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Advance-Purchase Financing of Projects with Few Buyers

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Marco Sahm

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Advance-Purchase Financing of Projects with Few Buyers

Abstract

I investigate a simple model of advance-purchase contracts as a mode of financing costly projects. The analysis can easily be reinterpreted as a model of the monopolistic provision of excludable public goods under private information. An entrepreneur has to meet some capital requirement in order to start production and sell the related good to a limited number of potential buyers who are privately informed about their willingness to pay. I find that advance-purchase arrangements enable more costly projects to be financed than traditional funding sources. The entrepreneur is able to use advance-purchase surcharges as a price discrimination device. However, the discriminatory power is limited by the problem of free-riding, which is exacerbated as the number of potential buyers increases. I apply the model to research and development activities in the health industry, discussing the availability of new drugs and vaccines in poor countries.

JEL-Code: D420, G320, H410, L120, L260, O310.

Keywords: pre-ordering, price discrimination, excludable public goods, monopolistic provision, crowdfunding, innovation and R&D.

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1 Introduction

There is a lively political debate in the public health sector about how the research and development (R&D) of new drugs and vaccines by private firms can be incentivized properly and how the resulting pharmaceuticals can also be made available in low-income countries. One proposal that would address both problems simultaneously, is the use of advance-purchase arrangements:¹ Negotiating with the producer, some national or supra-national health authorities may undertake to pre-order a drug and pay in advance (or promise to pay the prespecified price on delivery). Resolving the producer's uncertainty about how countries value the drug, the firm can then use these (promises of) advance payments to finance its R&D investments. Moreover, poorer countries could benefit from lower prices once the development of the drug has been financed by pre-orders placed by richer countries.

Despite being intuitively appealing, the proposal has not yet undergone a rigorous analysis based on a theoretical economic model. Is it really possible to finance more (costly) R&D activities based on advance-purchase contracts than relying on traditional funding like debt or equity? Does the use of advance-purchase financing instead of traditional funding actually improve the availability of pharmaceuticals in low-income countries? (Under which circumstances) does advance-purchase financing Pareto-dominate traditional funding?

In this paper, I examine these questions employing a simple model of advance-purchase financing. I consider an entrepreneur who must meet a certain capital requirement in order to start production. Once the fixed costs are covered, the entrepreneur has monopoly power and sells the related product to a limited number of potential buyers. Customers are privately informed about their willingness to pay, and buy either one or zero units of the good. Within this framework, I compare the allocations resulting from two different funding mechanisms: Under traditional (debt or equity) financing, the entrepreneur relies on standard uniform monopoly pricing, whereas advance-purchase financing can be modeled as a two-stage game. In the first stage, the entrepreneur offers to pre-order the good at a certain advance-purchase price. If the money collected from pre-orders fails to cover the fixed cost, then the advance purchasers will be reimbursed. If the money collected suffices, the game will move on to the second stage at which the good is produced, delivered to advance-purchasers, and offered to residual customers at a (possibly different) regular price.

¹Berndt and Hurvitz (2005) provide a comprehensive discussion of this proposal focusing on practical issues.

It transpires that this process enables the entrepreneur to discriminate between customers with different valuations: Agents with a high willingness to pay prefer to pre-order the good at the advance-purchase price because they fear the nonavailability of the product most and consider that their pre-order could be pivotal for its realization. In contrast, agents with a lower valuation prefer to wait, and perhaps purchase the good later at the regular price. This results in an advance-purchase surcharge, i.e. the entrepreneur optimally sets an advance-purchase price above the regular price. Note that the discriminatory power of the entrepreneur rests upon the threat of the potential nonavailability of the product, i.e. on the positive probability that the pre-order of a single agent may be pivotal. Since this probability decreases as the number of potential buyers increases, the problem that agents tend to free-ride on the advance payments of others limits the discriminatory power. Hence, the differential between the advance-purchase price and the regular price declines as the number of potential buyers increases and vanishes in the limit.

Comparing traditional funding and advance-purchase financing, I derive the following results: First, if fixed costs are sufficiently large, the entrepreneur will always prefer advance-purchase financing over traditional funding. Under traditional funding, projects will be realized if and only if they are ex ante profitable. However, ex post, they may turn out to be loss-making. By contrast, advance-purchase financing enables the entrepreneur to price-discriminate and run only projects that are ex post profitable. If the fixed costs that may be avoided this way are sufficiently large, these expected savings will outweigh the disadvantage that it will be impossible to realize some profitable projects under advance-purchase financing due to the problem of free-riding. Second, as this reasoning implies, under advance-purchase financing more costly projects can indeed be realized than under traditional funding. If the fixed costs exceed a certain threshold, projects will not be ex ante profitable and, hence, will definitely not be realized under traditional funding, whereas the probability of being ex post profitable and therefore realized under advance-purchase financing is strictly positive. Third, this shows that advance-purchase financing will actually Pareto-dominate traditional funding if fixed costs are sufficiently large.

Besides the introductory example from health economics, the model captures a series of stylized facts that are characteristic for many markets in which a single seller with increasing returns to scale deals with a limited number of potential buyers. Examples include the international defense industry where an arms manufacturer does business with a limited set of countries, and the international airline industry where a producer of jet engines can virtually only sell to two aircraft companies.

The remainder of this paper is organized as follows: In Section 2, I review the related literature. Section 3 presents the formal model. In Section 4, I derive the basic properties of the optimal advance-purchase contract and compare it to optimal pricing based on traditional funding. Section 5 illustrates the results for the examples of only one or two potential buyers with uniformly distributed valuations. In Section 6, I summarize and discuss the main findings.

2 Related Literature

In addition to its application to the field of R&D in international health economics (Berndt et al., 2007), this paper is closely related to three further strands of the economic literature.

