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## High and Low Activity Spells in Housing Markets

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Eric Smith

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# High and Low Activity Spells in Housing Markets

## Abstract

This paper demonstrates the way in which stock-flow matching with endogenous seller entry generates hot and cold spells in house sales. Potential sellers know the number of bidders remaining from the last house sale. If two or more bidders remain, the seller obtains the gains to trade through competitive bidding. The market is active. With one monopolistic bidder, the buyer captures the surplus and sellers become unwilling to enter. The market remains dormant until sellers think enough time has passed for buyer entry to have replenished the market and make entry profitable. The resulting pattern of trade matches up with observations from Wisconsin.

JEL-Codes: R310, D530.

Keywords: trading volume, stock-flow matching, house price dynamics.

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# 1 Introduction

Housing, labor and other markets with trading frictions often appear to experience prolonged spells of high and low turnover. Although details vary across markets and over time, the general finding is that during high volume periods, prices are high and time to trade short. When turnover becomes slack, prices are low if exchange occurs at all and traders spend a seemingly long time on the market.<sup>1</sup> As a result, prices and sales become variable, contemporaneously correlated and persistent.

This paper demonstrates the way in which such distinct hot and cold trading episodes can arise given a stock-flow matching process and evaluates the impact these fluctuating spells have on the residential property market. Stock-flow matching (see Taylor, 1995; Coles and Smith, 1998; Coles and Muthoo, 1998; Coles, 1999; Lagos, 2000; Gregg and Petrongolo, 2005) assumes that buyers and sellers do not search randomly. Instead, market participants have a good idea about where to look for suitable partners. They check public and private intermediaries such as real estate agencies and websites.

Although traders seek information about possible trading opportunities using large scale intermediaries, they look for very specific characteristics in their partners. Stock-flow buyers and sellers are heterogeneous and trade in distinct submarkets, differentiated by location and other characteristics in which there are no trading frictions.<sup>2</sup> In the housing market context, buyers look exclusively for a combination of rooms, acreage, local amenities and so on in a limited area. These submarkets are self-contained such that agents can only trade within their assigned submarket.

As buyers and sellers come and go in each submarket, the population fluctuates stochastically so that traders can be on either the long or the short side of their submarket. If lucky, an entrant is on the short side and finds one or more options immediately available in their submarket. If the entrant is unlucky and on the long side, there are no potential partners immediately at hand. In the event that no partners currently exist in the submarket, the entrant becomes a part of the stock or queue of traders on their side of the

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<sup>1</sup>For housing market evidence, see Stein (1995), Muelbauer and Murphy (1997), Ortalo-Magne and Rady (2006), Krainer (2008), Diaz and Jerez (2013), Ngai and Tenreyro (2014) and Ngai and Sheedy (2015). Case and Shiller (1989) document persistence in prices.

<sup>2</sup>Stock-flow also allows traders to submit multiple job applications as in Albrecht et al, (2003) who consider a directed search model with multiple applications by job seekers.

market and must wait to match from the flow of new entrants on the other side.<sup>3</sup>

The innovation introduced here is to allow endogenous seller entry.<sup>4</sup> To maintain a balanced market over time, the stock-flow literature typically assumes that buyers and sellers independently enter the market at the same exogenous Poisson rate. In this paper, sellers have a higher arrival rate than buyers but they have the option to decline the opportunity to enter the market and save the associated sunk cost of participation. To illustrate, consider a housing submarket in which buyers bid for available homes in a public, complete information auction. Suppose a potential house seller contemplates moving home or a developer weighs up the opportunity to build a new home. In either case, the seller is considered relaxed. The seller is not compelled or highly motivated to put a house on the market. Home owners have the option to wait and consider moving later. Developers will come across other opportunities in the future. If the seller knows that two or more bidders are willing to make offers, it will pay the cost of moving or of building the house knowing that Bertrand bidding by the buyers will result in the seller obtaining most of the gains to trade. On the other hand, with one or zero bidders present, the seller will face a monopolistic buyer (either immediately or in the future) who captures the majority of the gains to trade. Entry does not occur in this case.

Sellers know the number of bidders, to some extent, from the previous auction. If there were  $N$  bidders in the last auction, there must be at least  $N - 1$  for the next. As such, sellers will enter until only one known bidder remains. The market then goes quiet and house sellers forgo the opportunity to enter the market. As time passes, buyer entry will gradually replenish the market. Assuming buyer entry is not revealed until the next auction is held, the market remains dormant or fallow until sellers think enough time has passed to make it profitable to enter the market. When the market reopens, if new buyer entry has not occurred, the lone bidder left at the last auction pays a low price and the market becomes dormant again, even more so as replenishing now requires not one but two buyers. If buyer entry occurred,

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<sup>3</sup>Coles and Smith (1998) obtain compelling evidence in favor of this matching behavior. See also Gregg and Petrongolo (2005), Andrews, Bradley and Upward (2001), Shimer (2007), Coles and Petrongolo (2008) and Kuo and Smith (2009).

<sup>4</sup>Caplin and Leahy (2011) look at a related market and pricing structure but without entry. Their baseline model does not match up as well with the empirical regularities as the model analyzed in this paper.

the bidders offer high prices and entry remains active until the queue of buyers dwindles down again.

This pattern of trade is inefficient. When entry gets turned off with one bidder known to be waiting, gains to trade are passed over. A monopoly bidder exists but prospective sellers do not respond. In the housing market, a seller first pays a sunk entry cost which may be thought of as the cost of moving or building the home. If entry were to occur, the monopoly bidder would not compensate the seller for this sunk cost. Due to this hold-up problem, the outcome during this fallow period is therefore inefficient. Several factors - the rate of buyer entry, housing costs - naturally determine the duration of these active and inactive spells and hence the efficiency of the market.

Allowing for some exogenous entry enriches this framework. For a variety of reasons, some sellers in the housing market might be highly motivated or compelled to enter the market. Whether pushed by necessity or pulled by very low costs to put their house up for sale, these entrants help further shape the pattern of entry, the duration of these fallow and fertile spells, the volume of trade and prices paid. Moreover, as such motivated sellers list their homes for sale in submarkets without buyers, inventories build up.<sup>5</sup>

Simulations reveal that a rising proportion of motivated sellers increases the duration relaxed sellers will wait before they enter and decreases the payoff to buyers faced with the choice to buy or wait through a cold spell without trade. The model with both motivated and relaxed sellers also performs well when compared with the housing market of Dane County, Wisconsin home of the city of Madison. Variation, correlation and persistence in price, sales volume, new listings, inventories and time on the market from the model do well mimicking observed data. Moreover, data for Madison suggest hot and cold spells are prevalent and such spells are commonplace in the simulations. The cold spells in the model, however, under perform in generating the observed dispersion of sales due to their apparent short duration, suggesting omitted factors in the stylized trading specification are needed to account for sales dispersion.

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<sup>5</sup>Albrecht et. al. (2007) adopt the relaxed and motivated terminology for these types of traders in a model of the housing market with random matching and price bargaining.

## 2 Model

Individual buyers enter a small, specialized, island-like submarket for an indivisible good - a house or home - in continuous time at the constant Poisson rate  $\beta > 0$ . There are two potential types of sellers. Relaxed or discretionary sellers have the option to evaluate their prospects and decide whether to enter the submarket. For example, a relaxed household might like to trade up or to downsize their house but their immediate situation allows them the patience to wait for the right moment to capitalize on their sale. On the other hand, motivated sellers do not have any discretion. Their particular circumstances compel them to automatically enter.

