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Common and Country Specific Economic Uncertainty

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Common and country specific economic uncertainty*

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Abstract

We use a factor model with stochastic volatility to decompose the time-varying variance of Macroeconomic and Financial variables into contributions from country-specific uncertainty and uncertainty common to all countries. We find that the common component plays an important role in driving the time-varying volatility of nominal and financial variables. The cross-country co-movement in volatility of real and financial variables has increased over time with the common component becoming more important over the last decade. Simulations from a two-country DSGE model featuring Epstein Zin preferences suggest that increased globalisation and trade openness may be the driving force behind the increased cross-country correlation in volatility.

JEL Codes: C15,C32, E32

Key Words: FAVAR, Stochastic Volatility, Uncertainty Shocks, DSGE Model

1 Introduction

Recent turmoil in financial markets has led to a substantial increase in macroeconomic volatility across the industrialised world. This is clear from the simple calculation in figure 1 which shows the average of the rolling standard deviation of the main macroeconomic and financial variables for eleven OECD countries. This simple measure of economic volatility shows an increase across all countries over the post 2007 period highlighting the severity of the financial crisis. It is interesting to note that this high correlation of volatility is not just confined to the recent financial crisis but appears to be a prominent feature of this statistic over several episodes in the past. A casual examination of the figure suggests that this measure of volatility moved especially closely together during the mid-1970s, the early 1980s and then during the beginning and end of the last decade. The full sample correlation between these volatility measures is high, averaging across pairwise combinations at 50%.

The aim of this paper is to investigate the comovement in volatility from an empirical and theoretical perspective. Using a dynamic factor model with stochastic volatility, we decompose the movements in the volatility of real activity, inflation and financial series from these eleven OECD countries into the contributions from country specific and OECD wide uncertainty. We find that OECD wide uncertainty plays an important role in driving the variance of real and nominal variables and is especially important for the latter series. Moreover, we estimate that the contribution of common uncertainty has increased over the sample period and the *volatility* of key variables displays a higher correlation after the late 1990s.

We then build a two country model that features households with Epstein-Zin preferences. One of the key implications of the model set-up is the presence of heteroscedastic endogenous variables, with this feature induced by agents' preferences. We show that in a two country environment, shocks that lead to transfer of resources across countries imply comovement in the second moments of endogenous variables.

*The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.

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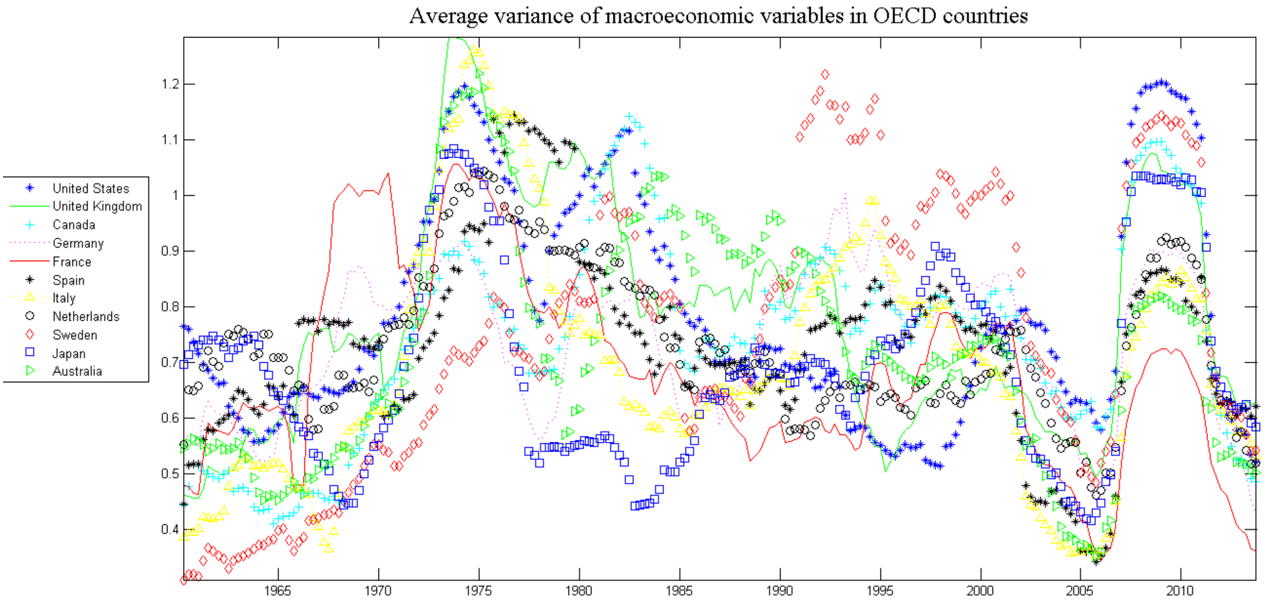

Figure 1: Two year rolling estimate of the average variance of macroeconomic variables in 11 OECD countries. The data set for each country is described in section XXX. For the purposes of this figure, the rolling standard deviation is calculated for each series and then the mean across the series is calculated. The data is standardised prior to this calculation.

This comovement rises as trade links in the model become stronger. In contrast, changes in monetary policy rule and/or the Phillips curve are unable to explain the increase in the importance of the common component in volatility.

This paper is closely related to two recent strands of the empirical Macro literature. The aim of the paper is similar in spirit to the work on international business cycles (see for example Kose *et al.* (2003)) and the research on inflation co-movements (see Mumtaz and Surico (2008)) that has sought to establish the importance of a common factor in explaining the movements in these variables. We focus on comovement in the second moment and show that this feature is important from an empirical and theoretical perspective. Our analysis is also closely related to the recent literature on uncertainty that has focussed on estimating proxies for economic uncertainty. However, we investigate the role of *common* movements in uncertainty, an aspect that has been relatively unexplored in this literature. One recent exception is Berger *et al.* (2014) who estimate uncertainty associated with real activity and inflation common factors, respectively. In contrast, our analysis explores common macroeconomic uncertainty that encompasses both real and financial variables and attempts to provide a theoretical explanation for the empirical findings.

Our results have important policy implications. In particular, they highlight potential consequences of globalisation that have been largely ignored in the literature–i.e. an increasing comovement in the time-varying volatility of output, inflation and stock returns. This suggests that policy makers may need to focus more on economic developments that cause uncertainty in countries that are linked through trade. This implication is especially relevent given the recent debt crisis in the Euro Area that has resulted in an increase in volatility in a number of countries.

The paper is organised as follows: Sections 2 and 3 introduce the empirical model and discuss the estimation method. The results from the empirical model are presented in Section 4. We introduce the DSGE model and present the model simulations in Section 5.

2 Common and country-specific uncertainty

In order to estimate country-specific and common (or 'world') specific measures of uncertainty, we use a dynamic factor model with time-varying volatility. The factor model is defined as

$$X_{it} = B_i^C F_t^C + B_i^W F_t^W + e_{it} \tag{1}$$

where X_{it} is a panel of macroeconomic and financial data for the set of OECD countries described below. This panel of data is summarised by three components: a set of K common or 'world' factors F_t^W, a set K country-specific factors F_t^C and idiosyncratic components e_{it}. The world and the country factors follow VAR processes:

$$F_t^W = \zeta + \sum_{j=1}^{P} p_j F_{t-j}^W + \Upsilon_t^{1/2} g_t \tag{2}$$

$$F_t^C = \alpha + \sum_{j=1}^{P} \rho_j F_{t-j}^C + \Omega_t^{1/2} v_t \tag{3}$$

while the idiosyncratic components have an AR transition equation

$$e_{it} = \sum_{j=1}^{J} \mu_{i,j} e_{it-j} + h_{it}^{1/2} \varepsilon_{it} \tag{4}$$

where $g_t, v_t, \varepsilon_{it}\tilde{\ } N(0,1)$. Note that the error terms in equations 2, 3 and 4 are heteroscedastic. The error covariance matrices in the VAR models 2 and 3 are defined as

$$\Upsilon_t = C^{-1} D_t C^{-1'} \tag{5}$$

$$\Omega_t = A^{-1} H_t A^{-1'} \tag{6}$$

where A and C are lower triangular and D_t and H_t are diagonal matrices defined as

$$\begin{aligned} D_t &= diag(s_k \gamma_t) \\ H_t &= diag(S_k \lambda_t) \end{aligned} \tag{7}$$

The time-varying volatility is captured by γ_t and λ_t with s_k and S_k the scaling factors for $k = 1, 2, ...K$. The overall volatilites evolve as an AR(1) process:

$$\ln \gamma_t = \bar{\alpha} + \bar{\beta} \ln \gamma_{t-1} + \bar{Q}^{1/2} \bar{\eta}_t \tag{8}$$

$$\ln \lambda_t = \tilde{\alpha} + \tilde{\beta} \ln \lambda_{t-1} + \tilde{Q}^{1/2} \tilde{\eta}_t \tag{9}$$

The structure defined by equations 5 and 6 suggests that the volatility specification is characterised by the following features: First, the specification captures the *overall* volatility in the orthogonalized residuals of the VAR models. As explained in Carriero *et al.* (2012), the common volatilities can be interpreted as the average of the variance of the shocks with equal weight given to each individual volatility. Note that the errors to these equations represent the shocks to 'world' and country factors. Thus, γ_t and λ_t capture the average volatility of the unpredictable part of the common component and the country-specific component. We interpret these volatilities as measures of uncertainty associated with OECD wide economic conditions and country specific economic conditions.

The variance of the shocks to the idiosyncratic components are also assumed to heteroscedastic with h_{it} evolving as a stochastic volatility process

$$\ln h_{it} = a_i + b_i \ln h_{it-1} + q_i^{1/2} n_{it} \tag{10}$$

The structure of the model implies that the unconditional variance of each series can be written as a function of Υ_t, Ω_t and h_t. In particular

$$var\left(X_{it}\right) = \left(B_i^C\right)^2 var\left(F_t^C\right) + \left(B_i^W\right)^2 var\left(F_t^W\right) + var\left(e_{it}\right) \tag{11}$$

where the variance terms on the RHS of equation 11 can be calculated using the standard VAR formula for the unconditional variance. Note that these variance terms are time-varying as they are functions of λ_t, γ_t and h_{it} respectively. The volatility of each series in our panel is thus driven by uncertainty that is common to all countries, uncertainty that is country-specific and a residual term that captures sectoral volatility and data uncertainty. Our framework, therefore, allows us to calculate how volatility of key series (such as GDP growth, CPI inflation and stock market returns) is driven by uncertainty that is common to all countries and uncertainty that is country and series-specific.

This empirical model is related to a number of contributions in the recent literature on measuring uncertainty. In particular, the spirit of our model is similar to the procedure used in Jurado *et al.* (2013) to estimate US economic uncertainty. The uncertainty measure in Jurado *et al.* (2013) is the average time-varying variance in the unpredictable component of a large set of real and financial time-series. The volatility specification in our factor model has a similar intepretation– it attempts to capture the average volatility in the *shocks* to the factors that summarise real and financial conditions. In contrast to Jurado *et al.* (2013), however, our model allows the estimation of uncertainty at the country and at the 'world' level.

3 Estimation and model specification

The factor model in equations 1 to 10 is estimated via Gibbs sampling. The technical appendix provides details of the priors and the conditional posterior distributions. In this section we summarise the main steps of the algorithm. The Gibbs algorithm samples through the following conditional posterior distributions

1. Given a draw for the factors F_t^C and F_t^W and the volatilities λ_t, γ_t and the matrices C, A, the coefficients of the VAR equations 2 and 3 have a standard conditional posterior once a GLS transformation is applied to the data. Similarly, conditional on e_{it} and h_{it}, equation 4 represents a series of AR models where the posterior for the coefficients of a linear regression applies.

2. Given a draw for the volatilities, the conditional posterior for the coefficients and the error variances of the transition equations 8, 9 and 10 is Normal-Inverse Gamma, and thus methods for linear regressions apply here as well.

3. Conditional on a draw for the VAR coefficients in equations 2 and 3, the volatilities λ_t, γ_t and the scaling factors S, s, the non-zero elements of the matrices C and A can be drawn using the method described in Cogley and Sargent (2005). Given the VAR residuals, C, A and the volatilities, the scaling factors S and s have an Inverse Gamma conditional posterior.

4. Given a draw for the transition equation parameters (equations 8, 9), the matrices C, A, the variances S, s and the coefficients of the VARs in equations 2 and 3, the stochastic volatilities λ_t, γ_t are drawn using the date by date Metropolis Hastings algorithm described in Cogley and Sargent (2005) and Jacquier *et al.* (1994) (see also Carlin *et al.* (1992)). This algorithm is also used to draw the state variable h_{it} in the univariate stochastic volatility models 10.

5. Given a value for the factors F_t^C and F_t^W, the AR coefficients $\mu_{i,j}$ and h_{it} the observation equation 1 is a sequence of linear regressions with heteroscedasticity and serial correlation. After applying a GLS transformation, the posterior for a linear regression applies for the *ith* equation and the factor loadings can be easily drawn.

6. Given, the factor loadings, the volatilities and the parameters of the transition equations 2, 3 and 4, the model can be cast in state-space form. The Carter and Kohn (2004) algorithm is then used to draw from the conditional posterior distribution of the factors.

In the benchmark specifications, we use 50,000 replications and base our inference on the last 5,000 replications. The recursive means of the retained draws (see technical appendix) show little fluctuation providing support for convergence of the algorithm.[1]

3.1 Model specification

In order to maintain parsimony, L the lag lengths in the VARs are fixed at 2. In addition, we allow for first order serial correlation in the idiosyncratic errors e_{it}. The number of common and county-specific factors is an important specification choice. We consider models with 2 to 5 common and country factors and select the model which minimises the Bayesian Deviance Information Criterion (DIC). [2]

Introduced in Spiegelhalter *et al.* (2002), the DIC is a generalisation of the Akaike information criterion – it penalises model complexity while rewarding fit to the data. As shown in the technical appendix, the DIC can be calculated as $DIC = \bar{D} + p_D$ where $\bar{D}$ measures goodness of fit and p_D approximates model complexity. A model with a lower DIC is preferred. Table 1 shows that the DIC is minimised for the model with 4 world and country factors and thus we set $K = 4$ in our benchmark model.

	DIC
2 factors	1.65×10^5
3 factors	1.75×10^5
4 factors	1.52×10^5
5 factors	1.64×10^5

Table 1: Model Comparison via DIC. Best fit indicated by lowest DIC

3.2 Data

As alluded to above, the model is estimated using quarterly data on eleven OECD countries. We consider data for the United States, United Kingdom, Canada, Germany, France, Spain, Italy, the Netherlands, Sweden, Japan and Australia. We limit our attention to these industrial countries mainly because of data availability – data on real and financial sectors of the economy is available for a reasonably long time span for these countries. Both of these features are essential for our analysis as we aim to incorporate uncertainty in a broad range of series and to also include periods which were subject to interesting fluctuations (such as the 1970s).

For each country the data runs from 1960Q1 to 2013Q3. The number of series included for each country varies according to data availability. However, we attempt to maintain a similar composition of macroeconomic and financial series. For each country, the data set includes real activity variables (for e.g. exports, imports, consumption, investment, production, GDP), measures of inflation and earnings, interest rates and term spreads, corporate bond spreads, exchange rates and stock prices. Where relevant and available, commodity prices are also included in the country specific data sets. Where necessary, the variables are log differenced to induce stationarity. Finally, all series are standardised. The technical appendix provides a list of the series used and the data sources.

[1] The technical appendix presents results from a small Monte-Carlo experiment that shows that this MCMC algorithm performs well.

[2] We vary the country and common factors in a symmetric manner in this exercise. In theory it is possible to consider combinations of a different number of these factors. Note, however, that as a computationally intensive particle filter algorithm (see appendix) is used to compute the likelihood for the purpose of DIC calculation, computational feasibility limits us to the symmetric approach.

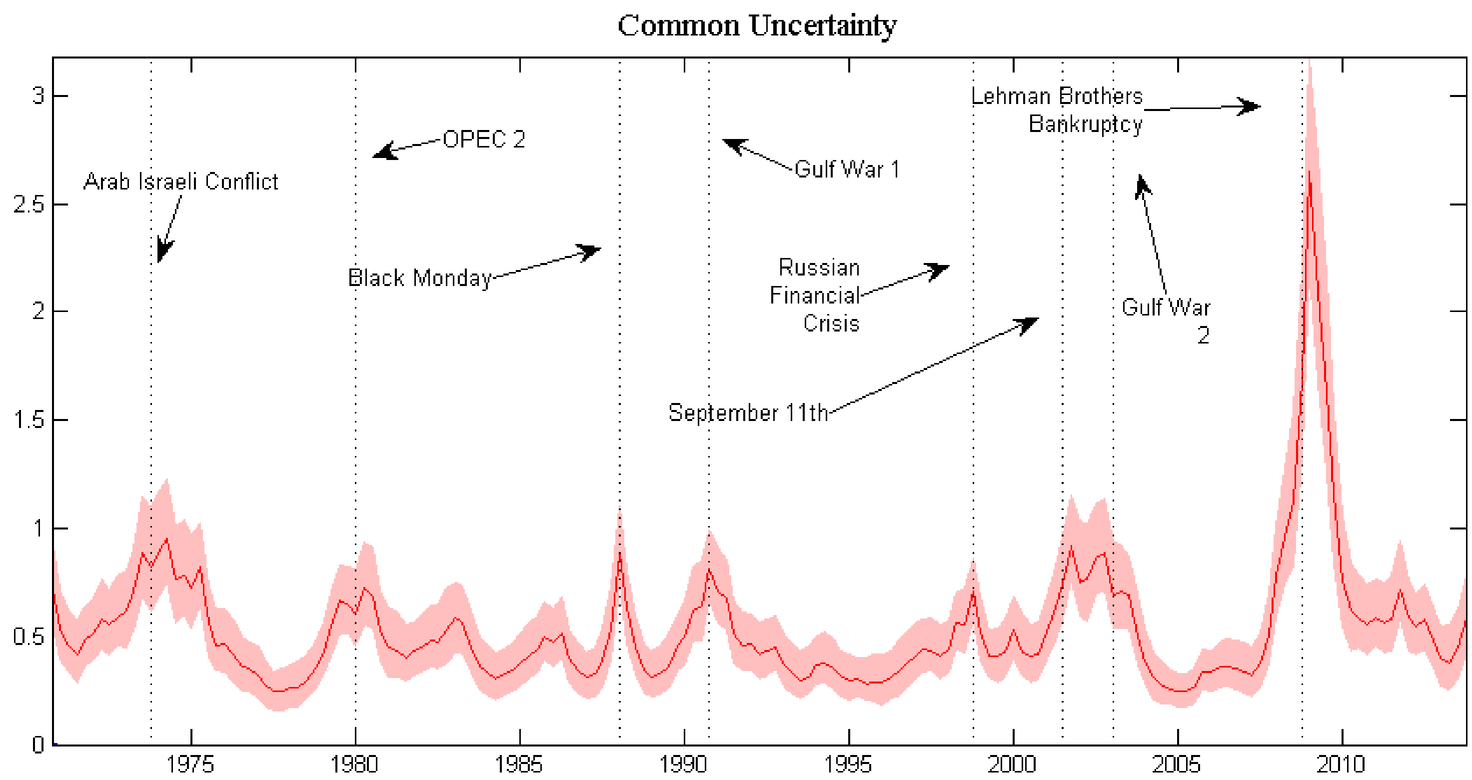

Figure 2: The estimate of the common standard deviation of shocks to the world factors.

