t_i is the year of observation. The variable $x = (G, k, S, F_1, F_2, F_3)$, where G represents operation costs, k is capital, S is stock and F_1, F_2 and F_3 indicate quality and size. These variables will be described in detail below. All monetary data are transformed into real value using the consumer price index (Danmarks Statistik 2011).

Operation cost G is the sum of wages, remuneration to active owners, fuel and lubrication, ice provision and storage, insurance and maintenance as given in the accounting statistics.

Capital k is the fishery assets at the beginning of the year as given in the accounting statistics.

The stock S is an indicator of the stock in which the fishing vessel is engaged:

$$S_i = \sum_l \theta_l \psi_{l,i} s_{l,t_i}$$

where l is the species and ICES area given in the landing statistics and $\psi_{l,i}$ is the catch from area and species l in observation i normalized to total landings; that is, $\psi_{l,i} = y_{l,i} / \sum y_{l,i}$, where $y_{l,i}$ is the aggregate catch corresponding to account observation i in the landing statistics for the species and ICES area l and s_{l,t_i} is the spanning stock biomass for the area and species l for the year t_i , as reported in ICES (2010b). The size-based model operates with the density of fish and assumes that the harvest is a function of the density. The ICES assessments are based on spanning stock biomass; the sea volumes covered by each species are very different and are relatively difficult to estimate. We take the approach of postulating the existence of a parameter θ_l that will transform the spanning stock biomass $s_{l,t}$ into a density, thereby rendering the densities comparable across species and additives. θ is found as the θ that maximizes the log likelihood of the model. Estimating all elements of θ will constitute an overparameterization, together with the coefficient on the main effect of the stock. Therefore, θ_1 is fixed at one, and the remaining values are found under the restriction $\theta_l > 0$.

Quality and size indicators F_1, F_2, F_3 . The production of a vessel given a particular set of input factors is dependent on the quality and size of the landed fish. The total spaces of quality and size have many dimensions; the qualities have seven categories (see Tab. 1) and are represented by their relative quantities in the landing statistics. The size distribution has been estimated for 14 classes based on the species in the landing statistics for which size sorting is conducted. For fish for reduction, the length distribution of sand eel landings (Rindoff 2011) is used. Because many of the dimensions are correlated, the quality and size of the variable space are transformed into three orthogonal dimensions F_1, F_2 and F_3 using `factanal()` (R Development Core Team 2011) to simplify and create a co-variable describing output.

The statistical model is a second-order approximation and is used to find a first-order approximation (main effects) for vessels that exhibit specific values of the input variables. In the present study, we are interested in finding values for two fleets, one fishing fish for reduction and the other fishing large fish for direct human consumption. These values are

TABLE 1: Quality labels in the landing statistics. The labels B, A and E are the standard labels for fish sold in auctions. The label I for fish for reduction is not a quality label *per se* but a landing state. However, in this analysis, the label I is treated as a quality label, overriding any other quality label given in the landing report.

Label	Quality
E	Quality Extra
A	Quality A
B	Quality B
K	Rejected
U	Unknown
X	Not graded
I	Fish landed for reduction

found as follows: The forage fish fleet comprises those vessels with $I > 0.9$, and the large fish fleet comprises those vessels with $I < 0.1$. The average \bar{G} , \bar{k} , \bar{F}_1 , \bar{F}_2 , \bar{F}_3 and \bar{S} are found for each fleet. The first-order parameters corresponding to the production function $y = aG^\alpha k^\beta S^\gamma$ are found for each fleet segment as follows:

$$\begin{aligned}
\alpha &= \frac{\partial \log y}{\partial \log G} = \alpha_G + \beta_{G,G} \log \bar{G} + \beta_{G,k} \log \bar{k} + \beta_{G,S} \log \bar{S} + \beta_{G,F_1} \log \bar{F}_1 + \\
&\quad \beta_{G,F_2} \log \bar{F}_2 + \beta_{G,F_3} \log \bar{F}_3 \\
\beta &= \frac{\partial \log y}{\partial \log k} = \alpha_k + \beta_{k,k} \log \bar{k} + \beta_{G,k} \log \bar{G} + \beta_{k,S} \log \bar{S} + \beta_{k,F_1} \log \bar{F}_1 + \\
&\quad \beta_{k,F_2} \log \bar{F}_2 + \beta_{k,F_3} \log \bar{F}_3 \\
\gamma &= \frac{\partial \log y}{\partial \log S} = \alpha_S + \beta_{S,S} \log \bar{S} + \beta_{G,S} \log \bar{G} + \beta_{k,S} \log \bar{k} + \beta_{S,F_1} \log \bar{F}_1 + \\
&\quad \beta_{S,F_2} \log \bar{F}_2 + \beta_{S,F_3} \log \bar{F}_3
\end{aligned}$$

The variance is found using the variance matrix from the model estimation. If the model is written as

$$E(\log y) = \mathbf{X}\mathbf{b}$$

where \mathbf{X} is the design matrix and \mathbf{b} the coefficient vector, the estimates above, for example, for α can be found as

$$\alpha = \mathbf{x}\mathbf{b}$$

TABLE 2: Model estimations of the first-order main effects. (Standard error is given in brackets.)

	Operation Cost	Capital	Stock
Total fleet	0.894 (0.028)	0.107 (0.020)	0.290 (0.014)
Forage fish fleet	1.120 (0.076)	0.113 (0.051)	0.175 (0.037)
Large fish fleet	0.846 (0.028)	0.104 (0.021)	0.280 (0.016)

TABLE 3: Summary of the account samples from Denmark for 2001-2009 as total numbers. The samples are the raw data obtained from the Danish Account Statistics. The cost of capital is calculated as the opportunity cost of capital set to 4% of the fishing capital.

	Unit	Forage fish	Large fish	All	Herring
Number	Companies	321	1985	2691	51
Production	ton	1 844 683	614 030	4 427 472	264 689
Revenue	k€	281 048	765 034	1 692 007	138 664
Operation Cost	k€	229 889	629 268	1 271 242	78 417
Contribution Margin I	k€	51 159	135 767	420 765	60 247
Depreciation	k€	47 251	99 745	256 764	20 601
Contribution Margin II	k€	3 908	36 022	164 001	39 646
Cost of capital	k€	21 953	53 793	143 181	18 058
Rent	k€	-18 045	-17 771	20 820	21 588

where the \mathbf{x} vector represents the partial deviates

$$\mathbf{x} = (0, 1, 0, 0, 0, \log \bar{G}, 0, 0, \dots)$$

The variance of the estimate is then found as

$$\text{var}(\alpha) = \mathbf{x} \text{var}(\mathbf{b}) \mathbf{x}^T$$

where $\text{var}(\mathbf{b})$ is the variance matrix of the estimated coefficients.

A.2. CALIBRATION OF COST MODEL

In section 4.2 of the main document, the unit cost model

$$\frac{C}{y} = AS^{-\gamma} \tag{1}$$

TABLE 4: Summary of the account samples from Denmark for 2001-2009 as unit prices. The samples are the raw data obtained from the Danish Account Statistics.

	Unit	Forage fish	Large fish	All	Herring
Price	€ ton ⁻¹	152.36	1245.92	382.16	523.88
Operation Cost	€ ton ⁻¹	124.62	1024.82	287.13	296.26
Net I	€ ton ⁻¹	27.73	221.11	95.04	227.61
Depreciation	€ ton ⁻¹	25.61	162.44	57.99	77.83
Net II	€ ton ⁻¹	2.12	58.67	37.04	149.78
Cost of capital	€ ton ⁻¹	11.9	87.61	32.34	68.23
Net III	€ ton ⁻¹	-9.78	-28.94	4.7	81.56

was established based on the production function. The function for unit costs (1) has two parameters, A and γ , and one variable, S . The values in Tab. 2 are applied for the parameter γ . The value of γ is independent of how S is measured as long as it is proportional to density. However, the value of A depends on the way in which S is measured, and there is no way to obtain a density measure from the spawning stock biomass. The approach taken is to calibrate the function to yield a unit cost similar to the cost observed from the data. Tab. 3 demonstrates that the average rents in Danish fisheries over the 2001-2009 period for the forage fish fleet, for the large fish fleet and in total are close to zero. The corresponding unit costs are given in Tab. 4. The reason for this situation can be ascribed to regulation of the fishing industry; Denmark's shift toward a transferable quota system has been gradual, and most of the data were therefore obtained under a system with bad incentives. In Denmark, transferable quotas were first introduced in the herring and mackerel fishery sector (such quotas were tested beginning in 2003 and made permanent in 2007). Today, the herring and mackerel sector is likely the most cost-efficient fishery in Denmark, and we assume that the other sectors can be as efficient if properly managed. The constant A is found by setting the rent to 15.57% of the revenue in a fishery that resembles today's fishery in the North Sea. Today's North Sea fishery is assumed to be 1 990 304 tons year⁻¹, which is the mean landing in the 2001-2009 period in the North Sea (ICES 2010a); this yield is assumed to be equally divided into forage fish and large fish.