Relaxed sellers receive the opportunity to participate in the submarket at the rate  $\sigma > 0$ . If a relaxed seller enters the submarket, the cost of entry is  $F$ . Motivated sellers enter at rate  $0 \leq \alpha < \beta$  such that  $\alpha + \sigma > \beta$ .<sup>6</sup> Since the motivated sellers are compelled to enter the submarket, it is immaterial whether they pay a sunk entry cost.

All agents are risk neutral - they maximize expected receipts - and discount at rate  $r$ . Idle agents receive and make no payments. Any seller, relaxed or motivated, who enters a submarket holds an auction for their good. An accepted bid at price  $p$  yields a payoff  $x - p$  to the buyer and revenue  $p$  to the seller. After a trade takes place the consummating buyer and seller both leave the market. Unsatisfied buyers and sellers remain behind to wait for the next trading opportunity.

Buyers and motivated sellers always participate whereas discretionary sellers choose to take advantage of an opportunity or not. Before entry occurs, a relaxed seller who receives an opportunity to trade knows the outcome of the previous auction, including the number of bidders and the date of the auction. This seller, however, does not know the outcome of buyer entry over the period since the last auction. Thus, at any given point in time, relaxed sellers who obtain an opportunity to enter a submarket decide whether to accept or decline this opportunity based on the number of bidders remaining from the last auction in that submarket as well as the duration since that event. On the other hand, a new relaxed seller contemplating entry observes whether there are existing homes for sale, that is any unsatisfied prior sellers

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<sup>6</sup>Without a relaxed seller entry decision, the model is balanced if and only if  $\alpha = \beta$  and  $\sigma = 0$  which is the assumed condition in the stock-flow literature. The alternative extreme case in which all buyers are relaxed ( $\alpha = 0$ ) is straightforward to consider and also yields hot cold spells.

who entered and did not sell, in a submarket.

Once entry occurs, there are no impediments to trade. Agents are perfectly informed about submarket conditions.

ASSERTION: Immediate trade occurs so that the submarket never simultaneously has unsatisfied buyers and unsatisfied sellers.

See Coles and Muthoo (1999) for a discussion of this assertion.

## 2.1 Hot and Cold Submarkets

In cold or dormant submarkets, relaxed home sellers with an entry option decline the opportunity to participate thereby saving the fee  $F$ . These sellers are waiting for buyer entry to replenish the pool of buyers and revitalize submarket entry.

Since buyer entry follows a known underlying Poisson process that is unobserved by sellers, the transition to an active or hot submarket can come in one of two ways. A motivated seller can enter automatically and trigger an auction. The outcome of this auction reveals the number of buyers who have entered during the cold phase of the submarket and hence resets the entry decision of potential relaxed sellers. If a sufficient number of bidders appear in the auction, the submarket entry of relaxed sellers becomes active. If not, the waiting decision resets itself to the beginning of the cold phase conditional on the number of bidders. If seller entry occurred but no sale followed, the inventory of available homes builds up. In this case, the submarket remains cold - no relaxed seller entry - until the stock of available homes is sold and then followed by an appropriate cold period to replenish buyers.

The cold phase may also end after a period of complete submarket inactivity, that is after a sufficiently long period without any motivated seller entry. After some length of time, seller expectations of (unobserved) buyer entry eventually improve enough to induce entry. These expectations and hence the potential duration of inactivity depend on the outcome of the previous auction, in particular the remaining number of bidders.

Now consider hot or active submarkets in which relaxed sellers accept the option to enter the submarket. These hot submarkets will eventually turn fallow once seller entry (which is faster than buyer entry) runs down the stock of buyers and becomes no longer profitable. Expected profits from an auction depend on the expected number of bidders found in the submarket. Since buyers exit only after consummating a trade, the expected number



of buyers depends on both the known number remaining from the previous auction and subsequent entry since that auction.

Assuming that the submarket becomes less profitable as the number of bidders decline, a hot submarket becomes dormant immediately following an auction with some threshold number of buyers. Directly after an auction, the number of potential bidders is known with certainty. The submarket will resume activity after a sufficient time elapses for expected turnover to revitalize the submarket or for the entry of an motivated seller to reveal sufficient buyer demand.

The previous auction may have any number of excess bidders from zero, one, or more. Monopolistic bidding submarkets - those with one or zero bidders left over from the last auction - allow buyers to capture most of trade surplus. Since market power in these cases rests with buyers, these submarkets are assumed to be cold. On the other hand, suppose that if there are two or more bidders left over. These Bertrand competitors are on the short side of their submarket once entry occurs and thus market power resides with the seller. In this case, immediate entry (if available) is assumed to be profitable and the submarket is hot. Conditions on entry fee levels will be derived below that deliver these assumptions.

## 2.2 Submarkets with Buyer Queues

Assuming that a sufficiently strong expectation of monopolistic bidding deters entry, a submarket becomes cold and dormant if the previous entrant found zero, one or two bidders available for the auction. A prospective seller knows that there exists one buyer right after an auction with two bidders and that there are no bidders immediately following an auction with one or zero bidders. Auctions without any bidders imply an excess supply of sellers which is discussed later. This section first discusses the submarket following an auction with single seller and at least one bidder.

### Buyer Payoffs

Let  $H(N)$  represent the payoff to a home buyer from being in a hot, active submarket where  $N \geq 1$  denotes the number of bidders (including the buyer) in the submarket waiting for the arrival of a seller. If  $N = 1$  and entry occurs, the single bidder has monopoly power. For  $N \geq 2$ , bidding is competitive. Similarly, let  $C(N, T)$  represented the expected payoff to a buyer in a cold submarket with  $N$  bidders who must wait a duration  $T$  before

entry of relaxed sellers becomes viable again, i.e. the remaining duration without any motivated seller entry before relaxed sellers with the option of entry become willing to pay  $F$  to visit the submarket.

Let  $P(N)$  represent the price resulting from an auction with  $N \geq 1$  bidders. In the Bertrand pricing game with more than one buyer, buyers bid prices up until the gains to trade from purchasing the currently available good equal the payoff of staying in the submarket and waiting for the next auction. For  $N \geq 2$ , the buyer is indifferent between paying  $P(N)$  and waiting for the next entrant, whether in a hot or cold submarket :

$$\begin{aligned} P(N) &= x - H(N - 1) & N \geq 3 \\ &= x - C(1, T_1) & N = 2 \end{aligned}$$

where  $T_1$  represents the duration relaxed sellers will wait before entering given there was one buyer in the submarket at the last auction when the submarket became cold.  $P(1)$ , the price paid when the submarket is balanced with one buyer and one seller, is discussed below. With zero flow costs from being on the market after the entry fee, the seller accepts the highest non-negative bid.

