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Abstract

Mortgages are prime examples of long-term nominal loans. As a result, under incomplete asset markets, monetary policy can affect household decisions through the cost of new mortgage borrowing and the value of payments on outstanding debt. These channels are distinct from the transmission through real interest rates. A stylized general equilibrium model incorporating these features is developed. Persistent monetary policy shocks, resembling the level factor in the nominal yield curve, have larger real effects than transitory shocks. The transmission is stronger under adjustable- than fixed-rate mortgages. Higher, persistent, inflation benefits homeowners under FRMs but hurts them under ARMs.

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1 Introduction

Most theories of how monetary policy affects the real economy rely on some form of nominal rigidity. The typical rigidities in this context come from product and labor markets in the form of prices or wages set for a number of periods ahead in nominal terms. As a result, nominal variables under the control of a monetary authority affect ex-ante real interest rates and relative prices. A somewhat overlooked nominal rigidity, with potentially important macroeconomic implications, however, lies at the heart of housing markets.

Specifically, standard mortgage loans require homeowners to make *nominal installments*—regular interest and amortization payments—during the life of the loan. The installments are set so as to guarantee that, given the mortgage interest rate, the principal is gradually repaid in full by the end of the mortgage term, typically 20 to 30 years. A standard fixed-rate mortgage (FRM), characteristic for instance for the United States, has a fixed nominal interest rate and constant nominal installments, set at origination, for the entire term of the loan. An adjustable-rate mortgage (ARM), typical for instance for the United Kingdom, sets nominal installments on a period-by-period basis so as, given the current short-term nominal interest rate, the loan is expected to be repaid in full during its remaining term. Most mortgage loans are variants of these two basic contracts.¹

This paper studies the macroeconomic and redistributive consequences of this particular aspect of mortgages. Our aim is to characterize the channels through which this feature of the contracts transmits monetary policy shocks to the real economy, especially to residential investment, and to investigate the relevance of these channels in general equilibrium. To provide a clear characterization and isolate the effects of this particular nominal rigidity,

¹Most countries have typically one of the two types dominating. In the United States, FRMs account on average (1982-2006) for 70% of mortgage originations (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10); before 1982, they were essentially the only contract available. Other countries in which FRMs—with interest rates fixed for at least 10 years—have traditionally dominated the mortgage market include Belgium, Denmark, and France; in most other advanced economies, either ARMs or FRMs with interest rates fixed for less than 5 years prevail; see Scanlon and Whitehead (2004) and European Mortgage Federation (2012a). Countries also differ in terms of prepayment penalties, costs of refinancing, recourse, and other details of the contracts. Research is still inconclusive on the causes of the cross-country heterogeneity (e.g., Miles, 2004; Green and Wachter, 2005; Campbell, 2012). We take the form of the contracts as given and abstract from many institutional details.

the paper abstracts from other nominal frictions, as well as other channels through which housing finance affects the macroeconomy.²

Some basic facts suggest potential importance of mortgages for monetary policy. Mortgage payments (interest and amortization) as a fraction of income—the so called ‘debt-servicing costs’—are nontrivial. Our estimates suggest that, on average over the past 30-40 years, they were equivalent to 15-22% of the pre-tax income of the 3rd and 4th quintiles of the U.S. wealth distribution, representing the typical homeowner (Campbell and Cocco, 2003). Hancock and Wood (2004) report that in the United Kingdom this ratio fluctuated between 15% and 20% over the period 1991-2001. In Germany, mortgage debt servicing costs are reported to be around 27% of disposable income (European Mortgage Federation, 2012b), in Denmark 36.5% (for first-time homeowners; European Mortgage Federation, 2012c), and in France 30% (for first-time homeowners; European Mortgage Federation, 2009). In terms of mortgage debt, the mortgage debt to (annual) GDP ratio in advanced economies has reached on average 70% in 2009, even though there is a substantial cross-country variation in this ratio (from 22% in Italy to 105% in the Netherlands; International Monetary Fund, 2011, Chapter 3). In some countries outstanding mortgage debt is even larger than government debt and its maturity is longer.³ Despite these facts, a recent survey article by Campbell (2012) reveals a glaring absence of mortgages—in the sense of long-term nominal debt contracts for house purchases—in macro models used for monetary policy analysis. The model developed in this paper aims to provide a step towards filling this gap.

The nominal rigidity in mortgages considered here leads to two channels of monetary policy transmission. One channel works through new borrowing (a price effect) and is *qualitatively the same* under both FRMs and ARMs. The other channel works through outstanding mortgage debt (current and expected future wealth effects) and is even *qualitatively different* under the two contracts. Both channels are distinct from standard transmissions through

²For instance, following Iacoviello (2005), a large literature focuses on the role of housing as a collateral facilitating borrowing for consumption purposes. We abstract from this channel.

³For instance, Hilscher, Raviv, and Reis (2014) show that in the United States government debt is predominantly of short-term maturity.

real interest rates—in our framework, monetary policy has real effects even if it has no direct effect on the ex-ante short-term real interest rate.⁴

Regarding the first channel, under incomplete financial markets, households value future cash flows differently than mortgage investors, for whom (in the absence of arbitrage opportunities) the present value of a new one-dollar loan is one dollar. As a result, even in the absence of movements in the ex-ante real interest rate, changes in the expected path of future short-term nominal interest rates, and thus inflation, affect the expected distribution of debt-servicing costs over the term of a new loan and hence the present value of its mortgage payments from households' perspective. House purchase decisions thus depend not only on the nominal interest rate and inflation between today and tomorrow, but on the entire path of these variables through out the term of the loan. We refer to this channel as the 'price effect', as it constitutes a part of the effective price of new housing. It is a dynamic version of the tilt/frontloading effect (e.g., Schwab, 1982) and resembles a 'taste shock' in the household's first-order condition for housing. Qualitatively, this effect is similar across FRMs and ARMs.

In the second channel, monetary policy affects the current and expected future real payments on outstanding mortgage debt and thus disposable income. We refer to these effects as current and future 'wealth effects' and the nature of the contract is critical for their direction. In the case of FRM, only the inflation rate affects the payments: a higher inflation rate reduces the real value of outstanding debt and thus the real value of the payments homeowners have to make. The strength of this effect increases with inflation persistence, amplifying the redistribution in favor of homeowners. In the case of ARM, the short-term nominal interest rate also matters. The immediate effect of an equiproportionate persistent increase in the two rates is an increase in the real value of interest payments—the effect of a higher nominal interest rate dominates the counterbalancing effect of higher inflation. As a result, mortgage payments increase *as if* monetary policy had a control over

⁴In our model, the real interest rate—equal to the marginal product of capital—responds to monetary policy only indirectly through general equilibrium effects.

the short-term real interest rate. Over time, however, the cumulated effect of persistently high inflation becomes stronger, gradually reducing the real value of the payments. Because most of the wealth effects are front-loaded under ARM but back-loaded under FRM, they are larger under ARM than under FRM when households discount future income. The form of the mortgage contract is thus critical for both the sign and size of the redistributive effects of inflation. In both cases, the quantitative effects depend also on the mortgage debt to GDP ratio.

These channels are first demonstrated analytically in a simple partial equilibrium model of a homeowner. After that they are embedded in a stylized general equilibrium model with incomplete asset markets. The aim of the model is to check if the main lessons learnt from the simple model carry over to general equilibrium. The model has a continuum of two agent types, homeowners and capital owners; there is a representative agent of each type. Motivated by Campbell and Cocco (2003), homeowners represent the 3rd and 4th quintiles of the wealth distribution, while capital owners represent the 5th quintile. As such, homeowners derive income from labor, which they supply elastically, and invest in housing capital, financing a given fraction of housing investment with mortgages. Capital owners do not work and invest in capital used in production, one-period nominal bonds, and mortgages, pricing the assets competitively by arbitrage (in an alternative version of the model homeowners can also access the noncontingent one-period bond market).⁵ The production side of the economy is standard, consisting of perfectly competitive producers and homebuilders. Monetary policy is characterized by an interest rate rule. As the two agent types do not trade a full set of state-contingent securities, their stochastic discount factors are not equalized. No-arbitrage pricing implies that capital owners are indifferent across the three assets and new mortgage lending is determined by housing demand, affected

⁵In the data, the 3rd and 4th quintiles have one major asset, a house, and one major liability, a mortgage. Their main source of income is labor income. In contrast, the 5th quintile hold almost the entire corporate equity in the economy and housing is a less important component of their asset composition; labor income is also a less important source of their income. The 1st and 2nd quintiles are essentially renters with no assets and little liabilities and are not included in the model. Our separation of the population into the two types with different assets has a flavor of Guvenen (2009).

by monetary policy, through the channels described above. Market incompleteness in this model is a necessary, but not sufficient, condition for monetary policy to have real effects. Without mortgages, monetary policy is close to neutral.⁶

Three key lessons, suggested by the partial equilibrium model, are confirmed by the general equilibrium analysis. First, the size of the real effects of monetary policy shocks increases with the persistence of the shocks: persistent monetary policy shocks that manifest themselves as the level factor in the nominal yield curve (they shift both short and long rates) have a larger impact than transitory shocks, manifesting themselves as the long-short spread. Second, monetary policy shocks have larger real effects under ARM than FRM. Broadly speaking, this is because the price and wealth effects reinforce each other under ARM, but tend to offset each other under FRM. And third, higher inflation redistributes income from capital owners to homeowners under FRM, but (at least initially) from homeowners to capital owners under ARM. For all three results, the real effects are stronger the more difficult it is for homeowners to smooth consumption over time through other financial assets.

A recent micro-level study by Di Maggio, Kermani, and Ramcharan (2014) provides empirical support for some of the predictions of the model. Specifically, the authors find that U.S. homeowners respond more to interest rate changes in counties with a larger fraction of ARMs than FRMs. Furthermore, households that are more financially constraint respond more. Even though the changes in nominal interest rates identified by their study may not necessarily occur due to inflation, as mentioned above, real mortgage payments under ARMs respond similarly to nominal interest rate shocks, at least initially, regardless of the source of the movements.

Two other studies, Ghent (2012) and Calza, Monacelli, and Stracca (2013), address the role of mortgages in the monetary transmission mechanism (other less directly related studies are covered in the next section). In both studies monetary policy transmits by affecting the

⁶By this we mean that if, instead of mortgages, housing investment is financed through one-period nominal loans, monetary policy has real effects only due to small current wealth effects (i.e., the realized real rate of return on the one-period loan). If housing loans are specified in real terms, monetary policy is superneutral even if markets are incomplete.

ex-ante short-term real interest rate. Ghent (2012) investigates the transmission mechanism in the presence of FRMs that are denominated in real—rather than nominal—terms. In our model, monetary policy is neutral when mortgages are set in real terms. Calza et al. (2013) extend the model of Iacoviello (2005) to distinguish between FRMs and ARMs. FRM is implemented in their model as a two-period loan while ARM is assumed to be a standard one-period loan. We demonstrate that shortening the term of the loan reduces the real effects in our model and that when mortgages are modeled as one-period loans, the real effects essentially disappear.⁷ In this sense, the channels studied here provide a novel mechanism for the real effects of monetary policy that is complementary to standard real interest rate channels, operating either through housing markets, as in the above studies, or the New-Keynesian mechanism. In addition, the channels are relevant for small open economies that take the real rate as given.⁸

The paper proceeds as follows. Section 2 relates the paper to a broader literature. Section 3 explains the nature of the nominal rigidity. Section 4 describes the general equilibrium model. Section 5 explains how the model can be mapped into data and calibrates the parameters. Section 6 reports the findings. Section 7 concludes and offers suggestions for future research. A supplemental material (for online publication only) contains (i) a list of the model’s equilibrium conditions, (ii) computation, (iii) a description of the data counterparts to the variables in the model, (iv) estimates of mortgage debt servicing costs for the United States, and (v) results demonstrating the robustness of the main findings to alternative parameterizations of the model.

⁷We also show that an extreme case of refinancing (prepayment)—if it occurs every period—is akin to one-period loans.

⁸There may be other housing market channels of monetary policy transmission, such as the effects of monetary policy on term premia, mortgage markups, or the primary-secondary spread. This paper abstracts from these channels.

2 Other related studies

The paper is related to different strands of the literature. First, following Iacoviello (2005), a number of studies focus on the interaction between sticky prices, borrowing constraints, and the collateral value of housing (Iacoviello, 2010, contains a brief summary of this line of research). Housing finance in these models takes the form of one-period loans and, due to sticky prices, monetary policy transmits to the real economy by affecting the ex-ante short-term real interest rate. The presence of borrowing constraints and housing as a collateral then further amplifies this mechanism.⁹ We abstract from both of these channels. On the other hand, this literature abstracts from mortgages and the nominal rigidity they contain.¹⁰

Second, a number of earlier studies investigate the effect of inflation on the housing market. This is done in the context of mortgage contract design (Lessard and Modigliani, 1975), a supply-demand econometric model (Kearl, 1979), and a consumer's optimal housing choice under different steady-state inflation rates (Schwab, 1982; Alm and Follain, 1984).¹¹ More recently, Brunnermeier and Julliard (2008) argue that inflation affects housing through money illusion, whereby households make home purchase decisions while ignoring the effects of inflation on future real mortgage payments. Aruoba, Davis, and Wright (2012) investigate the effect of inflation on housing investment in a money-search model with home production, to which housing is an important input.

Third, a few studies investigate the redistributive effects of monetary policy in economies with nominal debt contracts (Doepke and Schneider, 2006; Meh, Rios-Rull, and Terajima, 2010; Sheedy, 2013). In these models, higher inflation benefits borrowers. We show that, in the case of mortgage debt, the redistributive consequences critically depend, both qualita-

⁹Rubio (2011) extends the Iacoviello (2005) framework by considering one-period loans with a nominal interest rate evolving in a sluggish manner, as a weighted average of past interest rates. Graham and Wright (2007) study a similar setup, albeit without housing.

¹⁰Transmission into housing demand through the real interest rate on short-term loans is also studied outside of the Iacoviello (2005)-framework; i.e., abstracting from the collateral value of housing (see Edge, 2000; Li and Chang, 2004; Dressler and Li, 2009).

¹¹In addition, Poterba (1984) argues that, as the U.S. income tax brackets are set in nominal terms, mortgage finance and inflation interact due to the tax deductibility of mortgage interest payments. This feature, specific to the United States and a few other countries, adds an additional layer of nominal rigidity to a mortgage contract and is abstracted from in this paper.

tively and quantitatively, on whether mortgages are ARMs or FRMs. Auclert (2014) also considers a redistributive channel of monetary policy, but unlike the above studies and our paper, focusing on real interest rate effects. Redistributive effects of monetary policy are also at the heart of the transmission mechanism proposed by Sterk and Tenreyro (2013).

Fourth, the relationship between monetary policy and housing has been studied empirically in regression models (Kearl, Rosen, and Swan, 1975; Kearl, 1979) and more recently in structural VARs (e.g., Bernanke and Gertler, 1995; Iacoviello and Minetti, 2008; Calza et al., 2013). The more important ‘monetary policy shocks’ in our model, however, manifest themselves as the level factor in the nominal yield curve. As such, these shocks are not comparable to the standard monetary policy shocks in structural VARs. To mention just one difference, identification issues aside, the level factor is highly positively correlated with inflation, whereas the conventional response of inflation to a positive monetary policy shock in structural VARs is negative.

Finally, following Campbell and Cocco (2003), mortgages (or other long-term housing loans) have been considered in relation to public policies other than monetary policy. In particular, homeownership rates (Chambers, Garriga, and Schlagenhaut, 2009a,b) and foreclosures (Garriga and Schlagenhaut, 2009; Chatterjee and Eyigungor, 2011; Corbae and Quintin, 2011). The focus of these studies on steady-state equilibria allows the inclusion of various option-like features, such as the option to refinance or default, which our model abstracts from.¹² Mortgages also play a key role in the behavior of housing starts in a business cycle model of Kydland, Rupert, and Sustek (2014). But as the model has an estimated reduced-form process for short- and long-term nominal interest rates, inflation, and TFP, it does not allow the structural analysis of the effects of monetary policy shocks.

¹²An exception in this regard is Koijen, Van Hemert, and Van Nieuwerburgh (2009), who consider a mortgage choice problem with some option-like features in a model with aggregate shocks. Their agents and mortgages, however, live for only two periods and mortgages are priced by an exogenous process.

3 The nominal rigidity in mortgages

The nature of the nominal rigidity, and the resulting two channels of monetary policy transmission, are explained in partial equilibrium using a deterministic three-period model. An extension to an infinite horizon and uncertainty is straightforward but at the cost of extra notation and more complicated expressions. These extensions are carried out in the next section, which also endogenizes the variables that here are either held constant (real labor income and the real interest rate) or controlled exogenously (the one-period nominal interest rate). The main insights, however, can be gained from the simple model used here.