B. ESTIMATION OF THE PRICE MODEL

B.1. DATA

When a vessel in the EU lands fish, the event must be recorded in the landing statistics. The present study is based on Danish landing statistics provided by the Danish Ministry of Food, Agriculture and Fisheries. A landing from one vessel on a specific day can be sorted and graded; then, each sort, grade and species exists as a record in the statistics. One record, R , contains:

Date The date of the landing

Vessel A code for the vessel

Landing Because each vessel traditionally lands only once per day, the combination of vessel and date yields a landing L_i $i = 1, 2, 3, \dots, n$ where n is the total number of landings. The landing L is not in the record *per se* but is instead the interaction of Date and Vessel. When a vessel lands, the fish are sorted into categories according to species, size, quality and landing state and sold in batches equivalent to one record. The record R_{ij} is then indexed by i , the landing, and $j = 1, 2, 3, \dots, m_i$, the batch within landing.

Species Species a is straightforward; although approximately 100 species are landed in Denmark, approximately 25 species comprise 99 % of landings by quantity.

Sorting refers to separation of the fish according to size (weight). For example, cod is sold in categories 1, 2, 3, 4 and 5. The sorting categories are given in the Council Regulation (EC) No 2406/96¹; each category typically has a lower bound w_L and upper bound w_H .

Landing state The landing state refers to how the fish are handled between the catch and landing. There is large variation in processing, ranging from the simplest, 'I', where the fish are landed in bulk for industrial reduction, to fish landed in various categories for direct consumption; see Tab. 5.

¹Council Regulation (EC) No. 2406/96 of 26 November 1996, which sets common marketing standards for certain fishery products

TABLE 5: Landing state in the landing statistics

Mark	Meaning
1	Liver
2	Roe
3	Tails
4	Clubs
5	Fillet
6	Wings
7	Smoked
8	Milt
A	Frozen
B	Blocks
F	Fillet with skin and bones
H	Gutted without head
I	Industrial
K	Cooked in water, not frozen
L	Alive
R	Gutted with head
S	Species-specific
U	Untreated
X	Unspecified

Grading Grading refers to the quality of the landed fish. The quality is graded as A, B or E; each grade will bring a different price at auction. The price differs considerably according to grade. In this study, grade is not used as an explaining variable.

Quantity The quantity of fish in the landing state is measured in kilograms (kg).

Value. The value is measured in Danish kroner (kr). All nominal values are converted to real price using the consumer price index Danmarks Statistik (2011) and converted into euros by the conversion factor $7.45 \text{ kr } \text{€}^{-1}$.

B.2. CONVERSION TO WHOLE FISH EQUIVALENTS

When fish are processed before landing, the weight of the fish after processing is less than the living weight of the fish. For each combination of species and landing condition, a specific factor is used to convert the

landed weight into whole fish equivalents. These conversion factors follow Commission Regulation (EU) No. 404/2011

B.3. SIZE CLASSES

Each of the major fish species used for consumption is classified into species specific size categories. As an example, cod is separated into five classes where class 2, for example, is 4–7 kg per fish. If the landing condition is 'gutted with head', class 2 is 4–7 kg per gutted fish. Consequently, the size categories must also be converted to whole fish equivalents using the conversion factor.

The category for the largest fish is specified as fish above a given size; as an example, cod class 1 is greater than 7 kg per fish. To obtain an upper limit of the weight of the fish in class 1, the concept of asymptotic weight w_{∞} is used. Parameters for each species are collected from FishBase². Parameters for asymptotic length and length weight conversion are used to find the w_{∞} . FishBase has a warning: "Note that studies where L_{∞} is very different ($\pm 1/3$) from L_{max} are doubtful." Therefore, an alternative L_{∞} is calculated as the average of observations within $\pm 1/3$ relative to the observed maximum length. The upper limit of class 1 is set at the found w_{∞} if this value leads to an upper limit that is not less than 1.1 times the lower bound of the class. Alternatively, this value is replaced by the alternative calculated value unless that is smaller than 1.1 times the lower bound, in which case the alternative calculated value is replaced by 1.1 times the lower bound. As an example, the lower bound for cod of class 1 is 7 kg. The highest conversion factor found in the landings in Denmark is 1.7 for "Gutted without head". This yields a lower bound for the class of 11.9 kg per whole fish. In FishBase, the w_{∞} is found to be 11.12 kg; this value is clearly too small. Using the alternative calculated w_{∞} is no help in this case, as this value is also smaller than the weight of the landed fish. Instead, the upper boundary is set to 1.1 times the lower boundary for the category, that is, to 13.09 kg.

Some of the categories for smaller fish have open lower boundaries. For some fish, there is a legal minimum size for landing.³ If the size class for the smallest category is open in the lower boundary, the lower boundary is set to the minimum legal landing size. If there is no legal minimum

²URL: <http://www.fishbase.org>

³BEK nr 788 af 25/06/2010: Bekendtgørelse om mindstemål for fisk og krebsdyr i saltvand. Translates to: Ministerial order on minimum size for fish and crustaceans in salt water

TABLE 6: Statistics for weeding the dataset for the year 2010. Quantities and values are given relative to the full dataset with an absolute value of $8.208 \cdot 10^8$ kg and $4.028 \cdot 10^8$ €. The column marked ‘Records’ is the number of observations, and the column marked ‘Landing’ shows the number of landings. The comments pertain to the points in the text.

Records	Landings	Quantity	Value	Comments
841 469	88 546	1.000	1.000	Full data set
761 214	76 718	0.9468	0.8172	After 1.
733 901	76 453	0.9459	0.8135	After 2.
732 531	76 382	0.9459	0.8136	After 3.
732 271	76 382	0.9459	0.8136	After 4.
732 084	76 357	0.9447	0.8136	After 5.
732 083	76 357	0.9447	0.8136	After 6.
649 315	68 305	0.9155	0.7801	After 7.

size, the lower bound is set to 31 g, corresponding to the lower limit of the herring category.

B.4. LIVER ROE MILT

The value of liver, roe and milt should ideally be added to the value of the gutted fish to obtain the total value of the whole fish. Roe is landed primarily for cod and lumpfish. For cod, the share of the roe is 4% of the landed quantity and 3% of the landed value; for lumpfish, the ratios are 15% and 80%, respectively. For lumpfish, the roe is the most value part of the fish, and it is not uncommon to find landings of lumpfish roe without a corresponding landing of gutted lumpfish. Thus, it is impossible to assign the value of the roe to the corresponding gutted fish.

B.5. WEEDING

If the datasets contain observations that cannot be used in the analysis, these observations are deleted by the following procedure:

1. Species for which w_∞ is not obtained are deleted from the analysis. w_∞ was obtained for all major fish species covering 92% of landings. However, because w_∞ is obtained from FishBase, which only considers fish, crustaceans and mollusks are excluded; some of the latter species are highly valued (e.g., lobster), whereas others are of low value (e.g., blue mussel).

2. After the whole fish conversion factor is applied to the landed weight, some observations do not have a whole fish weight and are removed, as is the case for species that are not covered under Commission Regulation (EU) No. 404/2011.
3. Some observations have a non-positive value of whole fish weight; these observations are removed.
4. Some observations do not have a value and are removed.
5. Some observations have a negative value and are removed.
6. Only fish for reduction and fish for consumption with well-defined size categories are retained; that is, the landing conditions L, R, U, I and, for ray and skate, condition 6 (wings); see Tab. 5 for translation of the code.
7. At this stage, there are still some landings for direct consumption that do not have w_L and w_H values because they are not mentioned in Council Regulation (EC) No 2406/96. These observations are deleted.

In Tab. 6, the loss of data through the weeding process is documented for the year 2010.

B.6. METHOD AND RESULTS

Fish for direct consumption are sorted and landed in size categories with specific prices for each category, whereas fish for reduction are landed and sold in bulk without respect to size. Thus, there is no price–size relationship for fish for reduction and the two landing states are treated separately. The dataset is divided into two components, one with the landing state 'I', that is, for reduction, and the other with the fish for direct consumption.

B.6.1. Price of fish for direct human consumption

Fish are sorted into size classes when sold. Thus, information regarding the size distribution of the landed fish is hidden in the information about the quantity of fish landed in each class. From the data for each species, the empirical cumulative distribution values for the size values that limit the size categories, that is, all the w_L and w_H values, are calculated. If two or more size categories overlap, the density is assumed to be equal within

each size category. A Weibull cumulative distribution function

$$\Phi(w) = \begin{cases} 1 - \exp(-((w-u)v)^z) & w \geq u \\ 0 & \text{else} \end{cases} \quad (1)$$

is calibrated using a minimum least-squares estimate and the restriction that $u \geq 0$, $v > 0$, $z > 0$ and b is smaller than the second size value in the established empirical cumulative distribution. The estimate is controlled by plotting a histogram of the empirical density and plotting the Weibull density function

$$\phi(w) = \begin{cases} \frac{z}{v} \left(\frac{w-u}{v}\right)^{z-1} \exp(-((w-u)/v)^z) & w \geq u \\ 0 & \text{else} \end{cases} \quad (2)$$

with the estimated parameters on top. Fig. 1 illustrates the control plot for herring and cod in 2010.

