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The Changing Transmission of Uncertainty Shocks in the US: An Empirical Analysis

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The changing transmission of uncertainty shocks in the US: an empirical analysis*

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Abstract

This paper investigates if the impact of uncertainty shocks on the US economy has changed over time. To this end, we develop an extended Factor Augmented VAR model that simultaneously allows the estimation of a measure of uncertainty and its time-varying impact on a range of variables. We find that the impact of uncertainty shocks on real activity and financial variables has declined systematically over time. In contrast, the response of inflation and the short-term interest rate to this shock has remained fairly stable. Simulations from a non-linear DSGE model suggest that these empirical results are consistent with an increase in the monetary authorities' anti-inflation stance and a 'flattening' of the Phillips curve.

JEL Codes: C15,C32, E32

Key Words: FAVAR, Stochastic Volatility, Uncertainty Shocks, DSGE Model

1 Introduction

The recent financial crisis and ensuing recession have led to a renewed interest in the possible relationship between economic uncertainty and macroeconomic variables. A number of proxies for uncertainty have been proposed in the recent literature and several papers use VAR based analyses to estimate the impact of uncertainty shocks (see for example Bloom (2009) and Jurado *et al.* (2013)). In addition, a growing DSGE based literature has documented the transmission mechanism of these shocks from a theoretical point of view (see for example Fernandez-Villaverde *et al.* (2011)and Fernández-Villaverde *et al.* (2011)).

Overall, the empirical literature on this subject provides strong evidence that uncertainty shocks can have a significant adverse impact on the economy. For example, the analysis in Bloom (2009) suggests that a unit increase in uncertainty leads to a 1% decline in US industrial production and similar results are reported in related papers. However, the estimates reported in these papers are typically based on data that spans the last three or four decades and thus cover periods potentially characterised by changing dynamics, policy regimes and economic shocks.

There has been limited focus on exploring whether the impact of uncertainty shocks has changed over time and identifying the factors that can possibly explain any temporal shifts. An exception is Beetsma and Giuliodori (2012) who focus on shocks to US stock market volatility and show that the impact of these shocks on GDP has declined over time. However, the results in Beetsma and Giuliodori (2012) relate to stock market volatility rather than the impact of *macroeconomic* volatility. In addition, and perhaps more importantly, the authors do not provide a theoretical explanation for the identified change in the transmission mechanism.[1]

This paper attempts to fill these gaps and to introduce a general framework for exploring this issue. First, we propose an extended factor augmented VAR (FAVAR) model that allows the estimation of a measure of uncertainty that encompasses volatility from the real and financial sector of the economy and is a proxy for macroeconomic uncertainty. The proposed FAVAR allows for time-varying parameters and simultaneously provides an estimate of the time-varying response of macroeconomic variables to shocks to this uncertainty measure, thus allowing the investigation of temporal shifts in a coherent manner. We estimate this model using a comprehensive dataset for

*The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.

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[1] Benati (2014) uses a time-varying VAR to examine the importance of policy uncertainty shocks, but focusses on the great recession rather than structural changes over a longer time-period. Caggiano *et al.* (2014) and Alessandri and Mumtaz (2014) consider the possibility of non-linearities in the impact of uncertainty shocks but do not investigate if the impact may have changed gradually across time.

the US. Second, we use a non-linear DSGE model to explore the possible reasons behind the identified shifts in impulse responses and thus attempt to provide a structural explanation for the empirical results.

Our results suggest that impact of uncertainty shocks on measure of real activity, asset prices and indicators of financial conditions has declined systematically over time. For example, the magnitude of the impact of this shock on GDP and the corporate bond spread at the two year horizon over the current period is estimated to be half of that prevalent during the 1970s and the 1980s. In contrast, the estimated response of inflation and short-term interest rates has been fairly constant over time. We also find a negative co-movement between output and inflation conditional on uncertainty shocks which supports the conclusions reached in Fernández-Villaverde *et al.* (2011). Simulations from the DSGE model suggest that a possible explanation for these changes may be the structural shifts highlighted in Fernandez-Villaverde and Rubio-Ramirez (2008)– i.e. an increase in the Federal Reserve's anti-inflationary stance *and* a change in the parameters of the Phillips curve that imply a rise in price stickiness and a fall in indexation to past inflation. An increase in the magnitude of the Taylor rule inflation coefficient implies that inflation responds less to an increase in uncertainty as agents become less concerned about expected inflation. This in-turn allows the monetary authority to reduce interest rates more quickly than otherwise possible to tackle the adverse real activity effects of the uncertainty shock and thus reduces the magnitude of the decline in output and asset prices. The simultaneous increase in price stickiness and decrease in the degree of indexation in the model increases the positive response of inflation to the uncertainty shock (as agents hedge against being locked into a contract with an unfavourable price) and thus dampens the initial impact of a rise in the Fed's anti-inflation stance. This implies that the inflation and interest rate response remains fairly constant at short and medium horizons.

The analysis in the paper adds to the literature on uncertainty by systematically investigating how the impact of uncertainty has changed over time and provides a structural explanation for the estimated shifts. The empirical model proposed in the paper builds upon exisiting VAR and FAVAR models by simultaneously allowing the estimation of time-varying volatility and the time-varying impact of this volatility on the endogenous variables. In addition, the theoretical analysis in the paper contributes to the DSGE applications to this issue by showing how the impact of uncertainty shocks varies with the parameters of various key sectors in the model.

Our results have important implications. Our empirical findings suggest that uncertainty remains an important concern for inflation developments. This is particularly important in the current climate where uncertainty has been elevated after the financial crisis and this may go some way in explaining the puzzling persistence of inflation noted by Watson (2014). However, our results suggest that with the decline of the output response to uncertainty, the negative trade-off between inflation and output conditional on uncertainty has declined. This suggests that Fed has additional leeway in dealing with the adverse effects of an uncertainty shock than would be suggested by a fixed coefficient model.

The paper is organised as follows: Sections 2 and 3 introduce the empirical model and discuss the estimation method. The results from the empirical model are presented in Section 4. We introduce the DSGE model and present the model simulations in Section 5.

2 Empirical model

The core of the empirical model is the following time-varying parameter vector autoregression (TVP VAR):

$$Z_t = c_t + \sum_{j=1}^{P} \beta_{tj} Z_{t-j} + \sum_{j=0}^{J} \gamma_{tj} \ln \lambda_{t-j} + \Omega_t^{1/2} e_t \tag{1}$$

where Z_t is a matrix of endogenous variables that we describe below. The law of motion for the VAR coefficients is given by:

$$\begin{aligned} B &= vec([c;\beta;\gamma]) \\ B_t &= B_{t-1} + \eta_t, VAR(\eta_t) = Q_B \end{aligned} \tag{2}$$

As in Primiceri (2005), the covariance matrix of the residuals is defined as:

$$\Omega_t = A_t^{-1} H_t A_t^{-1'}$$

where A_t is lower triangular. Each non-zero element of A_t evolves as a random walk

$$a_t = a_{t-1} + g_t, VAR(g_t) = G \tag{3}$$

where G is block diagonal as in Primiceri (2005).

Following Carriero *et al.* (2012) the volatility process is defined as

$$\begin{aligned} H_t &= \lambda_t S \\ S &= diag(s_1, .., s_N) \end{aligned} \tag{4}$$

The overall volatility evolves as an AR(1) process

$$\ln \lambda_t = \alpha + F \ln \lambda_{t-1} + \bar{\eta}_t, VAR(\bar{\eta}_t) = Q_\lambda \tag{5}$$

The structure defined by equation 4 suggests that the specification is characterised by two features. First, the model does not distinguish between the common and idiosyncratic component in volatility and λ_t is a convolution of both components. While seperating these unobserved components may be interesting in its own right, it is not directly relevant for our application where the key aim is to estimate overall volatility which, by definition, is a combination of the two components. Secondly equation 4 implies that λ_t is a simple average of volatility of each shock with equal weights given to each individual volatility. As we show below, this simple scheme produces volatility estimates that are plausible from a historical perspective and compare favourably to non-parametric estimates of uncertainty recently suggested in the literature.

The formulation presented in equations 4 and 5 is related to a number of recent empirical contributions. For example, the structure of the stochastic volatility model used above closely resembles the formulations used in time-varying VAR models (see Cogley and Sargent (2005) and Primiceri (2005)). Our model differs from these studies in that it allows a direct impact of the volatilities on the level of the endogenous variables. The model proposed above can be thought of as a multivariate extension of the stochastic volatility in mean model proposed in Koopman and Uspensky (2000) and applied in Berument *et al.* (2009), Kwiatkowski (2010) and Lemoine and Mougin (2010). In addition, our model has similarities with the stochastic volatility models with leverage studied in Asai and McAleer (2009) and the non-linear model proposed in Aruoba *et al.* (2011). Finally, the model is based on the VAR with stochastic volatility introduced in Mumtaz and Theodoridis (n.d.). While Mumtaz and Theodoridis (n.d.) focus on the impact of volatility associated with the output shock, we focus on an overall measure of uncertainty that incorporates the variance of all shocks in the model. In addition the model proposed above incorporates time-variation, a feature missing from the studies that consider stochastic volatility in mean models.

In our application of this model, we attempt to incorporate a large number of macroeconomic and financial variables in Z_t. This allows us to account for the possibility of omitted variables and implies that the estimate of λ_t captures a broad range of economic and financial uncertainty. As is well known in the TVP-VAR literature, the stability of the VAR coefficients at each point in time is difficult to achieve when the number of endogenous variables exceeds 4 (see Koop and Potter (2011)). We deal with this issue by incorporating a factor structure into the model. In particular, we define an observation equation

$$X_{it} = \Lambda_t Z_t + R^{1/2} \varepsilon_{it} \tag{6}$$

In other words, Z_t are assumed to be a set of K unobserved factors that summarise an underlying dataset X_{it} via the factor loading matrix Λ_t. We allow for time-variation in the factor loadings as in Delnegro and Otrok (2005) which evolve as

$$\Lambda_t = \Lambda_{t-1} + \bar{\eta}_t, VAR\left(\bar{\eta}_t\right) = Q_\Lambda$$

The idiosyncratic components are defined by ε_{it} with a diagonal covariance matrix R. As described below, X_{it} contains key real activity variables, measures of inflation, short and long-term interest rates, money and credit growth and financial variables such as corporate bond spreads, stock market variables and other asset prices. Therefore, the factors Z_t contain a large amount of information and as a consequence, the measure of uncertainty λ_t spans the volatility across the key sectors of the US economy.

3 Estimation and model specification

The model defined in equation 1 and 6 is estimated using an MCMC algorithm. In this section we summarise the key steps of the algorithm and provide the details in the technical appendix.[2] The appendix also presents the details on the prior distributions which are standard. It is worth noting that we follow Cogley and Sargent (2005)

[2] The appendix presents a small Monte-Carlo experiment that shows that the algorithm displays a satisfactory performance.

in setting the prior for the variance of the shock to the transition equations for the time-varying parameters (Q_B and Q_λ). The prior for these covariance matrices is assumed to be inverse Wishart:

$$P(Q_B)\,\tilde{}\,IW(Q_{B,OLS} \times T_0 \times K, T_0)$$
$$P(Q_\lambda)\,\tilde{}\,IW(Q_{\lambda,OLS} \times T_0 \times K, T_0)$$

where $T_0 = 40$ is the length of the training sample and $Q_{B,OLS}$ and $Q_{\lambda OLS}$ are the OLS estimate of the coefficient covariances using the training sample where Z_t is approximated by principal components. The prior scale matrix is multiplied by the factor K which is set to 3.5×10^{-4} as in Cogley and Sargent (2005), but as shown in the sensitivity analysis, the key results also hold for smaller values of K. As noted in Bernanke *et al.* (2005), the FAVAR model is subject to rotational indeterminancy of the factors and factor loadings. Following Bernanke *et al.* (2005), we impose a normalisation under which the first $K \times K$ block of Λ_t is fixed to an identity matrix for all time periods.

The MCMC algorithm consists of the following steps:

1. Conditional on a draw for the stochastic volatility λ_t, the factors Z_t and the time-varying matrix A_t, and the variances S and Q_B equation (1) represents a VAR model with time-varying coefficients. The algorithm of Carter and Kohn (2004) is used to draw B_t from their conditional posterior density and rejection sampling is employed to ensure that the VAR coefficients are stable at each point in time. Conditional on B_t the covariance matrix Q_B can be drawn from the inverse Wishart (IW) density.

2. Conditional on a draw for the stochastic volatility Z_t, λ_t, B_t and G the time-varying elements of A_t are drawn equation by equation using the Carter and Kohn (2004) algorithm. Conditional on a_t, the respective blocks of G are drawn from the IW density.

3. Given A_t and λ_t, The elements of S have an inverse Gamma posterior and these parameters can be easily simulated from this distribution.

4. Conditional on λ_t, the constant α, autoregressive parameter F and variance Q_λ can be drawn using standard results for linear regressions.

5. Conditional on a draw for the factors Z_t, Q_Λ and R, the algorithm of Carter and Kohn (2004) is used to draw Λ_t. Conditional on Λ_t the covariance matrix Q_Λ can be drawn from the inverse Wishart (IW) density.

6. Conditional on a draw for the factors Z_t and the factor loadings Λ_t, standard results for linear regressions can be used to draw from the posterior distribution of the variance of the idiosyncratic components R.

7. Conditional on $Z_t, B_t, A_t, S, \alpha, F$ and Q_λ, the stochastic volatility λ_t is simulated using a date by date independence Metropolis step as described in Cogley and Sargent (2005) and Jacquier *et al.* (1994) (see also Carlin *et al.* (1992)).

8. Given the parameters of the observation equation 6 and the transition equation 1, the Carter and Kohn (2004) algorithm is used to draw from the conditional posterior distribution of the factors Z_t.

In the benchmark specifications, we use 500,000 replications and base our inference on the last 5,000 replications. The recursive means of the retained draws (see technical appendix) show little fluctuation providing support for convergence of the algorithm.

3.1 Model specification

We consider models with 2 to 4 factors and select the model which minimises the Bayesian Deviance Information Criterion (DIC). Note that for models with more than 4 factors it is difficult to ensure stability of the VAR coefficients at each point in time and step 1 of the algorithm described above becomes largely infeasible. Therefore the maximum number of factors is limited to 4. We show in the sensitivity analysis below that the key results remain the same across models with different number of factors.

Introduced in Spiegelhalter *et al.* (2002), the DIC is a generalisation of the Akaike information criterion – it penalises model complexity while rewarding fit to the data. As shown in the appendix, the DIC can be calculated as $DIC = \bar{D} + p_D$ where $\bar{D}$ measures goodness of fit and p_D approximates model complexity. A model with a lower DIC is preferred. Table 1 shows that the DIC is minimised for the model with 2 factors. Therefore, we select 2 factors in our benchmark model. We show in the sensitivity analysis below that the key results are preserved in a 3 factor model.

	DIC
2 factors	15447.5
3 factors	17873.9
4 factors	20589.5

Table 1: Model Comparison via DIC. Best fit indicated by lowest DIC

Following Cogley and Sargent (2005) and Primiceri (2005) the lag length P is set equal to 2 and we set $j = 2$ in the benchmark specification. This choice of P reflects the fact that as the number of lags increase, the stability of the VAR model is adversely affected. Given that we employ quarterly data, we allow the possibility of an impact of λ_t within a three-month period. We show in the sensitivity analysis below, that the key results are similar when longer lags are employed and when the contemporaneous impact of volatility is set to zero.

3.2 Data

The dataset is quarterly and runs from 1950Q1 to 2014Q2. We employ a panel of 39 variables listed in section 1.7 of the technical appendix. These variables include the key aggregates from the dataset of Stock and Watson (2012) that are available from 1950 onwards. We include real activity series such as consumption, investment, GDP, taxes, government spending, employment, unemployment, hours and surveys of economic activity. Data on prices is covered by CPI, consumption and GDP deflator and the producer price index. The dataset includes short-term and long term interest rates, various corporate bond spreads and series on money and credit growth. Finally, data on stock market variables, commodity prices and exchange rates is included. In summary, the dataset covers the key sectors of the US economy and incorporates a wide range of information.

4 Empirical results

4.1 Estimated volatility

Figure 1 plots the estimated volatility λ_t for the US. The figure also plots the uncertainty measure recently proposed in Jurado *et al.* (2013) for comparison. The estimated volatility is high during the early and the mid-1970s with a large peak during the early 1980s. The mid-1980s saw the onset of the great moderation and volatility declined and remained low until the recession during the early years of the last decade. The recent financial crisis saw a substantial increase in volatility with the level of λ_t during 2008/2009 matching the highs in volatility seen during the early 1980s. It is interesting to note that the estimate of λ_t is highly correlated with the measure of uncertainty proposed in Jurado *et al.* (2013). This reflects the fact that the underlying method of capturing uncertainty has a number of similarities with the calculation in Jurado *et al.* (2013). The uncertainty measure in Jurado *et al.* (2013) is the average time-varying variance in the unpredictable component of a large set of real and financial time-series. The volatility specification in equations 4 and 5 has a similar intepretation– it attempts to capture the average volatility in the *shocks* to Z_t where the factors summarise real and financial conditions. In the section below we consider how innovations to this measure affect the variables included in our dataset and whether the impulse responses display time-variation.