3.1 Three-period model

Time is denoted by $t = 1, 2, 3$. Each period a household is endowed with constant real income w and in $t = 1$ has no outstanding mortgage debt (outstanding debt is introduced later). In $t = 1$, the household makes a once-and-for-all housing investment decision, financing a fraction θ of the investment with a loan and a fraction $1 - \theta$ with income. The loan can be used only for housing investment and the house lasts for $t = 2, 3$. The life-time utility function of the household is $V = \sum_{t=1}^3 \beta^{t-1} u(c_t) + \sum_{t=2}^3 \beta^{t-1} g(h)$, where β is a discount factor, c_t is period- t nonhousing consumption, h is housing, and $u(\cdot)$ and $g(\cdot)$ have standard properties. The household maximizes utility with respect to c_1, c_2, c_3 , and h , subject to three per-period budget constraints: $c_1 + h = w + l/p_1$, $c_2 = w - m_2/p_2$, and $c_3 = w - m_3/p_3$, where $l = \theta p_1 h$ is the nominal value of the loan, m_2 and m_3 are nominal loan installments (to be specified below), and p_t is the aggregate price level (i.e., the price of goods in terms of an abstract unit of account; this section abstracts from the relative price of housing). Assume there is a financial market that prices assets by the no-arbitrage principle but in which the household does not participate due to, for instance, high entry costs (in the full model this assumption is partially relaxed). Assume also that monetary policy controls the one-period nominal interest rate i_t . The absence of arbitrage restricts i_t to satisfy $1 + r = (1 + i_t)/(1 + \pi_{t+1})$, where $1 + r$ is a gross rate of return on real assets, assumed to be constant and given by some

pricing kernel $\mu^* = (1 + r)^{-1}$, and $\pi_{t+1} \equiv p_{t+1}/p_t - 1$ is the inflation rate between periods t and $t + 1$.

3.2 Mortgages

Installments of a standard mortgage satisfy a general form $m_2 \equiv (i_2^M + \gamma)l$ and $m_3 \equiv (i_3^M + 1)(1 - \gamma)l$. Here, i_t^M denotes the mortgage interest rate (henceforth referred to as the ‘mortgage rate’). Under FRM, $i_2^M = i_3^M = i^F$; under ARM, i_2^M and i_3^M may be different. Further, γ is the amortization rate in the first period of the life of the mortgage, when the outstanding nominal debt is l . In the second period, the outstanding nominal debt is $(1 - \gamma)l$ and the amortization rate is equal to one (i.e., the mortgage is repaid in full).

FRM prescribes constant nominal installments: $m_2 = m_3$. The amortization rate therefore solves $i^F + \gamma = (i^F + 1)(1 - \gamma)$, which yields $\gamma = 1/(2 + i^F) \in (0, 0.5)$, for $i^F > 0$. As $d\gamma/di^F = -1/(2 + i^F)^2 \in (-0.25, 0)$, the installments m_2 and m_3 increase (for a given l) when i^F increases. Under ARM, $\gamma = 1/(2 + i_2^M) \in (0, 0.5)$, for $i_2^M > 0$. If $i_3^M > i_2^M$ then $m_3 > m_2$ and vice versa. It is also the case that $d\gamma/di_2^M \in (-0.25, 0)$ and therefore that m_2 increases when i_2^M increases.

3.2.1 Mortgage pricing and housing investment under FRM

In the absence of arbitrage, i^F has to satisfy

$$1 = Q_1^{(1)}(i^F + \gamma) + Q_1^{(2)}(1 - \gamma)(i^F + 1), \quad (1)$$

where $Q_1^{(1)} = (1 + i_1)^{-1}$ and $Q_1^{(2)} = [(1 + i_1)(1 + i_2)]^{-1}$ are the period-1 prices of one- and two-period zero-coupon bonds, determined according to the expectations hypothesis. Condition (1) states that the present value of installments for a mortgage of size one is equal to one. Notice that if $\gamma = 1$, the mortgage becomes a one-period bond and if $\gamma = 0$, the mortgage becomes a coupon bond. It is straightforward to show that, for $\gamma \in [0, 1)$, $i_1 < i_2$ implies $i_1 < i^F < i_2$ and vice versa.

The household's first-order condition is $u'(c_1)(1 + \tau_H) = \beta(1 + \beta)g'(h)$, where

$$\tau_H = -\theta \left\{ 1 - \left[\mu_{12} \frac{i^F + \gamma}{1 + \pi_2} + \mu_{12}\mu_{23} \frac{(1 + i^F)(1 - \gamma)}{(1 + \pi_2)(1 + \pi_3)} \right] \right\} \quad (2)$$

is a wedge between the marginal utility of period-1 nonhousing consumption and the marginal lifetime utility of housing, and where $\mu_{t,t+1} \equiv \beta u'(c_{t+1})/u'(c_t)$ is the household's 'stochastic' discount factor. Notice that the wedge works like an ad-valorem tax/subsidy on housing investment and that the expression within the square brackets is the present value of real installments from the household's perspective (i.e., the installments are evaluated at the household's stochastic discount factor, $\mu_{t,t+1}$, rather than the pricing kernel of the financial market, μ^*). The present value represents the cost of the mortgage to the household. Because the household does not trade in the financial market, in general, $\mu_{t,t+1} \neq \mu^*$ and the present value is different from one. Equation (2) shows that, in general, the wedge depends on nominal variables, i^F , π_2 , and π_3 . By controlling i_1 and i_2 —and thus, through the no-arbitrage conditions, i^F , π_2 , and π_3 —monetary policy affects τ_H and the household's optimal choice of h . This is the *price effect*. Subsection 3.3 contains numerical examples of the price effect for a 30-year FRM.

3.2.2 The role of market incompleteness and the term of the loan

When $\mu_{t,t+1} = \mu^*$, $\tau_H = 0$ and monetary policy is neutral (in fact superneutral, but the shorter 'neutral' is used through out the paper). Market incompleteness is thus a necessary condition for any real effects of monetary policy in this environment.

When $\mu_{t,t+1} \neq \mu^*$, neutrality results if $\gamma = 1$ (one-period loan). In this case, $\tau_H = -\theta \{1 - \mu_{12}[(1 + i_1)/(1 + \pi_2)]\}$, where $(1 + i_1)/(1 + \pi_2) = 1 + r = (\mu^*)^{-1}$, and μ_{12} is evaluated at $c_2 = w - \theta(1 + r)h$. Thus, while market incompleteness is a necessary condition, it is not sufficient for monetary policy to have real effects in this environment.

Neutrality also results under $\mu_{t,t+1} \neq \mu^*$ when the loan is a 2-period nominal zero-coupon bond; i.e., $m_2 = 0$ and $m_3 = (1 + i_1)(1 + i_2)l$. Here, implicitly, $\gamma = -i_1 < 0$.

It is straightforward to show that in this case $\tau_H = (\mu_{12}\mu_{23})/(\mu^*)^2$, where μ_{12} and μ_{23} are evaluated, respectively, at w and $w - (1+r)^2\theta h$. Thus, a long-term nature of the loan, by itself, also does not guarantee real effects.

The key ingredient needed for the existence of the price effect is that the long-term contract specifies periodic nominal payments. That is, $\gamma \in (-i_1, 1)$. In this case, the nominal variables in equation (2) do not cancel out and monetary policy affects τ_H . The value of γ controls the form of the nominal rigidity. For instance, in the case of $\gamma = 0$ (coupon bond), the nominal payments are concentrated in $t = 3$ and monetary policy works primarily by changing the real value of the repayment of the principal. In the case of a mortgage, the nominal rigidity is distributed evenly over the term of the loan.¹³

3.2.3 Mortgage pricing and housing investment under ARM

Under ARM, $i_2^M = i_1$ and $i_3^M = i_2$ guarantees the absence of arbitrage:

$$Q_1^{(1)}(i_2^M + \gamma) + Q_1^{(2)}(1 - \gamma)(i_3^M + 1) = \frac{i_1 + \gamma}{1 + i_1} + \frac{(1 - \gamma)}{(1 + i_1)} \left[\frac{(i_2 + 1)}{(1 + i_2)} \right] = 1.$$

The household's first-order condition takes the same form as under FRM, but with a wedge

$$\tau_H = -\theta \left\{ 1 - \left[\mu_{12} \frac{i_1 + \gamma}{1 + \pi_2} + \mu_{12}\mu_{23}(\mu^*)^{-1} \frac{1 - \gamma}{1 + \pi_2} \right] \right\}, \quad (3)$$

where we have used $(\mu^*)^{-1} = (1 + i_2)/(1 + \pi_3)$.¹⁴

The issues related to nonneutrality covered in the preceding subsection apply equally to the ARM case. As the ARM γ satisfies $\gamma \in (-i_1, 1)$, τ_H depends on nominal variables and monetary policy affects the household's optimal choice of h . For instance, a decline in i_1

¹³The focus of the paper is on mortgages, as opposed to coupon bonds typically issued by corporations, as long-term corporate assets are less debt-dependent than housing (long-term corporate assets are typically more than 75% financed through retained earnings and other forms of equity; Rajan and Zingales, 1995). For a model with nominal corporate debt see Gomes, Jermann, and Schmid (2013). The issues discussed here in relation to mortgages apply also to car loans. We abstract from car loans as mortgage debt has a longer term and makes up a much larger fraction of household debt than car loans.

¹⁴In actual economies, ARM rates are usually set as a constant mark up over a short-term government bond interest rate.

reduces the real installments in $t = 2$: through the Fisher effect, π_2 declines one for one with i_1 but—as $\gamma \in (0, 0.5)$ and $d\gamma/di_2^M \in (-0.25, 0)$ —the effect on the numerator in the first term in equation (3) is stronger than the effect on the denominator. The decline in π_2 , however, increases real mortgage installments in $t = 3$, thus increasing the second term in equation (3). If the household’s stochastic discount factor assigns a sufficiently large weight on payments in $t = 2$, the wedge declines. (Applying the same argument, the wedge declines also in the FRM case, as long as i^F falls enough in response to the decline in i_1 .) Subsection 3.3 contains some numerical examples of the price effect for a 30-year ARM.

3.2.4 Outstanding mortgage debt

Let us now abstract from the housing investment decision and focus instead on how monetary policy affects the real value of payments on outstanding mortgage debt. Suppose that in $t = 1$ the household has some outstanding mortgage debt l_0 , taken out in $t = 0$ and maturing in $t = 2$. The household’s budget constraint in $t = 1$ is $c_1 = w - \tilde{m}_1$, where $\tilde{m}_1 \equiv m_1/p_1 = [(i_1^M + \gamma)/(1 + \pi_1)]\tilde{l}_0$, with $\tilde{l}_0 \equiv l_0/p_0$. The mortgage rate i_1^M is predetermined in $t = 1$; it is equal to some i_0^F under FRM and to i_0 , the period-0 short rate, under ARM. Clearly, a lower π_1 generates a negative *current wealth effect* for the household in $t = 1$. This is the standard wealth effect considered in the literature reviewed in Section 2, which is present also in the case of one-period loans ($\gamma = 1$). In $t = 2$, the real payments on the 2-period loan are, respectively under FRM and ARM,

$$\tilde{m}_2 = \frac{i_0^F + 1}{(1 + \pi_1)(1 + \pi_2)}(1 - \gamma)\tilde{l}_0 \quad \text{and} \quad \tilde{m}_2 = \frac{1 + r}{1 + \pi_1}(1 - \gamma)\tilde{l}_0,$$

where in the second equation we have used $1 + r = (i_1 + 1)/(1 + \pi_2)$. Thus, a lower π_1 generates not only a negative wealth effect in $t = 1$, but also a negative *expected future wealth effect*, as it increases real payments in $t = 2$.

In the ARM expression for \tilde{m}_2 above, i_1 and π_2 get replaced by r due to the 2-period term of the loan considered here. To gain further insight into how monetary policy affects

the real payments on ARM debt, suppose that the loan has a 3-period term, maturing in $t = 3$. In $t = 2$, the real mortgage payments under both contracts are

$$\tilde{m}_2 = \frac{i_2^M + \gamma_2}{(1 + \pi_1)(1 + \pi_2)}(1 - \gamma_1)\tilde{l}_0, \quad (4)$$

where γ_2 is a period-2 amortization rate and $i_2^M = i_0^F$ under FRM and $i_2^M = i_1$ under ARM. As before, a lower π_1 generates a negative expected future wealth effect under both contracts. Now consider a decline in i_1 in $t = 1$ and the resulting equiproportionate decline in π_2 . This increases \tilde{m}_2 under FRM but reduces \tilde{m}_2 under ARM: it is straightforward to check that, as $\gamma_2 \in (0, 0.5)$ and $d\gamma_2/di_1 \in (-0.25, 0)$, a decline in i_1 , accompanied by an equiproportionate decline in π_2 , reduces \tilde{m}_2 as the effect on the numerator in equation (4) is greater than the effect on the denominator.¹⁵ In $t = 3$, the payments are

$$\tilde{m}_3 = \frac{i_3^M + 1}{(1 + \pi_1)(1 + \pi_2)(1 + \pi_3)}(1 - \gamma_2)(1 - \gamma_1)\tilde{l}_0,$$

where $i_3^M = i_0^F$ under FRM and $i_3^M = i_2$ under ARM, with $(i_2 + 1)/(1 + \pi_3) = 1 + r$. Under both contracts, the payments in $t = 3$ consist mainly of amortization (for plausible values of i_3^M) and a lower π_2 increases \tilde{m}_3 under both contracts and by similar magnitudes. Thus, in sum, while under FRM a lower i_1 increases real mortgage payments on outstanding debt in both $t = 2$ and $t = 3$, under ARM it increases the payments in $t = 3$ but reduces them in $t = 2$.

3.2.5 Inflation shocks as shocks to the real interest rate

Notice that in the case of ARM, in both the price and wealth effects, a decline in the inflation rate works in the first period of the life of the loan like a decline in the short-term real interest rate. In both effects, the real payments on a one-dollar loan are $(i_1 + \gamma)/(1 + \pi_2)$, where

¹⁵The properties of γ_2 listed here are derived from the equation $(i_1 + \gamma_2)(1 - \gamma_1) = (i_1 + 1)(1 - \gamma_2)(1 - \gamma_1)$, which states that the installments in periods 2 and 3 have to be equal, conditional on i_1 . This yields $\gamma_2 \approx (1 - \gamma_1)/(2 + i_1 - \gamma_1)$, which, for some $\gamma_1 \in (0, 1)$, is in the interval $(0, 0.5)$. Taking the derivative with respect to i_1 then confirms that $d\gamma_2/di_1 \in (-0.25, 0)$, for $\gamma_1 \in (0, 1)$.

$\gamma \in (0, 1)$. In the case of a 20-30 year mortgage, the amortization rate at the front end of the loan is in fact close to zero (in the numerical example below the initial amortization rate, for instance, is 0.00162). The above expression can thus be approximated as $i_t/(1 + \pi_{t+1})$. For a reasonably small π_{t+1} , this can be further approximated as i_t . Thus, at the front end of the loan, the movements in real mortgage payments occur essentially only due to movements in the nominal interest rate, regardless of whether the movements occur due to changes in the real rate (e.g., due to sticky prices) or the inflation rate, as in this case.

3.3 Numerical examples: 30-year mortgage

Figure 1 provides a numerical example to illustrate the price and expected future wealth effects in the case of a 30-year mortgage. Specifically, it plots debt-servicing costs, \tilde{m}_t/w , over the term of the loan under two alternative paths of i_t ; a constant $i_t = 4\%$ and a mean-reverting decline of i_t to 1% in period 1, which we refer to as ‘monetary policy easing’. The persistence of the decline is 0.95, which is the average autocorrelation of the short rate in the data. All the assumptions of the 3-period model—constant r and w and no-arbitrage pricing, with equation (1) extended to 120 quarters—are maintained here. The parameterization is $r = 1\%$ per annum and $\tilde{l} = 4w$. Indeed, in the model, the household chooses l optimally (by choosing h). The point here is simply to illustrate the size of the effects for one particular loan size.¹⁶

At the steady-state $i = 4\%$, debt-servicing costs are front-loaded and decline monotonically over the life of the mortgage, here from 29% to 6.5%. This is the standard ‘tilt effect’ (e.g., Schwab, 1982), which occurs due to a positive inflation rate (in this case 3%).¹⁷ The figure further shows that the price and expected future wealth effects associated with the

¹⁶The parameterization of the loan size is based on the average ratio, 1975-2010, of the median price of a new home (assuming a loan-to-value ratio of 76%) to the median household net income (assuming an income tax rate of 23.5%). The data on both house prices and incomes are from the U.S. Census Bureau. The loan-to-value ratio is the average ratio for single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10). The tax rate is an estimate discussed in Section 5.

