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## Misallocation of Talent in Competitive Labor Markets

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# Misallocation of Talent in Competitive Labor Markets\*

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## **Abstract**

We develop a model in which competition in the labor market may produce worker-firm matches that are inferior to those obtained in the absence of competition. This result contrasts with the conventional wisdom that competition among employers allocates scarce talent efficiently. In a model in which employers asymmetrically learn about the ability of their workers, we show that constraining labor market competition may be socially desirable precisely because it leads to better talent allocation. The model provides a cautionary counterpoint to one of the most popular arguments against the regulation of pay, i.e., the argument that price-distorting regulation leads to inefficient matches of workers and firms.

**Keywords:** Labor Markets, Asymmetric Employer Learning, Misallocation, Adverse Selection.

**JEL classification:** D82, J31, M5

## **1. Introduction**

Non-economists generally believe that increasing the compensation of highly-paid employees – such as executive compensation in general and banker compensation in particular – is often unfair and sometimes inefficient. Public outrage at high levels of executive pay has led to numerous calls for pay regulation, ranging from say-on-pay policies to proposals involving

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caps on salaries and bonuses.<sup>1</sup> There is more disagreement regarding the compensation of entertainers and celebrities, but it is also not difficult to find proposals for controlling the compensation of professional athletes.<sup>2</sup>

Conversely, most economists are less visceral about proposals for pay regulation. They often argue that pay regulation can reduce social welfare because price controls typically distort the allocation of human capital in the economy. Furthermore, pay regulation can also have unintended consequences, such as incentivizing highly talented professionals to move to unregulated jurisdictions or sectors, leading to brain drain losses in those industries and countries that introduce pay regulations.<sup>3</sup>

In this paper, we question the economist's folk wisdom. Using a reasonably conventional model of the labor market with asymmetric information as the main friction, we show that unconstrained supply and demand forces may lead to *misallocation of talent*. We also show that efficiency can be restored by restricting competition under certain conditions.

Our model belongs to the asymmetric employer learning literature, which was initiated by Waldman (1984) and Greenwald (1986). In such models, the current employer learns about the talent of her incumbent workers, while competing employers remain uninformed. This form of information asymmetry implies that competitors may learn about a worker's ability from the actions taken by the worker's employer, such as decisions involving promotion, retention, and termination. These models typically assume that workers accumulate firm-specific skills over time and are thus more valuable to their incumbent firms than to competing firms.<sup>4</sup>

We depart from the existing literature by introducing a specific form of firm heterogeneity: some firms are more productive/profitable than others.<sup>5</sup> This feature allows us to study

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<sup>1</sup>The Dodd-Frank Act made shareholder approval for executive compensation (*say-on-pay*) compulsory from 2011 onward for all publicly traded US companies. The European Union has placed caps on banker bonuses at a multiple of their fixed pay. In a referendum in November 2013, Swiss voters rejected a proposal to cap bosses' pay at a maximum of 12 times the pay of the lowest-paid worker.

<sup>2</sup>In the US, most professional sports leagues have salary caps. Conversely, most European professional soccer leagues do not have pay caps, and high pay levels are often debated in the European media as a consequence (see, for example, the debate regarding whether Gareth Bale's salary at Real Madrid is too high: "Is a salary of £300,000 a week too much?", available at <http://www.bbc.co.uk/news/uk-23931053> (October 2013)).

<sup>3</sup>For example, consider Murphy (2012) on the unintended consequences of regulating banker bonuses: "(...) *punitive restrictions on financial institutions will lead to both costly circumvention and a drain of talent from restricted to unrestricted sectors*" (p.62).

<sup>4</sup>Earlier papers in this literature include Lazear (1986), Milgrom and Oster (1987), Waldman (1990), Laing (1994), Chang and Wang (1996), Acemoglu and Pischke (1998, 1999), and Golan (2005), among others. Although some of the results in this early literature have been challenged on technical grounds, these models have nonetheless led to many important insights.

<sup>5</sup>Dispersion in profitability is widely documented, even within narrowly defined industries. A large strategy literature attributes profitability dispersion to monopoly profits that are explained by barriers to entry or ownership of unique resources (McGahan and Porter, 1997; Rumelt, 1991). Even in industries with free entry, equilibrium (ex post) profitability dispersion can be explained by the accumulation of organizational

worker poaching and job mobility in equilibrium. Under symmetric information (and supermodularity of firm and worker qualities) we obtain the standard result that the best firms are matched with the best workers in equilibrium, i.e., unconstrained competition promotes the socially optimal allocation of talent. By contrast, in an equilibrium with asymmetric employer learning, the socially optimal allocation of talent cannot generally be attained. Thus, misallocation of talent is a typical feature of such labor markets. We show that competition for talent often tends to exacerbate such misallocation problems rather than correct them. In other words, there could be too much – not too little – job mobility in equilibrium.

To illustrate this intuition, consider an employer who wants to retain a worker. The employer knows the worker’s general (i.e., portable) talent. By contrast, competing firms observe the wage offered by the incumbent employer but not the worker’s talent. A high wage is interpreted as a signal of high ability. To prevent the worker from being poached, the incumbent employer must offer a sufficiently high wage to the worker but will do so only if the worker is indeed very talented. Therefore, only the very best workers are retained. As a consequence, all workers who are poached are *mediocre*, i.e., workers with just about average ability.<sup>6</sup>

In an equilibrium with job mobility, departing mediocre workers have firm-specific skills that are then wasted. This situation is inefficient (from an allocational perspective) because these workers would be more productive with the incumbent firm. Conversely, some high-ability workers, who will remain employed by low-profitability firms in equilibrium, might be more productive if they were instead employed by high-profitability firms. Thus, the equilibrium is typically characterized by mismatch: high-ability workers are retained by low-profitability firms, while mediocre workers are either released or poached by high-profitability firms.

Our results may appear surprising in light of the original analysis of markets with asymmetric information by Akerlof (1970). In a lemons market in which the seller of an asset has private information, there is typically little or no trade. By analogy, one would expect that a labor market in which the current employer knows more about the quality of its worker than a competitor is likely to generate too little “trade,” i.e., insufficient worker mobility. However, this analogy is imperfect; the worker is not an asset that can be freely bought and sold. Assuming no slavery, the worker is free to work for the highest bidder, and the current employer typically receives no compensation if the worker is poached by another firm. In such a market, the incumbent can only retain the worker by paying more than a competitor.

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capital (Atkeson and Kehoe, 2005). For a recent review of the literature on productivity dispersion, see Syverson (2011).

<sup>6</sup>Firms that poach workers are not fooled in equilibrium and have correct beliefs about the abilities of the workers that they hire. Nonetheless, incumbent firms are unable to retain such workers at acceptable wages because any attempt to do so would trigger a higher offer from poachers, under reasonable off-the-equilibrium-path beliefs.

If we allow firms to design contracts that bind workers to firms by fining workers if they leave, our model would then be no different from an asset market model under private information.

We use our model to discuss policy interventions in the labor market. We show that there are conditions under which restricting labor market competition would unambiguously improve efficiency. These conditions are relatively simple and intuitive; if the expected benefit of matching talent to firm quality is small – either because firms are not too different from one another or because firm-specific skills are very important – then restricting competition can enhance welfare. For example, in a regulated industry in which firm heterogeneity is limited by regulators, it might also make sense to regulate and restrict the competition for talent among firms. Such policies might be successful (at least in principle) even when workers are free to move to unregulated sectors, provided that the regulated sector is sufficiently large.

After a brief discussion of the related literature, Section 2 presents a simplified version of our model with two firms, one incumbent worker, and three types. The example illustrates the main form of allocational inefficiency. For expositional simplicity, we keep the analysis informal. In Section 3, we present a more general model that features a continuum of firms, workers, and types, and we explain and discuss all the relevant definitions and assumptions. We present our main results using a one-period model. We then provide a discussion of the interpretation of our model in Section 4. In Section 5, we extend the model to an infinite horizon such that learning and its value to the firm can be explicitly modeled. With minor qualifications, the multi-period model yields similar qualitative implications as the simpler one-period model. Section 6 concludes with a discussion of our model’s empirical relevance.

### **Related Literature.**

Terviö (2009) also shows that competition for talent creates inefficiencies. In his model, a worker’s talent is revealed on the job but – unlike our model – this information is public. Terviö shows that in a competitive labor market, firms invest too little in talent discovery and over-recruit workers with mediocre abilities. There are two main results: mediocre workers receive large rents and labor turnover is inefficiently low. The key friction is the inability of workers to “purchase jobs” (i.e., pay entry fees) or sign long-term contracts binding them to firms.

Our model illustrates a different source of inefficiency in labor markets in which talent is discovered on the job. If learning is asymmetric, high-profitability firms over-recruit mediocre workers who would be better matched with low-profitability firms. Thus, as in Terviö’s model, large rents captured by mediocre workers are a symptom of inefficiency. To clarify that our results arise from a different friction, we show that our conclusions continue to hold even in an economy in which workers can pay for jobs (but still cannot write bonding contracts).

From a normative point of view, our paper is related to recent research that also empha-

sizes the welfare costs of fierce competition for talent, such as Bénabou and Tirole (2014) and Acharya, Pagano and Volpin (2013). These papers emphasize the deleterious effect that competition for talent can have on incentives (i.e., multitasking and/or project selection issues). By contrast, our focus is on the impact that competition has on how workers are allocated to firms.

Our analysis also shares certain ideas found in models of executive markets. As in firm-CEO assignment models, workers and firms are heterogeneous (Edmans, Gabaix, and Landier, 2009; Eisfeldt and Kuhnen, 2013; Gabaix and Landier, 2008; Terviö, 2008). As in Frydman (2013) and Murphy and Zabojnik (2004, 2006), workers are endowed with both firm-specific and general skills. As in Edmans and Gabaix (2011), the process of matching workers with firms is distorted by informational frictions.