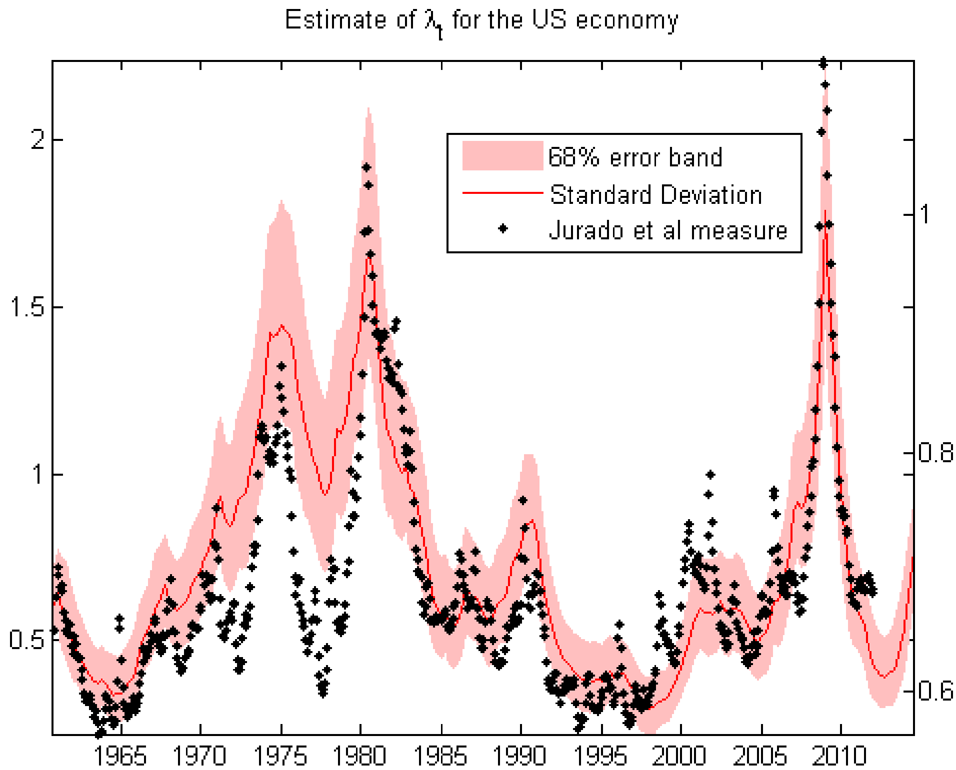

Figure 1: Estimated measure of uncertainty. The non-parametric measure proposed by Jurado *et al.* (2013) is plotted for comparison.

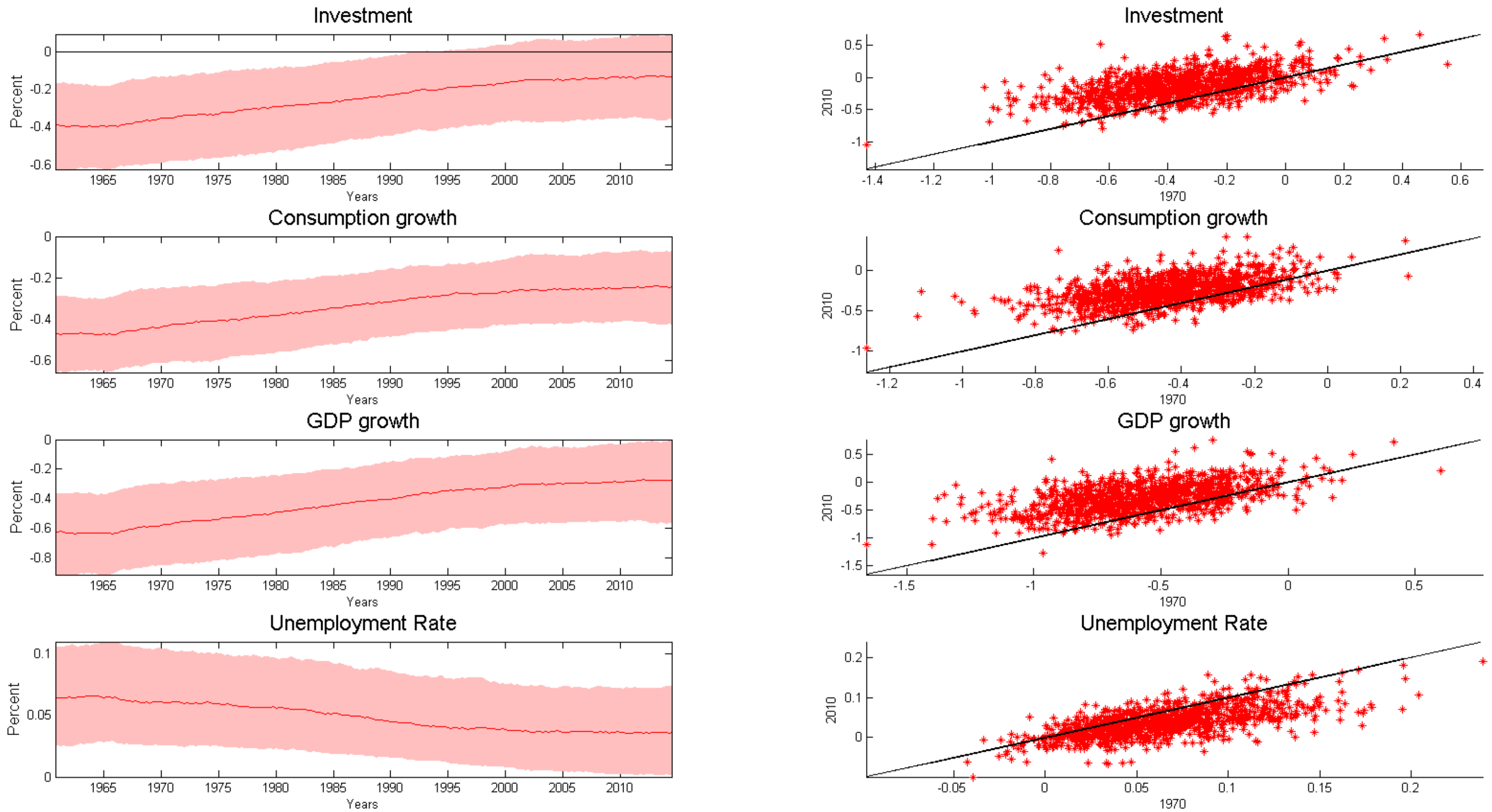

Figure 2: Impulse response of real activity series to a one standard deviation uncertainty shock. The left panels show the time profile of the cumulated response at the 2 year horizon. The right panels show the joint distribution of the cumulated response at the one year horizon in 1970 and 2010 along with the 45-degree line.

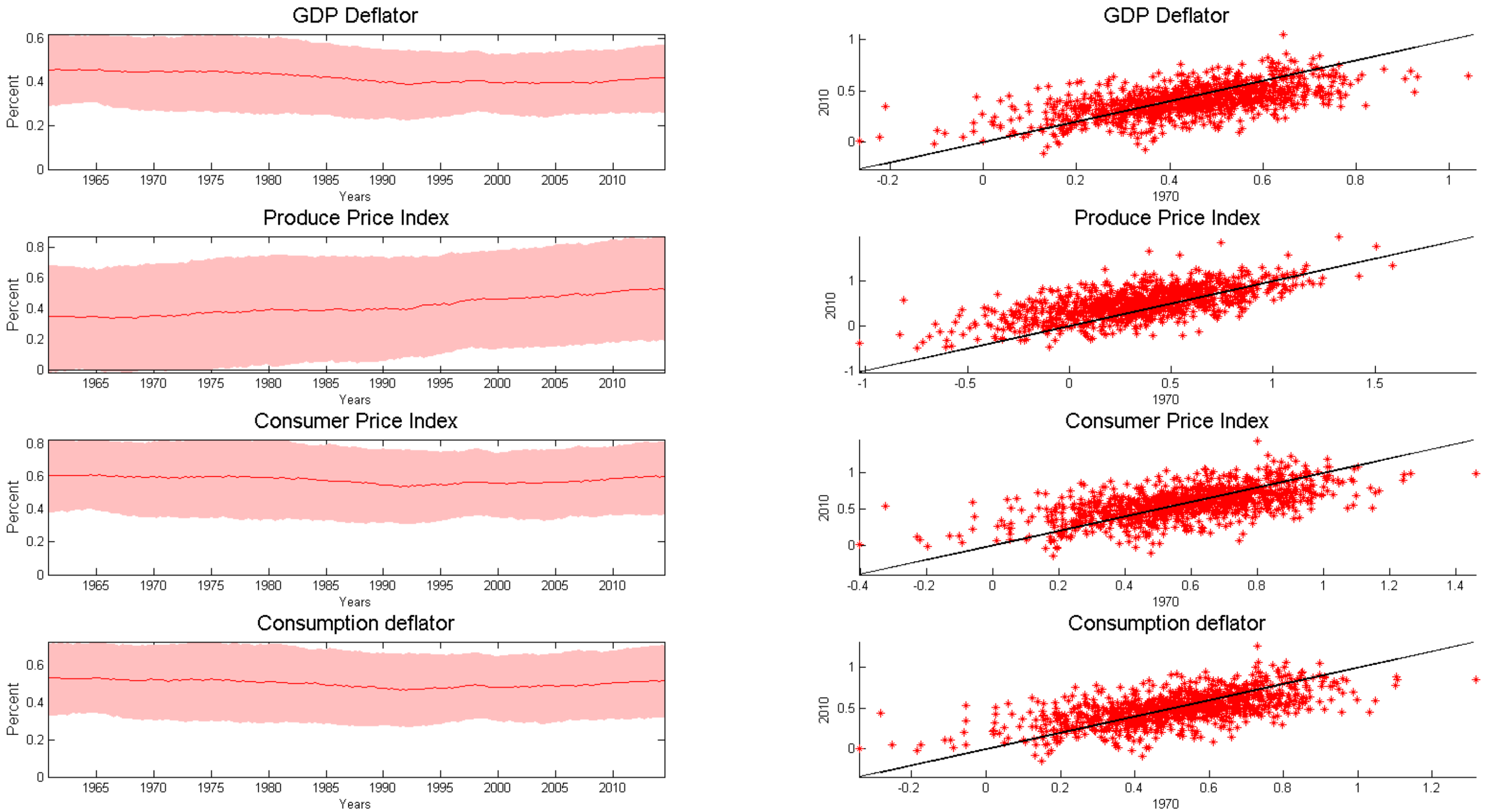

Figure 3: Impulse response of inflation series to a one standard deviation uncertainty shock. The left panels show the time profile of the cumulated response at the 2 year horizon. The right panels show the joint distribution of the cumulated response at the one year horizon in 1970 and 2010 along with the 45-degree line.

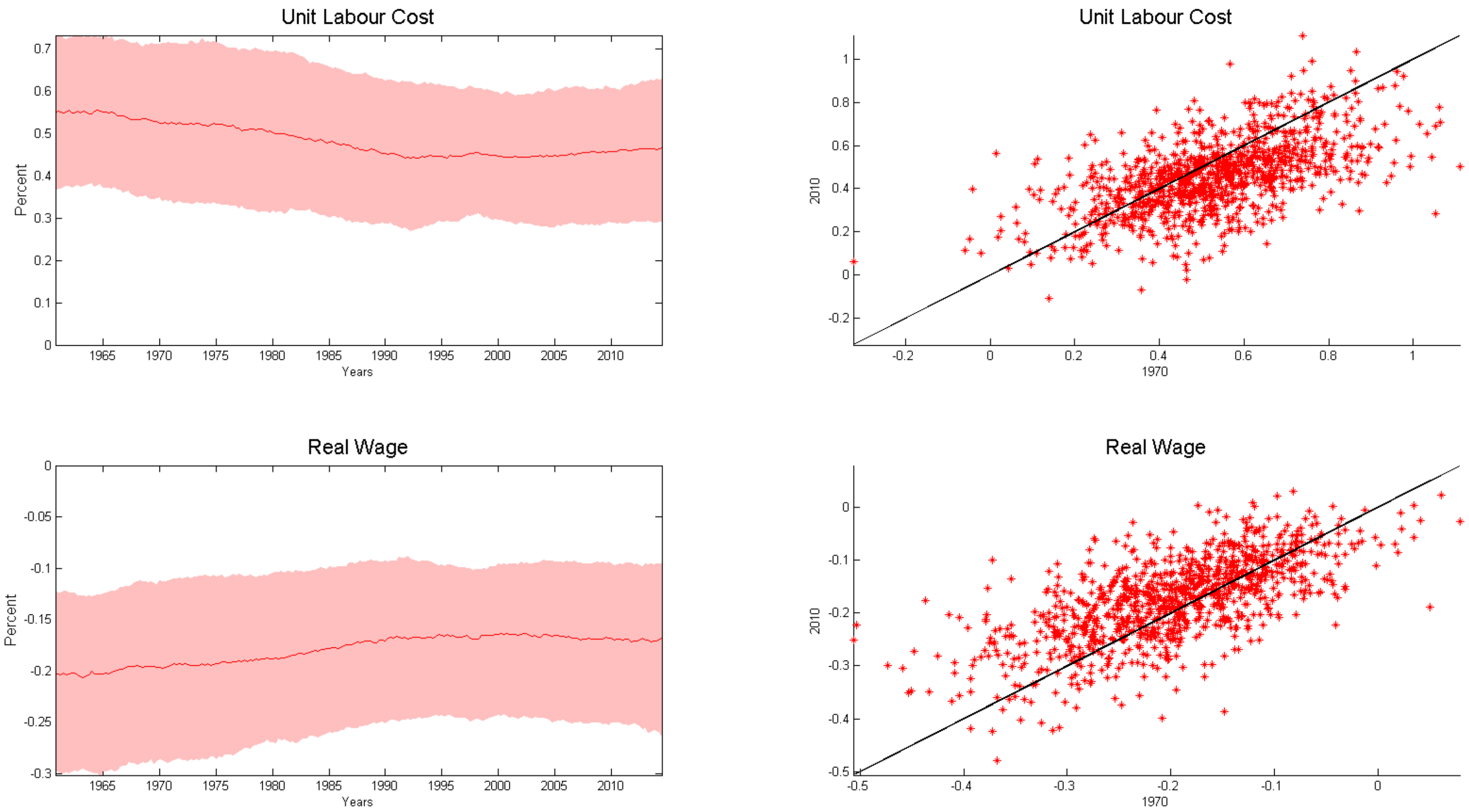

Figure 4: Impulse response of earnings series to a one standard deviation uncertainty shock. The left panels show the time profile of the cumulated response at the 2 year horizon. The right panels show the joint distribution of the cumulated response at the one year horizon in 1970 and 2010 along with the 45-degree line.

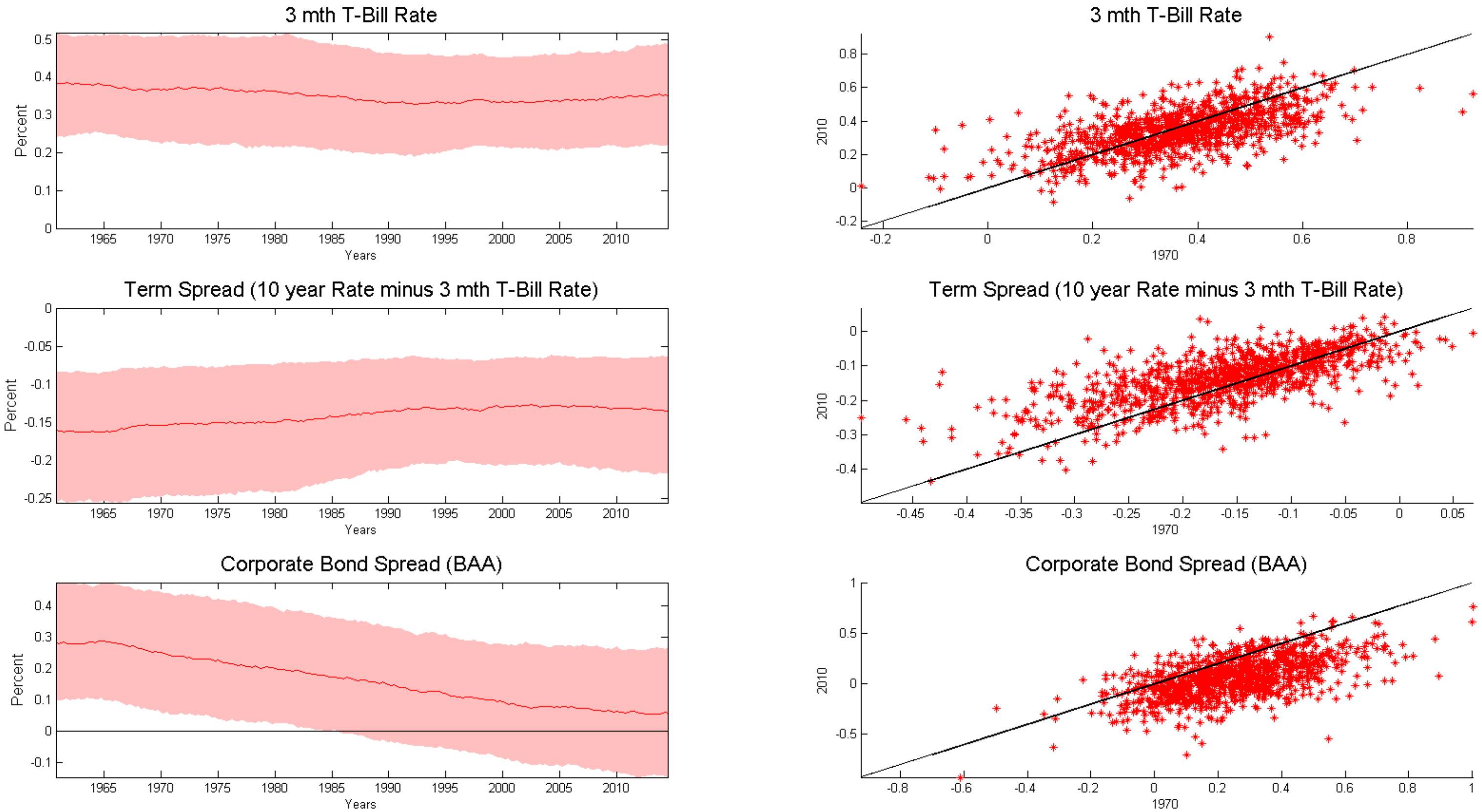

Figure 5: Impulse response of interest rate series to a one standard deviation uncertainty shock. The left panels show the time profile of the cumulated response at the 2 year horizon. The right panels show the joint distribution of the cumulated response at the one year horizon in 1970 and 2010 along with the 45-degree line.