In a few cases, the density function was not acceptable. For horse mackerel and blue witting, which are landed for consumption in rather small quantities, there were only sufficient data to establish a distribution function in 2007. The function for horse mackerel and blue witting established for 2007 was then applied to all other years. For the porbeagle, which accounts for 0.000029 of the quantity of fish landed, there were no years in which the data were sufficient to establish a distribution. Thus, the porbeagle data had to be disregarded.

In the data, each record belongs to a sorting category R , where R is defined by the species A_R , the lower bound of the category w_{RL} and the upper bound w_{RH} . If a record j from landing i belongs to the category, it follows that:

$$R_{ij} = R \iff w_{ijL} = w_{RL} \wedge w_{ijH} = w_{RH} \wedge a_{ij} = A_R$$

The quantity in each category can be aggregated as

$$Q_R = \sum_{(i,j)|R_{ij}=R} Q_{i,j}$$

and the value of each category aggregated as

$$V_R = \sum_{(i,j)|R_{ij}=R} V_{i,j}$$

The quantity density can then be defined as

$$\vartheta(R, w) = \begin{cases} \frac{Q_R \phi(A_R, w)}{\Phi(A_R, w_{RH}) - \Phi(A_R, w_{RL})} & w_{RL} < w < w_{RH} \\ 0 & \text{else} \end{cases} \quad (3)$$

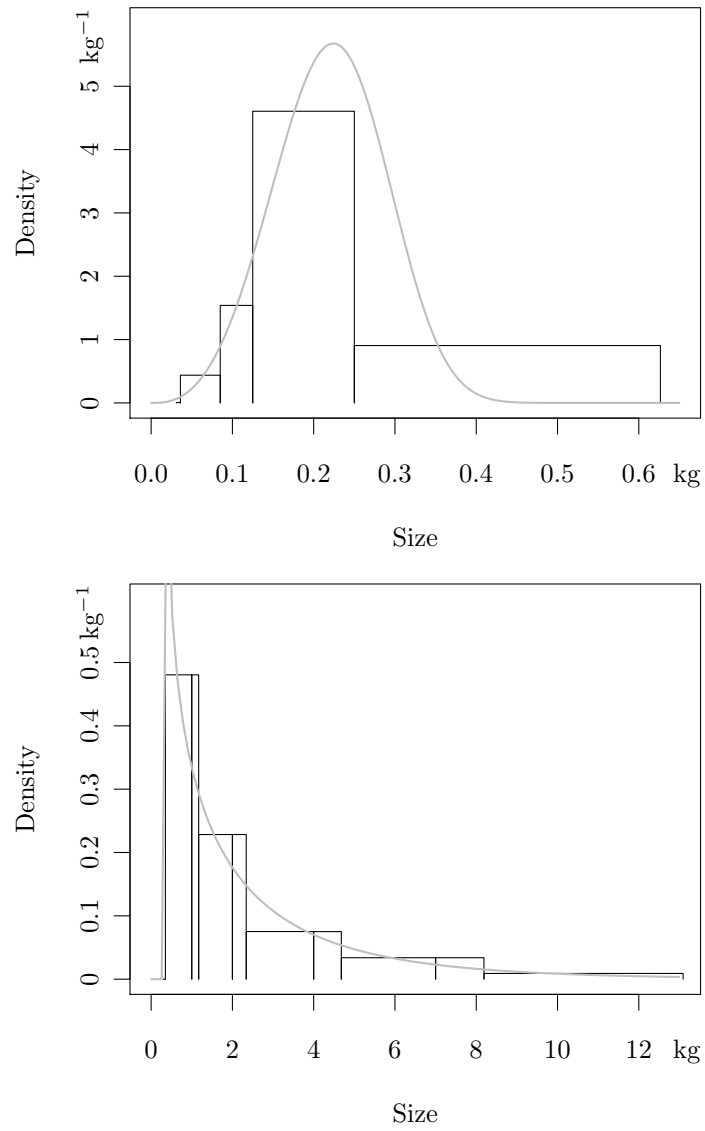


FIGURE 1: Histogram of density of landings in 2010, with herring in the upper panel and cod in the lower panel. The gray line is the estimated Weibull density function.

and the value density can be defined as

$$\theta(R, w) = \begin{cases} \frac{V_R \phi(A_R, w)}{\Phi(A, w_{RH}) - \Phi(A, w_{RL})} & w_{RL} < w < w_{RH} \\ 0 & \text{else} \end{cases} \quad (4)$$

Note that because

$$\int_{w_{RL}}^{w_{RH}} \phi(A_R, w) dw = \Phi(A, w_{RH}) - \Phi(A, w_{RL})$$

the formulation in (3) and (4) ensures that the quantity and value for the category is the same as the integral of the densities.

To summarize all of the categories' density functions to obtain a total density with respect to size, the size dimension is sampled at 1 000 discrete points on a logarithmic scale from 31 g to 13 kg; then, w is discrete in w_k , $k = 1, 2, \dots, 1000$. The total densities of quantity and value with respect to size are then

$$\begin{aligned} \vartheta_{\bullet, w_k} &= \sum_R \vartheta(R, w_k) \\ \theta_{\bullet, w_k} &= \sum_R \theta(R, w_k) \end{aligned}$$

In Fig. 2, $\vartheta_{\bullet, w}$ and $\theta_{\bullet, w}$ are plotted for the year 2009. The distance between the lines is observed to increase with size, indicating that quantity decreases faster than value with increasing size. That is, the fish increase in value with size. When the density of the value is divided by the density of the quantity, the quotient is the price with respect to size:

$$p_{w_k} = \frac{\theta_{\bullet, w_k}}{\vartheta_{\bullet, w_k}}$$

Fig. 3 illustrates the price with respect to size p_w for the year 2009. Although the pattern exhibits some variation, price clearly increases as a function of size. For each year investigated, there is a trend of increasing price with size, as shown for the year 2009; however, all of the curves exhibit minor variations. To establish a generic price model, we will capture the increasing trend with an increasing function. An appropriate model is the Weibull cumulative function:

$$p(w) = \begin{cases} q (1 - \exp(-(w - b)/a)^t) & w \geq b \\ 0 & \text{else} \end{cases} \quad (5)$$

The Weibull cumulative function is flexible and allows for both an asymmetric sigmoid functional form ($t > 1$) and concave form. The a

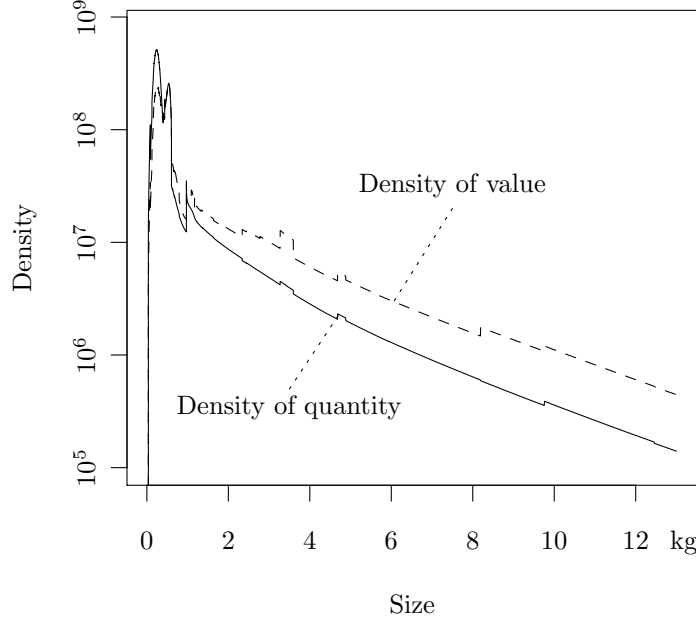


FIGURE 2: Density of landed quantity with respect to size $\vartheta_{\bullet,w}$ and density of landed value with respect to size $\theta_{\bullet,w}$ for 2009. The density axes are logarithmic. The value ‘density’ has the unit € kg^{-1} , and the density of quantity is unitless.

parameter scales the function along the w axis, whereas the q yields an asymptotic price when $w \rightarrow \infty$ and b shifts along the w axis. The gray line in Fig. 3 illustrates the estimated model. The model is estimated by the method of nonlinear least squares (`nls()`, R Development Core Team 2011); note that because the grid of w is equidistant on a logarithmic scale, landings of smaller sizes are given more weight in the estimation.

