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Compliance Technology and Self-Enforcing Agreements

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Abstract

We analyze a repeated game in which countries are polluting as well as investing in technologies. While folk theorems point out that the first best can be sustained as a subgame-perfect equilibrium when the players are sufficiently patient, we derive the second best equilibrium when they are not. This equilibrium is distorted in that countries over-invest in technologies that are “green” (i.e., strategic substitutes for polluting) but under-invest in adaptation and “brown” technologies (i.e., strategic complements to polluting). It is in particular countries which are small or benefit little from cooperation that will be required to strategically invest in this way. With imperfect monitoring or uncertainty, such strategic investments reduce the need for a long, costly punishment phase and the probability that punishment will be triggered.

JEL-Code: D860, F530, H870, Q540.

Keywords: climate change, environmental agreements, green technology, imperfect monitoring, policy instruments, repeated games, self-enforcing treaties.

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1 Introduction

To be successful, any environmental treaty must address two major challenges of climate change. First, in the absence of international enforcement body, any international treaty must be self-enforcing. In principle, sanctions could be imposed by threatening free-riders with trade barriers, the seizure of infrastructure, or armed conflicts, but such options are not on the table when climate negotiators meet. In the absence of such sanctions, one might hope that countries would follow the treaty in order to motivate other nations to cooperate in the future. This motivation, however, may not always be sufficiently strong. Australia did not ratify the Kyoto Protocol—the world’s only effective climate treaty—until 2007, ten years after it had been signed. Canada ratified the treaty in 2002, but in 2011 it simply withdrew.

The second challenge to confront climate change is to develop new and environmentally friendly technology. The importance of new and green technology is recognized in the treaties, but there has been no attempt to negotiate or quantify how much countries should be required to invest in these technologies. Instead, the negotiators focus on quantifying emissions or abatements and leave the investment decision to individual countries. Nevertheless, some countries do invest heavily in green technologies. The European Union aims for 20 percent of its energy to come from renewable sources by 2020, and to increase that number to 27 percent by 2030. China is a still larger investor in renewable energy and has invested heavily in wind energy and solar technology. Other countries have instead invested in so-called “brown” technology: Canada has developed its capacity to extract unconventional oil such as tar-sands, and Australia has continued to develop its coal-mining industry.

The interaction between the two challenges is poorly understood by economists as well as policymakers. To understand how treaties can address these challenges and how these challenges interact, we need a theory that allows technology investments as well as emission decisions to be made repeatedly. Since the treaty must be self-enforcing, strategies must constitute a subgame-perfect equilibrium (SPE).

There is no such theory in the literature, however, and many important questions have thus not been addressed. First, what is the best (i.e., Pareto optimal) SPE? Second, folk theorems have emphasized that even the first best can be sustained if the players are sufficiently patient, but what distortions occur if they are not? Third, will non-cooperative, selfish investments result in the optimal level of environmentally friendly technologies?

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1Chapter 16 of the Stern Review (2007) identified technology-based schemes as an indispensable strategy for tackling climate change. However, article 114 of the Cancun Agreement 2010 confirmed in Durban in 2011 states that "technology needs must be nationally determined, based on national circumstance and priorities”.

2On green energy, “Canada risks being left behind as green energy takes off” (The Globe and Mail, September 21st, 2009) and Australia “extends renewable energy investment ban to solar power” (The Guardian, July 17th, 2015).
Or are there reasons, beyond the traditional argument about technological spillovers, for including technology investments in the agreement? Which kinds of countries ought to invest the most, and in what kinds of technology?

To address these questions, we analyze a repeated extensive form game where countries invest in technology before deciding on emission levels. In the basic model, investments are selfish (i.e., there are no technological spillovers); this implies that equilibrium investments would have been first best if the countries had committed to the emission levels. The first best can also be achieved if the discount factor is sufficiently high, in line with standard folk theorems. For smaller discount factors, however, the best SPE is distorted. We show that the distortions take the form of over-investments in so-called “green” technologies, i.e., technologies that are strategic substitutes for pollution. Such over-investments reduce a country’s temptation to cheat by emitting more rather than less, and are necessary to satisfy the compliance constraint at the emission stage. For so-called “brown” technologies, including drilling technologies and other infrastructure investments that are strategic complements to fossil fuel consumption, investments must instead be less than the first-best amount to satisfy the compliance constraint. Our most controversial finding may be that countries should also be required to invest less than the first-best amount in adaptation, i.e., technologies that reduce the environmental harm in a country (and thus also the country’s benefit from continuing cooperation and less emissions).

Our analysis is positive if we believe that countries are able to coordinate on the best SPE, but normative if we think that they should. In any case, the comparative statics have important policy implications: Naturally, it is harder to motivate compliance if the discount factor is small, the environmental harm is small, or the investment cost is large. In these circumstances, the best SPE (i.e., the best self-enforcing treaty) requires countries to invest more in green technologies and less in adaptation or brown technologies. If countries are heterogeneous, the countries that are small and face less environmental harm are the most tempted to free-ride. Thus, for compliance to be credible, such countries must invest the most in green technologies or the least in adaptation and brown technologies. This advice contrasts the typical presumption that reluctant countries should be allowed to contribute less in order to satisfy their participation constraint. While the participation constraint requires that a country’s net gain of cooperating is positive, the compliance constraint requires that the net gain outweighs the positive benefit of free-riding for one period, before the defection is observed. The compliance constraint is therefore harder to satisfy than the participation constraint in this model, and, to satisfy it, the reluctant countries must invest more in green technologies and less in adaptation and brown technologies.

Simplicity and tractability are some of the assets of our workhorse model. The main results are derived in a pedagogical way with binary emission levels and without imperfect
monitoring, uncertainty, private information, renegotiation, technological spillovers, or policy instruments such as emission taxes or investment subsidies. These complicating factors are added in our extension section and our main results remain robust. The extensions can also be used to deepen our understanding of the interaction between technology and compliance.

When imperfect monitoring and uncertainty are added to the model, infinitely long punishments are not optimal since they may be triggered by mistake. Technology levels should then be chosen not only to motivate compliance, but also to allow the countries to reduce the duration of the punishment period without violating the compliance constraint. If a country’s cost of complying is stochastic, technologies should be chosen so as to increase the probability or the frequency of compliance. When renegotiation is possible, the realistic penalty declines, free-riding may become more attractive and, in response, the best renegotiation-proof SPE requires countries to invest even more in green technology and less in adaptation or brown technology. The necessity to invest strategically continues to hold if there are technological spillovers, but the investment-stage compliance constraint may then bind, perhaps making it impossible to sustain an equilibrium with less emissions. When we allow for continuous emission levels, it is also natural to discuss policy instruments such as emission taxes and investment subsidies. The first best requires an emission tax only, and no investment subsidy. If the discount factor is smaller, however, the emission tax that can be sustained in the repeated game is also smaller, and an investment subsidy should then be introduced.

It is widely accepted that international agreements must be self-enforcing. The literature on repeated games is thus the relevant one, but this literature has mostly been concerned with folk theorems and conditions under which the first best can be sustained if only the players are sufficiently patient. Our two contributions to this literature are that (i) we extend the standard repeated prisoner dilemma by allowing agents to take technology investment decisions in each period, and (ii) we derive the distortions that must occur when the discount factor is so small that the first best cannot be achieved. Note that each of these two extensions would be uninteresting in isolation: With high discount factors, the first best can always be sustained, also in a model with technology. Without technology and with small discount factors, only defect could be sustained in the repeated prisoner dilemma game.

This is the first paper, to the best of our knowledge, which studies the second best in

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3As Downs and Jones (2002) observed, “a growing number of international relations theorists and international lawyers have begun to argue that states’ reputational concerns are actually the principal mechanism for maintaining a high level of treaty compliance.”

4See, among others, Friedman (1971) and Fudenberg and Maskin (1986) for folk theorems which assume near perfect patience, Fudenberg et al. (1994) when monitoring is imperfect, and Mailath and Samuelson (2006) for an extensive review of the current state of the art for folk theorems. Parts of the more applied literature have allowed for smaller discount factors: On trade agreements, for example, see Bagwell and Staiger (1990), or the review by Maggi (2014).
a repeated game with technology choices. With this focus, our paper fills a gap between
the literature on repeated games and the one on green technology. Repeated games
have often been used to analyze self-enforcing environmental agreements (Barrett 1994;
1999; 2005), but these papers do not allow countries to invest in technologies along the
way. Investments in green technology are typically studied in models with just a few
stages. Dutta and Radner (2004; 2006) study a dynamic game with emissions as well
as technology choices. Like us, they refer to self-enforcing treaties as SPEs supported by
trigger strategies. However, technology in these papers is either exogenous or chosen as a
corner solution at the beginning of the game. The structure of our model is more similar
to those of Harstad (2012; 2015) and Battaglini and Harstad (2015), where countries
pollute and invest in green technologies in every period. These papers, however, assume
contractible emission levels and study Markov-perfect equilibria, while we focus on self-
enforcing agreements and subgame-perfect equilibria. This leads to a new strategic effect
of technology—namely that technology should be chosen so as to make future cooperation
credible.

The role of technology in our paper is somewhat similar to the role of capacity in
industrial organization, and the role of armament to sustain peace. In this literature,
investments tend to be irreversible, and thus affect the sustainability of collusion/peace
in two opposite ways. On the one hand, a reduction in production capacity as well as in
weapon stocks reduces the incentives to deviate, thereby reinforcing cooperation. On the
other hand, less capacity or arms weakens the severity of retaliation if one player deviates,
and this undermines cooperation. The total impact of technology on compliance is then
generally non-monotonic and depends on the specific features of the model. Relative to

5For example, in two-stage games, Golombek and Hoel (2005) show that environmental agreements
should be ambitious in order to induce R&D, while Hoel and de Zeeuw (2010) show that cooperation
on R&D can increase participation when R&D reduces the cost of technology adoption. Investments
are also permitted by Barrett (2006), studying the role of breakthrough technologies in environmental
agreements. In these contributions, the presence of technological spillover plays a crucial role. Buob
and Stephan (2011) allow for adaptation technology and point out that this is a strategic substitute to
mitigation. Acemoglu et al. (2012) present a dynamic model with pollution as well as investments in
clean and dirty technology, but there is a single economy only, and the focus is on imperfections in the
R&D market. For surveys and overviews, see Jaffe et al. (2003), Barrett (2005), and Calvo and Rubio
(2012).

6This contrasts to the strategic role of technology in the existing literature: when countries can
commit to emission levels, they will take into account that the commitments will influence the choices
of technology; and before negotiating these commitments, they anticipate a hold-up problem which
discourages them from investing. On the hold-up problem, see also Buchholz and Konrad (1994) or

7In their seminal contributions, Spence (1977) and Dixit (1980) study in a non-repeated setting how
firms can deter entry by modifying capacity limits. Fudenberg and Tirole (1984) discuss circumstances
under which strategic investment may lead the incumbent to exploit strategic complementarity and
accommodate entrants rather than exploit strategic substitutability and deter entry.

8Garfinkel (1990) is the first to study folk theorems for conflict models, establishing that peace can
be supported for sufficiently patient players. Jackson and Morelli (2009) study a coordination game of
war and peace where decisions of investments in weapons are taken in each period.

9In a setting where firms first collude on capacity and then engage in an infinitely repeated game
the literature on industrial organization and on conflicts, our contribution goes beyond developing these ideas in the context of environmental policy. More fundamentally, our model differs from those above in that (i) we allow countries to choose their technology level in every period,\textsuperscript{10} (ii) we allow for a general family of technologies and focus on what \textit{type} of technology countries should invest in, and (iii) we explicitly focus on the second best, that is, the best SPE that can be sustained when the discount factor is too small to sustain the first best.\textsuperscript{11,12}

The next section presents the stage game and discusses benchmark results. Section 3 derives a unique Pareto optimal SPE and discusses comparative statics. Using the basic model as a workhorse, five important extensions are analyzed in Section 4. The Appendix contains all proofs.

### 2 A Model of Compliance Technology

A repeated game consists of a stage game and a set of times when the stage game is played. While we focus on the dynamics and the subgame-perfect equilibria (SPEs) in the next section, we here present the stage game and discuss important benchmarks.

There are \( n \) players or countries, indexed by \( i \) or \( j \in N \equiv \{1, \ldots, n\} \). The average country size is normalized to one, although we can easily allow for heterogeneous country sizes \( s_i \leq 1 \). At the emission stage, the countries simultaneously decide between emitting more or less. Let \( b_i(\cdot) \) be the per capita benefit as an increasing and concave function of country \( i \)'s per capita emission \( g_i \in \{g, \bar{g}\} \), while \( c_i \sum_{j \in N} s_j g_j \) is the per capita environmental cost as a function of aggregate emissions. We assume that the countries’ emission decisions constitute a prisoner dilemma. That is, a country \( i \) benefits from emitting more of price competition, Benoit and Krishna (1987) find that all equilibria exhibit excess capacity. When firms are asymmetric, however, investment in capacity unambiguously hinders collusion (see Lambson, 1994, and Compte et al., 2002). Chassang and Padro i Miquel (2010) show that weapons unambiguously facilitate peace under complete information, but not under strategic risk.\textsuperscript{10}

This assumption is reasonable for long-run problems such as climate change, where the countries must expect to invest repeatedly partly to maintain the infrastructure and the capacity to produce renewable energy, but also to invest in research and development effort. See, for example, Dockner and Long (1993), Rubio and Casino (2002), Dutta and Radner (2004). If investments are more or less reversible in the long run, technology can weaken the temptation to deviate without affecting the severity of retaliation against deviators.\textsuperscript{11}

The idea that technology investments can relax compliance constraints is also present in the relational contracting literature. Halac (2015) explores this idea in a model with repeated trading, where, before trade starts, the principal can make a noncontractible irreversible investment. Baker et al. (2002) and Halonen (2002) investigate the sustainability of cooperation in a repeated relationship where different ownership structures can modify enforcement constraints and affect the parties’ ex-post incentive to renege.\textsuperscript{12}

From the authors’ point of view, this paper is the result of combining two independent and unrelated projects: Harstad (2015) studies green/brown technology as a way of partially committing to low emissions in the future in a setting where a single decision maker has time-inconsistent preferences; Lancia and Russo (2014) study how agents exert effort strategically to signal their willingness to cooperate in a stochastic overlapping-generations model.
for any fixed emission from the other countries, $g_{-i} ≡ \sum_{j \neq i} s_j g_j$, but every country would be better off if everyone emitted less instead of more.

$$b_i (g, r_i) - (s_i g + g_{-i}) c_i < b_i (\bar{g}, r_i) - (s_i \bar{g} + g_{-i}) c_i \text{ and}$$

$$b_i (g, r_i) - n g c_i > b_i (\bar{g}, r_i) - n \bar{g} c_i.$$

Variable $r_i \in \mathbb{R}_+$ is here capturing the fact that a country’s benefit depends on more than its emission levels. We will refer to $r_i$ as the country’s technology, but $r_i$ can actually be any variable which influences the benefit of emitting. In fact, we also allow $r_i$ to influence a country’s environmental cost by letting $c_i ≡ h_i c (r_i)$. It is reasonable that $c' (r_i) < 0$, if $r_i$ refers to a country’s adaptation technology, since more adaptation technology reduces the environmental cost of emissions.