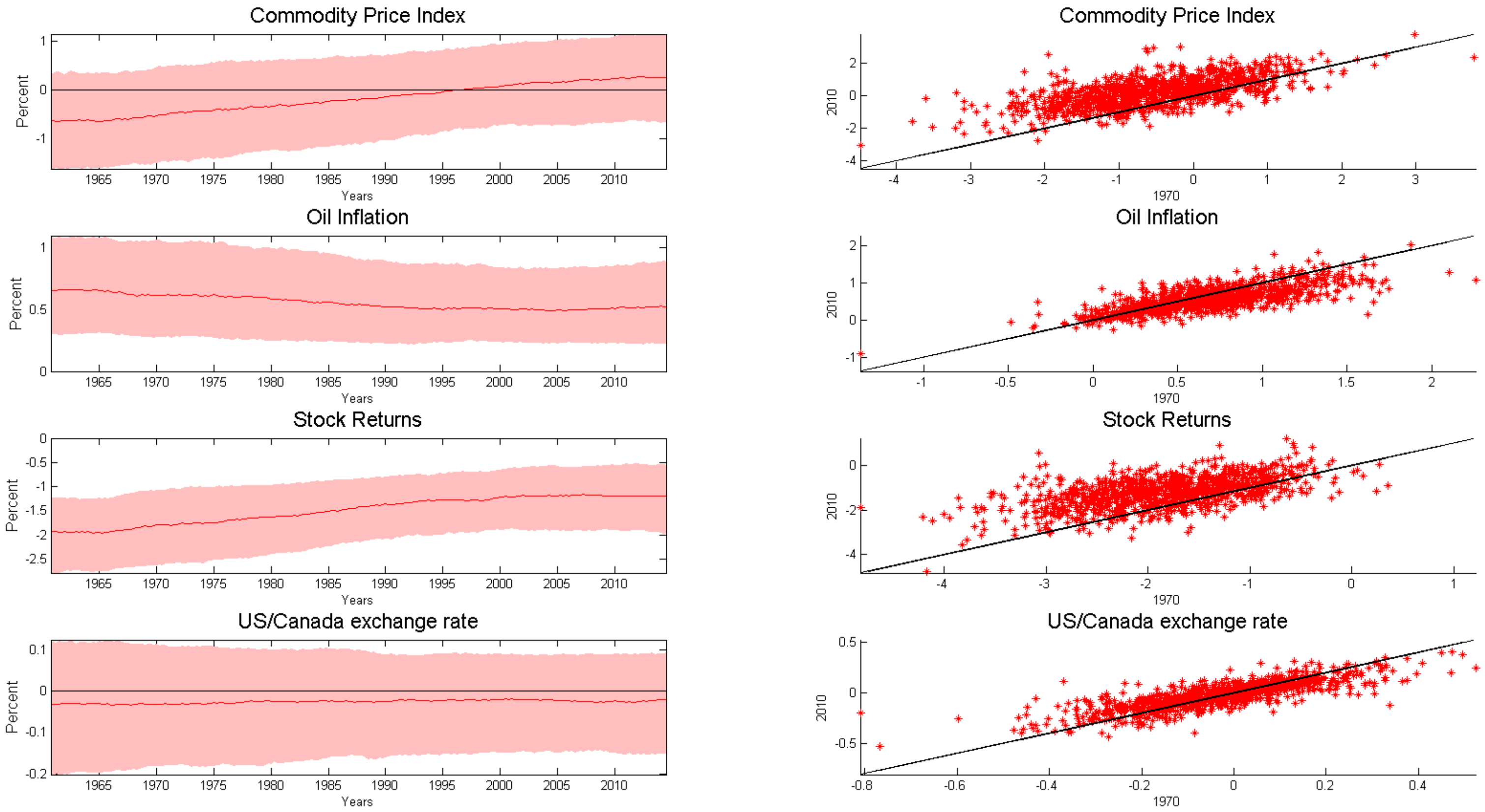

Figure 6: Impulse response of asset price series to a one standard deviation uncertainty shock. The left panels show the time profile of the cumulated response at the 2 year horizon. The right panels show the joint distribution of the cumulated response at the one year horizon in 1970 and 2010 along with the 45-degree line.

4.2 Impulse response to uncertainty shocks

Figure 2 shows the response of real activity variables to a 1 standard deviation positive shock to uncertainty. The left panels of the figure show the cumulated response at the two year horizon. The right panels display the joint distribution of this response in the early and later part of the sample and compare this with the 45-degree line. A systematic difference across time can be detected if the points on the scatter plot deviate from the 45-degree line.[3]

Figure 2 shows that the response of real activity to uncertainty has declined over time. Consider the response of GDP growth. GDP growth fell by 0.5%-0.6% in response to the uncertainty shock over the 1970s and the early 1980s. In contrast, after the mid-1990s, the decline in this variable in response to the uncertainty shock is closer to 0.3%. The right panel shows that that this decline in the magnitude of the response is systematic–the points in the joint distribution mostly lie above the 45-degree line suggesting that the response in 2010 was less negative than in the earlier period. Similarly, the response of consumption growth, investment and the unemployment rate shows a systematic decline after the mid-1980s.

Figure 3 shows the response of a number of inflation series to the uncertainty shock. Note that the response of inflation to the uncertainty shock is estimated to be positive and thus supports the existence of pricing bias channel postulated in Fernández-Villaverde *et al.* (2011). In contrast to the real activity responses, there is little evidence of any systematic decline in the magnitude of the response. The estimated joint distribution in 1970 and 2010 is clustered evenly around the 45-degree line. Similarly figure 4 shows that a similar conclusion holds for unit labour costs and earnings–the response of these variables is fairly stable across time.

Figure 5 plots the time-varying response of the short term interest rate and spreads. While the response of the short term rate and the the term spread is fairly constant over time, there appears to be a systematic decline in the response of the corporate bond spread to this shock with the cumulated response falling from about 0.3% in the early part of the sample to around 0.1% over the last decade. The bottom right panel provides some evidence that the change in this response is systematic.

The time-varying response of various asset prices to this shock is shown in figure 6. The response of stock returns at the 2 year horizon was about -2% during the 1970s and the 1980s. In contrast, stock returns declined by about 1% in response to the uncertainty shock after 2000. Similarly, the response of the commodity price index has declined over time. Note, however, that the error bands are wide for this response over the entire sample.

In summary, the estimates suggest that the response of real activity indicators (GDP growth, consumption growth, investment and unemployment) and some financial variables (corporate spread and stock returns) to uncertainty shocks has declined over time. In contrast, the the response of inflation and the short-term interest rate to this shock is estimated to have been fairly stable.

We show in section 1.6 of the technical appendix that these conclusions are robust to various changes in the specification of the empirical model. In particular, these results survive if the lag structure of the model is changed–we estimate versions of the model where (a) four lags of λ_t are allowed to affect the endogenous variables and (b) where the assumption that the volatility has a contemporaneous effect on the endogenous variables is relaxed. In both cases, we find that the response of real activity and financial variables declines while the response of inflation and the short-term rate is stable. Similarly, expanding the number of factors to 3 has little impact on these conclusions. Finally, we employ a tighter prior on the parameters governing the degree of time-variation in the coefficients (Q_B and Q_λ) and find that the conclusions reached above are largely unaffected.

The sensitivity analysis,therefore, supports the following main conclusion: There is evidence that the response of real activity and some financial indicators to the uncertainty shock has declined over time. In contrast, the response of inflation and interest rates to this shock has remained largely stable. We now turn to a DSGE model in order to explore the possible reasons behind the estimated temporal change in the impact of uncertainty shocks.

5 Explaining the results. A DSGE model

5.1 Summary of the model

The model used in this study is the one developed by Fernandez-Villaverde and Rubio-Ramirez (2008) (which in turn is a close relative to those developed by Christiano *et al.* (2005) and Smets and Wouters (2007)). Following Christiano *et al.* (2014), we augmented this model with Bernanke *et al.* (1999) type financial frictions. Briefly, the model features risk-averse consumers who supply labour to differentiated and sticky wage labour unions. There are risk-neutral entrepreneurs who borrow from perfectly competitive banks, build capital goods that they rent to the

[3] We focus on the two year horizon for simplicity. The full three-dimensional plots of the time-varying impulse responses can be found in the appendix to the paper.

imperfectly competitive (sticky price) producers of intermediate goods. Entrepreneurs, who are monitored by the banks, are subject to an idiosyncratic productivity shock. For an idiosyncratic shock below a threshold value, they declare bankruptcy and have everything taken from them. To prevent entrepreneurs from accumulating net-worth up to the point where the financial frictions become irrelevant, we assume that a fraction of them dies and the complementary fraction is born. There are perfectly competitive retailers selling the aggregated intermediate goods as a composite final good to the consumers. The final good is transformed to consumption and investment goods via linear technologies. However, the latter is subject to a non-stationary productivity shock. This stochastic trend is in addition to a labour augmented non-stationary shock. Government in this model runs a balanced budget in every period and the central bank sets monetary policy according to a Taylor-like rule. The model features a number of real (monopolistic competition, investment adjustment cost, capital utilisation and habits in consumption and asymmetric information between borrowers and lenders) and nominal (price and wage stickiness) frictions. In addition to the two non-stationary shocks, this economy is also subject to a preference, labour disutility and interest rate policy stationary shocks. These shocks are conditionally heteroscedastic and (consistently with the empirical model) subject to a common stochastic volatility shock. Note that we assume that this shock represents the uncertainty shock in the DSGE model.

The parametrization of the model is based on Fernandez-Villaverde and Rubio-Ramirez (2008) and Christiano *et al.* (2014). The model is solved using a third order perturbation and generalised impulse responses are computed. Details of model equations, the solution algorithm and impulse response calculation are provided in section 2 of the technical appendix.

The blue lines in figure 7 show the response to an uncertainty shock under the benchmark calibration of the model. We see in these simulations that as the uncertainty rises, agents respond by lowering (consumption and investment) demand and increase (precautionary) savings. Furthermore, agents expand their labour supply (see (Basu and Bundick, 2011)) pushing wages down and this offsets the increase in the rental rate of capital causing marginal cost to fall. Although, the marginal cost decreases, inflation rises because forward looking firms bias their pricing decision upwards in order to avoid supplying goods when demand and costs are high. Monetary authorities set policy according to a Taylor type rule and this constrains their ability to expand policy significantly and mitigate the adverse consequences from the uncertainty shock. This re-enforces agents' precautionary saving motives inducing a further reduction on demand. This in turn results in a fall in net worth and a rise in credit spreads.

5.2 DSGE interpretation of the empirical results

The empirical evidence suggests that the effect from an increase in the aggregate uncertainty on measures of real activity, asset prices and indicators of financial conditions has declined systematically over time. In contrast, the estimated response of inflation and short-term interest rates has been fairly constant. In this section we use the DSGE model discussed earlier to identify what 'constellation' of structural parameters that could be consistent with this pattern. We do this by considering a set of simulations under which key parameters in the model are changed and examine if the resulting shift in the response to uncertainty shocks matches the temporal pattern of impulse responses estimated using the FAVAR model. Note that this approach is similar in spirit to estimating model parameters by matching empirical and theoretical impulse responses, albeit less formal. We prefer this approach for two reasons. First, the non-linearity of the DSGE model implies that the computation of the (generalised) impulse responses is time-consuming and this hinders the use of numerical optimisation to minimise the distance between responses. Secondly, as argued forcefully in Canova and Sala (2009), weak or partial identification is a major issue in this approach and may lead to misleading results.

There are several existing DSGE studies that offer robust evidence regarding changes in the policy reaction function (see Lubik and Schorfheide (2004) and Davig and Leeper (2007) among others) and changes in the price and wage setting behavior of firms and households, respectively (see Fernandez-Villaverde and Rubio-Ramirez (2008) and Hofmann *et al.* (2012) among others). It seems, therefore, natural to initiate our investigation from those parameters and ask whether they could also explain changes in the pattern of uncertainty responses predicted by the empirical model. In addition we also consider the possibility that the economy has been subject to a process of financial liberalisation.

5.2.1 Hawkish Central Banker

We start by considering what happens when monetary policy authorities increase the weight placed on inflation (Figure 7). The dashed red line represents the scenario under which the policymaker's reaction coefficient to inflation (γ_π) in the monetary policy rule increases – γ_π rises from 1.01 to 1.5. The exercise suggests that the economic effects of an exogenous increase in uncertainty diminish as the policymaker fights inflation aggressively. When γ_π rises and authorities react strongly to inflation, future inflation is expected to be on target. This reduces firms' concerns about expected inflation and makes them less forward looking. In other words, the pricing bias decreases and the link between inflation and marginal cost is renewed. In this case authorities are able to cut the policy rate by more and for a longer period, which helps them to address the adverse effects from elevated uncertainty. The resulting amelioration in the fall in investment improves the entrepreneurs' leverage position and the increase in the credit spread is smaller.

The changes in the impulse responses predicted by the rise in γ_π go in the direction of the empirical results– the fall in the magnitude of the real activity and credit spread response is consistent with the estimates from the FAVAR. Note, however, that unlike the empirical estimates, the model simulations also predicts a decline in the response of inflation and the policy rate at all horizons.

