

Zhao, Kai

**Working Paper**

## Health insurance, annuities, and public policy

Research Report, No. 2013-1

**Provided in Cooperation with:**

Department of Economics, University of Western Ontario

*Suggested Citation:* Zhao, Kai (2013) : Health insurance, annuities, and public policy, Research Report, No. 2013-1, The University of Western Ontario, Department of Economics, London (Ontario)

This Version is available at:

<https://hdl.handle.net/10419/123306>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

**Health Insurance, Annuities, and Public  
Policy**

**by**

**Kai Zhao**

**Research Report # 2013-1**

**March 2013**



***Department of Economics  
Research Report Series***

Department of Economics  
Social Science Centre  
The University of Western Ontario  
London, Ontario, N6A 5C2  
Canada

This research report is available as a downloadable pdf file on our website  
<http://economics.uwo.ca/econref/WorkingPapers/departmenresearchreports.html>.

# Health Insurance, Annuities, and Public Policy\*

Kai Zhao<sup>†</sup>

University of Western Ontario

October 21, 2012

## Abstract

This paper studies the effects of health shocks on the demand for health insurance and annuities, precautionary saving, and the welfare implications of public policies in a simple life-cycle model. I show that when the health shock *simultaneously* increases health expenses and reduces longevity, the following results can be obtained via *closed-form* solutions. First, utility-maximizing agents would neither fully insure their uncertain health expenses nor fully annuitize their wealth, even in the absence of market frictions and bequest motives. Second, the effect of uncertain health expenses on precautionary saving may be smaller than what has been found in previous studies. Under certain conditions, uncertain health expenses may even reduce precautionary saving. Third, mandatory health insurance (e.g. public health insurance) tends to benefit the poor more, while mandatory annuitization (e.g. public pension) is more likely to favor the rich. A simple numerical application of the model to the US long term care (LTC) insurance market suggests that the simultaneous effect of health shock on health expenses and longevity is a quantitatively important reason why agents (especially the rich) do not purchase more private LTC insurance.

**Keywords:** Saving, Annuities, Health Insurance, Social Security, Medicare.

**JEL** Classifications:

---

\*I would like to thank Daniel Brou, Hugh Cassidy, Betsy Caucutt, Jim Davies, and Karen Kopecky for their helpful comments.

<sup>†</sup>Assistant Professor, Management and Organizational Studies, The University of Western Ontario, 4424 Social Science Centre, London, ON N6A 5C2, Canada (email: kzha06@uwo.ca).

# 1 Introduction

It is well known that health expenses in developed countries have risen dramatically over the last several decades and are projected to continue rising in the near future . For instance, the US aggregate health expenses rose from 5.2% of GDP in 1960 to 16% of GDP in 2007, are projected to be 25% in 2025 and 37 % in 2050.<sup>1</sup> Importantly, health expenses are extremely volatile and a significant portion of these expenses are not insured (out-of-pocket expenses). Meanwhile, a large cohort of baby boomers are retiring in the next decade. Hence, it is important to understand how health shocks affect retirees' lifetime financial planning. On the other hand, many developed countries have been considering policy reforms on their public pension and public health insurance programs, thus it is also important to understand the implications of health shocks for the welfare results of these public polices.

In this paper, I develop a simple life-cycle model and use it to study the effects of health shocks on the demand for health insurance and annuities, precautionary saving, and the welfare implications of public policies. In particular, I focus on health shocks that *simultaneously* increase health expenses and reduce longevity.

Many types of health shocks have simultaneous effects on health expenses and longevity. For instance, entering into long term care not only significantly increases health expenses, but also reduces survival probabilities to the future (Sinclair and Smetters (2004), Kopecky and Koreshkova (2009), etc.) Hurd, McFadden, and Merrill (2001) found that a variety of health conditions, e.g. cancer, heart disease, can reduce survival probabilities to the future. Similar results are found for the general concept of health shock, a health status change. De Nardi, French, and Jones (2010) documented in the AHEAD data that conditional on permanent income, gender, and age, people in good health status spend around 50% less on health care annually than those in bad health status, but they expect to live about 3 years longer than those in bad health.

In the model, I show via closed-form solutions that when health shocks can simultaneously increase health expenses and reduce longevity, several interesting results can be obtained. First, utility-maximizing agents would neither fully insure their uncertain health expenses nor fully annuitize their wealth, even when these insurance policies are actuarially-fair and there is no bequest motive. Second, in contrast to the existing literature, uncertain health expenses may even reduce precautionary saving under certain conditions. Third, mandatory health insurance (e.g. public health insurance) tends to benefit the poor more, while mandatory annuitization (e.g. public pension) is more likely to favor the rich.

---

<sup>1</sup>The 1960 and 2007 numbers are from OECD Health Data 2009. The projected numbers are from the Congressional Budget Office.

The intuition behind these results is simple. The simultaneous effect of health shocks on health expenses and longevity provides agents with a self insurance channel for both uncertain health expenses and uncertain longevity. When the agent is hit by a health shock (which simultaneously increases health expenses and reduces longevity), she can use the resources originally saved for consumption in the reduced period of life to pay for the increased health expenses. As a result, agents would neither fully insure their health expenses nor fully annuitize their wealth. Furthermore, when the resources freed up from a reduction in longevity are more than enough to pay for the simultaneous increase in health expenses, an increase in the uncertainty of health expenses may even improve consumption smoothing, and decrease precautionary saving. Note that the amount of resources freed up from a given reduction in longevity is usually higher for richer agents. Hence, mandatory health insurance is likely to be welfare-decreasing for the rich as the resources freed up from the reduction in longevity may be already more than enough to compensate for the increase in health expenses. In contrast, mandatory annuitization is more likely to be welfare-decreasing for the poor.

To assess the empirical relevance of the model implications, in section 6 I apply the model to the US long-term care (LTC) insurance market. Using numerical simulation techniques, I study to what extent the simultaneous effect of health shocks on health expenses and longevity can explain the puzzling fact in the US LTC insurance market. That is, why most Americans do not buy private LTC insurance? I find that the model can account for this LTC insurance puzzle (especially for the rich). This finding complements the existing findings on this topic, which say that Medicaid together with supply-side frictions can explain why Americans in the bottom two thirds of the wealth distribution do not buy private LTC insurance, but cannot explain why the rest of Americans (the rich) also do not buy LTC insurance.<sup>2</sup>

This paper is related to Sinclair and Smetters (2004) who have also studied the implications from the simultaneous effect of health shocks on health expenses and longevity. In a quantitative OLG model, they show that the simultaneous effect of health shocks on health expenses and longevity reduces the demand for annuities via numerical simulations. The same result is also found in this paper. However, in contrast to their paper, I provide closed-form solutions. Furthermore, I also derive new implications for the demand for health insurance, precautionary saving, and the welfare effects of public policies.

This paper contributes to several literatures. First, it contributes to the literature on the annuity puzzle by providing a new explanation for why people do not fully annuitize their wealth.<sup>3</sup> Second, this paper is also related to the recent literature that studies consumption

---

<sup>2</sup>See Brown and Finkelstein (2007, 2008), etc.