¹⁷Positive inflation deflates the real value of mortgage payments in later periods of the life of the loan, which means that mortgage investors have to be compensated by higher real payments at the beginning.

equiproportionate decline in the short rate and inflation are as expected.

Starting with the case of a new loan, under both ARM and FRM, debt-servicing costs decline at the front end, where they are the highest, and somewhat increase at the back end, where they are the smallest. The decline under FRM is smaller than under ARM because the FRM interest rate, due to the mean-reverting nature of the short rate in this example, declines by less than the short rate itself. The flattening of the path of debt-servicing costs results in smoother consumption and thus a decline in τ_H under a sufficiently concave utility function (and/or sufficiently small β). Using a log utility function and $\beta = 0.9883$, a baseline parameterization of the model of the next section, τ_H declines by 1.66 percentage points in the case of FRM and by 3.83 percentage points in the case of ARM. If the risk-aversion coefficient is increased from 1 to 2, the declines of τ_H are 3.2 and 7.76 percentage points, respectively. (Recall that the wedge has a direct interpretation as an ad-valorem tax on new housing.)

For the case of an existing loan, we consider a loan with 119 periods remaining (the magnitudes of the expected future wealth effects decline as the remaining term of the loan gets shorter). In the case of ARM, as the loan is only one period into its life, the expected path of debt-servicing costs is essentially the same as that for the new loan. That is, there is a sharp immediate decline in debt-servicing costs, followed by their increase several periods later. Under FRM, however, the persistently low inflation leads to a gradual increase in debt-servicing costs for the remainder of the term of the loan. Thus, while in the case of ARM, the price and expected future wealth effects work in the same direction, in the case of FRM they work in opposite directions.

Figure 2 plots the results of the same experiment for two alternative degrees of persistence of the decline in the short rate, 0.99 and 0.5. In the 0.99 case, the wedge declines by 5.42% under FRM and by 6.34% under ARM (log utility). Compared to the 0.95 case, the magnitudes are larger. They are also more similar to each other as the long rate drops almost as much as the short rate. The debt-servicing costs on the existing loans, however, diverge

further apart. When the persistence is 0.5, the wedge declines by only 0.02% under FRM, as the long rate hardly responds, and by 0.57% under ARM, as the short rate mean-reverts quickly. The expected future wealth effects are also much smaller than in the 0.95 case (like the price effect, hardly noticeable in the FRM case).

3.4 Housing finance without the nominal rigidity

Before moving on to the general equilibrium model, we briefly discuss in the context of the three-period model two housing finance arrangements, used in the literature, that are not subject to the nominal rigidity in mortgages.

3.4.1 Index-linked mortgage

An index-linked mortgage (considered by, e.g., Ghent, 2012), also known as a price-level adjusted or real mortgage, is a mortgage that adjusts the principal for changes in the price level. The nominal installments are thus $m_2 = (i_2^M + \gamma)[(1 + \pi_2)l]$ and $m_3 = (i_3^M + 1)(1 - \gamma)[(1 + \pi_2)(1 + \pi_3)l]$. Arbitrage imposes $i_2^M = i_3^M = r$. As a result, real installments are $\tilde{m}_2 = (r + \gamma)\tilde{l}$ and $\tilde{m}_3 = (r + 1)(1 - \gamma)\tilde{l}$, rendering monetary policy neutral. The wedge in this case is $\tau_H = -\theta \{1 - [\mu_{12}(\gamma + r) + \mu_{12}\mu_{23}(r + 1)(1 - \gamma)]\}$.¹⁸

3.4.2 Sequence of one-period loans and refinancing

Suppose the household can adjust h and l in $t = 2$. That is, the household chooses $l_t = \theta p_t h_t$ in periods $t = 1, 2$ and pays back $(1 + i_{t-1})l_{t-1}$ in periods $t = 2, 3$. This is a common assumption in the models with housing, borrowing constraints, and sticky prices reviewed in Section 2 (e.g., Iacoviello, 2005).¹⁹ Such arrangement is akin to period-by-period refinancing:

¹⁸Monetary policy can have real effects if it affects the real interest rate r . Frictions, such as sticky prices or liquidity effects, that make adjustments in prices sluggish in response to changes in i_1 would thus have an effect on τ_H . The strength of this channel will depend on the persistence of the real rate—or, equivalently, the sluggishness of the price adjustment—as the real rate in both $t = 2$ and $t = 3$ affects τ_H (generally, the entire path of the real rate matters).

¹⁹The constraint in these models is slightly different from our version of it. Usually, it takes the form $[(1 + i_t)/(1 + \pi_{t+1})](l_t/p_t) \leq \theta h_t$. That is, repayment of the one-period loan with interest, in real terms, must be less or equal to a fraction of the value of the house. The constraint is usually assumed to hold with

each period, an existing mortgage is fully prepaid (with interest) and a new mortgage—of a possibly different size and with a different interest rate—is taken out.²⁰ The sequence of loans results in wedges in periods $t = 1, 2$ given by $\tau_{Ht} = -\theta[1 - \mu_{t,t+1}(1 + r)]$, which are nonzero for $\mu_{t,t+1} \neq \mu^*$, but do not depend on nominal variables. Clearly, both period-by-period refinancing and holding the mortgage until maturity—an implicit assumption in our set up—are extreme cases. In reality, refinancing is an option, which the household may occasionally exercise. The nominal rigidity in mortgages in our model is thus at its extreme and the results are best viewed as upper bounds on the strength of the transmission mechanism under investigation.

4 General equilibrium model

The general equilibrium model extends the model of the previous section to an infinite horizon and uncertainty and endogenizes the variables that were either held constant (real labor income and the real interest rate) or were treated as exogenous (the short-term nominal interest rate). The purpose of this exercise is to check if the lessons learnt from the simple model carry over to general equilibrium and get a sense of the ballpark of the real effects.

4.1 Environment

The economy’s population is split into two groups, ‘homeowners’ and ‘capital owners’, with measures Ψ and $(1 - \Psi)$, respectively. Within each group, agents are identical. An aggregate production function combines capital and labor to produce a single good; houses require land in addition to structures. Capital owners own the economy’s capital stock, whereas homeowners supply labor and own the economy’s housing stock. Such abstraction is motivated by the cross-sectional observations discussed in the Introduction. Capital owners function as the mortgage investors kept outside of the three-period model. Real labor income is

equality in all states of the world. These details are unimportant for the point being made here.

²⁰If the size of the loan after refinancing is to stay the same and only the interest rate changes, period-by-period refinancing is akin to a variable-rate coupon bond, rather than a sequence of one-period loans.

endogenized by homeowners' labor supply decisions in competitive factor markets, the real interest rate is endogenized by the marginal product of capital, and the nominal interest rate is endogenized by a monetary policy feedback rule. We also relax the extreme assumption of no participation of homeowners in the financial market maintained in the three-period model by giving homeowners access, at a cost, to a one-period bond market. However, the case of no participation is considered as a benchmark. Where applicable, the notation is the same as in Section 3. Only new variables and functions are therefore defined. When a variable's notation is the same for both agent types, an asterisk (*) denotes the variable pertaining to capital owners.

4.1.1 Capital owners

A representative capital owner maximizes expected life-time utility

$$E_t \sum_{t=0}^{\infty} \beta^t u(c_t^*), \quad \beta \in (0, 1),$$

where $u(\cdot)$ has standard properties, subject to a sequence of budget constraints

$$c_t^* + x_{Kt} + \frac{b_{t+1}^*}{p_t} + \frac{l_t^*}{p_t} = [(1 - \tau_K)r_t + \tau_K\delta_K]k_t + (1 + i_{t-1})\frac{b_t^*}{p_t} + \frac{m_t^*}{p_t} + \tau_t^* + \frac{p_{Lt}}{1 - \Psi}. \quad (5)$$

Here, x_{Kt} is investment in capital, b_{t+1}^* is holdings of a one-period nominal bond between periods t and $t + 1$, τ_K is a capital income tax rate, $\delta_K \in (0, 1)$ is a depreciation rate, k_t is capital, and τ_t^* is a lump-sum transfer. In addition, $1/(1 - \Psi)$ is new residential land, which the capital owner receives each period as an endowment, and p_{Lt} denotes its price in terms of consumption. The capital stock evolves as

$$k_{t+1} = (1 - \delta_K)k_t + x_{Kt}. \quad (6)$$

As explained in the next section, the capital income tax rate, transfers, and tax deductible depreciation are included in order to allow a sensible calibration of the model to the data. Other than that, these elements of the model are unimportant. Land is also not essential for the analysis, but it allows deriving implications of the model for prices of new homes, as opposed to just new residential structures.

Mortgages in the economy are multi-period FRMs or ARMs, following the modeling approach of Kydland et al. (2014). Denoting by d_t^* the outstanding debt owed to the capital owner, the nominal mortgage payments received by the capital owner in period t are

$$m_t^* = (R_t^* + \gamma_t^*)d_t^*, \quad (7)$$

where R_t^* and γ_t^* are, respectively, the interest and amortization rates of the outstanding debt. The variables comprising m_t^* are state variables evolving as

$$d_{t+1}^* = (1 - \gamma_t^*)d_t^* + l_t^*, \quad (8)$$

$$\gamma_{t+1}^* = (1 - \phi_t^*) (\gamma_t^*)^\alpha + \phi_t^* \kappa, \quad (9)$$

$$R_{t+1}^* = \begin{cases} (1 - \phi_t^*)R_t^* + \phi_t^* i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \quad (10)$$

where $\phi_t^* \equiv l_t^*/d_{t+1}^*$ is the fraction of new loans in the outstanding debt next period and $\kappa, \alpha \in (0, 1)$ are parameters. Specifically, κ is the initial amortization rate of a new loan and α controls the evolution of the amortization rate over time. Notice that setting $\alpha = 0$ and $\kappa = 1$ implies $\gamma_t = 1 \forall t$. That is, l_t^* becomes a one-period loan. Setting $\alpha = 1$ results in a constant amortization rate $\gamma_t = \kappa$ and thus declining mortgage payments over the life of a loan. When $\kappa, \alpha \in (0, 1)$, the amortization rate increases (converging to one) as debt gets more and more amortized, thus approximating payments on standard mortgages loans, including the property that the share of amortization in mortgage payments increases over

the life of a loan.²¹ Notice that, even though new loans are extended every period, each new loan l_t^* (both FRM and ARM) is a long-term loan, starting with an amortization rate $\kappa < \gamma_t^*$. Furthermore, loans are never refinanced.

Under FRM, the first-order condition for l_t^* ensures that i_t^F is such that the capital owner is indifferent between new mortgages and rolling over the one-period bond from period t on. The first-order condition is an infinite-horizon counterpart to equation (1); see Appendix A. Under ARM, as equation (10) states, the current one-period interest rate i_t is applied to both new and outstanding mortgage loans, making the capital owner again indifferent between mortgages and rolling over the bond.

As, at given prices, the capital owner has to be indifferent across investing in mortgages, bonds, and capital, if all three assets are to be held in equilibrium, the capital owner's composition of period- t investment is pinned down by homeowners' demand for mortgages and the one-period bond.

4.1.2 Homeowners

A representative homeowner maximizes expected life-time utility

$$E_t \sum_{t=0}^{\infty} \beta^t v(c_t, 1 - n_t, h_t),$$

where n_t is labor and $v(., ., .)$ has standard properties. The maximization is subject to a sequence of constraints

$$c_t + p_{Ht}x_{Ht} - \frac{l_t}{p_t} + \frac{b_{t+1}}{p_t} = (1 - \tau_N)(w_t n_t - \tau) + (1 + i_{t-1} + \Upsilon_{t-1})\frac{b_t}{p_t} - \frac{m_t}{p_t} + \Omega_t, \quad (11)$$

$$\frac{l_t}{p_t} = \theta p_{Ht} x_{Ht}, \quad (12)$$

$$h_{t+1} = (1 - \delta_H)h_t + x_{Ht}. \quad (13)$$

²¹ κ and α can be chosen to approximate payments on loans of different maturities.

Here, x_{Ht} is newly purchased homes, p_{Ht} is their relative price, w_t is a real wage rate, τ_N is a labor income tax rate, τ is a pre-tax labor income deduction, and $\delta_H \in (0, 1)$ is a depreciation rate. Again, the tax rate and the income deduction are included purely for calibration purposes, as explained in the next section.²² Υ_{t-1} is a bond market participation cost, governed by a function $\Upsilon(-\tilde{b}_t)$, where $\tilde{b}_t \equiv b_t/p_{t-1}$ is the homeowner's real holdings of the bond. The function $\Upsilon(\cdot)$ is assumed to be increasing and convex and it allows us to control the extent to which homeowners can smooth consumption in the presence of time-variation in real mortgage payments.²³ In a nonstochastic steady state, $\tilde{b} = 0$. In order to avoid the cost affecting the definition of aggregate output, it is rebated to the homeowner as a lump-sum transfer Ω_t , which the homeowner takes as given. As in the case of the capital owner, mortgage payments are given by

$$m_t = (R_t + \gamma_t)d_t, \quad (14)$$

where

$$d_{t+1} = (1 - \gamma_t)d_t + l_t, \quad (15)$$

$$\gamma_{t+1} = (1 - \phi_t)(\gamma_t)^\alpha + \phi_t\kappa, \quad (16)$$

$$R_{t+1} = \begin{cases} (1 - \phi_t)R_t + \phi_t i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \quad (17)$$

with $\phi_t \equiv l_t/d_{t+1}$.

²²As in the three-period model, θ is a parameter. Chambers et al. (2009a) make a similar assumption and empirical evidence supports this assumption: over the period 1973-2006, there has been very little variation in the cross-sectional average of the loan-to-value ratio for single family newly-built home mortgages, despite large changes in interest rates and other macroeconomic conditions (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10).

²³It is further assumed that $\Upsilon(\cdot) = 0$ when $\tilde{b}_t = 0$, $\Upsilon(\cdot) > 0$ when $\tilde{b}_t < 0$ (the homeowner is borrowing), and $\Upsilon(\cdot) < 0$ when $\tilde{b}_t > 0$ (the homeowner is saving). We think of $\Upsilon(\cdot) > 0$ as capturing a premium for unsecured consumer credit, which is increasing in the amount borrowed. $\Upsilon(\cdot) < 0$ can be interpreted as capturing intermediation costs on household savings, which reduce the interest rate on savings below the market interest rate i_t . A technical role of the cost function is that, as in two-country business cycle models with incomplete markets, it prevents the one-period debt from becoming a random walk in a log-linear solution of the model.

4.1.3 Technology

An aggregate production function, operated by perfectly competitive producers, is given by $Y_t = A_t f(K_t, N_t)$, where K_t is the aggregate capital stock, N_t is aggregate labor, and $f(., .)$ has the standard neoclassical properties. Total factor productivity (TFP) evolves as $\log A_{t+1} = (1 - \rho_A) \log A + \rho_A \log A_t + \epsilon_{A,t+1}$, where $\rho_A \in (0, 1)$, A is the unconditional mean, and $\epsilon_{A_t} \sim iidN(0, \sigma_A)$. The real rate of return on capital, r_t , and the real wage rate, w_t , are determined by the marginal products of capital and labor, respectively. The resource constraint of the economy is $C_t + X_{Kt} + q_t X_{St} + G = Y_t$, where C_t is aggregate consumption, X_{Kt} is aggregate investment in capital, X_{St} is new residential structures, and G is (constant) government expenditures, introduced for calibration purposes only. Here, q_t is the marginal rate of transformation between new residential structures and the other uses of output, and hence the relative price of new residential structures. It is given by a strictly increasing convex function $q(X_{St})$, which makes the economy's production possibilities frontier concave in the space of $(C_t + X_{Kt} + G)$ and (X_{St}) —a specification akin to that of Huffman and Wynne (1999), a stand-in for the costs of moving factors of production across different sectors of the economy. The purpose of $q(\cdot)$ is to ensure realistic volatility of new residential structures in response to shocks; if the production possibilities frontier was linear, given the calibration of the shocks, the volatility would be way too high.