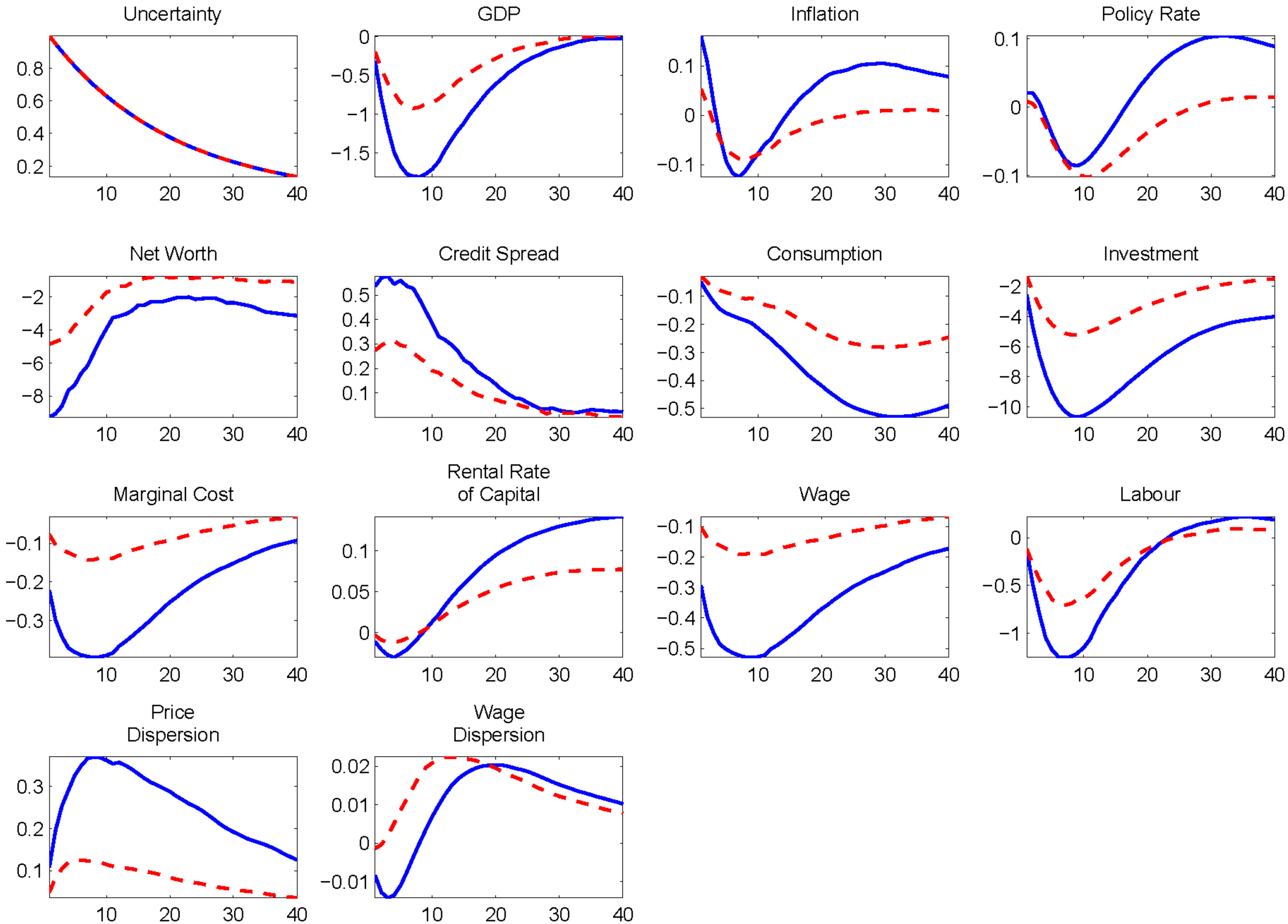

Figure 7: The solid (blue) line has been produced by using benchmark calibration. The dashed (red) line illustrates what happens when γ_π increases from 1.01 to 1.5.

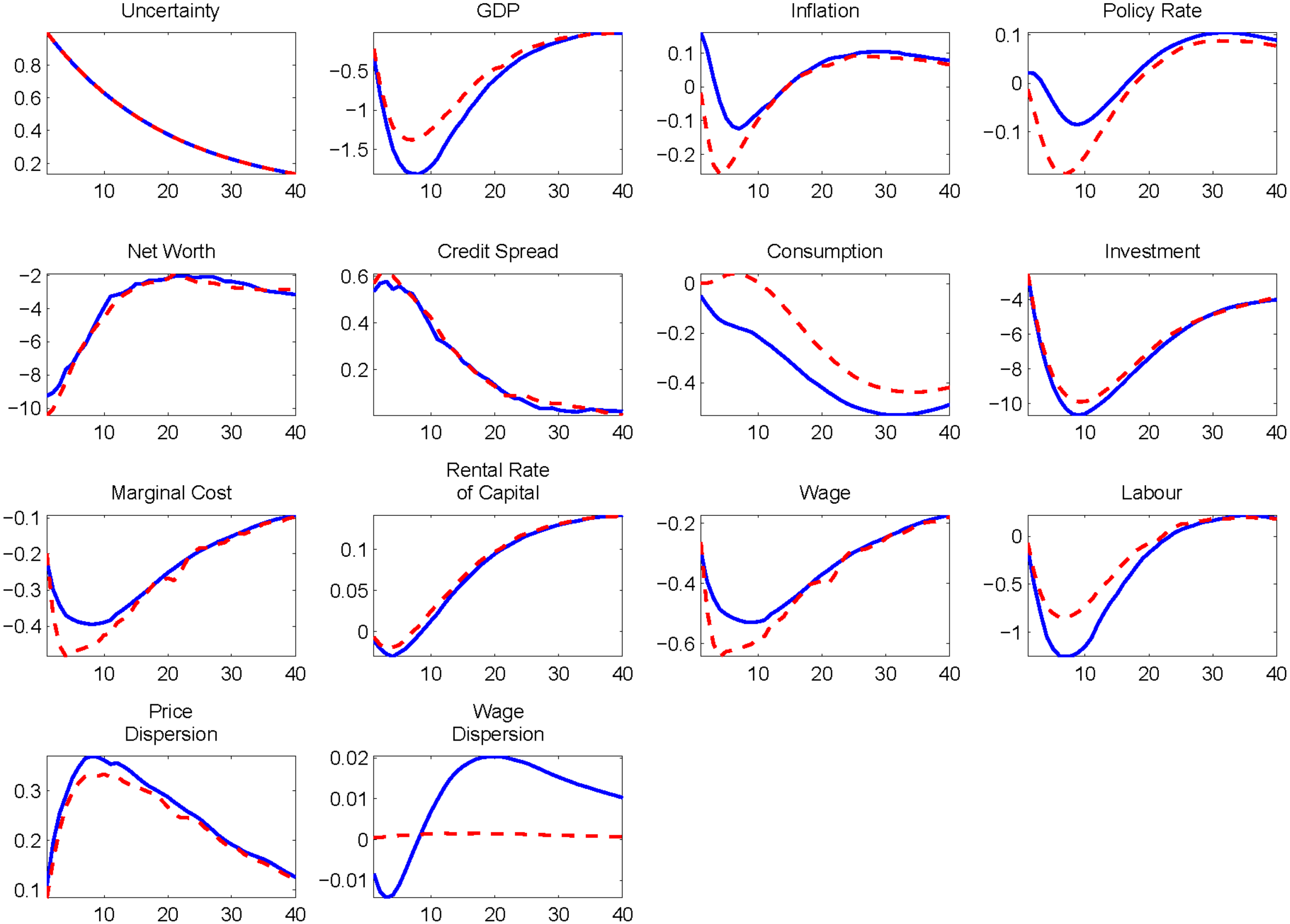

Figure 8: The solid (blue) line has been produced by using benchmark calibration. The dashed (red) line illustrates what happens when θ_w and χ_w decrease from 0.700 and 0.800 to 0.05 and 0, respectively.

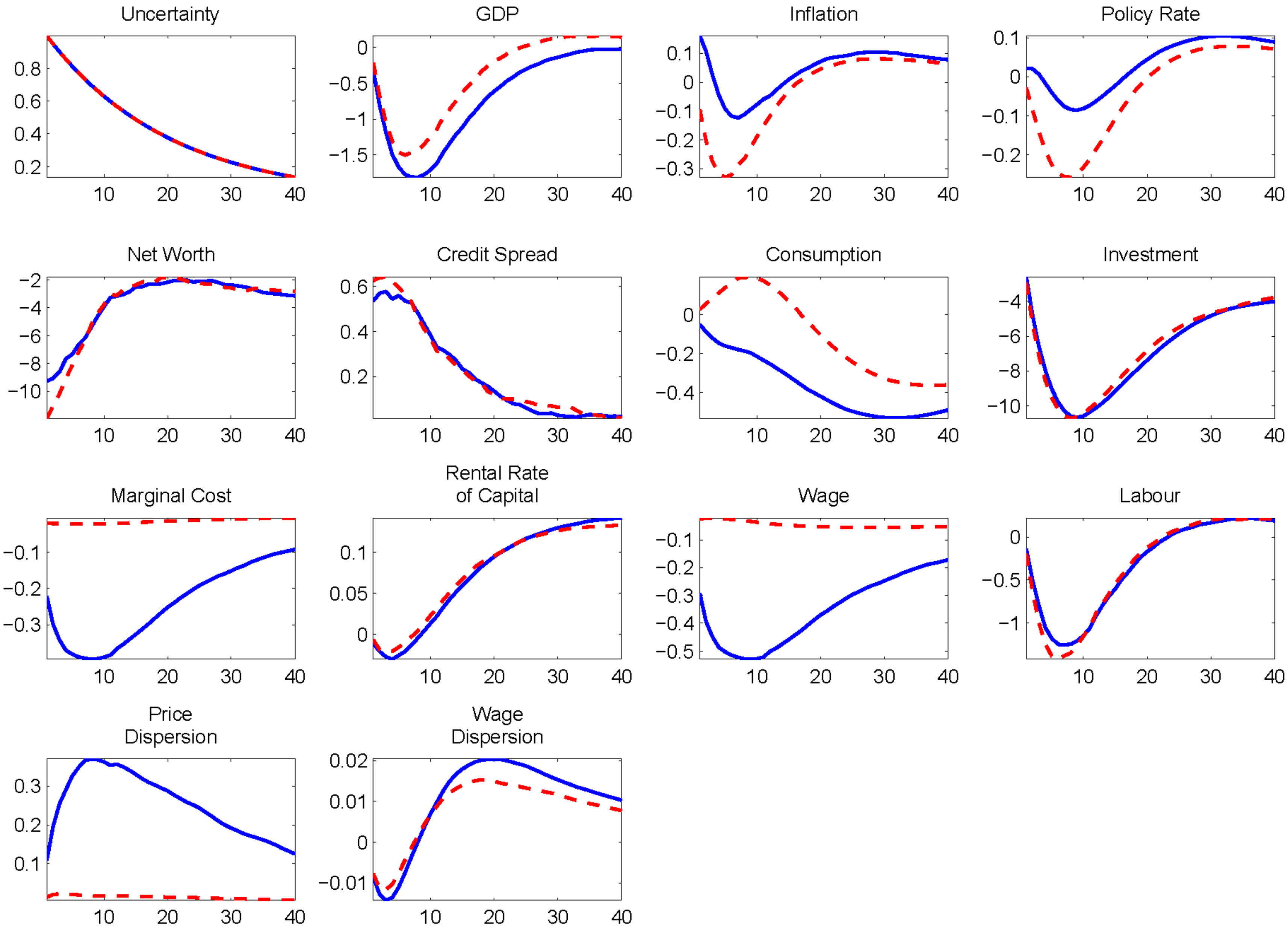

Figure 9: The solid (blue) line has been produced by using benchmark calibration.The dashed (red) line illustrates what happens when θ_p and χ decrease from 0.550 and 0.400 to 0.05 and 0, respectively.

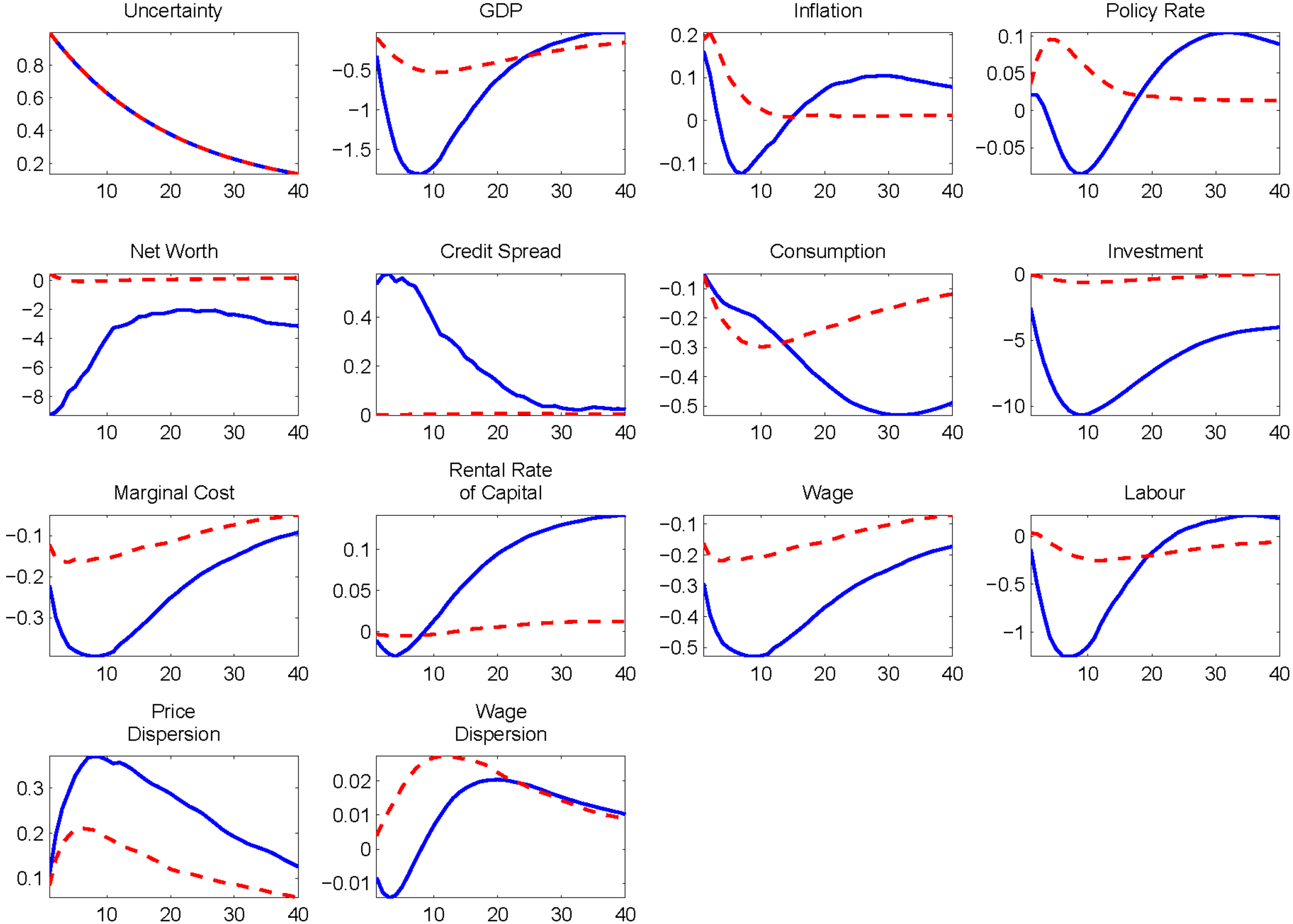

Figure 10: The solid (blue) line has been produced by using benchmark calibration.The dashed (red) line illustrates what happens when μ_E and the annual credit steady state spread decrease from 0.210 and 300bps to 0.05 and 50bps, respectively.

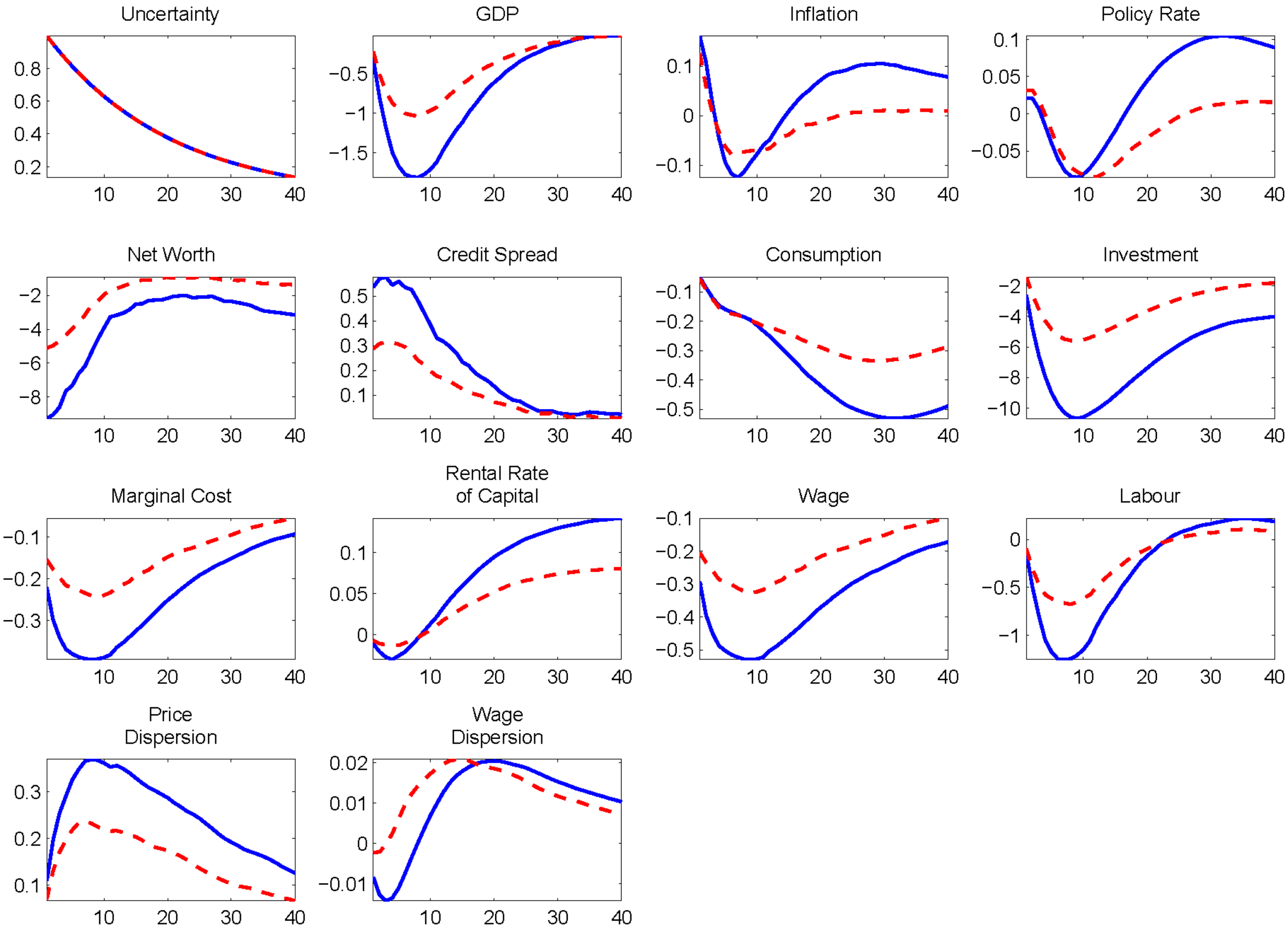

Figure 11: The solid (blue) line has been produced by using benchmark calibration. The dashed (red) line illustrates what happens when γ_π increases from 1.010 to 1.500, $\bar{\pi}$ decreases from 1.010 to 1.005, χ reduces from 0.400 to 0.100 and θ_p increases from 0.550 to 0.650.