To examine the persistence of the estimated price model, it is estimated for the previous 10 years. The estimated curves are shown in Fig. 4. In the upper panel of the figure, the 10 lines represent the estimated models; the four rectangular boxes are magnified in the four diagrams below, where the lines are labeled by year. In all cases, ι is clearly below one, resulting in a concave functional form; however, there is some variation in the shape of the curves. Examination of the year labels in the lower diagrams indicates

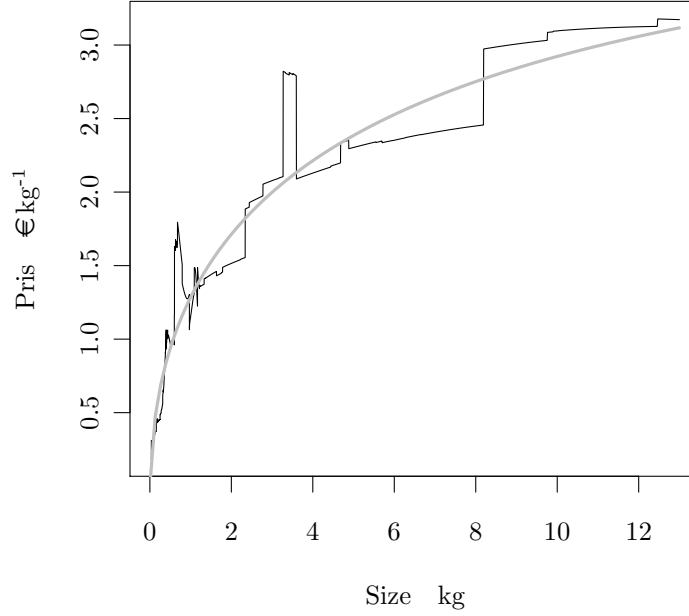


FIGURE 3: The denotes price with respect to size p_w for 2009 with black. The gray line is the accommodated price model.

that there does not appear to be a systematic trend in the price over time.

If the variation in the curves has its origin in random variations in the pattern of fishery, it can be smoothed by averaging over multiple years. Curves calculated for data aggregated over five-year intervals are presented in Fig. 5. There is some development in the shape of the curve in that the two most recent curves exhibit higher prices for the larger fish. The last two five-year periods resemble each other, and the last five-year period (2006—2010) is adopted as the price model. The estimated parameter is given in Tab. 7.

The procedure described above yields only a weighted mean price with respect to size. However, we would also like to know how precise this estimator is, i.e., to know the variation of the estimate. To obtain this information, the data are re-sampled. Here, each fishing trip is viewed as a sampling of the sea for fish. Thus, the re-sampling is conducted based on the landing L . First, a set Λ of n numbers is drawn with

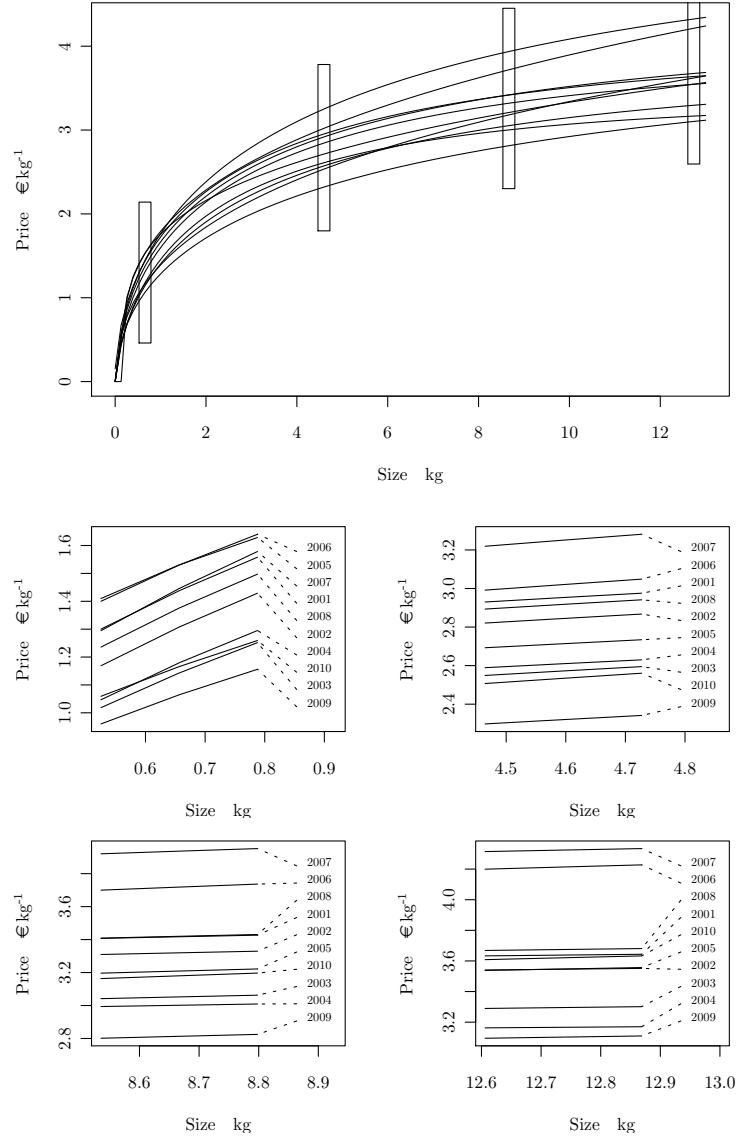


FIGURE 4: Estimated price models for the 2000-2010 period. The upper panel illustrates one line per year. The area in the boxes in the upper panel is magnified in the four lower panels, here with indications for the year.

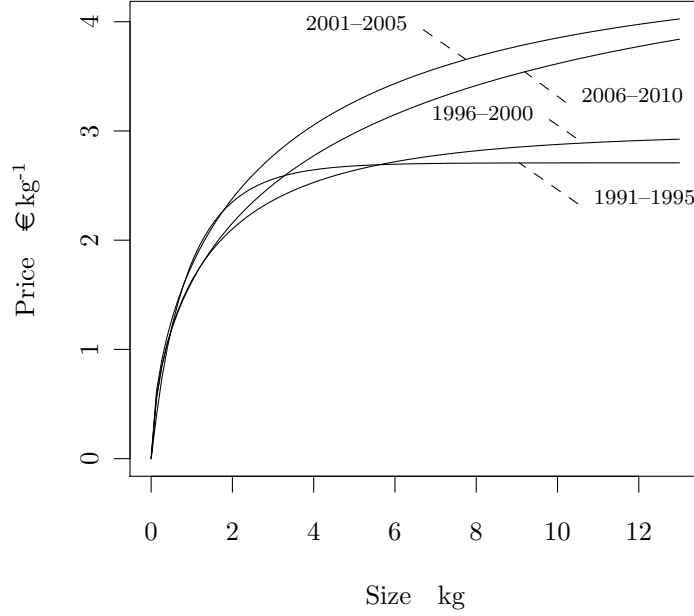


FIGURE 5: Estimated price models over five years for the 1991-1995, 1996-2000, 2001-2005 and 2006-2010 periods.

equal probability from the integers $1, 2, 3, \dots, n$ with replacement. A new dataset is then constructed with each landing consisting of the batches $B_{\Lambda_i, j}$, where $j = 1, 2, 3, \dots, m_{\Lambda_i}$. The parameters are then estimated using this new dataset. Over the five-year period, there is a change in the fishery toward fewer but larger landings. Because the fishing is changing over time, re-sampling is performed within each of the five-year periods. The five datasets are subsequently joined and aggregated. In total 1 000 re-samplings are performed.

In Tab. 7, the results of the re-sampling are presented as mean values, the standard error and a correlation matrix. As can be observed from the correlation matrix, there is high absolute correlation between the parameters, implying that the variation in the model values is actually smaller than indicated by the standard error. To estimate the variation in model values, the model value is calculated for 100 w values for each of the 1 000 re-samplings. At each grid point, the standard error and 95% confidence

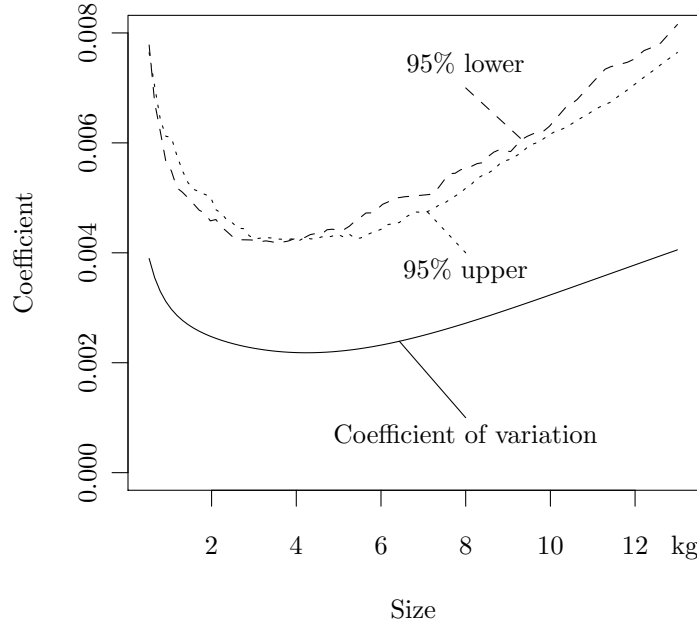


FIGURE 6: Coefficient of variation of the model values and 95% confidence interval as coefficient values, i.e., relative to the model value. Data based on 1 000 re-samplings.

interval are calculated by finding the 0.025 and 0.975 quantiles. In Fig. 6, the standard error and confidence interval are presented as coefficients, that is, relative to the mean. The coefficient of variation is below 0.004, and the 95% interval is within 0.008 of the mean on each side of the mean value.