As in Davis and Heathcote (2005), new homes consist of new residential structures and land and are produced by perfectly competitive homebuilders according to an aggregate production function $X_{Ht} = g(X_{St}, X_{Lt})$. Here, X_{Ht} is the aggregate number of new homes constructed in period t , X_{Lt} is the aggregate new residential land, and g has the standard neoclassical properties. Homebuilders choose X_{Ht} , X_{St} , and X_{Lt} to maximize profits $p_{Ht} X_{Ht} - q_t X_{St} - p_{Lt} X_{Lt}$, subject to the above production function.

4.1.4 Monetary policy shocks

A monetary authority follows an interest rate feedback rule with a stochastic inflation target (e.g., Ireland, 2007)

$$i_t = (i + \bar{\pi}_t - \bar{\pi}) + \nu_\pi(\pi_t - \bar{\pi}_t), \quad \nu_\pi > 1, \quad (18)$$

where i is the nonstochastic steady-state short-term nominal interest rate, and $\bar{\pi}_t$ is an inflation target. The inflation target follows an AR(1) process $\bar{\pi}_{t+1} = (1 - \rho_\pi)\bar{\pi} + \rho_\pi\bar{\pi}_t + \epsilon_{\pi,t+1}$, where $\rho_\pi \in [0, 1)$, $\bar{\pi}$ is the nonstochastic steady-state inflation rate, and $\epsilon_{\pi,t+1} \sim iidN(0, \sigma_\pi)$. Notice that the interest rate rule has a more standard representation: $i_t = i + \nu_\pi(\pi_t - \bar{\pi}) + \xi_t$, where $\xi_t \equiv -(\nu_\pi - 1)(\bar{\pi}_t - \bar{\pi})$ are mean-zero autocorrelated monetary policy shocks.²⁴

As shown in Section 4.3 below, when ρ_π is close to one, the inflation target shock works in equilibrium like a ‘level factor’, moving short and long rates equally, and allows the model to reproduce the observed volatility and persistence of the 30-year mortgage rate. A number of studies in finance document that the level factor accounts for over 90% of the volatility of nominal yields across maturities (see, e.g., Piazzesi, 2006). There has been little success so far in providing a structural interpretation of the level factor, though it is generally regarded to be related to the behavior of monetary authorities (see Atkeson and Kehoe, 2009, for a review). In the absence of an off-the-shelf theory, we follow Gallmeyer, Hollifield, Palomino, and Zin (2007) and Atkeson and Kehoe (2009) and simply model the level factor as arising from very persistent monetary policy shocks, taking the form of shocks to the inflation target. Experimentation with the persistence parameter, however, is carried out and less persistent, more traditional, monetary policy shocks are also considered. These shocks will affect the long-short spread, rather than the level factor.

²⁴Using an interest rate feedback rule, rather than treating i_t as exogenous, has the advantage that it endogenously determines the current inflation rate π_t , which otherwise would be undetermined (Woodford, 2003). An alternative strategy of assuming a money growth rule has the problem that money growth rules, in an environment like ours, fail to generate persistent movements in nominal interest rates and inflation, even in the presence of persistent money growth shocks (e.g., Gavin, Keen, and Pakko, 2005).

4.2 Equilibrium

The equilibrium concept is a recursive competitive equilibrium. First, let $z_t \equiv [\log A_t, \bar{\pi}_t, p_{t-1}]$ be the vector of exogenous state variables and the lagged endogenous variable p_{t-1} , $s_t^* \equiv [k_t, b_t^*, d_t^*, \gamma_t^*, R_t^*]$ the vector of the capital owner's state variables, $s_t \equiv [h_t, b_t, d_t, \gamma_t, R_t]$ the vector of the homeowner's state variables, and $S_t \equiv [K_t, H_t, B_t, D_t, \Gamma_t, \mathfrak{R}_t]$ the vector of aggregate endogenous state variables, where the elements are, respectively, aggregate capital, housing stock, bonds, outstanding mortgage debt, and its amortization and interest rates. Next, write the capital owner's optimization problem as

$$U(z, S, s^*) = \max_{[x_K, (b^*)', l^*]} \left\{ u(c^*) + \beta E[U(z', S', (s^*)') | z] \right\}, \quad (19)$$

where a prime denotes a value next period and the constraints (5)-(10) are thought to have been substituted in the utility and value functions. Similarly, write the homeowner's problem as

$$V(z, S, s) = \max_{[x_H, b', n]} \left\{ v(c, 1 - n, h) + \beta E[V(z', S', s') | z] \right\}, \quad (20)$$

where the constraints (11)-(17) are thought to have been substituted in the utility and value functions. Let $W_t \equiv [X_{Kt}, p_t, i_t^M, X_{Ht}, B_{t+1}, N_t]$ be the vector of aggregate decision variables and prices, where $i_t^M = i_t^F$ under FRM and $i_t^M = i_t$ under ARM. Define a function $W_t = W(z_t, S_t)$.

A recursive competitive equilibrium consists of the functions U , V , and W such that: (i) U and V solve (19) and (20), respectively; (ii) r_t and w_t are given by the respective marginal products of capital and labor, p_{Ht} and p_{Lt} are given by homebuilder's first-order conditions for structures and land, and $q_t = q(X_{St})$; (iii) i_t is given by the monetary policy rule (18); (iv) the bond, mortgage, housing, and land markets clear: $(1 - \Psi)b_{t+1}^* + \Psi b_{t+1} = 0$, $(1 - \Psi)(l_t^*/p_t) = \Psi \theta p_{Ht} x_{Ht}$, $\Psi x_{Ht} = g(X_{St}, X_{Lt})$, and $X_{Lt} = 1$; (v) aggregate consistency is ensured: $K_t = (1 - \Psi)k_t$, $X_{Kt} = (1 - \Psi)x_{Kt}$, $X_{Ht} = \Psi x_{Ht}$, $N_t = \Psi n_t$, $B_t = \Psi b_t$, $H_t = \Psi h_t$, $(1 - \Psi)m_t^* = \Psi m_t$, $(1 - \Psi)d_t^* = \Psi d_t = D_t$, $\gamma_t^* = \gamma_t = \Gamma_t$, $R_t^* = R_t = \mathfrak{R}_t$, and

$G + (1 - \Psi)\tau_t^* = \tau_K(r_t - \delta_K)K_t + \tau_N(w_t N_t - \tau\Psi) + \tau\Psi$; (vi) the exogenous state variables follow their respective stochastic processes and the endogenous aggregate state variables evolve according to aggregate counterparts to the laws of motion for the respective individual state variables; and (vii) the individual optimal decision rules of the capital owner (for x_K , $(b^*)'$, and l^*) and the homeowner (for x_H , b' , and n) are consistent with $W(z, S)$, once the market clearing conditions (iv) and the aggregate consistency conditions (v) are imposed.²⁵

It is straightforward to check that the goods market clears by Walras' Law: $C_t + X_{Kt} + q_t X_{St} + G = Y_t$, where $C_t = (1 - \Psi)c_t^* + \Psi c_t$. Equations characterizing the equilibrium are contained in Appendix A; a computational procedure resulting in log-linear approximation of $W(z, S)$ around the model's non-stochastic steady state is described in Appendix B.

4.3 The equilibrium nominal interest rate and inflation

The capital owner's first-order conditions for b_{t+1}^* and x_{Kt} yield the Fisher equation. In a linearized form: $i_t \approx E_t \pi_{t+1} + E_t r_{t+1}$, where (abusing notation) the variables are in percentage point deviations from steady state. Given a stochastic process for r_t , the Fisher equation and the monetary policy rule (18) determine i_t . For ρ_π close to one, excluding explosive paths for inflation (a common assumption), the resulting expression for i_t is

$$i_t \approx \sum_{j=0}^{\infty} \left(\frac{1}{\nu_\pi} \right)^j E_t r_{t+1+j} + \bar{\pi}_t. \quad (21)$$

The short rate is thus equal to the sum of the exogenous shock $\bar{\pi}_t$ and the endogenous expected future path of the real interest rate r_t , given by a linearized version of the marginal product of capital, $A_t f_K(K_t, N_t)$. Substituting i_t from equation (21) in the policy rule (18)

²⁵In the case of ARM, $i_t^M = i_t$ makes the capital owner indifferent between new mortgages and bonds and the first-order condition for l_t^* can be dropped from the description of the equilibrium. In the case of FRM, the first-order condition is needed to determine i_t^F .

gives the equilibrium inflation rate

$$\pi_t \approx \frac{1}{\nu_\pi} \sum_{j=0}^{\infty} \left(\frac{1}{\nu_\pi} \right)^j E_t r_{t+1+j} + \bar{\pi}_t. \quad (22)$$

Thus, under the assumption yielding the above expressions—that ρ_π is close to one—shocks to $\bar{\pi}_t$ generate highly persistent movements in i_t (subject to potential general equilibrium adjustments in r_t) and therefore move i_t and i_t^F approximately one for one. In addition, they also move π_t approximately one for one with the nominal interest rates. In this sense, $\bar{\pi}_t$ works like a level factor, moving all yields and inflation approximately equally.²⁶

5 Calibration

The model is quarterly and most parameter values are obtained by requiring the model to reproduce long-run averages of the data in a nonstochastic steady state. Some second moments are also used. As most of the required historical data are readily available for the United States, the calibration is based on U.S. data, even though the mechanism under investigation applies more generally (Appendix C contains the description of the U.S. data and their adjustments to conform with the notion of the variables in the model). We conduct sensitivity analysis with respect to some of the choices described here.

5.1 Functional forms

The capital owner’s per-period utility function is $u(c^*) = \log c^*$; the homeowner’s utility function is $v(\bar{c}, n) = \omega \log \bar{c} + (1 - \omega) \log(1 - n)$, where \bar{c} is a composite consumption good $\bar{c}(c, h) = c^\xi h^{1-\xi}$. The additive separability of the homeowner’s utility function facilitates a transparent interpretation of the results as marginal utilities are independent of the con-

²⁶In contrast, the first term in equation (21), mainly driven by shocks to A_t , is much less persistent (given the estimates of the persistence of A_t from the data). Therefore, it produces only temporary movements in i_t and thus smaller movements in i_t^F than in i_t . As a result, it moves the long-short spread, $i_t^F - i_t$. In this sense it works like a slope factor.

sumption of other goods. Further, the goods production function is $f(K, N) = K^\varsigma N^{1-\varsigma}$ and the housing production function is $g(X_S, X_L) = X_S^{1-\varphi} X_L^\varphi$. As in Kydland et al. (2014), $q(X_{St}) = \exp(\zeta(X_{St} - X_S))$, where $\zeta > 0$ and X_S is the steady-state ratio of new residential structures to output (Y is normalized to be equal to one in steady state). A similar functional form is used also for the bond market participation cost: $\Upsilon(-\tilde{B}) = \exp(-\vartheta\tilde{B}_t) - 1$, where $\vartheta > 0$ and $\tilde{B}_t = 0$ in steady state. It is straightforward to check that this function satisfies the properties set out in Section 4.1.2.

5.2 Debt-servicing costs

A particular difficulty in calibrating the model arises due to the need to match debt-servicing costs of homeowners, as this requires the model to be consistent with the cross-sectional distribution of income, in addition to standard aggregate ratios: $X_K/Y = 0.156$, $X_S/Y = 0.054$, $K/Y = 7.06$, $H/Y = 5.28$, $rK/Y = 0.283$, and $N = 0.255$. The last ratio is from the American Time-Use Survey 2003, population 16+, the others are averages for 1958-2006.

Official data for mortgage debt servicing costs are not published for the United States. Estimates, however, can be obtained from different sources (see Appendix D), resulting in long-run averages (1972-2006) in the ballpark of 18.5% of homeowners' pre-tax income. The model's steady-state counterpart to this aggregate ratio is $\tilde{M}/(wN - \tau\Psi)$, where $\tilde{M} = (R + \gamma)\tilde{D}/(1 + \pi)$, with \tilde{D} being real mortgage debt and $wN - \tau\Psi$ being homeowners' income, before the labor income tax rate is applied.

Consistency with the observed cross-sectional distribution of income is achieved through the transfer τ . Recall that homeowners in the model are an abstraction for the 3rd and 4th quintiles of the U.S. wealth distribution, while capital owners are an abstraction for the 5th quintile. In the data, the 5th quintile derive 40% of income from capital and 53% from labor; in the case of the 3rd and 4th quintiles, 81% comes from labor (SCF, 1998). As a result, if the only source of income of capital owners in the model was capital, and given that the model is required to match the observed average capital share of output ($rK/Y = 0.283$),

capital owners would account for too small fraction of aggregate income (28.3% in the model v.s. 48% in the data), while homeowners' share would be too large (71.7% v.s. 34%). As a result, the steady-state debt-servicing costs would be too low (or the debt-to-GDP ratio would have to be too high, thus being inconsistent with the observed loan-to-value ratio θ and amortization schedules). The parameter τ adjusts for this discrepancy by transferring, in a lump-sum way, some of the labor income from homeowners to capital owners so as to match the distribution of income, without affecting the model's ability to match the other calibration targets.

5.3 Parameter values

The baseline parameter values are listed in Table 1, where the parameters are organized into eight categories: Ψ (population); $\delta_K, \delta_H, \varsigma, A, \zeta, \varphi$ (technology); τ_K, τ_N, G, τ (fiscal); θ, α, κ (mortgages); ϑ (bond market); $\bar{\pi}, \nu_\pi$ (monetary policy); β, ω, ξ (preferences); and $\rho_A, \sigma_A, \rho_\pi, \sigma_\pi$ (stochastic processes). Most parameters can be assigned values without solving a system of steady-state equations, four parameters ($\omega, \xi, \tau_K, \tau$) have to be obtained jointly from such steady-state relations, and three parameters ($\zeta, \rho_\pi, \sigma_\pi$) are assigned values by matching second moments of the data. Calibration of the three types of parameters is described in turn.

In order to be consistent with the notion of homeowners and capital owners in the data, Ψ is set equal to $2/3$. The parameter ς corresponds to the share of capital income in output and is set equal to 0.283, an estimate obtained by Gomme and Rupert (2007) from National Income and Product Accounts (NIPA) for aggregate output close to our measure of output (see Appendix C). The share of residential land in new housing φ is set equal to 0.1, an estimate reported by Davis and Heathcote (2005). The depreciation rates δ_K and δ_H are set equal to 0.02225 and 0.01021, respectively, to be consistent with the average flow-stock ratios for capital and housing, respectively. The level of TFP, A , is set equal to 1.5321, so that steady-state output is equal to one. The stochastic process for TFP is

assigned $\rho_A = 0.9641$ and $\sigma_A = 0.0082$, estimates obtained by Gomme and Rupert (2007) for the Solow residual of a production function with the same ς and measurements of capital and labor inputs used here (see Appendix C). The labor income tax rate is derived from NIPA using a procedure of Mendoza, Razin, and Tesar (1994), yielding $\tau_N = 23.5\%$. The parameter G is set equal to 0.138, in order to correspond to our measure of government expenditures (see Appendix C). The loan-to-value ratio θ is set equal to 0.76, the average (1973-2006) of the cross-sectional mean of the loan-to-value ratio for single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10). Following Kydland et al. (2014), the amortization parameters are assigned $\kappa = 0.00162$ and $\alpha = 0.9946$. These values approximate a 30-year mortgage. The weight on inflation in the monetary policy rule ν_π is set equal to 1.35, which falls in the middle of the range of estimates reported by Woodford (2003), Chapter 1. The steady-state inflation rate $\bar{\pi}$ is set equal to 0.0113, the average (1972-2006) quarterly inflation rate. In steady state, the first-order condition for l_t^* constrains i^F to equal to i . The first-order condition for b_t^* then relates i and $\bar{\pi}$ to β . The above value of $\bar{\pi}$ and $i^F = 9.31\%$ per annum (the 1972-2006 average for 30-year FRM rate) imply $\beta = 0.9883$. For the participation cost function $\Upsilon(\cdot)$, the choice of ϑ is guided by available studies on prices of unsecured consumer credit. Setting ϑ equal to 0.035 gives similar interest premium schedule as in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Figure 6, white-collar workers.