To simplify, we use subscripts for derivatives whenever this is not confusing, and we abuse notation by writing $b''_{i,gr} ≡ \partial \left[ b_i (\bar{g}, r_i) - b_i (g, r_i) \right] / \partial r_i$. To illustrate the relevance of technologies, we will occasionally refer to the following special types:

**Definition 1.**

(A) **Adaptation technology** is characterized by $b''_{i,gr} = 0$ and $c' (r_i) < 0$.

(B) **Brown technology** is characterized by $b''_{i,gr} > 0$ and $c' (r_i) = 0$.

(C) **Clean technology** is characterized by $b''_{i,gr} < 0$ and $c' (r_i) = 0$.

Adaptation technologies refer to technologies which help a country to adapt to a warmer or more volatile climate. Such technologies include agricultural reforms or more robust infrastructure, and may even capture the effects of some geo-engineering practices that have strictly local effects. In other words, adaptation technology is useful because it helps the country to adapt to the emissions. Brown technology can be interpreted as drilling technology, infrastructure that is helpful in extracting or consuming fossil fuel, or other technologies that are complementary to fossil fuel consumption. Such technology is beneficial in part because it increases the marginal benefit of emitting. Clean technology, in contrast, is a strategic substitute for fossil fuel and reduces the marginal value of emitting another unit. This is the case for abatement technology or renewable energy sources, for example. Both brown and clean technology may be beneficial in that $\partial b_i (\cdot) / \partial r_i > 0$.

We assume that the emission game is a prisoner dilemma (1)-(2) for all relevant technology levels. Nevertheless, we now endogenize the technology levels by letting the countries simultaneously, non-cooperatively decide on their $r_i$’s at the investment stage, which is prior to the emission stage. We can without loss of generality assume that the investment cost is linear in $r_i$, so that the marginal investment cost is a constant $k_i > 0$, since $r_i$ can enter a country’s benefit function in arbitrary ways.\(^{13}\) It is also without loss if the investment cost were another function $\kappa_i (r_i)$, we could simply define $\tilde{b}_i (g_i, \kappa_i (r_i)) ≡ b_i (g_i, r_i)$.
of generality to assume there is no discounting between the investment stage and the emission stage.\footnote{If the discount factor between the two stages was $e^{-\rho \ell}$, where $\rho$ were the discount rate and $\ell$ the time between the two stages, we could refer to the investment cost as $\hat{k}_i \equiv e^{\rho \ell} k_i$, as evaluated at the time of the emission stage, and proceed with the analysis using $\hat{k}_i$ as the investment cost instead of $k_i$.} Thus, country $i$’s per capita utility is:

$$u_i = b_i (g_i, r_i) - h_i c (r_i) \sum_{j \in N} s_j g_j - k_i r_i. \quad (3)$$

Since investments are selfish, each country is voluntarily investing the socially optimal amount, conditional on the emission levels. To see this, note that the first best requires:

$$r^*_i (g) \equiv \arg \max_{r_i} b_i (g, r_i) - n g h_i c (r_i) - k_i r_i.$$  

Clearly, this coincides with the noncooperative choice of $r_i$ when country $i$ takes the emission levels as given. In other words, if the countries could solve their prisoner dilemma by committing to low emission levels in advance, then investments would be socially optimal and the first best would be implemented. These benchmark results provide some preliminary support for the presumption that it is not necessary to contract on investments in addition to emissions.

**Proposition 0.**

(i) In the first-best, $r^*_i \equiv r^*_i (g_i)$ and $g_i = g$.

(ii) In the unique SPE of the stage game, $r^*_i \equiv r^*_i (g_i)$ and $g_i = \bar{g}$.

(iii) If countries had committed to $g_i = \bar{g}$, the outcome, including the equilibrium investments, would be first best.

**Remark on assumptions and extensions.** In (3), we have assumed that technology investments are selfish in that such investments only affect the investing country’s technology. We have also abstracted away from uncertainty and policy instruments, and we permit only two possible emission levels. These assumptions allow us to derive key insights in a simple setting. Section 4 relaxes all these assumptions and shows that our main results continue to hold. The Appendix discusses time-varying parameters, rather than the stationary ones in our basic model.

**Remark on stocks and reversibility.** It is straightforward to reformulate this model and allow for stocks. Suppose the pollution stock accumulates over time and depreciates only at rate $q \in [0, 1]$. As long as the marginal cost of pollution is constant, the stock is payoff-irrelevant in that it does not influence future decisions, and the long-lasting cost of emission can already be accounted for today. To see this in the simplest way, let $c' (r_i) = 0$ and $\tilde{h}_i$ be the cost of a marginally larger pollution stock. Then, the present-discounted
cost of emitting another unit evaluated at the time of the emission is simply the constant 
\[ h_i \equiv \tilde{h}_i c(r_i) / (1 - \delta q^g). \]

Analogously, suppose a fraction \( q^*_i \in [0,1] \) of country \( i \)'s investments in technology survives to the next period. In this case, one benefit of investing today is that investments can be reduced in the next period. These cost-savings will not be payoff-relevant, however, in the sense that today’s choice of \( r_i \) will not influence the level of technology in the future; it will only reduce the cost of obtaining that level of technology. Thus, if \( \tilde{k}_i \) were the cost of adding to the technology stock, we can already account for the future cost-savings today and write the net marginal investment cost as \( k_i \equiv (1 - \delta q^*_i) \tilde{k}_i \).

If the \( q^*_i \)’s are small, then the analysis below is unchanged since countries do need to invest in every period (even off the equilibrium path). The investments are then, in effect, reversible. These assumptions are reasonable in the very long-run context of climate change, in our view. Furthermore, if the \( q^*_i \)’s were instead large, it would actually be easier to motivate countries to emit less, as we argue below.

By ignoring stocks and instead considering the one-period utilities given by (3), it is straightforward to interpret our dynamic game as a simple repeated game.

### 3 Self-enforcing Agreements

While the stage game is described above, we here assume that the stage game is played repeatedly in every period \( t \in \{1, 2, ..., \infty\} \). We let \( \delta \in [0,1) \) be the common discount factor and \( v^t_i = (1 - \delta) \sum_{\tau=t}^\infty \delta^{\tau-t} w^t_i \) measures country \( i \)'s continuation value at time \( t \) (normalized to per-period utility). The goal of this section is to characterize the “best” (that is, the Pareto optimal) subgame-perfect equilibrium (SPE). Since all parameters are invariant in time, the Pareto optimal SPE is stationary and we skip \( t \)-superscripts for simplicity. The Appendix allows for time-varying parameters and contains all proofs.

#### 3.1 The Worst Equilibrium

Note that there is a unique SPE in the one-period stage game described above. Given (1)-(2), more emissions at the emission stage are a dominant strategy for all countries; at the investment stage, emissions are individually optimally set to \( r_i = r^*_i (g) \). Clearly, these strategies also survive as an SPE in the infinitely repeated game in which the stage game is played in every period. In fact, in every SPE in which \( g_i = g \), we must have \( r_i = r^*_i (g) \). For any other equilibrium candidate \( r_i \), country \( i \) could benefit from deviating to \( r^*_i (g) \) without any risk of reducing \( v_i \). In other words, from country \( i \)'s point of view, emitting more is the other players’ worst strategy (i.e., the minmax strategy), and an SPE cannot be sustained with lower utilities. We refer to this equilibrium as the business-as-usual (BAU) equilibrium and label it with superscript \( b \).
Proposition 1. The worst SPE is BAU: \((r_i^b, g_i^b) = (r_i^*(\bar{g}), \bar{g})\). This equilibrium always exists.

Of course, the worst equilibrium might be used as a threat to enforce better equilibria. In fact, if a pair \((r_i, g_i)\) can be sustained in some SPE, then these actions can (also) be sustained in an SPE where any deviation requires the countries to revert to the worst possible SPE, i.e., BAU forever. Therefore, we can with no loss of generality focus on such simple trigger strategies.

Corollary 1. If \((r_i, g_i)\) can be sustained as an SPE, then it can be sustained as an SPE in which any deviation triggers an immediate reversion to BAU.

3.2 The Best Equilibrium

Corollary 1 implies that we can, without loss of generality, rely on SPEs that are enforced by simple trigger strategies. We are particularly interested in Pareto optimal SPEs with less emissions. When such an equilibrium is unique, we refer to it as “the best equilibrium”.

Definition 2. An equilibrium is referred to as best if and only if it is the unique Pareto optimal SPE satisfying \(g_i = \bar{g} \; \forall i \in N\).

Since there are two decision-stages in each period, we must consider the temptation to deviate at each of them. At the investment stage, a country must compare the continuation value \((v_i)\) it receives from complying with the SPE by investing \(r_i\), to the maximal continuation value it could possibly obtain by deviating. Since deviating at the investment stage implies that every country will emit more beginning from this period, the compliance constraint at the investment stage is the following:

\[
\frac{v_i}{1 - \delta} \geq \max_{r_i} b_i(\bar{g}, r_i) - h_i c(r_i) n\bar{g} - k_i r_i + \frac{\delta v_i^b}{1 - \delta}.
\]

The right-hand side of \((CC_i^r)\) is maximized when \(r_i = r_i^* (\bar{g})\), implying that the right-hand side is simply \(v_i^b\). Thus, \((CC_i^r)\) simplifies to \(v_i \geq v_i^b\). In other words, as long as every country prefers the SPE to BAU, the compliance constraint for the investment is trivially satisfied.

At the emission stage, the investment cost for this period is sunk and the compliance constraint becomes:

\[
b_i(\bar{g}, r_i) - h_i c(r_i) n\bar{g} + \frac{\delta v_i}{1 - \delta} \geq b_i(\bar{g}, r_i) - h_i c(r_i) (s_i \bar{g} + (n - s_i) \bar{g}) + \frac{\delta v_i^b}{1 - \delta},
\]

\((CC_i^g)\)
which implies that:

$$\delta \geq \hat{\delta}_i (r_i) \equiv 1 - \frac{v_i - v_i^b}{b_i (g, r_i) - b_i (g, r_i) - s_i h_i (r_i) (g - g) + v_i - v_i^b}. \tag{4}$$

In the limit as $\delta \to 1$, $\text{(CC}_g)$ approaches the condition $\text{(CC}_r)$, i.e., $v_i \geq v_i^b$. For any $\delta < 1$, however, $\text{(CC}_g)$ is harder to satisfy than $\text{(CC}_r)$ because of the free-riding incentive. It is not sufficient that the SPE is better than BAU. In addition, the discount factor must be large or the temptation to free-ride must be small.

As indicated in (4), the threshold for the discount factor generally depends on the equilibrium $r_i$. For first-best investments, $r_i^* \equiv r_i^*(g)$, the threshold is $\delta_i \equiv \hat{\delta}_i (r_i^*) < 1$. Thus, if $\delta \geq \delta_i$ holds for every $i \in N$, every $\text{(CC}_g)$ holds for first-best investment levels and the best SPE is simply the first best.

If $\delta < \delta_i$, however, $\text{(CC}_g)$ does not hold for $r_i = r_i^*$. To ensure that compliance constraint at the emission stage is satisfied, the temptation to free-ride must be reduced by requiring an $r_i$ so that $\hat{\delta}_i (r_i) \leq \delta$. This requires $r_i > r_i^*$ if $\hat{\delta}_i (r_i^*) < 0$, or $r_i < r_i^*$ if $\hat{\delta}_i (r_i^*) > 0$. It is easy to see that:

$$\hat{\delta}_i (r_i^*) < 0 \text{ if } b_{i,gr}^r < s_i h_i c' (r_i^*); \quad \text{(G}_i)$$

$$\hat{\delta}_i (r_i^*) > 0 \text{ if } b_{i,gr}^r > s_i h_i c' (r_i^*). \quad \text{(NG}_i)$$

Under condition $\text{(G}_i)$ for “green” technology, more investments relax the compliance constraint by reducing the lower threshold $\hat{\delta}_i (r_i)$. Above this threshold, less emission can be sustained as an equilibrium outcome. Under condition $\text{(NG}_i)$ for “non-green” technologies, less investments relax the compliance constraint.

As the discount factor $\delta < \delta_i$ declines further, $\text{(CC}_g)$ becomes even harder to satisfy and requires investment levels that increasingly differ from the first-best level. Once the discount factor is smaller than a lower threshold referred to as $\delta_i < \delta_i$, $g_i = g$ can no longer be sustained in an SPE. The thresholds are explained in the Appendix, which includes the proofs of the following results.

**Proposition 2.** An SPE exists in which $g_i = g \forall i \in N$ if and only if $\delta \geq \max_i \delta_i$. In this case, the Pareto optimal SPE is unique and it is characterized as follows:

(i) If $\delta \geq \delta_i$, then $r_i = r_i^*$ is first best.

(ii) If $\delta < \delta_i$, then:\footnote{In the following equations, the operators min and max are added since $\hat{\delta}_i^{-1} (\delta)$ is a correspondence and, of the two values of $\hat{\delta}_i^{-1} (\delta)$, it is optimal to select the one closest to $r_i^*$.}

$$r_i = \min \hat{\delta}_i^{-1} (\delta) > r_i^* \text{ under } (G_i);$$

$$r_i = \max \hat{\delta}_i^{-1} (\delta) < r_i^* \text{ under } (NG_i).$$
The result that the first best is achievable when the discount factor is sufficiently large is standard in the literature on repeated games. Thus, the contribution of Proposition 2 is to characterize the distortions that must occur if the discount factor is small. To understand the importance of this characterization, it is useful to once again refer to the special cases in Definition 1. Clearly, condition \((G_i)\) is satisfied for clean technology, while \((NG_i)\) is satisfied for adaptation and brown technology. In other words, if the first best cannot be achieved, countries are only motivated to comply with an agreement and emit less if they have, in advance, invested less in adaptation or brown technologies, or more in clean technologies. Intuitively, the temptation to free-ride is larger after investing in adaptation or brown technology, but smaller after investing in clean technology.