B.6.2. Price of fish for reduction

Fish for reduction are landed without sorting into size categories, and thus, their price cannot be related to their size. Thus, the price model will be a flat price estimated from the mean price, i.e., an aggregated value divided by an aggregated quantity. In Fig. 7, the average price for each year within the 2001-2010 period is provided. There is some variation in price, and the variation may be cyclical over the timespan. In Fig. 8, the average prices over several five-year periods are shown. The average price

TABLE 7: Parameter estimates and re-sampling statistics. The columns with headings Low 95% and High 95% indicate the 0.025 and 0.975 quantiles in the re-sampling, respectively. The parameters P and q have units € kg^{-1} , and a and b are in kilograms.

Parameter	Estimate	Re-sampling				
		Mean	Median	Std Error	Low 95%	High 95%
P	0.1610	0.1610	0.1610	0.00071	0.1597	0.1624
q	4.830	4.825	4.823	0.06857	4.6882	4.9602
b	0.0295	0.0292	0.0294	0.00096	0.0274	0.0306
a	5.38	5.36	5.35	0.25	4.89	5.87
ι	0.5230	0.5235	0.5232	0.0039	0.5159	0.5319

Correlation matrix based on re-sampling

	q	b	a	ι
q	1.00	0.16	0.99	-0.78
b	0.16	1.00	0.17	-0.39
a	0.99	0.17	1.00	-0.83
ι	-0.78	-0.39	-0.83	1.00

for the 2006-2010 period is adopted as the price model for the industrial landing state. In Tab. 7, the estimate is given with symbol P together with the re-sampling statistics.

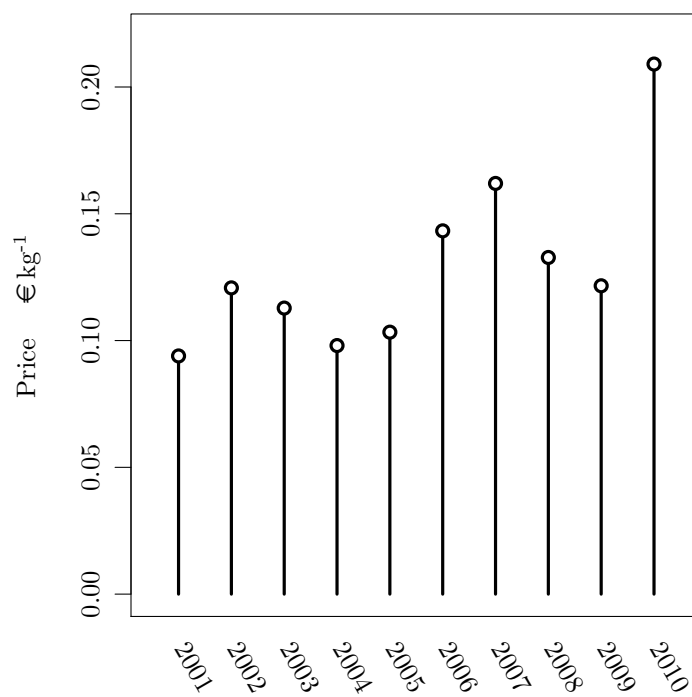


FIGURE 7: Average price for fish landed as 'T' 2000–2010.

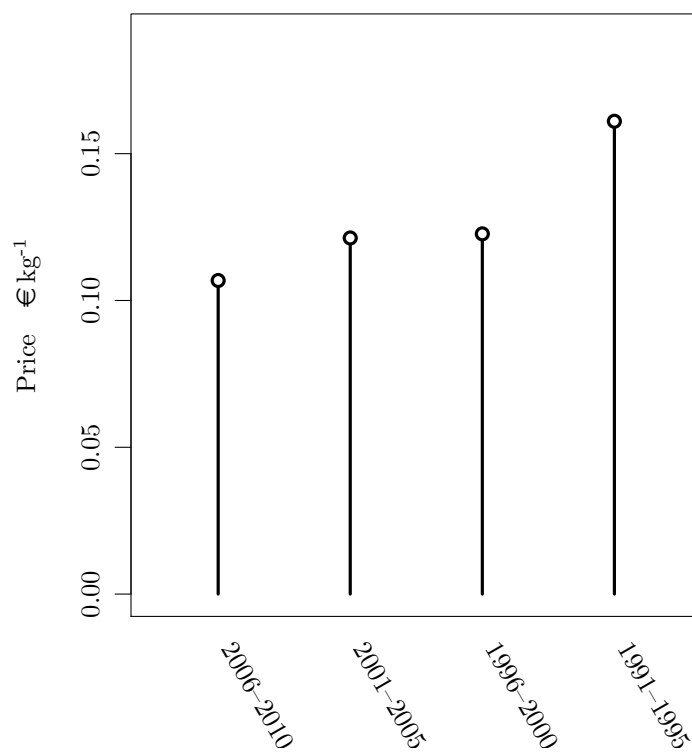


FIGURE 8: Average price for fish landed as 'T' for the 1991-1995, 1996-2000, 2001-2005 and 2006-2010 periods.

C. MODEL CONCEPTS AND ASSUMPTIONS

The model framework is based on two central assumptions and a number of standard assumptions. The first central assumption is that an individual can be characterized by its weight w and asymptotic weight W only. The aim of the model is to calculate the size- and trait-spectrum $\mathcal{N}(w, W)$, which is the density of individuals such that $\mathcal{N}(w, W) dw dW$ is the number of individuals in the interval $[w, w + dw]$ and $[W, W + dW]$. The dimensions of the size spectrum are numbers per weight per volume. In the numerical implementation, the trait-dimension is discretized, and the trait-spectrum $\mathcal{N}(w, W)$ is represented by a number of size spectra $N_i(w)$, each of which represent individuals with a range of trait values W_i . Scaling from individual-level processes of growth and mortality to the dynamic size spectrum of each trait group is achieved using the McKendrick-von Foerster equation, which is simply a conservation equation (1) in which the individual growth $g(w, W)$ and mortality rate $\mu(w, W)$ are determined by the availability of food and predation from the community size spectrum $N_c(w)$, which is the sum of all size spectra (2). The fishing mortality rate $\nu(w, W)$ is the sum of the sector's harvest (10). Each sector s of the fishery is associated with a selection function $\omega_s(w, W) \in [0, 1]$ and

TABLE 8: Model equations

Scaling:	
$\frac{\partial N_i}{\partial t} + \frac{\partial g N_i}{\partial w} = -\mu_i N_i - \nu_i N_i$	(1)
$N_c(w) = \sum_i N_i(w) + N_R(w)$	(2)
Food encounter and consumption:	
$\phi(w_p/w) = \exp [(-(\ln(w/(w_p\chi)))^2/(2\sigma^2))]$	(3)
$E_e(w) = \tau w^u \int N_c(w) \phi(w_p/w) w dw_p$	(4)
$f(w) = E_e/(E_e + hw^m)$	(5)
Growth:	
$\psi(w, W) = [1 + (w/(\eta W))^{-10}]^{-1} (w/W)^{1-m}$	(6)
$g(w, W) = (\delta f(w)hw^m - k_s w^d)(1 - \psi(w, W))$	(7)
Mortality:	
$\mu_p(w_p) = \int \phi(w_p/w)(1 - f(w))\tau w^u N_c(w) dw$	(8)
$\mu_{b,i} = \mu_0 W^{m-1}$	(9)
Fishing mortality:	
$\nu_i(w) = \sum_s \mathcal{F}_s \omega_{s,i}(w)$	(10)
$\omega_{s,i}(w) = \xi_i \left(1 + \exp \left(\zeta_s \left(w_{1/2,s}^{0.33} - w^{0.33}\right)\right)\right)^{-1}$	(11)

an overall fishing mortality scaling \mathcal{F}_s . The selection function is modeled as a trawl selection (11). The conservation equation is at weight w_0 and is supplemented by a boundary condition with a fixed $N_i(w_0)$.

The second central assumption is that food preference is determined by individual weight only, not by the trait-value or species identity of the prey. The preference for prey is described by the log-normal selection model (Ursin 1973), which ascribes prey preference in terms of the ratio between the weight of predators and the weight of prey, w and w_p , respectively (3), where χ is the preferred predator–prey mass ratio and σ is the width of the weight selection function.

The remainder the model formulation rests on a number of “standard” assumptions from ecology and fisheries science about how encounters between predators and prey lead to mortality of the prey $\mu(w)$ and consumption by and growth $g(w, W)$ of the predator.

The available food (mass per volume) for a predator of weight w is determined by integrating over the community size spectrum weighted by the size selection function (3): $\int N_c(w) \phi(w_p/w) w \, dw_p$. The food actually encountered E_e (mass per time) depends on the search rate (volume per time), which is assumed to scale with individual weight as τw^u (4). Encounters between predators and prey are only determined by their relative individual weights, not by the trait W . Thus, a 100 g cod will consume the same food as a 100 g herring.

The encountered food is consumed subjected to a standard Holling functional response type II (Holling 1959) to represent satiation. This response determines the feeding level $f(w)$, which is a dimensionless number between zero (no food) and one (fully satiated) (5) and where hw^m is the maximum consumption rate. The functional response is a concave increasing function of the available prey; it represents the declining marginal productivity with respect to the availability of prey.