Given the above parameter values, the second set of parameters $(\omega, \xi, \tau_K, \tau)$ is calibrated by forcing the model to replicate, in steady state, the observed average K/Y ratio, H/Y ratio, debt-servicing costs, and N . The relationship between the four parameters and the targets is given by the steady-state versions of the first-order conditions for x_{Kt} , x_{Ht} , and n_t , and the expression for steady-state debt-servicing costs noted above (see Appendix A for the first-order conditions). These restrictions yield $\omega = 0.2478$, $\xi = 0.6009$, $\tau_K = 0.3362$, and $\tau = 0.5886$.²⁷

²⁷In principle, τ_K can be measured from NIPA in the same way as τ_N . Such alternative parameterization, however, makes the model inconsistent with the observed capital-output ratio. This is because β is already pinned down by the first-order condition for bonds and thus cannot be used to match the

Finally, given the values of the first two sets of parameters, ζ , ρ_π , and σ_π are calibrated by matching certain second moments of the data. The parameters ρ_π and σ_π are obtained by matching the standard deviation (2.4%) and the first-order autocorrelation (0.97) of the 30-year FRM rate (annualized rate, unfiltered data).²⁸ This results in $\rho_\pi = 0.994$ and $\sigma_\pi = 0.0015$. The PPF parameter ζ controls the volatility of the expenditure components of output and is used to match the volatility of aggregate consumption, relative to the volatility of output. This has the advantage, compared to matching the volatility of one of the investment series, that approximately the same parameter value is obtained regardless of whether the model is simulated under FRM or ARM. The resulting value is $\zeta = 0.35$.

Table 2 lists the steady-state values of the model's endogenous variables implied by the above calibration and, where possible, the long-run averages of their data counterparts. As can be seen, despite the highly stylized nature of the model, the steady state is broadly consistent with a number of moments not targeted in calibration.²⁹

6 Findings

This section presents the general equilibrium findings. The results of sensitivity analysis are reported in Appendix E of the supplemental material.

6.1 Responses of key variables to $\bar{\pi}_t$ under FRM and ARM

The first set of findings is presented in Figures 3 and 4. The figures show responses of key variables to a 1 percentage point (annualized) increase in the level factor shock $\bar{\pi}_t$. Figure 3 is for the economy in which the homeowner has no access to the one-period bond market;

capital-output ratio. Nevertheless, τ_K implied by the model is not far from the NIPA tax rate obtained by Gomme, Ravikumar, and Rupert (2011): 33.62% in the model v.s. 40.39% in NIPA.

²⁸The 10-year government bond yield is actually used as a proxy for the 30-year mortgage rate. The two rates co-move closely for the period for which both series are available (from 1972), but the data for the 10-year yield are longer (1958-2007), thus providing a more accurate estimate of the parameters of the inflation target shock.

²⁹The model has a well-defined steady state even in the presence of one unit of new land being made available each period as each new unit of land is combined with the steady-state amount of new structures to form a steady-state number of new homes.

Figure 4 is for the economy in which the homeowner has access to the bond market at the calibrated cost. In each chart, the solid line is for the ARM economy and the dash line is for the FRM economy. The charts show responses for the first 40 periods (10 years). The responses of interest and inflation rates are expressed in (annualized) percentage point deviations from steady state; the responses of other variables are expressed as percentage deviations from steady state. Even though, for some variables, convergence back to the steady state may not be apparent from the figures, eventually all variables converge back to the steady state. This, however, takes longer than 40 periods. The immediate message from Figures 3 and 4 is that the responses of real variables are stronger under ARM than under FRM.

The first two upper-left charts in Figure 3 demonstrate the level factor nature of the shock: the short-term nominal interest rate and the inflation rate, and in the case of the FRM economy also the FRM rate, all increase more or less in parallel by approximately 1 percentage point. As a result, in accordance with our discussion in Section 3, real mortgage payments on outstanding debt display a persistent gradual decline under FRM, while under ARM the payments increase sharply almost immediately (one period after the shock). Under FRM, housing investment (here X_H is plotted but qualitatively the same applies to X_S) declines for the first few periods after the shock.³⁰ This reflects the price effect. Over time, however, due to the wealth effects generated by the gradual decline in real mortgage payments, housing investment increases above the steady-state level (before converging back to steady state). Under ARM, there is a quantitatively similar decline of housing investment in the first period as under FRM. But the major decline occurs in the second period, once the wealth effects kick in. The drop in housing investment in the second period dwarfs the decline under both contracts in the first period (about 6% v.s. about 0.5%). Over time, as the accumulated inflation sufficiently erodes real outstanding debt, negative wealth effects turn into positive wealth effects and housing investment increases above the steady state

³⁰In equilibrium, X_{Ht} and X_{St} are related through the production function of homebuilders as $\widehat{X}_{Ht} = (1 - \varphi)\widehat{X}_{St}$, where a hat denotes percentage deviations from steady state.

level (before converging back to steady state). The capital owner compensates the decline in the demand for new mortgage loans by increasing investment in productive capital. Capital investment thus follows an approximately opposite path to that of housing investment, increasing at first before falling below the steady-state level later on.

The dynamics of consumption reflect the redistribution of real income through mortgage payments. Thus, under FRM, consumption of homeowners gradually increases, while consumption of capital owners gradually declines. In contrast, under ARM, consumption of homeowners drops sharply in the second period, while consumption of capital owners increases (the increase is smooth as capital owners can use capital to smooth the anticipated increase in income). The behavior of output reflects predominantly the behavior of labor. In particular, output increases in the second period in the case of ARM as homeowners compensate the decline in their disposable income by working more. In the FRM case, a gradual decline in labor, due to the positive wealth effects, leads to a gradual decline in output.

Recall from equations (21) and (22) that, in principle, the effects of the $\bar{\pi}_t$ shock could be offset by adjustments in the path of the real interest rate r_t . Indeed, under FRM, the real rate persistently declines, thus working in the opposite direction of the $\bar{\pi}_t$ shock in its effect on the nominal interest rate and inflation. But this decline is clearly not sufficient to offset the increase in $\bar{\pi}_t$ as both the nominal interest rate and inflation increase in the upper-left charts about as much as $\bar{\pi}_t$. In the case of ARM, the response of the real rate even strengthens the effect of $\bar{\pi}_t$, at least initially (the real rate increases due to the effect of higher labor supply on the marginal product of capital). This is reflected by the more than one-for-one response of the nominal interest rate and inflation to the level factor shock in the first few periods after the shock. The final chart in Figure 3 (in the bottom right corner) shows the response of the price of new homes. As Appendix A of the supplemental material shows, this price is proportional (in logs) to housing investment and thus declines as housing investment declines. The magnitudes are however smaller.³¹

³¹From the first-order conditions of homebuilders: $\hat{p}_{Ht} = \hat{q}_t + \varphi \hat{X}_{St} = (1 - \varphi)^{-1}(\zeta X_S + \varphi) \hat{X}_{Ht}$, where a hat denotes percentage deviations from steady state and X_S denotes new residential structures in steady state.

In Figure 4, the economy with access to the bond market, the responses of real variables are basically smoother versions of the responses in Figure 3. The magnitudes are also smaller but not insignificant. For instance, under ARM, the maximum decline of housing investment is 3.5%, compared with 6.3% in the case of no access to the bond market. The smoothness and smaller magnitudes of the responses reflect the fact that homeowners can now partially undo the real effects of monetary policy on their disposable income by borrowing and saving in the bond market.

6.2 Discussion

Independent empirical support for some of these findings can be found in a household-level study by Di Maggio et al. (2014). Specifically, the authors find that homeowners' purchases of durable and nondurable goods respond more to interest rate changes in U.S. counties in which ARM debt dominates in the outstanding stock than in counties where FRM debt dominates.³² Furthermore, the authors find that households that are more financially constraint respond more. And even though our model does not have heterogenous households, the above two experiments show that the responses of the representative homeowner are larger when it has no access to the one-period bond market than when it does (Appendix E also shows that increasing mortgage payments as a fraction of income increases the responses, which could be interpreted as being broadly in line with the empirical finding that more indebted households respond more).³³

6.3 The role of persistence of the $\bar{\pi}_t$ shock

Figure 5 demonstrates the effects of reducing the persistence of the $\bar{\pi}_t$ shock, using the economy in which homeowners can access the bond market. Specifically, it compares the responses of housing investment for $\rho_\pi = 0.994$, the baseline value, with the responses

³²Even though the changes in nominal interest rates identified by their study may not necessarily occur due to inflation, real mortgage payments under ARMs respond initially to nominal interest rate shocks in a similar fashion, as discussed above, regardless of the source of the movements.

³³Indeed, this model is likely not the only one that may be consistent with the empirical findings.

for $\rho_\pi = 0.95$ and $\rho_\pi = 0.5$ (these values were also considered in the partial equilibrium experiments in Figures 1 and 2). The last of the three values is closer to persistence of standard monetary policy shocks in interest rate rules. In addition to housing investment, Figure 5 plots the responses of the long-short spread; i.e., $i_t^F - i_t$. The responses of the long-short spread show that, as the persistence of the shock declines, the shock starts to resemble the slope factor rather than the level factor. And in line with what one would expect from the partial equilibrium analysis in Figures 1 and 2, the responses of housing investment become also weaker and less persistent, at least in the case of ARM (in the case of FRM, the price effect becomes dominated by the wealth effects and the response of housing investment actually changes sign as the persistence of the shock declines).

6.4 The role of the term of the contract

Figure 6 then demonstrates (using again the economy in which homeowners can access the bond market) the effects of shortening the term of the loan from approximately 30 years to about 15 years ($\kappa = 0.0094$, $\alpha = 0.9912$ and steady-state $\gamma = 0.0313$) and then to one period ($\kappa = 1$, $\alpha = 0$ and steady-state $\gamma = 1$). Similarly to reducing the persistence of the shock, shortening the term of the loan from 30 to 15 years reduces the magnitude and persistence of the response of housing investment under ARM and leads to a change of the sign of the response of housing investment under FRM. When the term of the loan is just one period, the response becomes hardly visible (the response is positive as the only channel is a positive wealth effect for homeowners due to higher inflation in period 1).

6.5 Business cycle moments

From Figures 3 and 4 one could conclude that the model has counterfactual implications for the cyclical behavior of residential and nonresidential investment, as housing and capital investment in the two figures move in opposite directions. However, business cycles in a number of developed economies, including the United States, exhibit strong positive correlations

between the two types of investment on one hand and output on the other (even though their phase shifts in relation to output differ across countries, Kydland et al., 2014). As a final set of results, Table 3 therefore reports the cyclical properties of the model economy, once it is subjected to both TFP and the level factor shocks. The moments generated by the model are compared with corresponding moments of the U.S. business cycle.³⁴

The table shows that, with these two shocks, the model accounts for about half of the volatility of U.S. output (our measure of output is close to private sector output; see Appendix C for details). As in the data, all three expenditure side components of output in the model (C , X_S , and X_K) are positively correlated with output. The correlations are stronger than in the data, arguably due to the presence of only two shocks in the model, of which the TFP shock has the larger impact. The ranking of volatilities is also consistent with the data: residential investment is most volatile, followed by nonresidential investment, and then consumption. The volatility of residential investment is higher under FRM than ARM. This is because, as in the data, the nominal interest rate (and inflation) is pro-cyclical; in the model due to the slope factor part of equation (21). For the reasons discussed in the previous sections, the pro-cyclical movements of the short rate dampen the responses of X_{St} to TFP shocks more under ARM than under FRM.

Finally, notice that—as in the data—the long rate is almost acyclical, even though the short rate and inflation are pro-cyclical (the three variables are positively correlated with each other at lower than business cycle frequencies). As a result, the long-short spread is negatively correlated with output, as in the data. The model is also consistent with a pro-cyclical behavior of the relative price of new residential structures and new homes. The volatility of new home prices in the model is about 60 – 70% as high as in the data. The shortfall, and the relatively high correlation of new home prices with output, is due to the absence in the model of ‘housing supply shocks’.³⁵

³⁴U.S. moments are for HP-filtered series, post-Korean war data. The model moments are averages of moments for 150 runs of the model; the artificial series of each run have the same length as the data series and are HP filtered.

³⁵Notice that the volatility of the relative price of structures in the model is only about 30% as high as in the data and that its correlation with output is too high. Shocks to q_t , reflecting, for instance, shocks

7 Concluding remarks

A stylized model containing either ARM or FRM loans is constructed in order to investigate the real effects of the nominal rigidity inherent in standard mortgage contracts. The model economy has a population of homeowners and capital owners with selected key characteristics of each group observed in the data. Due to the nominal rigidity, and incomplete asset markets, monetary policy shocks transmit through the effective price of new housing and current and expected future wealth effects of outstanding mortgage payments.

Three key lessons emerge. First, the entire path of nominal interest rates matters for household decisions; monetary policy shocks that work like a level factor in the nominal yield curve have larger effects than transitory shocks. Second, the real effects are larger under ARM than FRM. And third, higher inflation—associated with shocks to the level factor—redistributes real income from lenders to borrowers under FRM, but from borrowers to lenders under ARM. While the redistribution under FRM is back-loaded, under ARM it is front-loaded. The latter effects are thus stronger, as households discount future income.

We have also demonstrated that inflation movements under ARM work as real interest rate movements in the initial periods of the shock. Further, that shortening the term of the loan reduces the real effects and that an extreme case of refinancing (prepayment)—when it occurs every period—eliminates the nominal rigidity. And finally that the effects are stronger the more costly it is for homeowners to smooth consumption over time through other financial assets.

Our aim was to develop a stylized model in order to isolate the channels under investigation and to describe their effects in a transparent way. The model therefore intentionally abstracts from other nominal frictions, as well as various real frictions and alternative channels through which housing affects the macroeconomy. The number of shocks was also limited. A natural extension, therefore, is to incorporate other relevant shocks, frictions, and aspects of the housing market to align the model further with the data. Such model could then be

specific to the construction industry, as in Davis and Heathcote (2005), may be a source of the remaining volatility of the price of structures and new homes.

used to carry out variance decomposition of main macro aggregates attributable to different shocks.

Another extension may involve replacing the representative homeowner and capital owner with heterogeneous agents facing realistic life-cycle choices over housing and nonhousing investments in an OLG framework with individual-level shocks. While this may provide a refinement of the quantitative results presented here for illustrative purposes, and a careful analysis of income redistribution across households with different individual-level characteristics, such extension is likely to face large computational costs in the presence of mortgage contracts and aggregate shocks.

Finally, an interesting normative question regards the design of optimal monetary policy in an environment like ours. Optimal monetary policy is likely to depend on the prevalent mortgage type, ARM or FRM, in the economy. Different countries may thus follow different policies depending on their institutional environment. This aspect of the optimal policy is likely to complicate matters further if a common monetary policy is to be conducted for an area with different mortgage markets, such as the Eurozone. Extending the model to allow for default and a banking sector may also generate interesting interactions between optimal monetary and macroprudential policies. All these issues are left for future research.

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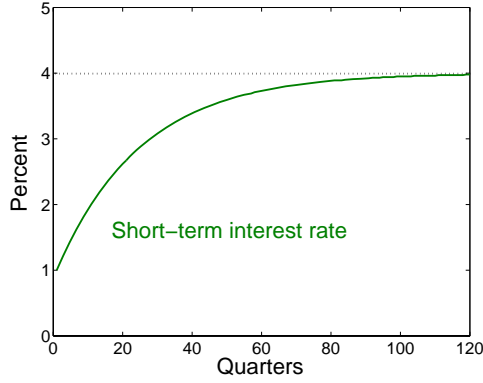
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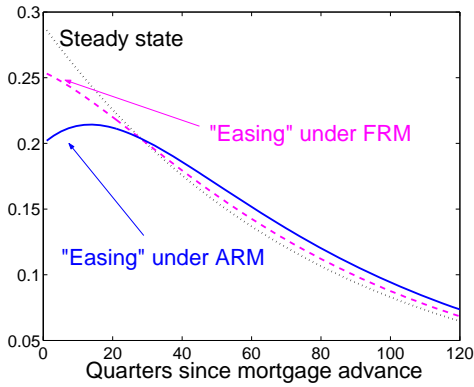
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MONETARY POLICY EASING
PERSISTENCE = 0.95



NEW LOAN
(120-PERIOD TERM)



EXISTING LOAN
(119 PERIODS REMAINING)

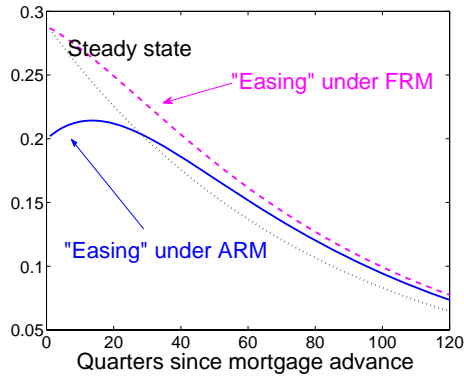
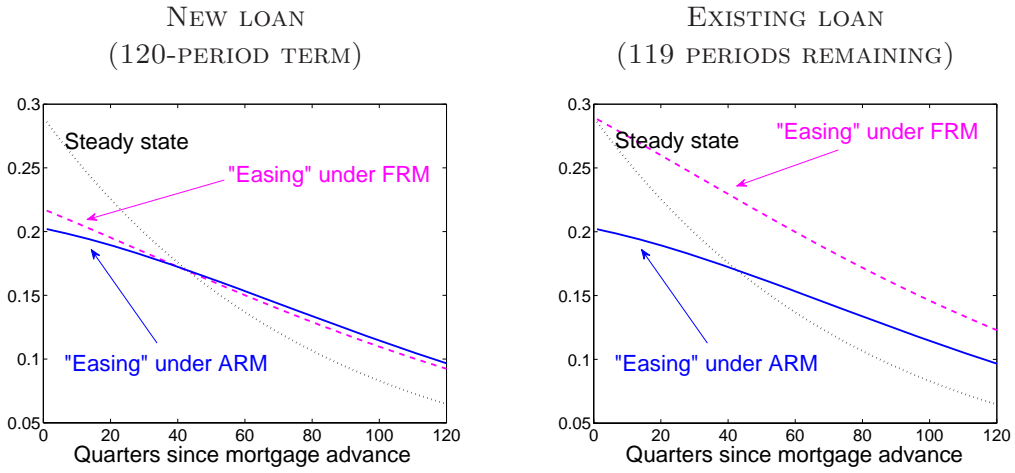


Figure 1: Illustration of price and expected future wealth effects. Debt-servicing costs over a term of a new and an existing 30-year mortgage under alternative paths of the short-term nominal interest rate. The label ‘steady-state’ refers to the case when the short rate is at its steady-state level of 4%. The mortgage is equal to four times the household’s income; the real interest rate is held constant at 1% per annum.