5.2.2 Flexible Wages or prices

Next we study the effects on the economy after an uncertainty shock when nominal wages are not subject to frictions (Figure 8). This simulation assumes that the Calvo probability of re-setting wages θ_w and the wage indexation parameter χ_w decrease from 0.700 and 0.800 to 0.05 and 0, respectively. Relative to the benchmark case we see that flexible wages do not lead to large changes in the impact of uncertainty on aggregate demand. Agents in the flexible wage case respond by supplying more labour causing wages to fall by more. This turns out to have an impact on the marginal cost and inflation as they fall in the first two years by more in the flexible wage case. However, the pricing bias channel is still present–inflation does not fall as much as the marginal cost and after two years inflation response closely tracks the benchmark inflation profile. This implies positive policy rates for a large period that offset the majority of the short lasting stimulus (via negative short term rates) and explain why demand contracts by a very similar amount under both scenarios.

Following Christiano *et al.* (2010) we investigate the economic effects after an uncertainty shock when prices are able to adjust freely (Figure 9)– the Calvo probability θ_p and indexation χ decrease from 0.550 and 0.400 to 0.05 and 0, respectively. Although the effects on (aggregate) demand and prices appear to be very similar to the flexible wages case, the transmission mechanism is very different. With flexible prices, the prices Philips curve drops from the system and real wages equal zero for all t. Thus price inflation becomes a function of the nominal wage inflation and not a function of the marginal cost. Given that real wages equal zero for all t, nominal wage inflation is just a function of the real marginal cost of work (namely consumption and labour demand) which is falling.

The change in real activity response implied by these simulations is consistent with what we find using the FAVAR model. Note, however, that these experiments suggest that an increase in wage or price flexibility has little impact on the magnitude of the credit spread response and leads to a change in the inflation response. The time-varying FAVAR responses offer little evidence to support these changes.

5.2.3 Financial liberalisation

In this simulation we assume that the asymmetry between borrower and lender has been reduced (Figure 10) – the entrepreneur auditing cost μ_E and the annual credit steady state spread decreases from 0.210 and 300*bps* to 0.05 and 50*bps*, respectively. After this change in these parameters, an uncertainty shock has almost no effect on credit spreads and net-worth. This is due to the low value of entrepreneur auditing cost ($\mu_E = 0$ implies no asymmetry between borrowers and lenders). As a consequence, investment does not collapse in this case as agents face a tiny external finance premium. Note also that the dynamics of inflation and the policy rate are quite different under financial liberalisation and this feature does not match the FAVAR results.

5.3 Discussion

The simulations presented above indicate that the estimated changes in the response of real activity and credit spreads to uncertainty shocks can be consistent with a shift in Taylor rule parameters, wage/price rigidity and easing of financial frictions. In contrast, it seems harder to replicate closely the empirical result that the response of inflation and the short-term rate has been stable over time. Comparing figures 7 to 10, it appears that one can get closest to the empirical results by increasing γ_π from 1.010 to 1.500. While, the increase in the Federal Reserve's anti-inflationary stance after the mid-1980s has been documented and supported by several studies (see Lubik and Schorfheide (2004)), it seems reasonable to suppose that the US economy has been subject to other structural changes at the same time. For example, using a time-varying DSGE model, Fernandez-Villaverde and Rubio-Ramirez (2008) provide evidence for a decrease in the inflation target and the 'flattening' of the Phillips curve on top of an increase in the Taylor rule inflation coefficient.

In figure 11 we consider changes in a number of parameters that match the findings of Fernandez-Villaverde and Rubio-Ramirez (2008). In particular, this figure compares the benchmark impulse responses from those obtained under the scenario where monetary authorities fight inflation more aggressively (γ_π increases from 1.010 to 1.500), the steady-state inflation is reduced ($\bar{\pi}$ decreases from 4% to 2%), firm rely less on indexation rules of thumb (χ reduces from 0.400 to 0.100) and reset prices less frequently due to price stability (θ_p increases from 0.550 to 0.650). Figure 11 shows that the changes in the impulse responses are close to the empirical results. Under the alternative scenario, the response of real activity and spreads to the uncertainty shocks is weaker. However, the response of inflation and the short-term interest rate is fairly similar, especially at short and medium term horizons. The increase in price stickiness and a decrease in indexation has an upward effect on the pricing bias which counteracts the decrease in this channel induced by the rise in γ_π. With inflation closer to a lower target, firms find it optimal not to rest prices very often. However, this reduction in price re-setting leads them to account more for the risk of being locked in a contractual agreement to supply goods at a price lower than the aggregate price.

In contrast, a combination of financial liberalisation and changes in the Phillips curve parameters outlined above would lead to large changes in the inflation and the interest rate response. Notice from figure 10 that as financial frictions ease, the response of inflation to uncertainty shock is larger in magnitude. This channel is further magnified with a fall in indexation and an increase in price stickiness.

Therefore, a change in Taylor rule *and* Phillips curve parameters provides a candidate explanation for the temporal shift in the responses estimated using the FAVAR model. As noted above, the fact that the change in these model parameters has been reported by other studies provides an argument for believing this explanation to be a plausible one.

6 Conclusions

This paper considers whether the impact of uncertainty shocks on the US economy has changed over time. Using an extended FAVAR model that allows the estimation of the time-varying impact of uncertainty shocks we find that the response of real activity series such as GDP growth and financial series such as the BAA credit spread to this shock has declined over time. In contrast, the estimated response of inflation and short-term interest rates has remained fairly constant over time. We use a non-linear DSGE model with stochastic volatility to gauge the possible factors behind these changes. The DSGE simulations suggest that the empirical results can be closely replicated when we incorporate an increase in the monetary authorities anti-inflation stance and simultaneously allow the degree of price stickiness to rise and indexation to fall. This highlights the importance of monetary policy and inflation dynamics in determining the role played by uncertainty and the importance of this shock for economic fluctuations.

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The changing transmission of uncertainty shocks in the US: an empirical analysis (Technical Appendix)*

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Abstract

Technical Appendix

1 Estimation of the FAVAR model

The TVP VAR model is defined as

$$Z_t = c_t + \sum_{j=1}^{P} \beta_{tj} Z_{t-j} + \sum_{j=0}^{J} \gamma_{tj} \ln \lambda_{t-j} + \Omega_t^{1/2} e_t \tag{1}$$

where Z_t is a matrix of endogenous variables describe below.

$$\begin{aligned} B &= vec([c;\beta;\lambda]) \\ B_t &= B_{t-1} + \eta_t, VAR(\eta_t) = Q_B \end{aligned} \tag{2}$$

$$\Omega_t = A_t^{-1} H_t A_t^{-1\prime}$$

where A_t is lower triangular. Each non-zero element of A_t evolves as a random walk

$$a_t = a_{t-1} + g_t, VAR(g_t) = G \tag{3}$$

where G is block diagonal as in Primiceri (2005).

Following Carriero *et al.* (2012) the volatility process is defined as

$$\begin{aligned} H_t &= \lambda_t S \\ S &= diag(s_1, .., s_N) \end{aligned} \tag{4}$$

The overall volatility evolves as an AR(1) process

$$\ln \lambda_t = \alpha + F \ln \lambda_{t-1} + \bar{\eta}_t, VAR(\bar{\eta}_t) = Q_\lambda \tag{5}$$

1.1 Priors and Starting values

1.1.1 Factors

We use a principal component estimator to calculate an initial value for the factors $\tilde{Z}_t$. The initial conditions for the Kalman filter employed in the Carter and Kohn (2004) step are set as $Z_0 \tilde{} N\left(\tilde{Z}_1, I\right)$.

*The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.

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1.1.2 Factor Loadings and error variances

The initial conditions for the factor loadings are obtained via an OLS estimate of the factor loadings using the first 40 observations of the sample period and employing the principal components $\tilde{Z}_t$. Let Λ^{ols} denote the OLS estimate of the factor loadings estimated using the pre-sample data described above. The prior is set as $\Lambda_0 \tilde{} N(\Lambda^{ols}, var(\Lambda^{ols}))$. The prior on Q_Λ is assumed to be inverse Wishart $Q_{\Lambda,0} \sim IW\left(\bar{Q}_{\Lambda,0}, T_0\right)$ where $\bar{Q}_{\Lambda,0}$ is assumed to be $T_0 \times var(\Lambda^{ols}) \times 10^{-4} \times 3.5$ and T_0 is the length of the sample used to for calibration. This follows Cogley and Sargent (2005).

The prior for the diagonal elements of R is assumed to be $IG\left(R_0, V_R\right)$ where the scale parameter $R_0 = 0.001$ and $V_R = 5$.

1.1.3 VAR Coefficients

The initial conditions for the VAR coefficients B_0 are obtained via an OLS estimate of a fixed coefficient VAR using the first 40 observations of the sample period. The VAR is estimated using the principal components $\tilde{Z}_t$. Let $\hat{B}^{ols}$and $\hat{v}^{ols}$ denote the OLS estimate of the VAR coefficients and the covariance matrix estimated on the pre-sample data described above. The prior for $B_0 \tilde{} N(\hat{B}^{ols}, var(\hat{B}^{ols}))$. The prior on Q_B is assumed to be inverse Wishart $Q_{B,0} \sim IW\left(\bar{Q}_{B,0}, T_0\right)$ where $\bar{Q}_{B,0}$ is assumed to be $T_0 \times var(\hat{B}^{ols}) \times 10^{-4} \times 3.5$ and T_0 is the length of the sample used to for calibration.

1.1.4 Elements of the A matrix

The prior for the off-diagonal elements A_t is $A_0 \sim N\left(\hat{a}^{ols}, V\left(\hat{a}^{ols}\right)\right)$ where $\hat{a}^{ols}$ are the off-diagonal elements of $\hat{v}^{ols}$, with each row scaled by the corresponding element on the diagonal. $V\left(\hat{a}^{ols}\right)$ is assumed to be diagonal with the elements set equal to 10 times the absolute value of the corresponding element of $\hat{a}^{ols}$. The prior distribution for the blocks of G is inverse Wishart: $G_{i,0} \sim IW(\bar{G}_i, K_i)$ where $i = 1..N-1$ indexes the blocks of S. $\bar{G}_i$ is calibrated using $\hat{a}^{ols}$. Specifically, $\bar{G}_i$ is a diagonal matrix with the relevant elements of $\hat{a}^{ols}$ multiplied by 10^{-3}.

1.1.5 Elements of S and the parameters of the transition equation

The elements of S have an inverse Gamma prior: $P(s_i) \tilde{} IG(S_{0,i}, V_0)$. The degrees of freedom V_0 are set equal to 1. The prior scale parameters are set by estimating the following regression: $\bar{\lambda}_{it} = S_{0,i}\bar{\lambda}_t + \varepsilon_t$ where $\bar{\lambda}_t$ is the first principal component of the stochastic volatilities $\bar{\lambda}_{it}$ obtained using a univariate stochastic volatility model for the residuals of each equation of a VAR estimated via OLS using the endogenous variables $\tilde{Z}_t$.

We set a normal prior for the unconditional mean $\mu = \frac{\alpha}{1-F}$. This prior is $N(\mu_0, Z_0)$ where $\mu_0 = 0$ and $Z_0 = 10$.The prior for Q_λ is $IG\left(Q_0, V_{Q0}\right)$ where Q_0 is the average of the variances of the transition equations of the initial univariate stochastic volatility estimates and $V_{Q0} = 5$. The prior for F is $N\left(F_0, L_0\right)$ where $F_0 = 0.8$ and $L_0 = 1$.

1.1.6 Common Volatility λ_t

The prior for the initial value of λ_t is defined as $\ln \lambda_0 \sim N(\ln \mu_0, I)$ where μ_0 is the initial value of $\bar{\lambda}_t$.

1.2 MCMC algorithm

The MCMC algorithm is based on drawing from the following conditional posterior distributions (Ξ denotes all other parameters):

1. $G(\Lambda_t \backslash \Xi)$. Given a draw for the factors the variances R and the variance of the shock to the transition equation Q_Λ, the following TVP regression applies for the ith X_{it} :

$$\begin{aligned} X_{it} &= \Lambda_{it} Z_t + R_i^{1/2} \varepsilon_{it} \\ \Lambda_{it} &= \Lambda_{it-1} + \bar{\eta}_{it}, VAR\left(\bar{\eta}_{it}\right) = Q_{\Lambda,i} \end{aligned}$$

As this is a linear and Gaussian state space model, the Carter and Kohn (2004) algorithm can be applied to draw from the conditional posterior of Λ_{it}. The distribution of the time-varying loadings conditional on all other parameters is linear and Gaussian: $\Lambda_{it} \backslash X_{it}, \Xi \sim N\left(\Lambda_{T\backslash T}, \bar{P}_{T\backslash T}\right)$ and $\Lambda_t \backslash \Lambda_{t+1}, X_{it}, \Xi \sim N\left(\Lambda_{t\backslash t+1,\Lambda_{t+1}}, \bar{P}_{t\backslash t+1,\Lambda_{t+1}}\right)$ where $t = T-1, ..1$, Ξ denotes a vector that holds all the other VAR parameters.

As shown by Carter and Kohn (2004) the simulation proceeds as follows. First we use the Kalman filter to draw $\Lambda_{T\backslash T}$ and $\bar{P}_{T\backslash T}$ and then proceed backwards in time using $\Lambda_{t|t+1,\Lambda_{t+1}} = \Lambda_{t|t} + \bar{P}_{t|t}\bar{P}^{-1}_{t+1|t}\left(\Lambda_{t+1} - \Lambda_{t|t}\right)$ and $\bar{P}_{t|t+1,\Lambda_{t+1}} = \bar{P}_{t|t} - \bar{P}_{t|t}\bar{P}^{-1}_{t+1|t}\bar{P}_{t|t}$. Note that in order to deal rotational indeterminancy of the factors and factor loadings, we fix the first K factor loadings where K is the number of factors. In particular, the first $K \times K$ block of Λ_{it} is equal to an identity matrix for all time periods (see Bernanke *et al.* (2005)).

2. $G(Q_{\Lambda,i}\backslash\Xi)$. Given a draw for Λ_{it}, the conditional posterior for $Q_{\Lambda,i}$ is inverse Wishart with scale matrix and degrees of freedom defined as: $IW\left(\bar{\eta}'_{it}\bar{\eta}_{it} + \bar{Q}_{\Lambda,0}, T + T_0\right)$.

3. $G(R\backslash\Xi)$. The diagonal elements of R have an inverse Gamma conditional posterior:

$$G(R_i\backslash\Xi)\tilde{\ } IG\left(\varepsilon'_{it}\varepsilon_{it} + R_0, T + V_R\right)$$

4. $G\left(Z\backslash\Xi\right)$. Given the parameters of the observation equation (Λ_t, R) and the transition equation (B_t, Ω_t), equations **??** and 1 constitute a linear Gaussian state space model and the Carter and Kohn (2004) algorithm can be employed to draw from the conditional posterior distribution of the factors. Carter and Kohn (2004) show that the conditional posterior is defined as $Z_T\backslash X_{it}, \Xi \sim N\left(Z_{T\backslash T}, \tilde{P}_{T\backslash T}\right)$ and $Z_t\backslash Z_{t+1}, X_{it}, \Xi \sim N\left(Z_{t\backslash t+1,Z_{t+1}}, \tilde{P}_{t\backslash t+1,Z_{t+1}}\right)$ where $t = T-1, ..1$, Ξ denotes a vector that holds all the other VAR parameters. A run of the Kalman filter delivers $Z_{T\backslash T}$ and $\tilde{P}_{T\backslash T}$ as the filtered states and its variance at the end of the sample. Then one proceeds backwards in time to obtain $Z_{t\backslash t+1,Z_{t+1}} = Z_{t|t} + \tilde{P}_{t|t}\tilde{F}'_t\tilde{P}^{-1}_{t+1|t}\left(Z_{t+1} - \tilde{\mu}_t - \tilde{F}Z_{t|t}\right)$ and $\tilde{P}_{t|t+1,Z_{t+1}} = \tilde{P}_{t|t} - \tilde{P}_{t|t}\tilde{F}'_tP^{-1}_{t+1|t}\tilde{F}_t\tilde{P}_{t|t}$. Note that $\tilde{F}_t$ and $\tilde{\mu}_t$ denote the coefficients on the lags and the coefficients on pre-determined variables in the transition equation 1 respectively in companion form.