<sup>3</sup>Yaari (1965), Kotlikoff and Spivak (1981), Sinclair and Smetters (2004), Pashchenko (2010), Lockwood (2011), etc.

and saving decisions when facing uncertain health expenditures.<sup>4</sup> The results of this paper suggest that the positive effect of uncertain health expenditures on saving may be overstated if the simultaneous effect of health shocks on health expenditures and longevity is not taken into account. Third, this paper is related to the recent public policy debate on health care reform in the US. An important motivation behind the recent “Obamacare” reform (the Patient Protection and Affordable Care Act) is to reduce the large number of uninsured Americans.<sup>5</sup> However, the results of this paper suggest that being uninsured may be the optimal choice for some households, thus the optimal public health insurance policy should not aim to insure everyone. Last, the finding from the numerical application of the model complements the existing literature on the LTC insurance puzzle by showing that the simultaneous effect of LTC shock on health expenses and longevity is a quantitatively important reason why people (especially the rich) do not buy LTC insurance in the US.

The rest of the paper is organized as follows. In section 2, I present a simple example to illustrate the intuition. In sections 3, 4, and 5, I present the full model and derive the analytical results. In section 6, I conduct a numerical application of the model to the demand for long term care insurance. I conclude in section 7.

## 2 A Simple Example

In this section, I present a simple numerical example to illustrate the intuition behind the main findings of this paper. Here I only look at the problem after retirement. Assume that an agent with endowment  $W$  faces the following two-period expected utility maximization problem,

$$\max_{C_1(h), C_2(h)} E[U(C_1(h)) + S(h)U(C_2(h))] \quad (1)$$

subject to

$$W - M(h) - C_1(h) = C_2(h), \forall h, \quad (2)$$

$$C_1(h) \geq 0 \text{ and } C_2(h) \geq 0, \forall h$$

Here the utility function,  $U(C)$ , is assumed to satisfy the Inada conditions. Consumption is denoted by  $C$ ,  $M$  is the health expense, and  $S$  is the survival probability to period 2. The agent receives a health shock,  $h$ , at the beginning of period 1. When it is a bad shock, i.e.  $h = h_b$ , the agent needs to pay health expenses  $M(h_b) = \frac{W}{2}$ , and she will *not* survive to period 2 for

---

<sup>4</sup>Jeske and Kitao (2009), De Nardi, French, and Jones (2010), Kopecky and Koreshkova (2009), and Yogo (2009), etc.

<sup>5</sup>For instance, according to US census bureau, approximately 49 millions Americans are without health insurance in 2010.

sure, i.e.  $S(h_b) = 0$ . When it is a good shock, the agent needs to pay no health expense, i.e.  $M(h_g) = 0$ , and she will survive to the second period for sure, i.e.  $S(h_g) = 1$ . For simplicity, the discounting factor and the gross interest rate are both equal to one.

Assuming that there are neither health insurance nor annuities available, the agent's optimal decision can be easily derived:  $C_1(h_g) = C_1(h_b) = \frac{W}{2}$ ,  $C_2(h_g) = \frac{W}{2}$ . Note that the agent faces both uncertain health expenses and uncertain longevity in this environment, but she is able to achieve perfect consumption smoothing over different states and time periods, even without any health insurance and annuities. The intuition behind this result is clear, the simultaneous effects of health shock on longevity and health expenses provide the agent with a self insurance channel for both uncertain longevity and uncertain health expenses. When the agent is hit by a bad shock, she uses the resources originally saved for consumption in period 2 to pay the increased health expenses.

### 3 The Model

There exist ex ante homogeneous agents of measure one. Again, here I only look at the problem after retirement. I will extend the model to include the working period in section 5. Each agent is initially endowed with asset  $W$ . At the beginning of time, she is hit by a health shock,  $h$ , which will determine her lifetime health expenses,  $M(h)$ , and longevity,  $T(h)$ . For simplicity, it is assumed that both the discounting factor and the gross interest rate are equal to one. To have a meaningful problem, I also assume that the expected health expenses are less than the initial endowment, i.e.  $E[M(h)] < W$ . Agents face the following expected utility maximizing problem.

$$\max_{\{C(t,h)\}_0^{T(h)}} E\left[\int_0^{T(h)} U(C(t,h))dt\right], \quad (3)$$

subject to

$$W - M(h) = \int_0^{T(h)} C(t,h), \forall h. \quad (4)$$

$$C(t,h) \geq 0, \forall h, t,$$

Here  $U(\cdot)$  satisfies the Inada Conditions, and  $C(t,h)$  represents the consumption at time  $t$ , conditional on the health shock,  $h$ . The health shock,  $h$ , has the following properties:  $h = h_g$  (good shock) with a probability of  $1 - P$ , and  $h = h_b$  (bad shock) with a probability of  $P$ . The lifetime health expenses and the longevity are determined by the health shock in the following way,  $M(h_g) = 0$ ,  $M(h_b) = M$ , and  $T(h_g) = T$ ,  $T(h_b) = \delta T$ , where  $0 < \delta < 1$ .

Since both the discounting factor and the gross interest rate are equal to one, it is obvious that rational agents will choose a flat consumption path after the health shock. That is,  $C(t,h) =$

$C(t', h)$ , for any  $t, t' \in [0, T(h)]$ . Using  $C(h)$  to represent the constant consumption per period, the above utility maximizing problem is simplified to the following problem,

$$\max_{C(h)} E[T(h)U(C(h))] \quad (5)$$

subject to

$$W - M(h) = T(h)C(h), \forall h \quad (6)$$

$$C(h) \geq 0, \forall h.$$

Assuming that neither annuities nor health insurance are available, the optimal solution for the above problem can be easily obtained. That is,  $C^*(h_g) = \frac{W}{T}$  and  $C^*(h_b) = \frac{W-M}{\delta T}$ . As can be seen, health insurance or annuities before the health shock is revealed can be welfare-improving as long as the following condition holds,

$$C^*(h_g) = \frac{W}{T} \neq \frac{W-M}{\delta T} = C^*(h_b) \quad (7)$$

thus,

$$M \neq W(1 - \delta) \quad (8)$$

### 3.1 Health Insurance

Now I consider agents' demand for health insurance in this model. Assume that the annuity market is closed, but agents have access to actuarially-fair health insurance. That is, the price of health insurance with a coinsurance rate of  $I$  is  $q_I = PIM$ . Agents maximize their expected lifetime utility by choosing the optimal coinsurance rate,  $I^*$ . That is, they face the following expected utility-maximizing problem,

$$\max_{C(h), I} E[T(h)U(C(h))] \quad (9)$$

subject to

$$W - M(h) - PIM + M(h)I = T(h)C(h), \forall h, \quad (10)$$

$$C(h) \geq 0, \forall h, \text{ and } I \geq 0.$$

Let us study this problem in two different scenarios.