First, it contributes to a series of articles that analyze the role of advance-purchase contracts as a means of price discrimination. Considering markets with a continuum of potential buyers, almost all of the seminal papers on this topic find advance-purchase discounts to characterize the optimal pricing scheme. The optimality of advance-purchase discounts may be due to limited production capacities and uncertainty about the aggregate level of demand (Dana, 1998, 1999, 2001, Gale and Holmes, 1992, 1993); it may also be due to different expected valuations among consumers (Nocke et al., 2011, Möller and Watanabe, 2010). Since in a continuum economy no single pre-order is pivotal for the availability of the respective product, in this part of the literature advance-purchase contracts are irrelevant for the financing decision; they are solely an instrument of price discrimination. By contrast, taking into account the strategic effects between a finite number of agents, the optimal advance-purchase contract in my model reflects the entrepreneur's simultaneous financing and pricing decision, yielding an advance-purchase surcharge.

The joint addressing of the financing and pricing decision is a feature that also relates my model to the recent literature on crowdfunding. Belleflamme et al. (2014) and Sahm et al. (2014) derive a similar result considering a crowdfunding model with a continuum of potential consumers. In their framework, however, the optimality of an advance-purchase surcharge originates from the behavioral assumption that consumers who pre-order experience community benefits, i.e. derive additional utility from belonging to the funding crowd. Independently from my own work, two recent working papers on crowdfunding also discuss the optimality of advance-purchase surcharges based on arguments of pivotality (Ellman and Hurkens, 2014, Kumar et al., 2015). In contrast to my study, however, both papers assume that the entrepreneur can commit to some arbitrary funding threshold (possibly

differing from the capital requirement), which obviously strengthens his discriminatory power. Consistent with my analysis, Ellman and Hurkens (2014) consider a finite set of potential buyers, but include only two possible valuations, whereas I allow for a continuum of valuations. By contrast, Kumar et al. (2015) consider a continuum of potential buyers and introduce some ad-hoc notion of pivotality which is not derived from limit considerations in some finite economy.²

Finally, as the advance payments from pre-orders can be understood as contributions to the realization and nonrival availability of the product, this paper also contributes to the literature on the monopolistic provision of excludable public goods under private information. Early work in this field focused on simple pricing mechanisms that put empirically motivated constraints on the class of admissible contracts (Brito and Oakland, 1980). The more recent contributions usually apply a general mechanism design approach in order to specify optimal contracts (Cornelli, 1996, Schmitz, 1997). Since they often find these optimal mechanisms to be rather complex, though, they raise the question how the prevailing use of much simpler contracts in practice can be explained from the viewpoint of contract theory. For example, both Schmitz (1997) and Norman (2004) show that the monopolist will indeed find it optimal to rely on simple contracts (such as average cost pricing) if the number of potential buyers becomes very large. By contrast, and more closely related to my paper, Cornelli (1996) focuses on the strategic effects within a small economy, emphasizing that the threat of nonproduction is a useful instrument of price discrimination between customers with different valuations. However, all of these articles assume that the monopolist can commit not to renegotiate with customers once they have been excluded. Although this may be a reasonable assumption for some instances, it seems to be violated for the introductory examples. In order to deviate from this assumption in the easiest way, I restrict my analysis to simple advance-purchase contracts with posted prices.

3 Analysis

3.1 Basic assumptions

A monopolistic entrepreneur seeks to finance a costly project with a commonly known capital requirement K . If the capital requirement is met, the

²Thus their findings conflict with the result that, in a large economy with a continuum of potential buyers, the behavioral assumption of community benefits is critical for crowd-funding to be more profitable than traditional funding (Belleflamme et al., 2014, Sahm et al., 2014).

entrepreneur will run the project and produce a related good of a fixed quality normalized to 1. Marginal costs of production are assumed to be zero.

There are $N \in \mathbb{N}$ potential buyers. Depending on their willingness to pay, each potential buyer purchases either zero or one unit of the good. The willingness to pay of buyer $i \in \{1, \dots, N\}$ is his private information, denoted θ_i . It is the realization of a random variable with some commonly known distribution. For the sake of concreteness, assume that all N random variables are independent and identically distributed on $[0, 1]$ according to the cumulative distribution function F with some continuously differentiable density f . Moreover, I impose the following regularity condition as in Myerson (1981).³

Definition 1 *F will be called regular if function v with $v(\theta) := \theta - \frac{1-F(\theta)}{f(\theta)}$ strictly increases in θ .*

If customer i buys one unit of the product at price p , he will realize the surplus $U_i = \theta_i - p$. The surplus from not buying is zero.⁴

The entrepreneur can choose between two mutually exclusive funding mechanisms: either traditional (debt or equity) financing (with opportunity costs of capital normalized to 0) or financing based on advance-purchase commitments. The latter refers to the case in which some customers pre-order the product and pay in advance. The advance payments are used to meet capital requirement K and realize the project.

I assume that the entrepreneur has the bargaining power to make take-it-or-leave-it price offers. This is common practice in the analysis of monopolies, and leads to a tractable screening model with the uninformed party proposing the contract. If the number of potential buyers is small, however, this assumption is debatable and will be discussed in more detail below.

3.2 Traditional funding

As a benchmark, consider the standard model of monopoly pricing in which the entrepreneur cannot commit to not running ex ante profitable projects.⁵ With traditional funding and asymmetric information about customers' preferences, the entrepreneur then relies on uniform pricing in order to maximize expected profits. Once the project is realized, the probability that a certain customer buys at price p is $1 - F(p)$. Hence, expected profits equal

$$E(\pi_0) = N[1 - F(p)]p - K.$$

³Examples of regularity include uniform, normal, and exponential distributions.

⁴The assumption of risk-neutral customers is particularly appropriate for B2B markets such as those in the introductory examples.

⁵Cornelli (1996) characterizes optimal selling procedures in a model with commitment.

The necessary condition for an optimal price p_0 implies $v(p_0) = 0$. For any regular F , the condition is also sufficient. Accordingly, with traditional funding, the entrepreneur will realize the project if and only if the capital requirement does not exceed the threshold $K_0 := N[1 - F(p_0)]p_0 < N$.