**Corollary 2.** Compared to the first-best, the Pareto optimal SPE requires the countries to:

(i) under-invest in adaptation technologies;
(ii) under-invest in brown technologies;
(iii) over-invest in clean technologies.

These strategic investment levels, which are clearly inefficient conditional on the emission levels, must be part of the self-enforcing agreement in the same way as are the small emission levels: any deviation must be triggered by a reversion to BAU.

Distorting the choice of technology in this manner reduces the temptation to deviate from the equilibrium. Note that it is not necessary to require so little or so much investment that emitting less becomes a dominant strategy: it is sufficient to ensure that the benefit of emitting more is smaller (although still positive) than the present discounted value of continuing cooperation.

### 3.3 Comparative Statics

We are finally ready to discuss important comparative statics. The compliance constraints are not only functions of technologies. They also depend on the other parameters of the model. Compliance is particularly difficult to motivate if the cost of reverting to BAU is small. The cost of BAU is small if relatively few countries are polluting (i.e., \(n\) is small), if the environmental harm \((h_i)\) is small, or if the countries heavily discount the value of cooperating in the future (i.e., \(\delta\) is small). In all these situations, a country \(i\) will not find it optimal to comply unless it is requested to invest less in adaptation and brown technologies, or more in clean technologies. The result that investments in clean technology are not sufficient to ensure compliance is due to Rubinstein and Wolinsky (1995), who show that the Fudenberg and Maskin (1986) folk theorem can be generalized.

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16 The result that folk theorems hold in repeated extensive-form games is due to Rubinstein and Wolinsky (1995), who show that the Fudenberg and Maskin (1986) folk theorem can be generalized.

17 If technology were long-lasting and not reversible, it would be easier to satisfy the compliance constraint. The reason is simply that the payoff after deviation would be less than the BAU payoff until the technology stock equaled \(r^* (\overline{y})\).

12
technologies should decline with the discount factor, for example, is certainly at odds with traditional results in economics.

Furthermore, we show that all investments should increase with the investment cost \( k_i \). For adaptation and brown technologies, we have \( r_i < r^b_i \). A larger \( k_i \) thus reduces the value of BAU \( (v^b) \) compared to cooperation, and makes the compliance constraint easier to satisfy. Thus, when \( k_i \) increases, \( r_i \) can increase towards \( r^*_i \) without violating \( (CC^g_i) \). For clean technologies on the other hand, we have \( r_i > r^b_i \), and a larger \( k_i \) again reduces the value of cooperating relative to BAU. The compliance constraint becomes harder to satisfy. As a response, countries must invest even more in clean technologies to satisfy \( (CC^g_i) \) when \( k_i \) increases.

**Proposition 3.** Suppose \( \delta \in [\max_j \delta_j, \overline{\delta}_i] \) and consider the Pareto optimal SPE.

(i) If \( k_i \) increases, then \( r_i \) increases.

(ii) If \( \delta \) or \( s_i \) decreases, then \( |r_i - r^*_i| \) increases.

(iii) If \( n \) or \( h_i \) decreases, then \( r_i \) increases for clean technologies, while \( r_i \) decreases for brown technologies, and, assuming \( (\epsilon')^2 / c'' < c \), also for abatement technology.\(^{18}\)

Note that the comparative statics are country-specific. When environmental harm is heterogeneous, countries subject to the least harm (i.e., those with the smallest \( h_i \)) are most tempted to emit more. These “reluctant” countries must be required to invest little in adaptation and brown technologies or more in green technologies. Similarly, small countries are tempted to emit more because they internalize less of the total harm. Small countries must thus be required to invest little in adaptation and brown technology or more in clean technology to counter their incentive to free ride.

**Corollary 3.** In the Pareto optimal SPE, the smallest and the most reluctant countries invest the least in adaptation and brown technology, and they invest the most in clean technology.

The result that countries which are small or have high investment costs ought to invest more in clean technology is in stark contrast to the idea that countries should contribute according to ability and responsibility.

The result that countries which are reluctant to cooperate (in that the harm \( h_i \) is small) ought to invest more is similarly in contrast to the intuition that such countries must be given a better deal to make them cooperate.

It is true, of course, that countries that are reluctant either because they are small or have high investment costs, or because they are subject to less harm, have participation constraints (i.e., the constraint \( v_i \geq v^b_i \)) that are more difficult to satisfy than for other

\(^{18}\)If, instead, \( (\epsilon')^2 / c'' > c \), investing in adaptation technology is so productive that if \( n, g \), or \( h_i \) increases, country \( i \)'s environmental harm \( ngh_i c(r_i) \) actually declines when the changes induce the country to invest more in adaptation technology. This is unrealistic, in our view.
countries. However, as we have shown above, the compliance constraint (CC) is more difficult to satisfy than the participation constraint. Although all countries must obviously benefit from cooperation compared to BAU, they must in addition benefit from cooperation at the stage when they face the possibility of free-riding one period before the others revert to BAU.

4 Extensions

In the following sections, we extend the basic model in several directions. We show that technology investments can have a new strategic role when private emissions cannot be observed or when private benefits from emissions and investments are uncertain. In the former case, strategic investments reduce the duration of the punishment period and, in the latter case, they reduce the probability of triggering the punishment, while keeping the incentives to comply at the emission stage. We also discuss how to relax some modeling assumptions. In particular, we allow for renegotiation, technological spillovers, continuous emission levels, and policy instruments such as emission taxes and investment subsidies. While the results of the basic model are robust to all these extensions, each extension deepens our understanding of the strategic role of technologies. The reader is free to jump directly to the extension of interest, since they all build directly on the basic model. To isolate the insight in each extension, we henceforth assume countries are symmetric. Then, conditions (G_i) and (NG_i), for example, simplify to:

\[ b'_{gr} < hc'(r); \]  \hspace{2cm} (G)

\[ b''_{gr} > hc'(r). \]  \hspace{2cm} (NG)

4.1 Imperfect Monitoring and Duration of Punishment

In the basic model, grim-trigger strategies with infinitely long penalties come at no cost, since they will never occur in equilibrium. The reality is less deterministic, however, and such a harsh punishment may be too risky. Even if every country has the best of intentions, there is some chance that emission levels will appear to be higher than agreed upon. With such a risk, it is desirable to reduce the punishment length. Since both the uncertainty and a shorter punishment length strengthen the compliance constraint, the best equilibrium must require even larger investments in clean technology, or even lower investments in adaptation and brown technologies. In other words, investments should be strategically chosen such as to reduce the need for a long and harsh punishment.

To capture real-world uncertainty, we let total emission be given by \( g = \sum_{i=0}^{n} g_i \), where \( g_0 \), drawn from the cdf \( F(\cdot) \) and i.i.d. over time, measures the net emission from Nature. In addition to the uncertain \( g_0 \), we also relax the assumption that the country-
specific emission levels are observable. Instead, only the aggregate \( g \) is observed. Note that neither of the two modifications would play any role if introduced in isolation: If the \( g_i \)'s were observable, the uncertain \( g_0 \) would play no role since the marginal cost of pollution is constant; if \( g_0 \) were deterministic or absent, it would be irrelevant whether the \( g_i \)'s were observable as long as the aggregate \( g \) could be observed. Together, however, the two modifications turn out to be important as well as realistic.

We restrict attention to the set of public perfect equilibria (PPEs). These are strategy profiles for the repeated game in which (i) each country’s strategy depends only on the public information, and (ii) no player wants to deviate at any public history.\(^{19}\) The best PPE \((r, g)\) can be sustained by the following class of grim-trigger strategies: Comply by investing \( r \) and emitting \( g \) as long as (i) no country has deviated at the investment stage and (ii) the observed pollution level has been \( g \leq \hat{g} \), for some threshold \( \hat{g} \), in every earlier period. As soon as \( g > \hat{g} \), play BAU in \( T \leq \infty \) periods before returning to the PPE. If one or more country deviates at the investment stage, play BAU forever after.\(^ {20}\)

The presence of uncertainty leads to two types of errors. First, we may have a type I error where cooperation ends even if every country polluted little. The probability for this to happen is \( q \equiv 1 - F (\hat{g} - n g) \). Second, we may alternatively have a type II error where cooperation continues even after a country deviates by polluting more. The probability for such type II error is given by \( 1 - p \), where \( p \) is the probability of a penalty being triggered because a country has violated the treaty by polluting more: \( p = 1 - F (\hat{g} - (\bar{g} + (n - 1) g)) \). We obviously have \( p > q \) when \( F \) is strictly increasing.

The emission-stage compliance constraint requires that the one-shot benefit of free-riding is smaller than the cost of risking the punishment with a larger probability:\(^{21}\)

\[
\begin{align*}
& b(g, r) - b(\bar{g}, r) - hc(r) (\bar{g} - g) \leq \frac{\delta (1 - \delta^T)}{1 - \delta} (p - q) (v - v^b), \quad \text{(CC}\_F^g) \\
& v = (1 - \delta) [b(g, r) - hc(r) n g - k r] + \delta [(1 - q) v + q (\frac{1 - \delta T}{\delta T}) v^b + \delta T v].
\end{align*}
\]

The last equation measures \( v \), the continuation value if the penalty is not triggered. Also, note that \( p - q < 1 \) is the increased likelihood that the penalty is triggered if, at the emission stage, a country emits more rather than less. Clearly, the compliance constraint is harder to satisfy than in the basic model. First, both errors \((p < 1 \text{ and } q > 0)\) mean that the benefit of emitting less declines: Penalties may be triggered in any case (when \( q > 0 \)), or they may not be triggered even if a country emits more (if \( p < 1 \)). Both errors

\(^{19}\)See Fudenberg and Tirole (1991) for a definition of this equilibrium concept.

\(^{20}\)The equilibrium strategy is along the lines of Green and Porter (1984), who show that with imperfect monitoring firms can create collusive incentives by allowing price wars to break out with positive probability. With binary actions on emissions such a strategy also sustain the optimal equilibrium. See Abreu et al. (1986) for a characterization of optimal symmetric equilibria under imperfect monitoring.

\(^{21}\)See the proof of Proposition 4 for the derivation of \((\text{CC}\_F^g)\) and the value function.
also reduce the continuation value, \( v \), which the countries hope to receive in the next period. Finally, a shorter punishment period \( T < \infty \) means that the countries have less to fear from the penalty.

If condition (CC\(^g\)\(_F\)) holds, then the compliance constraint at the investment stage, \( v \geq v^b \), is, as before, satisfied.

Condition (CC\(^g\)\(_F\)) can be written as \( \delta \geq \hat{\delta}(r, T) \), where \( \hat{\delta}(r, T) \) is the discount factor satisfying (CC\(^g\)\(_F\)) with equality. While \( r = r^* \) and \( T = 0 \) would maximize the continuation value \( v \), the compliance constraint at the emission stage may then be violated. At \( r = r^* \), condition (CC\(^g\)\(_F\)) is weakened, and compliance is easier to achieve, for a larger investment \( r > r^* \) (so \( \hat{\delta}'(r, T) < 0 \)) if and only if (G) holds. This strategic role of technology is the same as above.

The desire to reduce the punishment period, however, results in a new strategic role for technology. Starting at \( T = \infty \), equilibrium utility increases when \( T \) is reduced. However, a reduction in \( T \) makes (CC\(^g\)\(_F\)) harder to satisfy (so \( \hat{\delta}'(r, T) < 0 \)). To allow for a reduction in \( T \), without violating the compliance constraint, it is necessary to invest even more in green technology or less in adaptation or brown technology. In other words, technology can be strategically chosen so as to allow for a reduction in the punishment length.

We can solve a binding (CC\(^g\)\(_F\)) for \( \delta^T \) and insert it in the expression for \( v \), which then becomes:

\[
v = b(g, r) - hc(r) ng - kr - \frac{q}{p - q} \left[ b(\tilde{g}, r) - b(g, r) - hc(r) (\tilde{g} - g) \right]. \tag{5}\]

Clearly, the optimal emission cutoff level \( \tilde{g} \) is simply given by:

\[
\tilde{g}^* = \arg \min_{\tilde{g}} \frac{q}{p - q} = \arg \min_{\tilde{g}} \frac{1 - F(\tilde{g} - ng)}{F(\tilde{g} - ng) - F(\tilde{g} - (n + 1) g)}.
\]

which implies that \( p \) and \( q \) are only functions of \( F, n, g \) and \( \tilde{g} \), making them independent of any other parameter in the model.\(^2\)

The equilibrium investment level \( \tilde{r} \) is equal to \( \arg \max \) of (5). When \( q > 0 \), this implies \( \tilde{r} > r^* \) under (G), and \( r^* < \tilde{r} \) under (NG).

When \( \delta \) declines from 1, investment stays at \( \tilde{r} \), which is independent of \( \delta \), while \( T \) must increase to satisfy a binding (CC\(^g\)\(_F\)). The constraint \( \delta = \hat{\delta}(r, T) \) is implicitly defining \( T \) as a decreasing function of \( \delta \), i.e., \( T(\delta) \). At some threshold, \( \delta = \hat{\delta}(\tilde{r}, \infty) \), the required \( T \) reaches infinity and, for even smaller discount factors, the compliance constraint cannot be satisfied unless \( r \) is even larger than \( \tilde{r} \) under (G), or even lower than \( \tilde{r} \) under (NG). Therefore, at \( T = \infty \), a binding constraint \( \delta = \hat{\delta}(r, \infty) \) is now implicitly defining \( r \) as a

\(^2\)If \( T = \infty \), the derivation of the optimal \( \tilde{g}, p, \) and \( q \) is a bit more complicated, as shown in the proof of Proposition 4 in the Appendix.
function of $\delta$, i.e., $r(\delta)$. All this is proved in the Appendix.