Our model is also related to a family of labor market models in which workers and firms gradually learn about worker abilities (Gibbons and Murphy, 1992; Jovanovic, 1979a and 1979b; Harris and Holmström, 1982; MacDonald, 1982 and 1988; Murphy, 1986; Taylor, 2013). In our model, jobs are “experience goods;” in other words, a worker’s ability can only be learned by observation on the job for a period of time (Jovanovic, 1979a). In contrast with most papers in this literature, we assume that (i) workers’ abilities with respect to a particular job are only partially transferrable to different jobs and, (ii) most importantly, incumbent firms acquire non-public information about the worker’s ability.

More narrowly, our model belongs to the labor literature on asymmetric learning.

In a seminal paper, Waldman (1984) considers internal job assignment as a signal of employee ability. Homogeneous firms attempt to poach workers assigned to higher-level jobs by making offers corresponding to workers’ expected values. The resulting assignment of workers to jobs is inefficient; employees who would be more productive in higher-level jobs are not promoted. In a related paper, Milgrom and Oster (1987) show that employees whose abilities are observed only by their current employers tend to be promoted less often and paid lower wages than employees whose abilities are visible to other employers. Waldman’s promotion model has also been extended by Bernhardt and Scoones (1993) and Bernhardt (1995) to analyze a number of issues, such as turnover, compensation, demotions, and other labor market outcomes.

As in our paper, Greenwald (1986) focuses on the role played by asymmetric information on employee mobility. Employees may leave a firm either for exogenous reasons or because they are not retained. A key result is that there are few layoffs among the best workers (who only quit for exogenous reasons), and the stream of people changing jobs thus disproportionately consists of “bad” employees. A key difference in our model is that mobility is always endogenous and driven by firm heterogeneity. In a related paper, Laing (1994) considers a model in which the decision to retain or fire an employee is a signal of employee ability;

however, unlike our paper, firms are homogeneous and the focus is on the properties of the optimal contract for risk-averse employees.

Some papers show that variations of the key assumptions in these models can produce significantly different results. Ricart i Costa (1988) shows that if workers learn about their abilities and are able to choose from a menu of wage contracts, there is a separating equilibrium that resolves the “lemons” problem in Waldman’s (1984) model. In our model, we assume that workers do not know their types, and our main results will hold as long as employers have *some* informational advantage about *some* aspects of their workers’ abilities.

Golan (2005) challenges a different assumption in Waldman’s model: the timing of wage offers. This study shows that if the incumbent always has the option of matching outside offers, efficiency can be restored. In an earlier paper, Lazear (1986) makes a similar point. In our model, we show that inefficient poaching can occur in equilibrium even when the incumbent is able to match outside offers.

Another application of asymmetric learning models involves the problem of investing in general and/or firm-specific skills; these models are developed in Waldman (1990), Chang and Wang (1996), and Acemoglu and Pischke (1998, 1999), among others.<sup>7</sup>

## 2. A Simple Example

A firm called  $L$  (*the incumbent*) currently employs one worker  $i$  whose type is  $\tau \in \{0, \tau', \tau''\}$ , with  $\tau'' > \tau' > 0$ . Firm  $L$  perfectly observes  $\tau$ . The worker does not observe  $\tau$ . To retain the worker,  $L$  decides whether to make a wage offer. The worker accepts to work for any wage  $w \geq 0$ . Firm  $L$ ’s profit if the worker accepts the wage offer is  $\pi_L = \tau - w$ . If the worker does not accept the offer or if  $L$  decides not to make an offer,  $L$  may hire an unemployed agent whose expected type is  $\mu > 0$ . Unemployed agents always accept to work for a wage of zero. If it hires an unemployed worker,  $L$ ’s profit is  $\pi_L^u = \gamma\mu$ , where  $\gamma < 1$  reflects the loss in firm-specific skills from replacing an incumbent worker.

There is a second firm, called  $H$ , that has a vacant position. Because it needs a worker to operate its technology,  $H$  has two options: Either hire an unemployed agent or try to poach worker  $i$ . If it hires an unemployed agent, its profit is  $\pi_H^u = \theta\gamma\mu$ , where  $\theta > 1$  implies that firm  $H$  is more productive than firm  $L$  (e.g.,  $H$  is endowed with a superior technology). To poach worker  $i$ ,  $H$  makes a wage offer of  $w^p \geq 0$  after observing the incumbent’s offer,

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<sup>7</sup>There is also an important empirical literature on asymmetric employer learning. Gibbons and Katz (1991) provide empirical evidence that is compatible with the predictions of a model of layoffs with asymmetric employer learning. Pinkston (2009) constructs a model in which firms use bidding wars to compete for talent and finds empirical evidence of substantial asymmetric employer learning. Kahn (2013) also finds substantial evidence in favor of asymmetric learning. In contrast, Schönberg (2007) finds little evidence that employer learning is asymmetric.

$w$ . We assume that a worker who holds an offer of  $w$  from  $L$  accepts a poaching offer if and only if  $w^p > w$ .

Firm  $H$  does not know what type of worker  $i$  is. It believes that the unconditional probability distribution of types is  $(p, p', p'')$ , where  $p, p', p'' > 0$  and  $p + p' + p'' = 1$ . For simplicity, we assume that the expected type,  $p'\tau' + p''\tau''$ , is equal to the expected type of an unemployed worker,  $\mu$ . As  $H$  moves only after observing whether  $i$  has an offer  $w$  from  $L$ , it should use this information to update its beliefs about  $\tau$ . Let  $E[\tau | w]$  denote  $H$ 's expectation about the type of  $i$  that is conditional on observing  $w$ .

We assume that  $H$ 's objective function is  $h(m, x) = -[\theta\gamma(m - \mu) - x]^2$  if  $H$  hires a worker with expected type  $m$  for a wage of  $x$ . This is simply a reduced-form way of modeling Bertrand competition in which many firms similar to  $H$  compete for a limited number of workers<sup>8</sup> (we explicitly assume Bertrand competition in Section 3). If  $H$  hires an unemployed agent, it optimally chooses to offer a wage of zero, because  $h(\mu, 0) = 0$ . If  $H$  makes an offer, and worker  $i$  holds the offer  $w$ , then  $w^p = \theta\gamma(E[\tau | w] - \mu)$ .

In this example, we focus on a particular case to illustrate the intuition; we consider the general case in Section 3. We assume the following conditions: (i)  $\tau' > \mu$ , (ii)  $\tau'' - \theta\gamma(\tau'' - \mu) \geq \gamma\mu$ , and (iii)  $\tau' - \theta\gamma\left(\frac{\mu}{p'+p''} - \mu\right) < \gamma\mu$ . Under these assumptions, allocational efficiency dictates that type  $\tau'$  should remain with the incumbent firm, because

$$\tau' - \gamma\mu > \theta\gamma(\tau' - \mu), \quad (1)$$

where the left-hand side is the net surplus created if type  $\tau'$  remains with the incumbent and the right-hand side is the net surplus created if type  $\tau'$  works for  $H$ . The inequality above follows immediately from (ii).<sup>9</sup>

Under these assumptions, if information is symmetric (i.e., if both  $L$  and  $H$  know the worker's type), it is easy to use backward induction to show that the unique equilibrium is such that type  $\tau'$  indeed remains with the incumbent and allocational efficiency is achieved.

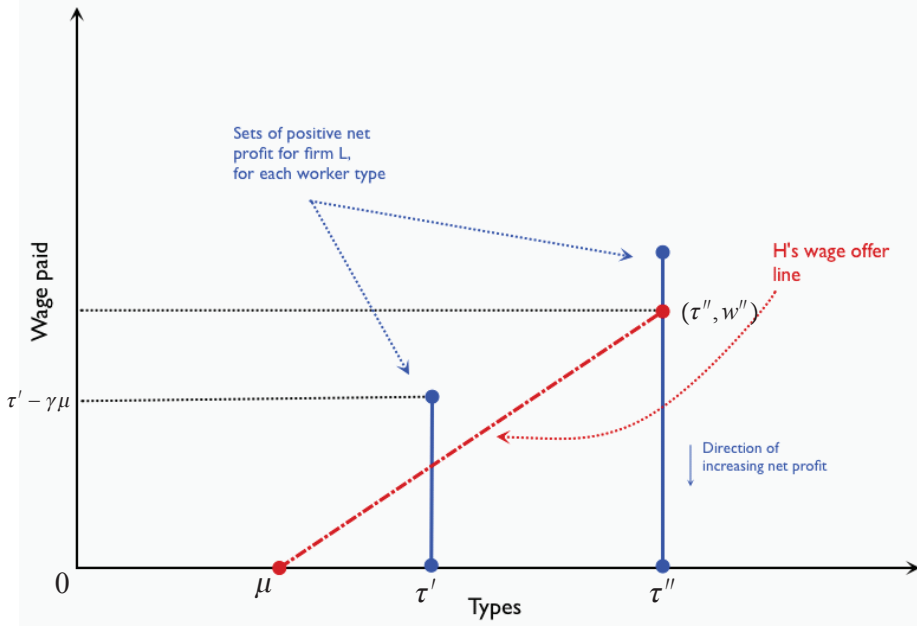
Now, we characterize the equilibrium under asymmetric information, using Perfect Bayesian Equilibrium (PBE) as the equilibrium concept. Under (iii), there is no equilibrium in which  $L$  retains type  $\tau'$  with probability 1. To illustrate this situation, suppose that  $L$  offers  $w$  to type  $\tau'$  and successfully retains this type.  $L$  should then also offer no more than  $w$  to type  $\tau''$ . Firm  $H$ , after observing  $w$ , infers that  $E[\tau | w] = \frac{p'\tau' + p''\tau''}{p'+p''} = \frac{\mu}{p'+p''}$  and then offers

$$w^p = \theta\gamma\left(\frac{\mu}{p'+p''} - \mu\right). \quad (2)$$

<sup>8</sup>See, e.g., Fudenberg and Tirole (1991), p. 329.