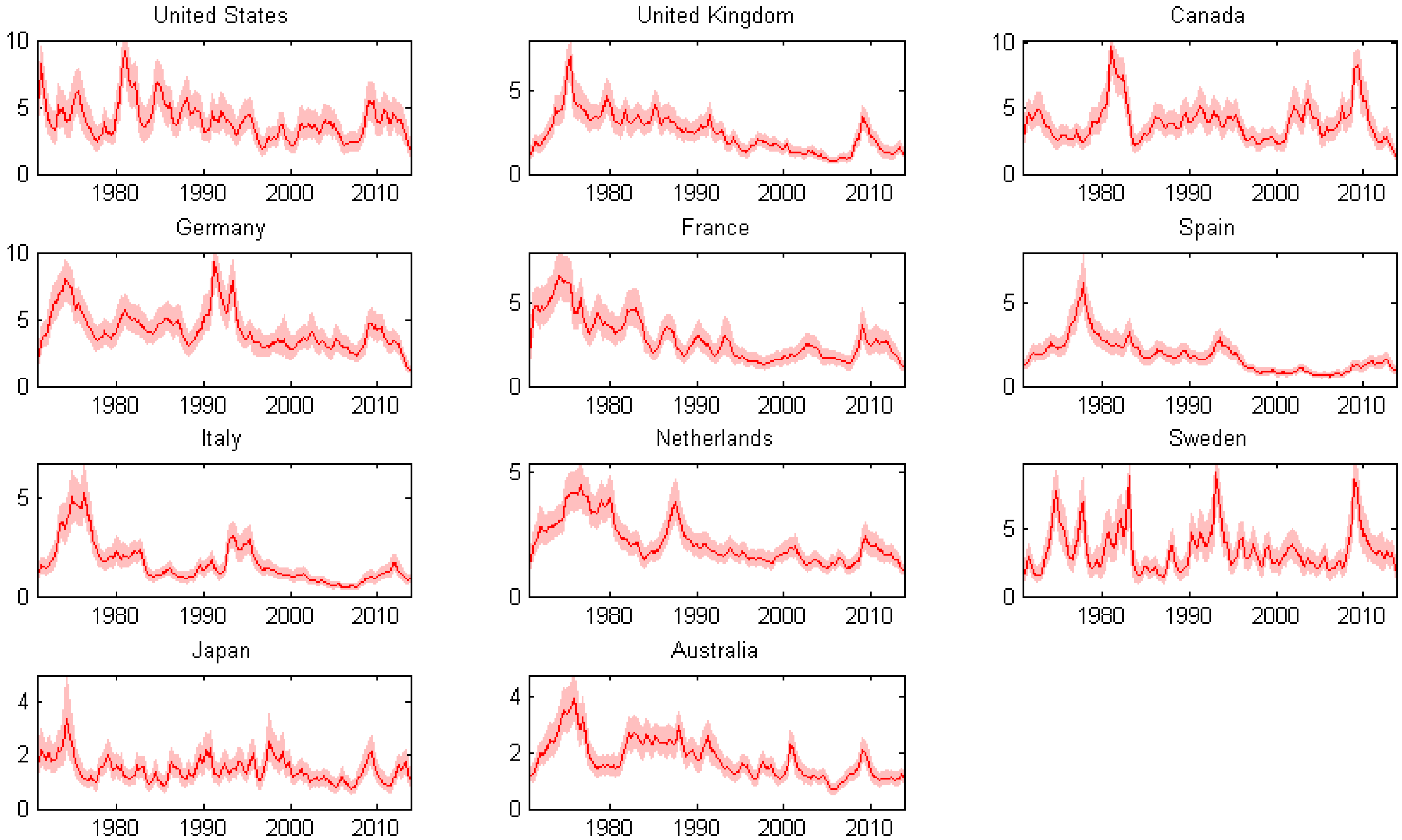

Figure 3: The estimate of the common standard deviation of shocks to the country factors.

4 Empirical results

4.1 The estimate of common and country-specific uncertainty

Figure 2 presents the posterior estimate of $\gamma_t^{1/2}$, the common standard deviation of the shocks to the 'world' factors. We interpret this as a measure of uncertainty that is common across our panel of countries. It is interesting to note that this measure of common or 'world uncertainty' was at its highest over the recent financial crisis in 2009, again emphasizing the scale of the recent recession and its wide-ranging effect on the countries in our sample. The vertical lines in the figure indicate that previous peaks in this measure have coincided with important events such as wars in the middle east, oil shocks, major financial crises and the September 11th terrorist attacks.

Figure 3 shows the estimated country specific uncertainty measures–i.e. $\lambda_{t,c}^{1/2}$ for countries $c = 1, 2, ...11$. The estimate country specific uncertainty for the United States reached its peak during the early 1980s that were characterised by a recession and change in the practice of monetary policy referred to as the 'Volcker Experiment'. The mid-1980s saw another increase in US-specific uncertainty corresponding to the savings and loan crisis in 1985-1986. The late 1980s and early 1990s were also characterised by periods of increased uncertainty with the US economy buffeted by a stock market crash, recession and the onset of a credit crunch and the war in Iraq. Uncertainty declined after the mid-1990s but rose following the September 11th terrorist attacks in 2001 remaining elevated until the middle of the decade. Uncertainty was also higher during the recent recession. Note that this estimate of US-specific uncertainty has a correlation of 0.5 with the measure proposed in Jurado *et al.* (2013). While the two measures coincide during some time periods, several peaks in the Jurado *et al.* (2013) uncertainty measure are classified as comon or 'world' uncertainty by our model.

The UK specific uncertainty measure peaked during the Sterling crisis of 1976. In the following years, this measure displays spikes during the recession of early 1980s, the exchange crisis in 1985, the ERM exit in 1992. The inflation targetting period was characterised by low volatility until the recent recession when the measure increased again. Canadian uncertainty peaked during 1981, when the country experienced one of its worst recessions. Prominent increases in volatility can also be seen during the early 2000s when the Canadian economy experienced a slowdown coinciding with a large appreciation of the Canadian dollar. Finally, Canada-specific uncertainty increased sharply during the recent recession. German uncertainty is at its highest during the period of re-unification and the ERM crisis in the early 1990s. Note also that the two strongest post-war German recessions in 1974 and 2008 are characterised by increases in uncertainty with the former rise estimated to be substantially larger and the latter episode largely captured by the rise in world-specific uncertainty. The uncertainty in the remaining Euro-Area countries follows a similar pattern, with highs estimated during the oil shock of the early 1970s, the stock market crash of the late1980s, the recessions following the ERM crisis and then during the great recession. It is interesting to note that uncertainty in Sweden appears to fluctuate more than the Euro-Area countries, with large increases coinciding with the oil shocks of the 1970s, the large depreciation in the exchange rate in 1982, the financial crisis of the early 1990s and finally during the downturn in 2008-2009.

Japanese uncertainty was high during the mid-1970s. It rose again with the bursting of the real-estate bubble in 1990-1991, in the aftermath of the Asian financial crisis and then in 2008/2009. In Australia, country-specific uncertainty was at its peak during the recession of the mid-1970s. Uncertainty remained elevated through the 1980s peaking after the stock market crash of 1987. Uncertainty rose again in 1991, when the Australian economy was embroiled in a deep recession. The slowdowns in 2001 and then in 2008/2009 also coincided with increasing uncertainty.

Table 2: Contribution of the common, country specific and idiosyncratic component to the variance of output growth, inflation, the short-term interest rate and stock returns.

	Common	Country	Idio.	Common	Country	Idio.	Common	Country	Idio.	Common	Country	Idio.
United States	1.6	94.9	3.1	50.7	28.8	15.9	13.4	40.4	39.5	23.5	55.2	17.4
United Kingdom	4.2	88.6	6.1	2.4	97.3	0.2	1.6	20.5	75.9	2.7	96.8	0.4
Canada	16.7	14.8	65.0	39.4	44.7	11.6	9.7	7.5	80.2	43.5	2.9	52.0
Germany	21.0	22.9	50.2	6.6	92.5	0.6	37.0	0.9	60.7	51.9	10.7	32.2
France	35.8	4.7	55.1	4.6	20.5	71.0	13.5	0.3	85.7	42.7	23.5	29.0
Spain	23.1	1.6	72.8	43.1	56.1	0.4				27.0	26.5	41.2
Italy	27.3	41.5	26.8	58.6	41.1	0.1	14.6	6.3	77.2	46.5	6.3	45.1
Netherlands	12.7	64.0	20.0	28.6	71.2	0.2	15.2	28.9	50.8	72.8	2.0	23.8
Sweden	10.5	22.5	62.1	19.2	80.6	0.2	8.6	2.0	86.3	44.0	24.9	24.6
Japan	18.1	49.3	26.7	14.0	84.8	1.0				14.7	84.7	0.4
Australia	9.7	89.5	0.6	19.9	80.0	0.0	31.1	0.5	68.0	26.2	73.2	0.3
Average	16.4	44.9	35.3	26.1	63.4	9.2	16.1	11.9	69.4	35.9	37.0	24.2

4.2 Variance decomposition

One of the key questions that we wish to address using our empirical model is to investigate how important the common uncertainty measure has been in driving the volatility of key macroeconomic and financial variables. We do this by decomposing the unconditional variance into contributions from λ_t, γ_t and h_{it} respectively. The structure of the model implies that these contributions can be calculated simply using equation 11. Note that as the variance in the model are time-varying, the implied decomposition changes over time as well.

Table 2 presents the contribution of of 'world' or common, country-specific and idiosyncratic uncertainty to the unconditional volatility of output growth, Inflation, the three month treasury bill rate and stock returns. The first three columns of the table show the decomposition for output growth. Output growth is defined as the quarterly growth of GDP for all countries except Sweden where this variable is unavailable and we report the results for the volatility of the growth of industrial production. The country-specific uncertainty is, in general, estimated to be more important for output volatility than 'world' uncertainty implying that domestic uncertainty shocks are largely responsible for driving this variable. However, the role played by common uncertainty is non-negligible. On average, this contribution ranges from a modest 2% to 5% for the United States and the United Kingdom to a high of around 35% for France. The average contribution is greater than 10% for countries other than United States and United Kingdom.

Columns 4 to 6 of table 2 decompose the unconditional volatility of CPI inflation. Across countries, common uncertainty is estimated to be more important for inflation volatility than the variance of output growth, with a cross-country mean contribution of 26%. Average contributions are around 40% to 50% for countries such as United States, Canada, Italy and Spain, with the contribution in Japan, Australia, Netherlands and Sweden ranging between 14% to 30%. In contrast, the importance of 'world' uncertainty is estimated to be small for inflation variance in the United Kingdom, Germany and France. For these countries, inflation variance is persistently high during the 1970s and the early 1980s with this pattern shared by country-specific uncertainty for the United Kingdom and Germany and by idiosyncratic uncertainty for France.

Columns 7 to 9 of the table show the decomposition of the unconditional variance of the three month Treasury bill rate. Note that this variable was not included in the data set for Japan and Spain because the available data over some sub-samples was characterised by discrete changes in this variable, with this feature resulting in estimation difficulties.[3]For countries such as the US, Germany, Netherlands and Australia, the contribution of common uncertainty to the volatility of this variable is higher than the estimate for output growth variance. It is interesting to note that idiosyncratic uncertainty plays a major role. If one interprets the idiosyncratic component of the interest rate as an approximation to the monetary policy shock, then this suggests that monetary policy uncertainty has played an important role in driving the variance of short-term rates.

'World' uncertainty also makes an important contribution to the variance of stock returns (columns 10 to 12 of table 2). For example, the average contribution of common uncertainty is around 25% for the United States and Australia and ranges from 25% to over 70% for Canada and the Euro-Area countries. On average, common uncertainty is estimated to more important for stock return variance than the remaining variables considered above.

[3]Discrete values for the interest rate introduces occasional zeros in the residuals leading to numerical problems in the stochastic volatility part of the MCMC algorithm.

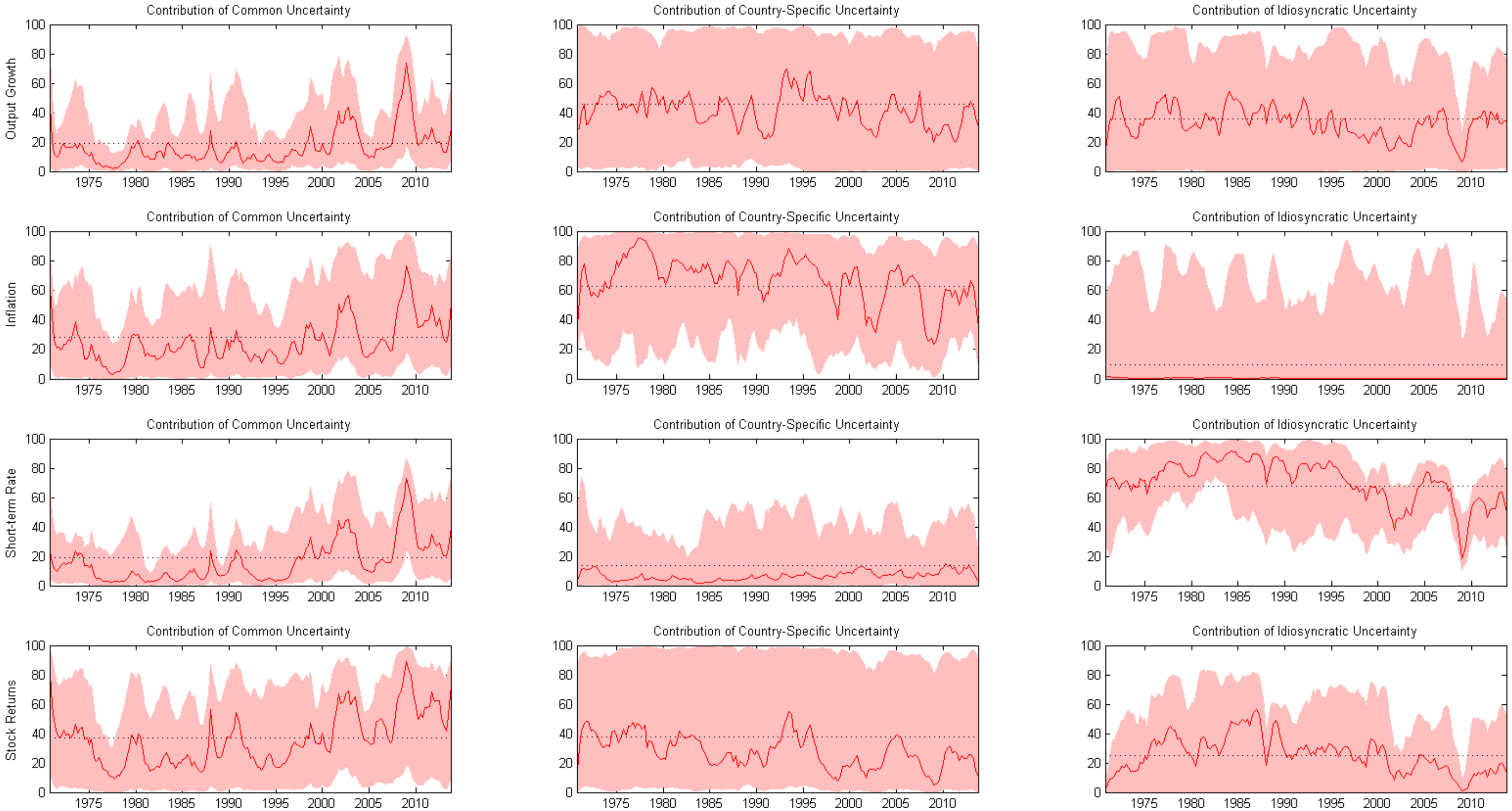

Figure 4: The contribution of common, country-specific and idiosyncratic uncertainty. The red line is the median contribution across countries, while the pink band represents the 5th and 95th percentiles. The dotted black line is the average contribution across countries and time.

4.2.1 Changes in the contributions over time

In figure 4, we consider the temporal evolution of the contribution of the common, country-specific and idiosyncratic uncertainty. The figure shows the median contributions across the 11 countries (red line) and the 5^{th} and the 95^{th} percentiles (pink shaded area). The overall average contribution is depicted as a dotted black line. The median contribution of the common component to output, inflation and interest rate volatility hovers around the 10% to 20% mark before the 2000s. As noted above, the contribution to stock return volatility is estimated to be higher relative to the other four variables. A key feature to note is the movement of this contribution after the late 1990s. For all variables, there is an increase in the contribution of the common component–i.e. the large increase in the volatility of these variables in early 2000s and 2008/2009 is driven to an important extent by common uncertainty.