**Proposition 4.** A PPE exists in which $g = g$ if and only if $\delta \geq \bar{\delta}$. In this case, the Pareto optimal PPE is unique and it is characterized as follows:

(i) If $\delta \geq \bar{\delta}$, then $T = T(\delta)$ with $T'(\delta) < 0$, and investments are given by:

\[
\hat{r} > r^* \text{ if } (G);
\hat{r} < r^* \text{ if } (NG).
\]

(ii) If $\delta \in [\underline{\delta}, \bar{\delta})$, then $T = \infty$, and investments are given by:

\[
r(\delta) > \hat{r} > r^* \text{ with } r'(\delta) < 0 \text{ if } (G);
\]

\[
r(\delta) < \hat{r} < r^* \text{ with } r'(\delta) > 0 \text{ if } (NG).
\]

The effect of $\delta$ on $T$ and $r$ are illustrated in Figure 1 for the case of green technology.

![Figure 1: Even for large discount factors, countries over-invest when (G) holds. This allows for a shorter punishment phase without violating the compliance constraint.](image)

Corollary 4. With imperfect monitoring, one strategic role of technology is to reduce the duration of punishment that is necessary to motivate compliance.
4.2 Technology and the Probability of Cooperation

The strategic choice of technology can also increase the probability of continuing cooperation and reduce the frequency at which a punishment is triggered. With a stochastic compliance cost, the temptation to emit more depends on the realization of the shock as well as on the technology. With more investment in green technology, or less investment in adaptation and brown technology, the temptation to emit more decreases, as does the set of shock-realizations which lead to non-compliance.

To illustrate this simply, suppose the benefit function is now given by $\theta_i b(g_i, r_i)$, where the privately observed shock $\theta_i$ is distributed with mean $\bar{\theta}$ and with strictly positive density everywhere on the support $\Theta \equiv [\bar{\theta} - \sigma, \bar{\theta} + \sigma]$. We continue to assume that the emission stage constitutes a prisoner dilemma game for every $\theta_i$. Let $\theta_i$ be i.i.d. in every period, and let its realization be learned by $i$ after the investment stage but before the emission stage. As in the previous subsection, we focus on PPEs.

In equilibrium, there will be an endogenous threshold $\hat{\theta} \in \Theta$, such that a country complies if and only if $\theta_i \leq \hat{\theta}$. Each country is thus complying with probability $\pi \equiv \Pr(\theta_i \leq \hat{\theta})$. The emission-stage compliance constraints (one for each $\theta_i \leq \hat{\theta}$) become:

$$\theta_i \left( b(\bar{g}, r_i) - b(g_i, r_i) \right) - hc(r_i) (\bar{g} - g) \leq \frac{\delta}{1 - \delta} \pi - \pi - 1 \left( \mathbb{E}v_i - v^b_i \right),$$

where $\pi^{-1}$, which is the probability that every other country complies, is replacing the term $p-q$ in condition (CCP), discussed in the previous subsection. The first best can be sustained if (CCP) holds for $\theta_i = \bar{\theta} + \sigma$ when $\pi = 1$, $r_i = r^*$, and $T = \infty$. In this case, let (CCP) bind at discount factor $\tilde{\delta}$. It is easy to see that $\tilde{\delta} < 1$.

When the discount factor falls below $\tilde{\delta}$, the first best cannot be achieved, and the equilibrium outcome will necessarily be distorted. But while two distortions are possible, one has first-order effects: if the compliance constraint is not satisfied for the highest realizations of $\theta_i$, then the punishment will be triggered with a strict positive possibility (so, $\pi < 1$). Alternatively, one may require a larger $r_i$ under (G), or a smaller $r_i$ under (NG), and still ensure that (CCP) holds for every $\theta_i \in \Theta$. This distortion has a second-order effect on utilities, since the utility is continuously differentiable in $r_i$. For this simple reason, it is always optimal to distort $r_i$ when $\delta$ is falling (marginally) below $\tilde{\delta}$, rather than letting $\hat{\theta}$ and $\pi$ fall.

Proposition 5. Suppose $\theta_i$ is distributed with strictly positive density on $\Theta$.

(i) A threshold $\delta$ exists such that the best PPE is first best if $\delta \geq \delta$.

(ii) When $\delta$ falls below $\tilde{\delta}$, the best PPE requires $r_i > r^*$ under (G), and $r_i < r^*$ under (NG).

23If the shocks were publicly observed, it would be optimal with “escape clauses” such as those that exist in trade agreements (Bagwell and Staiger, 1990).
(iii) The larger the uncertainty $\sigma$, the larger $\delta$, and the larger the necessary distortion $|r_i - r^*|$. 

The last part of the proposition requires countries to invest even more in green technology, or even less in adaptation and brown technology, if the compliance cost is highly uncertain. The proposition follows straightforwardly from $(\text{CC}_{\theta}^g)$ and the explanation above. In the Appendix, we also show that when $\delta$ continues to fall below $\bar{\delta}$, satisfying $(\text{CC}_{\theta}^g)$ requires strategic investments that eventually have first-order effects on the utilities. It may then be optimal to give up on the compliance constraint for the highest realizations of $\theta_i$. It continues to be true, of course, that one strategic role of choosing $r_i$ different from its first-best level is to satisfy the compliance constraint for a larger set of shocks.

**Corollary 5.** With stochastic compliance costs, one strategic role for technology is to raise the probability for continuing cooperation.

### 4.3 Renegotiation-Proofness and Compliance Technology

So far, the goal of our analysis has been to describe the best SPE. The game has included neither any negotiation, nor an explanation for how or why the countries are able to negotiate or coordinate on the best SPE. If we introduced such negotiations, it may also be natural to allow the countries to renegotiate later on. While there is no need to renegotiate when all countries comply with an agreement, countries do have an incentive to renegotiate as soon as a defection is observed, and before triggering a costly and long-lasting punishment phase. Why, after a defection, should the countries play BAU forever when everybody would be better of by returning to the best SPE?

Our results are strengthened (or unchanged) if we introduce renegotiation.\(^{24}\) Allowing for renegotiation can only reduce the effective penalty if a country defects by emitting more; thus, to satisfy the compliance constraint, the benefit of emitting more must be reduced as well. The benefit of emitting is reduced by investments in green technology, or by lower investments in adaptation and brown technology. Consequently, if renegotiation is feasible, countries will invest even more in green technology, and less in adaptation and brown technology, in the best SPE.

The mechanism is particularly simple to understand if we continue to assume that monitoring is imperfect, as in Section 4.1. With imperfect monitoring, it is impossible to determine which country defected, and the punishment must be collective. In the best SPE, BAU will be played in $T$ periods as soon as the observed emission stock is larger than some negotiated threshold. No matter how long the punishment period,

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\(^{24}\)The concept of a renegotiation-proof equilibrium used here is due to Farrell and Maskin (1989). An equilibrium is (weakly) renegotiation-proof if none of its continuation equilibria Pareto-dominate each other.
every country would benefit from starting again and pretending that the large emissions had not been observed. Thus, with imperfect monitoring, no punishment is possible and one cannot motivate less emissions under our assumption that the emission game is a prisoner dilemma. Instead, the only way of sustaining an equilibrium with less emission is to make this choice a dominant strategy. Let $r^D$ be defined as the $r$ closest to $r^*$ (i.e., $r^D = \arg \min_r |r - r^*|$) satisfying $b(g, r) - hc(r) g > b(\overline{g}, r) - hc(r) \overline{g}$. If such an $r^D$ exists, we clearly have $r^D > r^*$ under (G), and $r^D < r^*$ under (NG).

Another especially simple case arises under complete information if a defecting country has no bargaining power in the renegotiation game. Suppose that if a country $i \in N$ emits more, everyone will play BAU forever unless the countries renegotiate. If the coalition $N \setminus i$ of $n - 1$ countries has all the bargaining power in this renegotiation game, the coalition will ensure that country $i$ does not receive more than the BAU continuation value. If side payments are possible, this can be achieved by requesting the deviator to pay the other countries before cooperation is restored. If side payments are unavailable, the coalition $N \setminus i$ may request country $i$ to invest a particular amount at the next investment stage before cooperation continues.\textsuperscript{25} When free-riding one period leads to a continuation value identical to BAU for the deviator, the compliance constraint remains identical to the case without renegotiation. In this special case, allowing for renegotiation does not restrict the set of SPEs.

Alternatively, in the complete information case, the deviator can exercise some bargaining power $\alpha \in (0, 1)$, assumed to be constant over time, in the renegotiation game. This implies that if $i \in N$ deviates, it will receive more than its BAU continuation value beginning from the next period. The larger the bargaining power of $i$, the larger the continuation value $i$ will receive if $i$ defects by emitting more. This means that the compliance constraint is harder to satisfy in the case with renegotiation than in the case without renegotiation and, to satisfy it, $|r_i - r^*|$ must increase. The larger the bargaining power of a defecting country $i$ in the renegotiation game, the larger $r_i > r^*$ under (G) or the smaller $r_i < r^*$ under (NG), if $(g, r_i)$ is to be sustained as an SPE.

**Proposition 6.** Suppose that after a country deviates, the countries can renegotiate before triggering the penalty.

(i) If monitoring is imperfect, permitting renegotiation implies that $(g, r)$ can be sustained as an SPE only if less investments in adaptation or brown technology, and more in clean

\textsuperscript{25}That is, the coalition $N \setminus i$ may propose a take-it-or-leave-it offer to $i$ involving a large $r_i$ under (G), or a small $r_i$ under (NG), where the choice of $r_i$ is so costly for $i$ that $i$ is just receiving the outside option continuation value (which is BAU). For $N \setminus i$, this is one (out of several) optimal renegotiation offers when $r_i$ is selfish; with technological spillovers (discussed in the next subsection), requesting this particularly large level $r_i$ is strictly beneficial to $N \setminus i$ (under (G)).
technology are made:

\[ r = r^D > r^* \text{ under (G)}; \]
\[ r = r^D < r^* \text{ under (NG)}. \]

(ii) Under complete information, if a deviator has no bargaining power at the renegotiation stage (i.e., \(\alpha = 0\)), then permitting renegotiation does not alter the Pareto optimal SPE.

(iii) Under complete information, if a deviator has bargaining power at the renegotiation stage (i.e., \(\alpha \in (0, 1)\)), then \((g, r(\alpha))\) is the Pareto optimal renegotiation-proof SPE and it is characterized as follows:

(iii.1) If \(\delta \geq \bar{\delta}(\alpha)\), then \(r(\alpha) = r^*\) is first best;

(iii.2) If \(\alpha\) increases from zero, then \(r\) must increase under (G) but decrease under (NG): At \(\alpha \approx 0\), we have that for \(\delta \in (\bar{\delta}, \bar{\delta}(\alpha))\),

\[ r'(\alpha) > 0 \text{ under (G)}; \]
\[ r'(\alpha) < 0 \text{ under (NG)}. \]

Point (iii) of Proposition 6 is demonstrated in the Appendix.

**Corollary 6.** If renegotiation is possible, the strategic role of compliance technology is strengthened.

### 4.4 Technological Spillovers

One of the important results in this paper is that, even without technological spillovers, a tacit agreement on technology can be beneficial in a repeated game. Allowing for spillovers naturally strengthens the case for including technological investments in the agreement. But how does the need to internalize technological spillovers interact with the need to motivate less emissions, and what determines the optimal level of investments?

To address these questions, let \(e \in (0, 1)\) be the fraction of a country’s investment that benefits the others instead of the investor. Country \(i\)’s utility is:

\[ u_i = b(g_i, z_i(r_i, r_{-i})) - hc(z_i(r_i, r_{-i})) \sum_{j \in N} g_j - kr_i, \text{ where} \]
\[ z_i(r_i, r_{-i}) \equiv (1 - e) r_i + \frac{e}{n - 1} \sum_{j \neq i} r_j. \]

The first-best investment level \(r^*\) is invariant in \(e\). In BAU, in contrast, countries invest less the larger is \(e\). Thus, it is no longer true that countries will invest the optimal amount conditionally on \(g_i\). Since first-best investments are larger than non-cooperative
investments (conditional on \( g_i \)), countries may be tempted to deviate even at the investment stage. A country that deviates at the investment stage will not only enjoy its BAU continuation value, but it may also benefit if the other countries invest more than they would in BAU. The compliance constraint at the investment stage is then:

\[
\frac{v}{1 - \delta} \geq \frac{e}{1 - e} k (r - r^b) + \frac{v^b}{1 - \delta}.
\]

Condition \( (CC^*_e) \) is trivially satisfied if \( e = 0 \), but also if \( r < r^b \). As one may expect from the basic model (where \( e = 0 \)), we do have \( r^* < r^b \) under condition (NG) if \( e \) is below some threshold \( \tilde{e} \). If \( e > \tilde{e} \) or if condition (G) holds, however, it is optimal to implement \( r^* > r^b \). In this latter case, \( (CC^*_e) \) can be rewritten as:

\[
\delta \geq \frac{1}{1 - e} \frac{v - v^b}{r - r^b} k < 1.
\]

The compliance constraint at the emission stage is as in the basic model and therefore not reported on here, but we denote the threshold \( (4) \) by \( \hat{\delta}^g (r) \) to distinguish it from \( \hat{\delta}^r (r) \).

The first best \( (r^*, g) \) can be supported as an SPE if and only if both \( \delta \geq \delta^r \equiv \hat{\delta}^r (r^*) \) and \( \delta \geq \delta^g \equiv \hat{\delta}^g (r^*) \). The Appendix shows that with small spillovers (\( e < \bar{e} \) for some threshold \( \bar{e} > 0 \)) the emission-stage compliance constraint binds first (so, \( \delta^g > \delta^r \)). To satisfy it, one must require \( r > r^* \) for technologies satisfying (G) and \( r < r^* \) if the technology is characterized by (NG). However, if spillovers are large (\( e \geq \bar{e} \)), the investment-stage constraint binds first (so, \( \delta^g < \delta^r \)). This requires investments to be smaller than \( r^* \) when \( \delta < \delta^r \).

We can write the investment-stage compliance constraint as \( r \leq r^* (\delta) \equiv \hat{\delta}^{-1} (\delta) \), and the emission-stage compliance constraint as \( r \geq r^g (\delta) \equiv \hat{\delta}^{-1} (\delta) \) under (G), or as \( r < r^g (\delta) \) under (NG). The following figure shows how different levels of technological spillovers affect strategic investments in the case of green technology.

If \( \delta \geq \max \left\{ \delta^r, \delta^g \right\} \), then \( r^* \in [r^g (\delta), r^r (\delta)] \). However, with green technologies the minimum level \( r^g (\delta) \) increases as \( \delta \) decreases, while the maximal level \( r^r (\delta) \) declines with \( \delta \). So, for lower discount factors \( \delta < \max \left\{ \delta^r, \delta^g \right\} \), \( r \) must increase if \( e < \bar{e} \), as in the basic model, but decrease if \( e > \bar{e} \). For sufficiently small discount factors, \( \hat{\delta} \), the interval \( [r^g (\delta), r^r (\delta)] \) is empty. It is then impossible to sustain low emissions as an SPE.