<sup>9</sup>Assumption (ii) implies  $(1 - \theta\gamma)\tau'' \geq \gamma\mu(1 - \theta)$ . Because  $\theta > 1$ , this trivially holds strictly if  $\theta\gamma \leq 1$ , and this would hold also if  $\tau''$  is replaced by  $\tau'$ . If  $\theta\gamma > 1$ , then replacing  $\tau''$  with  $\tau'$  strictly increases the left-hand side of this inequality, thus proving that  $\tau' - \gamma\mu > \theta\gamma(\tau' - \mu)$ .





**Figure 1.** Positive net profit sets and the wage offer line.

For successful retention, we need  $w \geq w^p$ . However, if it pays  $w = w^p$  to a worker of type  $\tau'$ ,  $L$ 's profit is

$$\pi_L = \tau' - \theta\gamma \left( \frac{\mu}{p' + p''} - \mu \right), \quad (3)$$

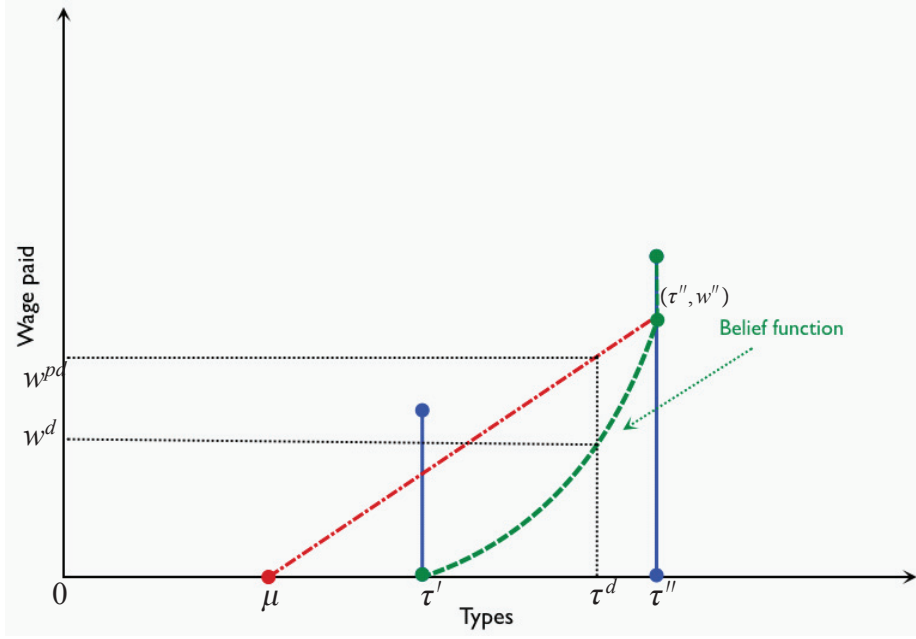
which is lower than  $\gamma\mu$  from Assumption (iii). Thus,  $L$  would not offer any  $w \geq w^p$  to type  $\tau'$ . As a consequence, any equilibrium must then involve some allocational inefficiencies.

We now describe what would be the most efficient equilibrium under Assumptions (i)-(iii). Define *net profit* for firm  $L$  as  $\tau - w - \gamma\mu$ . In other words, net profit is the profit that the firm would have if it paid wage  $w$  to type  $\tau$  minus its outside payoff  $\gamma\mu$ . Figure 1 shows the sets of positive net profits for each possible type. Within a particular set, net profit increases as wages fall.

If  $H$  believes that the worker has expected type  $m$ ,  $H$  offers wage  $w^p$  such that  $h(m, w^p) = 0$ . Figure 1 shows  $H$ 's *wage offer line*, which is the set of expected types and poaching wages  $(m, w^p)$ .

For notational convenience, we set  $w = -1$  for the case in which  $L$  dismisses type 0. The most efficient equilibrium (under the current assumptions) is as follows:  $L$  offers the wage schedule  $\{-1, 0, \theta\gamma[\tau'' - \mu]\}$  for types  $\{0, \tau', \tau''\}$ ,  $H$  offers wage  $w^p = \theta\gamma(m(w) - \mu)$  for  $w \in \{-1, 0, \theta\gamma[\tau'' - \mu]\}$ , and there is an equilibrium *belief function*  $m(w) = E[\tau | w]$ .

To see that this is indeed an equilibrium, first note that the pair  $(\tau'', w'')$  in Figure 1 lies in the interior of the positive net profit set if  $\tau = \tau''$  (which is implied by Assumption (ii)); thus, firm  $L$  is willing to pay  $w''$  for that type.



**Figure 2.** Beliefs Supporting an Equilibrium with Inefficient Poaching

If  $\tau = \tau'$ , we must assume the existence of an equilibrium belief function  $m(w)$  for  $H$  to show that  $L$  does not wish to deviate from offering  $w = 0$ . Figure 2 shows one possible belief function that is compatible with the separating equilibrium described above. Note that this function is depicted in inverse form, with  $w$  on the  $y$ -axis and  $m(w)$  on the  $x$ -axis. Note also that this belief attributes probability 1 to type  $\tau''$  if  $w''$  is observed and probability 1 to type  $\tau'$  if  $w = 0$  is observed, which indicates that it is Bayesian on the equilibrium path. To determine what this function implies off the equilibrium path, consider a possible deviation after  $L$  observes  $\tau'$ , in which  $w^d$  is offered (as in the figure). Firm  $H$  then believes that the *expected* type is  $m(w^d) = \tau^d$  (that is,  $H$  attributes some positive probability that the deviation came from type  $\tau''$ ), which implies that it should offer  $w^{pd} > w^d$ , and then poaching occurs with probability 1. Thus,  $L$  gains nothing from deviating and offering  $w^d$ . Further inspection shows that this argument holds for any deviation wage  $w^d$ , as long as the belief function lies strictly below the wage offer line for  $\tau \in (\tau', \tau'')$ .

The first-best outcome (under current assumptions) requires the retention of both  $\tau'$  and  $\tau''$ . Thus, the equilibrium exhibits too much poaching and results in misallocation of talent: Only type  $\tau''$  is retained in equilibrium. This equilibrium has a *Peter Principle Property*: Some mediocre workers (i.e., those of type  $\tau'$ ) are “promoted” to positions in better firms.

As this example shows, a competitive labor market might lead to excessive labor mobility and to the destruction of social surplus. In fact, a social planner can unambiguously increase total surplus by banning poaching, such that  $L$  retains  $\tau'$ .

In this example, we have ruled out by assumption the possibility that poaching may improve allocational efficiency. In the more general case that is analyzed in Section 3, a hypothetical social planner faces a trade-off: Competition increases welfare by assigning the most talented workers to the best firms but also results in misallocations due to excessive poaching of mediocre workers.

### 3. The Basic Model

We first present a simplified, one-period version of the model. In Section 5, we develop an infinite-horizon version of the model.

#### 3.1. Setup

There is a continuum of identical *incumbent firms* indexed by  $l \in L \equiv [0, L]$  (to save on notation, we use  $L > 0$  to denote both the set of incumbent firms and its mass). All such firms are paired with a worker, whose type (*talent*) is given by  $\tau \in [0, \bar{\tau}]$ . Each firm perfectly observes its incumbent worker's type.<sup>10</sup>

For the sake of simplicity, we assume that the worker does not observe  $\tau$ . This assumption rules out the possibility of workers signaling their types to potential employers. It also rules out the possibility of potential employers screening workers through a menu of contracts. We choose to rule out these possibilities in order to focus on the role of asymmetric information among employers. Our approach has the advantage of making clear precisely what informational assumptions are required for the results. By contrast, the related literature typically adopts a different approach that imposes exogenous restrictions on actions – and on the space of contracts – to eliminate screening and employee signaling.

Worker behavior is very simple. Workers have an outside option that gives them a payoff of zero. They always accept to work for the maximum wage that is offered to them,  $w$ , as long as  $w \geq 0$ .

A firm that successfully retains an incumbent worker of type  $\tau$  and pays wage  $w$  receives (expected) payoff  $\tau - w$ . However, the firm may choose to fire the incumbent worker and replace that worker with a randomly selected unemployed agent (we call the set of such agents the *outside pool*), whose type is distributed according to a differentiable cumulative distribution function (c.d.f.)  $F(\cdot)$  with support  $[0, \bar{\tau}]$ . Unemployed agents are in excess supply and thus accept all nonnegative wage offers. In this case, the firm's expected payoff is  $\gamma\mu$ , where  $\mu$  is the mean of  $F(\cdot)$ . Parameter  $\gamma \in (0, 1)$  represents the loss in firm-specific

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<sup>10</sup>In this one-period model, an incumbent firm knows the type of a worker from the outset. In the multi-period version in Section 5, the firm hires an inexperienced worker in the first period and learns the type of this worker in the second period.

skills that results when the incumbent worker is replaced by an outsider. Higher levels of  $\gamma$  mean that firm-specific skills are less important.

There is a second set of firms – called *poachers* – that do not have incumbent workers (this is for the sake of simplicity only; nothing substantial changes if some poachers begin the game with their own incumbent workers; see the multi-period version in Section 5). We call them poachers because they must hire someone from the outside, either by poaching from an incumbent firm or by hiring from the outside pool. There is a continuum of poachers indexed by  $h \in H \equiv [0, H]$ . All poachers are identical to one another but are different from incumbent firms. In particular, if a poacher hires a worker of type  $\tau$  for a wage of  $w^p$ , the poacher’s payoff is  $\theta\gamma\tau - w^p$ , where  $\theta > 1$  is the profitability advantage of poachers over incumbents. Intuitively, poachers are generally more productive than incumbents ( $\theta > 1$ ), but because they do not have an incumbent worker, they cannot benefit from firm-specific skills ( $\gamma < 1$ ). The poacher’s expected payoff from hiring from the outside pool is  $\theta\gamma\mu$  (the poacher’s *outside payoff*).