3.3 Advance-purchase financing

Project funding based on advance-purchase commitments can be described as a sequential game Γ with two stages: In the first stage, the entrepreneur offers all potential buyers the possibility to pre-order the product at price p_c and pay in advance. Individuals then simultaneously decide whether to pre-order at this price. If the money collected from pre-orders falls short of capital requirement K , advance payments are returned and the game ends. If instead capital requirement K is met, the project is realized and the game moves to the second stage. In the second stage, the entrepreneur sets the regular price p_r for buyers who did not pre-order. These residual customers then decide simultaneously whether to buy at this price. I normalize the discount rate to zero so all pay-offs can be treated as if they accrued at the end of stage 2.

The structure of the game reflects the implicit assumptions about the entrepreneur's bargaining power. Though being able to make take-it-or-leave-it price offers, it is limited in two ways. First, I assume that an advance-purchase contract specifies only the advance-purchase price p_c . In particular, the minimum number of pre-orders required to run the project cannot be contracted upon explicitly. Put differently, the funding threshold may not differ from capital requirement K . The idea is that the entrepreneur may not decide against running the project if the money collected from pre-orders meets the capital requirement, because then the expected profits from realizing the project are positive. Second, the above timing corresponds to the implicit assumption that the entrepreneur cannot commit to a regular price p_r ex ante. This lack of commitment is due to a problem of time-inconsistency similar to that for durable goods: as long as the market is not covered entirely by pre-orderers in stage 1, the entrepreneur always has an incentive to adjust the price in stage 2 in order to address additional buyers and make additional profits. As a consequence, in any buyer who pre-orders must have a higher willingness to pay for the good than any regular customer.

Lemma 1 *For any given prices p_c and p_r there is some $\theta_c \in \mathbb{R} \cup \{\pm\infty\}$ such that customer $i \in \{1, \dots, N\}$ will pre-order the product if and only if $\theta_i \geq \theta_c$.*

Proof. Denote by $m_c \in \{1, \dots, N\}$ the minimum number of pre-orders required to finance the project for the given advance-purchase price p_c , i.e.

$m_c p_c \geq K > (m_c - 1)p_c$. Let $\sigma(m)$ be the probability that the number of pre-orders among $N - 1$ potential buyers will be at least $m \in \mathbb{N}$. Trivially, $\sigma(m-1) \geq \sigma(m)$. Some customer with willingness to pay θ will weakly prefer to pre-order the product if and only if his expected utility from an advance-purchase, $\sigma(m_c-1)(\theta - p_c)$, is at least as high as that from a regular purchase, $\sigma(m_c)(\theta - p_r)$, i.e. if and only if

$$(\sigma(m_c - 1) - \sigma(m_c))\theta \geq \sigma(m_c - 1)p_c - \sigma(m_c)p_r. \quad (1)$$

For $\sigma(m_c - 1) = \sigma(m_c)$, nobody (everybody) will pre-order if $p_c > p_r$ ($p_c \leq p_r$), and $\theta_c := \infty$ ($\theta_c := -\infty$) has the stated property. For $\sigma(m_c - 1) > \sigma(m_c)$, set $\theta_c := \frac{\sigma(m_c - 1)p_c - \sigma(m_c)p_r}{\sigma(m_c - 1) - \sigma(m_c)}$.

□

In what follows, I solve the game by backward induction for its subgame-perfect equilibrium (SPE). Of course, only the non-trivial cases for which $\theta_c \in (0, 1)$ are of further interest. I refer to θ_c as the marginal willingness to pay since it expresses the willingness to pay of any buyer who is indifferent to whether or not he pre-orders.

Stage 2

Let n_c denote the actual number of pre-orders, and suppose that the capital requirement is met, i.e. $n_c p_c \geq K$. Then, the residual potential buyers in stage 2 are those for which $\theta \in [0, \theta_c]$. Since they actually purchase the good at price p_r if and only if $\theta - p_r \geq 0$, the conditional probability that such a customer will actually buy in stage 2 equals

$$\text{Prob}(\theta \geq p_r \mid \theta \leq \theta_c) = \frac{\text{Prob}(p_r \leq \theta \leq \theta_c)}{\text{Prob}(\theta \leq \theta_c)} = \frac{F(\theta_c) - F(p_r)}{F(\theta_c)}.$$

Hence, the entrepreneur maximizes his additional conditional expected profits from stage 2

$$E(\pi_{2nd} \mid n_c) = (N - n_c) \cdot \frac{F(\theta_c) - F(p_r)}{F(\theta_c)} \cdot p_r \quad (2)$$

by the choice of p_r .

Lemma 2 *Any optimal regular price satisfies $p_r < \theta_c$.*

Proof. Any $p_r \geq \theta_c$ would imply $E(\pi_{2nd} \mid n_c) \leq 0$.

□

The first-order condition for an optimal solution implies

$$p_r - \frac{F(\theta_c) - F(p_r)}{f(p_r)} = 0. \quad (3)$$

The condition is also sufficient for any regular F . To see this, note that

$$v_{\theta_c}(\theta) := \theta - \frac{F(\theta_c) - F(\theta)}{f(\theta)}$$

strictly increases for all $\theta \in [0, \theta_c)$ since

$$v'_{\theta_c}(\theta) = 2 + \frac{[F(\theta_c) - F(\theta)]f'(\theta)}{[f(\theta)]^2} > 0.$$

This is obvious for $f'(\theta) \geq 0$; for $f'(\theta) < 0$ it follows from the fact that v strictly increases in θ and thus

$$v'_{\theta_c}(\theta) = 2 + \frac{[F(\theta_c) - F(\theta)]f'(\theta)}{[f(\theta)]^2} > 2 + \frac{[1 - F(\theta)]f'(\theta)}{[f(\theta)]^2} = v'(\theta) > 0.$$