(1)  $M \leq W(1 - \delta)$ . As shown in equations (7) and (8), even without any health insurance, agents already have a higher consumption per period after a bad health shock than after a good shock. Therefore, in this scenario, agents do not need any health insurance, i.e.  $I^* = 0$  (corner solution). The intuition behind this result is simple. If the health expenses (i.e.  $M$ ) are not



larger than the resources freed up from a reduction in longevity (i.e.  $W(1-\delta)$ ), health insurance is not needed, as the self insurance channel itself is enough to insure against the risk.

(2)  $M > W(1-\delta)$ . In this scenario, there exists an interior solution for  $I$ . After substituting the budget constraint into the objective function, the following First Order Condition (FOC) can be obtained,

$$-(1-P)TU'\left(\frac{W-PIM}{T}\right)\frac{PM}{T} - P\delta TU'\left(\frac{W-M-PIM+IM}{\delta T}\right)(M-PM)\frac{1}{\delta T} = 0 \quad (11)$$

Rearranging the above equation and solving for  $I$ ,

$$I^* = \frac{1 - \frac{W}{M}(1-\delta)}{1 - P(1-\delta)} \quad (12)$$

The above equation describes the optimal solution for  $I^*$ . From this equation, the following propositions can be obtained,

**Proposition 1:** (1) The optimal health coinsurance rate,  $I^*$ , is less than 1. In other words, agents do not choose to fully insure their health expense risk. (2) The optimal health coinsurance rate,  $I^*$ , decreases as the reduction in life expectancy increases, i.e.  $\frac{\partial I^*}{\partial \delta} < 0$ ,  $I^*$  increases as the probability of getting a bad shock increases, i.e.  $\frac{\partial I^*}{\partial P} > 0$ , and  $I^*$  decreases the endowment increases,  $\frac{\partial I^*}{\partial W} < 0$ .

**Proof:** As for statement (1), since the expected health expense is less than the initial endowment, the following inequation holds,  $E[M(h)] = PM < W$ . Rearranging and multiplying both sides of this inequation by  $(1-\delta)$ , I obtain  $P(1-\delta) < \frac{W}{M}(1-\delta)$ . As a result,  $1 - \frac{W}{M}(1-\delta) < 1 - P(1-\delta)$ , and thus  $I^* = \frac{1 - \frac{W}{M}(1-\delta)}{1 - P(1-\delta)} < 1$ . Statement (2) can be simply obtained by taking the first order derivative of equation (12) with respect to  $\delta$ ,  $P$ , and  $W$ , respectively.

### 3.2 Annuities

Now I consider agents' demand for annuities in this model. Assume that the health insurance market is closed, but agents have access to actuarially-fair annuities. That is, the price of an annuity policy that pays  $A$  per period while alive is,  $q_A = P\delta TA + (1-P)TA$ . Note that rational agents never spend more than  $W - M$  on annuities, otherwise they will not have resources for consumption after a bad health shock. That is, they never choose an annuity level  $A > \frac{W-M}{P\delta T + (1-P)T}$ . Therefore, the optimal annuity level,  $A^*$ , solves the following problem,

$$\max_A E[T(h)U(C(h))], \quad (13)$$

subject to

$$W - M(h) - (P\delta TA + (1 - P)TA) + T(h)A = T(h)C(h), \forall h \quad (14)$$

$$C(h) \geq 0, \forall h, \text{ and } A \geq 0.$$

Again, I analyze the problem in two cases.

(1)  $M \geq W(1 - \delta)$ . In this case, even without purchasing any annuity, the agent would already have a higher consumption per period when she happens to live longer than expected. Therefore, agents do not need any annuity, i.e.  $A^* = 0$ . The intuition behind this result is simple. If the health expense saved is larger than the resources needed for the extra years of life, no annuity is needed.

(2)  $M < W(1 - \delta)$ . In this case, agents need annuities to insure against the risk of outliving their resources (interior solution). After substituting the budget constraints into the objective function, the following FOC can be obtained,

$$(1 - P)TU' \left( \frac{W}{T} - (P\delta A + (1 - P)A) + A \right) [1 - (P\delta + (1 - P))] + \\ P\delta TU' \left( \frac{W - M}{\delta T} - (PA + \frac{(1 - P)A}{\delta}) + A \right) [1 - (P + \frac{(1 - P)}{\delta})] = 0$$

Rearranging the above equation and solving for A,

$$A^* = \frac{W - \frac{M}{1 - \delta}}{P\delta T + (1 - P)T} \quad (15)$$

The above equation describes the optimal annuity level, and the price of this annuity policy is,

$$q_A^* = (P\delta T + (1 - P)T)A^* = W - \frac{M}{(1 - \delta)} \quad (16)$$

As can be seen, the annuitized wealth (measured by  $q_A^*$ ) is less than the total wealth available after the health shock ( $W$  or  $W - M$ ). In other words, agents do not fully annuitize their wealth. I summarize the main properties of the optimal annuitization decision in the following proposition.

**Proposition 2:** (1) Agents do not fully annuitize their wealth, i.e.  $q_A^* < W - M$  or  $W$ . (2) The annuitized wealth (measured by  $q_A^*$ ) increases as the initial wealth increases, i.e.  $\frac{\partial q_A^*}{\partial W} > 0$ . (3) The annuitized wealth decreases as the health expenses increase, i.e.  $\frac{\partial q_A^*}{\partial M} < 0$ . (4) The annuitized wealth increases as the reduction in life expectancy increases, i.e.  $\frac{\partial q_A^*}{\partial (1 - \delta)} > 0$ .

**Proof:** Statement (1) is from the assumption  $0 < \delta < 1$ . Statements (2)-(4) can be easily obtained by taking the first order derivative of equation (16) with respect to  $W$ ,  $M$ , and  $1 - \delta$ , respectively.

It is worth mentioning that people may also be interested in another measure of annuitization, the fraction of wealth that is annuitized, which can be measured by  $\frac{q_A^*}{W} = 1 - \frac{M}{(1-\delta)W}$ . By taking the first order derivative of  $\frac{q_A^*}{W}$  with respect to  $W$ ,  $M$ , and  $1 - \delta$ , respectively, it is easily to see that statements (2)-(4) in proposition 2 would still hold if  $\frac{q_A^*}{W}$  is used as the measure of annuitization instead of  $q_A^*$ .<sup>6</sup>

### 3.3 Annuities and Health Insurance

Now I study a version of the model in which agents have access to both actuarially-fair health insurance and actuarially-fair annuities. It is worth noting that here annuities and health insurance are linked together by the simultaneous effect of health shocks on longevity and health expenses. In fact, these two products are insuring against the same risk in the model, but in the opposite direction. This finding has an interesting implication, that is, when both products are available in the market, the optimal demand for annuities and health insurance is indeterminate. The reason for that is agents can always increase their holdings of both annuities and health insurance simultaneously and still achieve perfect consumption smoothing because the extra annuities and health insurance offset each other. I rule out this possibility for the sake of reality in the following analysis.<sup>7</sup>

Based on the analysis in sections 3.1 and 3.2, it is obvious that agents with endowment  $W > \frac{M}{1-\delta}$ , only need annuities, while agents with endowment  $W < \frac{M}{1-\delta}$ , only need health insurance. These results are summarized in proposition 3 below.