With probability  $\alpha e^{-\alpha t} dt$ , a motivated seller enters the submarket during the cold period after a duration  $t$  and triggers an auction with the existing bidders and any other buyers who might have entered during the cold period up to time  $t$ . In this environment, the buyer's expected payoff in a cold submarket can be written as

$$\begin{aligned} C(N, T) &= \int_0^T \alpha e^{-(r+\alpha)t} \sum_{i=0}^{\infty} \pi_i(t) [x - P(i + N)] dt \\ &\quad + e^{-(r+\alpha)T} \sum_{i=0}^{\infty} \pi_i(T) H(i + N) \end{aligned} \tag{1}$$

where  $\pi_i(t)$  denotes the probability that  $i$  buyers enter the submarket after a duration  $t$  in which case  $N + i$  bidders await an incoming seller. Since buyers enter at Poisson rate  $\beta$ , the probability of  $i$  entrants after duration  $t$  is given by

$$\pi_i(t) = \frac{e^{-\beta t} (\beta t)^i}{i!}$$

Now consider the bidder payoff in a hot submarket. Suppose the buyer

is alone in the submarket. With a monopoly position ( $N = 1$ )<sup>7</sup>, the buyer receives  $x - P(1)$  if seller entry occurs and  $H(2)$  if buyer entry occurs. Accounting for arrival rates, this expected payoff can be written as the linear difference equation:

$$H(1) = \frac{1}{1 + rdt} [(\alpha + \sigma)dt(x - P(1)) + \beta dt H(2) + (1 - (\alpha + \sigma + \beta)dt)H(1)].$$

With one other bidder ( $N = 2$ ), seller entry results in an auction that will leave one known, unsatisfied bidder remaining for the next auction. This outcome stops entry of relaxed sellers and leaves the unsatisfied bidder in a cold submarket with expected payoff  $C(1, T_1)$ . Since the successful bid with two bidders leaves them indifferent between buying and remaining, the payoff to a buyer in an active submarket with two bidders is given by

$$H(2) = \frac{1}{1 + rdt} [(\alpha + \sigma)dt C(1, T_1) + \beta dt H(3) + (1 - (\alpha + \sigma + \beta)dt)H(2)]$$

For  $N \geq 3$ , competitive bidding makes buyers indifferent between purchasing and waiting for the next auction with one less competitor. Hence,

$$H(N) = \frac{1}{1 + rdt} [(\alpha + \sigma)dt H(N - 1) + \beta dt H(N + 1) + (1 - (\alpha + \sigma + \beta)dt)H(N)]$$

For  $N > 1$ , the solution to these difference equations is given by

$$H(N) = C(1, T_1)\eta^{N-1}$$

where

$$\eta = \frac{r + \alpha + \sigma + \beta - [(r + \alpha + \sigma + \beta)^2 - 4(\alpha + \sigma)\beta]^{1/2}}{2\beta}$$

After substituting for  $H(2)$  in  $H(1)$ , the payoff to a lone monopolistic buyer in an active submarkets becomes

$$H(1) = \frac{(\alpha + \sigma)(x - P(1)) + \beta\eta C(1, T_1)}{r + \alpha + \sigma + \beta}$$

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<sup>7</sup>With one bidder remaining from the previous auction ( $N = 1$ ), a one bidder auction can arise after entry of a motivated seller (and no buyer entry) or after entry of a relaxed seller following a cold period in which no buyer entry occurred.

### Relaxed Seller Entry

Following a spell of duration  $t$  without seller entry, expected revenue less the entry fee for a relaxed seller contemplating entry into the submarket with  $N \geq 1$  bidders remaining from the last auction is given by

$$R(N, t) = \sum_{i=0}^{\infty} \pi_i(t) P(i + N) - F$$

Consider the monopolistic bidding cases. If one bidder remains from the previous auction, then the expected revenue from entry is the expected sales less the cost of entry. If zero new buyers have entered since the previous auction, the monopolistic buyer bids the monopolistic price  $P(1)$ . With two bidders, the seller receives  $x - C(1, T_1)$ . With three or more bidders, the price offered and paid is  $P(i) = x - H(i - 1)$ . Since  $\pi_i(t)$  gives the probability of observing  $i + 1$  bidders, expected profit is given by

$$R(1, t) = \pi_0(t) P(1) + \pi_1(t) [x - C(1, T_1)] + \sum_{i=2}^{\infty} \pi_i(t) [x - H(i)] - F$$

Substitution and manipulation yields

$$R(1, t) = e^{-\beta t} P(1) + (1 - e^{-\beta t}) x + e^{-\beta(1-\eta)t} [1 - e^{-\beta\eta t}] C(1, T_1) / \eta - F$$

Entry occurs if and only if  $R(1, t) \geq 0$  hence the critical duration of entry,  $T_1$ , for relaxed sellers aware of only one known bidder satisfies

$$x - e^{-\beta T_1} (x - P(1)) + e^{-\beta(1-\eta)T_1} [1 - e^{-\beta\eta T_1}] C(1, T_1) / \eta - F = 0 \quad (2)$$

A similar procedure reveals that revenue after a duration  $T_0$  following an auction with zero bidders left over is given by

$$\begin{aligned} & \beta e^{-\beta T_0} [T_0 + 1 / (r + \alpha + \beta - \alpha\lambda)] P(1) \\ & + (1 - e^{-\beta T_0} - \beta T_0 e^{-\beta T_0}) x \\ & + e^{-\beta(1-\eta)T_0} [1 - e^{-\beta\eta T_0} - \beta\eta T_0 e^{-\beta\eta T_0}] C(1, T_1) / \eta^2 - F = 0 \end{aligned} \quad (3)$$

Firm entry is assumed for  $N \geq 2$ . If entry occurs immediately after an auction with  $N = 3$  bidders, the entering sellers receives  $P(2) = x - C(1, T_1)$ . The entry assumption thus holds provided entry cost is sufficiently small, that is if and only if

$$x - C(1, T_1) \geq F.$$

### 2.3 Excess Sellers and Balanced Trade

Entry of motivated sellers occurs during both active and dormant submarkets. Even though motivated sellers enter at a slower rate than buyers, from time to time the realization of the entry processes will be such that more motivated sellers than buyers enter and cold submarkets will experience having excess sellers. An inventory of unsold homes builds up. In addition, the entry of relaxed sellers can cause an inventory of one seller. If the previous auction had a single bidder and no entry of buyers or sellers occurs, eventually the submarket becomes active and a relaxed seller will enter but not find a bidder.

Relaxed sellers will not enter submarkets with excess supply until all of the previous sellers who entered consummate trades. They observe all unsatisfied trade and hence do not enter if another seller already exists in the submarket. Motivated sellers, however, may enter to cause additional excess supply. Really cold submarkets, those with excess sellers, remain cold until balance is restored.