5. $G(B_t\backslash\Xi)$. Given a draw for the factors and variances Ω_t, Q_B, 1 and 2 constitute a VAR with time-varying parameters and the Carter and Kohn (2004) algorithm can again be applied to draw from the conditional posterior of the VAR coefficients. The distribution of the time-varying VAR coefficients B_t conditional on all other parameters is linear and Gaussian: $B_t\backslash Z_t, \Xi \sim N\left(B_{T\backslash T}, P_{T\backslash T}\right)$ and $B_t\backslash B_{t+1}, Z_t, \Xi \sim N\left(B_{t\backslash t+1,B_{t+1}}, P_{t\backslash t+1,B_{t+1}}\right)$ where $t = T-1, ..1$, Ξ denotes a vector that holds all the other VAR parameters. As shown by Carter and Kohn (2004) the simulation proceeds as follows. First we use the Kalman filter to draw $B_{T\backslash T}$ and $P_{T\backslash T}$ and then proceed backwards in time using $B_{t|t+1,B_{t+1}} = B_{t|t} + P_{t|t}P^{-1}_{t+1|t}\left(B_{t+1} - B_{t|t}\right)$ and $P_{t|t+1,B_{t+1}} = P_{t|t} - P_{t|t}P^{-1}_{t+1|t}P_{t|t}$. Rejection sampling is used to ensure that the draws satisfy stability at each point in time.

6. $G(Q_B\backslash\Xi)$. The draw for Q_B is standard with conditional distribution $IW\left(\eta'_t\eta_t + \bar{Q}_{B,0}, T + T_0\right)$.

7. $G(A_t\backslash\Xi)$. Given a draw for the VAR parameters and the model can be written as $A'_t\left(v_t\right) = e_t$ where v_t denotes the VAR residuals. This is a system of linear equations with time-varying coefficients and a known form of heteroscedasticity. The jth equation of this system is given as $v_{jt} = -a_{jt}v_{-jt} + e_{jt}$ where the subscript j denotes the jth column of v while $-j$ denotes columns 1 to $j-1$. Note that the variance of e_{jt} is time-varying and given by $\lambda_t s_j$. The time-varying coefficient follows the process $a_{jt} = a_{jt-1} + g_{jt}$ with the shocks to the jth equation g_{jt} uncorrelated with those from other equations. In other words the covariance matrix $var\left(g\right)$ is assumed to be block diagonal as in Primiceri (2005). With this assumption in place, the Carter and Kohn (2004) algorithm can be applied to draw the time varying coefficients for each equation of this system seperately.

8. $G(S\backslash\Xi)$. Given a draw for the VAR parameters the model in can be written as $A'\left(v_t\right) = e_t$. The jth equation of this system is given by $v_{jt} = -a_{jt}v_{-jt} + e_{jt}$ where the variance of e_{jt} is time-varying and given by $\lambda_t s_j$. Given a draw for λ_t this equation can be re-written as $\bar{v}_{jt} = -a_{jt}\bar{v}_{-jt} + \bar{e}_{jt}$ where $\bar{v}_{jt} = \frac{v_{jt}}{\lambda_t^{1/2}}$ and the variance of $\bar{e}_{jt}$ is s_j. The conditional posterior is for this variance is inverse Gamma with scale parameter $\bar{e}'_{jt}\bar{e}_{jt} + S_{0,j}$ and degrees of freedom $V_0 + T$.

9. $G(\lambda_t\backslash\Xi)$.Conditional on the VAR parameters, and the parameters of the transition equation, the model has a multivariate non-linear state-space representation. Carlin *et al.* (1992) show that the conditional distribution of the state variables in a general state-space model can be written as the product of three terms:

$$\tilde{h}_t\backslash Z_t, \Xi \propto f\left(\tilde{h}_t\backslash\tilde{h}_{t-1}\right) \times f\left(\tilde{h}_{t+1}\backslash\tilde{h}_t\right) \times f\left(Z_t\backslash\tilde{h}_t, \Xi\right) \qquad (6)$$

where Ξ denotes all other parameters and $\tilde{h}_t = \ln \lambda_t$. In the context of stochastic volatility models, Jacquier *et al.* (1994) show that this density is a product of log normal densities for λ_t and λ_{t+1} and a normal density for Z_t.Carlin *et al.* (1992) derive the general form of the mean and variance of the underlying normal density for $f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi\right) \propto f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}\right) \times f\left(\tilde{h}_{t+1} \backslash \tilde{h}_t\right)$ and show that this is given as

$$f\left(\tilde{h}_t \backslash \tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi\right) \sim N\left(B_{2t} b_{2t}, B_{2t}\right) \tag{7}$$

where $B_{2t}^{-1} = Q_\lambda^{-1} + F' Q_\lambda^{-1} F$ and $b_{2t} = \tilde{h}_{t-1} F' Q_\lambda^{-1} + \tilde{h}_{t+1} Q_\lambda^{-1} F$. Note that due to the non-linearity of the observation equation of the model an analytical expression for the complete conditional $\tilde{h}_t \backslash Z_t, \Xi$ is unavailable and a metropolis step is required. Following Jacquier *et al.* (1994) we draw from 6 using a date-by-date independence metropolis step using the density in 7 as the candidate generating density. This choice implies that the acceptance probability is given by the ratio of the conditional likelihood $f\left(Z_t \backslash \tilde{h}_t, \Xi\right)$ at the old and the new draw. To implement the algorithm we begin with an initial estimate of $\tilde{h} = \ln \bar{\lambda}_t$ We set the matrix $\tilde{h}^{old}$ equal to the initial volatility estimate. Then at each date the following two steps are implemented:

(a) Draw a candidate for the volatility $\tilde{h}_t^{new}$ using the density 6 where $b_{2t} = \tilde{h}_{t-1}^{new} F' Q_\lambda^{-1} + \tilde{h}_{t+1}^{old} Q_\lambda^{-1} F$ and $B_{2t}^{-1} = Q_\lambda^{-1} + F' Q_\lambda^{-1} F$

(b) Update $\tilde{h}_t^{old} = \tilde{h}_t^{new}$ with acceptance probability $\frac{f\left(Z_t \backslash \tilde{h}_t^{new}, \Xi\right)}{f\left(Z_t \backslash \tilde{h}_t^{old}, \Xi\right)}$ where $f\left(Z_t \backslash \tilde{h}_t, \Xi\right)$ is the likelihood of the VAR for observation t and defined as $|\Omega_t|^{-0.5} - 0.5 \exp\left(\tilde{e}_t \Omega_t^{-1} \tilde{e}_t'\right)$ where $\tilde{e}_t = Z_t - \left(c_t + \sum_{j=1}^{P} \beta_{tj} Z_{t-j} + \sum_{j=0}^{J} \gamma_{tj} \ln \lambda_{t-j}\right.$ and $\Omega_t = A_t^{-1}\left(\exp(\tilde{h}_t) S\right) A_t^{-1'}$

Repeating these steps for the entire time series delivers a draw of the stochastic volatilties.[1]

7. $G(\alpha, F \backslash \Xi)$.We re-write the transition equation in deviations from the mean

$$\tilde{h}_t - \mu = F\left(\tilde{h}_{t-1} - \mu\right) + \bar{\eta}_t \tag{8}$$

where the elements of the mean vector μ_i are defined as $\frac{\alpha_i}{1-F_i}$. Conditional on a draw for $\tilde{h}_t$ and μ the transition equation 8 is a simply a linear regression and the standard normal and inverse Gamma conditional posteriors apply. Consider $\tilde{h}_t^* = F \tilde{h}_{t-1}^* + \bar{\eta}_t, VAR\left(\bar{\eta}_t\right) = Q_\lambda$and $\tilde{h}_t^* = \tilde{h}_t - \mu, \tilde{h}_{t-1}^* = \tilde{h}_{t-1} - \mu$. The conditional posterior of F is $N\left(\theta^*, L^*\right)$ where

$$\begin{aligned}
\theta^* &= \left(L_0^{-1} + \frac{1}{Q_\lambda} \tilde{h}_{t-1}^{*\prime} \tilde{h}_{t-1}^*\right)^{-1} \left(L_0^{-1} F_0 + \frac{1}{Q_\lambda} \tilde{h}_{t-1}^{*\prime} \tilde{h}_t^*\right) \\
L^* &= \left(L_0^{-1} + \frac{1}{Q_\lambda} \tilde{h}_{t-1}^{*\prime} \tilde{h}_{t-1}^*\right)^{-1}
\end{aligned}$$

The conditional posterior of Q_λ is inverse Gamma with scale parameter $\bar{\eta}_t' \bar{\eta}_t + Q_0$ and degrees of freedom $T + V_{Q0}$.

Given a draw for F, equation 8 can be expressed as $\bar{\Delta} \tilde{h}_t = C\mu + \bar{\eta}_t$ where $\bar{\Delta} \tilde{h}_t = \tilde{h}_t - F \tilde{h}_{t-1}$ and $C = 1 - F$. The conditional posterior of μ is $N\left(\mu^*, Z^*\right)$ where

$$\begin{aligned}
\mu^* &= \left(Z_0^{-1} + \frac{1}{Q_\lambda} C'C\right)^{-1} \left(Z_0^{-1} \mu_0 + \frac{1}{Q_\lambda} C' \bar{\Delta} \tilde{h}_t\right) \\
Z^* &= \left(Z_0^{-1} + \frac{1}{Q_\lambda} C'C\right)^{-1}
\end{aligned}$$

Note that α can be recovered as $\mu\left(1 - \theta\right)$

[1] In order to take endpoints into account, the algorithm is modified slightly for the initial condition and the last observation. Details of these changes can be found in Jacquier *et al.* (1994).

1.3 A Monte-Carlo experiment

In order to evaluate the MCMC algorithm we conduct a simple Monte Carlo experiment. 400 observations are generated from the following DGP with the number of variables $N = 40$ and the number of factors $k = 2$. The first 100 observations are discarded to remove the impact of initial conditions.

$$X_{it} = \Lambda_t F_{kt} + u_{it}, u_{it}\tilde{} N(0,1)$$

$$Z_t = \beta_t Z_{t-1} + \gamma_t \ln \lambda_t + c_t + \Omega_t^{1/2} e_t, e_t\tilde{} N(0,1)$$

$$\begin{aligned}
H_t &= \lambda_t S \\
S &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
\lambda_t &= -0.1 + 0.75\lambda_{t-1} + (0.5)^{\frac{1}{2}} v_t \\
\beta_t &= \begin{pmatrix} \beta_{11,t} & \beta_{12,t} \\ \beta_{21,t} & \beta_{22,t} \end{pmatrix}, \gamma_t = \begin{pmatrix} \gamma_{11,t} \\ \gamma_{21,t} \end{pmatrix}
\end{aligned}$$

where λ_t is generated once using $v_t\tilde{} N(0,1)$ and fixed for all iterations of the experiment. Following Gamble and LeSage (1993) we assume that a one time shift defines the change in the factor loadings, the VAR coefficients and the non-zero element of A_t. During the first 100 observations the VAR coefficients equal $\beta_t = \begin{pmatrix} 0.5 & 0.05 \\ 0.05 & 0.5 \end{pmatrix}, \gamma_t = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}$ and $A = -1$. The factor loading matrix Λ_t is equal to $\Lambda_1\tilde{}\ N(0,1)$. During the next 300 observations, the coefficients change to $\beta_t = \begin{pmatrix} 0.5 & 0.05 \\ 0.05 & 0.5 \end{pmatrix}, \gamma_t = \begin{pmatrix} -1.5 \\ 1.5 \end{pmatrix}$, $A = 1$ and $\Lambda_t = \Lambda_2\tilde{}\ N(0,1)$. Note that the factor loadings are generated once and held fixed over the Monte-Carlo iterations.

The data is generated 100 times. For each replication, the MCMC algorithm described above is run using 5000 iterations and the last 1000 draws are used to compute the impulse response to a one standard deviation shock to the volatility λ_t. Note that we use the first 20 observations to calibrate priors and starting values.

Figure 1 plots the median estimate of the cumulated impulse response of X_{it} (for $i = 1, 2, ..40$) at the 4-period horizon across Monte-Carlo replications and compares these with the true underlying values of the response (solid black lines). The figure shows that in the case of most variables, the Monte-Carlo estimates of the shift in the response matches the change in the response assumed in the DGP.

1.4 DIC Calculation

In practical terms, the DIC can be calculated as: $DIC = \bar{D} + p_D$. The first term is defined as $\bar{D} = E\left(-2\ln L\left(\Xi_i\right)\right) = \frac{1}{M}\sum_i \left(-2\ln L\left(\Xi_i\right)\right)$ where $L\left(\Xi_i\right)$ is the likelihood evaluated at the draws of all of the parameters Ξ_i in the MCMC chain. This term measures goodness of fit. The second term p_D is defined as a measure of the number of effective parameters in the model (or model complexity). This is defined as $p_D = E\left(-2\ln L\left(\Xi_i\right)\right) - \left(-2\ln L\left(E(\Xi_i)\right)\right)$ and can be approximated as $p_D = \frac{1}{M}\sum_i \left(-2\ln L\left(\Xi_i\right)\right) - \left(-2\ln L\left(\frac{1}{M}\sum_i \Xi_i\right)\right)$. The likelihood function of the model is evaluated using a particle filter with 2000 particles.

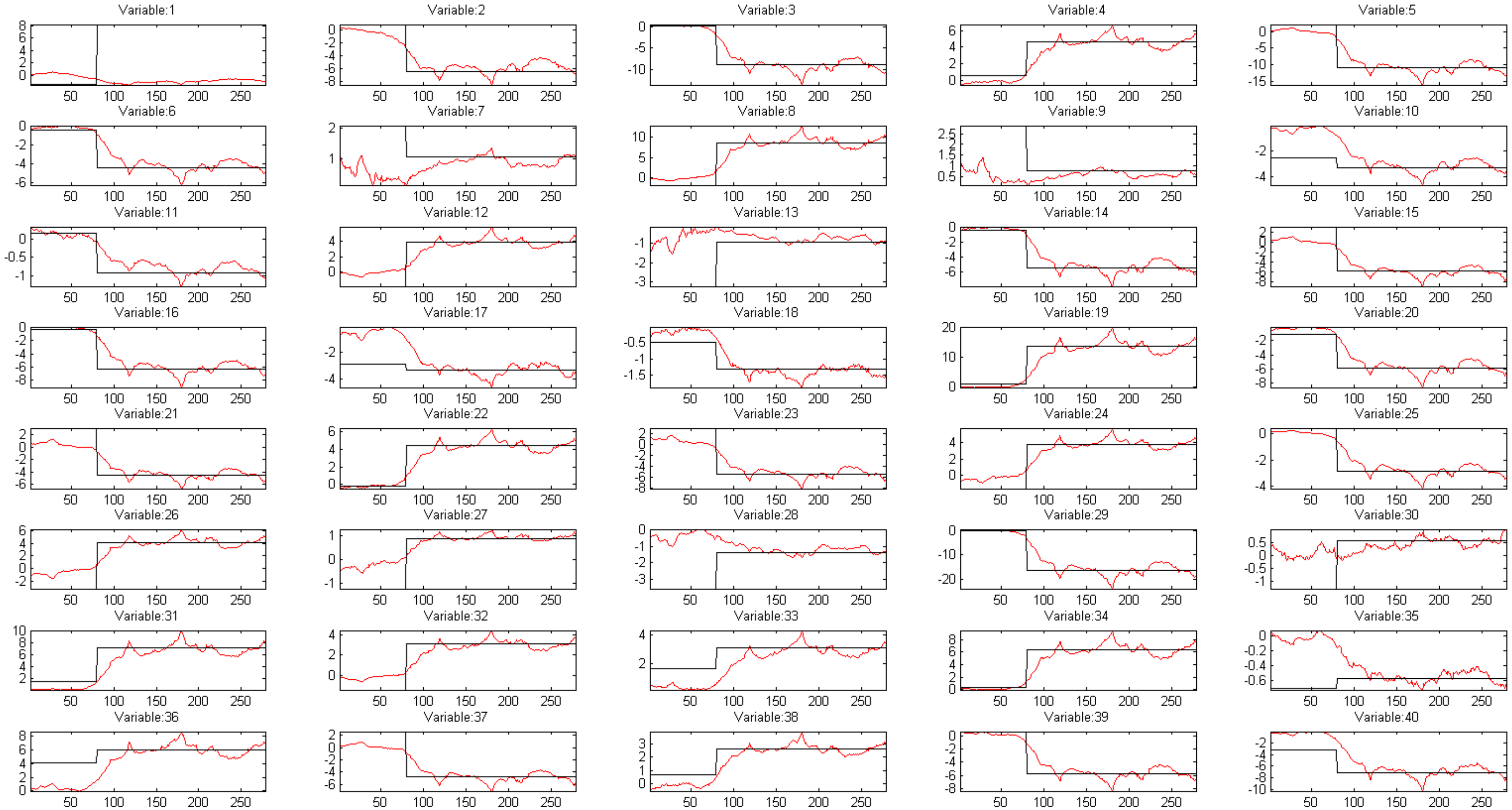

Figure 1: Estimates of the cumulated impulse response at the 4-period horizon. The figure presents the median (red line) across the 100 Monte-Carlo replications. The black line represents the true time-varying cumulated response at the 4-period horizon.