Stage 1

By setting the advance-purchase price, the entrepreneur implicitly determines the marginal willingness to pay as well as the minimum number of pre-orders required to finance the project. To see this, let p_n be the advance-purchase price. Then, $n \in \{1, \dots, N\}$ with $np_n \geq K > (n-1)p_n$ is the corresponding minimum number of pre-orders required. By definition, for any customer with marginal willingness to pay θ_n , the expected payoff from advance-purchase

$$A(n) := \left[\sum_{i=0}^{N-n} \binom{N-1}{n-1+i} [1 - F(\theta_n)]^{n-1+i} [F(\theta_n)]^{N-n-i} \right] (\theta_n - p_n) \quad (4)$$

equals the expected payoff from regular purchase

$$R(n) := \left[\sum_{i=1}^{N-n} \binom{N-1}{n-1+i} [1 - F(\theta_n)]^{n-1+i} [F(\theta_n)]^{N-n-i} \right] (\theta_n - p_r). \quad (5)$$

Note that the probability that the project will be realized if the customer pre-orders in (4) and the probability that the project will be realized if he does not pre-order in (5) differ only by the probability that his own pre-order

will be pivotal for meeting the capital requirement

$$P(n) := \binom{N-1}{n-1} [1 - F(\theta_n)]^{n-1} [F(\theta_n)]^{N-n}. \quad (6)$$

The equality of (4) and (5) characterizes the relation between the marginal willingness to pay θ_n and the advance-purchase price

$$p_n = \frac{P(n)\theta_n + \left[\sum_{i=1}^{N-n} \binom{N-1}{n-1+i} [1 - F(\theta_n)]^{n-1+i} [F(\theta_n)]^{N-n-i} \right] p_r}{\sum_{i=0}^{N-n} \binom{N-1}{n-1+i} [1 - F(\theta_n)]^{n-1+i} [F(\theta_n)]^{N-n-i}}, \quad (7)$$

where p_r is implicitly defined by (3) with $\theta_c = \theta_n$.

Now suppose that $n \in \{1, \dots, N\}$ is the minimum number of pre-orders required. If the realized number of pre-orders is $i \in \{0, \dots, N\}$, the entrepreneur's profit will be 0 for $i < n$ and $ip_n - K + E(\pi_{2nd} \mid i)$ for $i \geq n$. Applying (2), the entrepreneur's expected profit $E(\pi_n)$ equals

$$\sum_{i=n}^N \binom{N}{i} [1 - F(\theta_n)]^i [F(\theta_n)]^{N-i} \left[ip_n - K + (N - i) \frac{F(\theta_n) - F(p_r)}{F(\theta_n)} p_r \right], \quad (8)$$

where p_n is given by (7) and p_r is implicitly defined by (3) with $\theta_c = \theta_n$. The problem of finding the advance-purchase price p_c that maximizes expected overall profits can therefore be solved in two steps:

Step 1: For any $n \in \{1, \dots, N\}$ choose the marginal willingness to pay θ_n that maximizes expected overall profits $E(\pi_n)$ given by (8) subject to the constraint that n is indeed the minimum number of pre-orders required to finance the project for the corresponding advance-purchase price p_n given by (7), i.e. such that $np_n \geq K > (n-1)p_n$.

Step 2: Choose $p_c := p_{n^*}$ (or equivalently $\theta_c := \theta_{n^*}$) with $n^* \in \arg \max_{n \in \{1, \dots, N\}} E(\pi_n)$.

4 Results

Before taking these steps in order to solve the entrepreneur's problem explicitly for some simple examples in Section 5, I record the general properties of the optimal pricing scheme under advance-purchase financing and compare the respective allocation with the allocation under optimal pricing based on traditional funding.

4.1 General properties of the optimal pricing scheme

Proposition 1 *Any SPE of game Γ with $N \in \mathbb{N}$ potential buyers and a regular distribution of valuations F has the following properties.*

- (a) $p_c > p_r$, i.e. the advance-purchase price exceeds the regular price.
- (b) If $n^* = N$, then $p_c = \theta_c$ else $p_c < \theta_c$, i.e. the advance-purchase price falls short of the marginal willingness to pay unless it is optimal to realize the project only if all potential buyers pre-order.

Proof. As demonstrated in the previous section, there is some $n^* \in \arg \max_{n \in \{1, \dots, N\}} E(\pi_n)$ such that $p_c = p_{n^*}$ and $\theta_c = \theta_{n^*}$.

- (a) Suppose to the contrary that $p_c = p_{n^*} \leq p_r$. Then the equality of (4) and (5) implies

$$\begin{aligned}
 A(n^*) &= \left[\sum_{i=0}^{N-n^*} \binom{N-1}{n^*-1+i} [1 - F(\theta_{n^*})]^{n^*-1+i} [F(\theta_{n^*})]^{N-n^*-i} \right] \\
 &\quad \cdot (\theta_{n^*} - p_{n^*}) \\
 &\geq \left[\sum_{i=0}^{N-n^*} \binom{N-1}{n^*-1+i} [1 - F(\theta_{n^*})]^{n^*-1+i} [F(\theta_{n^*})]^{N-n^*-i} \right] \\
 &\quad \cdot (\theta_{n^*} - p_r) \\
 &= A(n^*) + P(n^*)(\theta_{n^*} - p_r),
 \end{aligned}$$

a contradiction since both $P(n^*)$ and $\theta_{n^*} - p_r$ are positive by equation (6) and Lemma 2, respectively.

- (b) If $n^* = N$, then the equality of (4) and (5) will imply

$$0 = R(N) = A(N) = [1 - F(\theta_N)]^{N-1}(\theta_N - p_N)$$

and thus $p_N = \theta_N$; else, by Lemma 2, it will imply $0 < R(n^*) = A(n^*)$ and thus $p_{n^*} < \theta_{n^*}$.