Like buyers in submarkets with excess bidders, sellers in submarkets with excess goods accept bids that make them indifferent between taking the bid and waiting for the next auction. Since  $\alpha$  alone governs the arrival rate of motivated sellers, the payoff to a lone seller in the submarket ( $M = 1$ ) awaiting for the arrival of buyer is given by

$$Z(1) = \frac{1}{1 + rdt} [\alpha dt Z(2) + \beta dt P(1) + (1 - \alpha dt - \beta dt)Z(1)].$$

With other sellers waiting the arrival of a buyer ( $M = 2$ ), sellers are willing to accept a bid that makes them indifferent between selling and waiting in the submarket, the payoff to  $Z(M)$  is given by

$$Z(M) = \frac{1}{1 + rdt} [\alpha dt Z(M + 1) + \beta dt Z(M - 1) + (1 - \alpha dt - \beta dt)Z(M)].$$

The solution to these difference equations is given by

$$Z(M) = Z(1)\lambda^{M-1}$$

for  $M > 1$ , and

$$Z(1) = \frac{\beta P(1)}{r + \alpha + \beta - \alpha\lambda}$$

where

$$\lambda = \frac{r + \alpha + \beta - [(r + \alpha + \beta)^2 - 4\alpha\beta]^{1/2}}{2\alpha}$$

From time to time, entry from one side or the other of the submarket will occur such that the auction has one bidder and one seller. In this auction, the buyer's offer again makes the seller indifferent between waiting and accepting. Given that buyer entry or seller entry will unbalance the submarket, the equilibrium bid can be written as

$$P(1) = \frac{1}{1 + rdt} [\alpha dt Z(1) + \beta dt [x - C(1, T_1)] + (1 - \alpha dt - \beta dt)P(1)]$$

Substitution for  $Z(1)$  gives

$$P(1) = \frac{\beta(r + \alpha + \beta - \alpha\lambda)[x - C(1, T_1)]}{(r + \alpha + \beta)^2 - \alpha\lambda(r + \alpha + \beta) - \alpha\beta} \quad (4)$$

### 3 Equilibrium

An equilibrium is a set of prices  $P(N)$  and accompanying entry decisions,  $T_0$  and  $T_1$ , that terminate cold submarkets such that buyers and sellers willingly trade whenever possible and relaxed seller entry is profitable. If a buyer and a seller are simultaneously present in a submarket, trade takes place immediately at a price that makes the long side of the submarket indifferent between trading and waiting.

Two sets of difference equations emerge,  $H(N)$  for buyers and  $Z(M)$  for sellers  $N, M = 1, 2, 3, \dots$ . Given the payoff to waiting at the onset of a cold submarket  $C(1, T_1)$ , these equations can be recursively solved. Prices  $P(N)$  follow accordingly. Thus, equation (1) reduces to an implicit equation in  $C(1, T_1)$  and  $T$ :

$$\begin{aligned} C(1, T_1) = & \left[ \frac{\alpha(1 - e^{-(r+\alpha+\beta)T_1})}{r + \alpha + \beta} + \frac{(\alpha + \sigma)e^{-(r+\alpha+\beta)T_1}}{r + \alpha + \sigma + \beta} \right] (x - P(1)) \quad (5) \\ & + \frac{\alpha}{\eta} \left[ \frac{1 - e^{-(r+\alpha+\beta(1-\eta))T_1}}{r + \alpha + \beta(1-\eta)} - \frac{1 - e^{-(r+\alpha+\beta)T_1}}{r + \alpha + \beta} \right] C(1, T_1) \\ & e^{-(r+\alpha+\beta)T_1} \left[ 1 - e^{-\beta\eta T_1} + \frac{\beta\eta}{r + \alpha + \sigma + \beta} \right] C(1, T_1) \end{aligned}$$

where (4) reveals that  $X - P(1)$  is a linear function of  $C(1, T_1)$ . The entry decisions of relaxed sellers, captured through  $T_1$  and  $T_0$  and encapsulated

in (2) and (3) depend on expected revenue from these prices and buyer entry. Since equation(2) contains only  $C(1, T_1)$  and  $T_1$  (along with exogenous parameters) an equilibrium follows from a positive solution of equations (2) and (5) in these two unknowns. Equation (3) then gives  $T_0$ . The payoffs and prices follow accordingly.

Equilibria with hot and cold cycles are inefficient due to a familiar hold up problem.<sup>8</sup> With one buyer known to exist in the submarket ( $N = 1$ ), a social planner would want entry. Otherwise, there are unexploited gains to trade. Declined entry by relaxed sellers during a  $T_1$  cold spell is therefore suboptimal and due to the inability of relaxed sellers to recoup the sunk costs of entry. When there are no known buyers in the submarket ( $N = 0$ ), entry may or may not be efficient. If no known buyer exists, relaxed seller entry has a positive return if and only if

$$\sum_{i=1}^{\infty} \pi_i(t)x > F$$

which simplifies to

$$t > \frac{\ln[1 - F/x]}{\beta}$$

Analytic solutions are in general unavailable so it is hard to compare this figure to  $T_0$ . Simulations reveal the unsurprising result that  $T_0$  is generally greater than the planner's solution.

Simulated results also help gauge the impact of the composition of seller types. Queues in each submarket wax and wane with the arrival of buyers and sellers, both voluntary and involuntary. The composition of relaxed and motivated sellers, which reflects the economic context, thus affects economic performance. For example, cities differ in the range and number of jobs available. If the willingness of a home owner to sell is linked with job opportunities within and across cities, housing markets will contain different numbers of motivated sellers.

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<sup>8</sup>It is possible to establish an equilibrium exists with  $T_0 > T_1 > 0$ . The cumbersome algebra involved with characterizing the two equations in two unknowns is unexceptional and omitted here.

**Table 1. Composition of Relaxed and Motivated Sellers**

$\alpha$	0.00	0.01	0.02	0.03	0.04	0.05
$\alpha/(\alpha + \sigma)$	0.0	0.1	0.2	0.3	0.4	0.5
<u>Fallow Intervals</u>						
$T_1$	9.96	9.99	10.03	10.05	10.08	10.10
$T_0$	30.95	31.28	31.62	31.97	32.32	32.66
$C(1, T_1)$	0.245	0.217	0.190	0.163	0.138	0.112
<u>Summary Statistics</u>						
Average Price	0.751	0.734	0.686	0.581	0.384	0.085
Average Sales	223	224	225	224	226	226
Average Inventory	26	29	40	87	323	2,018
Relative S.D. $\sigma_q/\sigma_p$	3.407	3.041	1.938	1.347	0.593	0.320
Correlation $\sigma_{p,q}$	0.493	0.351	0.319	0.258	-0.0857	0.1045
Persistence $\rho_{-1}$	0.269	0.112	0.386	0.331	0.547	0.598

Table 1 lists basic statistics for a simulated market - details are provided below - for varying values of  $\alpha$ , keeping the total entry rate of sellers,  $\alpha + \sigma = 2\beta$ , constant.<sup>9</sup> The lengths of cold spells, measured by  $T_1$  and  $T_0$ , increase with  $\alpha$ . A lower expected utility  $C(1, T)$  of a buyer waiting in the submarket at the start of a cold period accompanies these increased durations. As the proportion of exogenous entry increases, the contribution of  $T_1$  and  $T_0$  to the actual duration of cold spells and the pattern of trade changes. Realized periods without relaxed entry also depend on the outcome following automatic entry. When the entry of a motivated seller reveals a buyer queue, relaxed seller entry is switched on. A motivated seller that reveals a dearth of buyers resets and prolongs the absence of relaxed entry.

As motivated sellers become more prevalent, fewer buyers wait in queues. Indeed, as motivated sellers becomes highly likely, many of the homes for sale from motivated sellers appear to wind up on the other side of the market in

<sup>9</sup>Given the restriction that  $\alpha < \beta$ , it must be the case that  $\alpha/(\alpha + \sigma)$  is less than a half so the last column is calculated using a value just below  $\alpha = 0.05$ . This has no apparent impact on the results but this specification is not equivalent to the standard exogenous, balanced entry model as  $\sigma = 0.05$  and not zero. The other parameters for Table 1 simulations are the same as those used below with one exception. In order to keep have sellers willing to trade with two buyers, the interest rate for Table 1 is increased from  $r = 0.025$  to  $r = 0.035$ .



long queues. As a result, the average price of a home in Table 1 declines considerably whereas the average quantity sold for the most part does not vary substantially. Average sales are ultimately tied to the average number of buyers determined by  $\beta$ . Average inventories unsurprisingly rise as more sellers enter exogenously. These figures together suggest that the net effect of less relaxed entry and a higher proportion of motivated sellers will result in more advantageous trade for the buyers. Without further analysis on the distribution across submarkets, however, it is inconclusive whether these submarkets necessarily experience smoother trading and shorter dormant periods.