**Proposition 3:** When both annuity and health insurance markets are open, the optimal solution for agents with  $W > \frac{M}{1-\delta}$  is,  $A^* = \frac{W - \frac{M}{1-\delta}}{P\delta T + (1-P)T}$ , and  $I^* = 0$ . The optimal solution for agents with  $W \leq \frac{M}{1-\delta}$  is,  $A^* = 0$ , and  $I^* = \frac{1 - \frac{W}{M}(1-\delta)}{1-P(1-\delta)}$ .

The intuition behind proposition 3 is simple. Since the relatively rich (agents with  $W > \frac{M}{1-\delta}$ ) tend to consume more per period than the relatively poor (agents with  $W \leq \frac{M}{1-\delta}$ ), for the same reduction in longevity, the resources freed up for the relatively rich are usually more than those for the relatively poor, and thus are more likely to be enough to compensate for the simultaneous increase in health expenses. In other words, the simultaneous effects of health shocks on health expenses and longevity has differential effects across the income distribution. It provides more insurance against uncertain health expenses for the relatively rich, and more insurance against uncertain longevity for the relatively poor.

As can be easily seen, the finding in proposition 3 has direct implications for the distributional

<sup>6</sup>Note that the same result is obtained if  $\frac{q_A^*}{W-M}$  is used instead of  $\frac{q_A^*}{W}$ .

<sup>7</sup>In reality, this result is not very likely to occur because there are entry costs and administrative costs in both markets.

effects of relevant public policies, which will be considered in the next section.

## 4 Welfare Effects of Public Policy

In this section, I study the welfare effects of two relevant public policies, i.e. mandatory annuitization (e.g. public pension) and mandatory health insurance (e.g. public health insurance). I do so in a version of the economy without private annuity and health insurance markets.<sup>8</sup> Assume that the economy contains an actuarially-fair mandatory annuity program and an actuarially-fair mandatory health insurance program. The annuity program pays  $A$  per year while alive and collects  $q_A = P\delta TA + (1 - P)TA$  at the beginning of time. The health insurance program provides a coinsurance rate of  $I$  to health expenses and collects  $q_I = PIM$  at the beginning of time. To have a meaningful problem, I also assume that  $W - q_A - q_I > (1 - I)M$ , that is, the initial endowment after taxes is enough to cover the out-of-pocket health expense after a bad shock. As a result, agents face the following expected utility maximizing problem,

$$\max_{\{C(t,h)\}_0^{T(h)}} E\left[\int_0^{T(h)} U(C(t,h))dt\right], \quad (17)$$

subject to

$$W - q_A - q_I - M(h) + IM(h) + \int_0^{T(h)} A = \int_0^{T(h)} C(t,h), \forall h. \quad (18)$$

$$C(t,h) \geq 0, \forall h, t,$$

Since both the discounting factor and the gross interest rate are equal to one, it is obvious that rational agents will choose a flat consumption path after the health shock. That is,  $C(t,h) = C(t',h)$ , for any  $t, t' \in [0, T(h)]$ . Using  $C(h)$  to represent the constant consumption per period, its optimal value,  $C^*(h)$ , can be expressed as follows,

$$C^*(h) = \frac{W - q_A - q_I - M(h) + IM(h) + T(h)A}{T(h)}, \forall h. \quad (19)$$

Thus, agents' maximized expected lifetime utility can be simply expressed as follows,

$$\bar{U} = ET(h)U(C^*(h)). \quad (20)$$

---

<sup>8</sup>One justification for this assumption is that these markets are very thin in most developed countries. Note that when endogenous private markets are included, the effects of public policies may be smaller than what have been found here because of the crowding-out effects of public insurance on private insurance. I leave this issue for future research.

## 4.1 Mandatory Annuitization

To study the welfare effect of mandatory annuitization, I simply consider how a marginal increase in annuity payment  $A$  affects individual welfare.<sup>9</sup> This can be captured by the first order derivative of the maximized expected lifetime utility  $\bar{U}$  w.r.t.  $A$ ,

$$\frac{\partial \bar{U}}{\partial A} = (1 - P)TU' \left( \frac{W - q_A - q_I + TA}{T} \right) (P - P\delta) - PTU' \left( \frac{W - q_A - q_I - (1 - I)M + \delta TA}{\delta T} \right) (1 - P)(1 - \delta)$$

As can be seen, the sign of  $\frac{\partial \bar{U}}{\partial A}$  depends on whether the first term on the right-hand side is larger than the second term. After some simple algebra, the following results about the welfare effect of mandatory annuitization can be obtained.<sup>10</sup>

**Proposition 4:** (1) When  $\frac{(1-I)M}{1-\delta} + q_A + q_I < W$ , mandatory annuitization increases individual welfare, i.e.  $\frac{\partial \bar{U}}{\partial A} > 0$ . (2) When  $\frac{(1-I)M}{1-\delta} + q_A + q_I > W$ , mandatory annuitization reduces individual welfare, i.e.  $\frac{\partial \bar{U}}{\partial A} < 0$ . (3) The marginal effect of mandatory annuitization on welfare is nil when  $\frac{(1-I)M}{1-\delta} + q_A + q_I = W$ .

This proposition says that there exists a cut-off level for initial endowment,  $\frac{(1-I)M}{1-\delta} + q_A + q_I$ , such that mandatory annuitization is welfare-improving for agents with initial endowment above the cut-off level, but it is welfare-reducing for those with initial endowment below this cut-off level. In other words, mandatory annuitization is more likely to benefit the rich.

It is worth noting that after rearranging, the condition,  $\frac{(1-I)M}{1-\delta} + q_A + q_I = W$ , becomes  $q_A = (W - q_I) - \frac{(1-I)M}{1-\delta}$ , which is in fact corresponding to the solution for optimal annuity level derived in section 3.2 (equation (16)). Here  $W - q_I$  is the wealth (net of health insurance premium), and  $(1 - I)M$  is the out-of-pocket health expenses after a bad shock. Therefore, an alternative interpretation of proposition 4 is that mandatory annuitization is welfare-improving if the mandatory annuity level is still lower than the optimal annuity level, and it is welfare-decreasing if the mandatory annuity level is above the optimal level.

## 4.2 The Welfare Effect of Mandatory Health Insurance

To study the welfare effect of mandatory health insurance, I consider how a marginal increase in health coinsurance rate  $I$  affects individual welfare.<sup>11</sup> This can be captured by the first order

<sup>9</sup>Note that the corresponding annuity tax  $q_A$  is also adjusted so that the annuity program is self-financing.

<sup>10</sup>The algebra is available from the author upon request.