For non-green technology satisfying (NG), both compliance constraints define upper boundaries for \( r \), namely \( r \leq r^r (\delta) \) and \( r \leq r^g (\delta) \). Regardless of which constraint is binding, \( r \) must fall whenever \( \delta \) falls. For a sufficiently small discount factor, \( \hat{\delta} \), no \( r \) can satisfy both compliance constraints and at the same time satisfy \( v \geq v^b \).

**Proposition 7.** An SPE exists in which \( g_i = g \ \forall i \in N \) if and only if \( \delta \geq \hat{\delta} \). In this

\[26\] The threshold level \( \tilde{e} \) is derived in the Appendix.
Figure 2: With small spillovers (left panel), the emission-stage compliance constraint (dashed line) will bind first and over-investments may be necessary. With large spillovers (right panel), the investment-stage compliance constraint (dotted line) becomes tougher to satisfy, and investments may be suboptimally small.

In this case, the Pareto optimal SPE is unique and it is characterized as follows:

(i) If $\delta \geq \max\{\delta^r, \delta^g\}$, then $r = r^*$, and the SPE is first best.

(ii) If $\delta \leq \max\{\delta^r, \delta^g\}$, then:

$$r = r^g(\delta) > r^* \text{ if } e \leq \bar{e} \text{ under } (G),$$

$$r = r^r(\delta) < r^* \text{ if } e > \bar{e} \text{ under } (G),$$

$$r = \min\{r^g(\delta), r^r(\delta)\} < r^* \text{ under } (NG).$$

Note that the large spillover is always harmful since it imposes a constraint on the investment levels that can be sustained as SPEs. Specifically, requiring a high level of investment in clean technology to motivate compliance at the emission stage may not be possible if the spillover is large. Thus, with a policy that reduces the spillover, for example by strengthening intellectual property rights, the countries can require more investments in green technology without fearing that the investment-level compliance constraint will be violated.

**Corollary 7.** Stronger intellectual property rights may be necessary to sustain a self-enforcing treaty.

### 4.5 Policy Instruments and Continuous Emission Levels

In this section, we study the optimal use of policy instruments, and we permit the emission level to be a continuous variable. It is natural to make these two extensions at the same
time, since we cannot pin down a unique emission tax if the emission level continues to be a binary variable. (For example, any sufficiently large emission tax ensures that $g_0$ is preferred to $g > g_0$.)

We assume that country $i$’s investment subsidy, $\varsigma_i$, is set by $i$ just before the investment stage in each period, and it is observable by all countries. The actual investment is made by private investors who receive the subsidy $\varsigma_i$ in addition to the price paid by the consumers. The emission tax, $\tau_i$, is set just before the emission stage, and it represents the cost of polluting paid by the consumers. If the taxes are collected and the subsidies are paid by the national governments, they do not represent actual costs or revenues—from the government’s perspective—and their only effect is to influence the decisions $g_i$ and $r_i$. The agreement between the countries then amounts to setting domestic taxes/subsidies such that the desired SPE is implemented.

Allowing for a continuous $g_i$ complicates the analysis. To proceed, we restrict attention to the case in which $g_i$ and $r_i$ are perfect substitutes in a linear-quadratic utility function:

$$u_i = -\frac{B}{2} (\bar{y} - (g_i + r_i))^2 - \frac{K}{2} r_i^2 - c \sum_{j \in N} g_j,$$

where $B$ and $K$ are positive constants. Here, $\bar{y}$ is a country’s bliss level for consumption, and consumption is the sum of $g_i$ (energy from fossil fuels) and $r_i$ (energy from renewable energy sources). Since $\partial^2 u_i / \partial g_i \partial r_i < 0$, we explicitly consider only clean technology. We can easily reformulate the utility function such that the investment cost becomes linear, although there is no need to do so here.

Since the emission tax is the only cost of consuming fossil fuel, $g_i$ is chosen by the consumers to satisfy the first-order condition:

$$B (\bar{y} - (g_i + r_i)) = \tau_i.$$

The left-hand side is also equal to the consumer’s willingness to pay for green technology, so private investors invest according to the first-order condition:

$$K r_i = B (\bar{y} - (g_i + r_i)) + \varsigma_i = \tau_i + \varsigma_i.$$

(6)

Note that the first-best outcome is

$$r^* = \frac{cn}{K} \quad \text{and} \quad g^* = \bar{y} - \frac{cn}{B} - r^*.$$

27This utility function is also considered in Battaglini and Harstad (2014), who do not study SPEs, but instead the Markov-perfect equilibria when countries can commit to the emission levels. The first best and the BAU equilibrium are as in that paper, of course.

28To see this, simply define $\tilde{r}_i = r_i^2 / 2$ and rewrite to $u_i = -\frac{B}{2} (\bar{y} - (g_i + \sqrt{2\tilde{r}_i}))^2 - K \tilde{r}_i - c \sum_{j \in N} g_j$. 

24
which coincides with the equilibrium when the tax and the subsidy are equal to their first-best values:

\[ \varsigma^* = 0 \quad \text{and} \quad \tau^* = cn. \]

In the first best, the emission tax is set at the Pigouvian level and there is no need to additionally regulate investments, since the investors capture the entire surplus associated with their technology investments.

The BAU equilibrium is (the unique SPE in the one-period game):

\[ r^b = \frac{cK}{cK} \quad \text{and} \quad g^b = \bar{y} - \frac{c}{B} - r^b, \]

which is equivalent to

\[ \varsigma^b = 0 \quad \text{and} \quad \tau^b = c. \]

Thus, the investment subsidy is zero in the first best as well as in BAU.

To follow the same line of reasoning as in the rest of the paper, we here only consider SPEs enforced by the threat of reverting to BAU, despite the fact that BAU is not the harshest penalty when \( g < g^b \) is possible. Furthermore, we consider only symmetric SPEs, despite the fact that there can also be asymmetric SPEs that are Pareto optimal.

Naturally, the first best can be achieved when the discount factor is sufficiently large. When \( \delta \) falls, however, each country finds it tempting to introduce a smaller emission tax than the first-best one. Once \( \delta \) falls to some threshold, \( \delta^* \), the emission-stage compliance constraint starts to bind. For smaller discount factors, the emission tax must be allowed to fall to satisfy the compliance constraint. The associated increase in emissions can be mitigated by introducing an investment subsidy.

Note that the investment-stage compliance constraints will never bind first. As soon as one country deviates by setting a different investment subsidy, investors in all countries anticipate that cooperation will break down and demand for their technology at the emission stage will be reduced. This lowers investments everywhere, not only in the deviating country. Deviating at the investment stage immediately gives the deviator the BAU payoff, plus the benefit of the other countries’ larger investments induced by their subsidies. These subsidies are zero for \( \delta \geq \delta^* \) and are small for discount factors close to \( \delta^* \). Consequently, some \( \delta < \delta^* \) exists such that the compliance constraint at the investment stage is not binding when \( \delta \in (\delta^*, 1) \). (The proof in the Appendix derives both thresholds.)

**Proposition 8.** Consider the symmetric Pareto optimal SPE sustained by the threat of reverting to BAU if a country deviates.

(i) If \( \delta \geq \delta^* \), the equilibrium is first best: \( \tau = cn \) and \( \varsigma = 0 \).
(ii) If $\delta \in [\underline{\delta}, \overline{\delta})$, the equilibrium is:

\[ \tau = cn - \phi(\delta) \quad \text{and} \quad \zeta = \phi(\delta), \quad \text{where} \]
\[ \phi(\delta) \equiv c(n-1) \left( 1 - \delta - \sqrt{\delta^2 + \delta B/K} \right) \geq 0. \]

Figure 3: When the discount factor falls and free-riding becomes more tempting, the emission tax must be allowed to fall, but the investment subsidy must increase.

The function $\phi(\delta)$ decreases toward zero when $\delta$ increases to $\overline{\delta}$. Thus, as illustrated in Figure 3, when the discount factor is smaller, the equilibrium tax is also smaller. The investment subsidy, however, is accordingly larger.\textsuperscript{29}

**Corollary 8.** The sum of the equilibrium emission tax and the investment subsidy is, for every $\delta \geq \underline{\delta}$, equal to $nc$, the first-best Pigouvian tax level.

\textsuperscript{29}The proposition implies that the equilibrium investment level, $r_i$, given by (6), stays unchanged as the discount factor falls. On the one hand, the fact that a larger $g$ must be tolerated implies that it becomes optimal to invest less in clean technology. On the other hand, the countries can dampen the increase in $g$ by requesting countries to invest more in green technology upfront. These two effects cancel each other out when $g$ and $r$ are perfect substitutes. Relative to the ex post optimal level, however, it is clear that $r - r^*(g)$ is positive and increases as $\delta$ falls, just as the equilibrium investment subsidy. The optimal investment level, conditional on the emission level $g_i$, is decreasing in $g_i$ and given by:

\[ r^*(g_i) = \frac{B(\bar{y} - (g_i + r_i))}{K} = \frac{B(\bar{y} - g_i)}{B + K}. \]
5 Conclusions

To confront global climate change, an environmental treaty must address two primary challenges. The treaty must be self-enforcing and must lead to the development of green technology. This paper analyzes these challenges in a joint framework and uncovers interesting interactions between them. Specifically, we demonstrate that when free-riding is tempting and cooperation difficult to sustain, the best self-enforcing treaty requires countries to over-invest in “green” technology, reducing the temptation to pollute, or under-invest in adaptation or “brown” technology that would have made free-riding more attractive. When countries are heterogeneous, it is particularly countries that are small or reluctant to cooperate (because their environmental harm is relatively small, for example) that are most tempted to pollute. To ensure that compliance by these countries is credible, small or reluctant countries must invest the most in green technology, or the least in adaptation and brown technology.

In a time when the world struggles to develop and reach an agreement on a global climate change treaty, it is natural that the motivation for our analysis has mainly been normative. We believe that the international community has not yet implemented the best possible self-enforcing treaty, and thus do not expect that our predictions are directly observable or consistent with the facts of today. That said, our theory is testable: we do provide a number of predictions that may eventually be compared to the data. Our assumptions are also in line with the facts: Policymakers do have few sanctions available (meaning a treaty must be self-enforcing) and they do consider the development of technology to be of great importance. Furthermore, some of the countries that have invested the most in green technology (notably the European Union) are also the ones that have complied with the Kyoto Protocol to the largest extent.30 Other countries that have instead invested in brown technology (notably Canada and Australia) ended up not complying or increased emissions. Future research should empirically clarify the interaction between technology, emissions, and compliance to test the model’s predictions.

Theoretical research should also continue. Our simple workhorse model has proven to be sufficiently tractable to be extended in many ways, but our approach is still only a first cut. We have simplified tremendously by abstracting away from payoff-relevant stocks of pollution or technologies. We have focused exclusively on the Pareto optimal subgame-perfect equilibrium, although the actual transition toward such a treaty appears to be characterized by high transaction costs and multiple wars of attritions. By focusing on the best subgame-perfect equilibrium, we have also abstracted from the possibility of opting out of the negotiations at the beginning of the game. When countries are heterogeneous, it may actually be optimal, even for the countries that cooperate, to exclude certain

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reluctant countries, since these countries may, with some probability, cheat and thus trigger a costly and long-lasting punishment phase. One of the goals with this project has been to provide a tractable workhorse model that can be developed along these lines in future research.
6 Appendix

Proofs of Propositions 0-1.

These proofs are in the text.

The proofs of the basic results, Propositions 2-3, allow for time-varying parameters.

Proof of Proposition 2.

Since investments are selfish, the Pareto optimal SPE satisfying \( g_i = g \) is for each \( i \in N \) specifying the \( r_i^t \) closest to \( r_i^{*,t} \) satisfying the compliance constraints. That is, the Pareto optimal SPE solves:

\[
\max_{r_i^t} u^t_i = b^t_i (r_i^t, g) - n^t c_i^t (r_i^t) g - k_i^t r_i^t \quad \text{s.t.} \quad v^t_i - v^{b,t}_i \geq 0, \quad \Delta^t_i \equiv -b^t_i (g, r_i^t) + b^t_i (g, r_i^t) + s_i^t h_i^t c (r_i^t) (g - g) + \frac{\delta (v^{t+1}_i - v^{b,t+1}_i)}{1 - \delta} \geq 0. \quad (CC^r_i)
\]

(i) Since \( v^t_i > v^{b,t}_i \) at \( r_i^{*,t} \), both conditions hold if \( \delta \) is close to 1. A binding \( (CC^g_i) \) defines \( \hat{\delta}^t_i (r_i^t) \) implicitly, and if \( \delta \geq \hat{\delta}^t_i (r_i^{*,t}) \), then \( r_i^{*,t} \) satisfies both compliance constraints.