1.5 Impulse responses from the benchmark model (full 3-D figures)

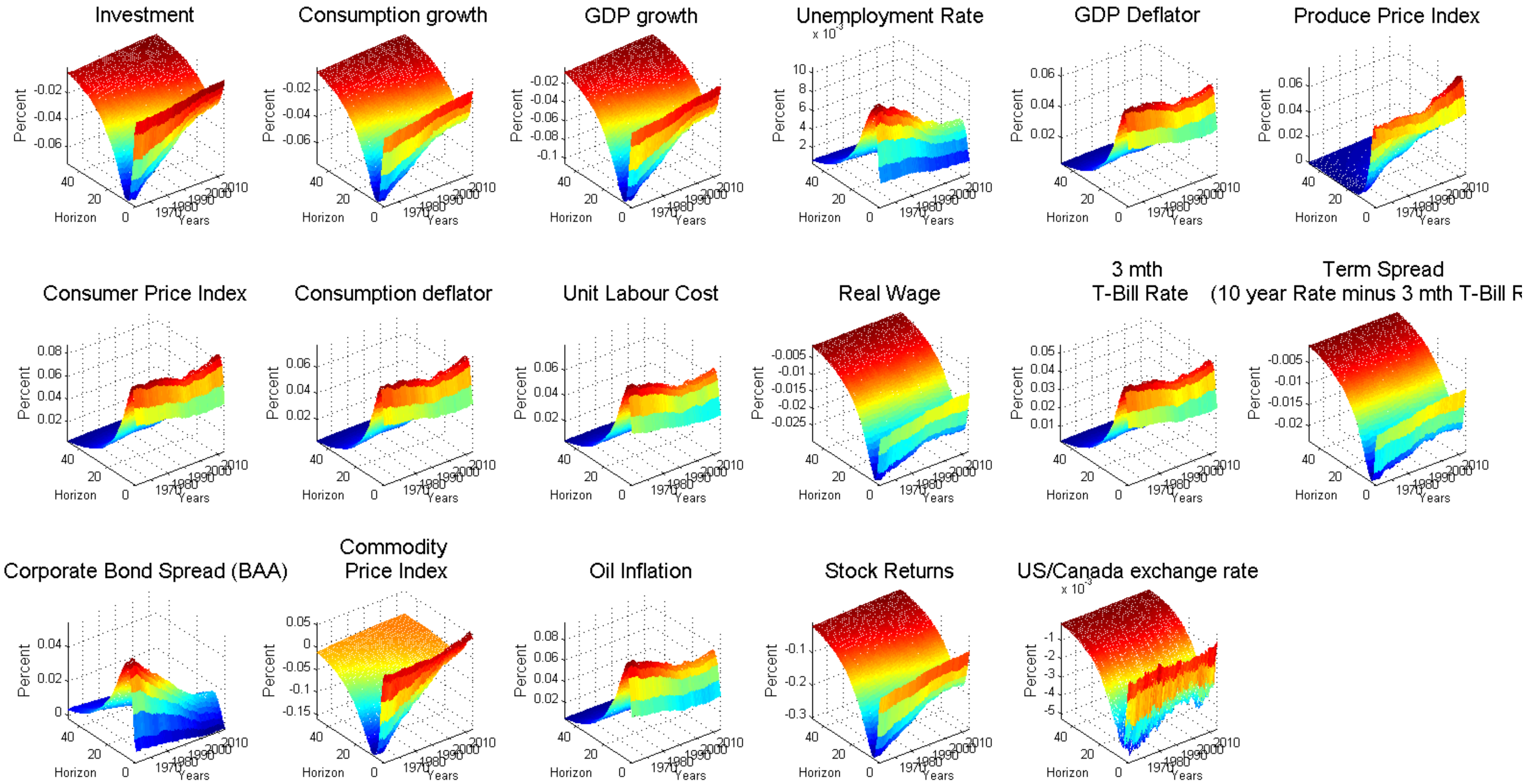

Figure 2: Three dimensional version of the impulse responses from the benchmark model

1.6 Sensitivity analysis

The top row of figure 3 in section 1.6 presents the impulse response of key series to a 1 standard deviation uncertainty shock using a version of the benchmark model where the four lags of λ_t are allowed to affect the endogenous variables. As in the benchmark case, the response of GDP the corporate bond spread and stock market returns show a decline. In contrast, the decline in the inflation and the short term interest rate response is estimated to be weaker.

The second row of figure 3 shows the impulse response from a version of the benchmark model where the assumption that the volatility has a contemporaneous affect on the endogenous variables is relaxed. In contrast, only the coefficients on lagged values of λ_t are allowed to have non-zero coefficients. The temporal pattern of the estimated impulse responses support the benchmark results– while the response of GDP growth and the financial variables declines over time, the response of inflation and the short-term rate is fairly stable.

The third row of the figure shows that the responses from a three factor model support the key conclusions reached using the benchmark model.

The bottom row of the figure considers a version of the benchmark model where an alternative prior imposed on the variance of the shock to the transition equations for the time-varying parameters (Q_B and Q_λ). In the benchmark model, the prior for these covariance matrices is assumed to be inverse Wishart:

$$P\left(Q_B\right) \tilde{} IW\left(Q_{OLS} \times T_0 \times K, T_0\right)$$

where $T_0 = 40$ is the length of the training sample and Q_{OLS} is the OLS estimate of the coefficient covariance using the training sample. The prior scale matrix is multiplied by the factor K which is set to 3.5×10^{-4} following Cogley and Sargent (2005). In the alternative specification, we set $K = 1 \times 10^{-4}$ and thus incorporate a belief of lower time-variation in the VAR coefficients and factor loadings. The bottom panel of figure 3 shows that while the change in impulse responses is smoother, there is evidence that the estimated response of GDP growth, the corporate bond spread and stock price index declines over time. In contrast, the response of inflation and interest rates remains largely constant. Thus, the results from this model with a tighter prior support the benchmark conclusions.

In summary, the benchmark results and the sensitivity analysis suggests the following main conclusion: There is evidence that the response of real activity and some financial indicators to the uncertainty shock has declined over time. In contrast, the response of inflation and interest rates to this shock has remained largely stable. We now turn to a DSGE model in order to explore the possible reasons behind the estimated change in the impact of uncertainty shocks.

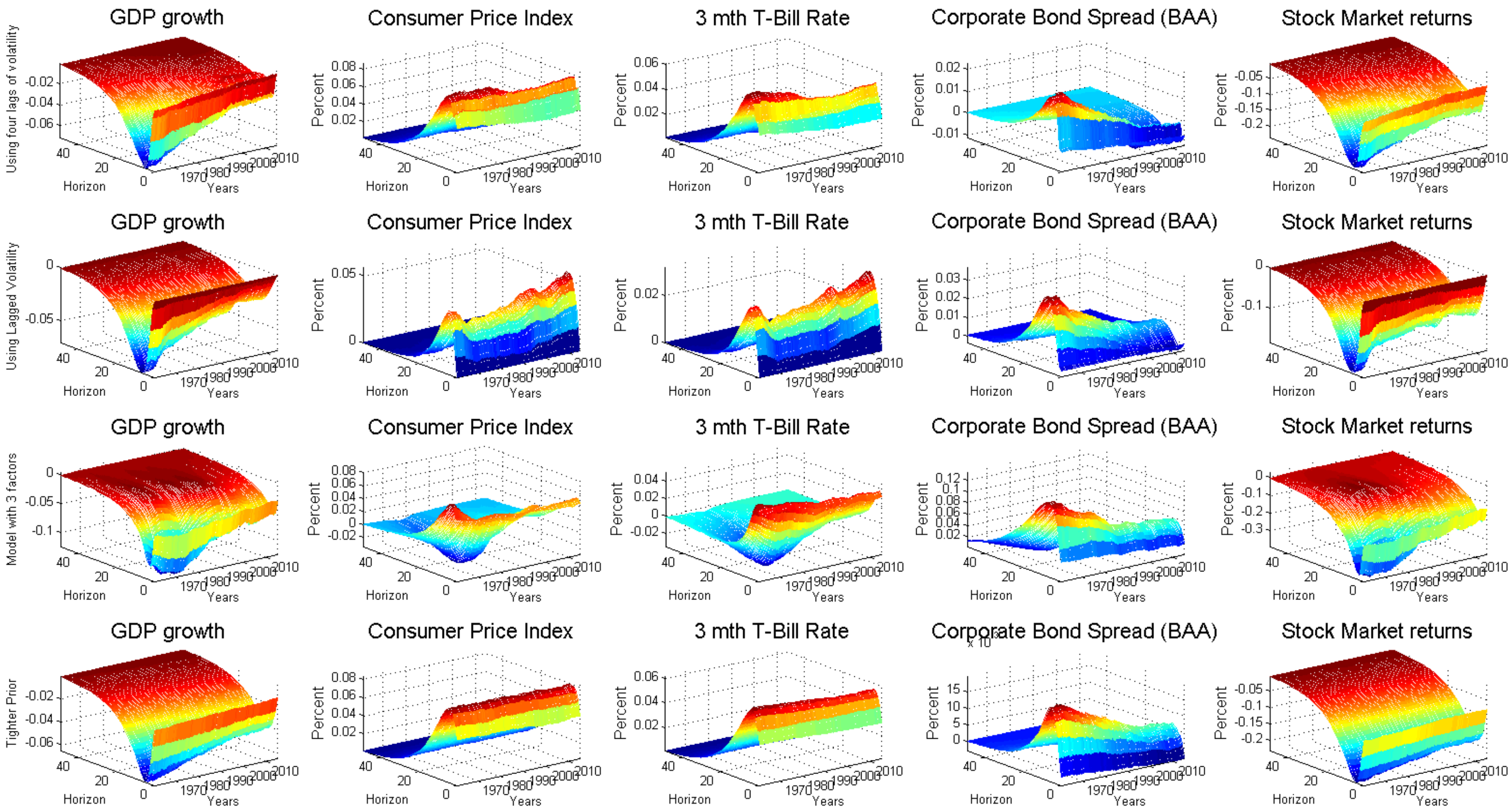

Figure 3: Sensitivity Analysis

1.7 Data

Table 1: List of Data series. GFD refers to Global Financial data. FRED is the St Louis Fed database. LD denotes 100 times the log difference while N denotes no transformation

No.	Variable	Source	Code	Transformation
1	Industrial Production	FRED	INDPRO	LD
2	Dow Jones Industrial Total returns index	GFD	_DJITRD	LD
3	GDP Deflator	FRED	GDPDEF	LD
4	ISM Manufacturing: New Orders Index	FRED	NAPMNOI	N
5	ISM Manufacturing: Inventories Index	FRED	NAPMII	N
6	ISM Manufacturing: Supplier Deliveries Index	FRED	NAPMSDI	N
7	All Employees: Total nonfarm	FRED	PAYEMS	LD
8	Business Confidence Index	GFD	BCUSAM	N
9	Real Imports	FRED	IMPGSC96	LD
10	Real Exports	FRED	EXPGSC1	LD
11	Government Spending to GDP ratio	BEA	see Mumtaz and Surico (2013)	LD
12	Net Taxes to GDP ratio	BEA	seeMumtaz and Surico (2013)	LD
13	Real Gross Private Domestic Investment	FRED	GDPIC96	LD
14	Real Personal Consumption Expenditures	FRED	PCECC96	LD
15	Real GDP	FRED	GDPC96	LD
16	Unemployment Rate	FRED	UNRATE	N
17	Average Hours	FRED	AWHMAN	LD
18	Civilian Labour Force	FRED	CLF16OV	LD
19	Civilian Labor Force Participation Rate	FRED	CIVPART	LD
20	Nonfarm Business Sector: Unit Labor Cost	FRED	ULCNFB	LD
21	Nonfarm Business Sector: Real Compensation Per Hour	FRED	COMPRNFB	LD
22	M2 Money Stock	Fred	M2	LD
23	Total Consumer Credit Owned and Securitized, Outstanding	FRED	TOTALSL	LD
24	Producer Price Index	GFD	WPUSAM	LD
25	CPI	FRED	CPIAUCSL	LD
26	Personal Consumption Expenditures: Chain-type Price Index	FRED	PCECTPI	LD
27	3-Month Treasury Bill: Secondary Market Rate	FRED	TB3MS	N
28	10 year Govt Bond Yield minus 3 mth yield	GFD	IGUSA10D (minus TB3MS)	N
29	6-month Treasury bill minus 3 mth yield	GFD	ITUSA6D (minus TB3MS)	N
30	1 year Govt Bond Yield minus 3 mth yield	GFD	IGUSA1D (minus TB3MS)	N
31	5 year Govt Bond Yield minus 3 mth yield	GFD	IGUSA5D (minus TB3MS)	N
32	Reuters/Jeffries-CRB Total Return Index (w/GFD extension)	GFD	_CRBTRD	LD
33	West Texas Intermediate Oil Price	GFD	_WTC_D	LD
34	BAA Corporate Spread	GFD	MOCBAAD (minus IGUSA10D)	N
35	AAA Corporate Bond Spread	GFD	MOCAAAD (minus IGUSA10D)	N
36	S&P500 Total Return Index	GFD	_SPXTRD	LD
37	NYSE Stock Market Capitalization	GFD	USNYCAPM	LD
38	S&P500 P/E Ratio	GFD	SYUSAPM	N
39	US Canada exchange rate]	GFD	USDCAD	LD

1.8 Recursive means

Figure 4 presents the mean of the Gibbs draws for key model parameters calculated every 100 draws. These are fairly stable which provides evidence of convergence of the Gibbs algorithm.

2 Details on the DSGE model

2.1 Calibration

The benchmark calibration is listed in table 3. The parametrization of the non-financial block of the of the model is based on Fernandez-Villaverde and Rubio-Ramirez (2008), while the calibration of the financial block of the model relies on Christiano *et al.* (2014). In both studies full information Bayesian estimation techniques have been used to decide about the structural parameter and this is what drives our selection. However, for the purposes of the simulation experiments discussed in the main text we have changed the values of γ_π, $\bar{\pi}$, θ_p, χ, Λ_A, η and ϵ relative to the numbers reported by Fernandez-Villaverde and Rubio-Ramirez (2008) (Table 2.1). To be precise, the discount factor is set equal to 0.999 and combined with the inflation target $\bar{\pi} = 1.010$, the growth rates of the investment specific technological change $\Lambda_\mu = 1.010$ and of the neutral technology $\Lambda_A = 1.0013$, implies that the steady-state value of the annual real rate is 6.40%. The degree of habit persistence is 0.88, this value is higher than the estimates reported by Smets and Wouters (2007) and Justiniano *et al.* (2010), however, due to log consumption preferences a high degree of habit persistence is needed so demand does not display excess sensitivity to the real interest rate (via the Euler consumption equation). Similar to Smets and Wouters (2007) the inverse Frisch elasticity of labour supply is equal to 1.36 and the investment adjustment is equal to 7.68 suggesting very little response of investment to changes in Tobin's q. The Calvo parameters imply that prices and wages are reset every 2.22 and 3.33 quarters respectively, while households rely on indexation ($\chi_w = 0.80$) more heavily than firms ($\chi = 0.40$). The elasticities of substitution ϵ for firms and η for households imply an average markup of around 5% and 30%, respectively. The Taylor rule parameters are $\gamma_\pi = 1.010$, $\gamma_y = 0.190$ and $\gamma_R = 0.79$. The steady-state probability of defaults is 2.24% and slightly smaller than the value 3% used in the literature (see Bernanke *et al.* (1999)). The value of entrepreneur auditing cost is 0.21 and the fraction of survival entrepreneurs is 0.985, again these values are higher than those used in the literature (3% and 0.976 Bernanke *et al.* (1999)).

2.2 Solution

The model is solved using third-order perturbation methods (see Judd (1998)) as for any order lower than three, uncertainty shocks (our main objects of interest) do not enter into the decision rule as independent components. One difficulty of using these higher-order solution techniques is that paths simulated by the approximated policy function often explode. This is because regular perturbation approximations are polynomials that have multiple steady states and could yield unbounded solutions (Kim *et al.*, 2008). This means that the approximation is valid only locally and along the simulation path we may enter into a region where its validity is not preserved anymore.

To avoid this problem Kim *et al.* (2008) suggest to 'prune' all those terms that have an order that is higher than the approximation order, while Andreasen *et al.* (2013) show how this logic can be applied to any order. Although there are studies that question the legitimacy of this approach (see Haan and Wind (2010)), it has by now been widely accepted as the only reliable way to get the solution of n^{th} order approximated DSGE model (where $n > 1$).