Incumbent firms move first. After observing their incumbent workers’ types, each firm simultaneously commits to a wage offer  $w \in \mathbb{R}$  to their incumbent workers. We permit strictly negative wage offers, as these offers will not be accepted, which implies that a negative wage offer is equivalent to dismissing the incumbent worker.

Let  $w_l$  denote firm  $l$ ’s action (i.e., the wage firm  $l$  offers to its incumbent worker). Define the set of retention wages as  $W = \{w : w = w_l \text{ for some } l \in L\}$ . The set of all possible  $W$  is the set  $\mathcal{W}$ .

Poachers only move after observing some  $W \in \mathcal{W}$ . Importantly, poachers do not observe the types of the incumbent workers. Instead, they form beliefs regarding these types after observing  $W$ . Poachers believe that the unconditional distribution of  $\tau$  is  $F(\cdot)$ .<sup>11</sup> We assume that all poachers share the same beliefs, whether on or off any equilibrium path, which is usual in sequential games with incomplete information that use Perfect Bayesian Equilibrium (PBE) as a solution concept. Thus, we denote by  $F^W(\tau_l | w_l)$  the common belief about  $\tau_l$  that poachers hold after observing that worker  $l$  has an offer of  $w_l$  when the set of all offers is  $W$ .

Let  $\omega_w$  denote the set of workers who hold an offer  $w$  from their firms, that is,  $\omega_w \equiv \{l \in L : w_l = w\}$ . We assume that each poacher can make an offer (a *poaching wage*)  $w^p(w)$  to all workers in  $\omega_w$ . We will also write the poaching wage as  $w^p(w, W)$  whenever we wish to emphasize that poaching wages are equilibrium strategies and, as such, they depend on the set of all observed wages  $W$ .

We assume that poachers compete among themselves in Bertrand fashion. In other words,

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<sup>11</sup>This assumption is made for the sake of simplicity; nothing important changes if the unconditional c.d.f. for incumbent workers is  $G(\cdot) \neq F(\cdot)$ .

no poacher can have a payoff larger than the outside payoff  $\theta\gamma\mu$ . We assume that this is true both on and off any equilibrium path. After observing the set of offers,  $W$ , a poacher offers

$$w^p(w, W) = \theta\gamma \left( \int_0^{\bar{\tau}} \tau dF^W(\tau | w) - \mu \right) \quad (4)$$

to all workers in the set  $\omega_w$ . As above, if  $w^p(w, W) < 0$ , the offer is not accepted, which means that a negative poaching wage offer is equivalent to no offer.

We will keep the following two simplifying assumptions for most of this paper:

**Assumption A1** A worker in  $\omega_w$  accepts all offers where  $w^p(w) > w$  and rejects all offers where  $w^p(w) \leq w$ .

In other words, if indifferent, a worker stays with her current employer, which is a standard assumption in the literature (see e.g., Waldman, 1984). However, this assumption entails some loss of generality because it eliminates a number of equilibria in mixed strategies. Thus, we consider Assumption A1 as an equilibrium selection criterion with intuitive properties: Workers may have a small bias against changing jobs because of unmodeled costs. In Subsection 3.3.4, we relax this assumption and discuss the additional equilibria that arise.

**Assumption A2**  $H > [1 - F(\gamma\mu)]L$ .

In other words, Assumption A2 indicates that the measure of poachers is always greater than the measure of incumbent workers whose talent is above  $\gamma\mu$ . This assumption implies that poachable workers are always in “short supply,” which is the most interesting case to analyze. Below, we keep A2 for most of the analysis, but in some applications we also briefly consider the case in which A2 does not hold.

We now formally define the timing of the game.

**Timing.**

*Date 0.* Each firm  $l \in L$  learns the type  $\tau_l \in [0, \bar{\tau}]$  of its incumbent worker.

*Date 1.* Each firm  $l \in L$  independently chooses a wage  $w_l \in \mathbb{R}$ .

*Date 2.* For a given  $W$ , each poacher  $h \in H$  simultaneously makes offers  $w^p(w, W)$ , as in (4), to all workers in  $\omega_w$  for all  $w \in W$ .

*Date 3.* A worker from firm  $l$  in  $\omega_w$  accepts all offers such that  $w^p(w_l, W) > w_l$  and rejects all offers such that  $w^p(w_l, W) \leq w_l$  (as described in Assumption A1).

*Date 4.* All incumbent firms and poachers that do not have a worker at this date randomly select one agent from the outside pool. Outside pool agents accept to work for a wage of zero.

*Date 5.* Payoffs are realized.

**Comments on timing and assumptions.** The current timing assumes that incumbent firms move before poachers. Changing the timing such that incumbent firms move after poachers and make the final offer makes retention easier, as we show in Subsection 4.3 below, but does not fundamentally affect the qualitative properties of the equilibrium.

There are only two dates when meaningful decisions are made: Dates 1 and 2, i.e., when incumbent firms and poachers choose their actions, respectively.<sup>12</sup> We only consider pure strategies at Date 1, but this is without loss of generality; the continuum assumption allows for mixing at the population level. The assumption that poachers only play pure strategies at Date 2 is also without loss of generality because of Assumption A1 (we return to this point when we relax this assumption).

The assumption that the incumbent firm makes an offer to the worker is meant to imply that workers have no bargaining power vis-à-vis incumbent firms; they either accept their offers or move elsewhere. Alternatively, there could also be multiple rounds of offers and counter-offers by incumbents and poachers. We assume a single round as a simple way of introducing costs of delayed negotiations.<sup>13</sup>

We assume that the continuation game beginning at Date 3 must be in equilibrium regardless of the history of play and that if  $w^p(w, W) > w$ , each poacher is matched with a worker with equal probability (i.e., we assume random rationing).

### 3.2. Benchmark: Symmetric Information

In this subsection, we briefly discuss the benchmark case of symmetric information. Suppose that poachers have the same information as incumbent firms. In other words, modify the timing of the game such that at Date 1, when firm  $l$  learns the type  $\tau_l \in [0, \bar{\tau}]$  of its incumbent worker, all poachers  $h$  also learn  $\tau_l$  for all  $l \in L$ .<sup>14</sup>

Poachers compete à la Bertrand for each type  $\tau_l \in [0, \bar{\tau}]$ . Their profits must equal their outside payoff,  $\theta\gamma\mu$ . Thus, the poaching wage offered to type  $\tau_l$  is given by

$$w^{pS}(\tau_l) = \theta\gamma(\tau_l - \mu), \quad (5)$$

---

<sup>12</sup>Date 0 is just a reminder that incumbent firms know the types of their workers. At Date 4, there is no meaningful choice because both incumbent firms and poachers are strictly better off by hiring a worker from the outside pool than keeping a job post unfilled. Date 3 is when workers who hold (potentially multiple) offers choose whether to stay or leave. This decision is not strategic.

<sup>13</sup>Alternatively, we could also consider a situation in which there are potentially infinite rounds of offers and counter offers, in which each additional round introduces a cost  $c$  paid by the incumbent (equivalently, the incumbent discounts the future). Poachers are competitive, thus the incumbent may face a different bidder for its worker in each round. In this modified game, the incumbent would immediately offer either the wage that would retain the worker or any wage that would not lead to retention.

<sup>14</sup>Whether firm  $l \in L$  also learns the type of  $\tau_{l'}$ ,  $l' \neq l$ , is immaterial for what follows. Similarly, workers may or may not know their types.

where the superscript  $S$  denotes symmetric information.

In a subgame perfect equilibrium, incumbent firm  $l$  solves  $\max_{w \in \mathbb{R}} \pi_l(w)$ , where

$$\pi_l(w) = \begin{cases} \tau_l - w & \text{if } w \geq \max\{\theta\gamma(\tau_l - \mu), 0\} \\ \gamma\mu & \text{otherwise} \end{cases}. \quad (6)$$

Suppose first that  $\tau_l \leq \mu$ . In this case, firm  $l$  does not have to worry about poaching and will pay  $w_l = 0$  if  $\tau_l \in [\gamma\mu, \mu]$  and some  $w_l < 0$  if  $\tau_l < \gamma\mu$  (in other words, it dismisses the worker).

If instead  $\tau_l > \mu$  and firm  $l$  wants to retain a worker, then the firm must offer at least as much as the poacher, that is,  $w_l$  must be equal to or greater than  $\theta\gamma(\tau_l - \mu) > 0$ . Then,  $l$ 's profit is  $\pi_l = \tau_l - \theta\gamma(\tau_l - \mu)$ , which implies that this is an optimal choice if and only if  $\tau_l - \theta\gamma(\tau_l - \mu) \geq \gamma\mu$ . If  $\theta\gamma \leq 1$ , then this condition is true for any  $\tau_l > \mu$  (recall that  $\theta > 1$ ). If  $\theta\gamma > 1$ , this condition holds for any  $\tau_l \leq (\theta - 1)\gamma\mu / (\theta\gamma - 1)$ .

Define

$$\tau^\# = \begin{cases} \bar{\tau} & \text{if } \theta\gamma \leq 1 \\ \min\{(\theta - 1)\gamma\mu / (\theta\gamma - 1), \bar{\tau}\} & \text{if } \theta\gamma > 1 \end{cases}. \quad (7)$$

This reasoning implies that firm  $l$ 's optimal strategy is to offer

$$w_l^S = \begin{cases} \text{any } w < 0 & \text{if } \tau_l \leq \gamma\mu \\ 0 & \text{if } \tau_l \in [\gamma\mu, \mu] \\ \theta\gamma(\tau_l - \mu) & \text{if } \tau_l \in [\mu, \tau^\#] \\ \text{any } w < w^{pS}(\tau_j) & \text{if } \tau_l \geq \tau^\# \end{cases}. \quad (8)$$

Here we make no assumption about the boundary cases (i.e.,  $\tau_l = \gamma\mu$ ,  $\tau_l = \mu$ , and  $\tau_l = \tau^\#$ ), as the properties of the equilibrium are unaffected by what happens in these cases. Thus, we always use closed intervals to denote the equilibrium sets of types.