The counterpart to the increasing importance of the common component is a decline in the median contribution of the country-specific and idiosyncratic components for output growth, inflation and stock returns and a decline in the role of the idiosyncratic uncertainty in the case of the short-term interest rate. This is in contrast to the earlier decades where these components seem to be largely dominant.

In summary, the variance decomposition exercise suggests three results. First, we find that the role played by 'world' uncertainty in driving the variance of real and nominal variables is non-trivial on average. Second, common uncertainty appears to be, on average, more important for the variance of inflation and stock returns than output growth. Finally, the contribution of common uncertainty is estimated to be higher after the mid and late-1990s and the co-movement in volatility of key variables has increased. In the next section we test if these results are robust to changes in the specification of the benchmark model.

4.3 Sensitivity Analysis

In order to test the robustness of these results, we carry out key sensitivity checks. While our model features time-varying variances, the parameters of the observation and transition equations are assumed to be fixed. To investigate the impact of this assumption, we re-estimate the model over the post-1985 period. This period corresponds roughly to a shift in macroeconomic dynamics for OECD countries, with lower volatility and lower inflation signalling the start of the 'Great Moderation'. The resulting variance decomposition is shown in figure 2 in the technical appendix. The figures show that the key results are preserved. In particular, common uncertainty is estimated to be important in driving the variance of output growth, inflation, interest rates and stock returns with the average contribution increasing after the late 1990s. As a second check, we re-estimate the benchmark model restricting the sample to 2006Q4. This is done to account for the possibility of large parameter changes over the recent financial crisis. Figures 3 in the technical appendix again show that the main features of the variance decomposition are preserved in the this alternative specification.

5 Explaining the empirical results. A DSGE model

The empirical analysis in the paper suggests that the time-varying volatility of key macroeconomic and financial variables is characterised by a common component across countries and that this co-movement has become more important over time. In this section, we build a DSGE model to provide a theoretical explanation for these results. The proposed New Keynesian model features two-countries with recursive preferences (Epstein and Zin (1989) and Weil (1989, 1990)) and long-run risk (Bansal and Yaron (2004), Rabanal *et al.* (2011) and Rabanal and Rubio-Ramirez (2015)). To the best of our knowledge, this is the first paper that introduces recursive preferences and long-run risk into a two-country New Keynesian model.

5.1 Summary of the model

Households in each country form Epstein-Zin preferences, consume, supply labour and invest on a internationally traded riskless bond. Part of the consumption is produced domestically and the rest is imported from the foreign economy. On the supply side, there is a continuum of monopolistically competitive firms that produce an intermediate good by combining labour and (fixed) physical capital. The output of this process is used for the production of the final good, which can be either consumed domestically or exported, by a perfectly competitive sector. Similarly, the foreign consumption good is imported by a perfectly competitive sector. Prices in the intermediate sector are based on Calvo (1983) schemes, while import prices obey the law of one price. Monetary authorities set policy based on a Taylor type rule (Taylor (1993)). Each economy is disturbed by a non-stationary productivity, a stationary productivity and a financial shock. The two stochastic trends are assumed to be co-integrated to ensure the existence of a balanced path. Furthermore, the financial shock is the one used by Smets and Wouters (2007)–i.e. it decreases the value of the bond exogenously. In short, our theoretical setup can be viewed as the two-country version of the model developed by Rudebusch and Swanson (2012) and Swanson (2015). Technical details on the model variables and equations can be found in the technical appendix of the paper. The appendix also provides details on the calibration of the parameters. The calibration is largely standard and uses the parameter values proposed in previous studies.

5.2 Key implications from the model

5.2.1 Heteroscedasticity

It is interesting to note that this model implies that the volatility of endogenous variables is time-varying without the explicitly need for heteroscedastic shocks. As explained by Rudebusch and Swanson (2012) and Swanson (2015), this heteroscedasticity is a consequence of the nonlinearity of the lifetime preference function as well as the additive separability of consumption in the period utility function. According to the authors the additive separability of consumption makes the model non-homogeneous which induces a small degree of conditional heteroscedasticity, which is further enhanced by the high risk aversion.

The economic intuition behind these two technical conditions is as follows. The specific life time utility functional form breaks the link between the intertemporal elasticity of substitution parameter and the coefficient of relative risk aversion (induced by standard preferences) and agents in these economies prefer an early resolution of uncertainty over future consumption.[4] As explained in Colacito and Croce (2013, Appendix A), this creates an endogenous trade-off between expected utility and variance utility, meaning that agents are happy to give away some expected future consumption in order to mitigate the risks of future expected consumption. This feature combined with the additive separability of consumption makes agents' responses depend on the state of the economy (i.e. the level of output and consumption). This implies that when the current level of output/consumption is low, consumption uncertainty is higher. This reflects agents' elevated concerns about future shocks. This is because an adverse shock that lowers output further is going to induce a proportionally larger reduction in consumption relative to the case where the initial level of consumpton/output was high.

We illustrate this feature of the model in figure 5. The figure plots the realised volatility of domestic and foreign output, inflation, policy rate and equity returns. These simulations confirm that the data generated by the model with homoscedastic shocks display time-varying variance.[5]

[4] This is true when the degree of risk aversion exceeds the intertemporal elasticity of substitution parameter, which is the case in this paper.

[5] The volatility is calculated via simulation. The model is simulated for 100000 periods. The first 50000 observations are discarded to eliminate the effects of the initial conditions. The remaining 50000 periods are used to calculate the realised standard deviation of these series based on a 40-quarter rolling window.A very similar picture is obtained when a univariate stochastic volatility model is used to estimate the realised volatility of the simulated data.

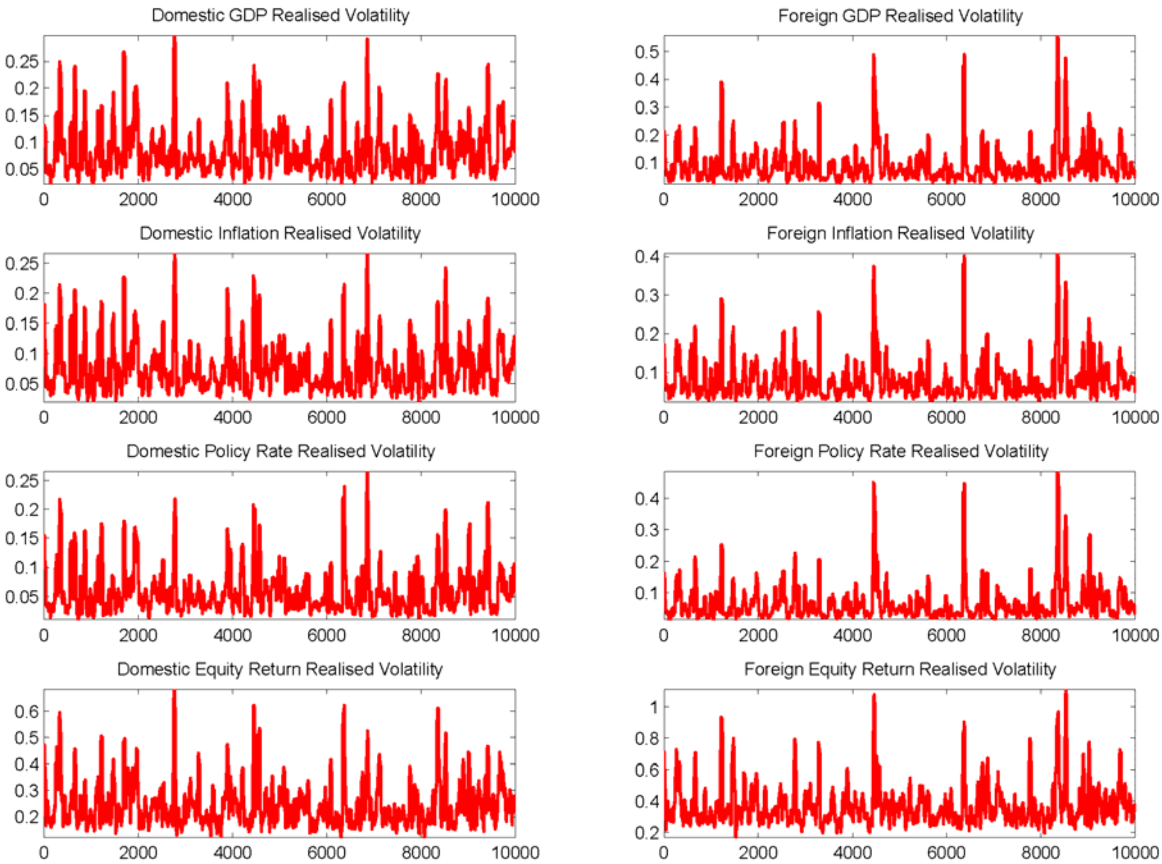

Figure 5: Realised volatility of key model variables.

5.2.2 Comovement in volatility

The model also implies cross-country comovement in the volatility of endogenous variables. First, as the two economies are symmetric, agents in both countries display precautionary behaviour in response to shocks. Furthermore, shocks originating in the domestic economy are transmitted to the foreign one via asset links (UIP condition) or/and good (imported consumption) trade links. These channels: (i) magnify the precautionary behaviour (see Colacito and Croce (2013, Appendix A)) (ii) cause higher moments to be correlated across the two economies. For example, consider a situation where a positive shock increases the supply of the domestic good and thus reduces domestic uncertainty. As the home economy marginal utility of consumption drops home economy agents find it optimal to transfer resources to the foreign country. This transfer of resources has an ameliorating impact on the volatility of foreign variables and thus induces a correlation between the second moments across countries.

Table 3: Correlation amongst the time-varying variance of output (σ_t^Y), inflation (σ_t^π), interest rates (σ_t^R) and equity returns (σ_t^{QE}) across countries. The superscript $*$ denotes the foreign country.

Moments	Benchmark	Strong Home Bias	Smaller response to inflation	Lower Price/wage rigidity
$Corr\left(\sigma_t^Y, \sigma_t^{Y^*}\right)$	0.60	0.28	0.63	0.80
$Corr\left(\sigma_t^\pi, \sigma_t^{\pi^*}\right)$	0.77	0.36	0.69	0.94
$Corr\left(\sigma_t^R, \sigma_t^{R^*}\right)$	0.68	0.35	0.64	0.86
$Corr\left(\sigma_t^{Q^E}, \sigma_t^{Q^{E,*}}\right)$	0.83	0.46	0.75	0.77

In order to demonstrate this feature, we calculate the correlations across countries of the simulated volatilities shown in figure 5. The first column of table 3 displays the results under the benchmark calibration.

The estimated correlations capture a number of features highlighted by the empirical results. First, the estimated correlations are non-negligible and show that the volatility co-moves across countries. Second, the correlation of the volatility of output is lower than the correlation of the variance of inflation and the financial variables. This feature is driven by the precautionary mechanism described earlier, where agents with recursive preferences are keen to give up expected future consumption in order to ensure a less volatile future consumption profile and this leads to a transfer of consumption across countries. For this transfer to occur, prices and especially the exchange rate need to adjust in order for the markets to clear. Since domestic prices are staggered (Calvo contracts) most of the variability of the CPI inflation is driven by import prices and, as the law of one price holds for importing firms, by the exchange rate variability. Due to the symmetry of the model, exchange rate uncertainty has very similar effects on CPI inflation in both countries and this induces the higher moments of CPI inflation across countries to co-move strongly. Strong cross-country CPI inflation volatility co-movement implies that the interest rate variance across counties is correlated as monetary authorities stabilise inflation by adjusting the policy rate. In contrast, the equity price volatility correlation is driven by the stochastic discount factor. Equity prices are functions of the stochastic discount factor. As the UIP condition attempts to equalise the stochastic discount factor across countries (risk sharing), this results in a closer co-movement of volatility of this variable.

The second column of table 3 presents second moment correlations estimated assuming a stronger home bias. A comparison with the benchmark correlations make it clear that as the degree of home bias falls, the second moment correlations rise dramatically. As the economy becomes more open, agents are more willing to hedge against future risks about expected utility by transferring resources abroad. Therefore, globalisation and an increase in trade provides one explanation for the empirical result that the role of the common uncertainty component has increased over time and consequently, the volatility of output, inflation and financial variables has become more correlated. Figure 6 shows that the ratio of total trade to GDP, a measure of trade intensity, rose sharply in the countries in our panel after 1990.

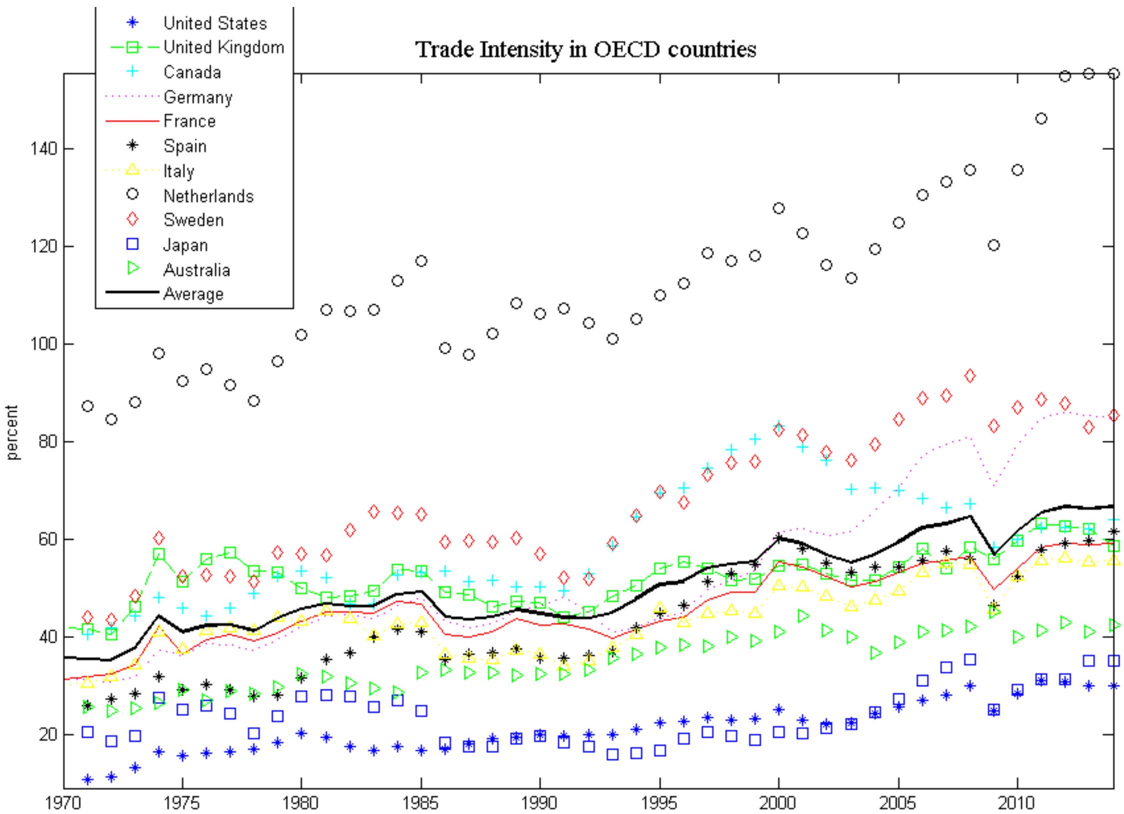

Figure 6: Ratio of total trade to GDP. Source: World Bank

This date broadly coincides with the increase in the contribution of common uncertainty evident in figure 4 above and again points to the role of globalisation in driving the second moment co-movement estimated by the empirical model.

Of course, a number of other structural changes also took place in the OECD and globalisation is not the only possible explanation for the increased importance of common uncertainty. However, it is worth noting that simulations from the DSGE model allow us to largely rule out two alternative explanations for this result. First, numerous studies have shown that after the mid-1980s a number of countries in our panel changed their practice of monetary policy and adopted a more anti-inflationary stance (see for example Clarida *et al.* (1998)). In order to consider the role of systematic policy, we re-calculate the cross-country volatility correlation under the assumption that the coefficient on inflation in the Taylor rule in both countries falls from 1.5 to 1.01. As monetary authorities do not target inflation aggressively, price dispersion rises, magnifying the resource costs associated with sticky prices. This increases the unconditional variance of real and nominal variables in each country but, as shown in the third column of table 3, does not change the second moment correlations relative to the benchmark case. This result suggests that as long as preferences of policy makers regarding inflation are symmetric across countries, any change on the weight placed on inflation does not trigger a change in the way that agents try to ensure against future utility risks. Given that anti-inflationary monetary policy was in place across our panel of countries after the mid-1990s, a change in the practice of monetary policy is less likely to be an explanation for the increasing role of common uncertainty. Fernandez-Villaverde and Rubio-Ramirez (2008) and Hofmann *et al.* (2012) present evidence showing that the degree of price and wage nominal rigidities increased during the Great Moderation. The fourth column of table 3 presents the estimated second moment correlations assuming a lower degree of nominal rigidities than the benchmark case. A comparison of these results with the benchmark correlations suggest that a rise in nominal rigidities is associated with decreased comovement in some of the volatilities thus ruling out this structural shift as a factor behind the increased role played by the common uncertainty component.

6 Conclusions

This paper uses a factor model with stochastic volatility to decompose the time-varying volatility of output, inflation, interest rates and stock returns into contributions from country-specific uncertainty and uncertainty common to all countries. We find that the common component plays an important role in driving the time-varying volatility implying that the second moments are correlated across countries. The empirical results suggest that the role of the common component has increased after the mid-1990s and the second moments of real and financial variables have become more correlated in the recent past. The correlations are estimated to be larger for nominal and financial variables.