The consumed food $f(w)hw^m$ is assimilated by an efficiency δ and used to fuel the need for standard metabolism and activity $k_s w^d$. The remaining available energy, $\delta f(w)hw^m - k_s w^d$, is divided between growth and reproduction by a function of weight (6), varying between zero around the weight of maturation to one at the asymptotic weight at which all available energy is used for reproduction, leading to an equation for growth (7). The form of the allocation function is chosen such that the growth curve approximates a von Bertalanffy growth curve if the feeding level is constant (see Hartvig, Andersen, and Beyer 2011, for details about the derivation). The actual growth curves emerging from the model depends on the amount of food available.

The model described above explicitly allocates energy for reproduc-

tion. Our objective in this study is to investigate the trade-offs of fishing various species of fish in the context of a trophic system. Thus, we do not include feedback from the mature fish allocation for reproduction to the number of eggs because this variable would make the model less traceable. Instead, the model has a fixed boundary value for the smallest fish in the model w_0 .

The mortality rate (not including fishery) of an individual $\mu(w)$ has two sources: predation mortality $\mu_p(w)$ and a constant background mortality $\mu_{b,i}(w)$. Predation mortality is calculated such that all that is eaten translates into corresponding predation mortalities on the ingested prey individuals ((8); see Hartvig, Andersen, and Beyer (2011, App. A) for derivation). When the food supply does not meet metabolic requirements $k_s w^d$, growth stops, i.e., no negative growth, and the individual is subjected to starvation mortality. Starvation mortality is assumed to be proportional to the energy deficiency $k_s w^d - \delta f(w) h w^m$ and inversely proportional to lipid reserves, which are assumed to be proportional to body weight. Starvation does not appear in the simulations presented here.

Mortality from sources other than predation is assumed to be constant within a trait and inversely proportional to generation time (Peters 1983) (9). Background mortality is needed to remove the largest individuals, who do not experience predation mortality.

Food items for the smallest individuals (smaller than χw_0) are represented by a resource spectrum $N_R(w)$. The temporal evolution of each size group in the resource spectrum is described using semi-chemostatic growth, where $r_0 w^{d-1}$ is the population regeneration rate (Fenchel 1974; Savage et al. 2004) and $\kappa_R w^{-\lambda} = \kappa_R w^{-2-u+m}$ is the carrying capacity.

The model is implemented with two fleets: a forage fish fleet targeting small fish for reduction and a large fish fleet targeting larger fish for direct consumption. The two fleets target different species; the forage fish fleet targets forage fish species, and the large fish fleet targets piscivorous species. Forage fish species are species with $W < 512$ g, and piscivorous species have $W \geq 512$ g. The overall fishing mortality rate of the two fleets ($\mathcal{F}_F, \mathcal{F}_L$) is the control variable in the model. The selection function (11) is a logistic function of the length of the fish. This function is normally used to model trawling; the parameters are provided in Tab. 10.

The model system presented in Tab. 8 is implemented in a 25×106 grid representing $W \times w$ in a logarithmic space. The partial differential equation (1) for modeling the time dimension is implemented numerically with an upwind implicit schema, as described in Ravn-Jonsen (2011), with time step $\Delta t = 0.25$ year. The simulations are performed in Matlab, and the source code is available upon request.

TABLE 9: Parameters of the system. For a detailed explanation of the determination of the values, see Hartvig, Andersen, and Beyer (2011, App. E).

Symbol	Value	Units	Description
Resource spectrum			
κ_R	$5 \cdot 10^{-3}$	$\text{g}^{\lambda-1}\text{m}^{-3}$	Magnitude of the resource spectrum
λ	2.05	-	Exponent of resource spectrum ($= 2 - m + u$)
r_0	4	$\text{g}^{1-p}\text{yr}^{-1}$	Constant for regeneration rate of resources
w_{cut}	1	g	Upper weight limit of the resource spectrum
Individual growth			
f_0	0.6	-	Initial feeding level
δ	0.6	-	Assimilation efficiency
h	20^a	$\text{g}^{1-n}\text{yr}^{-1}$	Constant for maximum food intake
m	0.75	-	Exponent for maximum food intake
k_s	2.4^a	$\text{g}^{1-p}\text{yr}^{-1}$	Constant for standard metabolism and activity
d	0.75^b	-	Exponent of standard metabolism
χ	100	-	Preferred predator-prey mass ratio
σ	1.3^c	-	Width of size selection function
τ	1459	$\text{g}^{-q}\text{yr}^{-1}$	Constant for the volumetric search rate
u	0.8	-	Exponent for the volumetric search rate
Mortality			
μ_0	2^a	$\text{g}^{1-n}\text{yr}^{-1}$	Constant for the background mortality
Reproduction and recruitment			
w_0	0.001	g	Offspring weight
η	0.25	-	Weight at maturation divided by W

^aAdjusted to a different value than in (Hartvig, Andersen, and Beyer 2011) to yield growth rates similar to growth rates of species in the North Sea.

^bLaboratory experiments on fish indicate that the exponent of standard metabolism should be higher, approximately $d = 0.82$ (Killen et al. 2007).

^cThe width of the selection function is chosen to be larger in the trait-based model than in the species-based model (Hartvig, Andersen, and Beyer 2011) to emulate the diversity in prey-preferences of the species within a trait-class. The practical implication of enlarging σ is that the model is more stable (fewer oscillations) (Datta et al. 2010).

TABLE 10: Parameters for the fishery selection function. The parameter for trait target ζ is separation of species into fish maturing as small, forage fish species and into fish maturing as large, piscivorous species. The steepness and inflection point parameters are chosen such that the harvest resembles the Danish landing in mean size.

Symbol	Value	Description
ζ_F	$\begin{cases} 1 & W < 512 \text{ g} \\ 0 & W \geq 512 \text{ g} \end{cases}$	Trait Target
ζ_L	$\begin{cases} 0 & W < 512 \text{ g} \\ 1 & W \geq 512 \text{ g} \end{cases}$	
ζ_F	10	Steepness
ζ_L	2	
$w_{1/2F}$	1 g	Inflection point
$w_{1/2L}$	100 g	

D. SENSITIVITY OF COST PARAMETERS

The unit cost function

$$\frac{C}{y} = AS^{-\gamma} \quad (1)$$

has two parameters for each fleet, the exponent γ and the coefficient A . The sensitivity of the results with respect to these parameters is evaluated by repeating the same analysis as in the main paper with these parameters changed.

D.1. SENSITIVITY WITH RESPECT TO THE EXPONENT

The values of the exponents γ are found empirically; however, in absolute value, they are smaller than the typical values. To observe the consequences of overly small exponents, the exponents are increased by a factor of 1.5 in the experiments. Because the cost function is calibrated so that the rent in present fishery yields 15.57% of the revenue in rent, the coefficient A also changes:

	Original	Sensitivity	
γ_F	0.175	0.263	
γ_L	0.280	0.420	
A_F	0.05748	0.03738	€ kg ⁻¹
A_L	0.2759	0.1476	€ kg ⁻¹

The results of the experiments are presented in Fig. 10 and 9 as diagrams equivalent to Figs. 4 and 5 in the main paper.

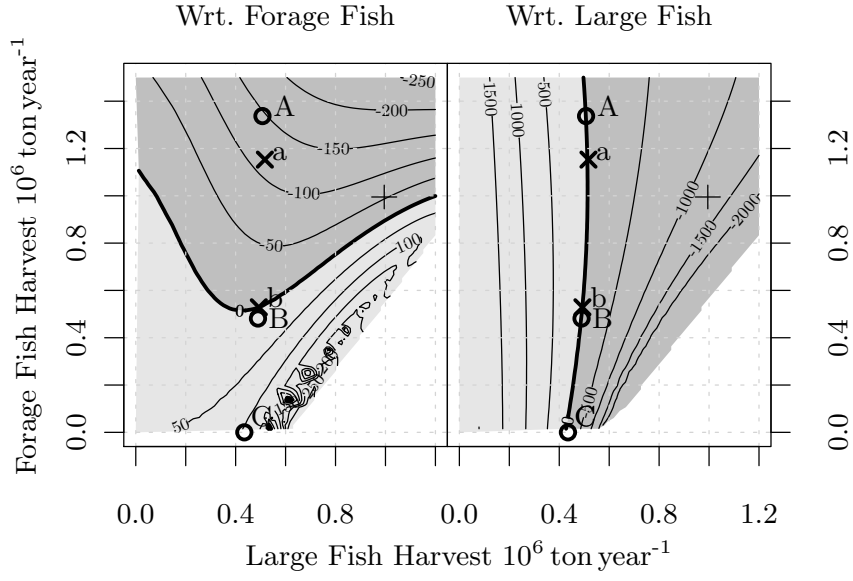


FIGURE 9: Sensitivity with respect to γ : Total benefit indicator (€ ton^{-1}) for the forage fish fleet $B_{\bullet/\text{F}}$ (left) and the large fish fleet $B_{\bullet/\text{L}}$ (right). The figure is equivalent to Fig. 5 in the main paper. The points with letters A, B and C are the same as in the original, whereas the \times marked 'a' and 'b' indicate the same solutions as A and B but under the test parameters. The plus sign is the current state of the North Sea; in this point $B_{\bullet/\text{F}} = -54 \text{ € ton}^{-1}$ and $B_{\bullet/\text{L}} = -1\,215 \text{ € ton}^{-1}$.