□

To get some intuition for these results, note that the product will be available under advance-purchase financing if and only if the entrepreneur collects enough money from pre-orders to run the project. Since agents with a high willingness to pay fear the possible non-availability of the product most,

the offered advance-purchase contract serves as a price discrimination device that attracts only customers with the highest valuations. In order to reach additional customers after the project has been realized, the entrepreneur must lower the price and choose $p_r < p_c$. However, unless all of the N potential buyers have to purchase in advance to meet the capital requirement, any agent's probability of being pivotal for the availability of the product will be smaller than 1. Put differently, from the perspective of any single agent, there is a positive probability that the product will be available later at some reduced price p_r . The agent's willingness to pay must therefore exceed the advance-purchase price p_c by some strictly positive amount for pre-ordering to be attractive to him. The difference $\theta_c - p_c > 0$ can be interpreted as the minimum information rent the entrepreneur has to leave to agents with high valuations in order to make them reveal their willingness to pay.

Note that any single customer's probability of being pivotal for the availability of the product becomes increasingly small as the number of potential buyers rises. Consequently, the incentives to free-ride on the pre-orders of others increase. The growing problem of free-riding narrows the entrepreneur's possibility to claim a surcharge for pre-orders such that the price differential vanishes in the limit. With a large customer base, price discrimination by means of advance-purchase surcharges is no longer feasible.⁶

Proposition 2 *Consider the SPE of game Γ with $N \in \mathbb{N}$ potential buyers, a regular distribution of valuations F , and some fixed capital requirement K . As the number of potential buyers increases, the difference between the advance-purchase price and the regular price converges to zero, i.e. $\lim_{N \rightarrow \infty} p_c - p_r = 0$.*

Proof. For any $N \in \mathbb{N}$ there is some $N^* \in \arg\max_{n \in \{1, \dots, N\}} E(\pi_n)$ such that $p_c = p_{N^*}$ and $\theta_c = \theta_{N^*}$. Without loss of generality, assume that $\lim_{N \rightarrow \infty} \theta_{N^*} \in (0, 1)$ if existent; otherwise, either everybody or no one pre-orders and no discrimination takes place in the limit.

First assume that N^* has no upper bound as N increases. In this case, the requirement $(N^* - 1)p_{N^*}^* < K$ implies $\lim_{N \rightarrow \infty} p_{N^*} = 0$ and thus $\lim_{N \rightarrow \infty} p_{N^*} - p_r = 0$ since $0 \leq p_r \leq p_{N^*}$ for all N .

Now suppose that N^* is bounded from above by some $\bar{N} \in \mathbb{N}$. The equality of (4) and (5) implies $A(N^*) - R(N^*) = 0$, which is equivalent to

$$p_{N^*} - p_r = \frac{P(N^*)(\theta_{N^*} - p_{N^*})}{\sum_{i=1}^{N-N^*} \binom{N-1}{N^*-1+i} [1 - F(\theta_{N^*})]^{N^*-1+i} [F(\theta_{N^*})]^{N-N^*-i}}. \quad (9)$$

⁶This result is in stark contrast to the findings of Kumar et al. (2015) who base their analysis of a large economy on some nonstandard notion of pivotality which is not derived from limit considerations of growing finite economies.

The denominator of the fraction on the right-hand side of equation (9) is bounded from below by

$$\sum_{i=1}^{N-\bar{N}} \binom{N-1}{\bar{N}-1+i} [1 - F(\theta_{N^*})]^{\bar{N}-1+i} [F(\theta_{N^*})]^{N-\bar{N}-i} > 0.$$

The nominator of the fraction on the right-hand side of equation (9) is bounded from above by

$$\binom{N-1}{(N-1)/2} \left(\frac{1}{2}\right)^{N-1}.$$

To finish the proof, I show that this last term converges to zero. Substituting $n := (N-1)/2$ and applying Stirling's formula $\lim_{n \rightarrow \infty} \frac{n!}{\sqrt{2\pi n} \left(\frac{n}{e}\right)^n} = 1$ yields

$$\begin{aligned} \lim_{n \rightarrow \infty} \binom{2n}{n} \left(\frac{1}{2}\right)^{2n} &= \lim_{n \rightarrow \infty} \frac{(2n)!}{(n!)^2 \cdot 4^n} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{2\pi 2n} \left(\frac{2n}{e}\right)^{2n}}{[\sqrt{2\pi n} \left(\frac{n}{e}\right)^n]^2 \cdot 4^n} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\pi n}} = 0. \end{aligned}$$

□

4.2 Comparison: traditional funding vs. advance-purchase financing

Under traditional funding, only projects with fixed costs $K \leq K_0 < N$ will be realized. Under advance-purchase financing, in contrast, the probability of realization is positive for all projects with fixed costs $K < N$. Put differently, more costly projects can be realized based on advance-purchase financing than based on traditional funding.

To see this, consider the entrepreneur's strategy to choose an advance-purchase price p_N that makes each potential buyer pivotal for running the project. Obviously, the corresponding expected profit

$$E(\pi_N) = [1 - F(p_N)]^N (Np_N - K) \quad (10)$$

is a lower bound for the entrepreneur's optimal profit under advance-purchase financing. The optimal p_N maximizes (10) subject to the constraints $Np_N \geq K > (N-1)p_N$. For any regular F , the unconstrained solution to this

problem can be derived from the first-order condition and is implicitly given by

$$Np_N - \frac{1 - F(p_N)}{f(p_N)} = K.$$

Since $v(p_N)$ strictly increases in p_N , so does the left-hand side of this equation, rising from some negative value $-1/f(0)$ to N as p_N increases from 0 to 1. The equation thus has an interior solution for all $K < N$. This solution will satisfy the first constraint $Np_N - K = \frac{1 - F(p_N)}{f(p_N)} > 0$. Moreover, it will meet the second constraint

$$K - (N - 1)p_N = p_N + K - Np_N = p_N - \frac{1 - F(p_N)}{f(p_N)} > 0$$

as well if and only if $p_N > p_0$. In this case, the expected profit equals $E(\pi_N) = [1 - F(p_N)]^{N+1} > 0$. Otherwise the entrepreneur can set p_N arbitrarily close to $\frac{K}{N-1} < p_0 < 1$. The corresponding expected profit is then given by

$$E(\pi_N) = [1 - F\left(\frac{K}{N-1}\right)]^N \left(\frac{K}{N-1}\right) > 0.$$