In equilibrium, incumbent firms retain all types in  $[\gamma\mu, \tau^\#]$ . Types that are lower than  $\gamma\mu$  are fired and not poached. Types higher than  $\tau^\#$  are poached in equilibrium. There might be cases in which  $\tau^\# = \bar{\tau}$  and no one is poached.

To verify whether the equilibrium outcome of this game is efficient, we consider what a social planner trying to maximize total surplus would choose. The surplus in the symmetric information equilibrium is

$$\mathcal{S} = H\theta\gamma\mu + L \left\{ F(\gamma\mu)\gamma\mu + \int_{\gamma\mu}^{\tau^\#} \tau dF(\tau) + \int_{\tau^\#}^{\bar{\tau}} [\theta\gamma(\tau - \mu) + \gamma\mu] dF(\tau) \right\}. \quad (9)$$

To understand this expression, note that the first term on the right-hand side represents the total payoffs of poachers, which are given by their outside payoffs (due to the Bertrand

competition assumption). The second term (the first term inside the brackets) represents the profit of all firms that dismiss workers with talents lower than  $\gamma\mu$ . The third term is the aggregate revenue generated by all firms that successfully retain their workers in equilibrium. This revenue is split between firms and workers, as determined by the optimal strategy in (8). The fourth term is the aggregate wage of all workers that are poached plus the outside profits of all firms that have lost their workers to poachers.

We say that an equilibrium leads to a (*Kaldor-Hicks*) *efficient allocation of talent* when it maximizes total surplus. A social planner trying to achieve allocative efficiency would assign worker  $\tau_l$  to a poacher only if

$$\theta\gamma\tau_l - \theta\gamma\mu \geq \tau_l - \gamma\mu. \quad (10)$$

In other words, worker  $\tau_l$  is a better match with a poacher when the incremental surplus to the poacher is larger than the net loss to the incumbent firm (assuming, for simplicity, that both firms pay the same wage). Condition (10) implies that poaching should occur when

$$(\theta\gamma - 1)\tau_l \geq (\theta - 1)\gamma\mu. \quad (11)$$

It is straightforward to verify that the condition above is identical to the condition that allows poaching to occur in equilibrium (see (7) and (8)). Thus, the equilibrium outcome in this benchmark case yields an efficient allocation of talent. This result will be useful because we can infer allocative inefficiencies in equilibria under asymmetric information by comparing the equilibrium match of workers and firms with that of the benchmark case.

### 3.3. Asymmetric Information

#### 3.3.1. Equilibrium: Definition

We now define the equilibrium conditions. Poachers' strategies are given by the function  $w^p(w, W)$ , as defined in (4). We denote an incumbent firm  $l$ 's strategy by  $w_l \in \mathbb{R}$ , and a given set of such strategies is denoted by  $\tilde{w} \equiv \{w_l : l \in L\}$ .

Recall that we defined the c.d.f.  $F^W(\tau | w)$  as the common belief of poachers about the type of worker who is offered  $w$  when  $W$  is observed. Beliefs are given by a family of functions  $F^W(\tau | w)$  defined for each  $W \in \mathcal{W}$ . We denote such a family of functions simply by  $F^W$ .

Let  $\pi_l(w_l, w^p(w, W))$  denote the expected payoff to firm  $l$  if it chooses to offer  $w_l$  to its worker, while poachers play strategy  $w^p(w, W)$ . Note first that this payoff does not depend directly on the strategies of other incumbent firms or on poachers' beliefs  $F^W$ ; the poaching wage  $w^p(w, W)$  is sufficient for firm  $l$  to forecast its payoff. Second, note that firm



$l$  can compute  $\pi_l(w_l, w^p(w, W))$  with no ambiguity because we assume that poaching wages  $w^p(w_l, W)$  are given by (4) and are common knowledge.

Finally, because there is a continuum of firms  $l \in L$  and a continuum of types  $\tau \in [0, \bar{\tau}]$ , for each set of pure strategies  $\tilde{w}$  there is a unique  $W$ , which happens with probability 1. The difference between  $\tilde{w}$  and  $W$  is that the former keeps track of which firm  $l \in L$  made which offer, while the latter only contains those offers made without distinguishing among the firms that made such offers. We denote the set of wages  $W$  induced by strategy  $\tilde{w}$  by  $W(\tilde{w})$ .

**Definition 1** *A strategy profile  $(\tilde{w}, w^p(w, W))$  and a family of belief functions  $F^W$  constitute an equilibrium of the game if*

- (i) *for each  $l \in L$ ,  $w_l \in \tilde{w}$  only if  $w_l \in \arg \max_{w \in \mathbb{R}} \pi_l(w, w^p(w, W(\tilde{w})))$ ;*
- (ii) *poaching wages  $w^p(w, W)$  are given by (4); and*
- (iii) *all poachers hold identical beliefs  $F^W(\tau | w)$  for all  $w \in W$  and all  $W \in \mathcal{W}$ . These beliefs must be consistent with Bayes's rule for all  $w \in W(\tilde{w})$ . Poachers believe that the incumbent firms behave independently of one another, which specifically implies that, if  $l \neq l'$ ,  $F^W(\tau_l, \tau_{l'} | w_l, w_{l'}) = F^W(\tau_l | w_l) \cdot F^W(\tau_{l'} | w_{l'})$  for all  $W \in \mathcal{W}$ .*

This definition is equivalent to a Perfect Bayesian Equilibrium. Conditions (i)-(ii) are the standard requirement that the equilibrium strategies are best responses to one another.

Condition (iii) not only requires that beliefs are updated by Bayes's rule whenever possible but also imposes some additional weak restrictions on beliefs off the equilibrium path. As usual in PBE definitions with many players, we require all poachers to hold the same beliefs both on and off the equilibrium path. We also require that beliefs depend only on  $W$ , which is mostly for tractability. This is a slightly stronger restriction because it implies  $F^W(\tau_l | w) = F^W(\tau_{l'} | w)$  for any  $l, l' \in L$ . In particular, note that we assume that beliefs are independent of  $l \in L$  despite the fact that a strategy  $w_l$  may indeed depend on  $l \in L$  (and not only on the type  $\tau_l$ ). One interpretation is that all incumbent firms are observationally identical; thus, they cannot be differentiated by poachers when these firms play the same wage  $w$  in equilibrium.

### 3.3.2. Equilibrium: Characterization

We begin by making two additional simplifying assumptions. We first assume that an incumbent would never make an offer that is weakly dominated by making no offer:

**Assumption E1** Incumbent  $l$  offers  $w_l \geq 0$  only if  $\tau_l - w_l \geq \gamma\mu$ .

We also assume the following:

**Assumption E2** (*Divinity*) After observing an off-the-equilibrium-path wage  $w'$ , poachers believe that the probability that type  $\tau' \geq w' + \gamma\mu$  deviates is no less than the probability that type  $\tau'' > \tau'$  deviates.

Assumption E2 is a technical assumption that restricts the set of admissible off-the-equilibrium-path beliefs. This assumption is an adaptation to our setup of the Divinity Criterion from Banks and Sobel (1987). Assumption E2 is not particularly restrictive and is compatible with (infinitely) many off-the-equilibrium beliefs; thus, it does not eliminate equilibrium multiplicity. None of our main conclusions depends on this assumption.<sup>15</sup>

The role of Assumptions E1 and E2 is to restrict the set of equilibria; thus, they may be interpreted as equilibrium selection criteria. They simplify the analysis significantly, although they do not eliminate equilibrium multiplicity.

We now state some preliminary results.

**Lemma 1** *In any equilibrium, all retained workers are offered the same wage.*

This important result has a very simple proof. Suppose that there are two types,  $\tau'$  and  $\tau''$ , where  $\tau'' > \tau'$ . Suppose that the incumbent wishes to retain both types. Suppose also that  $w'' > w'$  (the argument is analogous if  $w'' < w'$ ). This situation cannot be an equilibrium because there is a profitable deviation for an incumbent with worker  $\tau''$ . Indeed, the incumbent prefers to offer  $w'$  to a worker of type  $\tau''$ . Type  $\tau''$  would nonetheless be retained, but at a lower wage.

**Lemma 2** *Any equilibrium must have a threshold property: If type  $\tau'$  is retained by an incumbent in equilibrium, type  $\tau'' > \tau'$  must also be retained in equilibrium.*

This is again easily proven: For a given retention wage,  $w$ , if it is optimal to retain  $\tau'$  ( $\tau' - w \geq \gamma\mu$ ), then it is also optimal to retain any  $\tau$  such that  $\tau \geq \tau'$ .

We now state and prove (in Appendix 1.1) our main proposition:<sup>16</sup>

**Proposition 1** *The set of equilibria is non-empty. All equilibria have the following properties:*

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<sup>15</sup>The intuition for Assumption E2 is as follows. For concreteness, suppose that type  $\tau''$  is retained in equilibrium with wage  $w''$ , while type  $\tau' < \tau''$  is not retained (the intuition for the other cases is analogous to this example). An incumbent with a worker of type  $\tau''$  who deviates and offers this type a wage  $w' < w''$  can only benefit from the deviation if poachers offer  $w^p(w') \leq w'$ . However, for this set of poaching wages, type  $\tau'$  would also benefit from a deviation. On the other hand, type  $\tau''$  would be worse off if  $w^p(w') > w'$ , whereas type  $\tau'$  would not be worse off. Thus, the logic of Banks and Sobel's Divinity Criterion requires that the probability of  $\tau'$  deviating should be no less than that of  $\tau''$  deviating.