In an attempt to provide an economic explanation for these results, we build a two-country DSGE model featuring Epstein Zin preferences. One of the key implications of this preference set-up is that the volatility of endogenous states in the model is time-varying. Moreover, agents are keen to give up expected future consumption in order to ensure a less volatile future consumption profile and this leads to a transfer of consumption across countries. These transfers and risk-sharing result in a close co-movement in second moments across countries. Simulations from the model suggest that an increase in trade openness leads to a closer movement in volatilties and thus provides one explanation for the increasing importance of the common uncertainty factor suggested by the empirical model.

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Technical Appendix: Common and country specific economic uncertainty*

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July 30, 2015

Abstract

JEL Codes: C15,C32, E32
Key Words: FAVAR, Stochastic Volatility, Uncertainty Shocks, DSGE Model

1 Model

The factor model is defined as

$$
\begin{aligned}
X_{it} &= B_i^C F_t^C + B_i^W F_t^W + e_{it} && (1)\\
F_t^C &= \alpha + \sum_{j=1}^{P} \rho_j F_{t-j}^C + \Omega_t^{1/2} v_t && (2)\\
F_t^W &= \zeta + \sum_{j=1}^{P} p_j F_{t-j}^W + \Upsilon_t^{1/2} g_t && (3)\\
e_{it} &= \sum_{j=1}^{P} \mu_{i,j} e_{it-j} + h_{it}^{1/2} \varepsilon_{it} && (4)\\
R_t &= diag(h_{1t}, ..h_{Nt}) && (5)\\
\Omega_t &= A^{-1} H_t A^{-1'} && (6)\\
\Upsilon_t &= C^{-1} D_t C^{-1'} && (7)\\
H_t &= diag(S_k \lambda_t) && (8)\\
D_t &= diag(s_k \gamma_t) && (9)\\
\ln \lambda_t &= \tilde{\alpha} + \tilde{\beta} \ln \lambda_{t-1} + \tilde{Q}^{1/2} \tilde{\eta}_t && (10)\\
\ln \gamma_t &= \bar{\alpha} + \bar{\beta} \ln \gamma_{t-1} + \bar{Q}^{1/2} \bar{\eta}_t && (11)\\
\ln h_{it} &= a_i + b_i \ln h_{it-1} + q_i^{1/2} n_{it} && (12)\\
& \varepsilon_{it}, v_t, \tilde{\eta}_t, \bar{\eta}_t, n_{it} \tilde{} N(0,1) && (13)
\end{aligned}
$$

2 Estimation

In order to deal with rotational indeterminancy we impose the condition that the top $n \times n$ block of the factor loading matrix for each country is diagonal with positive diagonal elements (see Geweke and Zhou (1996)). n denotes the total number of factors. Following Delnegro and Otrok (2005) we fix the initial conditions for the the stochastic volatilities λ_0, γ_0 and h_{i0} to fix the scale of the factors.

*The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.

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2.1 Priors

2.1.1 Factor loadings

The prior on $\tilde{B}_i = [B_i^C; B_i^W]$ is normal and is assumed to be $N(B_{i,0}, V_B)$ where $B_{i,0}$ is set equal to the loadings obtained using a principal component estimate of $F_t = [F_t^C, F_t^W]$. The variance V_B is assumed to be equal to 1. The initial estimate of the factors F_t^{PC} provides the initial value of the factors $F_{0\backslash 0}$ with the initial variance set equal to the identity matrix.

2.1.2 VAR Coefficients

Following Banbura *et al.* (2010) we introduce a natural conjugate prior for the VAR parameters via dummy observations. In our application, the prior means are chosen as the OLS estimates of the coefficients of an AR(1) regression estimated for each endogenous variable using a training sample. As is standard for US data, we set the overall prior tightness $\tau = 0.1$. We use the same prior for the AR coefficients of the idiosyncratic errors

2.1.3 Elements of S, A, s, C and the parameters of the common volatility transition equation

The elements of S have an inverse Gamma prior: $P(S_i)\tilde{}IG(S_{0,i}, V_0)$. The degrees of freedom V_0 are set equal to 1. The prior scale parameters are set by estimating the following regression: $\bar{\lambda}_{it} = S_{0,i}\bar{\lambda}_t + \varepsilon_t$ where $\bar{\lambda}_t$ is the first principal component of the stochastic volatilities $\bar{\lambda}_{it}$ obtained using a univariate stochastic volatility model for the orthogonalised residuals of each equation of the VARs in equations 2 and 3 estimated via OLS using the principal components F_t^{PC}.

The prior for the off-diagonal elements A is $A_0 \sim N\left(\hat{a}^{ols}, V\left(\hat{a}^{ols}\right)\right)$ where $\hat{a}^{ols}$ are the off-diagonal elements of the inverse of the Cholesky decomposition of $\hat{v}^{ols}$, with each row scaled by the corresponding element on the diagonal. These OLS estimates are obtained using the initial VAR models described above. $V\left(\hat{a}^{ols}\right)$ is assumed to be diagonal with the elements set equal to 10 times the absolute value of the corresponding element of $\hat{a}^{ols}$.

The prior for s and C is set exactly as the priors for S and A described above.

We set a normal prior for the unconditional mean $\tilde{\mu} = \frac{\tilde{\alpha}}{1-\tilde{\beta}}$ and $\bar{\mu} = \frac{\bar{\alpha}}{1-\bar{\beta}}$. This prior is $N(\mu_0, Z_0)$ where $\mu_0 = 0$ and $Z_0 = 1$.The prior for $\tilde{Q}$ and $\bar{Q}$ is $IG(Q_0, V_{Q0})$ where Q_0 is the average of the variances of the shocks to the transition equations using the initial univariate stochastic volatility estimates described above and $V_{Q0} = 5$. The prior for $\tilde{\beta}$ and $\bar{\beta}$ is $N(F_0, L_0)$ where $F_0 = 0.8$ and $L_0 = 1$.

2.1.4 Parameters of the idiosyncratic shock volatility transition equation

We set a normal prior for the unconditional mean $\breve{\mu} = \frac{a}{1-b}$. This prior is $N(\mu_0, Z_0)$ where $\mu_0 = 0$ and $Z_0 = 1$.The prior for q_i is $IG(q_0, V_{q0})$ where$q_0 = 0.01$ and $V_{q0} = 5$. The prior for b is $N(F_0, L_0)$ where $F_0 = 0.8$ and $L_0 = 1$.

2.2 Gibbs algorithm

The Gibbs algorithm involves a draw from the following conditional distributions (Ξ denotes all other parameters) :

1. $G(\alpha, \rho_j\backslash\Xi)$.Given a draw of λ_t, the left and the right hand side variables of the VAR models in equations 2 can be transformed to remove remove the heteroscedasticity in the following manner: Let $y_t = F_t^C$ and $x_t = [1, F_{t-1}^C, F_{t-2}^C, ..F_{t-j}^C]$. Then the following transformation can be applied

$$\tilde{y}_t = \frac{y_t}{\lambda_t^{1/2}}, \tilde{x}_t = \frac{x_t}{\lambda_t^{1/2}}$$

Then the conditional posterior distribution for the VAR coefficients is standard and given by

$$N(\tilde{b}^*, \bar{\Omega} \otimes (X^{*\prime}X^*)^{-1})$$

where $\tilde{b}^* = (X^{*\prime}X^*)^{-1}(X^{*\prime}Y^*)$ and Y^* and X^* denote the transformed data appended with the dummy observations. The covariance matrix is $\bar{\Omega} = A^{-1}diag(S)A^{-1\prime}$.

2. $G(\zeta, p_j \backslash \Xi)$.Given a draw of γ_t, the left and the right hand side variables of the VAR models in equations 3 can be transformed to remove remove the heteroscedasticity in the following manner: Let $y_t = F_t^W$ and $x_t = [1, F_{t-1}^W, F_{t-2}^W, ..F_{t-j}^W]$. Then the following transformation can be applied

$$\tilde{y}_t = \frac{y_t}{\gamma_t^{1/2}}, \tilde{x}_t = \frac{x_t}{\gamma_t^{1/2}}$$

Then the conditional posterior distribution for the VAR coefficients is standard and given by

$$N(\tilde{b}^*, \bar{\Omega} \otimes (X^{*\prime}X^*)^{-1})$$

where $\tilde{b}^* = (X^{*\prime}X^*)^{-1}(X^{*\prime}Y^*)$ and Y^* and X^* denote the transformed data appended with the dummy observations. The covariance matrix $\bar{\Omega} = C^{-1}diag(s)C^{-1\prime}$.

3. $G(A\backslash\Xi)$. Given a draw for the VAR parameters α, ρ_j the model can be written as $A'(v_t) = \tilde{e}_t$ where $v_t = F_t^C - \left(\alpha + \sum_{j=1}^P \rho_j F_{t-j}^C\right)$ and $VAR(\tilde{e}_t) = H_t$. This is a system of linear equations with a known form of heteroscedasticity. The conditional distributions for a linear regression apply to each equation of this system after a simple GLS transformation to make the errors homoscedastic. The jth equation of this system is given as $v_{jt} = -\check{\alpha}v_{-jt} + \tilde{e}_{jt}$ where the subscript j denotes the jth column while $-j$ denotes columns 1 to $j-1$. Note that the variance of $\tilde{e}_{jt}$ is time-varying and given by $\lambda_t S_j$. A GLS transformation involves dividing both sides of the equation by $\sqrt{\lambda_t S_j}$ to produce $v_{jt}^* = -\check{\alpha}v_{-jt}^* + \tilde{e}_{jt}^*$ where * denotes the transformed variables and $var\left(\tilde{e}_{jt}^*\right) = 1$. The conditional posterior for $\check{\alpha}$ is normal with mean and variance given by M^* and V^* :

$$\begin{aligned} M^* &= \left(V\left(\hat{a}^{ols}\right)^{-1} + v_{-jt}^{*\prime}v_{-jt}^*\right)^{-1}\left(V\left(\hat{a}^{ols}\right)^{-1}\hat{a}^{ols} + v_{-jt}^{*\prime}v_{jt}^*\right) \\ V^* &= \left(V\left(\hat{a}^{ols}\right)^{-1} + v_{-jt}^{*\prime}v_{-jt}^*\right)^{-1} \end{aligned}$$

4. $G(C\backslash\Xi)$.The conditional posterior distribution is identical to that described in step 3 above using the residuals of equation 3.

5. $G(S\backslash\Xi)$. Given a draw for the VAR parameters α, ρ_j the model can be written as $A'(v_t) = \tilde{e}_t$. The jth equation of this system is given by $v_{jt} = -\check{\alpha}v_{-jt} + \tilde{e}_{jt}$ where the variance of e_{jt} is time-varying and given by $\lambda_t S_j$. Given a draw for λ_t this equation can be re-written as $\bar{v}_{jt} = -\check{\alpha}\bar{v}_{-jt} + \bar{e}_{jt}$ where $\bar{v}_{jt} = \frac{v_{jt}}{\lambda_t^{1/2}}$ and the variance of $\bar{e}_{jt}$ is S_j. The conditional posterior is for this variance is inverse Gamma with scale parameter $\bar{e}_{jt}'\bar{e}_{jt} + S_{0,j}$ and degrees of freedom $V_0 + T$.

6. $G(s\backslash\Xi)$. This draw is done in exactly the same manner as step 5 above using the residuals of equation 3, the matrix C and the volatility γ_t.

7. Elements of λ_t. Conditional on the VAR coefficients, and the parameters of the volatility transition equation, the model in equations 2 and 10 has a multivariate non-linear state-space representation. Carlin *et al.* (1992) show that the conditional distribution of the state variables in a general state-space model can be written as the product of three terms:

$$\tilde{h}_t \backslash Z_t, \Xi \propto f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}\right) \times f\left(\tilde{h}_{t+1} \backslash \tilde{h}_t\right) \times f\left(Z_t \backslash \tilde{h}_t, \Xi\right) \tag{14}$$

where Ξ denotes all other parameters, Z_t denotes the endogenous variables in equation 2 and $\tilde{h}_t = \ln \lambda_t$. In the context of stochastic volatility models, Jacquier *et al.* (1994) show that this density is a product of log normal densities for λ_t and λ_{t+1} and a normal density for Z_t.Carlin *et al.* (1992) derive the general form of the mean and variance of the underlying normal density for $f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi\right) \propto f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}\right) \times f\left(\tilde{h}_{t+1} \backslash \tilde{h}_t\right)$ and show that this is given as

$$f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi\right) \sim N\left(B_{2t}b_{2t}, B_{2t}\right) \tag{15}$$

where $B_{2t}^{-1} = Q^{-1} + \bar{F}'Q^{-1}\bar{F}$ and $b_{2t} = \tilde{h}_{t-1}\bar{F}'Q^{-1} + \tilde{h}_{t+1}Q^{-1}\bar{F}$. Here $\bar{F}$ denotes the autoregressive coefficient of the transition equation and Q is the variance of the shock to the transition equation in companion form. Note that due to the non-linearity of the observation equation of the model an analytical expression for the complete conditional $\tilde{h}_t \backslash Z_t, \Xi$ is unavailable and a metropolis step is required. Following Jacquier *et al.* (1994) we draw from 14 using a date-by-date independence metropolis step using the density in 15 as the candidate generating density. This choice implies that the acceptance probability is given by the ratio of the conditional likelihood $f\left(Z_t \backslash \tilde{h}_t, \Xi\right)$ at the old and the new draw. To implement the algorithm we begin with an initial estimate of $\tilde{h} = \ln \bar{\lambda}_t$ We set the matrix $\tilde{h}^{old}$ equal to the initial volatility estimate. Then at each date the following two steps are implemented:

(a) Draw a candidate for the volatility $\tilde{h}_t^{new}$ using the density 14 where $b_{2t} = \tilde{h}_{t-1}^{new} \bar{F}' Q^{-1} + \tilde{h}_{t+1}^{old} Q^{-1} \bar{F}$ and $B_{2t}^{-1} = Q^{-1} + \bar{F}' Q^{-1} \bar{F}$

(b) Update $\tilde{h}_t^{old} = \tilde{h}_t^{new}$ with acceptance probability $\frac{f\left(Z_t \backslash \tilde{h}_t^{new}, \Xi\right)}{f\left(Z_t \backslash \tilde{h}_t^{old}, \Xi\right)}$ where $f\left(Z_t \backslash \tilde{h}_t, \Xi\right)$ is the likelihood of the VAR for observation t and defined as $|\Omega_t|^{-0.5} - 0.5 \exp\left(\tilde{e}_t \Omega_t^{-1} \tilde{e}_t'\right)$ where $\tilde{e}_t = F_t^c - \left(\alpha + \sum_{j=1}^P \rho_j F_{t-j}^C\right)$ and $\Omega_t = A^{-1}\left(\exp(\tilde{h}_t) S\right) A^{-1'}$

Repeating these steps for the entire time series delivers a draw of the stochastic volatilties.[1]

8. The draw for γ_t is carried out using the procedure described in step 7 above.

9. $G\left(h_{it} \backslash \Xi\right)$: Given a draw for the factors, the parameters of the transition equation 12 and the factor loadings $\tilde{B}_i = [B_i^C; B_i^W]$ and the autoregressive coefficients $\mu_{i,j}$, a univariate stochastic volatility model applies for each i:

$$\begin{aligned} X_{it}^* &= B_i F_t^* + h_{it}^{1/2} \varepsilon_{it} \\ \ln h_{it} &= a_i + b_i \ln h_{it-1} + q_i^{1/2} n_{it} \end{aligned}$$

where $X_{it}^* = X_{it} - \sum_{j=1}^P \mu_{i,j} X_{it-j}$ and $F_t^* = F_t - \sum_{j=1}^P \mu_{i,j} F_{it-j}$.The algorithm of Jacquier *et al.* (1994) (described above) is used to draw h_{it}.

10. $G(\tilde{\alpha}, \tilde{\beta}, \tilde{Q} \backslash \Xi)$.We re-write the transition equation in deviations from the mean

$$\tilde{h}_t - \tilde{\mu} = \tilde{\beta}\left(\tilde{h}_{t-1} - \tilde{\mu}\right) + \eta_t \tag{16}$$

where $\tilde{h}_t = \ln \lambda_t$ and the elements of the mean vector $\tilde{\mu}$ are defined as $\frac{\tilde{\alpha}}{1-\tilde{\beta}}$. Conditional on a draw for $\tilde{h}_t$ and $\tilde{\mu}$ the transition equation 16 is a simply a linear regression and the standard normal and inverse Gamma conditional posteriors apply. Consider $\tilde{h}_t^* = \tilde{\beta} \tilde{h}_{t-1}^* + \eta_t, VAR\left(\eta_t\right) = \tilde{Q}$ and $\tilde{h}_t^* = \tilde{h}_t - \mu, \tilde{h}_{t-1}^* = \tilde{h}_{t-1} - \mu$. The conditional posterior of $\tilde{\beta}$ is $N\left(\theta^*, L^*\right)$ where

$$\begin{aligned} \theta^* &= \left(L_0^{-1} + \frac{1}{\tilde{Q}} \tilde{h}_{t-1}^{*\prime} \tilde{h}_{t-1}^*\right)^{-1} \left(L_0^{-1} F_0 + \frac{1}{\tilde{Q}} \tilde{h}_{t-1}^{*\prime} \tilde{h}_t^*\right) \\ L^* &= \left(L_0^{-1} + \frac{1}{\tilde{Q}} \tilde{h}_{t-1}^{*\prime} \tilde{h}_{t-1}^*\right)^{-1} \end{aligned}$$

The conditional posterior of $\tilde{Q}$ is inverse Gamma with scale parameter $\eta_t' \eta_t + Q_0$ and degrees of freedom $T + V_{Q0}$.

Given a draw for $\tilde{\beta}$, equation 16 can be expressed as $\bar{\Delta} \tilde{h}_t = C\mu + \eta_t$ where $\bar{\Delta} \tilde{h}_t = \tilde{h}_t - \tilde{\beta} \tilde{h}_{t-1}$ and $C = 1 - \tilde{\beta}$. The conditional posterior of $\tilde{\mu}$ is $N\left(\mu^*, Z^*\right)$ where

$$\begin{aligned} \mu^* &= \left(Z_0^{-1} + \frac{1}{\tilde{Q}} C'C\right)^{-1} \left(Z_0^{-1} \mu_0 + \frac{1}{\tilde{Q}} C' \bar{\Delta} \tilde{h}_t\right) \\ Z^* &= \left(Z_0^{-1} + \frac{1}{\tilde{Q}} C'C\right)^{-1} \end{aligned}$$

Note that $\tilde{\alpha}$ can be recovered as $\tilde{\mu}\left(1 - \tilde{\beta}\right)$.