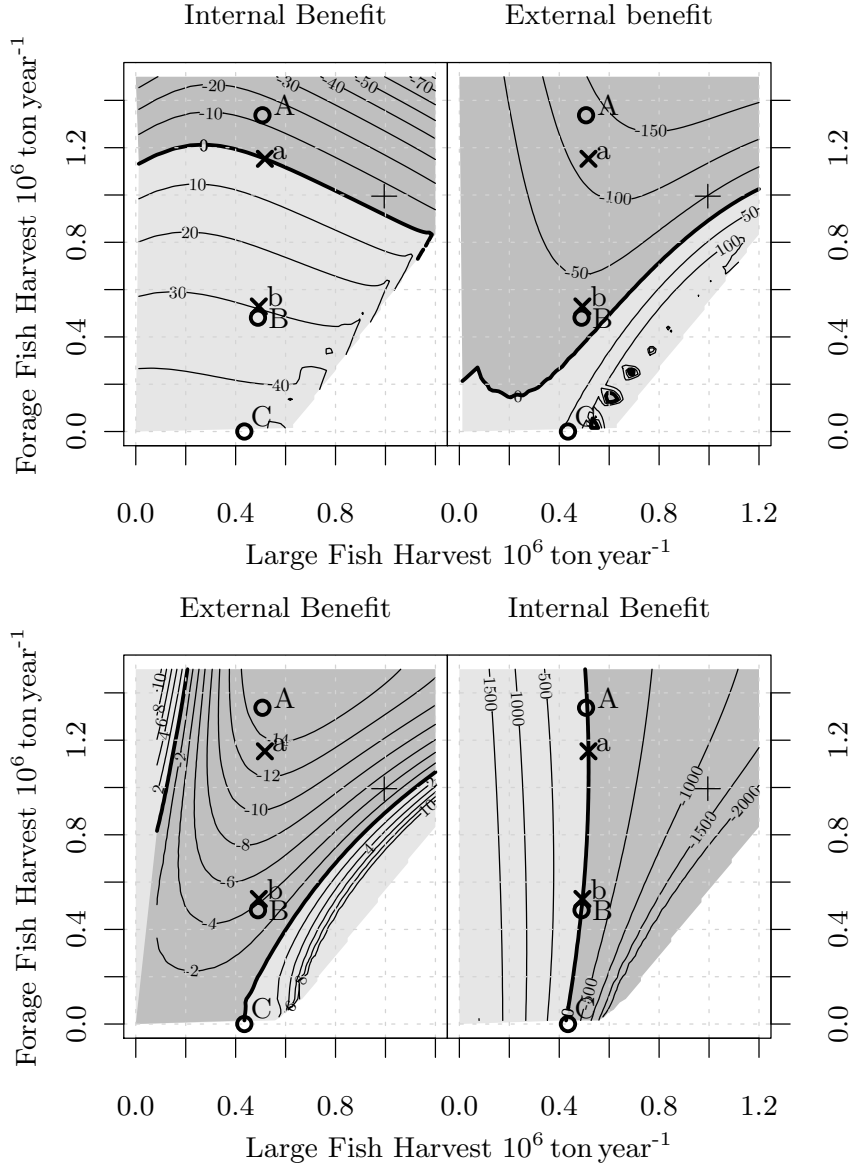


FIGURE 10: Sensitivity with respect to γ : The benefit indicators (€ ton^{-1}) for the North Sea forage fish fleet (top) and large fish fleet (bottom) divided into internal benefit $B_{i/i}$ and external benefit $B_{i/j}$. The figure is equivalent to Fig. 4 in the main paper. The points marked are the same as in Fig. 9. The plus sign is the current state of the North Sea; at this point, $B_{F/F} = -6$, $B_{L/F} = -121$, and $B_{F/L} = -4$ and $B_{L/L} = -1211$ (all in in € ton^{-1}).

D.2. SENSITIVITY WITH RESPECT TO THE COEFFICIENT

The value for the coefficient A is found through calibration from the assumption that a well-regulated fishery will yield a rent comparable to that of the best-performing fleet segment in Denmark. The herring and mackerel fishery, which has been under provisional ITQ regulation since 2003 and under permanent regulation since 2007, is today probably the most cost-efficient fishery in Denmark. We assume that the other sectors can be as efficient if properly managed. The constant A is found by setting the rent to 15.57% of the revenue in a fishery that resembles today's fishery in the North Sea. To test the influence of this assumption on the results, the experiments are performed using a calibration in which the rent in today's fishery is zero. This calibration will increase A by a factor of $1/(1-0.1557)$

	Original	Sensitivity	
A_F	0.05748	0.06809	€ kg ⁻¹
A_L	0.2759	0.3268	€ kg ⁻¹

The results of the experiments are presented in Figs. 12 and 11 as diagrams equivalent to Figs. 4 and 5, respectively, in the main paper.

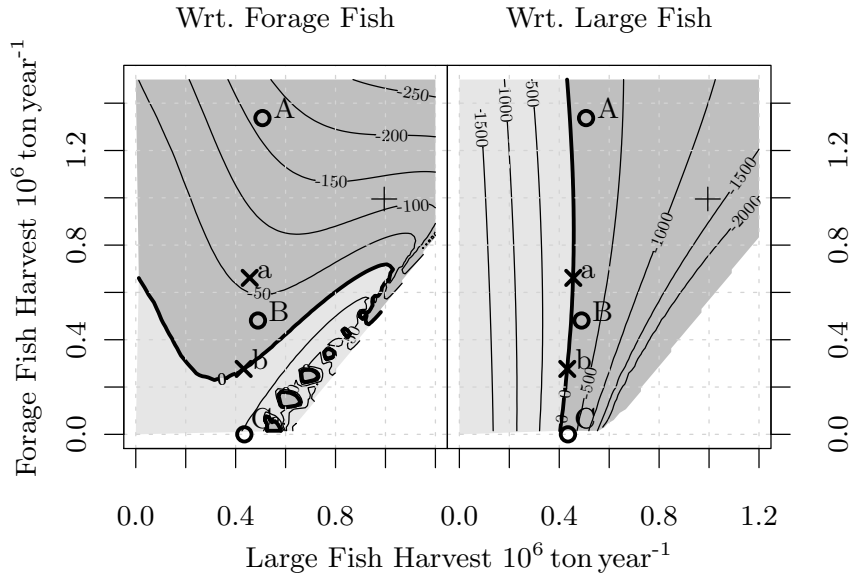


FIGURE 11: Sensitivity with respect to A : Total benefit indicator (€ ton^{-1}) for the forage fish fleet $B_{\bullet/\text{F}}$ (left) and the large fish fleet $B_{\bullet/\text{L}}$ (right). The figure is equivalent to Fig. 5 in the main paper. The points with letters A , B and C are the same as in the original, whereas the \times marked 'a' and 'b' indicates the same solution as A and B but under the test parameter. The plus sign is the current state of the North Sea; at this point, $B_{\bullet/\text{F}} = -116 \text{ € ton}^{-1}$ and $B_{\bullet/\text{L}} = -1316 \text{ € ton}^{-1}$.