<sup>16</sup>In what follows, for simplicity, we define all equilibrium sets of types as closed intervals. That is, we refrain from specifying what happens in equilibrium in the knife-edge cases in which an incumbent is indifferent between retaining or not retaining a type. The equilibrium is unaffected by what happens in these cases.

1. There is a unique  $\tilde{\tau} \in [\gamma\mu, \bar{\tau}]$  such that all types  $\tau \geq \tilde{\tau}$  are retained. Threshold  $\tilde{\tau}$  is the same for all equilibria and is either  $\bar{\tau}$  or the least element of the set of fixed points of

$$H(\tau) \equiv \tau - \theta\gamma \left( \int_{\tau}^{\bar{\tau}} x dF(x | x \geq \tau) - \mu \right) - \gamma\mu, \quad (12)$$

that is,  $\tilde{\tau}$  is the lowest of all  $\tau$  such that  $H(\tau) = 0$ .

All retained workers are offered the wage

$$w^* = \theta\gamma \left( \int_{\tilde{\tau}}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tilde{\tau}) - \mu \right). \quad (13)$$

2. All types  $\tau \in [0, \gamma\mu]$  are fired in equilibrium.
3. There is a subset of types  $P \subseteq [\gamma\mu, \tilde{\tau}]$  that are poached in equilibrium and a subset of types  $S \subseteq [\gamma\mu, \tilde{\tau}]$  that are fired in equilibrium, with  $S \cup P = [\gamma\mu, \tilde{\tau}]$ .

This proposition shows that, in equilibrium, incumbent workers will find themselves in one of the following three situations: unemployed, employed by the incumbent firm, or employed by a poacher. Part 1 of Proposition 1 implies that the very best workers will typically be retained by the incumbent firm, which implies that if workers are retained at all, they must be the best workers. By contrast, in the symmetric information benchmark discussed in Subsection 3.2, retained workers are not the best workers. Thus, the equilibrium will often be inefficient.

Part 2 of Proposition 1 implies that workers with sufficiently low abilities always end up unemployed, which is not surprising. Part 3 implies that only those workers with talent in the interval  $[\gamma\mu, \tilde{\tau}]$  may be poached in equilibrium. With some abuse of language, we call these workers *mediocre workers*, although in some cases this interval will also contain the very best workers (e.g., if  $\tilde{\tau}$  is close to or equal to  $\bar{\tau}$ ).

Proposition 1 also reveals that equilibria differ from one another (meaningfully) only because the sets  $P$  and  $S$  may differ.<sup>17</sup> Because we focus on the efficiency properties of the equilibria, it is natural to select the most efficient equilibrium as the focal equilibrium. The next proposition characterizes the most efficient equilibrium:

**Proposition 2** *There is a most-efficient equilibrium in which  $P = [\mu, \tilde{\tau}]$  and  $S = [\gamma\mu, \mu]$ .*

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<sup>17</sup>Two observationally equivalent equilibria with the same  $P$  and  $S$  may also differ from one another because they are sustained by different beliefs off the equilibrium path and may display different retention wages for types in  $P$ .

In the most efficient equilibrium, the equilibrium outcome changes monotonically with  $\tau$ : As  $\tau$  increases, outcomes change from unemployment to poaching and then from poaching to retention.

To focus on the most interesting case, in the remainder of this paper, we assume that the following condition holds:

**Condition H**  $\max_{\tau \in [0, \bar{\tau}]} H(\tau) > 0$ .

Under Condition H, there is an interior  $\tilde{\tau} < \bar{\tau}$  such that all types  $\tau \in [\tilde{\tau}, \bar{\tau}]$  are retained by incumbents. Condition H always holds for any set of parameters if  $\bar{\tau} \rightarrow \infty$ . If Condition H does not hold, incumbent firms never retain any worker in equilibrium, i.e.,  $\tilde{\tau} = \bar{\tau}$ .

### 3.3.3. Equilibrium: Efficiency

The most efficient equilibrium implies that types  $\tau \in [\tilde{\tau}, \bar{\tau}]$  are retained by incumbents,  $\tau \in [\mu, \tilde{\tau}]$  are poached, and  $\tau \in [0, \mu]$  are fired. By contrast, allocational efficiency requires types  $\tau \in [\tau^\#, \bar{\tau}]$  to be poached (recall the definition of  $\tau^\#$  in (7)),  $\tau \in [\gamma\mu, \tau^\#]$  to be retained, and  $\tau \in [0, \gamma\mu]$  to be fired. Thus, the most efficient equilibrium does not lead to an efficient allocation of talent, which is formally stated in the next corollary.

**Corollary 1** *The most efficient equilibrium under asymmetric information is talent-allocation inefficient. In particular, there are three different sources of misallocation of talent:*

1. **Excessive firing:** *Types  $\tau \in [\gamma\mu, \mu]$  are fired but should have been retained.*
2. **Excessive poaching of mediocre types:** *Types  $\tau \in [\mu, \min\{\tau^\#, \tilde{\tau}\}]$  are poached but should have been retained.*
3. **Insufficient poaching of high types:** *Types  $\tau \in [\max\{\tau^\#, \tilde{\tau}\}, \bar{\tau}]$  are retained but should have been poached.*

The corollary above demonstrates the main qualitative ideas that we want to discuss. The possibility of poaching creates three distortions relative to the first-best scenario. Incumbent firms do not try to retain some workers who are potential poaching targets, which leads to excessive turnover. Such turnover results in misallocation of talent because some workers who have acquired firm-specific skills are either inefficiently fired (Case 1) or inefficiently poached by high-profitability firms (Case 2). Thus, the equilibrium displays a “Peter Principle Property”; mediocre workers are “promoted” to positions in better firms, whereas the best workers stay with their current employers. Finally, firms might be too successful in retaining workers who would otherwise be matched with better firms in the first-best allocation. In other words, there might be too little poaching in equilibrium (Case 3).

### 3.3.4. Mixed-strategy Equilibria

We have focused thus far on pure-strategy equilibria. In this subsection, we consider the possibility of mixed-strategy equilibria.

We now temporarily drop Assumption A1 to allow for the possibility of mixed-strategy equilibria. In a mixed-strategy equilibrium, a type- $\tau$  worker who is indifferent between accepting or rejecting a poaching offer (i.e., an offer such that  $w^p(w) = w$ ) rejects the poaching offer with probability  $p(w)$ . We then obtain the following result:

**Lemma 3** *In any equilibrium,  $p(w)$  is non-decreasing in  $w$ .*

Lemma 3 implies that higher types are more likely to be retained in any equilibrium. Because the first-best allocation implies that only the best types should be poached (if anyone should be poached at all), Lemma 3 implies that mixed-strategy equilibria are also talent-allocation inefficient. Furthermore, mixed-strategy equilibria also typically involve the inefficient poaching of mediocre workers. Thus, allowing for mixed-strategy equilibria does not restore efficiency, and our qualitative results are not affected by Assumption A1. In Appendix 1.2, we fully characterize equilibria involving strictly mixed strategies.

### 3.3.5. Comparative Statics

To perform comparative statics, we focus on two parameters with intuitive interpretations. The first is  $\theta$ , which could be interpreted as the (cross-sectional) dispersion in firm profitability. Because firm profitability in reality may be positively related to firm survival and growth,  $\theta$  can also be interpreted as a measure of heterogeneity in firm sizes. The second parameter,  $\gamma$ , measures the importance of general skills relative to firm-specific skills. Alternatively, an increase in  $\gamma$  can also be interpreted as a decrease in the cost of recruiting a new worker (e.g., search costs).

Under Condition H,  $\tilde{\tau}$  is the least fixed point of  $H(\tau)$ . Because  $H(0) < 0$ ,  $H'(\tilde{\tau}) > 0$ . By the implicit function theorem

$$\frac{d\tilde{\tau}}{d\theta} = \gamma \frac{\int_{\tilde{\tau}}^{\bar{\tau}} \tau f(\tau) d\tau - [1 - F(\tilde{\tau})] \mu}{[1 - F(\tilde{\tau})] H'(\tilde{\tau})} > 0. \quad (14)$$

In other words, the retention threshold increases with the profitability dispersion parameter  $\theta$ . Intuitively, as poachers and incumbent firms become more heterogeneous, incumbents find it increasingly more difficult to retain workers and are thus only able to retain the very best workers. This result also means that job mobility increases with  $\theta$  because the set of poached workers  $[\mu, \tilde{\tau}]$  increases with  $\tilde{\tau}$ . This increase in mobility can be either efficient or inefficient. For example, if  $\tilde{\tau} < \tau^\#$ , then increasing  $\theta$  leads to more inefficient poaching.

The effect of the importance of general skills relative to firm-specific skills is also easily inferred from

$$\frac{d\tilde{\tau}}{d\gamma} = \theta \frac{\int_{\tilde{\tau}}^{\bar{\tau}} \tau f(\tau) d\tau - [1 - F(\tilde{\tau})] \mu \left( \frac{\theta-1}{\theta} \right)}{[1 - F(\tilde{\tau})] H'(\tilde{\tau})} > 0. \quad (15)$$

The retention threshold increases with the relative importance of general skills  $\gamma$ . Again, this result is intuitive: There is more poaching when general skills are more important (i.e., if skills are more portable). Therefore, an increase in  $\gamma$  also increases job mobility. As above, an increase in poaching may be either efficient or inefficient.

## 4. Interpretation and Discussion

In this section, we consider possible solutions to the talent misallocation problem. Our focus is on the most novel form of inefficiency, i.e., the excessive poaching of mediocre types. In Subsection 4.1, we consider direct interventions in the labor market. In Subsection 4.2, we consider voluntary contractual solutions (those that we have assumed away until now). In Subsection 4.3, we consider changes in the structure of the competition for talent, such as the timing of the offers.