These considerations imply the following

Proposition 3 *For any number of potential buyers $N \in \mathbb{N}$, any capital requirement $K \in \mathbb{R}$, and any regular distribution of valuations F the following statements hold:*

- (a) *For any $K \in (K_0, N)$, the project is not realized under traditional funding but has a strictly positive probability of realization under advance-purchase financing.*
- (b) *There is some $K_1 \in [0, K_0]$ such that the entrepreneur strictly prefers advance-purchase financing over traditional funding for all $K \in (K_1, N)$.*
- (c) *There is some $K_2 \in [K_1, K_0]$ such that the allocation under advance-purchase financing Pareto-dominates the allocation under traditional funding for all $K \in (K_2, N)$.*

As stated in Proposition 3, advance-purchase financing leads to welfare improvements in at least all those cases in which the capital requirement cannot be met under traditional funding. For sufficiently high capital requirements, advance-purchasing contracts are in the interest of all agents. For small capital requirements, however, this is unambiguously true only for customers with valuations below the uniform price p_0 . For all other agents, the incidence of advance-purchase financing is less clear.

Proposition 4 *Consider a situation with $N \in \mathbb{N}$ potential buyers, some regular distribution of valuations F , and some capital requirement $K \leq K_0$. Compared to the allocation under traditional funding, in the allocation under advance-purchase financing*

- (a) *the regular price p_r falls short of the uniform price p_0 ,*
- (b) *consumers with valuations $\theta \in [0, p_r]$ are equally well-off,*
- (c) *consumers with valuations $\theta \in (p_r, p_0)$ are strictly better off,*
- (d) *consumers with valuations $\theta \in [\theta_c, 1]$ will be worse off if $p_c > p_0$.*

Proof.

- (a) The first-order condition (3) implies $v(p_r) < 0$. Since v strictly increases and $v(p_0) = 0$, this implies $p_r < p_0$.
- (b) Since $p_r < p_0$, consumers with $\theta \in [0, p_r]$ will not buy in either case.
- (c) Since $p_r < p_0$, consumers with $\theta \in (p_r, p_0)$ will definitely not buy under traditional funding but may possibly buy at the regular price and thus derive a positive expected utility under advance-purchase financing.
- (d) Under advance-purchase financing, consumers with $\theta \in [\theta_c, 1]$ pre-order and will thus possibly buy at a higher price if $p_c > p_0$. Moreover, they face a positive probability that the product will not be available.

□

Whether the remaining consumers profit from advance-purchase financing is, in general, ambiguous. On the one hand, they have to pay less under advance-purchase financing. On the other hand, however, there is a positive probability that the product will not be available, whereas it will be produced for sure under traditional funding. As the examples of the next section will show, the incidence of advance-purchasing contracts on the entrepreneur also depends on the exact capital requirement.

5 Examples

In this section I will explicitly solve the game for the examples of one and two potential buyers with uniformly distributed valuations. These two cases illustrate the most relevant aspects: The example with only one potential buyer highlights how advance-purchase contracts enable more costly projects to be financed than traditional funding methods. The example with two

potential buyers shows how advance-purchase contracts can be used for price discrimination and why this use is limited by free-riding arising from the public goods character of pre-orderers' contributions to the realization of the project.

Note that with $F(\theta) = \theta$ and $f(\theta) = 1$ for all $\theta \in [0, 1]$, the optimal uniform price under traditional funding equals $p_0 = 1/2$ and the corresponding expected profit equals $E(\pi_0) = N/4 - K$, i.e. projects are realized if and only if $K \leq K_0 = N/4$. Moreover, it is easy to compute that the regular price under advance-purchase financing equals $p_r = \theta_c/2$ and the corresponding expected profit from additional sales in stage 2 equals $E(\pi_{2nd} | n_c) = (N - n_c) \cdot \theta_c/4$.

5.1 Example: $N = 1$

I first consider the case with one potential buyer only. With traditional funding, the project can be realized if and only if $K \leq 1/4$. In this case, the customer buys the product if and only if he has a willingness to pay of at least $p_0 = 1/2$, yielding an expected profit of $E(\pi_0) = 1/4 - K$.

With funding based on an advance-purchase contract, the project is realized if and only if the potential buyer pre-orders at the advance-purchase price p_c . Accordingly, the entrepreneur maximizes his expected profit

$$E(\pi_1) = (1 - p_c)(p_c - K)$$

by the choice of p_c subject to the constraint that $p_c \geq K$. The unconstrained solution to this problem is given by $p_c = \frac{1+K}{2}$. It is feasible for all $K \leq 1$.