<sup>11</sup>The corresponding health insurance tax  $q_I$  is also adjusted so that the health insurance program is self-financing.

derivative of the maximized expected lifetime utility  $\bar{U}$  w.r.t.  $I$ ,

$$\frac{\partial \bar{U}}{\partial I} = -(1-P)TU' \left( \frac{W - q_A - q_I + TA}{T} \right) \frac{PM}{T} + P\delta TU' \left( \frac{W - q_A - q_I - (1-I)M + \delta TA}{\delta T} \right) \frac{(1-P)M}{\delta T}.$$

Again, the sign of  $\frac{\partial \bar{U}}{\partial I}$  depends on whether the first term on the right-hand side is larger than the second term. After some simple algebra, the following results about the welfare effect of mandatory health insurance can be obtained.<sup>12</sup>

**Proposition 5:** (1) Mandatory health insurance increases individual welfare, i.e.  $\frac{\partial \bar{U}}{\partial I} > 0$ , when  $\frac{(1-I)M}{1-\delta} + q_A + q_I > W$ . (2) It decreases individual welfare, i.e.  $\frac{\partial \bar{U}}{\partial I} < 0$ , when  $\frac{(1-I)M}{1-\delta} + q_A + q_I < W$ . (3) Its marginal effect on individual welfare is nil when  $\frac{(1-I)M}{1-\delta} + q_A + q_I = W$ .

As can be seen, the cut-off wealth level for mandatory health insurance is the same as mandatory annuitization, but its welfare effects are exactly the opposite of the welfare effects of mandatory annuitization. In other words, mandatory health insurance is more likely to be welfare-improving for the poor, while it can be welfare-reducing for the rich.

Rearranging the condition,  $\frac{(1-I)M}{1-\delta} + q_A + q_I = W$ , I obtain  $I = \frac{1 - \frac{W - q_A}{M}(1-\delta)}{1 - P(1-\delta)}$ , which corresponds to the solution for optimal health coinsurance rate derived in section 3.1 (equation (12)). Here  $W - q_A$  is the wealth (net of the annuity premium). Therefore, an alternative interpretation of proposition 5 is that mandatory health insurance is welfare-improving if the coinsurance rate is still lower than the optimal health coinsurance rate, and it is welfare-decreasing if the coinsurance rate is already above the optimal rate.

### 4.3 The Interaction between Mandatory Annuitization and Mandatory Health Insurance

It is also interesting to see whether mandatory annuitization and mandatory health insurance interact with each other. That is, does mandatory health insurance (mandatory annuitization) change the welfare effect of mandatory annuity (mandatory health insurance)? To answer this question, I simply look at  $\frac{\partial \bar{U}}{\partial I \partial A}$ , which is derived in the following,

$$\frac{\partial \bar{U}}{\partial I \partial A} = -(1-P)TU'' \left( \frac{W - q_A - q_I + TA}{T} \right) P(1-\delta) \left( \frac{PM}{T} \right) - PTU'' \left( \frac{W - q_A - q_I - (1-I)M + \delta TA}{\delta T} \right) (1-P)(1-\delta) \left( \frac{M - PM}{\delta T} \right)$$

<sup>12</sup>The algebra is available from the author upon request.

From the above equation, it is obvious to see that  $\frac{\partial \bar{U}}{\partial T \partial A}$  is positive because  $U'' < 0$ . Therefore, the following result can be obtained,

**Proposition 6:** Mandatory annuitization and mandatory health insurance positively interact with each other. That is, mandatory health insurance (mandatory annuitization) increases the marginal effect of mandatory annuity (mandatory health insurance) on individual welfare.

This result has a very clear policy implication. That is, reforming one policy has a spill-over effect on the welfare result of the other policy, which should be taken into account in any future related policy studies.

## 5 Uncertain Health Expenses and Precautionary Saving

Economists have long argued that uncertain out-of-pocket health expenses generate precautionary saving. Recently, there has been a growing literature arguing that the precautionary saving for uncertain health expenses is large and is quantitatively important for accounting for the US wealth inequality.<sup>13</sup> However, most studies in the literature do not take into account the fact that uncertain health expenses are correlated with uncertain longevity, and how it implies for precautionary saving.<sup>14</sup>

In this section, I show that when uncertain health expenses are correlated with uncertain longevity, the effect of uncertain health expenses on precautionary saving may be different from what has been found in previous studies.<sup>15</sup> Under certain conditions, the effect may even be negative (especially for richer households).

To study the saving behavior, I extend the model by introducing a working period before retirement. The length of the working period is normalized to one, so the length of retirement,  $T(h)$ , measures the relative length of retirement compared to work. In the working period, each agent receives income  $Y$ , and after that she chooses consumption for that period,  $C_1$ , and savings for retirement,  $W = Y - C_1$ . In the retirement period, agents face the same problem as in the benchmark model. It is assumed that private markets for annuities and health insurance are closed. Agents face the following expected utility maximizing problem,

$$\max_{W, C_1, C_2(h)} U(C_1) + E[T(h)U(C_2(h))], \quad (21)$$

---

<sup>13</sup>Hubbard, Skinner, and Zeldes (1995), De Nardi, French, and Jones (2010), Kopecky and Koreshkova (2009), etc.

<sup>14</sup>Note that a few studies do partially capture the correlation between uncertain health expenses and uncertain longevity in their models. For instance, De Nardi, French, and Jones (2010) partially capture the correlation through health status. Kopecky and Koreshkova (2009) partially capture the correlation through nursing home shock. However, all these papers do not explore its implication for precautionary saving.

<sup>15</sup>Note that the findings here are also related to the papers that study the saving effects of uncertain lifetime, such as Davies (1981) and Leung (1994).

subject to

$$Y - W = C_1 \quad (22)$$

$$W = M(h) + T(h)C_2(h), \forall h \quad (23)$$

Substituting the budget constraints into the objective function, the first order condition w.r.t.  $W$  can be derived,

$$U'(Y - W^*) = EU'(\frac{W^* - M(h)}{T(h)}) \quad (24)$$

The above equation describes the optimal saving decision,  $W^*$ .

For the convenience of the analysis, I assume that health expenses and longevity have the following properties:  $M(h_g) = M - \frac{P}{1-P}\Delta_M$ ,  $M(h_b) = M + \Delta_M$ , and  $T(h_g) = T + \Delta_T$ ,  $T(h_b) = T - \frac{1-P}{P}\Delta_T$ . This assumption implies that the expected health expense (equal to  $M$ ) is independent of  $\Delta_M$  and the expected longevity (equal to  $T$ ) is independent of  $\Delta_T$ . As a result, the equation determining the optimal saving decision (equation (24)) becomes,

$$U'(Y - W^*) = (1 - P)U'(C_2^*(h_g)) + PU'(C_2^*(h_b)), \quad (25)$$

where  $C_2^*(h_g) = \frac{W^* - M + \frac{P}{1-P}\Delta_M}{T + \Delta_T}$ , and  $C_2^*(h_b) = \frac{W^* - M - \Delta_M}{T - \frac{1-P}{P}\Delta_T}$ .