Finally, we follow Fernández-Villaverde *et al.* (2011) and generate the responses of model variables to stochastic volatility shocks using generalised impulse responses developed by Koop *et al.* (1996).

2.3 IRFs

The exact simulation steps to produce the impulse responses reported here are as follows:

1. We draw 5×1040 structural shocks $\omega_{j,t}$ from the standard normal distribution (where $j = 1,..,5$ and $t = 1,..,1040$)

2. We simulate the model using the shocks from step 1, we denote the simulated data by y_t

3. We simulate the model using again the structural shocks $\omega_{j,t}$ from step 1 but now we increase the value of the structural shock of interest in period 1001 by an amount necessary to rise uncertainty by 1 times the standard devition of the uncertainty shock, namely

$$\tilde{\omega}_{j,1001} = \omega_{j,1001} + x \tag{9}$$

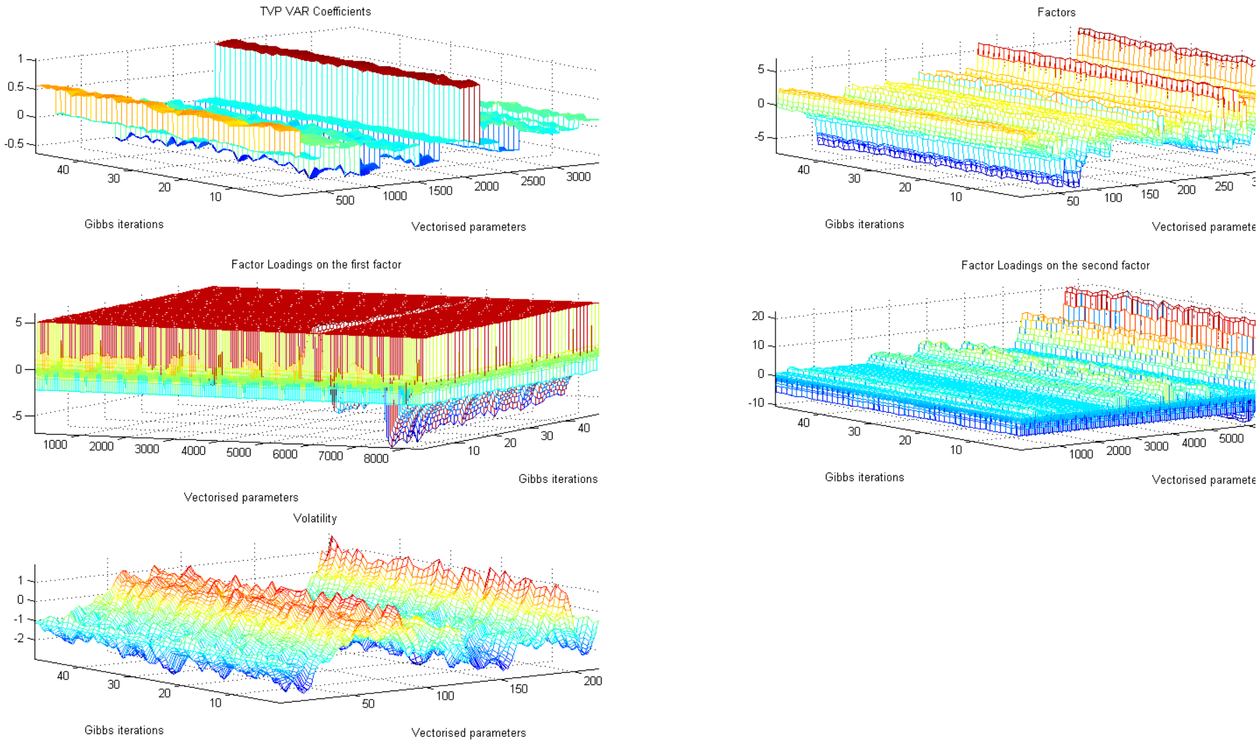

Figure 4: Recursive means of the Gibbs draws calculated at every 100 draws.

We denote the data obtained from this simulation by $\tilde{y}_t$

4. Steps 1 to 3 are repeated 1000 times
5. The IRF is calculated as follows

$$IRF = \frac{1}{1000} \sum_{i=1}^{1000} \left(\tilde{y}_t^i - y_t^i \right) \tag{10}$$

All calculations have been produced using Dynare 4.4.2 and Matlab 2012b.

Table 2: DSGE Model Variables

Description	Mnemonic
d_t	Preference Shock
c_t	Consumption
$\mu_{z,t}$	Trend Growth Rate of the Economy
$\mu_{I,t}$	Growth rate of Investment-Specific Technology Growth
$\mu_{A,t}$	Growth rate of Neutral Technology
λ_t	Lagrange multiplier
R_t	Nominal Interest rate
π_t	Inflation
r_t	Rental Rate of Capital
x_t	Investment
u_t	Capacity Utilization
q_t	Tobin's Marginal Q
f_t	Variable for Recursive Formulation of Wage Setting
l_t^d	Aggregate Labor Demand
w_t	Real Wage
w_t^*	Optimal Real Wage
$\pi_t^{w,*}$	Optimal Wage Inflation
π_t^*	Optimal Price Inflation
g_t^1	Variable 1 for Recursive Formulation of Price Setting
g_t^2	Variable 2 for Recursive Formulation of Price Setting
y_t^d	Aggregate Output
mc_t	Marginal Cost
k_t	Capital
v_t^p	Price Dispersion Term
v_t^w	Wage Dispersion Term
l_t	Aggregate Labor Bundle
ϕ_t	Labor Disutility Shock
F_t	Firm Profits
σ_t	Aggregate Uncertainty
$\bar{\omega}_t$	Bankruptcy Cutoff Value
R_t^k	Return of Capital
n_t	Net worth

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Table 3: DSGE Model Parameters

Description	Mnemonic	Value
Λ_μ	Steady State Growth Rate of Investment-Specific Technology	1.010
Λ_A	Steady State Neutral Technology Growth	1.005
β	Discount Factor	0.999
h	Consumption Habits	0.877
γ_1	Capital Utilization, Linear Term	0.039
γ_2	Capital Utilization, Quadratic Term	0.001
δ	Depreciation Rate	0.015
κ	Capital Adjustment Cost Parameter	7.679
η	Elasticity of Substitution between Labor Varieties	4.20
ϵ	Elasticity of Substitution between Goods Varieties	21.00
ψ	Labor Disutility Parameter	9.340
γ	Inverse Frisch Elasticity	1.359
χ_w	Wage Indexation Parameter	0.800
χ	Price Indexation	0.400
θ_p	Calvo Probability Prices	0.550
θ_w	Calvo Probability wages	0.700
α	Capital Share	0.255
$\bar{\pi}$	Steady State Inflation	1.010
γ_R	Interest Smoothing Coefficient Taylor Rule	0.790
γ_π	Feedback Inflation Coefficient Taylor Rule	1.010
γ_y	Feedback Output Coefficient Taylor Rule	0.190
Φ	Firms Fixed Cost	0.025
	Financial Friction Parameters	
$F(\bar{\omega})$	Steady State Probability of Default	0.006
γ	Fraction of Survival Entrepreneurs	0.985
μ_E	Entrepreneur Auditing Cost	0.210
$\bar{\omega}$	Steady State Bankruptcy Cutoff Value	0.568
σ_ω	Standard Deviation Entrepreneur's Idiosyncratic Productivity Shock	0.214
Θ	Fraction of Assets Consumed During Exit	0.005
	Shock Process Parameters	
ρ_d	Autocorrelation Preference Shock	0.951
ρ_ϕ	Autocorrelation Labor Disutility Shock	0.942
ρ_σ	Autocorrelation Uncertainty Shock	0.950
σ_d	standard deviation preference shock	0.060
σ_ϕ	Standard Deviation labor Disutility Shock	0.070
σ_μ	Standard Deviation Investment-Specific Technology	0.151
σ_A	Standard Deviation Neutral Technology	0.070
σ_m	Standard Deviation Policy shock	0.003
σ_σ	Standard Uncertainty Shock	1.000

Notes: The parametrization of the model is based on Fernandez-Villaverde and Rubio-Ramirez (2008), while the values of the financial friction parameters are those estimated by Christiano *et al.* (2014). We refer to this version of the model in the text as 'benchmark' and for the purposes of the simulation experiments discussed in Section xxx we have changed the values of γ_π, $\bar{\pi}$, θ_p, χ, Λ_A, η and ϵ relative to the numbers reported by Fernandez-Villaverde and Rubio-Ramirez (2008) (Table 2.1). γ_1 is selected to deliver a steady-state (annual) credit spread is equal to 300bps, ψ ensures the steady-state value of labour is equal to 1/3, Φ is selected so the steady-state value of profits is equal to zero, $\bar{\omega}$ and σ_ω have been also selected in order to be consistent with $F(\bar{\omega}) = 0.0056$ and $\mu_E = 0.210$.

Table 4: DSGE Model Equations

Description	Equation
Marginal Utility of Consumption	$d_t\left(c_t - h\frac{c_{t-1}}{z_t}\right)^{-1} - h\beta E_t d_{t+1}\left(c_{t+1}z_{t+1} - hc_t\right)^{-1} = \lambda_t$
Euler Equation	$\lambda_t = \beta E_t\left(\frac{\lambda_{t+1}R_t}{z_{t+1}\pi_{t+1}}\right)$
Rental Rate of Capital	$r_t = \alpha'(u_{t+1})$
Return of Capital	$\frac{R_t^k}{\pi_t} \equiv \frac{r_t u_t + (1-\delta)q_t - a(u_t)}{q_{t-1}\mu_t}$
Investment Equation	$1 = q_t\left\{1 - S\left(\frac{x_t z_t}{x_{t-1}}\right) - S'\left(\frac{x_t z_t}{x_{t-1}}\right)\frac{x_t z_t}{x_{t-1}}\right\}$
	$+\beta E_t \frac{\lambda_{t+1}}{\lambda_t z_{t+1}} S'\left(\frac{x_{t+1}z_{t+1}}{x_t}\right)\left(\frac{x_{t+1}z_{t+1}}{x_t}\right)^2$
Wages Equation 1	$f_t = \frac{\eta-1}{\eta}(w_t^*)^{(1-\eta)}\lambda_t w_t^{\eta} l_t^d + \beta\theta_w E_t\left(\frac{\pi_t^{\chi_w}}{\pi_{t+1}}\right)^{(1-\eta)}\left(\frac{w_{t+1}^* z_{t+1}}{w_t^*}\right)^{(\eta-1)} f_{t+1}$
Wages Equation 2	$f_t = \psi d_t \phi_t (\pi_t^{w,*})^{-\eta(1+\gamma)}(l_t^d)^{1+\gamma}$
	$+\beta\theta_w E_t\left(\frac{\pi_t^{\chi_w}}{\pi_{t+1}}\right)^{-\eta(1+\gamma)}\left(\frac{w_{t+1}^* z_{t+1}}{w_t^*}\right)^{\eta(1+\gamma)} f_{t+1}$
Wages Equation 3	$1 = \theta_w\left(\frac{\pi_{t-1}^{\chi_w}}{\pi_t}\right)^{1-\eta}\left(\frac{w_{t-1}}{w_t z_t}\right)^{1-\eta} + (1-\theta_w)(\pi_t^{w,*})^{1-\eta}$
Prices Equation 1	$g_t^1 = \lambda_t mc_t y_t^d + \beta\theta_p E_t\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\epsilon} g_{t+1}^1$
Prices Equation 2	$g_t^2 = \lambda_t \pi_t^* y_t^d + \beta\theta_p E_t\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\epsilon}\frac{\pi_t^*}{\pi_{t+1}^*} g_{t+1}^2$
Prices Equation 3	$\epsilon g_t^1 = (\epsilon-1)g_t^2$
Prices Equation 4	$1 = \theta_p\left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{1-\epsilon} + (1-\theta_p)(\pi_t^*)^{1-\epsilon}$
Demand for Capital	$\frac{u_t k_{t-1}}{l_t^d} = \frac{\alpha}{1-\alpha}\frac{w_t z_t \mu_t}{r_t}$
Marginal Cost	$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} w_t^{1-\alpha} r_t^{\alpha}$
Taylor Rule	$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R}\left\{\left(\frac{\pi_t}{\bar{\pi}}\right)^{\gamma_\pi}\left(\frac{y_t^d z_t}{y_{t-1}^d \Lambda_A^{\frac{1}{1-\alpha}}\Lambda_\mu^{\frac{\alpha}{1-\alpha}}}\right)^{\gamma_y}\right\}^{1-\gamma_R} e^{m_t}$
Goods Market Clearing	$y_t^d = \frac{\frac{1}{z_t}A_t(u_t k_{t-1})^{\alpha}\left(l_t^d\right)^{1-\alpha} - \Phi}{v_t^p}$
Aggregate Demand	$y_t^d = c_t + x_t + \frac{\alpha(u_t)k_{t-1}}{z_t\mu_t} + \frac{\mu_E G(\bar{\omega}_t)R_t^k q_{t-1}k_{t-1}}{z_t\pi_t} + \Theta\frac{1-\gamma}{\gamma}(n_t - w_E)$
Labour Market Clearing	$l_t = v_t^w l_t^d$
Price Dispersion	$v_t^p = \theta_p\left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{-\epsilon} + (1-\theta_p)(\pi_t^*)^{-\epsilon}$
Wage Dispersion	$v_t^w = \theta_w\left(\frac{w_{t-1}}{w_t z_t}\frac{\pi_{t-1}^{\chi_w}}{\pi_t}\right)^{-\eta} + (1-\theta_w)\left(\pi_t^{w,*}\right)^{-\eta}$
Capital Accumulation	$k_t = (1-\delta)\frac{k_{t-1}}{z_t\mu_t} + \left(1 - S\left(\frac{x_t z_t}{x_{t-1}}\right)\right)x_t$
Bank zero profit condition	$\left[\Gamma(\bar{\omega}_t) - \mu_E G(\bar{\omega}_t)\right]R_t^k\frac{q_{t-1}k_{t-1}}{n_{t-1}} = R_{t-1}\left(\frac{q_{t-1}k_{t-1}}{n_{t-1}} - 1\right)$
Optimality loan contract condition	$E_t\left\{\left[1-\Gamma(\bar{\omega}_{t+1})\right]\frac{R_{t+1}^k}{R_t} + \frac{\frac{\partial\Gamma(\bar{\omega}_{t+1})}{\partial\bar{\omega}_{t+1}}\left[\left[\Gamma(\bar{\omega}_{t+1}) - \mu_E G(\bar{\omega}_{t+1})\right]\frac{R_{t+1}^k}{R_t} - 1\right]}{\left[\frac{\partial\Gamma(\bar{\omega}_{t+1})}{\partial\bar{\omega}_{t+1}} - \mu_E\frac{\partial G(\bar{\omega}_{t+1})}{\partial\bar{\omega}_{t+1}}\right]}\right\} = 0$
Net-worth accumulation	$n_t = \gamma\frac{R_{t-1}}{\pi_t z_t}n_{t-1} + \frac{\gamma\left[R_t^k - \left(R_{t-1} + \mu_E G(\bar{\omega}_t)R_t^k\right)\right]}{\pi_t z_t}q_{t-1}k_{t-1} + w_E$
Exogenous States	
Preference shock	$\log(d_t) = (1-\rho_d)\,log(d) + \rho_d\log(d_{t-1}) + \sigma_t\sigma_d\omega_t^d$
Labour disutility shock	$\log(\phi_t) = \left(1-\rho_\phi\right)log(\phi) + \rho_\phi\log\left(\phi_{t-1}\right) + \sigma_t\sigma_\phi\omega_t^\phi$
Policy Shock	$log(m_t) = \sigma_t\sigma_m\omega_t^m$
TFP Shock	$\log(A_t) = log(\Lambda_A) + \sigma_t\sigma_A\omega_t^A$
IST Shock	$\log(\mu_t) = log(\Lambda_\mu) + \sigma_t\sigma_\mu\omega_t^A$
Uncertainty Shock	$\log(\sigma_t) = \rho_\sigma\log(\sigma_{t-1}) + \sigma_\sigma\omega_t^\sigma$

Notes: $G(\omega,\sigma_\omega) = 1 - \Phi\left(\frac{0.5\sigma_\omega - \log\omega}{\sigma_\omega}\right)$, $\Gamma(\bar{\omega}_t) = \bar{\omega}_t\left[1 - F(\bar{\omega}_t)\right] + G(\bar{\omega}_t)$, where Φ is the CDF of a normal distribution. $\frac{\partial\Gamma(\bar{\omega}_t)}{\partial\bar{\omega}_t}$ and $\frac{\partial G(\bar{\omega}_t)}{\partial\bar{\omega}_t}$ denote the partial derivatives of $\Gamma(\bar{\omega}_t)$ and $G(\bar{\omega}_t)$ with respect to $\bar{\omega}_t$.

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