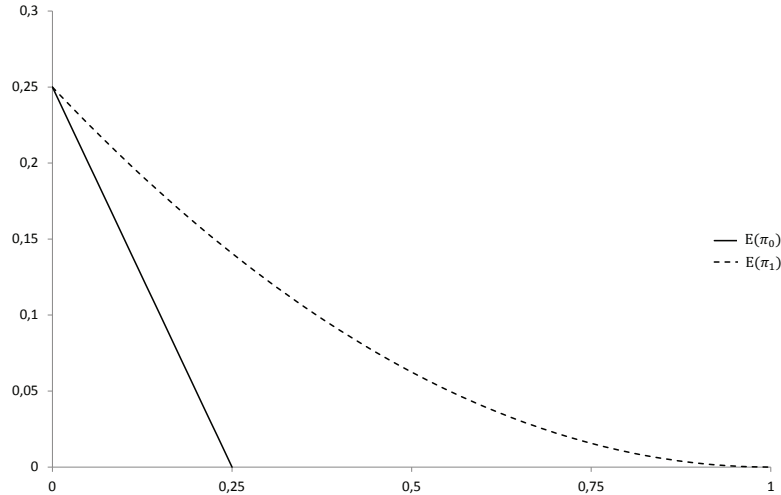


Figure 1: Graphs of $E(\pi_0)$ and $E(\pi_1)$ as functions of K

The expected profits related to the two alternative financing schemes are depicted in Figure 1. A comparison shows that, with funding based on an advance-purchase contract, more costly projects can be realized than with traditional funding. Under the requirement of pre-ordering, the entrepreneur will incur the fixed costs only if the purchase actually takes place. This eliminates the possibility of making losses and leads to a stricter policy of exclusion: the entrepreneur raises the advance-purchase price above the monopoly price under traditional funding. Though this decreases the probability of realizing the project and selling the product, the effect of increased profits in case of realization dominates.

For $1/4 < K < 1$, the probability of realization is zero under traditional funding but strictly positive under advance-purchase financing. Hence, the latter Pareto-dominates the former for sufficiently large fixed costs. For $0 \leq K \leq 1/4$, however, any type of customer would prefer traditional funding and uniform pricing over financing based on advance-purchase contracts because $p_c > p_0$. This raises the question whether the entrepreneur can commit to rely exclusively on the latter funding method. If the entrepreneur lacks such commitment power, only projects with $1/4 < K \leq 1$ can be realized based on advance-purchase contracts, whereas all projects with $0 \leq K \leq 1/4$ have to be financed traditionally.

5.2 Example: $N = 2$

To illustrate the strategic effects among customers, I now consider the case with two potential buyers. With traditional funding, the project can be realized if and only if $K \leq 1/2$. In this case, some customer buys the product if and only if he has a willingness to pay of at least $p_0 = 1/2$, yielding an expected profit of $E(\pi_0) = 1/2 - K$.

To finance the project based on advance-purchase contracts, the entrepreneur can address either one or both potential buyers. Capital requirement K determines which of the two strategies depicted below yields higher expected profits.

First suppose that the entrepreneur sets the advance purchase price to p_1 relying on at least one pre-order. The customer who is indifferent to whether or not to pre-order is characterized by the marginal willingness to pay θ_1 for which the utility from pre-ordering $\theta_1 - p_1$ equals the expected utility from a possible regular purchase $(1 - \theta_1)(\theta_1 - p_r)$. Remember that the regular price is anticipated to equal $p_r = \theta_c/2$. Hence, using equation (7), the marginal

willingness to pay is characterized by

$$p_1 = \theta_1 \left(1 - \frac{1}{2}(1 - \theta_1) \right) = \frac{1}{2}\theta_1(1 + \theta_1).$$

The entrepreneur's problem can therefore be stated as follows: choose θ_1 in order to maximize the expected profit

$$\begin{aligned} E(\pi_1) &= (1 - \theta_1)^2 \cdot (2p_1 - K) + 2(1 - \theta_1)\theta_1 \cdot \left(p_1 + \frac{\theta_1 - p_r}{\theta_1} p_r - K \right) \\ &= -\frac{3}{2}\theta_1^3 + \left(\frac{1}{2} + K \right) \theta_1^2 + \theta_1 - K \end{aligned}$$

subject to the constraint that one pre-order is sufficient to finance the project, i.e. $p_1 = \frac{1}{2}\theta_1(1 + \theta_1) \geq K$. The unconstrained solution to this problem is derived from the necessary condition $\partial E(\pi_1)/\partial \theta_1 = 0$, yielding

$$\theta_1 = \frac{1}{9} \left(\sqrt{18 + (1 + 2K)^2} + 1 + 2K \right). \quad (11)$$

It will satisfy the constraint $p_1 \geq K$ if and only if capital requirement K is below a certain threshold $\bar{K} \approx 0.76$. Otherwise, the solution is given by $p_1 = K$ and $\theta_1 = \sqrt{2K + 1/4} - 1/2$, which is feasible for all $K \leq 1$. The entrepreneur's resulting expected profit from financing the project by at least one pre-order $E(\pi_1)$ is depicted in Figure 2.

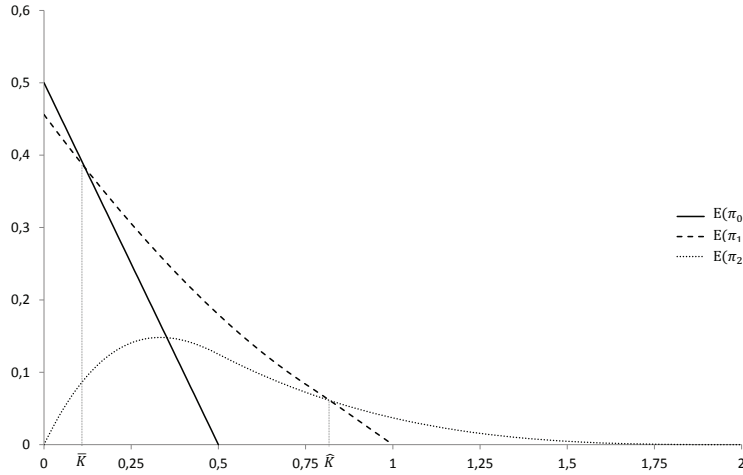


Figure 2: Graphs of $E(\pi_0)$, $E(\pi_1)$, and $E(\pi_2)$ as functions of K

Now consider a minimum number of two pre-orders. The project can

then be realized if and only if both potential buyers purchase in advance. The entrepreneur's problem can therefore be stated as follows: choose p_2 in order to maximize the expected profit

$$\begin{aligned} E(\pi_2) &= (1 - p_2)^2 \cdot (2p_2 - K) \\ &= 2p_2^3 - (4 + K)p_2^2 + (2 + 2K)p_2 - K \end{aligned}$$

subject to the constraints that two pre-orders are sufficient but one pre-order is insufficient to finance the project, i.e. $2p_2 \geq K > p_2$. The unconstrained solution to this problem is derived from the necessary condition $\partial E(\pi_2)/\partial p_2 = 0$, yielding $p_2 = (K + 1)/3$. It will satisfy the constraints $2p_2 \geq K > p_2$ if and only if $1/2 < K \leq 2$. It then yields the expected profit $E(\pi_2) = \left(\frac{2-K}{3}\right)^3$. For $K \leq 1/2$, no solution exists unless there is a smallest monetary unit μ . As $\mu \rightarrow 0$, the optimal price p_2 converges to K , yielding the asymptotic expected profit $E(\pi_2) = (1 - K)^2 K$. The entrepreneur's resulting expected profit from financing the project by at least two pre-orders $E(\pi_2)$ is also depicted in Figure 2.