To understand how uncertain health expenses affect saving in the model, I simply look at the effect of a marginal increase in  $\Delta_M$  on  $W^*$ , i.e.  $\frac{\partial W^*}{\partial \Delta_M}$ . To derive  $\frac{\partial W^*}{\partial \Delta_M}$ , I use the implicit function theorem. Setting  $F = -U'(Y - W^*) + (1 - P)U'(C_2^*(h_g)) + PU'(C_2^*(h_b))$ ,

$$\frac{\partial W^*}{\partial \Delta_M} = -\frac{\partial F / \partial \Delta_M}{\partial F / \partial W} = -\frac{(1 - P)U''(C_2^*(h_g))\frac{P}{(T + \Delta_T)(1 - P)} - PU''(C_2^*(h_b))\frac{1}{T - \frac{1-P}{P}\Delta_T}}{U''(Y - W^*) + \frac{(1 - P)U''(C_2^*(h_g))}{T + \Delta_T} + \frac{PU''(C_2^*(h_b))}{T - \frac{1-P}{P}\Delta_T}}. \quad (26)$$

Since  $U'' < 0$ , the denominator in the above equation is negative. Hence, the sign of  $\frac{\partial W^*}{\partial \Delta_M}$  is equivalent to the sign of the numerator,

$$(1 - P)U''(C_2^*(h_g))\frac{P}{(T + \Delta_T)(1 - P)} - PU''(C_2^*(h_b))\frac{1}{T - \frac{1-P}{P}\Delta_T}. \quad (27)$$

After some simple algebraic manipulation, the following proposition can be obtained.

**Proposition 7:** (1) Uncertain health expenses reduce precautionary saving, i.e.  $\frac{\partial W^*}{\partial \Delta_M} < 0$ , if the following condition holds,

$$U''(C_2^*(h_g))\left[\frac{T - \frac{1-P}{P}\Delta_T}{(T + \Delta_T)}\right] - U''(C_2^*(h_b)) < 0. \quad (28)$$

Note that when the condition (28) does not hold, the effect of uncertain health expenses



on precautionary saving would be either positive or nil, which is the standard answer in the literature.

To better understand the intuition behind the negative effect of uncertain health expenses on precautionary saving (described in proposition 7), I consider the following two scenarios in which condition (28) holds:

(1)  $U''(C_2^*(h_g)) < U''(C_2^*(h_b))$ . In this case,  $C_2^*(h_g) < C_2^*(h_b)$  as  $U'''() > 0$ . That is, the second-period consumption after a bad health shock is even higher than that after a good health shock. The reason for that is the resources originally saved for the reduced period of life are more than enough cover the extra health expenses after a bad health shock. As a result, an increase in the uncertainty of health expense even improves consumption smoothing across states, thus reducing precautionary saving.

(2)  $U''(C_2^*(h_g)) \geq U''(C_2^*(h_b))$  but  $[\frac{T-\frac{1-P}{P}\Delta_T}{(T+\Delta_T)}] < \frac{U''(C_2^*(h_b))}{U''(C_2^*(h_g))}$ . In this case, the intuition behind the negative effect of uncertain health expenses on precautionary saving is less obvious. As can be seen,  $C_2^*(h_g) > C_2^*(h_b)$  in this case as  $U'''() > 0$ . That is the second-period consumption after a bad health shock is lower than that after a good health shock, and thus an increase in the uncertainty of health expense should reduce consumption smoothing across states. However, why does it not increase precautionary saving? The intuition for that is as follows. According to the existing literature on precautionary saving (e.g. Leland (1968), Kimball (1990)), precautionary saving increases (decreases) when the shock increases (decreases) the expected marginal utility function in the second period ( $EU'$ ). In a standard framework (in which the health shock does not also affect longevity), the first order derivative of the expected marginal utility function with respect to the uncertainty of health expenses (measured by  $\Delta_M$ ) is  $P[U''(C_2^*(h_g)) - U''(C_2^*(h_b))]$ . Thus, as long as  $C_2^*(h_g) > C_2^*(h_b)$ , an increase in the uncertainty of health expenses increases precautionary saving. However, in this model, the first order derivative of the expected marginal utility function with respect to  $\Delta_M$  is  $P[U''(C_2^*(h_g))\frac{1}{T+\Delta_T} - U''(C_2^*(h_b))\frac{1}{T-\frac{1-P}{P}\Delta_T}]$ . As  $\frac{1}{T+\Delta_T} < \frac{1}{T-\frac{1-P}{P}\Delta_T}$ , even when  $C_2^*(h_g) > C_2^*(h_b)$ , the first order derivative of the expected marginal utility function may be negative. Specifically, when  $[\frac{T-\frac{1-P}{P}\Delta_T}{(T+\Delta_T)}] < \frac{U''(C_2^*(h_b))}{U''(C_2^*(h_g))}$ , the health shock decreases the expected marginal utility in the second period, although it improves consumption smoothing across states. As a results, uncertain health expenses reduce precautionary saving.

It is worth noting that everything else being equal, condition (28) is more likely to hold for richer households. In other words, the effect of uncertain health expenses on precautionary saving is more likely to be negative for them. This result can be easily obtained by looking at the marginal effect of  $W$  on the left-hand side of condition (28). Substituting  $C_2^*(h_g) = \frac{W^*-M+\frac{P}{1-P}\Delta_M}{T+\Delta_T}$  and  $C_2^*(h_b) = \frac{W^*-M-\Delta_M}{T-\frac{1-P}{P}\Delta_T}$  into the left-hand side of condition (28) and taking its first order derivative w.r.t.  $W$ , I find that the left-hand side of condition (28) decreases in

$W$ . This implies that condition (28) is more likely to hold for agents with a higher value of  $W$ . This implication is of particular interest because a major portion of aggregate saving is from the rich. As a result, the negative saving effect of uncertain health expenses may be not only a qualitative result, but also quantitatively relevant for understanding the effect of health expenses on aggregate capital accumulation.

A recent quantitative literature has found that uncertain health expenses have a large positive effect on saving and are quantitatively important for understanding capital accumulation in the US.<sup>16</sup> However, no studies in this literature consider health shocks that simultaneously affect health expenses and longevity. According to the findings in this section, these studies may have overestimated the effect of uncertain health expenses on saving (especially for the rich). As a major portion of aggregate saving is from the rich, the effect of uncertain health expenses on aggregate capital accumulation may also be significantly smaller than what has been found in these studies.

## 6 A Numerical Application To Long Term Care Insurance

To assess the empirical relevance of the mechanisms emphasized in this paper, in this section I apply the model to the US long term care (LTC) insurance market, and use numerical techniques to study whether these mechanisms are quantitatively important for understanding the stylized facts about the US private LTC insurance market.

Long term care is one of the largest uninsured financial risks facing elderly Americans. Long term care expenditures were over \$200 billion (approximately 1.4% of GDP) in 2008, and are projected to increase significantly further in the near future due to population aging. However, only about 14% of elderly Americans (aged 60+) have any private long term care insurance (Brown and Finkelstein (2011)). Why do not Americans buy more private LTC insurance? This question has attracted considerable attention in the literature.<sup>17</sup> Two main findings have been obtained so far. First, the supply-side problems (e.g. markups due to asymmetric information) are not enough to explain why Americans do not buy private LTC insurance. Second, Medicaid can explain why poor Americans do not buy private LTC insurance. However, to the best of my knowledge, no study has explained why the rich also do not buy private LTC insurance. Though the data show that richer Americans buy more private LTC insurance, LTC insurance ownership rates are still low for the rich, i.e. only a quarter of Americans in the top wealth quintile hold private LTC insurance. In this section, I attempt to fill this gap by assessing whether the mechanisms emphasized in this paper can explain why richer Americans do not buy

---

<sup>16</sup>E.g. De Nardi, French, and Jones (2010), Kopecky and Koreshkova (2009).