Table 1 also presents dispersion, correlation and persistence statistics. For very low values of  $\alpha$ , sales are roughly three time more variable than prices ( $\sigma_q/\sigma_p$ ) but become relatively less variable as the proportion of motivated sellers increases. The contemporaneous co-movement of price and sales ( $\sigma_{p,q}$ ) also generally declines as motivated sellers become more commonplace. As motivated sellers become the norm (high  $\alpha$  with long seller queues in some submarkets), the average price is very low and displays virtually no connection with quantity sold across all submarkets. On the other hand, price persistence (lagged one period and denoted by  $\rho_{-1}$ ) occurs for all mixtures of relaxed and motivated sellers and exhibits a general U-shape as this proportion increases.

## 4 Benchmark Evidence

To establish an empirical benchmark, this section presents commonly used housing market statistics from Dane County, Wisconsin (pop. 426,526 in 2000). Approximately half the population of Dane County lives in the city of Madison. The South Central Wisconsin Multiple Listing Service (SCWMLS) provides monthly data on average prices, sales, new listings and end of month inventory of unsold homes from January 1997 onwards. The SCWMLS also lists average time on market for unsold inventory until December 2002. To bypass the upheaval of the 2008 recession, the sample considers these series until December 2007.

Table 1 presents the mean values of these variables (in levels) along with their minimum and maximum values. To facilitate comparison with a stationary model, Table 1 also presents the standard deviation of their logged,

detrended and seasonally adjusted values.<sup>10</sup> As seen in Table 1, the variation in sales over this period is approximately two and a half times higher than the variation in price and nearly identical to the variation in listings. Inventory exhibits the most variation over time. The standard deviation in time on market lies between the variation of price and sales but this figure is derived from far fewer observations.

**Table 2. Means and Standard Deviations**

	<i>Dane County SCWMLS Data</i>					<b>Model</b>	
	<i>Mean</i>	<i>Min</i>	<i>Max</i>	<i>Std Dev</i>	<i>N</i>	<b>Mean</b>	<b>Std Dev</b>
Price	<i>209,258</i>	<i>116,674</i>	<i>292,462</i>	<i>0.047</i>	<i>132</i>		<b>0.0285</b>
Sales	<i>423</i>	<i>157</i>	<i>792</i>	<i>0.114</i>	<i>132</i>	<b>225</b>	<b>0.0617</b>
Listings	<i>636</i>	<i>238</i>	<i>1241</i>	<i>0.119</i>	<i>132</i>	<b>225</b>	<b>0.0634</b>
Inventory	<i>1847</i>	<i>1228</i>	<i>3118</i>	<i>0.241</i>	<i>132</i>	<b>29</b>	<b>0.1793</b>
Time on market	<i>66</i>	<i>42</i>	<i>98</i>	<i>0.081</i>	<i>72</i>	<b>21</b>	<b>0.3477</b>

*Notes:* SCWMLS data in *italics*. Simulated data in **bold face**

The lower diagonal half of Table 2 reports the observed contemporaneous correlations among these (logged, detrended and seasonally adjusted) variables. Table 3 contains monthly autocorrelations. From the lower half of Table 2, price exhibits positive correlation with sales and with listings. This price-sales correlation is lower in Dane County than found elsewhere from more aggregated, longer but less frequent, and sometimes trending series.<sup>11</sup> The correlations of price with inventory and of price with time on market are both negative but weak. All figures involving time on market presented here should, however, be viewed cautiously given the relatively low number of observations from just six years. Several authors document and emphasize a robust negative relationship between price and time on market or with time to sell.<sup>12</sup> Krainer (2008), Díaz and Jerez (2013) and Ngai and Sheedy

<sup>10</sup>OLS regressions with a linear trend and monthly dummies are used to detrend and seasonally adjust all logged variables. The general picture of relative standard deviations is similar but less pronounced when the variables are not detrended and seasonally adjusted.

<sup>11</sup>See Stein (1995), Ortalo-Magne and Rady (2006), Diaz and Jerez (2013) and Ngai and Sheedy (2015). An exception is Head et al (2014) with a negative price-sales relationship.

<sup>12</sup>Average time on market, which measures the average duration a home in the unsold stock available has been on the market, differs from the average time to sell, which measures how long a house took to sell among the flow of houses leaving the market.

(2015) describe negative price-duration correlations linked with hot and cold markets.

Sales and inventory are inversely related whereas listings and inventory correlate positively and prominently. Given that sales and new listings are weakly correlated, the new listing-inventory relationship makes basic accounting sense. If sales (or new listings) surge, inventory will drop (rise) without a contemporaneous surge in new homes listed for sale (sold). On the other hand, the weak link between sales and new listings appears somewhat at odds with the stock-flow interpretation of trade. The frequency of the SCWMLS data may play a role in depressing observed sales-new listings comovements. If it takes a few weeks between agreeing to exchange and actually completing the deal rather than sales occurring instantaneously, the observed link between sales and the flow of new homes for sales will be depressed in monthly data.

**Table 3. Correlation Coefficients**

	Price	Sales	Listings	Inventory	Time on Market
Price	1.0	<b>0.5769</b>	<b>0.6405</b>	<b>-0.2981</b>	<b>-0.2767</b>
Sales	<i>0.2188</i>	1.0	<b>0.9272</b>	<b>-0.5800</b>	<b>0.2229</b>
Listings	<i>0.2665</i>	<i>.1026</i>	1.0	<b>-0.5139</b>	<b>0.0072</b>
Inventory	<i>-0.0921</i>	<i>-0.4264</i>	<i>0.5677</i>	1.0	<b>-0.3433</b>
Time on market	<i>-0.0851</i>	<i>0.3222</i>	<i>0.2739</i>	<i>0.0537</i>	1.0

*Notes:* SCWMLS data in *italics*. Simulated data in **bold face**

As documented in Table 3, persistence is positive for both prices and sales with a more pronounced and enduring pattern in sales than in prices. Caplin and Leahy (2011) emphasize the inverse relationship between the inventory of homes for sale and future prices.<sup>13</sup> The third column of Table 3 therefore reports the correlations for price and lagged inventory. Consistent with Caplin and Leahy, this relationship is negative in Dane County. It begins modestly after a month and increases with duration of the lagged inventory.

<sup>13</sup>On the other hand Diaz and Jerez (2013) very weak support for a link between prices and lagged inventory.