[1] In order to take endpoints into account, the algorithm is modified slightly for the initial condition and the last observation. Details of these changes can be found in Jacquier *et al.* (1994).

11. $G(\bar{\alpha}, \bar{\beta}, \bar{Q}\backslash\Xi)$.The draw for these parameters is carried out as in step 10 above.

12. $G(a_i, b_i, q_i\backslash\Xi)$. Given a draw for h_{it}, the conditional posterior distributions for the parameters of the transition equations 12 are as described in step 10.

13. $G(B_i\backslash\Xi)$: Given a draw for the factors, the autoregressive coefficients $\mu_{i,j}$ and the variance of the idiosyncratic component, a seperate heteroscedastic linear regression model applies to each X_{it} and the standard formulae for linear regressions apply. In particular, the model for each i is

$$X_{it} = B_i F_t + e_{it}$$

The model can be transformed to remove serial correlation and heteroscedasticity by creating $X^*_{it} = \frac{\left(X_{it} - \sum_{j=1}^{P} \mu_{i,j} X_{it-j}\right)}{\sqrt{h_{it}}}$, $\tilde{F}^*_t$ $\frac{\left(F_t - \sum_{j=1}^{P} \mu_{i,j} F_{it-j}\right)}{\sqrt{h_{it}}}$ The conditional posterior is: $N\left(B^*_i, \Lambda_B\right)$

$$\begin{aligned} B^*_i &= \left(V_B^{-1} + \tilde{F}^{*\prime}_t \tilde{F}^*_t\right)^{-1} \left(V_B^{-1} B_{i,0} + \tilde{F}^{*\prime}_t X^*_{it}\right) \\ \Lambda_B &= \left(V_B^{-1} + \tilde{F}^{*\prime}_t \tilde{F}^*_t\right)^{-1} \end{aligned}$$

Note that the factors and factor loadings are not identified seperately and the model suffers from the usual rotational indeterminancy problem. In order to deal with this we impose the condition that the top $n \times n$ block of the factor loading matrix for each country is diagonal with positive diagonal elements (see Geweke and Zhou (1996)). n denotes the total number of factors.

14. $G\left(\mu_{ij}\backslash\Xi\right)$. Given a draw for the factors and the factor loadings, equation 4 represents a series of linear regressions with heteroscedastic errors. Given h_{it}, the left and the right hand side can be transformed to remove heteroscedasticity (by dividing by $\sqrt{h_{it}}$). Then the conditional posterior of μ_{ij} is normal with mean and variance given by the standard formulae for the linear regression model.

15. $G(F_t\backslash\Xi)$: Given a draw for all other parameters, the algorithm of Carter and Kohn (2004) is used to sample from the conditional posterior distribution of the factors F_t. The conditional posterior is: $F_t\backslash X_{it}, \Xi \sim N\left(F_{T\backslash T}, P_{T\backslash T}\right)$ and $F_t\backslash F_{t+1}, X_{it}, \Xi \sim N\left(F_{t\backslash t+1, F_{t+1}}, P_{t\backslash t+1, B_{t+1}}\right)$ where $t = T-1, ..1$. As shown by Carter and Kohn (2004) the simulation proceeds as follows. First we use the Kalman filter to draw $F_{T\backslash T}$ and $P_{T\backslash T}$ and then proceed backwards in time using $F_{t|t+1} = F_{t|t} + P_{t|t} f' P^{-1}_{t+1|t}\left(F_{t+1} - fF_{t\backslash t} - \mu\right)$ and $P_{t|t+1} = P_{t|t} - P_{t|t} f' P^{-1}_{t+1|t} f P_{t|t}$. Here f denotes the autoregressive coefficients of the transition equations 2 and 3 in companion form, while μ denotes the pre-determined regressors in that equation in companion form.

2.3 A Monte-Carlo experiment

In order to examine the performance of this algorithm, we consider a small Monte-Carlo experiment

2.3.1 Data Generating Process

We generate data from the following FAVAR model with 2 world and country factors. We assume 4 countries with 10 series per country:

$$\begin{aligned} X_{it} &= B^C_i F^C_t + B^W_i F^W_t + e_{it} \\ e_{it} &= 0.8 e_{it-1} + h^{1/2}_{it} \varepsilon_{it} \end{aligned}$$

where the factor loadings B_i are drawn from $N(0, 0.5)$ and $i = 1, 2, ...40$. The stochastic volatility process h_{it} is assumed to follow the process

$$\ln h_{it} = -0.1 + 0.9 \ln h_{it-1} + (0.5)^{\frac{1}{2}} k_{it}, k_{it}\tilde{} N(0,1)$$

The dynamics of the world and country factors are defined as

$$\begin{pmatrix} F_{1t} \\ F_{2t} \end{pmatrix} = \begin{pmatrix} 0.3 & 0.05 \\ -0.05 & 0.4 \end{pmatrix} \begin{pmatrix} F_{1t-1} \\ F_{2t-1} \end{pmatrix} + \begin{pmatrix} 0.2 & 0.05 \\ -0.05 & 0.2 \end{pmatrix} \begin{pmatrix} F_{1t-2} \\ F_{2t-2} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix}, var \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix} = \Omega_t$$

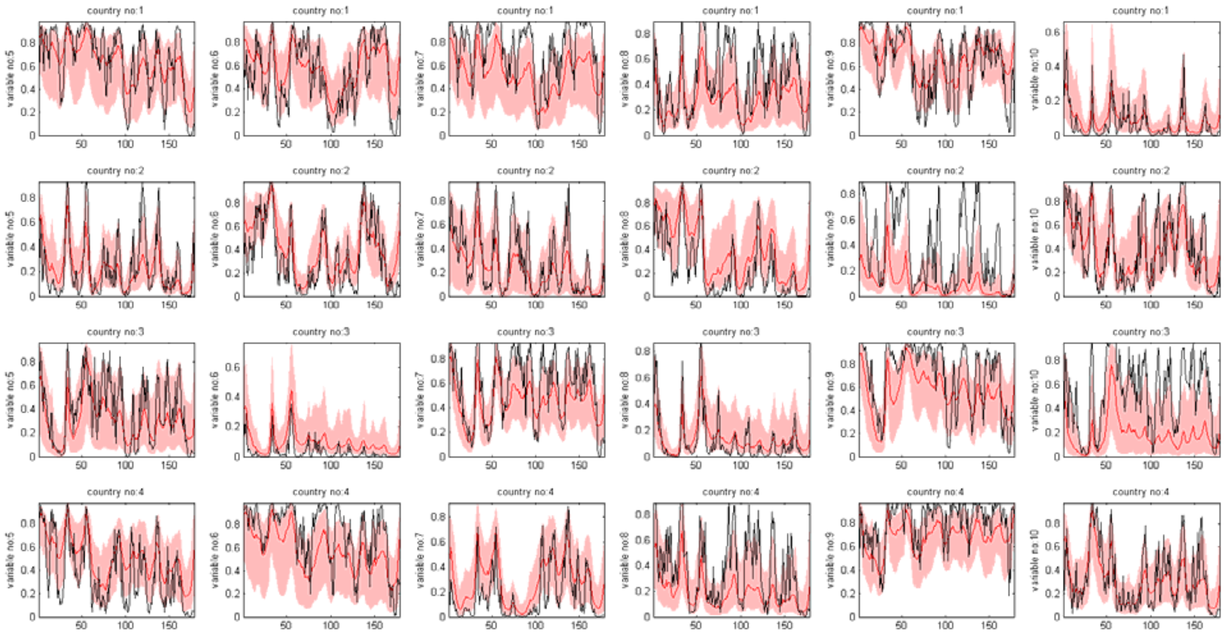

Figure 1: The contribution of the variance of F_t^W to the variance of X_{it}. The figure shows the estimates for the series where the factor loadings are freely estimated. The first four series for each country are not considered as the factor loadings are fixed to identify the sign of the factors.

The variance process for both the world and the country factor VARs is defined as

$$\begin{aligned}
\Omega_t &= A^{-1}\left(S\lambda_t\right)A^{-1'} \\
A &= \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \\
S &= \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \\
\ln\lambda_t &= -0.1 + 0.9\ln\lambda_{t-1} + (0.5)^{\frac{1}{2}}\, v_t, v_t\tilde{\ } N(0,1)
\end{aligned}$$

We generate 300 observations for X_{it} and drop the first 100 observations to reduce the influenceof initial conditions. Note that the state variables are generated once and then kept fixed. The experiment is repeated 100 times. At each iteration, the model is estimated using the MCMC algorithm described above using 5000 iterations with a burn-in of 4000 observations. The retained draws are used to calculate the contribution of variance of the world factor to the unconditional variance of each variable in the model. The results are shown in figure 1. The Monte-Carlo estimates of the contribution of $var\left(F_t^W\right)$ to each series tracks the true values quite closely suggesting that the algorithm works well.

3 Recursive Means

Figure 2 presents the recursive means of retained draws. These are calculated for every 50 draws. The estimates are fairly stable providing evidence in favour of convergence of the algorithm.

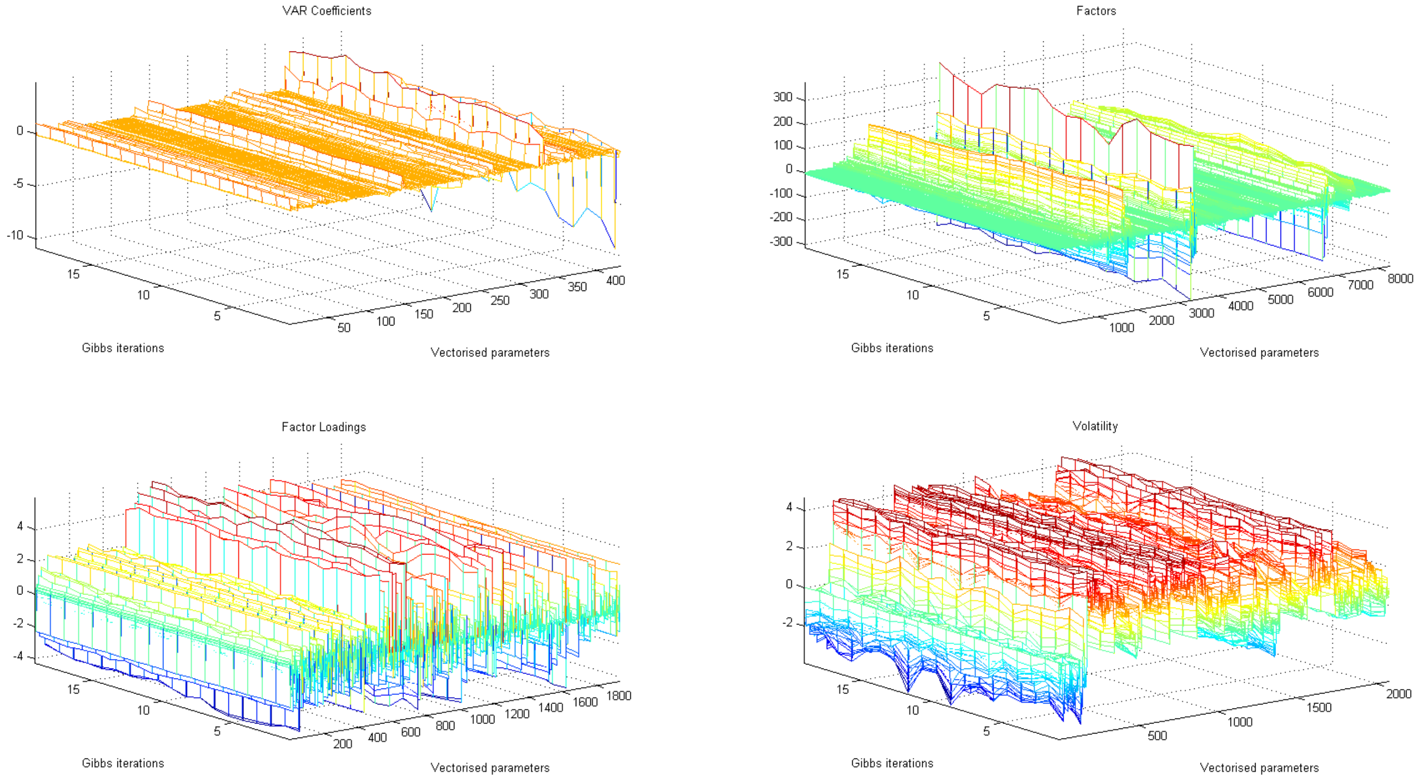

Figure 2: Recursive Means of Retained Draws

4 Calculation of the DIC

The DIC is defined as

$$DIC = \bar{D} + p_D.$$

The first term $\bar{D} = E\left(-2\ln L\left(\Xi_i\right)\right) = \frac{1}{M}\sum_i \left(-2\ln L\left(\Xi_i\right)\right)$ where $L\left(\Xi_i\right)$ is the likelihood evaluated at the draws of all of the parameters Ξ_i in the MCMC chain. This term measures goodness of fit. The second term p_D is defined as a measure of the number of effective parameters in the model (or model complexity). This is defined as $p_D = E\left(-2\ln L\left(\Xi_i\right)\right) - \left(-2\ln L\left(E(\Xi_i)\right)\right)$ and can be approximated as $p_D = \frac{1}{M}\sum_i \left(-2\ln L\left(\Xi_i\right)\right) - \left(-2\ln L\left(\frac{1}{M}\sum_i \Xi_i\right)\right)$.[2] Prior distributions on the parameters in our model and the presence of latent variables implies that the number of parameters (as used in the calculation of the Akaike and Schwarz information criterion) do not necessarily represent model complexity. The definition of the effective number of parameters used in the computation of the DIC avoids this problem. Note that the model with the lowest estimated DIC is preferred. Calculation of the DIC requires the calculation of the likelihood of the VAR model. The likelihood function of the model is calculated using a particle filter using 1,000 particles. We employ the Rao-Blackwellized particle filter described in section 2.5.7 of Creal (2009). In particular, given the stochastic volatility the remaining states in the model are linear and Gaussian (i.e. the factors). This version of the filter, thus simulates particles for the non-linear states and evaluates the linear states via the Kalman filter.

5 Sensitivity Analysis

5.1 Estimation using post 1985 data

Figures 3 presents the variance decomposition from the version of the model estimated using post 1985 data. The estimates show that common uncertainty makes the largest contribution to inflation and stock returns. The contribution of common uncertainty increases after the late 1990s. These features are very similar to the benchmark model.

5.2 Estimation on pre Great Recession data

Figure 4 presents the variance decomposition from the version of the model estimated using data up to 2006Q4. As above, the key features of the results are preserved: The common component of uncertainty makes a larger contribution to nominal variables such as inflation and stock prices and this contribution is larger over the last decade in the sample.

[2] The first term in this expression is an average of -2 times the likelihood function evaluated at each MCMC iteration. The second term is (-2 times) the likelihood function evaluated at the posterior mean.

Figure 3: The contribution of common, country-specific and idiosyncratic uncertainty using post-1985 data. The red line is the median contribution across countries, while the pink band represents the 5th and 95th percentiles. The dotted black line is the average contribution across countries and time.

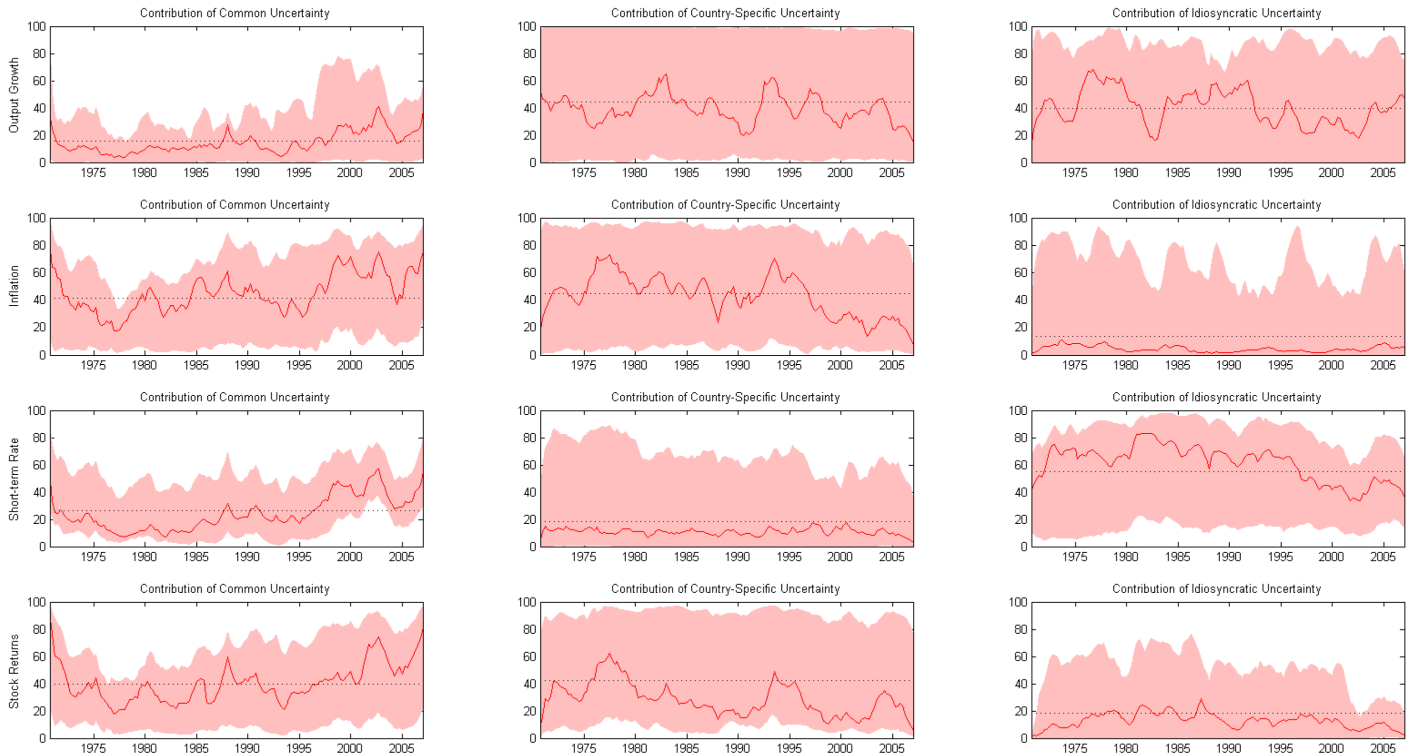

Figure 4: The contribution of common, country-specific and idiosyncratic uncertainty using pre-2007 data. The red line is the median contribution across countries, while the pink band represents the 5th and 95th percentiles. The dotted black line is the average contribution across countries and time.