A. High persistence (0.99) of the short rate decline



B. Low persistence (0.5) of the short rate decline

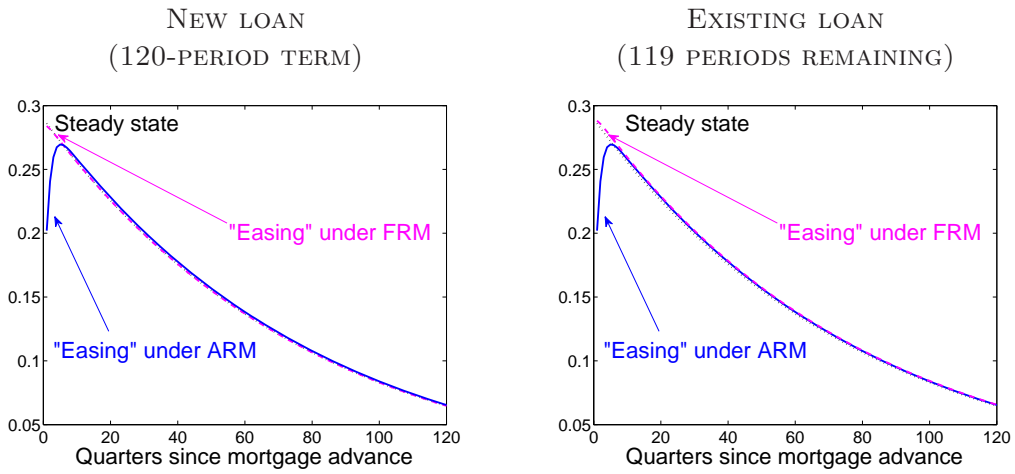


Figure 2: Illustration of price and expected future wealth effects for high and low persistence of the mean-reverting short rate decline by 3 percentage points.

Table 1: Calibration

Symbol	Value	Description
Population		
Ψ	2/3	Share of homeowners
Technology		
A	1.5321	Steady-state level of TFP
ς	0.283	Capital share of output
δ_K	0.02225	Depreciation rate of capital
δ_H	0.01021	Depreciation rate of housing
ζ	0.35	Curvature of PPF
φ	0.1	Land share of new housing
Fiscal		
G	0.138	Government expenditures
τ_N	0.235	Labor income tax rate
τ_K	0.3362	Capital income tax rate
τ	0.5886	Labor income transfer
Preferences		
β	0.9883	Discount factor
ω	0.2478	Cons. composite's share in utility
ξ	0.6009	Share of market cons. in composite
Mortgages		
θ	0.76	Loan-to-value ratio
κ	0.00162	Initial amortization rate
α	0.9946	Amortization adjustment factor
Bond market		
ϑ	0.035	Participation cost function
Monetary policy rule		
ν_π	1.35	Weight on inflation
$\bar{\pi}$	0.0113	Steady-state inflation rate
Exogenous processes		
ρ_A	0.9641	Persistence of TFP shocks
σ_A	0.0082	Std. of TFP innovations
ρ_π	0.994	Persistence of infl. target shocks
σ_π	0.0015	Std. of infl. target innovations

Table 2: Nonstochastic steady state and long-run averages of data

Symbol	Model	Data	Description
Normalized:			
Y	1.0	N/A	Output
Targeted in calibration:			
K	7.06	7.06	Capital stock
H	5.28	5.28	Housing stock
X_K	0.156	0.156	Capital investment
X_S	0.054	0.054	New housing structures
N	0.255	0.255	Hours worked
$\widetilde{M}/(wN - \Psi\tau)$	0.185	0.185	Debt-servicing costs (pre-tax)
i^M	0.0233	0.0233	Mortgage rate
Not targeted:			
Aggregate mortgage variables			
\widetilde{D}	1.61	2.35 [†]	Mortgage debt
γ	0.0144	0.0118 [‡]	Amortization rate
Capital owner's variables			
$(1 - \tau_K)(r - \delta_K)$	0.012	0.013 [§]	Net (post-tax) rate of return on capital
$[(r - \delta)k + \widetilde{m}^*]/[(r - \delta)k + \widetilde{m}^* + \tau^*]$	0.33	0.39 ^{¶,§§}	Income from assets to total income
Homeowner's variables			
τ_H	0	N/A	Housing wedge
$\widetilde{m}/[(1 - \tau_N)(wn - \tau)]$	0.24	N/A	Debt-servicing costs (post-tax)
$(wn - \tau)/(wn - \tau)$	1.00	0.81 [¶]	Income from labor to total income
Distribution of wealth			
$(K + \widetilde{D})/(K + H)$	0.71	0.82 [¶]	Capital owners
$(H - \widetilde{D})/(K + H)$	0.29	0.18 [¶]	Homeowners

Note: Rates of return and interest and amortization rates are expressed at quarterly rates; capital owners = the 5th quintile of the SCF wealth distribution; homeowners = the 3rd and 4th quintiles of the SCF wealth distribution.

[†] Upper bound for the mortgage debt in the model due to the presence in the data of equity loans, second mortgages, and mortgages for purchases of existing homes.

[‡] For a standard 30-year mortgage.

[§] NIPA-based estimate of Gomme et al. (2011).

[¶] 1998 SCF; the model counterpart is defined so as to be consistent with the definition in SCF.

^{§§} The sum of capital and business income.

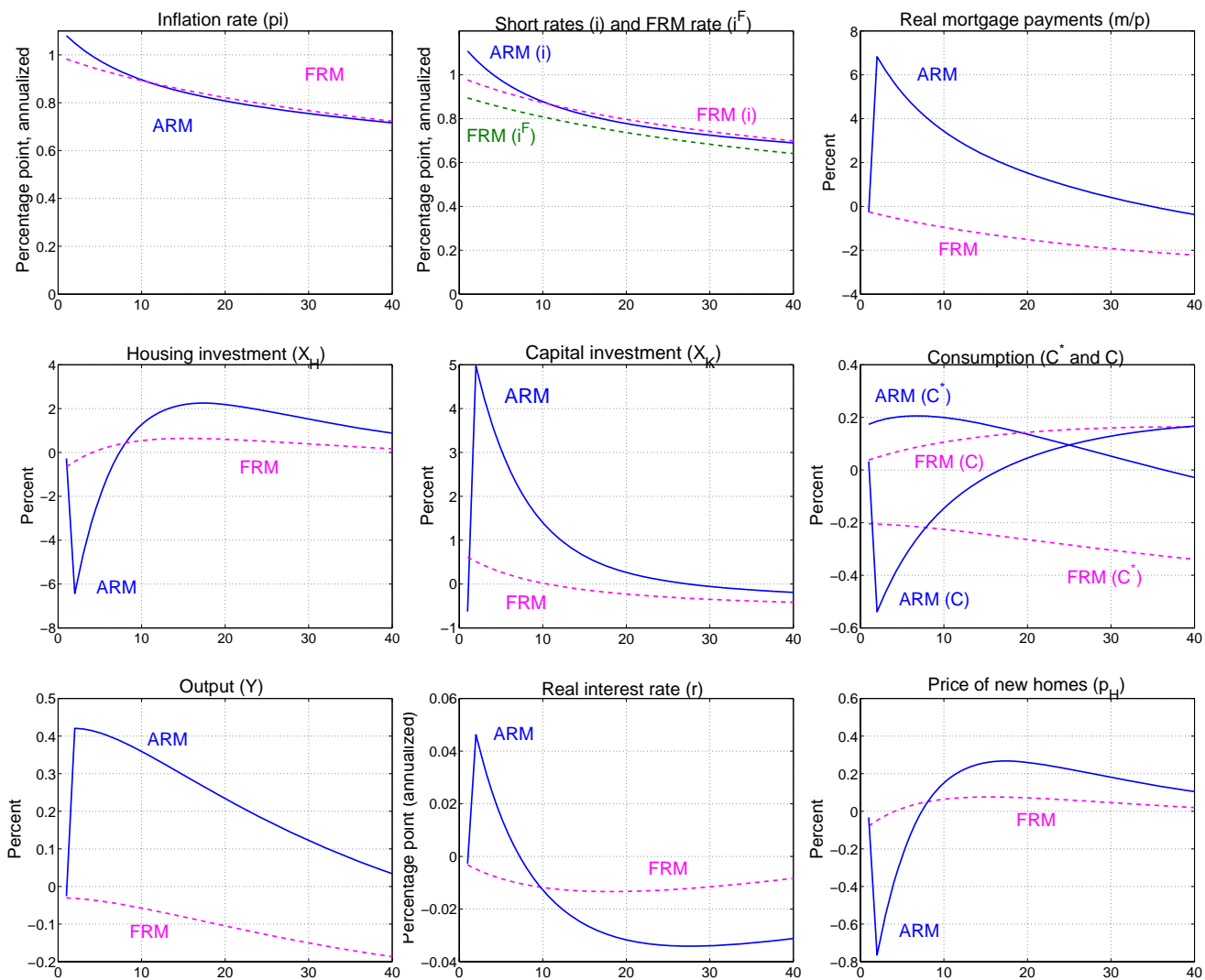


Figure 3: General equilibrium responses to 1 percentage point (annualized) increase in $\bar{\pi}_t$ in period 1; version with no access of homeowners to the 1-period bond market.

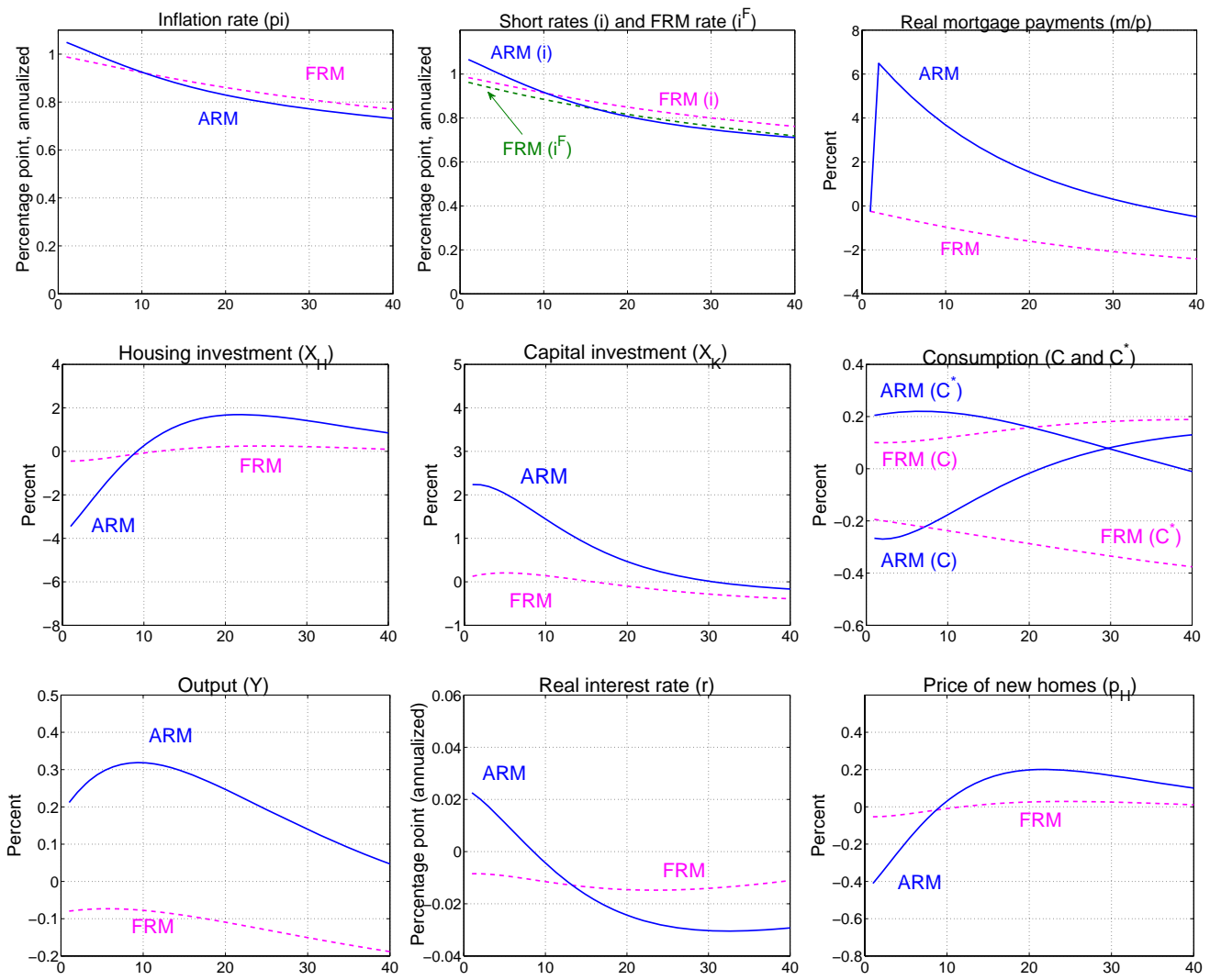


Figure 4: General equilibrium responses to 1 percentage point (annualized) increase in $\bar{\pi}_t$ in period 1; version with access of homeowners to the 1-period bond market.