To preview our conclusions, we show that both private and government-mandated solutions that place restrictions on outsiders' ability to poach workers can sometimes restore efficiency. We argue however that such solutions are often imperfect, may entail costs, and are sometimes infeasible or unrealistic.

### 4.1. Labor market interventions

Corollary 1 implies that labor market interventions that restrict poaching can increase or decrease welfare. Thus, one cannot generally say whether such policies are desirable from an allocational point of view. However, the model does offer hints about when such interventions are more likely to improve efficiency. To illustrate this point, consider the following result:<sup>18</sup>

**Proposition 3** *Restrictions on poaching, such as wage caps, unambiguously improve the allocation of talent in the economy if*

$$\tilde{\tau} < \tau^\# = \frac{\gamma\mu(\theta-1)}{\theta\gamma-1}. \quad (16)$$

If Condition 16 holds, if poaching occurs in equilibrium, it is welfare-reducing. Thus, restrictions on poaching would unambiguously improve welfare. Our comparative statics results show that welfare-reducing poaching is more likely when  $\theta$  is low. In that case,  $\tilde{\tau}$  is

<sup>18</sup>The proof of this result is immediate and thus omitted.

low and  $\frac{\gamma\mu(\theta-1)}{\theta\gamma-1}$  is large, which slacks condition (16). Intuitively, if firms are not too different from one another in terms of profitability, then worker mobility is unlikely to improve talent allocation because the benefits of matching good firms with high-talent workers are offset by the loss of firm-specific skills. An analogous result follows for  $\gamma$ : When general skills are not very important (low  $\gamma$ ), equilibrium poaching is more likely to be inefficient.

As a possible application of these ideas, consider the case of regulated industries. In such industries, regulation would typically reduce the profitability differential across firms. Thus, our analysis suggests that most poaching activity is likely to be inefficient. As a consequence, it might make sense to restrict poaching in such industries. A good example is the banking industry, where high salaries are typically justified as a normal consequence of fierce competition for talent. Our model illustrates that competition does not imply efficiency. In particular, if banks are not too different from one another, our model suggests that competitive poaching may contribute to an inefficient allocation of talent in the industry.

Under condition (16), a simple policy to improve welfare is as follows: The planner imposes a maximum wage of  $\bar{w} = 0$ . The incumbent firm would then retain all workers with talent equal to or above  $\gamma\mu$ . Because poachers cannot offer a higher wage, all such workers would successfully be retained by the incumbent, and this equilibrium is more efficient than the competitive equilibrium.

The wage-cap policy also has redistributive consequences. Incumbent firms (i.e., their shareholders) are the big winners because they retain talented workers at low cost and enjoy profits that are unambiguously larger than those in the unconstrained-competition equilibrium. High-talent workers are the big losers because they now capture nothing of the surplus that they help create. Finally, poachers neither gain nor lose.

A typical problem under pay regulation is the possibility that high-ability workers might choose to migrate to jobs in unregulated firms. For example, bankers who face constraints on their pay might move to other sectors – such as hedge funds or private equity – in which pay regulation is not (yet) in place. Alternatively, they may move to other countries or jurisdictions in which there is no pay regulation. Thus, the consequences of pay regulation would simply be a (possibly inefficient) reallocation of workers to alternative jobs with little or no actual reduction in the average wage in the economy.

To think about such possibilities, we first calculate the average wage in an unconstrained-competition equilibrium:

$$\begin{aligned}
E(w^u) &= [F(\tilde{\tau}) - F(\mu)] \theta \gamma \left( \int_{\mu}^{\tilde{\tau}} \frac{\tau f(\tau)}{F(\tilde{\tau}) - F(\mu)} d\tau - \mu \right) + \\
&\quad + [1 - F(\tilde{\tau})] \theta \gamma \left( \int_{\tilde{\tau}}^{\bar{\tau}} \frac{\tau f(\tau)}{1 - F(\tilde{\tau})} d\tau - \mu \right) \\
&= \theta \gamma \left( \int_{\mu}^{\bar{\tau}} \tau f(\tau) d\tau - (1 - F(\mu)) \mu \right). \tag{17}
\end{aligned}$$

Consider now a wage cap  $\bar{w} = 0$  that applies to all incumbent firms but only to a small subset of poachers (i.e., there is a large set of poachers in unregulated jurisdictions). Consider the following equilibrium: The incumbent tries to retain all workers with talent equal to or above  $\mu$  but does not succeed because these workers would all be poached by the unregulated poachers at wage

$$w' = \theta \gamma \left( \int_{\mu}^{\bar{\tau}} \frac{\tau f(\tau)}{1 - F(\mu)} d\tau - \mu \right), \tag{18}$$

which implies an average wage of  $E(w^r) = [1 - F(\mu)] w'$  for the regulated equilibrium, which is identical to  $E(w^u)$ . Thus the regulation has no effect on the average wage (although it decreases the dispersion of pay across workers). However, the wage cap increases the labor mobility rate from  $[F(\tilde{\tau}) - F(\mu)]$  to  $[1 - F(\mu)]$ , which has an ambiguous effect on welfare. In sum, the wage cap policy does not achieve its intended goal because average pay does not fall. However, the wage cap may have an unintended consequence because there is now more turnover that might lead to greater misallocation of talent.

However, these conclusions are sensitive to the assumption that there is a large set of poachers in unregulated jurisdictions. Let  $H = H_1 + H_2$ , where  $H_1$  is the mass of regulated poachers and  $H_2$  is the mass of unregulated poachers. Assumption A2 implies that  $H_1 + H_2 > [1 - F(\gamma\mu)] L$ . But suppose that the mass of unregulated poachers is not too large and that  $H_2 < [1 - F(\gamma\mu)] L$ . Now, the regulated equilibrium can be described as follows. The incumbent attempts to retain all workers with talent equal or above  $\gamma\mu$ . Assume for the sake of simplicity that  $E[\tau \mid \tau \geq \gamma\mu] > \mu$  (which is always true for  $\gamma$  sufficiently close to 1). Then, all firms in  $H_2$  will try to poach incumbent workers. However, because there are more talented workers than there are poachers ( $H_2 < [1 - F(\gamma\mu)] L$ ), the workers are now competing for jobs, which implies that unregulated poachers can hire some workers for a wage that is arbitrarily close to zero. The remaining  $[1 - F(\gamma\mu)] L - H_2$  incumbent workers are retained by their firms.

In this case, the average wage after the regulation is zero. In other words, the regulation succeeds in its goal of reducing wages, despite the fact that some workers move to unregulated



firms. The effect of the regulation on turnover – and thus on allocational efficiency – is again ambiguous. Before the regulation, a fraction  $[F(\tilde{\tau}) - F(\mu)]$  of incumbent workers are inefficiently poached. After the regulation, the fraction of poached workers is  $H_2/L$ . For small  $H_2$ , the regulation may also succeed at improving allocational efficiency. Importantly, this conclusion holds even when the regulator cares only about the surplus generated by the regulated firms, as would be the case when the unregulated firms are based in foreign countries.

We conclude that “brain drain” does not necessarily undermine the effectiveness of pay regulations. The typical argument assumes that there is unlimited demand for workers abroad (i.e., in unregulated jurisdictions), there are few or no mobility costs, and there are no efficiency gains from distorting prices. In reality, unregulated labor demand is limited, there are mobility costs, and price distortions may improve efficiency, as our model illustrates. Another often forgotten effect is that pay regulation increases the supply of workers available to unregulated firms. As our model illustrates, this supply effect may lead to lower worker bargaining power and wages may fall as a consequence – even in the unregulated sector.

## 4.2. Potential Contractual Solutions

Suppose that condition (16) holds and job mobility is unambiguously inefficient. Are there voluntary contractual arrangements that restrict mobility and thus restore efficiency? In other words, what are the types of contracts that we are ruling out by assumption? In this subsection, we consider a set of alternatives and explain why some would work and others would not.

### Fines and Bonding Contracts

Under condition (16), efficiency is restored when the firm is allowed to impose large fines on workers who leave voluntarily. Such fines might be set high enough so that no worker would choose to leave voluntarily. Under such a contract, it is easy to see that the incumbent firm would retain all workers with talent higher than  $\gamma\mu$  at wage zero, and no poaching would thus occur in equilibrium.

As an alternative to fines, the firm could instead request that its workers put some money into an account, and this money is given back to the workers at the end of the game only if they still work for the firm or if they are fired. Such bonding contracts are conceptually equivalent to fines for quitting and would thus achieve the same outcome.

We assume away all such forms of bonding contracts. The absence of bonding contracts is a realistic institutional friction in real-world labor markets. In practice, an excessively large fine on workers who quit would be akin to slavery, and smaller fines or other bonding arrangements may be inadequate or infeasible under limited liability and borrowing constraints. For such contracts to work, either limited liability constraints are not binding (and

workers can pay large fines) or borrowing is costless (and workers can have sufficient funds ex ante to enter into bonding contracts).

The debate on the realism of models in which workers effectively commit themselves to firms (through bonding contracts or other mechanisms) is not new. For example, Becker (1962) explains why long-term contracts that insure firms against employees quitting are rare by arguing that “(...) *courts have considered them a form of involuntary servitude. Moreover, any enforceable contract could at best specify the hours required on a job, not the quality of performance. Since performance can vary widely, unhappy workers could usually “sabotage” operations to induce employers to release them from contracts.*” Other examples from both sides of this debate include Carmichael (1985), Shapiro and Stiglitz (1985), and Dickens, Katz, Lang, and Summers (1989), among others.

### **Transfer Fees**

Consider now a contract in which a poacher must pay a “transfer fee” to an incumbent firm when an incumbent worker is poached. Alternatively, a worker could be charged this fee when she/he quits voluntarily to work for another firm. In the latter case, the fee could be set as a fraction of the wage offered by the poacher, such that the fee system is “self-financed” because the worker does not need to put money into the system through borrowing or personal savings.