6 DSGE Model

The section provides a detailed discussion of the theoretical model. The novel feature of the model developed here is that agents in both economies form recursive preferences (Epstein and Zin (1989), Weil (1989, 1990)) and they do so in a production economy with nominal frictions (New-Keynesian) and long-run risks. To best of our knowledge this is the first study that introduces Epstein and Zin preferences in a two-country New-Keynesian model. The work Colacito and Croce (2013), Kollmann (2015) and Gourio *et al.* (2013) has also employed recursive preferences and long-run risks in order to understand the correlation patterns between assets prices and macroeconomic aggregates observed in the data across countries. The model presented here can be viewed as an extension of this previous work along several dimensions, either by modelling supply properly (production economy), or/and adding nominal frictions.

From the work of Rudebusch and Swanson (2012) and Swanson (2015) it is known that when additively separable period preferences are combined with Epstein and Zin lifetime preferences, then the second moments of the endogenous state vector of the economy are conditionally heteroskedastic even if the the shocks in the model are homoskedastic. We find this setup attractive as it can generate time-varying second moments without relying on stochastic volatility shocks (see, Fernandez-Villaverde *et al.* (2011), Fernández-Villaverde *et al.* (2011) and Mumtaz and Theodoridis (2015), among others). Although stochastic volatility shocks are a useful tool to model an exogenous increase in the macroeconomic uncertainty, they are not always intuitive and, consequently, not easy to motivate.

To be able to understand how uncertainty affects the nominal variables in the data, we proceed by adopting standard New-Keynesian features such as monopolistic competition and nominal rigidities in order for inflation to exist in our theoretical framework.

Finally, the model consists of two equal size countries denoted by H (home) and F (foreign), furthermore, quantities and prices in country F are denoted by asterisks, while those in country H without asterisks.

6.1 Firms

Two types of firms are operated in each economy. The intermediate monopolistically competitive domestic firms that use labour supplied by households and (fixed) capital to produce a differentiated good that is sold to a final good producer who employs a continuum of these differentiated goods in her constant elasticity of substitution – CES – production to deliver the final good. The competitive importing firms use a costless technology and turn a homogenous good – bought in the world market – into a differentiated good, which is then sold to the domestic consumers.

Domestic Firms The final good producer's CES production function is given by

$$\tilde{Y}_t = \left[\int_0^1 \tilde{Y}_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon-1}{\varepsilon}} \tag{17}$$

$$\tilde{Y}_t^* = \left[\int_0^1 \tilde{Y}_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon-1}{\varepsilon}} \tag{18}$$

where ε denotes the elasticity of substitution between the differentiated goods produced in each country. The final good producer's demand curve for $y_{i,t}$ arises from the profit minimisation problem – $\max_{\tilde{Y}_t(h)} \Big\{ P_{H,t} \left[\int_0^1 \tilde{Y}_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon-1}{\varepsilon}} - \int_0^1 P_{H,}$

$$\tilde{Y}_t(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\varepsilon} \tilde{Y}_t \tag{19}$$

$$\tilde{Y}_t^*(f) = \left(\frac{P_{F,t}^*(f)}{P_{F,t}^*}\right)^{-\varepsilon} \tilde{Y}_t^* \tag{20}$$

The final good price index is obtained by combining (17) and (19)

$$P_{H,t} = \left[\int_0^1 P_{H,t}(h)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}} \tag{21}$$

$$P_{F,t}^* = \left[\int_0^1 P_{F,t}^*(f)^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}} \tag{22}$$

Intermediate good producers use the following production function

$$\tilde{Y}_t(h) = A_t \tilde{Z}_t (L_t(h))^{1-\phi} K^{\phi} \tag{23}$$

$$\tilde{Y}_t^*(f) = A_t^* \tilde{Z}_t^* (L_t^*(f))^{1-\phi} (K^*)^{\phi} \tag{24}$$

where

$$\log(A_t) = \rho_A \log(A_{t-1}) + \sigma_A \epsilon_{A,t} \tag{25}$$

$$\log(A_t^*) = \rho_{A^*} \log\left(A_{t-1}^*\right) + \sigma_{A^*} \epsilon_{A^*,t} \tag{26}$$

is a stationary exogenous technological process, $\tilde{Z}_t$ is the non-stationary exogenous technological process with $\Gamma_t = \frac{\tilde{Z}_t}{\tilde{Z}_{t-1}} \left(\Gamma_t^* = \frac{\tilde{Z}_t^*}{\tilde{Z}_{t-1}^*}\right)$

$$\log(\Gamma_t) = \kappa \left[\log\left(\tilde{Z}_{t-1}\right) - \log\left(\tilde{Z}_{t-1}^*\right)\right] + \rho_\Gamma \log(\Delta\Gamma_{t-1}) + \sigma_\Gamma \epsilon_{\Gamma,t} \tag{27}$$

$$\log(\Gamma_t^*) = -\kappa \left[\log\left(\tilde{Z}_{t-1}\right) - \log\left(\tilde{Z}_{t-1}^*\right)\right] + \rho_{\Gamma^*} \log\left(\Gamma_{t-1}^*\right) + \sigma_{\Gamma^*} \epsilon_{\Gamma^*,t} \tag{28}$$

$L_t(h)$ is the amount of homogeneous labour rented by the firm and K denotes the amount of (fixed) physical capital. The intermediate firm select $L_t(h)$ in order to minimise its production cost

$$\min_{L_t(h)} \tilde{W}_t L_t(h) + MC_t P_{H,t} \left[\tilde{Y}_t(h) - A_t \tilde{Z}_t (L_t(h))^{1-\phi} K^{\phi}\right] \tag{29}$$

The real marginal cost for the intermediate firms is given by the first order condition of (29) with respect to $L_t(h)$ is

$$MC_t(h) = \frac{\tilde{W}_t L_t(h)}{\bar{P}_{H,t}(1-\phi)\tilde{Y}_t(h)} \tag{30}$$

$$MC_t^*(f) = \frac{\tilde{W}_t^* L_t^*(f)}{\bar{P}_{F,t}^*(1-\phi)\tilde{Y}_t^*(f)} \tag{31}$$

Domestic intermediate good producers are subject to Calvo-type price setting (Calvo (1983)), meaning that only a fraction – $(1-\xi_H)$ – of firms who receives a random signal are allowed to optimally reset their prices

$$\max_{\breve{P}_{H,t}(h)} E_t \sum_{j=0}^{\infty} M_{t,t+j} \xi_H^j \left[\left\{\frac{\breve{P}_{H,t}(h)}{P_{H,t+j}} - MC_{t+j}(h)\right\} \tilde{Y}_{t+j}(h)\right]$$

subject to

$$\tilde{Y}_t(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\varepsilon} \tilde{Y}_t \tag{32}$$

The first-order condition is expressed as system of difference equations

$$K_{H,t} = MC_t \tilde{Y}_t^d + \beta\xi_H E_t M_{t+1} \left(\frac{1}{\Pi_{H,t+1}}\right)^{-\varepsilon} K_{H,t+1} \tag{33}$$

$$F_{H,t} = \tilde{Y}_t^d + \beta\xi_H E_t M_{t+1} \left(\frac{1}{\Pi_{H,t+1}}\right)^{1-\varepsilon} F_{H,t+1} \tag{34}$$

$$\bar{\Pi}_{H,t} = \frac{\varepsilon}{\varepsilon-1} \frac{K_{H,t}}{F_{H,t}} \tag{35}$$

$$1 = \xi_H \left(\frac{1}{\Pi_{H,t+1}}\right)^{1-\varepsilon} + (1-\xi_H)\bar{\Pi}_{H,t}^{1-\varepsilon} \tag{36}$$

$$
\begin{aligned}
K^*_{F,t} &= MC^*_t \tilde{Y}^{d,*}_t + \beta \xi_H E_t M^*_{t+1} \left(\frac{1}{\Pi^*_{F,t+1}} \right)^{-\varepsilon} K^*_{F,t+1} && (37)\\
F^*_{F,t} &= \tilde{Y}^{d,*}_t + \beta \xi_H E_t M^*_{t+1} \left(\frac{1}{\Pi^*_{F,t+1}} \right)^{1-\varepsilon} F^*_{F,t+1} && (38)\\
\bar{\Pi}^*_{F,t} &= \frac{\varepsilon}{\varepsilon - 1} \frac{K^*_{F,t}}{F^*_{F,t}} && (39)\\
1 &= \xi_H \left(\frac{1}{\Pi^*_{F,t+1}} \right)^{1-\varepsilon} + (1-\xi_H) \left(\bar{\Pi}^*_{F,t} \right)^{1-\varepsilon} && (40)
\end{aligned}
$$

where $\bar{\Pi}_{H,t} \equiv \frac{\check{P}_{H,t}}{P_{H,t}}$ and $\bar{\Pi}^*_{F,t} \equiv \frac{\check{P}^*_{F,t}}{P^*_{F,t}}$.

Market clearing condition in the domestic sector

$$
\begin{aligned}
\tilde{Y}_t &= \int_0^1 \left(\frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\varepsilon} dh \tilde{Y}^d_t = \Delta_{H,t} \tilde{Y}^d_t && (41)\\
\tilde{Y}^*_t &= \int_0^1 \left(\frac{P^*_{F,t}(f)}{P^*_{F,t}} \right)^{-\varepsilon} df \tilde{Y}^{d,*}_t = \Delta^*_{F,t} \tilde{Y}^{d,*}_t && (42)
\end{aligned}
$$

where $\Delta_{H,t} = \int_0^1 \left(\frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\varepsilon} dh$ and $\Delta^*_{F,t} = \int_0^1 \left(\frac{P^*_{F,t}(f)}{P^*_{F,t}} \right)^{-\varepsilon} df$ ae the price dispersion terms and they are given by

$$
\begin{aligned}
\Delta_{H,t}^{\frac{1}{1-\phi}} &= (1-\xi_H) \bar{\Pi}_{H,t}^{-\frac{\varepsilon}{1-\phi}} + \xi_H \left(\frac{1}{\Pi_{H,t}} \right)^{-\frac{\varepsilon}{1-\phi}} \Delta_{H,t-1}^{\frac{1}{1-\phi}} && (43)\\
\left(\Delta^*_{F,t} \right)^{\frac{1}{1-\phi}} &= (1-\xi_H) \left(\bar{\Pi}^*_{F,t} \right)^{-\frac{\varepsilon}{1-\phi}} + \xi_H \left(\frac{1}{\Pi^*_{F,t}} \right)^{-\frac{\varepsilon}{1-\phi}} \left(\Delta^*_{F,t-1} \right)^{\frac{1}{1-\phi}} && (44)
\end{aligned}
$$

Importing firms The import sector is much simpler, it consists of a continuum of competitive firms that buy a homogenous good from the foreign economy $\tilde{C}^*_{F,t}$ ($\tilde{C}_{H,t}$) at price $P^*_{F,t}$ (P_{H_t}). These firms have access to a costless technology and transform the homogenous good into a differentiated product $\tilde{C}_{F,t}(h)$ ($\tilde{C}^*_{H,t}(f)$) consumed by domestic households. Perfect competition and zero production cost imply the low of one price holds meaning

$$
\begin{aligned}
P_{F,t} &= S_t P^*_{F,t} && (45)\\
P^*_{H,t} &= \frac{P_{H,t}}{S_t} && (46)
\end{aligned}
$$

where S_t is the nominal exchange rate.

6.2 Households

The domestic economy is populated by a continuum of households that attain utility from consumption $\tilde{C}_t(h)$ $\left(\tilde{C}^*_t(f)\right)$ and leisure $1 - L_t(h)$ $(1 - L^*_t(f))$. Household's preference preferences are separable

$$
\begin{aligned}
u\left(\tilde{C}_t(h), \tilde{Z}_t, L_t(h)\right) &= \frac{\tilde{C}_t(h)^{1-\sigma_C}}{1-\sigma_C} - \chi_0 \tilde{Z}_t^{1-\sigma_C} \frac{L_t(h)^{1+\sigma_L}}{1+\sigma_L} && (47)\\
u\left(\tilde{C}^*_t(f), \tilde{Z}^*_t, L^*_t(f)\right) &= \frac{\tilde{C}^*_t(f)^{1-\sigma_C}}{1-\sigma_C} - \chi_0 \left(\tilde{Z}^*_t\right)^{1-\sigma_C} \frac{L^*_t(f)^{1+\sigma_L}}{1+\sigma_L} && (48)
\end{aligned}
$$

where σ_L the inverse of the Frisch elasticity and σ_C the inverse of intertemporal elasticity of substitution. Furthermore, households have recursive preferences (Epstein and Zin (1989), Weil (1989, 1990))

$$
\begin{aligned}
V_t(h) &= u\left(\tilde{C}_t(h), \tilde{Z}_t, L_t(h)\right) + \beta \left(E_t V_{t+1}(h)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} && (49)\\
V^*_t(f) &= u\left(\tilde{C}^*_t(f), \tilde{Z}^*_t, L^*_t(f)\right) + \beta \left[E_t \left(V^*_{t+1}(f) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} && (50)
\end{aligned}
$$

The attractive feature of the Epstein-Zin preference is that it breaks the link between the intertemporal elasticity parameter and the coefficient of relative risk aversion, which is now controlled by the risk parameter γ. Aggregate consumption is function of domestically produced and imported consumption

$$\tilde{C}_t(h)^{\frac{\theta-1}{\theta}} = (1-n)^{\frac{1}{\theta}} \tilde{C}_{H,t}(h)^{\frac{\theta-1}{\theta}} + n^{\frac{1}{\theta}} \tilde{C}_{F,t}(h)^{\frac{\theta-1}{\theta}} \tag{51}$$

$$\tilde{C}_t^*(f)^{\frac{\theta-1}{\theta}} = (1-n)^{\frac{1}{\theta}} \left(\tilde{C}_{F,t}^*(f)\right)^{\frac{\theta-1}{\theta}} + n^{\frac{1}{\theta}} \left(\tilde{C}_{H,t}^*(f)\right)^{\frac{\theta-1}{\theta}} \tag{52}$$

The elasticity of substitution between domestic and foreign goods in given by θ and n measures the 'trade openness'. The maximisation of (51) subject to the budget constraint $P_t\tilde{C}_t(h) = P_{H,t}\tilde{C}_{H,t}(h) + P_{F,t}\tilde{C}_{F,t}(h)$ delivers the following demand functions

$$\tilde{C}_{H,t}(h) = (1-n)\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} \tilde{C}_t(h) \tag{53}$$

$$\tilde{C}_{F,t}(h) = n\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} \tilde{C}_t(h) \tag{54}$$

$$\tilde{C}_{F,t}^*(f) = (1-n)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\theta} \tilde{C}_t(f) \tag{55}$$

$$\tilde{C}_{H,t}^*(f) = n\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\theta} \tilde{C}_t^*(f) \tag{56}$$

Plugging (53) and (54) into the budget constraint we obtain the definition of the consumer price index – CPI

$$P_t = \left[(1-n)P_{H,t}^{1-\theta} + nP_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{57}$$

$$P_t^* = \left[(1-n)\left(P_{F,t}^*\right)^{1-\theta} + n\left(P_{H,t}^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{58}$$

Household's budget constraint is given by

$$\tilde{C}_t(h) + \frac{Q_t\tilde{D}_t(h)}{B_t} = \tilde{W}_tL_t(h) + \left[\frac{\tilde{D}_{t-1}(h)}{\Pi_t} - \Phi\left(\tilde{D}_t(h), \tilde{Z}_{t-1}\right)\right] + \Xi_t(h) \tag{59}$$

$$\tilde{C}_t^*(f) + \frac{Q_t^*\tilde{D}_t^*(f)}{B_t^*} = \tilde{W}_t^*L_t^*(f) + \frac{\tilde{D}_{t-1}^*(h)}{\Pi_t^*} + \Xi_t^*(f) \tag{60}$$

where $\tilde{D}_t(h)$ denotes the holding of the internationally traded riskless bond, Q_t is its prices, $\Xi_t(h)$ represents the dividends distributed by the intermediate goods producers,

$$\Phi\left(\tilde{D}_t(h), \tilde{Z}_{t-1}\right) = \frac{\phi}{2}\tilde{Z}_{t-1}\left(\frac{\tilde{D}_t(h)}{\tilde{Z}_{t-1}}\right)^2 \tag{61}$$

is an adjustment cost function that ensures balanced growth and

$$\log(B_t) = \rho_B \log(B_{t-1}) + \sigma_B\epsilon_{B,t} \tag{62}$$

$$\log(B_t^*) = \rho_{B^*} \log\left(B_{t-1}^*\right) + \sigma_{B^*}\epsilon_{B^*,t} \tag{63}$$

is an exogenous premium shock in the return to bonds (see Smets and Wouters (2007)).