A. $\rho_\pi = 0.994$

B. $\rho_\pi = 0.95$

C. $\rho_\pi = 0.50$

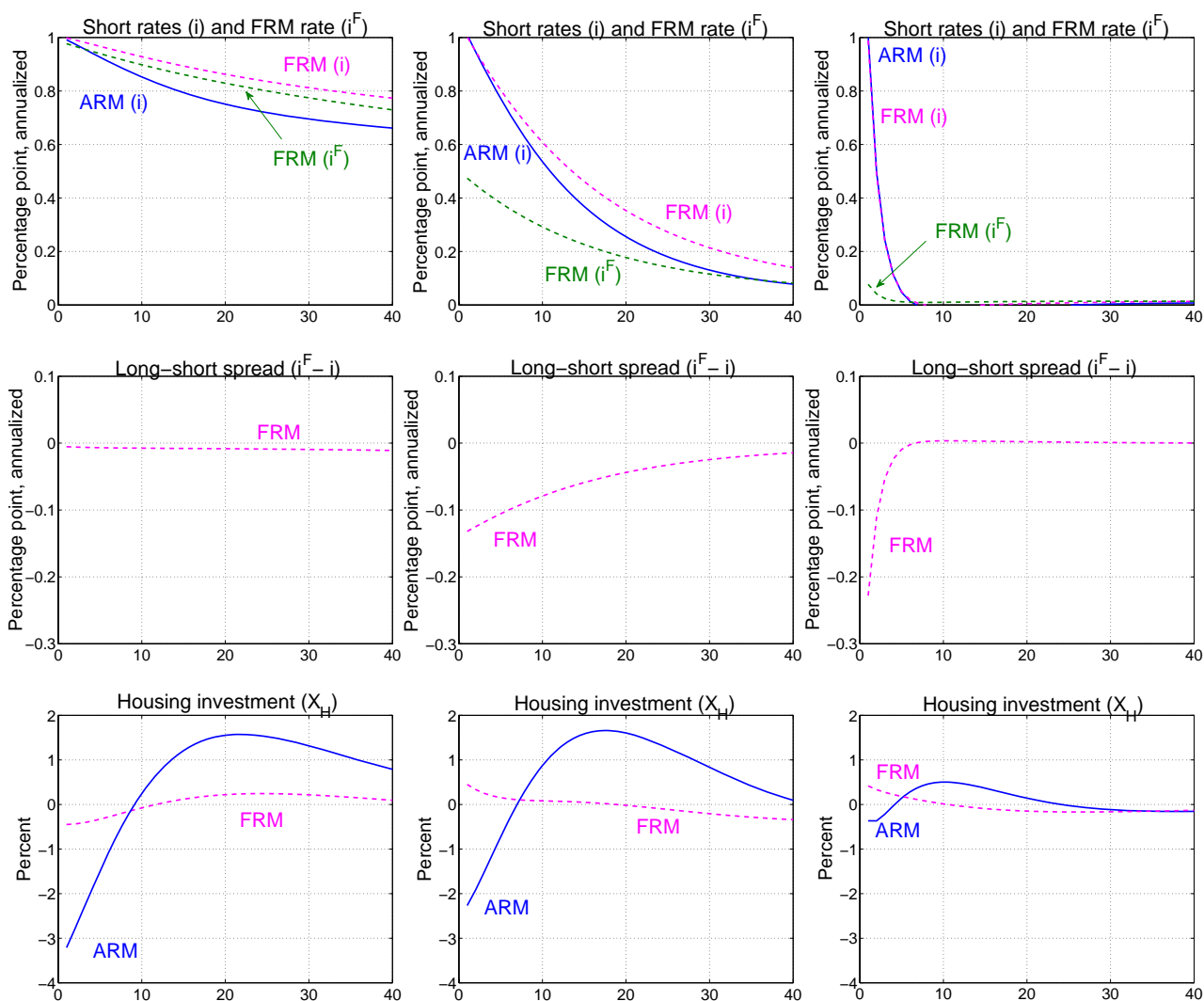


Figure 5: The effect of the persistence of the short-term nominal interest rate. General equilibrium responses to an increase in $\bar{\pi}_t$ in period 1 scaled so as to generate 1 percentage point increase in the short rate in period 1; version with access of homeowners to the 1-period bond market.

A. approx. 120 periods (30yrs)

B. approx. 60 periods (15yrs)

C. 1 period

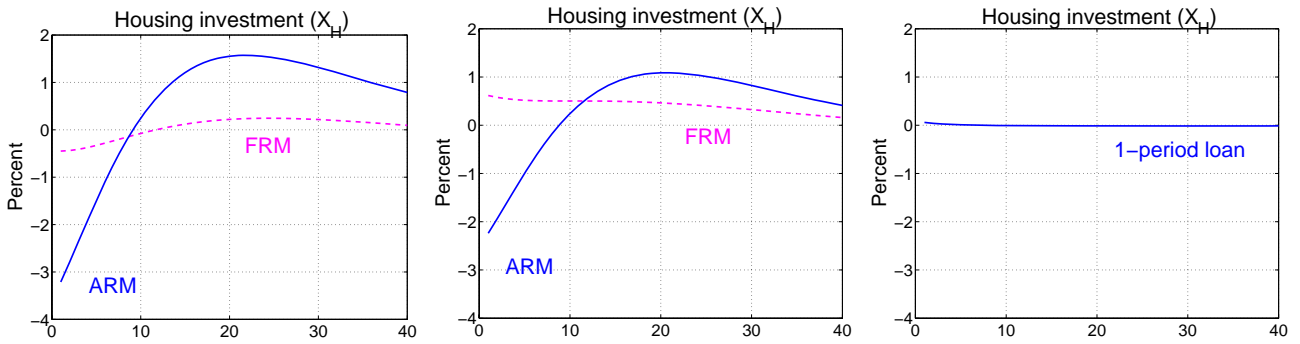


Figure 6: The effect of the term of the loan. General equilibrium responses to 1 percentage point (annualized) increase in $\bar{\pi}_t$ in period 1; version with access of homeowners to the 1-period bond market. Shock persistence is $\rho_\pi = 0.994$.

Table 3: Business cycle properties

	US data	Model			US data	Model	
		FRM	ARM			FRM	ARM
Std(Y)	1.92	0.94	1.04				
Rel. std					Corr with Y		
Y	1.00	1.00	1.00		Y	1.00	1.00
C	0.42	0.42	0.35		C	0.79	0.88
X_S	6.94	9.48	8.20		X_S	0.60	0.99
X_K	2.45	1.76	3.01		X_K	0.73	0.92
π	0.58	0.85	0.81		π	0.14	0.23
i	0.58	0.85	0.85		i	0.36	0.32
i^F	0.35	0.77	N/A		i^F	0.01	0.09
$i^F - i$	0.42	0.21	N/A		$i^F - i$	-0.49	-0.98
q	0.58	0.18	0.15		q	0.41	0.99
p_H	1.57	1.13	0.97		p_H	0.55	0.99

Note: All U.S. moments are for HP-filtered series, post-Korean war data. Interest and inflation rates are annualized. The 10-year government bond yield is used as a proxy for i_t^F due to its longer time availability; the inflation rate of the GDP deflator is used for π_t ; the 3-month T-bill yield is used for i_t ; the ratio of the residential investment deflator to the GDP deflator is used for q_t ; the ratio of the average price of new homes sold (Census Bureau) and the GDP deflator is used for p_{Ht} (1975-2006). The model moments are averages of moments for 150 runs of the model; the artificial series of each run have the same length as the data series and are HP filtered.

Supplemental material—appendices

Appendix A: Equilibrium conditions

This appendix lists the conditions characterizing the equilibrium defined in Section 4.2. Throughout, the notation is that, for instance, u_{ct} denotes the first derivative of the function u with respect to c , evaluated in period t . Alternatively, v_{2t} , for instance, denotes the first derivative of the function v with respect to the second argument, evaluated in period t .

Capital owner's optimality

The first-order conditions with respect to, respectively, x_{Kt} , b_{t+1}^* , and l_t^* :

$$\begin{aligned} 1 &= E_t \left\{ \beta \frac{u_{c,t+1}}{u_{ct}} [1 + (1 - \tau_K)(r_{t+1} - \delta_K)] \right\}, \\ 1 &= E_t \left[\beta \frac{u_{c,t+1}}{u_{ct}} \left(\frac{1 + i_t}{1 + \pi_{t+1}} \right) \right], \\ 1 &= E_t \left\{ \beta \frac{\widehat{U}_{d,t+1}}{u_{ct}} + \beta \frac{U_{\gamma,t+1}}{u_{ct}} \zeta_{Dt}^* [\kappa - (\gamma_t^*)^\alpha] + \beta \frac{U_{R,t+1}}{u_{ct}} \zeta_{Dt}^* (i_t^F - R_t^*) \right\}. \end{aligned}$$

In the first-order condition for l_t^* , which—as discussed in the text—applies only in the FRM case, $\widehat{U}_{dt} \equiv p_{t-1}U_{dt}$ is a normalization to ensure stationarity in the presence of positive steady-state inflation and U_{dt} , $U_{\gamma t}$, and U_{Rt} are the derivatives of the capital owner's value function with respect to d_t^* , γ_t^* , and R_t^* , respectively. These derivatives are given by the Benveniste-Scheinkman (BS) conditions:

$$\widetilde{U}_{dt} = u_{ct} \frac{R_t^* + \gamma_t^*}{1 + \pi_t} + \beta \frac{1 - \gamma_t^*}{1 + \pi_t} E_t \left\{ \widetilde{U}_{d,t+1} + \zeta_{lt}^* [(\gamma_t^*)^\alpha - \kappa] U_{\gamma,t+1} + \zeta_{lt}^* (R_t^* - i_t^F) U_{R,t+1} \right\},$$

$$\begin{aligned} U_{\gamma t} &= u_{ct} \left(\frac{\widetilde{d}_t^*}{1 + \pi_t} \right) - \beta \left(\frac{\widetilde{d}_t^*}{1 + \pi_t} \right) E_t \widetilde{U}_{d,t+1} \\ &\quad + \beta \left(\frac{\widetilde{d}_t^*}{1 + \pi_t} \right) \left\{ \zeta_{lt}^* [\kappa - (\gamma_t^*)^\alpha] + \frac{(1 - \gamma_t^*)^\alpha (\gamma_t^*)^{\alpha-1}}{\frac{1 - \gamma_t^*}{1 + \pi_t} \widetilde{d}_t^* + \widetilde{l}_t^*} \right\} E_t U_{\gamma,t+1} \\ &\quad + \beta \left(\frac{\widetilde{d}_t^*}{1 + \pi_t} \right) \zeta_{lt}^* (i_t^F - R_t^*) E_t U_{R,t+1}, \end{aligned}$$

$$U_{Rt} = u_{ct} \left(\frac{\widetilde{d}_t^*}{1 + \pi_t} \right) + \beta \left(\frac{\frac{1 - \gamma_t^*}{1 + \pi_t} \widetilde{d}_t^*}{\frac{1 - \gamma_t^*}{1 + \pi_t} \widetilde{d}_t^* + \widetilde{l}_t^*} \right) E_t U_{R,t+1}.$$

In these expressions, $\tilde{d}_t^* \equiv d_t^*/p_{t-1}$, $\tilde{l}_t^* \equiv l_t^*/p_t$,

$$\zeta_{Dt}^* \equiv \frac{\tilde{l}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t} \tilde{d}_t^* + \tilde{l}_t^*\right)^2} \in (0, 1),$$

and

$$\zeta_{Dt}^* \equiv \frac{\frac{1-\gamma_t^*}{1+\pi_t} \tilde{d}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t} \tilde{d}_t^* + \tilde{l}_t^*\right)^2} \in (0, 1).$$

Notice that for a once-and-for-all mortgage loan ($l_t^* = l^*$ in period t and $l_t^* = 0$ thereafter) and no outstanding mortgage debt ($d_t^* = 0$ in period t), we have $\zeta_{Dt}^* = 0$ and $\zeta_{l,t+j}^* = 0$, for $j = 1, 2, \dots$. In this case, the first-order condition for l_t^* and the BS condition for \tilde{U}_{dt} simplify, as the terms related to $U_{\gamma t}$ and U_{Rt} drop out. Once combined, the two optimality conditions result in an equation that is a straightforward infinite-horizon extension of the mortgage-pricing equation (1) in the two-period mortgage example of Section 3:

$$1 = E_t [Q_{1t}^* (i_t^F + \gamma_{t+1}^*) + Q_{2t}^* (i_t^F + \gamma_{t+2}^*) (1 - \gamma_{t+1}^*) + \dots],$$

where

$$Q_{Jt}^* \equiv \prod_{j=1}^J \beta \frac{u_{c,t+j}}{u_{c,t+j-1}} \frac{1}{1 + \pi_{t+j}} \quad J = 1, 2, \dots$$

The terms related to $U_{\gamma t}$ and U_{Rt} in the general form of the optimality conditions arise because the mortgage payment m_t^* entering the budget constraint of the capital owner pertains to payments on the entire outstanding mortgage debt, not just the new loan. In this case, the terms related to $U_{\gamma t}$ and U_{Rt} capture the marginal effect of l_t^* on the average interest and amortization rates of the outstanding debt, and thus the marginal effect of l_t^* on the mortgage payments on the outstanding debt.

The capital owner's constraints:

$$c_t^* + k_{t+1} + \tilde{b}_{t+1}^* + \tilde{l}_t^* = [1 + (1 - \tau_K)(r_t - \delta_K)] k_t + (1 + i_{t-1}) \frac{\tilde{b}_t^*}{1 + \pi_t} + \tilde{m}_t^* + \tau_t^* + \frac{pL_t}{1 - \Psi},$$

$$\tilde{m}_t^* = (R_t^* + \gamma_t^*) \frac{\tilde{d}_t^*}{1 + \pi_t},$$

$$\tilde{d}_{t+1}^* = \frac{1 - \gamma_t^*}{1 + \pi_t} \tilde{d}_t^* + \tilde{l}_t^*,$$

$$\gamma_{t+1}^* = (1 - \phi_t^*) (\gamma_t^*)^\alpha + \phi_t^* \kappa,$$

$$R_{t+1}^* = \begin{cases} (1 - \phi_t^*) R_t^* + \phi_t^* i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases}$$

where $\phi_t^* \equiv \tilde{l}_t^*/\tilde{d}_{t+1}^*$ and $\tilde{b}_t^* \equiv b_t^*/p_{t-1}$.

Homeowner's optimality

The first-order conditions with respect to, respectively, n_t , x_{Ht} , and b_{t+1} :

$$v_{ct}(1 - \tau_N)w_t = v_{2t},$$

$$v_{ct}(1 - \theta)p_{Ht} = \beta E_t \left\{ V_{h,t+1} + p_{Ht}\theta \left[\tilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \gamma_t^\alpha)V_{\gamma,t+1} + \zeta_{Dt}(i_{t+1}^M - R_t)V_{R,t+1} \right] \right\},$$

$$1 = E_t \left[\beta \frac{v_{c,t+1}}{v_{ct}} \left(\frac{1 + i_t + \Upsilon_t}{1 + \pi_{t+1}} \right) \right],$$

where $\tilde{V}_{dt} \equiv p_{t-1}V_{dt}$ and V_{ht} , V_{dt} , $V_{\gamma t}$, and V_{Rt} are the derivatives of the homeowner's value function. Further, $i_{t+1}^M = i_t^F$ in the FRM case and $i_{t+1}^M = i_t$ in the ARM case. Analogously to the case of the capital owner,

$$\zeta_{Dt} \equiv \frac{\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t}{\left(\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t + \tilde{l}_t\right)^2} \in (0, 1).$$

The derivatives of the value function with respect to d_t , γ_t , and R_t are given by BS conditions, which take similar forms as those of the capital owner:

$$\tilde{V}_{dt} = -v_{ct}\frac{R_t + \gamma_t}{1 + \pi_t} + \beta\frac{1 - \gamma_t}{1 + \pi_t}E_t \left[\tilde{V}_{d,t+1} + \zeta_{lt}(\gamma_t^\alpha - \kappa)V_{\gamma,t+1} + \zeta_{lt}(R_t - i_{t+1}^M)V_{R,t+1} \right],$$

$$V_{\gamma t} = -v_{ct} \left(\frac{\tilde{d}_t}{1 + \pi_t} \right) - \beta \left(\frac{\tilde{d}_t}{1 + \pi_t} \right) E_t \tilde{V}_{d,t+1}$$

$$+ \beta \left(\frac{\tilde{d}_t}{1 + \pi_t} \right) \left[\zeta_{lt}(\kappa - \gamma_t^\alpha) + \frac{(1 - \gamma_t)\alpha\gamma_t^{\alpha-1}}{\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t + \tilde{l}_t} \right] E_t V_{\gamma,t+1}$$

$$+ \beta \left(\frac{\tilde{d}_t}{1 + \pi_t} \right) \zeta_{lt}(i_{t+1}^M - R_t)E_t V_{R,t+1},$$

$$V_{Rt} = -v_{ct} \left(\frac{\tilde{d}_t}{1 + \pi_t} \right) + \beta \left(\frac{\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t}{\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t + \tilde{l}_t} \right) E_t V_{R,t+1},$$

where

$$\zeta_{lt} \equiv \frac{\tilde{l}_t}{\left(\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t + \tilde{l}_t\right)^2} \in (0, 1).$$

In addition, there is a BS condition for the derivative with respect to h_t :

$$V_{ht} = v_{ht} + \beta(1 - \delta_H)E_t V_{h,t+1}.$$

Rearranging the first-order condition for x_{Ht} yields

$$v_{ct}p_{Ht}(1 + \tau_{Ht}) = \beta E_t V_{h,t+1},$$

where the wedge τ_{Ht} is given by

$$\tau_{Ht} \equiv -\theta E_t \left[1 + \beta \frac{\tilde{V}_{d,t+1}}{v_{ct}} + \zeta_{Dt}(\kappa - \gamma_t^\alpha) \beta \frac{V_{\gamma,t+1}}{v_{ct}} + \zeta_{Dt}(i_{t+1}^M - R_t) \beta \frac{V_{R,t+1}}{v_{ct}} \right].$$

For the same reasons as in the case of the mortgage-pricing equation of the capital owner, the wedge is more complicated than in the case of the two-period mortgage. Again, it becomes a straightforward infinite-horizon extension of either equation (2) or (3) in the main text if the housing investment decision is once-and-for-all and there is no outstanding mortgage debt ($\Rightarrow \zeta_{Dt} = 0$ and $\zeta_{l,t+j} = 0$, for $j = 1, 2, \dots$):

$$\tau_{Ht} \equiv -\theta E_t \left\{ 1 - [Q_{1t}(i_{t+1}^M + \gamma_{t+1}) + Q_{2t}(i_{t+2}^M + \gamma_{t+2})(1 - \gamma_{t+1}) + \dots] \right\},$$

where

$$Q_{Jt} \equiv \prod_{j=1}^J \beta \frac{v_{c,t+j}}{v_{c,t+j-1}} \frac{1}{1 + \pi_{t+j}}.$$

The constraints pertaining to the homeowner are:

$$c_t + p_{Ht}x_{Ht} - \tilde{l}_t + \tilde{b}_{t+1} = (1 - \tau_N)(w_t n_t - \tau) + (1 + i_{t-1} + \Upsilon_{t-1}) \frac{\tilde{b}_t}{1 + \pi_t} - \tilde{m}_t + \Omega_t,$$

where

$$\begin{aligned} \tilde{m}_t &= (R_t + \gamma_t) \frac{\tilde{d}_t}{1 + \pi_t}, \\ \tilde{l}_t &= \theta p_{Ht} x_{Ht}, \\ x_{Ht} &= h_{t+1} - (1 - \delta_H) h_t. \end{aligned}$$

While the homeowner takes Ω_t as given, in equilibrium

$$\Omega_t = -\Upsilon_{t-1} \frac{\tilde{b}_t}{1 + \pi_t}.$$

Due to the aggregate consistency conditions $(1 - \Psi)\tilde{d}_t^* = \Psi\tilde{d}_t$, $\gamma_t^* = \gamma_t$, and $R_t^* = R_t$, it is not necessary to include the homeowners laws of motion for the mortgage variables among the equations characterizing the equilibrium.