**Table 4. Autocorrelations**

	Price	Sales	Price-Inventory
L(-1)	<i>0.3318</i>	<i>0.5106</i>	<i>-0.1112</i>
L(-2)	<i>0.2748</i>	<i>0.4310</i>	<i>-0.1330</i>
L(-3)	<i>0.3078</i>	<i>0.5181</i>	<i>-0.1761</i>
L(-4)	<i>0.2661</i>	<i>0.3663</i>	<i>-0.2115</i>
L(-5)	<i>0.2420</i>	<i>0.2910</i>	<i>-0.2292</i>
L(-6)	<i>0.3258</i>	<i>0.2165</i>	<i>-0.2653</i>
L(-12)	<i>0.1301</i>	<i>0.2584</i>	<i>-0.3847</i>
<b>Model L(-1)</b>	<b>0.3765</b>	<b>0.2209</b>	<b>-0.3174</b>

*Notes:* SCWMLS data in *italics*. Simulated data in **bold face**

This picture of home sales in Dane county is generally consistent with the evidence emphasized in the literature. In Dane and elsewhere

- sales vary more than prices
- prices correlate with contemporaneous sales
- prices and sales exhibit persistence
- prices are negatively correlated with lagged inventory

The notable anomaly in Dane County is the absence of a strong inverse link between price and time on market which as noted may be due to small sample size.

Before comparing this picture with the performance of the model, it is worth looking further into the underlying data for Madison. Hendel et al (2009) and Ortalo-Magné (2011) use the raw data generating the Dane County series for the period 1998-2005 to assess the housing market in Madison. Ortalo-Magné (2011) looks in detail at a particular homogeneous sub-market in Madison - large houses in a single elementary school district - which conforms closely to the specification of a submarket in this paper. Figure 1 reproduces Ortalo-Magne's observations of price premia<sup>14</sup> and the number of available houses on the market during the period January 1999

<sup>14</sup>The price premium is the ratio of the transaction price to a hedonic regression adjusted price minus one.

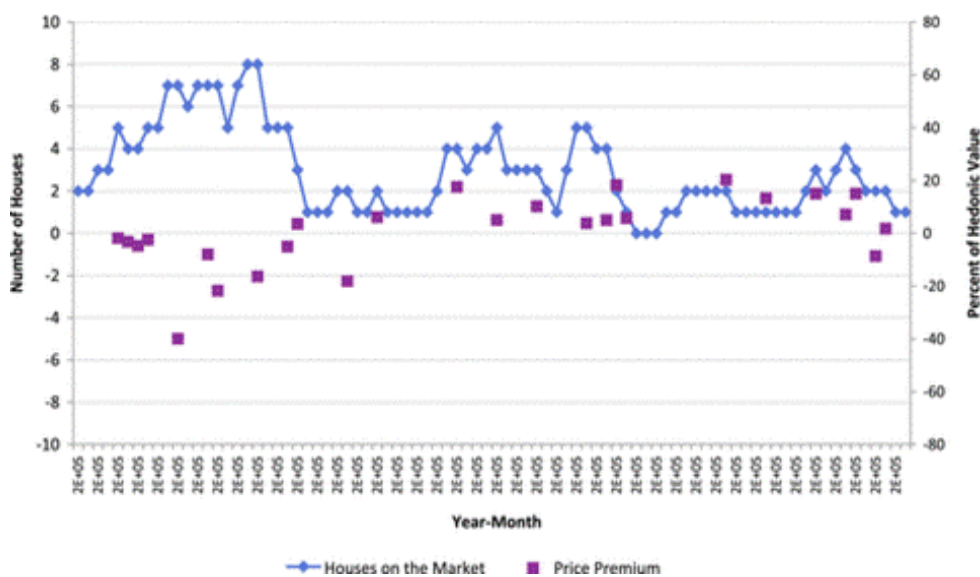


Figure 1:

until December 2004. Viewed through the stock-flow model of trade, this plot is intriguing. Two episodes stand out given the focus on hot and cold spells. Between December 2000 and December 2001, the Ortalo-Magné figure displays only two price premia both of which occur shortly after the addition of a new home for sale, one after a month and the other after two months. It looks as if a seller entered a cold market and was able to sell to an eager buyer. In the eight months between July 2001 and March 2002, no sales appear to take place. In the second episode from September 2003 until January 2005, again only two price premia appear. Within this period, no sales take place in the ten months between September 2003 and June 2004 during which time two house were added to the inventory for sale. It looks as if in this instance sellers entered during a dry patch in turnover but did not find a buyer willing to trade quickly.

Hendel et al (2009) look at individual prices and sales across all sub-markets in Madison. It is possible to inspect their data for January 1999-December 2005 for hot and cold spells. Most of the observed homes sold belong to five distinct quality classes. There are 29 identifiable elementary school districts. Adopting a quality class of home in an elementary school district approach as in Ortalo-Magné (2011), there are potential 145 submar-

kets. Some elementary districts do not have homes of all five quality classes. Eliminating the quality class  $\times$  elementary submarkets with no sales leaves 139 different sized submarkets as determined by average sales.

A memoryless Poisson process for sales over time would not exhibit hot or cold spells but would generate a distribution of sales in which the mean equals the variance. Table 4 therefore reports the average and maximum number of sales for each month along with the standard deviation across markets. Unsurprisingly, June and July are the busiest months and January and February the slowest.<sup>15</sup> Although the size of these markets varies across markets and with time, the high degree of variation in sales suggests hot and cold spells exists. A more explicit test derives from a negative binomial regression which nests the memoryless Poisson model. Given month, year, quality class and average market size controls, the specification test rejects the Poisson specification ( $N = 11,676, \chi^2 = 2482.42$ ). The Poisson specification is also rejected using a wide variety of more homogeneous subsamples. Restricting attention to submarkets within a quarter standard deviation of the mean sales and looking at sales for June and July, the test statistic is  $\chi^2 = 18.27$  with  $N = 476$ .

A simple graphic approach illustrates the same excessive variation. The top panel in Figure 2 graphs the distribution of sales in June and July for submarkets within a quarter of a standard deviation of the mean number of sales. A Poisson distribution with the same average sales is graphed in the middle panel. This figure reveals an overabundance of no sales and of a high number of sales. Stated simply, there is too much variation for a market which does not experience hot and cold spells.

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<sup>15</sup>Ngai and Tenreiro (2014) investigate this seasonality component.

**Table 5. Average Monthly Submarket Sales**

	Mean	Standard Deviation	Max
January	<i>0.7133</i>	<i>1.3190</i>	<i>13</i>
February	<i>0.8582</i>	<i>1.6557</i>	<i>17</i>
March	<i>1.1840</i>	<i>2.2144</i>	<i>31</i>
April	<i>1.4573</i>	<i>2.4032</i>	<i>25</i>
May	<i>1.8335</i>	<i>2.9213</i>	<i>25</i>
June	<i>2.1891</i>	<i>3.4161</i>	<i>32</i>
July	<i>2.1244</i>	<i>3.1691</i>	<i>28</i>
August	<i>1.7955</i>	<i>2.7615</i>	<i>21</i>
September	<i>1.2508</i>	<i>1.9774</i>	<i>16</i>
October	<i>1.1716</i>	<i>1.9606</i>	<i>19</i>
November	<i>0.9764</i>	<i>1.6312</i>	<i>14</i>
December	<i>1.0349</i>	<i>1.7510</i>	<i>13</i>
Average	<i>1.3820</i>	<i>2.4010</i>	<i>32</i>

## 5 Simulated Outcomes

To numerically simulate the model suppose there are  $N = 150$  identical but isolated submarkets in which buyers and sellers arrive according to independent continuous time Poisson processes. Let the daily arrival rate of buyers in each submarket be

$$\beta = 0.05$$

which implies a buyer arrives on average once every 20 days. Over a month, this rate (1.5) is marginally higher than the average (1.4) for the quality class  $\times$  elementary school submarkets in Madison. When a new buyer arrives, either trade with an existing seller in the submarket occurs immediately after entry or the queue of potential buyers in the submarket increases by one.