6.3 Monetary policy

The monetary authority sets its instrument short-term interest rate according to a Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left(\frac{\Pi_t}{\Pi}\right)^{(1-\rho_R)\zeta_\Pi}\left(\frac{Y_t}{Y}\right)^{(1-\rho_R)\zeta_{Y^d}} e^{\sigma_R\epsilon_{R,t}} \tag{64}$$

$$\frac{R_t^*}{R} = \left(\frac{R_{t-1}^*}{R}\right)^{\rho_R}\left(\frac{\Pi_t^*}{\Pi}\right)^{(1-\rho_R)\zeta_\Pi}\left(\frac{Y_t^*}{Y}\right)^{(1-\rho_R)\zeta_{Y^d}} e^{\sigma_{R^*}\epsilon_{R^*,t}} \tag{65}$$

In other words, the policymaker adjusts the nominal interest rate in response to its lag value, to inflation deviations from the target – $\Pi = 1$ – and to output deviations from the long-run equilibrium – Y^d.

6.4 Market Clearing Conditions

The model is closed with the following market clearing conditions in the final good (domestic and foreign) markets:

$$\tilde{Y}_t^d = \tilde{C}_{H,t} + \tilde{C}_{H,t}^* \tag{66}$$
$$\tilde{Y}_t = \Delta_{H,t}\tilde{Y}_t^d \tag{67}$$

$$\tilde{Y}_t^{d.*} = \tilde{C}_{F,t}^* + \tilde{C}_{F,t} \tag{68}$$
$$\tilde{Y}_t^* = \Delta_{F,t}^*\tilde{Y}_t^{d,*} \tag{69}$$

and in the international bond market

$$\tilde{D}_t^* = -\tilde{D}_t \tag{70}$$

After some algebra the evolution of the net foreign asset position is given by

$$\frac{Q_t D_t}{B_t} = \left[\frac{D_{t-1}}{\Gamma_{t-1}} - \frac{\psi}{2\Gamma_t}\left(D_t\Gamma_t\right)^2\right] + C_{H,t}^* - \frac{\bar{P}_{F,t}}{\bar{P}_{H,t}} C_{F,t}\Omega_t \tag{71}$$

where $\Omega_t = \frac{\tilde{Z}_t^*}{\tilde{Z}_t}$

6.5 Steady-States

After the stochastic trends are removed (the stationary equations can be found in the main text), the steady-states can be derived analytically. Since the model is symmetric these values are going to be the same for both economies. Given ε then the marginal cost is given

$$MC = \frac{\varepsilon - 1}{\varepsilon} \tag{72}$$

We assume that $L = 1/3$ and $Y = 1$ and we derive the steady-state of capital from the production function

$$K^{\phi} = \left(\frac{Y}{L^{1-\phi}}\right)^{\frac{1}{\phi}} \tag{73}$$

We assume further that $\Pi_H = \bar{\Pi}_H = 1$ and this implies that $\Delta_H = 1$, which helps to pin down the values of Y^d and W:

$$Y = \Delta_H Y^d$$
$$W = \frac{MC\,(1-\phi)\,Y^d}{L} \tag{74}$$

We also assume that $\frac{P_H}{P} = \frac{P_F}{P} = \frac{P_F^*}{P^*} = \frac{P_H^*}{P^*} = 1$ and this implies

$$\Upsilon = 1$$

$$\bar{P}_H = \bar{P}_{F,t} = 1$$
$$\frac{C_H}{C_F} = \frac{1-n}{n} \tag{75}$$

There is no debt in the steady-state, meaning that $D = 0$ and this help to pin down:

$$C_H^* = C_F \tag{76}$$

$$Y = C \tag{77}$$

$$\chi_0 = \frac{W}{L^{\sigma_L} C^{\sigma_C}} \tag{78}$$

$$u\left(C, L\right) = \frac{C^{1-\sigma_C}}{1-\sigma_C} - \chi_0\frac{L^{1+\sigma_L}}{1+\sigma_L} \tag{79}$$

$$V = \frac{u(C,L)}{1-\beta\Gamma^{1-\sigma_C}} \tag{80}$$

$$\breve{V} \equiv \left(V\Gamma^{1-\sigma_C}\right)^{1-\gamma}$$

$$M = \Gamma^{-\sigma_C} \tag{81}$$

$$Q = \beta M B \tag{82}$$

$$K_H = \frac{MCY^d}{1-\beta\xi_H\Gamma^{1-\sigma_C}} \tag{83}$$

$$F_H = \frac{Y^d}{1-\beta\xi_H\Gamma^{1-\sigma_C}} \tag{84}$$

6.6 Calibration and Solution

The two economies are treated symmetrically and we, therefore, discuss only one set of structural parameters. Commonly, the time discount rate (β) and steady-state productivity growth (Γ) have been set equal to 0.99 and 1.005 respectively, which imply an annul interest rate of 6%. The (inverse) intertemporal substitution (σ_C) and labour supply (σ_L) elasticities equal to 2 and 3 respectively, a choice consistent with Rudebusch and Swanson (2012) and Fernández-Villaverde *et al.* (2011). The share of capital in the production (ϕ) has been calibrated to 0.36, a number typically used in the literature (Christiano *et al.* (2005), Trabandt and Uhlig (2011) and Jermann and Quadrini (2012)). Following Rudebusch and Swanson (2012) γ is -148.30 and this delivers a coefficient of relative risk aversion equal to 75.[3] Similar to Smets and Wouters (2007) and Christiano *et al.* (2005) the steady-state value of domestic producers' markup is 20% ($\epsilon = 6$). The Calvo probability of not resetting prices (ξ_H) equals 0.75 and this value lies between the estimates reported by Smets and Wouters (2007), Christiano *et al.* (2005) ($\xi_H = 0.65$) and Justiniano *et al.* (2010) ($\xi_H = 0.84$). The calibration of the Taylor rule is quite standard, namely the smoothing parameter has been set equal to 0.75, the inflation and output reaction parameters to 1.50 and 0.125, respectively. The elasticity of substitution between home-country and foreign-country consumption (θ) is 1.5, a value used by Rabanal and Rubio-Ramirez (2015) and estimated by Chin *et al.* (2015). In the benchmark calibration the home bias parameter $1-n$ is set equal to 0.3. Finally, the parameters that govern the non-stationary productivity process are taken from the work of Rabanal and Rubio-Ramirez (2015), while the calibration of the stationary, financial and policy shock rely on the estimates reported by Chin *et al.* (2015).

The model is solved using third-order perturbation methods. To avoid explosive solutions we follow Kim *et al.* (2008) and Andreasen *et al.* (2013) and we 'prune' all those terms that have an order that is higher than the approximation order.[4]

7 Data

The table below lists the variables used in the analysis. In terms of the data sources GFD refers to Global Financial Database, FRED is the Federal Reserve Bank of St Louis database and ONS refers to the Office of National Statistics. LD denotes the log difference transformation, while N denotes no transformation.

[3] The coefficient of relative risk aversion is a function of σ_L, γ and the steady-state value of labour see Rudebusch and Swanson (2012) and Swanson (2012).

[4] All the calculations have implemented using Dynare 4.4.2. The model and replication files can be found here.

Table 1: Model Variables

Description	Domestic	Foreign
Domestic Relative Price	$\bar{P}_{H,t}$	$\bar{P}^*_{F,t}$
Import Relative Price	$\bar{P}_{F,t}$	$\bar{P}^*_{H,t}$
Domestic Consumption	$C_{H,t}$	$C^*_{F,t}$
Import Consumption	$C_{F,t}$	$C^*_{H,t}$
Total Consumption	C_t	C^*_t
Asset Price	Q_t	
Total Consumption	C_t	C^*_t
Domestic Inflation	$\Pi_{H,t}$	$\Pi^*_{F,t}$
Utility	V_t	V^*_t
Wage	W_t	W^*_t
Price Dispersion	$\Delta_{H,t}$	$\Delta^*_{F,t}$
Aggregate Demand	Y^d_t	Y^{*d}_t
Relative Optimal Price	$\bar{\Pi}_{H,t}$	$\bar{\Pi}^*_{F,t}$
Marginal Cost	MC_t	MC^*_t
Phillips Curve Term	$K_{H,t}$	$K^*_{F,t}$
Phillips Curve Term	$F_{H,t}$	$F^*_{F,t}$
Real Interest Rate	Υ_t	
Debt	D_t	D^*_t
CPI Inflation	Π_t	Π^*_t
Policy Rate	R_t	R^*_t
Stochastic Discount Factor	M_t	M^*_t
Stochastic Trend Differential	Ω_t	
Stationary Productivity Process	A_t	A^*_t
Non Stationary Productivity Process	Γ_t	Γ^*_t
Financial Shock Process	B_t	B^*_t

Table 2: Model Structural Parameters

Description	Mnemonic	Value
Steady State Productivity Growth	Γ, Γ^*	1.005
Steady State Output	Y, Y^*	1.00
Steady State Labour	L, L^*	1/3
Steady State Real Exchange Rate	Υ	1.00
Steady State CPI Inflation	Π, Π^*	1.00
Steady State Domestic Inflation	Π_H, Π^*_F	1.00
Steady State Domestic Relative Prices	$\bar{P}_H, \bar{P}^*_F$	1.00
Home Bias	$1-n$	0.70
E.o.S between Domestic and Foreign Consumption	θ	1.50
Time Discount	β	0.99
Intertemporal Substitution Elasticity	σ_C	2.00
Labor Supply Frisch Elasticity	σ_L	3.00
Risk Preference	γ	-148.30
Production Capital Share	ϕ	0.36
Calvo Non Reset Price Probability	ξ_H	0.75
E.o.S between Intermediate Goods	ε	11.00
Exchange Rate Risk Premium	ψ	0.05
Policy Smoothness	ρ_R	0.75
Policy Inflation Reaction	ζ_Π	1.50
Policy Output Reaction	ζ_{Y^d}	0.25

Table 3: Model Shock Parameters

Description	Mnemonic	Domestic Value	Foreign Value
Stationary Productivity Process Persistence	ρ_A	0.90	0.90
Stationary Productivity Shock Uncertainty	$100\sigma_A$	0.87	0.87
Non Stationary Productivity Process Persistence	ρ_Γ	0.35	0.35
Non Stationary Productivity Shock Uncertainty	$100\sigma_\Gamma$	0.88	0.88
Error Correction	κ	-0.007	0.007
Financial Process Persistence	ρ_B	0.90	0.90
Financial Shock Uncertainty	$100\sigma_B$	0.30	0.30
Policy Shock Uncertainty	$100\sigma_R$	0.24	0.24

Table 4: Domestic Economy

Name	Equation
CPI Definition	$1 = (1-n)\,\bar{P}_{H,t}^{1-\theta} + n\bar{P}_{F,t}^{1-\theta}$
Domestic Consumption Demand	$C_{H,t} = (1-n)\,\bar{P}_{H,t}^{-\theta}C_t$
Foreign Consumption Demand	$C_{F,t}\Omega_t = \bar{P}_{F,t}^{-\theta}C_t$
Asset Pricing Equation	$\frac{Q_t}{B_t} = \beta E_t\left\{M_{t+1}\frac{\bar{P}_{H,t+1}}{\bar{P}_{H,t}}\right\} - \psi D_t\Gamma_t$
Utility Function	$u\left(C_t, L_t\right) = \frac{C_t^{1-\sigma_C}}{1-\sigma_C} - \chi_0\frac{L_t^{1+\sigma_L}}{1+\sigma_L}$
Utility Continuation Value	$\breve{V}_t \equiv E_t\left(V_{t+1}\Gamma_{t+1}^{1-\sigma_C}\right)^{1-\gamma}$
Preferences Expected Term	$\dddot{V}_t \equiv \breve{V}_t^{\frac{1}{1-\gamma}}$
Recursive Preferences	$V_t = u\left(C_t, L_t\right) + \beta\dddot{V}_t$
Labour Supply	$\chi_0 L_t^{\sigma_L}C_t^{\sigma_C} = W_t$
Stochastic Discount Factor	$M_{t+1} = \left[\frac{V_{t+1}}{\dddot{V}_t}\Gamma_{t+1}^{1-\sigma_C}\right]^{-\gamma}\left(\frac{C_t}{C_{t+1}\Gamma_{t+1}}\right)^{\sigma_C}$
Production Function	$Y_t = A_t K^{\phi}L_t^{1-\phi}$
Marginal Cost	$MC_t = \frac{W_t L_t}{\bar{P}_{H,t}(1-\phi)Y_t^d}$
Domestic Producer Phillips Curve	$K_{H,t} = MC_t Y_t^d + \beta\xi_H E_t M_{t+1}\left(\frac{1}{\Pi_{H,t+1}}\right)^{-\varepsilon}K_{H,t+1}\Gamma_{t+1}$
Domestic Producer Phillips Curve	$F_{H,t} = Y_t^d + \beta\xi_H E_t M_{t+1}\left(\frac{1}{\Pi_{H,t+1}}\right)^{1-\varepsilon}F_{H,t+1}\Gamma_{t+1}$
Domestic Producer Phillips Curve	$\bar{\Pi}_{H,t} = \frac{\varepsilon}{\varepsilon-1}\frac{K_{H,t}}{F_{H,t}}$
Domestic Producer Phillips Curve	$1 = \xi_H\left(\frac{1}{\Pi_{H,t}}\right)^{1-\varepsilon} + (1-\xi_H)\,\bar{\Pi}_{H,t}^{1-\varepsilon}$
Price Dispersion	$\left(\Delta_{H,t}\right)^{\frac{1}{1-\phi}} = (1-\xi_H)\left(\bar{\Pi}_{H,t}\right)^{-\frac{\varepsilon}{1-\phi}} + \xi_H\left(\frac{1}{\Pi_{H,t}}\right)^{-\frac{\varepsilon}{1-\phi}}\left(\Delta_{H,t-1}\right)^{\frac{1}{1-\phi}}$
Import Relative Prices	$\bar{P}_{F,t} = \Upsilon_t\bar{P}_{F,t}^{*}$
Net Foreign Assets	$\frac{Q_t D_t}{B_t} = \left[\frac{D_{t-1}}{\Gamma_{t-1}} - \frac{\psi}{2\Gamma_t}\left(D_t\Gamma_t\right)^2\right] + C_{H,t}^{*} - \frac{\bar{P}_{F,t}}{\bar{P}_{H,t}}C_{F,t}\Omega_t$
Aggregate Demand	$Y_t^d = C_{H,t} + C_{H,t}^{*}$
Market Clearing Condition	$Y_t = \Delta_{H,t}Y_t^d$
UIP Condition	$E_t\left\{\frac{M_{t+1}}{\Pi_{t+1}}\right\}B_t - E_t\left\{\frac{M_{t+1}^{*}}{\Pi_{t+1}^{*}}\frac{\Upsilon_t}{\Upsilon_{t+1}}\Omega_t\right\}B_t^{*} = \psi D_t\Gamma_t$
Policy Rule	$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left(\frac{\Pi_t}{\Pi}\right)^{(1-\rho_R)\zeta_\Pi}\left(\frac{Y_t^d}{Y^d}\right)^{(1-\rho_R)\zeta_{Y^d}}e^{\sigma_R\epsilon_{R,t}}$
Price-Yield Relation	$R_t = \frac{1}{Q_t}$
CPI Inflation	$\frac{\bar{P}_{H,t}}{\bar{P}_{H,t-1}} = \frac{\Pi_{H,t}}{\Pi_t}$
Financial Process	$\log(B_t) = \rho_B\log(B_{t-1}) + \sigma_B\epsilon_{B,t}$
Productivity Stationary Process	$\log(A_t) = \rho_A\log(A_{t-1}) + \sigma_A\epsilon_{A,t}$
Productivity Non Stationary Process	$\log\left(\frac{\Gamma_t}{\Gamma}\right) = \kappa\log(\Omega_t) + \rho_\Gamma\log\left(\frac{\Gamma_{t-1}}{\Gamma}\right) + \sigma_\Gamma\epsilon_{\Gamma,t}$
Stochastic Trend Differential	$\log(\Omega_t) = \log\left(\frac{Z_t^{*}}{Z_t}\right) = \log(\Omega_{t-1}) + \log\left(\frac{\Gamma_t^{*}}{\Gamma_t}\right)$