The optimum expected profit from advance-purchase financing is given by the upper envelope of $E(\pi_1)$ and $E(\pi_2)$ as functions of K . As Figure 2 illustrates, it will be better to use contracts based on at least one (two) pre-order(s) if capital requirement K is below (above) a certain threshold $\bar{K} \approx 0.82$.

Observe that, with two potential buyers, the use of advance-purchase contracts as a device of price discrimination is limited in the following sense. While advance-purchase financing is still a profitable measure for sufficiently high fixed costs, the entrepreneur prefers traditional funding for capital requirements K below a certain threshold $\bar{K} \approx 0.11$. The reason for this limitation is a problem of free-riding arising from the public goods character of the customers' advance payments as contributions to the fixed costs. First, if financing is based on at least one pre-order, the agents' probability of being pivotal for the realization of the project will be smaller than one. This induces an advance-purchase price p_1 below the marginal willingness to pay θ_1 , i.e. a positive information rent even for the marginal pre-order. Second, if financing is based on at least two pre-orders, the advance-purchase price p_2 will be limited by the incentive compatibility constraint that one single pre-order must not be sufficient to cover the fixed costs. Both effects undermine the discriminatory power of advance-purchase contracts, reducing the gap between the advance-purchase price p_c and the regular price p_r .

Discussing the welfare effects for consumers, note once more that for

$1/2 < K < 2$, the probability of realization is zero under traditional funding but strictly positive under advance-purchase financing. Hence, the latter also Pareto-dominates the former in this range of fixed costs. For fixed costs $\bar{K} \leq K \leq 1/2$, for which the entrepreneur prefers advance-purchase financing over traditional funding, customers who pre-order, i.e. with valuations $\theta \geq \theta_c = \theta_1$ as given by equation (11), would actually prefer traditional funding and uniform pricing over financing based on advance-purchase contracts since, in this range, $p_c = p_1 > 1/2 = p_0$. Customers who do not pre-order, i.e. with valuations $\theta < \theta_c = \theta_1$ as given by equation (11), prefer advance-purchase financing over traditional funding if and only if

$$(1 - \theta_c)(\theta - p_r) \geq \theta - p_0 \quad \Leftrightarrow \quad \theta \leq \bar{\theta} := \frac{1 - \theta_c(1 - \theta_c)}{2\theta_c}.$$

Summing up, for $\bar{K} \leq K \leq 1/2$, customers with valuations below $\bar{\theta}$ prefer advance-purchase financing, whereas customers with valuations above $\bar{\theta}$ prefer traditional funding. As $\bar{\theta}$ decreases in θ_c and $\theta_c = \theta_1$ increases in K , threshold $\bar{\theta}$ decreases in K .

Similar to the case of one potential buyer only, the fact that pre-orderers prefer traditional funding here again raises the question whether the entrepreneur can commit to rely exclusively on advance-purchase financing. If the entrepreneur lacks such commitment power, only projects with $1/2 < K \leq 2$ can be realized based on advance-purchase contracts whereas all projects with $\bar{K} \leq K \leq 1/2$ have to be financed traditionally.

6 Conclusion

I studied a simple model of advance-purchase financing in which a monopolist has to meet some capital requirement in order to start production. He then sells the related good to a limited number of potential buyers who are privately informed about their willingness to pay. In contrast to most of the previous literature, I have shown that advance-purchase surcharges may arise as an optimal strategy of price discrimination. The discriminatory power, though, is limited by the problem of free-riding, which is exacerbated as the number of potential buyers increases.

The setting considered in this paper can easily be reinterpreted as a model of the monopolistic provision of excludable public goods under private information. Indeed, the introductory example of *R&D* in the public health sector allows for this reinterpretation. I have shown that advance-purchase arrangements enable more costly projects to be financed than traditional funding sources. Thus, the former mode of financing will Pareto-dominate

the latter if the capital requirement is sufficiently large. Besides efficiency concerns, following the *ability-to-pay-principle*, advance-purchase financing may also improve on the fairness of the allocation under traditional funding and uniform pricing: If the buyers' different valuations stem from differences in income and if the good under consideration is normal, richer customers will pay the advance-purchase surcharges. In contrast, poorer customers may be able to buy at the regular price below the uniform price under traditional funding. In the context of *R&D* in the international health sector, this means that, in many instances, advance purchase contracts could indeed improve the availability of new drugs and vaccines in poorer countries.

The analysis presented here assumes that the bargaining power is on the seller side, though it is limited by the lack of commitment to not renegotiate with initially excluded customers and the inability to commit to some funding goal above the actual capital requirement. These limitations are captured by the specific structure of the sequential game I consider. The corresponding constraints on the set of admissible contracts are motivated by their relevance in practise. With the uninformed side of the market proposing the contract, this leads to a tractable screening model. However, further limitations of the entrepreneur's bargaining power could be considered. For example, think of commitment problems with respect to the exclusive use of advance-purchase contracts for funding the project as already discussed above. With more and more restrictions on the entrepreneur's bargaining power and the number of buyers being small, one may prefer to switch to a signaling model in which the informed side of the market makes proposals.

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