Assume potential sellers arrive at twice the buyer rate,  $\alpha + \sigma = 2\beta$ . Let one in every eight potential sellers be ( $\alpha$ ) motivated or compelled to sell so that

$$\begin{aligned}\alpha &= \beta/4 = 0.0125; \\ \sigma &= 2\beta - \alpha = 0.0875.\end{aligned}$$

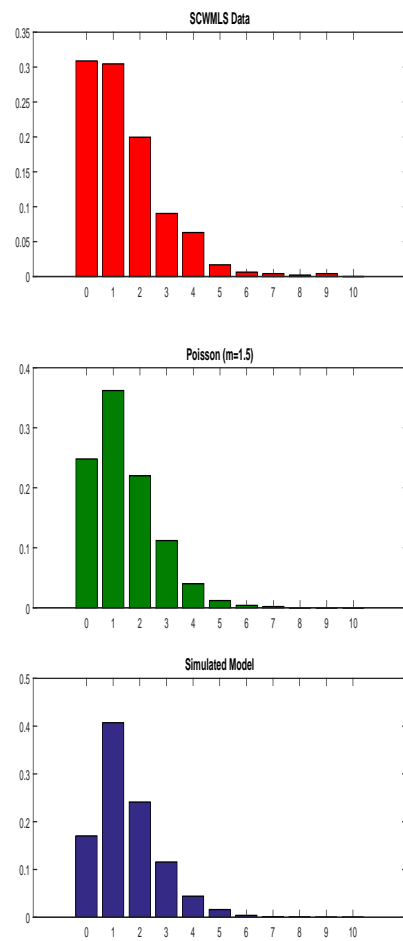


Figure 2: Sales Distributions



When new, unmotivated ( $\sigma$ ) sellers arrive, they first decide whether or not to enter the submarket based on the number of traders at the last transaction. If entry occurs, again either trade with an existing buyer occurs immediately or the queue of potential sellers increases by one.

The buyer's utility to owning a home is normalized to one and the sunk cost to a relaxed seller is three fourths this value :

$$\begin{aligned}x &= 1 \\ F &= .75.\end{aligned}$$

The discount rate

$$r = 0.025$$

is set high in part to account for the many costs of trading homes abstracted away in the model. The high rate of time preference also plays a critical role in yielding a buyer payoff at the start of a cold market that satisfies the assumption in the model that sellers are willing to trade with just two bidders  $x - F > C(1, T)$ .

The six parameters  $(\alpha, \beta, \sigma, x, F, r)$  describe the submarket. Numerically solving for the endogenous variables gives

$$\begin{aligned}T_1 &= 9.11 \\ T_0 &= 29.38 \\ C(1, T) &= 0.2479\end{aligned}$$

The duration of a cold spell with one known buyer in a submarket is a little less than a week and a half. The duration of a cold spell when there are no known buyers is about one month. Although neither spell lasts very long, they are long enough to replenish the submarket. Given  $\beta = 0.05$ , the probability that at least one new buyer arrives after a  $T_1$  spell is more than one third (0.3659). After a duration of  $T_0$ , the probability that there will be two or more buyers equals 0.4317 whereas the probability of at least one buyer is more than three quarters (0.7698). As a result, sellers with the option of entry will infrequently find themselves without at least one buyer and therefore the stock of unsold homes derives primarily from motivated sellers entering cold markets.

Poisson arrival rates imply that the associated waiting times between buyers and sellers are distributed exponentially so that more than one trade does not take place at any one instance. The organization of simulated

events, however, reflects observed data in that events are recorded in discrete intervals during which more than one sale can occur. In particular, the model is simulated for 240 intervals or discrete periods of 30 days each. The first 100 periods are dropped. Summing over the month and over submarkets yields marketwide monthly average prices, total sales, new listings, inventory of unsold homes and time on the market.

To begin evaluating the performance of the model against the SCWMLS benchmark, the last column in Table 1 reports the means and standard deviations in the simulated data. The arrival rate of buyers was chosen to realize the approximate number of sales in Madison which as noted is half the size of Dane County. New listings, inventory and time on market are all lower than the observed data as are the standard deviations of these simulated statistics. Missing factors in the model potentially account for this discrepancy. The more relevant and fundamental point is that the relative variations compare well with the data. As observed in Dane County, the simulated standard deviation in price is less than half the standard deviation in sales. Díaz and Jerez (2013) and Ngai and Sheedy (2015) report similar relative magnitudes. The rankings and ratios for variation of new listings and inventory are likewise reasonable.<sup>16</sup> The exception is the variation in time on market which displays much higher dispersion in the simulated data than in the limited SCWMLS data. Díaz and Jerez (2013) find that the time on market variation is four times the price variation. Ngai and Sheedy (2015) find that time to sale - a different but related measure - is twice the price variation.

Correlations and persistence are critical in evaluating the model. The upper diagonal half of Table 2 and the bottom row of Table 3 report the contemporaneous and one month autocorrelations from the simulations. Given the limited number of parameters, the pattern of correlations from the model fits well. The model, in particular, fits two important stylized facts emphasized in the literature - the co-movement in price and sales and their persistence. The simulations also yield the inverse price-lagged inventory relationship highlighted by Caplin and Leahy (2011).<sup>17</sup> Except for the persistence

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<sup>16</sup>Ngai and Sheedy (2015) report several figures similar to the ones reported here. Using seasonally adjusted, trending quarterly data, they find new listings are more volatile than sales whereas the Dane County and model figures are very close. Ngai and Sheedy also report time to sell (not time on market) and new listings figures constructed observed sales and inventories. Díaz and Jerez (2013) also present a number of overlapping measures based on filtered, quarterly figures.

<sup>17</sup>In their baseline model which shares aspects of this model but does not have hot and

in sales, the simulated correlations for price and sales tend to be higher than SCWMLS correlations which themselves tend to be below figures reported elsewhere. Indeed, in most cases, the simulations generally exhibit stronger correlations than observed in Dane County which may in part reflect the impact of detrending and seasonally adjusting the SCWMLS data.

An important relationship emphasized in the literature is the link between price and time on market. Here the model differs from the SCWMLS data. In the model the correlation is negative and prominent but weakly negative in Dane County data. Again, time on market in the SCWMLS is calculated using a short time period. Moreover, as noted above, the absence of this co-movement in the SCWMLS data is out of line with the relationship obtained elsewhere. The pronounced inverse relationship in the simulations conforms with evidence of a strong inverse relationship perceived as stylized fact. See Ngai and Sheedy (2015), Díaz and Jerez (2013), Krainer (2008), Novy-Marx (2009).

The results discussed so far match the key components highlighted in the previous section. Correlations involving sales, new listings and inventories in the simulated data also deserve consideration. Inventories are inversely related to price in the model and in the data as one might plausibly anticipate when many sellers are waiting for buyers. In the model, new listings correlate very closely with sales and closely with price. In the SCWMLS data, however, new listings and sales correlate very weakly whereas new listings and price exhibit less co-movement than in the model. In the model a new listing will often result in an immediate sale (and frequently with more than one bidder) thereby generating the strong positive co-movements. The decision to enter rests on a high probability of a sale in a competitive auction. In the model, the sale occurs immediately hence the tight link between new listings and sales. In practice, it takes time to carry out a home sale and such delays potentially generate such a discrepancy even if the model captures the essential trading process. Using quarterly data Ngai and Sheedy (2015) report correlations of 0.602 and 0.850 for new listings with price and with sales respectively which match up well with the model. Moreover, when the listing does not sell immediately, sales are lower so inventory rises. In both the data and the simulations, inventory and sales are inversely correlated with similar magnitudes.