Table 5: Foreign Economy

Name	Equation
CPI Definition	$1 = (1-n)\,\bar{P}_{F,t}^{*1-\theta} + n\bar{P}_{H,t}^{*1-\theta}$
Domestic Consumption Demand	$C_{F,t}^{*} = (1-n)\,\bar{P}_{F,t}^{*-\theta} C_t^{*}$
Foreign Consumption Demand	$C_{H,t}^{*}\Omega_t = \bar{P}_{F,t}^{*-\theta} C_t^{*}$
Asset Pricing Equation	$\frac{1}{R_t^{*}} = \beta E_t \left\{ M_{t+1}^{*} \frac{\bar{P}_{F,t+1}^{*}}{\bar{P}_{F,t}^{*}} \right\} B_t^{*}$
Utility Function	$u\left(C_t^{*}, L_t^{*}\right) = \frac{C_t^{*1-\sigma_C}}{1-\sigma_C} - \chi_0 \frac{L_t^{*1+\sigma_L}}{1+\sigma_L}$
Utility Continuation Value	$\breve{V}_t^{*} \equiv E_t \left(V_{t+1}^{*} \Gamma_{t+1}^{*1-\sigma_C} \right)^{1-\gamma}$
Preferences Expected Term	$\ddddot{V}_t^{*} \equiv \breve{V}_t^{*\frac{1}{1-\gamma}}$
Recursive Preferences	$V_t^{*} = u\left(C_t^{*}, L_t^{*}\right) + \beta \ddddot{V}_t^{*}$
Labour Supply	$\chi_0 L_t^{*\sigma_L} C_t^{*\sigma_C} = W_t^{*}$
Stochastic Discount Factor	$M_{t+1}^{*} = \left[\frac{V_{t+1}^{*}}{\ddddot{V}_t^{*}} \Gamma_{t+1}^{*1-\sigma_C} \right]^{-\gamma} \left(\frac{C_t^{*}}{C_{t+1}^{*} \Gamma_{t+1}^{*}} \right)^{\sigma_C}$
Production Function	$Y_t^{*} = A_t^{*} K^{*\phi} L_t^{*1-\phi}$
Marginal Cost	$MC_t^{*} = \frac{W_t^{*} L_t^{*}}{\bar{P}_{F,t}^{*}(1-\phi) Y_t^{*d}}$
Domestic Producer Phillips Curve	$K_{F,t}^{*} = MC_t^{*} Y_t^{*d} + \beta \xi_H E_t M_{t+1}^{*} \left(\frac{1}{\Pi_{F,t+1}^{*}} \right)^{-\varepsilon} K_{F,t+1}^{*} \Gamma_{t+1}^{*}$
Domestic Producer Phillips Curve	$F_{F,t}^{*} = Y_t^{*d} + \beta \xi_H E_t M_{t+1}^{*} \left(\frac{1}{\Pi_{F,t+1}^{*}} \right)^{1-\varepsilon} F_{F,t+1}^{*} \Gamma_{t+1}^{*}$
Domestic Producer Phillips Curve	$\bar{\Pi}_{F,t}^{*} = \frac{\varepsilon}{\varepsilon-1} \frac{K_{F,t}^{*}}{F_{F,t}^{*}}$
Domestic Producer Phillips Curve	$1 = \xi_H \left(\frac{1}{\Pi_{F,t}^{*}} \right)^{1-\varepsilon} + (1-\xi_H)\, \bar{\Pi}_{F,t}^{*1-\varepsilon}$
Price Dispersion	$\left(\Delta_{F,t}^{*}\right)^{\frac{1}{1-\phi}} = (1-\xi_H) \left(\bar{\Pi}_{F,t}^{*}\right)^{-\frac{\varepsilon}{1-\phi}} + \xi_H \left(\frac{1}{\Pi_{F,t}}^{*} \right)^{-\frac{\varepsilon}{1-\phi}} \left(\Delta_{F,t-1}^{*}\right)^{\frac{1}{1-\phi}}$
Import Relative Prices	$\bar{P}_{H,t}^{*} = \frac{\bar{P}_{H,t}}{\Upsilon_t}$
Net Foreign Assets	$D_t^{*} = -D_t$
Aggregate Demand	$Y_t^{*d} = C_{F,t}^{*} + C_{F,t}$
Market Clearing Condition	$Y_t^{*} = \Delta_{F,t}^{*} Y_t^{*d}$
Policy Rule	$\frac{R_t^{*}}{R} = \left(\frac{R_{t-1}^{*}}{R} \right)^{\rho_R} \left(\frac{\Pi_t^{*}}{\Pi} \right)^{(1-\rho_R)\zeta_\Pi} \left(\frac{Y_t^{*d}}{Y^d} \right)^{(1-\rho_R)\zeta_{Y^d}} e^{\sigma_R \epsilon_{R^*,t}}$
CPI Inflation	$\frac{\bar{P}_{F,t}^{*}}{\bar{P}_{F,t-1}^{*}} = \frac{\Pi_{F,t}^{*}}{\Pi_t^{*}}$
Financial Process	$\log(B_t^{*}) = \rho_B \log(B_{t-1}^{*}) + \sigma_B \epsilon_{B^*,t}$
Productivity Stationary Process	$\log(A_t^{*}) = \rho_A \log(A_{t-1}^{*}) + \sigma_A \epsilon_{A^*,t}$
Productivity Non Stationary Process	$\log\left(\frac{\Gamma_t^{*}}{\Gamma} \right) = -\kappa \log(\Omega_t) + \rho_\Gamma \log\left(\frac{\Gamma_{t-1}^{*}}{\Gamma} \right) + \sigma_\Gamma \epsilon_{\Gamma^*,t}$

Table 6: Data for the factor model.

Variable Number	Country	Variable Name	Source	Transformation	
1	United States	Industrial Production	FRED	LD	
2	United States	Dow Jones industrial index	GFD	LD	
3	United States	GDP Deflator	FRED	LD	
4	United States	New Orders Index	FRED	N	
5	United States	Inventories	FRED	N	
6	United States	Suppliers Deliveries Index	FRED	N	
7	United States	Employment	FRED	LD	
8	United States	Business Conditions Index	GFD	N	
9	United States	Real Imports	FRED	LD	
10	United States	Real Exports	FRED	LD	
11	United States	Government Spending	BEA	LD	
12	United States	Net Taxes	BEA	LD	
13	United States	Real Gross Private Domestic Investment	FRED	LD	
14	United States	Real Personal Consumption Expenditure	FRED	LD	
15	United States	Unemployment Rate	FRED	N	
16	United States	Average Hours	FRED	LD	
17	United States	Civilian Labour Force	FRED	LD	
18	United States	Civilian Labor Force Participation Rate	FRED	LD	
19	United States	Nonfarm Business Sector: Unit Labor Cost	FRED	LD	
20	United States	Nonfarm Business Sector: Real Compensation Per Hour	FRED	LD	
21	United States	M2 Money Stock	FRED	LD	
22	United States	Total Consumer Credit Owned and Securitized, Outstanding	FRED	LD	
23	United States	Producer Price Index	FRED	LD	
24	United States	Personal Consumption Expenditures: Chain-type Price Index	FRED	LD	
25	United States	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
26	United States	6-month Treasury bill minus 3 mth yield	GFD	N	

Table 6: Data for the factor model.

27	United States	1 year Govt Bond Yield minus 3 mth yield	GFD	N	
28	United States	5 year Govt Bond Yield minus 3 mth yield	GFD	N	
29	United States	Reuters/Jeffries-CRB Total Return Index	GFD	LD	
30	United States	West Texas Intermediate Oil Price	GFD	LD	
31	United States	BAA Corporate Spread	GFD	N	
32	United States	AAA Corporate Bond Spread	GFD	N	
33	United States	NYSE Stock Market Capitalization	GFD	LD	
34	United States	S&P500 P/E Ratio	GFD	N	
35	United States	Dividend Yield	GFD	N	
36	United States	US Canada Exchange Rate	GFD	LD	
37	United States	Nominal Effective Exchange Rate	GFD	LD	
38	United States	Real Effective Exchange Rate	GFD	LD	
39	United States	Real GDP	FRED	LD	
40	United States	CPI	FRED	LD	
41	United States	3 month T-Bill rate	FRED	N	
42	United States	S&P500 Total Return Index	FRED	LD	
43	United Kingdom	Industrial Production	GFD	LD	
44	United Kingdom	UK FT-Actuaries 500 index (Non-Financials)	GFD	LD	
45	United Kingdom	Retail Price Index	GFD	LD	
46	United Kingdom	Composite Leading Indicator	GFD	N	
47	United Kingdom	Real Exports	GFD	LD	
48	United Kingdom	Real Imports	GFD	LD	
49	United Kingdom	Government Spending	ONS	LD	
50	United Kingdom	Government Consumption	ONS	LD	
51	United Kingdom	Gross Capital Formation	GFD	LD	
52	United Kingdom	Consumption Expenditure	GFD	LD	
53	United Kingdom	GDP Deflator	GFD	LD	
54	United Kingdom	Wage	GFD	LD	
55	United Kingdom	20 year Govt Bond Yield minus 3 mth yield	GFD	N	
56	United Kingdom	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
57	United Kingdom	5 year Govt Bond Yield minus 3 mth yield	GFD	N	

Table 6: Data for the factor model.

58	United Kingdom	Brent Oil Price	GFD	N	
59	United Kingdom	Corporate Bond Spread	GFD	N	
60	United Kingdom	Real House Prices	GFD	N	
61	United Kingdom	Dividend Yield	GFD	N	
62	United Kingdom	FT Actuaries P/E Ratio	GFD	N	
63	United Kingdom	Pounds to DollarExchange Rate	GFD	LD	
64	United Kingdom	Pounds to Euro Exchange Rate	GFD	LD	
65	United Kingdom	Pounds to Yen Exchange Rate	GFD	LD	
66	United Kingdom	Nominal Effective Exchange Rate	GFD	LD	
67	United Kingdom	Real Effective Exchange Rate	GFD	LD	
68	United Kingdom	Real GDP	GFD	LD	
69	United Kingdom	CPI	GFD	LD	
70	United Kingdom	3 month T-Bill rate	GFD	N	
71	United Kingdom	FTSE All share Index	GFD	LD	
72	Canada	Industrial Production	GFD	LD	
73	Canada	Consumption Deflator	GFD	LD	
74	Canada	Producer Price Index	GFD	LD	
75	Canada	Composite Leading Indicator	GFD	N	
76	Canada	Real Exports	GFD	LD	
77	Canada	Real Imports	GFD	LD	
78	Canada	Government Consumption	GFD	LD	
79	Canada	Gross Capital Formation	GFD	LD	
80	Canada	Consumption Expenditure	GFD	LD	
81	Canada	Unemployment Rate	GFD	N	
82	Canada	GDP Deflator	GFD	LD	
83	Canada	M1 Money Stock	GFD	LD	
84	Canada	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
85	Canada	5 year Govt Bond Yield minus 3 mth yield	GFD	N	
86	Canada	3 year Govt Bond Yield minus 3 mth yield	GFD	N	
87	Canada	S&P TSX Oil and Gas index	GFD	LD	
88	Canada	Dividend Yield	GFD	N	
89	Canada	S&P TSX P/E Ratio	GFD	N	
90	Canada	Canadian Dollar to US Dollar exchange rate	GFD	LD	
91	Canada	Canadian Dollar to Euro Rate	GFD	LD	

Table 6: Data for the factor model.

92	Canada	Canadian Dollar to Yen Rate	GFD	LD	
93	Canada	Nominal Effective Exchange Rate	GFD	LD	
94	Canada	Real Effective Exchange Rate	GFD	LD	
95	Canada	Real GDP	GFD	LD	
96	Canada	CPI	GFD	LD	
97	Canada	3 month T-Bill rate	GFD	N	
98	Canada	S&P/TSX-300 Total Return Index	GFD	LD	
99	Germany	Industrial Production	GFD	LD	
100	Germany	DAX Price Index	GFD	LD	
101	Germany	Unemployment Rate	GFD	N	
102	Germany	Composite Leading Indicator	GFD	N	
103	Germany	Real Exports	GFD	LD	
104	Germany	Real Imports	GFD	LD	
105	Germany	Government Consumption	GFD	LD	
106	Germany	Gross Capital Formation	GFD	LD	
107	Germany	Consumption Expenditure	GFD	LD	
108	Germany	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
109	Germany	5 year Govt Bond Yield minus 3 mth yield	GFD	N	
110	Germany	Corporate Bond Spread	GFD	N	
111	Germany	Dividend Yield	GFD	N	
112	Germany	P/E Ratio	GFD	N	
113	Germany	Nominal Effective Exchange Rate	GFD	LD	
114	Germany	Real Effective Exchange Rate	GFD	LD	
115	Germany	Real GDP	GFD	LD	
116	Germany	CPI	GFD	LD	
117	Germany	3 month T-Bill rate	GFD	N	
118	Germany	CDAX Total Return Index	GFD	LD	
119	France	Industrial Production	GFD	LD	
120	France	GDP Deflator	GFD	LD	
121	France	CAC All-Tradable Total Return Index	GFD	LD	
122	France	Unemployment Rate	GFD	N	
123	France	Composite Leading Indicator	GFD	N	
124	France	Real Exports	GFD	LD	
125	France	Real Imports	GFD	LD	
126	France	Government Consumption	GFD	LD	

Table 6: Data for the factor model.

127	France	Gross Capital Formation	GFD	LD	
128	France	Consumption Expenditure	GFD	LD	
129	France	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
130	France	Corporate Bond Spread	GFD	N	
131	France	Dividend Yield	GFD	N	
132	France	Nominal Effective Exchange Rate	GFD	LD	
133	France	Real Effective Exchange Rate	GFD	LD	
134	France	Real GDP	GFD	LD	
135	France	CPI	GFD	LD	
136	France	3 month T-Bill rate	GFD	N	
137	France	Paris CAC-40 index	GFD	LD	
138	Spain	Manufacturing Output	GFD	LD	
139	Spain	Madrid General Index	GFD	LD	
140	Spain	Unemployment Rate	GFD	N	
141	Spain	Composite Leading Indicator	GFD	N	
142	Spain	Real Exports	GFD	LD	
143	Spain	Real Imports	GFD	LD	
144	Spain	Industrial Production	GFD	LD	
145	Spain	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
146	Spain	Nominal Effective Exchange Rate	GFD	LD	
147	Spain	Real Effective Exchange Rate	GFD	LD	
148	Spain	Real GDP	GFD	LD	
149	Spain	CPI	GFD	LD	
150	Spain	Barcelona SE-30 Return Index	GFD	LD	
151	Italy	Production excluding construction	GFD	LD	
152	Italy	Banca Commerciale Italiana Index	GFD	LD	
153	Italy	Total Car Registrations	GFD	LD	
154	Italy	Real Exports	GFD	LD	
155	Italy	Real Imports	GFD	LD	
156	Italy	Industrial Production	GFD	LD	
157	Italy	Unemployment Rate	GFD	N	
158	Italy	Manufacturing Production	GFD	LD	
159	Italy	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
160	Italy	5 year Govt Bond Yield minus 3 mth yield	GFD	N	

Table 6: Data for the factor model.

161	Italy	3 year Govt Bond Yield minus 3 mth yield	GFD	N	
162	Italy	Corporate Bond Spread	GFD	N	
163	Italy	Dividend Yield	GFD	N	
164	Italy	Nominal Effective Exchange Rate	GFD	LD	
165	Italy	Real Effective Exchange Rate	GFD	LD	
166	Italy	Real GDP	GFD	LD	
167	Italy	CPI	GFD	LD	
168	Italy	3 month T-Bill rate	GFD	N	
169	Italy	BCI Global Return Index	GFD	LD	
170	Netherlands	Manufacturing Production	GFD	LD	
171	Netherlands	Netherlands All Share Return Index	GFD	LD	
172	Netherlands	GDP Deflator	GFD	LD	
173	Netherlands	Composite Leading Indicator	GFD	N	
174	Netherlands	Gross Capital Formation	GFD	LD	
175	Netherlands	Retail Trade	GFD	LD	
176	Netherlands	Industrial Production	GFD	LD	
177	Netherlands	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
178	Netherlands	Nominal Effective Exchange Rate	GFD	LD	
179	Netherlands	Real Effective Exchange Rate	GFD	LD	
180	Netherlands	Dividend Yield	GFD	N	
181	Netherlands	P/E Ratio	GFD	N	
182	Netherlands	Unemployment Rate	GFD	N	
183	Netherlands	Real GDP	GFD	LD	
184	Netherlands	CPI	GFD	LD	
185	Netherlands	3 month T-Bill rate	GFD	N	
186	Netherlands	Netherlands All Share Price Index	GFD	LD	
187	Sweden	Manufacturing Production	GFD	LD	
188	Sweden	OMX Stockholm Gross Index	GFD	LD	
189	Sweden	Real Exports	GFD	LD	
190	Sweden	Real Imports	GFD	LD	
191	Sweden	Composite Leading Indicator	GFD	N	
192	Sweden	Nominal Effective Exchange Rate	GFD	LD	
193	Sweden	Real Effective Exchange Rate	GFD	LD	
194	Sweden	Dividend Yield	GFD	N	
195	Sweden	Unemployment Rate	GFD	N	
196	Sweden	Industrial Production	GFD	LD	

Table 6: Data for the factor model.

197	Sweden	CPI	GFD	LD	
198	Sweden	3 month T-Bill rate	GFD	N	
199	Sweden	OMX General Index	GFD	LD	
200	Japan	Industrial Production	GFD	LD	
201	Japan	Manufacturing Production	GFD	LD	
202	Japan	Tokyo SE Price Index	GFD	LD	
203	Japan	Producer Price Index	GFD	LD	
204	Japan	Composite Leading Indicator	GFD	N	
205	Japan	Real Exports	GFD	LD	
206	Japan	Real Imports	GFD	LD	
207	Japan	Government Consumption	GFD	LD	
208	Japan	Gross Capital Formation	GFD	LD	
209	Japan	Consumption Expenditure	GFD	LD	
210	Japan	Hours	GFD	LD	
211	Japan	Unemployment Rate	GFD	N	
212	Japan	Earnings	GFD	LD	
213	Japan	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
214	Japan	& year Govt Bond Yield minus 3 mth yield	GFD	N	
215	Japan	Nikko Securities Composite Total Return Index	GFD	LD	
216	Japan	Dividend Yield	GFD	N	
217	Japan	P/E Ratio	GFD	N	
218	Japan	Nominal Effective Exchange Rate	GFD	LD	
219	Japan	Real Effective Exchange Rate	GFD	LD	
220	Japan	Real GDP	GFD	LD	
221	Japan	CPI	GFD	LD	
222	Japan	Topix Total Return Index	GFD	LD	
223	Australia	Composite Leading Indicator	GFD	N	
224	Australia	ASX All-Ordinaries Composite	GFD	LD	
225	Australia	Consumption Deflator	GFD	LD	
226	Australia	Retail Trade	GFD	LD	
227	Australia	Real Exports	GFD	LD	
228	Australia	Real Imports	GFD	LD	
229	Australia	Gross Capital Formation	GFD	LD	
230	Australia	Government Consumption	GFD	LD	
231	Australia	Consumption	GFD	LD	

Table 6: Data for the factor model.

232	Australia	GDP Deflator	GFD	LD	
233	Australia	Compensation	GFD	LD	
234	Australia	10 year Govt Bond Yield minus 3 mth yield	GFD	N	
235	Australia	ASX 200 Composite	GFD	LD	
236	Australia	Dividend Yield	GFD	N	
237	Australia	P/E Ratio	GFD	N	
238	Australia	Real Effective Exchange Rate	GFD	LD	
239	Australia	Nominal Effective Exchange Rate	GFD	LD	
240	Australia	Real GDP	GFD	LD	
241	Australia	CPI	GFD	LD	
242	Australia	3 month T-Bill rate	GFD	N	
243	Australia	ASX Accumulation Index-All Ordinaries	GFD	LD	

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