(ii) As soon as \( \delta \) declines below the level \( \hat{\delta}^t_i \), \( (CC^g_i) \) binds (before \( (CC^r_i) \) does). The problem above is then a Kuhn-Tucker maximization problem which can be written as:

\[
\max_{r_i^t} u^t_i + \lambda^t_i \left( -b^t_i (g, r_i^t) + b^t_i (g, r_i^t) + s_i^t h_i^t c (r_i^t) (g - g) + \frac{\delta (v^{t+1}_i - v^{b,t+1}_i)}{1 - \delta} \right),
\]

where \( \lambda^t_i > 0 \) is the shadow value of satisfying a strictly binding \( (CC^g_i) \). The first-order condition for an interior \( r_i^t \) is:

\[
0 = \frac{\partial u^t_i}{\partial r_i^t} + \lambda^t_i \frac{\partial}{\partial r_i^t} \left[ -b^t_i (g, r_i^t) + b^t_i (g, r_i^t) + s_i^t h_i^t c (r_i^t) (g - g) \right], \quad (7)
\]

while the second-order condition is satisfied for \( u^t_i \) sufficiently concave. Since \( r_i^{*,t} \) is defined by \( \partial u^t_i / \partial r_i^t = 0 \), \( r_i^t \) must increase above \( r_i^{*,t} \) to satisfy this first-order condition if the second term in (7) is positive, i.e., if:

\[
\frac{\partial}{\partial r_i^t} \frac{b^t_i (g, r_i^t) - b^t_i (g, r_i^t)}{g - g} < s_i^t h_i^t c' (r_i^t), \quad (G^t_i)
\]
while \( r_i^t \) must decrease below \( r_i^*,t \) if

\[
\frac{\partial}{\partial r_i^t} \left( b_i^t(\bar{g}, r_i^t) - b_i^t(g, r_i^t) \right) > s_i^t h_i^t c'(r_i^t),
\]

(NG\(_i^t\))

As \( \delta \) declines further, (CC\(_i^{g,t}\)) can only be satisfied if \(|r_i^t - r_i^*,t|\) increases more. Eventually, \( \delta \) becomes so small than either (i) \(|r_i^t - r_i^*,t|\) becomes so large that (CC\(_i^{g,t}\)) is violated, (ii) \( u_i^t \) and thus \( v_i^t \) is reduced so much that (CC\(_i^{g,t'}\)) is violated at some earlier date \( t' < t \), or (iii) \( \delta \) reaches zero. We let \( \delta_i^t \) measure the maximum of these three thresholds. Clearly, \( \delta_i^t \in [0, \delta_i^\infty) \). QED

**Proof of Proposition 3.**

If \( \delta < \delta_i^t \), the Pareto optimal SPE satisfying \( g_i^t = \bar{g} \) ensures that \( r_i^t \) solves \( \hat{\delta}_i^t(r_i^t) = \delta \), so that (CC\(_i^{g,t}\)) binds. As long as (CC\(_i^{g,t}\)) binds, we can simply differentiate the left-hand side of (CC\(_i^{g,t}\)) to learn how \( r_i^t \) must change with the other parameters at time \( t \):

\[
\begin{align*}
\frac{\partial r_i^t}{\partial h_i^t} &= -\frac{\Delta_i^{t,h}}{\Delta_i^{t,r}} = -\frac{s_i^t c(r_i^t)(\bar{g} - g)}{\Delta_i^{t,r}}, \\
\frac{\partial r_i^t}{\partial s_i^t} &= -\frac{\Delta_i^{t,s}}{\Delta_i^{t,r}} = -\frac{h_i^t c(r_i^t)(\bar{g} - g)}{\Delta_i^{t,r}}, \text{ where} \\
\Delta_i^{t,r} &= \partial \left[ -b_i^t(\bar{g}, r_i^t) + b_i^t(g, r_i^t) \right] / \partial r_i^t + s_i^t h_i^t (\bar{g} - g) c'(r_i^t).
\end{align*}
\]

(8)

Thus, \( \partial r_i^t / \partial h_i^t \) and \( \partial r_i^t / \partial s_i^t \) are both negative if (G\(_i^t\)) holds (then, \( \Delta_i^{t,r} > 0 \)), and otherwise they are positive.

If we could write \( b_i^t(\cdot) = \theta_i^t b(\cdot) \), where the importance of consumption, \( \theta_i^t \), could be particularly high at times of recessions, then we could show that countries ought to invest more at such times in clean technologies, and less in adaptation or brown technologies, for compliance to be credible:

\[
\frac{\partial r_i^t}{\partial \theta_i^t} = -\frac{b(\bar{g}, r_i^t) - b(g, r_i^t)}{\Delta_i^{t,r}}.
\]

If we differentiated \( \Delta_i^t \) with respect to \( k_i^t \) or \( n_i^t \) and \( r_i^t \), we would clearly get \( \partial r_i^t / \partial k_i^t = \partial r_i^t / \partial n_i^t = 0 \). The explanation for \( \partial r_i^t / \partial k_i^t = 0 \), for example, is that \( r_i^t \) must be set to satisfy (CC\(_i^{g,t}\)), and the investment cost at time is sunk and thus irrelevant at the emission stage in period \( t \). Of course, a larger \( k_i^t \) changes \( v_i^t \) and \( v_i^{h,t'} \) with \( t' \leq t \), and therefore the compliance constraints at the earlier periods. Similarly, changes in \( h_i^t \) or \( s_i^t \) will influence earlier investments as well as \( r_i^t \). To illustrate the total effects without unnecessary notations we henceforth assume that all parameters are time-invariant, as in the model and the text above, and we thus skip the subscripts \( t \).
Stationary parameters: We must now take into account the effects on the continuation value, and we write:

\[
\Delta_{i,r} = \frac{\partial}{\partial r_i} \left[ -b_i (g, r_i) + b_i (g, r_i) \right] + s_i h_i (\bar{g} - g) c' (r_i) + \frac{\delta}{1 - \delta} \frac{\partial u_i}{\partial r_i}.
\]

Under (G), \( \Delta_{i,r} > 0 \) at \( r_i^* \) and \( \Delta_{i,r} \) remains positive as long as a larger \( r_i \) weakens (CC\(_g^i\)) (that is, for \( \delta > \bar{\delta}_i \)). Similarly, under (NG), \( \Delta_{i,r} < 0 \) at \( r_i^* \) and \( \Delta_{i,r} \) remains negative as long as a smaller \( r_i \) weakens (CC\(_g^i\)) (that is, for \( \delta > \bar{\delta}_i \)).

(i) Effect of \( k_i \): The compliance constraint depends on \( k_i \) because:

\[
v_i - v_i^b = - \left[ b_i (g, r_i^b) - b_i (g, r_i) \right] + nh_i \left[ c (r_i^b) \bar{g} - c (r_i) g \right] - k_i \left[ r_i - r_i^b \right].
\]

Suppose, as a start, that \( r_i \) does not change in \( k_i \). Then, \( \partial u_i / \partial k_i = -r_i \). Furthermore, from the Envelope theorem, \( \partial u_i^b / \partial k_i = -r_i^b \). Thus:

\[
\frac{dr_i}{dk_i} = - \frac{\Delta_{i,k}}{\Delta_{i,r}} = \frac{\delta}{1 - \delta} \frac{r_i - r_i^b}{\Delta_{i,r}}.
\] (9)

To see the sign of \( r_i - r_i^b \), assume, for a moment, that \( r_i \) remains at \( r_i^* \). If so, all investment levels are given by the first-order condition:

\[
\frac{\partial b_i (g, r_i)}{\partial r_i} - h_i c' (r_i) n g = k_i,
\]

which we can differentiate to get

\[
\frac{dr_i}{dg} = \frac{-\frac{\partial^2 b_i (g, r_i)}{\partial r_i \partial g} + h_i c' (r_i) n}{\frac{\partial^2 b_i (g, r_i)}{\partial (r_i)^2} - h_i c'' (r_i) g n},
\] (10)

where the denominator is simply the second-order condition with respect to \( r_i \), which must be negative. With this, we have:

\[
r_i^* - r_i^b = \int_0^g \left( \frac{\partial^2 b_i (g, r_i)}{\partial r_i \partial g} + h_i c' (r_i) n - h_i c'' (r_i) g n \right) dg.
\] (11)

Thus, for adaptation and brown technologies, \( r_i^* < r_i^b \). For such technologies we also have \( r_i \leq r_i^* < r_i^b \) and \( \Delta_{i,r} < 0 \), so from (9) we have that \( dr_i / dk_i > 0 \). For clean technologies, Eq. (11) gives \( r_i^* > r_i^b \). For clean technologies, we also have \( r_i \geq r_i^* > r_i^b \) and \( \Delta_i > 0 \), so from (9) we again have that \( dr_i / dk_i > 0 \).

(ii) Effect of \( s_i \) and \( \delta \): The effect of \( s_i \) on \( r_i \) is exactly as in the case with time-dependent variables (8). The effect of \( \delta \) is trivial. Note that \( r_i^* \) does not depend on \( s_i \) or \( \delta \).
(iii) Effect of \( n \): Consider first the case where \( c'(r_i) = 0 \) (brown or clean technologies). In this case, \( \partial (v_i - v_i^b) / \partial n = (\bar{g} - g) h_i c > 0 \), so

\[
\frac{d r_i}{d n} = - \frac{\Delta_i,n}{\Delta_i,r} = - \frac{\delta}{1 - \delta} \frac{(\bar{g} - g) h_i c}{\Delta_i,r},
\]

which has the opposite sign of \( \Delta_i,r \). Note that \( r_i^* \) does not depend on \( n \).

Consider next adaptation technologies, where \( c'(\cdot) < 0 \) but \( \partial b_i (\cdot) / \partial r_i = 0 \). If \( r_i \) were invariant in \( n \), we would have \( \partial (v_i - v_i^b) / \partial n = h_i \left[ c (r_i^b) \bar{g} - c (r_i) g \right] \), but \( r_i \) and \( r_i^b \) may differ since \( r_i^* (g) \) depends on \( g \). If \( c(r_i^*(g)) \) increased in \( g \), then we would have \( c (r_i^* (g)) \bar{g} > c (r_i^* g) \). If we use Eq. (10), we can show that \( c (r_i^*(g)) \) increases in \( g \) if:

\[
c (r_i^*(g)) - \left[ c' (r_i^*(g)) \right]^2 / \left[ c'' (r_i^*(g)) \right] > 0. \tag{12}
\]

So, under this condition, \( \partial (v_i - v_i^b) / \partial n \) and \( \Delta_i,n \) would be positive for \( r_i \) close to \( r_i^* \) and, then, \( dr_i / dn > 0 \).

Effect of \( h_i \): This effect is derived in a similar way. For adaptation technologies, if \( r_i \) were invariant in \( h_i \), we would have \( \partial (v_i - v_i^b) / \partial h_i = n \left[ c (r_i^b) \bar{g} - c (r_i) g \right] \), where the bracket is, as before, positive under condition (12). Then, \( \Delta_i,h > 0 \) and, therefore, \( dr_i / dh_i > 0 \) for adaptation technologies. For brown technologies we have \( \Delta_i,h > 0 \) and \( dr_i / dh_i > 0 \) while for clean technologies we have \( \Delta_i,h < 0 \) and \( dr_i / dh_i < 0 \), even though \( r_i^* \) is independent of \( h_i \). QED

Proof of Proposition 4.

Let first determine the compliance constraint at the emission stage. In the cooperation phase, the country’s intertemporal value is

\[
v = (1 - \delta) \left[ b (\bar{g}, r) - h c (r) n g - k r \right] + \delta \left[ (1 - q) v + q \left( (1 - \delta^T) v^b + \delta^T v \right) \right].
\]

Each country has incentive to cooperate at the emission stage if and only if the following constraint is satisfied:

\[
v \geq (1 - \delta) \left[ b (\bar{g}, r) - h c (r) \left( (n - 1) g + \bar{g} \right) - k r \right] + \delta \left[ (1 - p) v + p \left( (1 - \delta^T) v^b + \delta^T v \right) \right],
\]

which implies condition (CC\(_F\)) in the text.

(i) Inserting \( \delta^T \) from (CC\(_F\)) into \( v \) gives the indirect value (5). Let \( \bar{r} \) be the solution of the first-order condition when maximizing (5) with respect to \( r \):

\[
\frac{\partial b (\bar{g}, \bar{r})}{\partial r} - h c' (\bar{r}) n g - k - \frac{q}{p - q} \frac{\partial}{\partial r} \left[ b (\bar{g}, \bar{r}) - b (g, \bar{r}) \right] - h c' (\bar{r}) (\bar{g} - g) = 0.
\]

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The second-order condition is clearly satisfied. By inspecting the first-order condition, it is easy to see that \( \hat{r} < r^* \) under condition (NG) and \( r > r^* \) under condition (G) insofar as \( q > 0 \). Note that \( \hat{r} \) does not depend on \( \delta \). Given \( r = \hat{r} \), a binding (CC\( _F^g \)) implicitly defines \( T(\delta) \). Differentiating \( T(\delta) \) with respect to \( \delta \) gives \( T'(\delta) < 0 \). The minimum level of \( \delta \) which satisfies (CC\( _F^g \)) is \( \delta = \lim_{T \to \infty} \hat{\delta}(\hat{r}, T) \). For any \( \delta \geq \delta \), the Pareto optimal PPE that sustains low emissions is then characterized by \( r = \hat{r} \) and \( T = T(\delta) \).

(ii) For \( \delta < \delta \), \( T = \infty \) and (CC\( _F^g \)) can be written as
\[
\Delta \equiv b(\bar{g}, r) - hc(r)n_g - kr - v^b - \frac{1 - \delta (1 - q)}{\delta (p - q)} (b(\bar{g}, r) - b(g, r) - hc(r)(\bar{g} - g)) \geq 0.
\]
In this case, the equilibrium \( r \) is implicitly defined by \( \Delta = 0 \). By differentiating this equation, we get:
\[
\frac{dr}{d\delta} = \frac{-\Delta g}{\Delta r} = -\frac{b(\bar{g}, r) - b(g, r) - hc(r)(\bar{g} - g)}{\Delta r}
\]
and
\[
\frac{dr}{dg} = \frac{-\Delta g}{\Delta r} = -\frac{\partial}{\partial g} \left[ \frac{1 - \delta F(\bar{g} - n_g)}{\delta (F(\bar{g} - n_g) - F(\bar{g} - n_g - (\bar{g} - g)))} \right],
\]
where, by the monotone likelihood ratio property \( p - q \) is nondecreasing in \( \hat{g} \), which implies that \( \Delta g \) is positive. For \( r \) close to \( \hat{r} \), \( \Delta_r \) is positive under (G) and negative under (NG). Therefore, both \( dr/d\delta \) and \( dr/d\hat{g} \) have the opposite sign of \( \Delta_r \). As before, the value function \( v \) under a binding constraint (CC\( _F^g \)) yields (5). The optimal \( \hat{g} \) is now the solution of the first-order condition:
\[
\frac{\partial}{\partial \hat{g}} \left[ \frac{1 - F(\bar{g} - n_g)}{F(\bar{g} - n_g) - F(\bar{g} - n_g - (\bar{g} - g))} \right] = \frac{1}{b(\bar{g}, r) - b(g, r) - hc(r)(\bar{g} - g)} \left[ \frac{\partial}{\partial \hat{g}} \left( \frac{\partial}{\partial \hat{g}} b(g, r) - hc'(r) n_q - k \right) \right] \frac{dr}{d\hat{g}}.
\]
Under (G), \( dr/d\hat{g} < 0 \) and \( r > \hat{r} \) for \( \delta < \delta \). Whereas, under (NG), \( dr/d\hat{g} > 0 \) and \( r < \hat{r} \) for \( \delta < \delta \). This implies that the right-hand side of (13) is always positive. Therefore, as \( \delta < \delta \) declines further, (CC\( _F^g \)) can only be satisfied if \( |r - r^*| \) increases more. In turn, the right-hand side of (13) also increases. As a consequence, the optimal \( \hat{g} \) must increase as well (which partially weakens the increase of \( |r - r^*| \)). Eventually, \( \delta \) becomes so small than either (i) the compliance constraint at the investment stage is violated, (ii) \( \hat{v} \) is reduced so much that (CC\( _F^g \)) is violated for any level of \( r \), or (iii) \( \delta \) reaches zero. We let \( \delta \) measure the maximum of these three thresholds. Clearly, \( \delta \in (0, \delta) \). \textit{QED
Proof of Propositions 5.