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cold spells, Caplin and Leahy are unable to explain positive autocorrelation in prices or a negative correlation between price and lagged inventory.

Now consider new listings and inventory which are positively correlated in the data but negatively in the simulations. In the model, if inventories become high, many submarkets will experience cold spells without relaxed seller entry, hence the negative co-movement between listings and inventory. Cold spells accompanying high inventories and low entry will have lower sales thereby contributing to the negative inventory-sales relationship. In practice, sales do not correlate - at least immediately - with listings as in the model, which, as discussed in the previous section, yields a more direct accounting relationship driving the observed listings-inventory data.

The relationships between time on market and the non-price variables in the model involve a number of interactions that also deserve discussion even if the time on market from the SCWMLS data is limited. To organize ideas, first suppose inventories are below average indicating there are relatively few submarkets experiencing cold spells of declined entry. New listings will tend to be high as relaxed buyers are more likely to enter. Sales too will tend to be above normal but these sales will not greatly affect time on market. Sales of homes that occur immediately after seller entry have no effect on the time on the market as these homes come and go without effect. Likewise, a sale from the existing stock of homes does not lower the average time on the market. If such a sale is a random selection, it removes an average duration home and hence has no immediate impact on the average time on the market. After the sale, however, average time on market in existing stock continues to increase. If new listings from entry of relaxed sellers are high due to low inventories and resulting sales are high, time on market can still rise and result in a positive and non-negligible sales-time on market co-movement as observed in the simulated as well as in the SCWMLS data.<sup>18</sup>

Now suppose inventories of unsold homes are high which will deter entry of many relaxed sellers. A composition effect from the exogenous motivated seller entry potentially explains the inventory-time on market inverse co-movement found in the model. Average time on the market falls (or rises more slowly) as new listings from such sellers do not find an existing buyer and are added to the stock of homes for sale. As entry without an immediate sale is essentially exogenous and scattered across all submarkets, it is more common when inventories are high. With an above normal number of unsold

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<sup>18</sup>Here the distinction between time on market and time to sale might be particularly relevant. Ngai and Sheedy (2015) find a strong strong negative link between new listings and time to sell. They further report very little correlation between inventories and new listings.

homes, new listings by motivated sellers become more likely not to sell immediately driving down average time on market. The process in the observed data may be more involved as actual trade takes some time and new listings that sell quickly are sometimes around long enough to affect time on market as well as inventory measures.

Given that there are only six parameters with one ( $x$ ) normalized and another ( $\beta$ ) chosen to match the mean number of sales, the variation, correlation and persistence in the model compare favorably to benchmark data. On the other hand, dispersion in sales across submarkets, despite frequent cold spells of no entry, does match the Madison quality class  $\times$  elementary school dispersion. The bottom panel of Figure 2 displays the distribution of sales across submarkets. Relative to the SCWMLS data, there are not only too few large sale months but also too few months with zero sales. Yet submarkets frequently go cold. The proportion of months that experience at least one relaxed seller not entering during a  $T_1$  cold spell is 0.2037. The proportion for  $T_0$  spells is 0.4291. Although declined entry slows the transition from one to no homes for sale, when the transition to zero inventory occurs, the ensuing absence of willing entrants lasts longer and typically spans two different months.

Cold spells are essentially too short to accumulate enough potential buyers to trigger the observed within month sales dispersion. Sellers, either motivated or relaxed, enter quickly after slow periods so that large numbers of buyers for sustained high frequency sales volume rarely accumulate. Likewise, cold spells are over and potential entry revived too quickly to create enough months without a sale. As a result, the structure of the model leads to a high number of months with just one sale.

Although the spell lengths are not highly elastic, parameters can be found to enhance the length of  $T_1$  and  $T_0$ . More enduring cold spells come at the cost of extreme parameter values and a less convincing qualitative and quantitative correspondence with the data. An alternative approach inducing more protracted cold spells would be to incorporate aspects of the housing market currently unaccounted for in the model. For example, risk aversion, spillovers and mobility across submarkets, trading frictions, search/waiting and transaction costs, taxes, idiosyncratic or heterogeneous buyer preferences, dispersion in seller costs  $F$ , counter-offer bargaining and the rental market are all abstracted factors that potentially further delay seller entry. Similarly, as the simulated buyers value function is close to deterring relaxed entry ( $x - F \approx C(1, T_1)$ ), it may also be the case that sellers without a sale

likely need more than one bidder for profitable entry.<sup>19</sup> The list is varied, the implications diverse and incorporating many of these factors will undermine transparency and tractability.

## 6 Conclusion

Profit attracts entry. In perfectly competitive markets with full information, instantaneous erosion pins down the timing and number of entrants as well as the price and quantity sold. In markets with frictions, entry and the subsequent pattern of trade may not be as immediate or straightforward. Depending on market structure, it may take time to uncover profitable opportunities which in turn affects the ability and willingness of agents to exchange goods and services.

This paper investigates the way in which entry of this sort affects housing markets with stock-flow matching. Home buyers compete in complete information auctions for homes brought to market one by one. To attract home seller entry, there must be sufficient competition among these bidders. Bertrand-like offers from two bidders are sufficient to induce entry. One monopolistic buyer is not.

Given that house sellers can enter more rapidly than home buyers, markets will alternate between periods of inactive and active entry. In cold markets, prospective sellers pass up production opportunities as they wait for the (unobserved) arrival of buyers to replenish the market. Once they think enough time has passed, entry resumes and reveals the profits to be made in the market. Seller entry continues until it exhausts the existing demand.

The periods of inactivity with one willing but monopolistic bidder are inefficient. Gains to trade exist but are passed over because sellers do not obtain a sufficient share of this payoff. In particular, sellers will want an upfront sunk cost which a monopolistic bidder cannot commit to paying before entry takes place. With Bertrand numbers, buyers are compelled to bid above the sunk cost making entry profitable.

Depending on household circumstances, some sellers may be more compelled to enter than others. The interactions between motivated and relaxed sellers has a profound impact on the entry decision and subsequent trade. The composition of relaxed and motivated sellers affects the duration of hot

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<sup>19</sup>Caplin and Leahy (2011) explore this possibility.

and cold spells as well as the basic picture of trading patterns. Simulations suggest that prices fall, relative volatility and co-movement with sales and persistence remains constant as motivated sellers become more common.

Simulated outcomes with both types of sellers present also fit empirical evidence from Dane County, Wisconsin as well as stylized facts established in the literature. With only a small number of parameters, the simulated variation, correlation and persistence in price, sales volume, new listings, inventories and time on market broadly match up well with the observed data. The observed data are also consistent with the existence of having hot and cold spells. Such spells frequently occur in the simulated model. These cold spells are, however, not sufficiently long enough to generate the observed dispersion of sales. A number of omitted factors could potentially extend the duration of these cold spells and thereby properly account for sales dispersion.

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