Production

The producer's first-order conditions:

$$r_t = A_t f_1((1 - \Psi)k_t, \Psi n_t),$$

$$w_t = A_t f_2((1 - \Psi)k_t, \Psi n_t).$$

Output:

$$Y_t = A_t f((1 - \Psi)k_t, \Psi n_t).$$

The relative price of structures (i.e., the curvature of the production possibilities frontier):

$$q_t = q(\Psi x_{St}).$$

Homebuilding

Land market clearing:

$$X_{Lt} = 1.$$

The production function and the first-order conditions of homebuilders (for the Cobb-Douglas production function) after imposing the land market clearing condition:

$$x_{St} = \frac{1}{\Psi} (\Psi x_{Ht})^{\frac{1}{1-\varphi}},$$

$$p_{Ht} = q_t \frac{(\Psi x_{St})^\varphi}{1 - \varphi},$$

$$p_{Lt} = p_{Ht} \varphi (\Psi x_{St})^{1-\varphi}.$$

For a given x_{Ht} , the first equation determines x_{St} , the second p_{Ht} , and the third p_{Lt} . Notice that when $\varphi = 0$, $x_{Ht} = x_{St}$ and $p_{Ht} = q_t$.

Monetary policy and the government

The monetary policy rule:

$$i_t = (i - \bar{\pi} + \bar{\pi}_t) + \nu_\pi (\pi_t - \bar{\pi}_t).$$

The government budget constraint:

$$G + (1 - \Psi)\tau_t^* = \tau_K(r_t - \delta_K)(1 - \Psi)k_t + \tau_N(w_t \Psi n_t - \tau \Psi) + \tau \Psi.$$

Market clearing

The labor and capital market clearing conditions have already been imposed in the production sector. And the land and structures market clearing conditions have already been imposed in the homebuilding sector. The remaining market clearing conditions are for the bond market:

$$(1 - \Psi)\tilde{b}_t^* + \Psi\tilde{b}_t = 0;$$

and mortgage market:

$$(1 - \Psi)\tilde{l}_t^* = \Psi\tilde{l}_t.$$

It is straightforward to verify that the Walras' law holds (i.e., the goods market clears and national accounts hold):

$$(1 - \Psi)c_t^* + \Psi c_t + (1 - \Psi)x_{Kt} + q_t \Psi x_{St} + G = Y_t = r_t(1 - \Psi)k_t + w_t \Psi n_t.$$

Stochastic processes

TFP:

$$\log A_{t+1} = (1 - \rho_A) \log A + \rho_A \log A_t + \epsilon_{A,t+1}, \quad \text{where } \epsilon_{A,t+1} \sim iidN(0, \sigma_A).$$

Inflation target:

$$\bar{\pi}_{t+1} = (1 - \rho_\pi) \bar{\pi} + \rho_\pi \bar{\pi}_t + \epsilon_{\pi,t+1}, \quad \text{where } \epsilon_{\pi,t+1} \sim iidN(0, \sigma_\pi).$$

Appendix B: Computation

The recursive competitive equilibrium (RCE) is computed using a linear-quadratic (LQ) approximation method for distorted economies with exogenously heterogeneous agents (see Hansen and Prescott, 1995), adjusted along the lines of Benigno and Woodford (2006) to take into account nonlinear constraints of the agents. The centering point of the approximation is the nonstochastic steady state and the LQ approximation of the Bellman equations is computed using numerical derivatives. All variables in the approximation are either in percentage deviations or percentage point deviations (for rates) from the steady state. Before computing the equilibrium, the model is made stationary by expressing all nominal variables in real terms and replacing ratios of price levels with the inflation rate, as in Appendix A.

The nonlinearity in the constraints of the agents comes from the laws of motion for the mortgage variables. The nonlinearity means that these equations cannot be substituted out into the per-period utility function, as required by the standard LQ approximation procedure. For this reason, as noted above, the method is modified along the lines of Benigno and Woodford (2006). This involves forming a Lagrangian, consisting of the per-period utility function and the laws of motion for the mortgage variables. The Lagrangian is then used as the return function in the Bellman equation being approximated. This adjustment is necessary to ensure that second-order cross-derivatives of the utility function and the constraints are taken into account in the LQ approximation. This modification, as applied to the homeowner, is described in detail by Kydland et al. (2014). The specification for the capital owner is analogous. We therefore refer the reader to that paper for details.

An alternative procedure—implemented, for instance, by Dynare—would be to log-linearize the model's equilibrium conditions in Appendix A and use a version of the Blanchard-Kahn method to arrive at the equilibrium decision rules and pricing functions. As is well known, this method yields the same linear equilibrium decision rules and pricing functions as the adjusted LQ approximation; i.e., the same approximation to the set of functions $W(z, S)$.

Appendix C: Data counterparts to variables

This appendix describes the data used to calculate the aggregate ratios employed in calibrating the model. Adjustments to official data are made to ensure that the data correspond conceptually more closely to the variables in the model. To start, for reasons discussed by Gomme and Rupert (2007), the following expenditure categories are taken out of GDP: gross housing value added, compensation of general government employees, and net exports. In addition, we also exclude expenditures on consumer durable goods (as our ‘home capital’ includes only housing) and multifamily structures (which are owned by corporate entities and rented out to households mainly in the 1st and 2nd quintiles of the wealth distribution). With these adjustments, the data counterparts to the expenditure components of output in the model are constructed from BEA’s NIPA tables as follows: consumption (C) = the sum of expenditures on nondurable goods and services less gross housing value added; capital investment (X_K) = the sum of nonresidential structures, equipment & software, and the change in private inventories; housing structures (X_S) = residential gross fixed private investment less multifamily structures; and government expenditures (G) = the sum of government consumption expenditures and gross investment less compensation of general government employees. Our measure of output ($Y = C + X_K + X_S + G$) accounts, on average (1958-2006), for 74% of GDP.

BEA’s Fixed Assets Tables and Census Bureau’s M3 data provide stock counterparts to capital and housing investment: capital stock (K) = the sum of private nonresidential fixed assets and business inventories; housing stock (H) = residential assets less 5+ unit properties.³⁶ Federal Reserve’s Flow of Funds Accounts provide data on mortgages and we equalize mortgage debt in the model (D) with the stock of home mortgages for 1-4 family properties. The Flow of Funds data, however, include mortgage debt issued for purchases of existing homes, second mortgages, and home equity loans. In contrast, the model speaks only to first mortgages on new housing. The data thus provide an upper bound for D in the model.

Appendix D: Estimation of mortgage debt servicing costs

A key measurement for calibrating the model concerns the mortgage debt servicing costs of homeowners. Unfortunately, such information for the United States is not readily available. Four different procedures are therefore used to arrive at its estimate. The four procedures exploit the notion that the homeowners in the model correspond to the 3rd and 4th quintiles of the U.S. wealth distribution. Some of these estimates arguably overestimate the debt servicing costs, while others underestimate it. Nevertheless, all four procedures yield estimates in the ballpark of 18.5% of pre-tax income, the value used to calibrate the model.

The first procedure, for FRM (1972-2006) and ARM (1984-2006), combines data on income from the Survey of Consumer Finances (SCF) and the model’s expression for debt servicing costs. Suppose that all mortgage debt is FRM. The model’s expression for steady-state debt-servicing costs, $(R + \gamma)[D/(pwN - p\tau\Psi)]$, can then be used to compute the average debt-servicing costs of homeowners. The various elements of this expression are

³⁶Separate stock data on 2-4 unit properties are not available, but based on completions data from the Census Bureau’s Construction Survey, 2-4 unit properties make up only a tiny fraction of the multifamily housing stock.

mapped into data in the following way: $D/(pwN - p\tau\Psi)$ corresponds to the average ratio of mortgage debt (for 1-4 unit structures) to the combined personal income (annual, pre-tax) of the 3rd and 4th quintiles, equal to 1.56; R corresponds to the average FRM annual interest rate for a conventional 30-year mortgage, equal to 9.31%; and γ corresponds to the average amortization rate over the life of the mortgage, equal to 4.7% per annum. This yields debt-servicing costs of 22%. This estimate is likely an upper bound as some of the outstanding mortgage debt in the data is owed by the 5th quintile (the 1st and 2nd quintiles are essentially renters) and the effective interest rate on the stock in the data is likely lower than the average FRM rate due to refinancing. When all mortgage debt is assumed to be ARM, this procedure yields 17.5% (based on the average Treasury-indexed 1-year ARM rate for a conventional 30-year mortgage).

The second estimate is based on Federal Reserve's Financial Obligation Ratios (FOR) for mortgages (1980-2006). FOR report all payments on mortgage debt (mortgage payments, homeowner's insurance, and property taxes) as a fraction of NIPA's share of disposable income attributed to homeowners. For our purposes, the problem with these data is that members of the 5th quintile of the wealth distribution are also counted as homeowners in the data (as long as they own a home), even though they do not represent the typical homeowner in the sense of Campbell and Cocco (2003). To correct for this, we apply the shares of the aggregate SCF personal income attributed to the 3rd, 4th, and 5th quintiles of the wealth distribution to disposable income from NIPA. This gives us an estimate of NIPA disposable income attributed to these three quintiles. This aggregate is then multiplied by the financial obligation ratio to arrive at a time series for total mortgage payments. Assuming again that all mortgage payments are made by the 3rd and 4th quintiles, the total mortgage payments are divided by NIPA personal (pre-tax) income attributed to just these two quintiles (using the SCF shares). This procedure yields average debt-servicing costs of 20%.

Third, we use the ratio of all debt payments to pre-tax family income for the 50-74.9 percentile of the wealth distribution, reported in SCF for 1989-2007. The average ratio is 19%. About 80% of the payments are classified as residential by the purpose of debt, yielding an average ratio of 15.2%. A key limitation of this procedure is that the data exclude the 1970s and most of the 1980s—periods that experienced almost twice as high mortgage interest rates, on average, than the period covered by the survey. Another issue is that the information reported in the survey is not exactly for the 3rd and 4th quintiles.

The fourth procedure is based on the Consumer Expenditure Survey (CEX), 1984-2006. This survey reports the average income and mortgage payments (interest and amortization) of homeowners with a mortgage. To the extent that homeowners without a mortgage are likely to belong to the 5th quintile of the wealth distribution—they have 100% of equity in their home and thus have higher net worth than homeowners with a mortgage—the survey's homeowners with a mortgage should closely correspond to the notion of homeowners used in this paper (CEX does not contain data on wealth). The resulting average, for the available data period, for mortgage debt servicing costs of this group (pre-tax income) is 15%. Given that the data do not cover the period of high mortgage rates of the late 1970s and early 1980s, like the third estimate, this estimate probably also underestimates the debt servicing costs for the period used in calibrating the model.

Taken together, the four procedures lead us to use 18.5%, a value in the middle of the range of the estimates, as a target in calibration.

Appendix E: Sensitivity analysis

See Figure A1. The figure is for the economy with no access of homeowners to the one-period bond market so as to prevent them from partially undoing the effects of the shocks. This gives alternative parameterization the best chance to matter. The focus is only on parameterizations that are directly related to the two channels of monetary policy transmission.

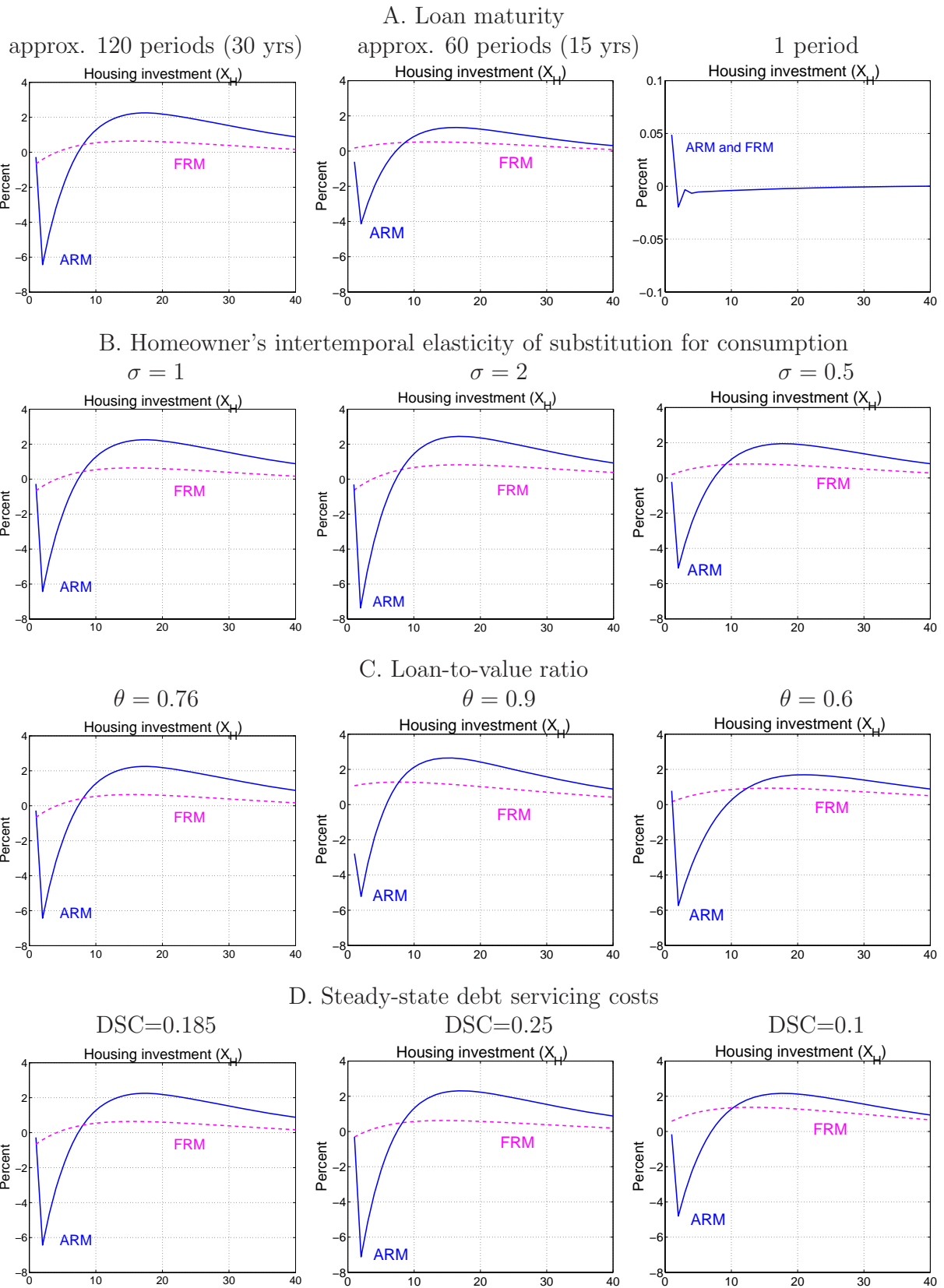


Fig A1. Sensitivity analysis; version with no access of homeowners to the 1-period bond market.

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