With stochastic compliance costs, the country \(i\)'s per capita utility is equal to:

\[
u_i = \theta_i b(g_i, r_i) - h c(r_i) (g_i + \mathbb{E}g_{-i}) - k r_i,
\]

where \(\mathbb{E}g_{-i}\) is the aggregate expected emission by the other countries conditional on the realization of their uncertain private benefits with \(\partial \mathbb{E}g_{-i}/\partial \theta < 0\). Let \(\mathbb{E}v_i = \pi v_i^c + (1 - \pi) v_i^d\) be the expected continuation value, where \(v_i^c\) represents the value in the case of cooperation and \(v_i^d\) represents the value in the case of deviation. Suppose that \(\theta_i \leq \hat{\theta}\), which implies that country \(i\) cooperates. In the cooperation phase, its intertemporal value function is:

\[
\frac{v_i^c}{1 - \delta} = \theta_i b(g_i, r_i) - h c(r_i) (g_i + \mathbb{E}g_{-i}) - k r_i
\]

\[
+ \frac{\delta}{1 - \delta} \left[ \pi v_i^c + (1 - \pi) v_i^d - (1 - \pi^{n-1}) (1 - \delta^T) (\pi v_i^c + (1 - \pi) v_i^d - v_i^b) \right].
\]

Country \(i\) deviates if \(\theta_i > \hat{\theta}\). In this case, the intertemporal value function is:

\[
\frac{v_i^d}{1 - \delta} = \theta_i b(g_i, r_i) - h c(r_i) (g_i + \mathbb{E}g_{-i}) - k r_i
\]

\[
+ \frac{\delta}{1 - \delta} \left[ \pi v_i^c + (1 - \pi) v_i^d - (1 - \delta^T) (\pi v_i^c + (1 - \pi) v_i^d - v_i^b) \right].
\]

Each country has then incentive to cooperate if and only if the following condition \((CC_g^\theta)\). Using Eqs. (14) and (15), we get:

\[
\mathbb{E}v_i = \pi v_i^c + (1 - \pi) v_i^d
\]

\[
= \frac{(1 - \delta)}{1 - \delta (\delta^T + (1 - \delta^T) \pi^n)} \left[ \pi \left( \theta_i b(g_i, r_i) - h c(r_i) (g_i + \mathbb{E}g_{-i}) - k r_i \right) \right. \]

\[
+ (1 - \pi) \left( \theta_i b(g_i, r_i) - h c(r_i) (g_i + \mathbb{E}g_{-i}) - k r_i \right)
\]

\[
+ \frac{\delta}{1 - \delta (\delta^T + (1 - \delta^T) \pi^n)} \pi v_i^c + (1 - \pi) v_i^d - v_i^b.
\]

Inserting Eq. (16) into \((CC_g^\theta)\), we have:

\[
0 \leq \Delta \equiv \theta_i b(g_i, r_i) - h c(r_i) (g_i + \mathbb{E}g_{-i}) - k r_i - v_i^b
\]

\[
- \frac{1 - \delta^{T+1}}{\delta (1 - \delta^T) \pi^{n-1}} \left[ \theta_i [b(g_i, r_i) - b(g_i, r_i)] - h c(r_i) (g_i - \bar{g}) \right].
\]

The endogenous threshold \(\hat{\theta} = \hat{\theta}(T, r_i, \delta)\) is then equal to the solution of the following
non-linear equation:

\[
0 = \hat{\theta} b (\overline{g}, r_i) - hc (r_i) (\overline{g} + \mathbb{E} g_{-i}) - kr_i - v_i^b
- \frac{1 - \delta^{T+1}}{\delta (1 - \delta^T) \pi^{n-1}} \left[ \hat{\theta} b (\overline{g}, r_i) - b (g, r_i) \right].
\] (17)

The function (CC) is convex in \( \hat{\theta} \). Hence, the equilibrium \( \hat{\theta} \) which satisfies the condition \( \Delta \geq 0 \) for \( \theta_i \leq \hat{\theta} \) is necessarily the smallest solution of (17) provided that \( \hat{\theta} \in \Theta \). This implies that \( \Delta_{\theta_{\hat{\theta}=\hat{\theta}}} < 0 \). By differentiating (CC) with respect to \( \delta \), we have:

\[
\Delta_s \simeq - (1 - (T + 1) \delta^T) (1 - \delta) \left[ \frac{hc (r_i) (\overline{g} - g)}{\hat{\theta}} - b (\overline{g}, r_i) - b (g, r_i) \right].
\]

Since \( \hat{\theta} > hc (r_i) (\overline{g} - g) / [b (\overline{g}, r_i) - b (g, r_i)] \), \( \Delta_s > 0 \) and, in turn, \( d\hat{\theta}/d\delta > 0 \). The derivative with respect to \( T \) is

\[
\Delta_T \simeq \log (\delta) \left[ \frac{hc (r_i) (\overline{g} - g)}{\hat{\theta}} - b (\overline{g}, r_i) - b (g, r_i) \right].
\]

which implies that \( d\hat{\theta}/dT > 0 \). Finally, the derivative with respect to \( r \) is

\[
\Delta_r = \hat{\theta} \frac{\partial b (\overline{g}, r_i)}{\partial r_i} - hc^c (r_i) (\overline{g} + \mathbb{E} g_{-i}) - k
- \frac{1 - \delta^{T+1}}{\delta (1 - \delta^T) \pi^{n-1}} \left[ \hat{\theta} \frac{\partial}{\partial r_i} [b (\overline{g}, r_i) - b (g, r_i)] - hc^c (r_i) (\overline{g} - g) \right].
\]

Therefore, \( \Delta_r > 0 \) under (G) and \( \Delta_r < 0 \) under (NG). Inserting (CC) into Eq. (14), we obtain the indirect value:

\[
v_i^c = \frac{1}{\pi^{n-1}} \left[ \hat{\theta} b (g, r_i) - hc (r_i) (g + \mathbb{E} g_{-i}) - kr_i \right]
+ \left( 1 - \frac{1}{\pi^{n-1}} \right) \left[ \hat{\theta} b (\overline{g}, r_i) - hc (r_i) (\overline{g} + \mathbb{E} g_{-i}) - kr_i \right],
\] (18)

where \( \hat{\theta} = \hat{\theta} (T, r_i, \delta) \).

We notice two facts. First, \( v_i^c \) is not directly affected by the length of punishment. The variable \( T \) enters the value only indirectly through \( \hat{\theta} (T, r_i, \delta) \). Second, the value \( v_i^c \) is convex in \( \hat{\theta} \). Therefore, it is maximized at the boundary levels of \( \hat{\theta} \). Since the function \( \hat{\theta} (T, r_i, \delta) \) is monotonic increasing in \( T \), these two facts jointly imply that the optimal \( T \) is either 1, if the value is maximized at \( \hat{\theta}^{\min} (r_i, \delta) \equiv \hat{\theta} (1, r_i, \delta) \), or infinity, if the value is maximized at \( \hat{\theta}^{\max} (r_i, \delta) \equiv \hat{\theta} (\infty, r_i, \delta) \). Let \( \overline{\delta} \equiv \hat{\theta}^{\max} (\overline{\theta} + \sigma)_{|r=r^*}^{-1} \) be the threshold level of discount factor such that if \( r_i = r^* \) for any \( \delta \geq \overline{\delta} \), then \( \hat{\theta}^{\max} (r^*, \delta) \) equals the maximum
of the support Θ, implying π = 1. In the absence of uncertainty, deviations are triggered with infinite punishment off-the-equilibrium path. For a continuity argument then it must be that, for \( r_i = r^* \) and \( \delta \geq \overline{\delta} \), the value \( v_i^\tau \) is maximized at \( \hat{\theta}^\max (r^*, \overline{\delta}) \) rather than \( \hat{\theta}^\min (r^*, \delta) \). This directly implies that the optimal \( T \) is equal to \( \infty \). Therefore, if \( \delta \geq \overline{\delta} \), then the best SPE is first best and the compliance constraint (CC_\theta^g) simplifies to:

\[
\theta_i \left[ b(\overline{g}, r_i) - b(g, r_i) \right] - hc(r_i) (\overline{g} - g) \leq \frac{\delta (E v_i - v_i^b)}{1 - \delta},
\]

which holds for \( r_i = r^* \) and for every \( \theta_i \in \Theta \). For \( \delta < \overline{\delta} \), however, \( \hat{\theta}^\max (r^*, \delta) < \overline{\theta} + \sigma \), since \( \hat{\theta} (T, r_i, \delta) \) is monotonic increasing in \( \delta \). For \( \delta \) close to \( \overline{\delta} \), \( v_i^\tau \) is still maximized when \( \hat{\theta} = \hat{\theta}^\max (r_i, \delta) \) and, in turn, \( T = \infty \). To determine the optimal \( r_i \) when \( \delta \) is close to \( \overline{\delta} \), we can then maximize the value (18) with respect to \( r_i \) subject to the constraint \( \hat{\theta} = \hat{\theta}^\max (r_i, \delta) \). Taking the limit for \( r_i \to r^* \) and \( \delta \to \overline{\delta} \) of \( \partial v_i^\tau / \partial r_i \) yields:

\[
\frac{\partial \pi}{\partial \theta_i |_{\hat{\theta} = \overline{\theta} + \sigma}} \frac{\partial \hat{\theta}^\max (r^*, \overline{\delta})}{\partial r_i} \left[ \hat{\theta}^\max (r^*, \overline{\delta}) \left[ b(\overline{g}, r^*) - b(g, r^*) \right] - hc(r^*) (\overline{g} - g) \right].
\]

Given that \( \partial \hat{\theta}^\max (r^*, \overline{\delta}) / \partial r_i \) is positive under (G) and negative under (NG), and that \( \hat{\theta}^\max (r^*, \overline{\delta}) \left[ b(\overline{g}, r^*) - b(g, r^*) \right] - hc(r^*) (\overline{g} - g) > 0 \), then we conclude that \( r_i > r^* \) under (G) and \( r_i < r^* \) under (NG). If \( \delta \) decreases further it can be that for a given \( r_i \) the value \( v_i^\tau \) is maximized at \( \hat{\theta} = \hat{\theta}^\min (r_i, \delta) \) rather than at \( \hat{\theta} = \hat{\theta}^\max (r_i, \delta) \). In this scenario, the optimal \( T \) is equal 1 and \( r_i \) must decline instead of increase under (G) and increase instead of decline under (NG). We assume that \( \delta \) is larger than a lower threshold \( \delta \) such that (i) the value \( v_i^\tau \) is always maximized at \( \hat{\theta} = \hat{\theta}^\max (r_i, \delta) \) rather than at \( \hat{\theta} = \hat{\theta}^\min (r_i, \delta) \), (ii) the compliance constraint at the investment stage is never violated, (iii) (CC_\theta^g) is not violated for any level of \( r_i \), or (iv) \( \delta \) does not reach zero. QED

**Proof of Propositions 6.**

The proofs of parts (i) and (ii) can be found in the text. Consider the case with complete information where the deviant country \( i \in N \) has some bargaining power at the renegotiation stage \( \alpha \in (0, 1) \). Suppose that side payments are possible at any stage and consider renegotiation possibilities both at the emission and investment stages. If country \( i \) deviates at the emission stage, renegotiation can be achieved by requesting the deviator to pay a monetary transfer \( f^g > 0 \) to the coalition \( N \setminus i \) of \( n - 1 \) countries before cooperation is restored. Hence, the country \( i \)’s intertemporal value is:

\[
b(r, \overline{g}) - hc(r) ((n - 1) \overline{g} + \overline{g}) - kr + \delta \left[ -f^g + \frac{1}{1 - \delta} \left[ b(r, \overline{g}) - hc(r) n \overline{g} - kr \right] \right]. \tag{19}
\]
The monetary transfer that the deviator must pay yields a continuation value equal to
\[-fg + \frac{1}{1-\delta} \left[ b(r,g) - hc(r) ng - kr \right] = \frac{v^b}{1-\delta} + \frac{\alpha}{n} S^g, \tag{20}\]
where
\[S^g \equiv n \left[ \frac{b(r,g) - hc(r) ng - kr}{1-\delta} - \frac{v^b}{1-\delta} \right] \]
is the aggregate surplus generated by the opportunity of renegotiation after the emission stage. Clearly, if \(\alpha = 0\), then the deviator cannot extract any surplus, implying that the continuation value cannot be larger than the BAU continuation value, as argued in the text. Using Eq. (20) and the expression of \(S^g\) yields the endogenous transfer:
\[f^g = (1 - \alpha) \left[ \frac{b(r,g) - hc(r) ng - kr}{1-\delta} - \frac{v^b}{1-\delta} \right], \tag{21}\]
which is smaller the stronger is the bargaining power of the deviating country. Plugging Eq. (21) into Eq. (19), the compliance constraint at the emission stage is equal to:
\[0 < \Delta^g \equiv \delta (1 - \alpha) \left[ \frac{b(r,g) - hc(r) ng - kr}{1-\delta} - \frac{v^b}{1-\delta} \right] - \left[ b(r,\bar{g}) - b(r,g) - hc(r) (\bar{g} - g) \right], \tag{CC^g_R}\]
Let \(\tilde{\delta}^g \equiv \tilde{\delta}^g (r^*) be the threshold for the discount factor at which \((CC^g_R)\) is binding for \(r = r^*:\)
\[\tilde{\delta}^g (r^*) \equiv 1 - \frac{1 - (1 - \alpha) (b(r^*,g) - hc(r^*) ng - kr^* - v^b)}{(1 - \alpha) (b(r^*,g) - hc(r^*) ng - kr^* - v^b) + b(r^*,\bar{g}) - b(r^*,g) - hc(r^*) (\bar{g} - g)}.\]
The threshold \(\tilde{\delta}^g (\alpha)\) increases strictly in \(\alpha\) from \(\tilde{\delta}^g (0) = \tilde{\delta}\) to \(\tilde{\delta}^g (1) = 1.\) This occurs because an increase of the relative bargaining power of the deviator reduces the penalty, thereby increasing the country’s temptation to deviate. If \(\delta < \tilde{\delta}^g\), then countries must invest more in green technology or less in brown and adaptation to satisfy compliance at the emissions stage. If a country deviates at the investment stage, then renegotiation can be achieved by requesting the deviator to pay the other countries a monetary transfer \(f^r > 0\) before the emission stage in the same period. Starting from the next period, cooperation is then restored. Note that if a country deviates from the cooperative level of investment, then such a country will also deviate at the subsequent emission stage. This is because individually optimal investments make it more profitable for countries to deviate also on emissions. This is anticipated by the other countries, who rationally decide to play cooperatively conditioned on the monetary transfer be paid by the deviating country. In the case of deviation at the investment stage, the deviating country’s intertemporal
value is therefore equal to:

\[ b(\tilde{r}, \bar{g}) - hc(\tilde{r}) \left( (n - 1) \bar{g} + \bar{g} \right) - k\tilde{r} - f^*(\tilde{r}) + \frac{\delta}{1 - \delta} \left[ b(r, g) - hc(r) n g - kr \right], \] (22)

where \( \tilde{r} \) is the individually optimal investment level, which also affects the equilibrium penalty and will be determined later. For a country with \( \alpha \) bargaining power, the monetary transfer \( f^*(\tilde{r}) \) will be set such that the intertemporal utility (22) equates the value

\[
\left[ b(\tilde{r}, \bar{g}) - hc(\tilde{r}) n \bar{g} - k\tilde{r} + \frac{\delta v^b}{1 - \delta} \right] + \frac{\alpha}{n} S^r(\tilde{r}),
\]

where the first bracket represents the worst payoff the deviating country can get given the per-period’s sunk investment \( \tilde{r} \). Note that if \( \alpha = 0 \), then such an intertemporal value simply corresponds to \( v^b \) and \( \tilde{r} = r^b \). The aggregate surplus generated by the opportunity of renegotiation after the investment stage is \( S^r(\tilde{r}) \), which is equal to

\[
S^r(\tilde{r}) \equiv (n - 1) \left( - (b(r, \bar{g}) - b(r, g)) + (n - 1) hc(r) (\bar{g} - g) \right)
+ (n - 1) hc(\tilde{r}) (\bar{g} - g) + \delta S^g(r).
\]

The first component is the current surplus attached to the coalition \( N \setminus i \) of \( n - 1 \) punisher countries. The second component is the current surplus attached to the deviator. Finally, the third component is the aggregate surplus generated from the next period onward. Using the expression of \( S^r(\tilde{r}) \), we get the endogenous monetary transfer:

\[
f^*(\tilde{r}) = -\frac{\alpha}{n} (n - 1) \left[ - (b(r, \bar{g}) - b(r, g)) + (n - 1) hc(r) (\bar{g} - g) \right]
+ (n - 1) \left( \frac{n - \alpha}{n} \right) hc(\tilde{r}) (\bar{g} - g) + \delta (1 - \alpha) \left[ \frac{b(r, g) - hc(r) n g - kr}{1 - \delta} - \frac{v^b}{1 - \delta} \right].
\]

Plugging \( f^*(\tilde{r}) \) into Eq. (22), the compliance constraint at the investment stage is equal to:

\[
0 < \Delta^r \equiv \delta (1 - \alpha) \left[ \frac{b(r, g) - hc(r) n g - kr}{1 - \delta} - \frac{v^b}{1 - \delta} \right] \quad \text{(CC)_{R}}
- \left[ b(\tilde{r}, \bar{g}) - hc(\tilde{r}) \left( \bar{g} + (n - 1) \left( \frac{\alpha}{n} g + \frac{n - \alpha}{n} \bar{g} \right) \right) \right] - k\tilde{r} - \left[ b(r, g) - hc(r) n g - kr \right]
- \frac{\alpha}{n} (n - 1) \left[ - (b(r, \bar{g}) - b(r, g)) + (n - 1) hc(r) (\bar{g} - g) \right].
\]

\( ^{31} \)Using Eq. (22), the individual optimal investment level, which also takes into account the effect on the equilibrium monetary transfer \( f^*(\tilde{r}) \), corresponds then to \( \tilde{r} = \arg\max_r \left( b(r, \bar{g}) - hc(r) (\bar{g} + (n - 1) \left( \frac{\alpha}{n} g + \frac{n - \alpha}{n} \bar{g} \right)) - kr \right). \)
Note that if $\alpha = 0$, then $\text{CC}_R$ simplifies to:

$$
\frac{\delta}{1-\delta} \left[ b(r, g) - hc(r) \frac{n g - kr}{n} - \frac{\nu_b}{1-\delta} \right] \geq \nu_b - \left[ b(r, g) - hc(r) \frac{n g - kr}{n} \right] < 0,
$$

which is satisfied for any discount factor. From $\text{CC}_R$, we obtain the threshold for the discount factor $\tilde{\delta}_r(r^*):$

$$
\tilde{\delta}_r(r^*) \equiv 1 - \frac{\frac{1}{1-\delta} \left[ b(r^*, g) - hc(r^*) \frac{n g - kr^*}{n} - \nu_b \right]}{\frac{1}{1-\alpha} \left[ b(r^*, g) - hc(r^*) \frac{n g - kr^*}{n} - \nu_b \right] + \Omega(r^*)},
$$

where

$$
\Omega(r^*) \equiv \left[ b(\hat{r}, \hat{g}) - hc(\hat{r}) \left( \frac{\alpha}{n} + \frac{n - \alpha}{n} \hat{g} \right) \right] - \left[ b(r^*, \hat{g}) - hc(r^*) \frac{n g - kr^*}{n} \right] + \frac{\alpha}{n} (n - 1) \left[ - b(r^*, \hat{g}) - b(r^*, g) \right] + (n - 1) hc(r^*) \left( \hat{g} - \bar{g} \right).
$$

The threshold $\tilde{\delta}_r(\alpha)$ is also strictly increasing in $\alpha$, implying that compliance at the investment stage is harder to satisfy when the relative bargaining power of the deviant country is stronger. From $\text{CC}_R$, we notice that if $\delta < \delta^*$, then countries must invest less in green technology or more in brown and adaptation to satisfy compliance at the investment stage, contrarily to what is required in order to satisfy compliance at the emissions stage. Under the condition $b(r^*, \bar{g}) - b(r^*, g) - hc(r^*) \left( \bar{g} - g \right) \geq \Omega(r^*)$, the thresholds are $\tilde{\delta}_R(\alpha) > \tilde{\delta}_v(\alpha)$. In this scenario, $(???)$ binds first compared with $\text{CC}_R$ as $\delta$ declines starting from $\delta = 1$. Given the above discussion, it is clear that $\tilde{\delta}(\alpha) \equiv \tilde{\delta}_R(\alpha) = \max \{ \tilde{\delta}_R(\alpha), \tilde{\delta}_v(\alpha) \}$ for $\alpha \approx 0$. Therefore, for $\delta < \tilde{\delta}(\alpha)$, $r$ must increase under $(G)$ but decrease under $(NG)$ if $\alpha$ increases from zero. $QED$

**Proof of Proposition 7.**

With technological spillovers, the Pareto optimal SPE is obtained by maximizing the intertemporal value $v$ subject to the compliance constraints at the investment stage $(\text{CC}_R^e)$ and at the emission stage $(\text{CC}_e^t)$.

(i) Since $v > \nu_b$ at $r^*$, both conditions hold if $\delta$ is close to one. Binding $(\text{CC}_R^e)$ and $(\text{CC}_e^t)$ implicitly define $\tilde{\delta}_e(r)$ and $\tilde{\delta}_v(r)$, respectively, such that if $\delta \geq \max \{ \tilde{\delta}_e^t, \tilde{\delta}_v^t \}$ with $\tilde{\delta}_e^t \equiv \tilde{\delta}_e(r^*)$ and $\tilde{\delta}_v^t \equiv \tilde{\delta}_v(r^*)$, then $r^*$ satisfies both compliance constraints.

(ii) We will first establish the condition under which the compliance constraint at the emission or at the investment stage binds first. If $r^* < r_b$, then $(\text{CC}_e^t)$ can never bind.
Otherwise, if \( r^* > r^b \), then the inequality \( \delta r \leq (>) \delta^g \) holds if and only if:

\[
b (g, r^b) - h c (r^b) n g - k r^b + k \frac{1}{1 - e} (r^* - er^b) \leq (>) b (g, r^*) - h c (r^*) (g + (n - 1) g),
\]

which follows when comparing \((CC^r_e)\) and \((CC^g_i)\). The right-hand side of Eq. (23) is not affected by changes in \( e \), whereas the left-hand side varies as follows:

\[
\frac{\partial}{\partial e} \left( b (g, r^b) - h c (r^b) n g - k r^b + k \frac{1}{1 - e} (r^* - er^b) \right) = k \frac{r^* - r^b}{(1 - e)^2} > 0.
\]

Thus \( \delta r \leq (>) \delta^g \) when \( e \leq (>) \bar{e} \), where \( \bar{e} \) is the unique level that satisfies Eq. (23) with equality.

Consider the case of green technologies satisfying (G). If \( e \leq \bar{e} \), condition \((CC^g_i)\) can be written as \( r \geq r_g (\delta) \equiv \hat{\delta}^g - 1 (r) \), which as before (Proposition 2) is decreasing in \( \delta \), and \( r^g (\delta) > r^* \) when \( \delta < \tilde{\delta}^g \). When \( e > \bar{e} \) and \( \delta \) declines, \((CC^r_e)\) binds first and the optimal investment is \( r = r^r (\delta) \equiv \hat{\delta}^r - 1 (r) \). The Pareto optimal SPE is then solving:

\[
\max_r v + \eta \left( v - v^b - k \frac{(1 - \delta) e}{1 - e} (r - r^b) \right),
\]

where \( \eta > 0 \) is the shadow value of satisfying a strictly binding \((CC^r_e)\). The first-order condition for an interior \( r \) is:

\[
0 = \frac{\partial v}{\partial r} + \eta \frac{\partial}{\partial r} \left[ v - \frac{k (1 - \delta) e}{1 - e} r \right],
\]

while the second-order condition is satisfied for \( v \) sufficiently concave. Since \( r^* \) is defined by \( \frac{\partial v}{\partial r} = 0 \), \( r \) must necessarily be below \( r^* \) to satisfy this first-order condition. Therefore, \( r^r (\delta) < r^* \). Moreover, as \( \delta \) declines further, \((CC^r_e)\) can only be satisfied if \( |r - r^*| \) increases more.

In the case of non-green technology satisfying (NG), as soon as \( \delta \) declines below \( \max \{ \tilde{\delta}^r, \tilde{\delta}^g \} \), either \((CC^r_e)\) binds first and then \( r = r^r (\delta) \) (if \( e > \bar{e} \)) or \((CC^g_i)\) is the hardest to satisfy and then \( r = r^g (\delta) \) (if \( e \leq \bar{e} \)). In both cases, however, as \( \delta \) declines further, the binding constraint can only be satisfied if \( |r - r^*| \) increases more.

Finally, no \( r \) can enforce compliance with the agreement if \( \delta \) is so small than either (i) \((CC^g_i)\) is violated, (ii) \( v \) is reduced so much that \((CC^r_e)\) is violated for any level of \( r \), or (iii) \( \delta \) reaches zero. We let \( \tilde{\delta} \) measure the maximum of these three thresholds. Clearly, \( \tilde{\delta} \in [0, \tilde{\delta}] \). QED

**Proof of Proposition 8.**

If we define \( d_i \equiv g_i - r_i \) to be the decrease in consumption relative to the bliss level \( g \), the first-best emission (and consumption) level is simply given by \( d^* = cn/B \),
while, in BAU, \( d^b = c/B \). We can write the continuation value as:

\[
v = d (nc - Bd/2) + r (nc - Kr/2) - cn\bar{y}.
\]

(i) The compliance constraint at the emission stage can be written as:

\[
\frac{v}{1 - \delta} \geq nc \left( r - r^b \right) - \frac{K}{2} \left( r^2 - (r^b)^2 \right) + c (n - 1) (d - d^b) + \frac{v^b}{1 - \delta},
\]

which implies that:

\[
\delta \geq \hat{\delta} (g, r) \equiv 1 - \frac{-cn\bar{y} + d (nc - Bd/2) + r (nc - Kr/2) - v^b}{nc \left( r - r^b \right) - \frac{K}{2} \left( r^2 - (r^b)^2 \right) + c (n - 1) (d - d^b)},
\]

so

\[
\delta \equiv \hat{\delta} (g^*, r^*) = 1 - \frac{(n - 1)^2 (1/2B + 1/2K)}{(n - 1)^2/2K + (n - 1)^2/B} = \frac{K}{B + 2K}.
\]

(ii) Note that \( r_i = r^* \) is both maximizing \( v \) and weakening \((CC^g_c)\). Given this \( r^* \), the optimal \( d \) is the largest \( d \) satisfying \((CC^g_c)\). Substituting for \( v \) and then solving \((CC^g_c)\) for the largest \( d \), we get:

\[
 dB = nc - \phi,
\]

where

\[
\phi (\delta) \equiv c (n - 1) \left[ 1 - \delta - \sqrt{\delta^2 + \delta B/K} \right] = c (n - 1) \left[ 1 - \delta - \sqrt{(1 - \delta)^2 - \left( \bar{\delta}^g - \delta \right)/\bar{\delta}^g} \right],
\]

where \( \phi (\delta) \) decreases from \( c (n - 1) \) to 0 as \( \delta \) increases from 0 to \( \bar{\delta}^g \). This \( d \) is implemented by the emission tax \( nc - \phi \). To ensure \( r_i = r^* \), the subsidy must be \( \phi \).

Note that the investment-stage compliance constraint can be written as

\[
\frac{v}{1 - \delta} \geq \varsigma (n - 1) \frac{c}{K} + \frac{v^b}{1 - \delta},
\]

which always holds when \( \varsigma = \phi (\delta) \to 0 \). When \( \delta \) falls, \( v \) declines and \( \varsigma = \phi (\delta) \) increases. The threshold \( \hat{\delta} \) is defined implicitly by requiring \((CC^r_c)\) to hold with equality at \( \hat{\delta} \). **QED**
References