<sup>17</sup>Brown and Finkelstein (2007, 2008), etc.

more private LTC insurance.

Consider the problem (after retirement) studied in section 3. I assume that the health shock  $h$  in the model represents the risk of entering long term care, and agents can buy private LTC insurance to insure the possible LTC expenses,  $M$ . The coinsurance rate of the LTC insurance policy is  $I$ , and the insurance premium is  $q_I = \sum_{i=1}^n P_i I M(h_i)$ , where  $n$  is the number of possible states. The discounting factor and the gross interest rate are both assumed to equal to one. Then, agents are facing the following utility-maximizing problem,

$$\max_{C(h), I} E[T(h)U(C(h))] \quad (29)$$

subject to

$$W - M(h) - q_I + IM(h) = T(h)C(h), \forall h \quad (30)$$

Deriving the first order condition (FOC),

$$E \left[ U'(C(h)) \left( M(h) - \sum_{i=1}^n P_i M(h_i) \right) \right] = 0. \quad (31)$$

The above equation determines the optimal  $I^*$ , which can be solved by numerical techniques after parameterization.

## 6.1 Parameterization

I assume that the LTC shock  $h$  has four possible values: (1) no LTC ( $h = h_1$ ), (2) one year in LTC ( $h = h_2$ ), (3) three years in LTC ( $h = h_3$ ), (4) five years in LTC ( $h = h_4$ ). According to the estimates in Brown and Finkelstein (2008), the probability distribution of the shock  $h$  is set as in the second column of Table 1.<sup>18</sup>

Table 1: Properties of The LTC Shock  $h$

| LTC shock $h$ | probability $Pr(h)$                 | health expenses $M(h)$ | longevity $T(h)$      |
|---------------|-------------------------------------|------------------------|-----------------------|
| $h = h_1$     | $Pr(h_1) = 0.6$                     | $M(h_1) = 0$           | $T(h_1) = T$          |
| $h = h_2$     | $Pr(h_2) = 0.4 \times 0.63 = 0.252$ | $M(h_2) = L$           | $T(h_2) = \delta T$   |
| $h = h_3$     | $Pr(h_3) = 0.4 \times 0.2 = 0.08$   | $M(h_3) = 3L$          | $T(h_3) = \delta^3 T$ |
| $h = h_4$     | $Pr(h_4) = 0.4 \times 0.17 = 0.068$ | $M(h_4) = 5L$          | $T(h_4) = \delta^5 T$ |

<sup>18</sup>Brown and Finkelstein (2008) report the probability of ever using LTC, and among users, the probabilities of using LTC more than 1 year, 3 years, and 5 years. Here I set  $Pr(h_1)$  to be one minus the probability of ever using LTC,  $Pr(h_2)$  to be the probability of ever using LTC, but less than 3 years,  $Pr(h_3)$  to be the probability of using LTC more than 3 years, but less than 5 years, and  $Pr(h_4)$  to be the probability of using LTC more than 5 years.

Assuming that  $L$  is one year of LTC expenses, and that the agent's longevity is discounted by  $1 - \delta$  for every year in LTC, the health expenses  $M(h)$  and longevities  $T(h)$  can be expressed as in the third and fourth columns of Table 1.

According to Kopecky and Koreshkova (2009), one year of nursing home care expenses in a semi-private room are about \$64240 in 2005, and 37% of these expenses are paid out-of-pocket. Thus, I set  $L = \$64240 \times 37\% = \$23769$ .<sup>19</sup> Kopecky and Koreshkova (2009) also estimate that conditional survival probability to the next year drops by 8.1% if entering into nursing home care. Thus, I set  $\delta = 1 - 0.081 = 0.919$ . The parameter,  $T$ , represents the life expectancy at age 65 for people who will never enter LTC in the rest of life, which is calibrated to match the average life expectancy at age 65 in 2005, that is, 18.2 years (CDC/NCHS).

Utility function  $U(C)$  takes the CRRA form, i.e.  $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$ , where  $\sigma$  is set to 2 based on the existing literature. The following table summarizes the parameterization results.

Table 2: Parameterization

| Parameter | Description               | Value   |
|-----------|---------------------------|---------|
| $L$       | one year of LTC expenses  | \$23769 |
| $\delta$  | longevity shock parameter | 0.919   |
| $T$       | maximum longevity         | 18.2    |
| $\sigma$  | CRRA utility parameter    | 2       |

With the parameterization described above, the first order condition for  $I^*$  (equation (31)) can be rewritten as follows,

$$Pr(h_1)U'\left(\frac{W - q_I}{T}\right)(-p) + Pr(h_2)U'\left(\frac{W - (1 - I)L - q_I}{\delta T}\right)(L - p) + Pr(h_3)U'\left(\frac{W - (1 - I)3L - q_I}{\delta^3 T}\right)(3L - p) + Pr(h_4)U'\left(\frac{W - (1 - I)5L - q_I}{\delta^5 T}\right)(5L - p) = 0$$

Using numerical techniques, I can solve the above equation for the optimal LTC coinsurance rate  $I^*$  at any given level of wealth,  $W$ . Table 3 presents the optimal LTC coinsurance rate  $I^*$  at some selected levels of wealth.

As can be seen from the second column of Table 3, most agents do not choose to fully insure against uncertain LTC expenses, even when LTC insurance is actuarially-fair. Furthermore, as the level of wealth increases, agents choose to have less LTC insurance. Agents with \$50,000 wealth in age 65 only need a LTC insurance policy with a coinsurance rate of 91%. Agents with a level of wealth above \$340,000 do not need any LTC insurance. The intuition behind these

<sup>19</sup>Note that nursing home care is the most important type of long term care, but long term care also includes assisted living facilities care and home health care which are usually less expensive than nursing home care. Thus, the value of  $L$  used here may be on the higher side.

Table 3: Results: the Demand for LTC Insurance ( $I^*$ )

| wealth (\$) | LTC insurance<br>(actuarially-fair) | wealth (\$) | LTC insurance<br>(with 0.18 load) |
|-------------|-------------------------------------|-------------|-----------------------------------|
| 50,000      | 91%                                 | 50,000      | 88%                               |
| 100,000     | 74%                                 | 100,000     | 66%                               |
| 200,000     | 43%                                 | 200,000     | 21%                               |
| 340,000     | 0                                   | 250,000     | 0                                 |

results is simple. Entering LTC reduces longevity, and thus frees up the resources (originally saved for the consumption in these extra years). Agents can use these resources to cover part of the LTC expenses. As the level of wealth increases, the amount of resources freed up from a given reduction in longevity increases. Therefore, richer agents need less LTC insurance.