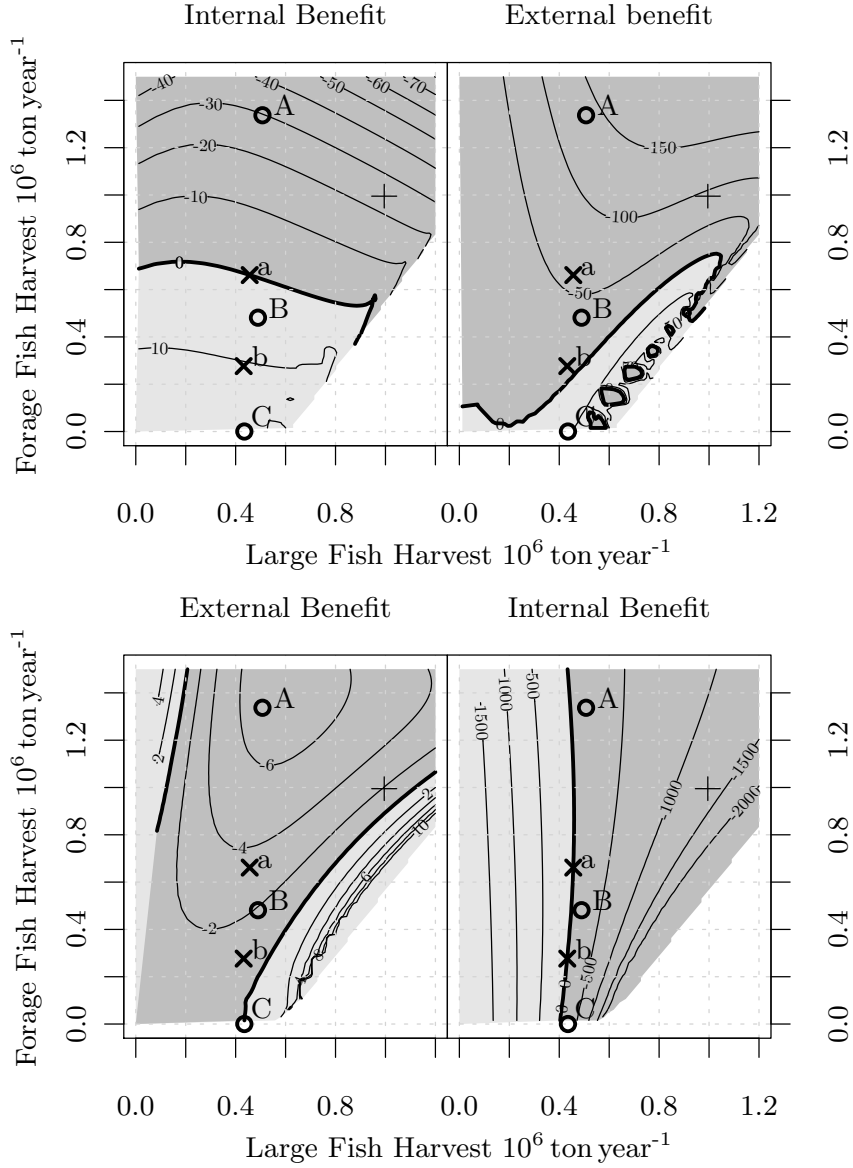


FIGURE 12: Sensitivity with respect to A: The benefit indicators (€ ton^{-1}) for the North Sea forage fish fleet (top) and large fish fleet (bottom) divided into internal benefit $B_{i/i}$ and external benefit $B_{i/j}$. The figure is equivalent to Fig. 4 in the main paper. The points marked are the same as in Fig. 11. The plus sign is the current state of the North Sea; at this point, $B_{F/F} = -24$, $B_{L/F} = -92$, and $B_{F/L} = -2$ and $B_{L/L} = -1314$ (all in € ton^{-1}).

E. SENSITIVITY OF CONTROL VARIABLE

The definition of the benefit indicator in section 3 of the main paper depends on the control variable. The tradition of using fishing mortality as the control variable stems partly from the undesirable consequence of using harvest as the control variable in a traditional logistic function; the consequence is that there will be two equilibrium points, one stable and one unstable. In contrast, by using fishing mortality as the control variable, there is one stable equilibrium. The size spectrum model differs in this respect because use of the harvest as the control variable leads to stable equilibria for harvests within certain limits. This result is because the model system has the ability to react in the size dimension. A demand for increased harvest will lead to smaller size. In this analysis, *continue as usual* is defined as keeping a constant Y and *an action* is to change the Y . The experiment is conducted by keeping the harvest of fishery j at Y_j while changing the harvest in fishery i by setting $Y_i(A) = (1 - \epsilon)Y_i$ and $Y_i(B) = (1 + \epsilon)Y_i$. The benefit indicator is calculated as

$$B_{j/i} \approx \frac{\Delta V_j}{\Delta Y_i}$$

The results of the experiments are presented in Fig. 14 and 13 as diagrams equivalent to Figs. 4 and 5 in the main paper.

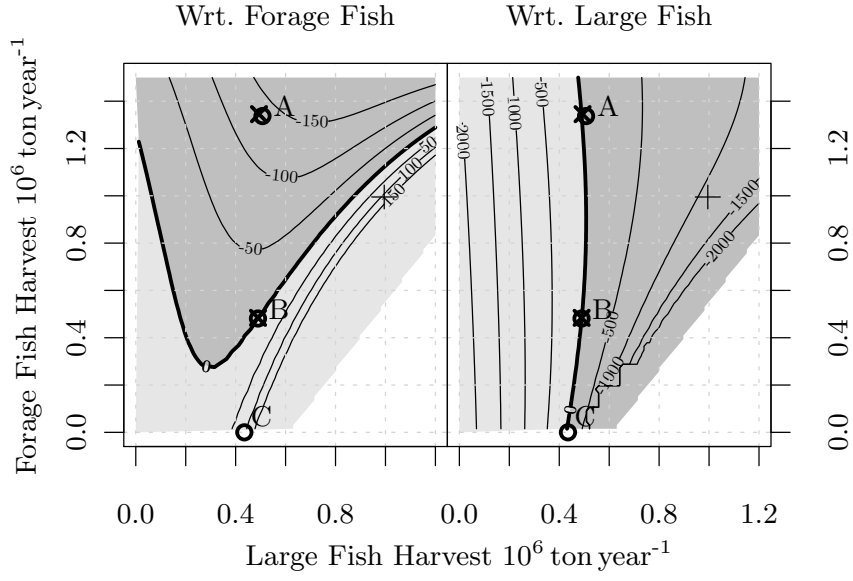


FIGURE 13: Sensitivity with respect to control variable: Total benefit indicator (€ ton^{-1}) for the forage fish fleet $B_{\bullet/F}$ (left) and large fish fleet $B_{\bullet/L}$ (right). The figure is equivalent to Fig. 5 in the main paper. The circle points with letters A, B and C are the same as in the original, whereas the \times indicates the same solution but under the test parameter. The plus sign is the current state of the North Sea; at this point, $B_{\bullet/F} = 115 \text{ € ton}^{-1}$ and $B_{\bullet/L} = -1316 \text{ € ton}^{-1}$.

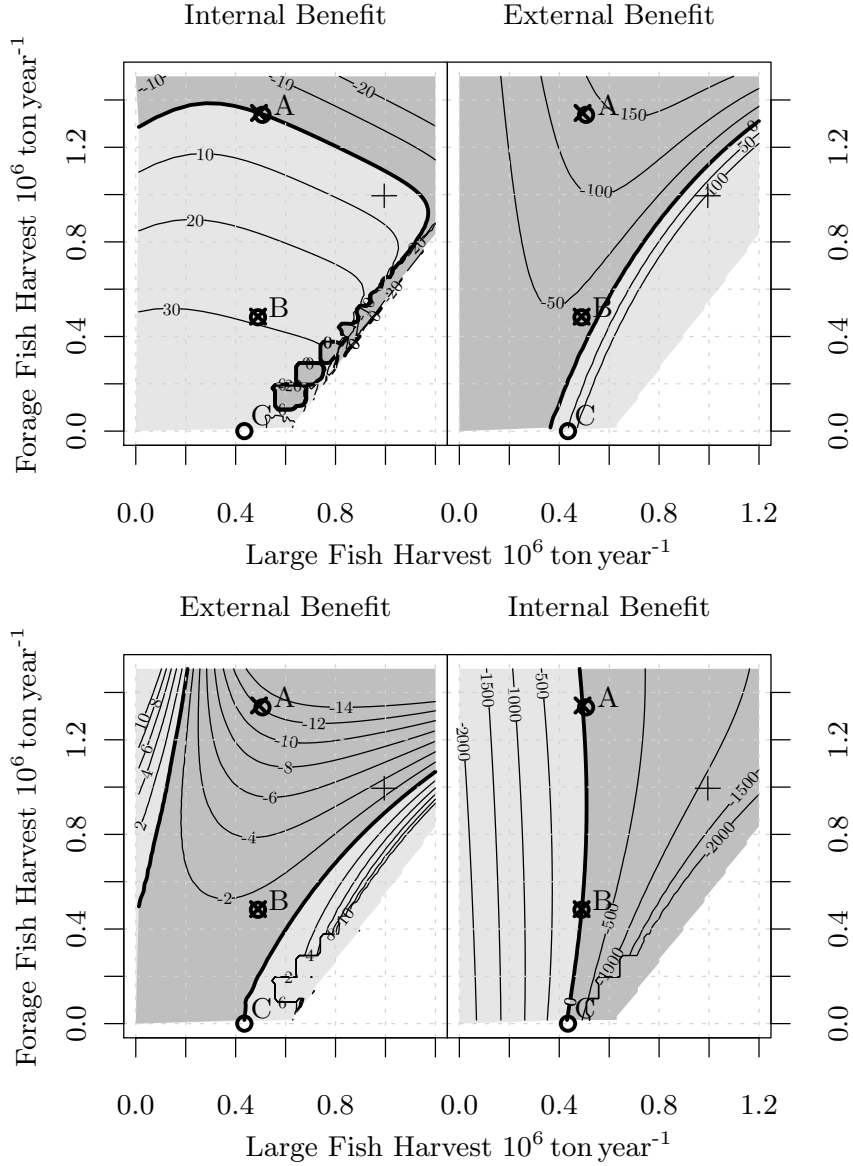


FIGURE 14: Sensitivity with respect to control variable: The benefit indicators (€ ton^{-1}) for the North Sea forage fish fleet (top) and large fish fleet (bottom) divided into internal benefit $B_{i/i}$ and external benefit $B_{i/j}$. The figure is equivalent to Fig. 4 in the main paper. The points marked are the same as in Fig. 13. The plus sign is the current state of the North Sea; at this point, $B_{F/F} = 5$, $B_{L/F} = 100$, and $B_{F/L} = -3$ and $B_{L/L} = -1089$ (all in € ton^{-1}).

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