It is worth noting that the US private LTC insurance market features serious supply-side frictions. According to Brown and Finkelstein (2007), the available LTC insurance policies on average have a load of 0.18. To investigate how these supply-side frictions affect the demand for LTC insurance, I add a markup to the prices of LTC insurance policy and then recompute the optimal condition for  $I^*$ . The results are reported in the fourth column of Table 3. As can be seen, all the qualitative results remain, but the optimal demand for LTC insurance decreases for any level of wealth. For instance, agents with a level of wealth above \$250,000 do not need to buy any LTC insurance.

So far I have assumed that agents can choose the coinsurance rate of the LTC insurance policy,  $I$ . However, the choice facing most agents in reality is usually just whether or not to buy an insurance policy with a specific coinsurance rate. Denoting the specific coinsurance rate by  $\hat{I}$ , agents are facing the following discrete-choice utility-maximizing problem,

$$\max_{C(h), I \in \{0, \hat{I}\}} E[T(h)U(C(h))] \quad (32)$$

subject to

$$W - M(h) - q_I + IM(h) = T(h)C(h), \forall h \quad (33)$$

As estimated by Brown and Finkelstein (2007), a typical LTC insurance policy in 2002 has a daily benefit of \$100 while a semi-private nursing home room costs \$143 on average. Thus, I set  $\hat{I}$  to be 0.7 (that is  $\frac{100}{143}$ ). Table 4 reports the optimal choice for LTC insurance at some selected levels of wealth when buying LTC insurance is a discrete choice. As can be seen, agents with a level of wealth above \$181,000 would not buy a typical LTC insurance policy available in the US market. According to Hendricks (2007), the mean wealth at retirement in the US is

Table 4: Results: the Demand for LTC Insurance (Discrete Choice)

| wealth (\$) | LTC insurance<br>(actuarially-fair) | wealth (\$) | LTC insurance<br>(with 0.18 load) |
|-------------|-------------------------------------|-------------|-----------------------------------|
| < 235,000   | buy                                 | < 181,000   | buy                               |
| > 235,000   | not buy                             | > 181,000   | not buy                           |

approximately \$358,500 for couples, and \$147,900 for singles (in 1994 dollars). These numbers suggest that the cut-off wealth derived here, \$181,000, is close to the average retirement wealth in the US. In other words, the numerical result obtained here suggests that Americans with above-average retirement wealth would not like to buy a typical LTC insurance policy available in the private market.

It is worth noting that previous studies on LTC insurance have found that Medicaid (together with supply-side frictions) can explain why Americans in the bottom two thirds of the wealth distribution do not buy private LTC insurance (see Brown and Finkelstein (2008)). However, none of them has not explained why the rich (e.g. the rest of Americans in the distribution) also do not buy private LTC insurance. Therefore, the numerical results obtained in this section complement the existing findings by providing an explanation for the LTC insurance puzzle for the rich.

## 7 Conclusion

In this paper, I study a simple life-cycle model with health shocks that simultaneously increase health expenses and longevity. Via closed-form solutions, I show that the model has several novel and interesting implications. First, utility-maximizing agents would neither fully insure their uncertain health expenses nor fully annuitize their wealth, even in the absence of market frictions and bequest motives. Second, the effect of uncertain health expenses on precautionary saving may be smaller than what has been found in previous studies. Under certain conditions, uncertain health expenses may even reduce precautionary saving. Third, mandatory health insurance (e.g. public health insurance) tends to benefit the poor more, while mandatory annuitization (e.g. public pension) is more likely to favor the rich.

The numerical application of the model conducted in the last section suggests that the mechanisms studied here are quantitatively important, and may have the potential to account for some puzzling facts observed in the data, e.g. the LTC insurance puzzle. However, to fully understand the quantitative implications of these mechanisms, the model needs to be extended to incorporate more quantitatively-relevant elements, such as multiple-period overlapping gen-

erations, idiosyncratic income shocks, and general equilibrium elements. I leave this for future research.

## References

- BROWN, J., AND A. FINKELSTEIN (2007): “Why Is the Market for Long Term Care Insurance so Small,” *Journal of Public Economics*, 91(10), 1967–1991.
- (2008): “The Interaction of Public and Private Insurance: Medicaid and the Long-Term Insurance Market,” *American Economic Review*, 98(3), 1083–1102.
- (2011): “Insuring long-term care in the U.S.,” *Journal of Economic Perspectives*, 25(4).
- DAVIES, J. B. (1981): “Uncertain Lifetime, Consumption, and Dissaving in Retirement,” *Journal of Political Economy*, 89, 561–577.
- DE NARDI, M., E. FRENCH, AND J. JONES (2010): “Why Do the Elderly Save? The Role of Medical Expenses,” *Journal of Political Economy*, 118(1), 39–75.
- HENDRICKS, L. (2007): “Retirement Wealth and Lifetime Earnings,” *International Economic Review*, 48(2).
- HUBBARD, R. G., J. SKINNER, AND S. P. ZELDES (1995): “Precautionary Saving and Social Insurance,” *Journal of Political Economy*, 103(2).
- HURD, M., D. MCFADDEN, AND A. MERRILL (2001): “Predictors of Mortality among the Elderly,” NBER Chapters, in: *Themes in the Economics of Aging*, 171–198.
- JESKE, K., AND S. KITAO (2009): “U.S. tax policy and health insurance demand: Can a regressive policy improve welfare?,” *Journal of Monetary Economics*, 56(2).
- KIMBALL, M. (1990): “Precautionary Saving in the Small and in the Large,” *Econometrica*, pp. 53–73.
- KOPECKY, K. A., AND T. KORESHKOVA (2009): “The Impact of Medical and Nursing Home Expenses and Social Insurance Policies on Savings and Inequality,” unpublished manuscript, University of Western Ontario.
- KOTLIKOFF, L. J., AND A. SPIVAK (1981): “The Family as an Incomplete Annuities Market,” *Journal of Political Economy*, 89(2).

- LELAND, H. E. (1968): "Saving and Uncertainty: The Precautionary Demand for Saving," *Quarterly Journal of Economics*, 82, 465–473.
- LEUNG, S. F. (1994): "Uncertain Lifetime, the Theory of the Consumer, and the Life Cycle Hypothesis," *Econometrica*, 62(5).
- LOCKWOOD, L. M. (2011): "Bequest Motives and the Annuity Puzzle," *Review of Economic Dynamics*, forthcoming.
- PASHCHENKO, S. (2010): "Accounting for Non-Annuitization," Federal Reserve Bank of Chicago WP 2010-03.
- SINCLAIR, S. H., AND K. A. SMETTERS (2004): "Health Shocks and the Demand for Annuities," 2004-9, Congressional Budget Office.
- YAARI, M. E. (1965): "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *Review of Economic Studies*, 32(2), 137–150.
- YOGO, M. (2009): "Portfolio Choice in Retirement: Health Risk and the Demand for Annuities, Housing, and Risky Assets," NBER working paper 15307.