If designed properly, such transfer fee arrangements would restore efficiency because they would effectively bind workers to firms as would fines and bonding contracts. Transfer fee arrangements might also seem easier to implement in practice, as they might work even under limited liability and borrowing constraints.

In reality, transfer fees are common in the sports and entertainment industries, but uncommon in most other industries. In sports, transfer fees are typically paid by the poaching firms and not by the athletes. Contracts are frequently of limited duration; thus, after some years under a given contract, athletes can move at no cost. A problem with such contracts is that athletes may shirk and underperform if the incumbent team refuses to sell them to another team (as is illustrated in Becker’s quote above), which may explain why teams often sell their players (reluctantly) for less than the contractual transfer fee.

Although such transfer fees might work in our model, their practical implementation is imperfect due to the reasons discussed above.

### **Non-compete Clauses**

Contractual clauses that forbid workers to quit and work for competitors are common in a number of high-skill occupations. Such non-compete clauses typically impose a quarantine period before a worker can join another firm in the same industry and/or geographical region.

Non-compete clauses only offer an imperfect solution to the problem because they are of limited duration and because the definitions of both the applicable industry and geographical

area may be fuzzy in practice. Furthermore, non-compete clauses are controversial and have often been challenged in courts.<sup>19</sup>

### Contingent Deferred Compensation

Suppose that a firm could defer all compensation to be paid to a worker until the end of the game, conditional on the worker not (voluntarily) quitting the firm. Thus, such contingent compensation schemes could in principle bind workers to firms and restore efficiency. Furthermore, such schemes are appealing because they do not require the worker or other firms to make transfers to the incumbent and do not impose bans on mobility. Here we show that such schemes, even when feasible, may not be voluntarily adopted by firms (at least not as a solution to the inefficient poaching problem), because some of the efficiency gains produced by such schemes would be captured by workers, which could make firms worse off.

To illustrate this result, consider the following contract: Before the incumbent firm learns its worker's type (i.e., at "Date -1"), the firm commits to a fixed wage  $\bar{w}$  to be paid at the end of the game, but only if the worker remains with the firm or if the worker is fired. Under condition (16), retention of all workers with talent greater than  $\gamma\mu$  is welfare-improving, and those workers with talent lower than  $\gamma\mu$  should be fired. To retain types  $\tau \geq \gamma\mu$ , the lowest wage that must be offered is  $\bar{w} = w^p(\bar{w}) = \theta\gamma \left( \int_{\gamma\mu}^{\bar{\tau}} \frac{\tau f(\tau)}{1-F(\gamma\mu)} d\tau - \mu \right)$ . Under commitment to  $\bar{w}$ , expected profit to the incumbent is thus

$$E[\pi_c] = F(\gamma\mu)\gamma\mu + [1 - F(\gamma\mu)] \int_{\gamma\mu}^{\bar{\tau}} \frac{\tau f(\tau)}{1 - F(\gamma\mu)} d\tau - \bar{w}. \quad (19)$$

Without commitment, we know that the equilibrium implies that the incumbent chooses some  $\tilde{\tau} \geq \gamma\mu$ , and thus its expected profit is

$$E[\pi_{nc}] = F(\tilde{\tau})\gamma\mu + [1 - F(\tilde{\tau})] \left[ \int_{\tilde{\tau}}^{\bar{\tau}} \frac{\tau f(\tau)}{1 - F(\tilde{\tau})} d\tau - \theta\gamma \left( \int_{\tilde{\tau}}^{\bar{\tau}} \frac{\tau f(\tau)}{1 - F(\tilde{\tau})} d\tau - \mu \right) \right]. \quad (20)$$

It can be shown, through simple examples, that  $E[\pi_{nc}] \leq E[\pi_c]$  depending on the parameters. Deferred compensation schemes (such as, e.g., restricted shares or vesting of stock options) are costly to the firm because some workers who are fired are still paid  $\bar{w}$ , which leaves rents to dismissed workers. Thus, the expected excess cost of such a scheme is  $F(\gamma\mu)\bar{w}$ . Without such a scheme, the overall surplus is lower but the profit could still be larger. Hence, deferred compensation contracts may not be chosen by firms even when they are feasible.

Formally, our model rules out all contracts that effectively give the firm some form of "ownership" over the employee. Fines, bonding contracts, transfer fees, non-compete clauses,

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<sup>19</sup>For example, in California non-compete clauses are considered void and non-enforceable, except in a small set of cases.

and deferred compensation contracts are all restrictions on free labor mobility. Free labor mobility is what distinguishes our model from models of trading of physical assets under asymmetric information. Any mechanism that results in the ownership of labor resembling the ownership of physical assets would eliminate the inefficient mobility result.

### 4.3. Changing the Timing of the Offers

Under the current assumptions about the timing of the game, the uninformed party (the poacher) moves last. In this subsection, we now introduce the case in which the informed party (the incumbent) moves last.

We modify the original timing slightly by adding a date between Dates 2 and 3:

*Date*  $2\frac{1}{2}$ . Each firm  $l$  independently makes a counter offer  $w_l^c$ .

At Date 3, a worker from firm  $l$  who holds an initial offer  $w_l$ , a poaching offer  $w^p(w_l, W)$ , and a counter offer  $w_l^c$ , accepts the poaching offer if and only if  $w^p(w_l, W) > \max\{w_l, w_l^c\}$ . We now characterize the equilibrium under this modified timing. For the sake of brevity, we focus only on the equilibrium that displays the maximum amount of retention by the incumbent firm.<sup>20</sup> First, define the set  $Y \equiv \{y \in Y : G(y) = 0\}$  where

$$G(y) \equiv y - \theta\gamma \left( \frac{\int_{\gamma\mu}^y \tau dF(\tau)}{F(y) - F(\gamma\mu)} - \mu \right) - \gamma\mu. \quad (21)$$

We then have the following result:<sup>21</sup>

**Proposition 4** *The (maximum-retention) equilibrium has the following properties:*

1. *There is a unique  $\tilde{\tau}' \in [\gamma\mu, \bar{\tau}]$  such that all types  $\tau \geq \tilde{\tau}'$  are retained. Threshold  $\tilde{\tau}'$  is given by*

$$\tilde{\tau}' = \begin{cases} \text{the largest element in } \{\gamma\mu\} \cup Y & \text{if } G(\bar{\tau}) \geq 0 \\ \bar{\tau} & \text{if } G(\bar{\tau}) \leq 0 \end{cases}. \quad (22)$$

*All retained workers are offered wage*

$$w^{*l} = \max \left\{ \theta\gamma \left( \frac{\int_{\gamma\mu}^{\tilde{\tau}'} \tau dF(\tau)}{F(\tilde{\tau}') - F(\gamma\mu)} - \mu \right), 0 \right\}. \quad (23)$$

<sup>20</sup>In the original game, the most-efficient equilibrium is also the equilibrium that maximizes retention. By contrast, in the modified game, these two properties (“most-efficient” and “maximum-retention”) may not lead to the same equilibrium. For comparing the two games, we choose the maximum retention criterion as the most natural. However, our conclusions are not sensitive to using alternative equilibrium-selection criteria.

<sup>21</sup>The proofs are in Appendix 1.3.

2. All types  $\tau \in [0, \gamma\mu]$  are fired in equilibrium.
3. All types  $\tau \in [\gamma\mu, \tilde{\tau}']$  are poached in equilibrium.

The equilibrium outcome is qualitatively similar to the outcome in Proposition 1: All types above a threshold are retained, and only mediocre types are poached. Thus, our main result that asymmetric information creates inefficiencies in talent allocation does not depend on whether the informed party moves last or not. In particular, we note that not only inefficient retention is possible, but also that inefficient poaching will often occur because at least a subset of types in  $[\gamma\mu, \tilde{\tau}']$  should be retained in the first-best allocation.

An important property of this equilibrium is as follows:

**Proposition 5** *In the modified game in which the incumbent moves last, fewer types are poached in equilibrium:*

$$\tilde{\tau}' \leq \tilde{\tau}. \tag{24}$$

This proposition demonstrates that when the incumbent has the option to make the last offer, it is able to retain the worker more often. This result is unsurprising because this modified timing gives more market power to the incumbent. One interpretation for this timing of offers is that if the worker accepts the incumbent's offer at date  $2\frac{1}{2}$ , this offer becomes binding and the worker can no longer accept a poaching offer.

Because of (24), the modified game is more likely to display inefficient retention than the original game. The modified game is less likely to display inefficient poaching than the original game for the same reason. Thus, by giving the incumbent the option to make a final binding offer, poaching inefficiencies can be reduced and sometimes eliminated. Such an option works as the wage cap discussed in Subsection 4.1 and the bonding arrangements discussed in Subsection 4.2 because all of these mechanisms restrict competition for talent. In fact, it can be shown that once the incumbent is given the option of making a final binding offer, the wage cap policy would never increase welfare. Thus, these two mechanisms of restricting competition are substitutes, to a certain extent.

Again, we conclude that restricting competition for talent may have positive welfare consequences.

## 5. A Dynamic Model

We now extend the analysis to more than one period, which allows for a more natural interpretation of the model. In particular, unlike the one-period model in which incumbents learn their workers' types immediately, in the multi-period version, we can explicitly model the process of employer learning over time. In addition, we can now treat both types of

firms symmetrically; we allow both types of firms to have incumbent workers and to become poachers if they choose.

The economy is populated with many infinitely-lived firms. Again, firms can be of one of two types,  $L$  or  $H$ , and these represent both the type and the mass of firms of each type. To simplify the notation and the exposition, we denote a representative firm of each type by  $i \in \{l, h\}$ , which also denotes the profitability parameter, i.e.,  $h = \theta$  and  $l = 1$ , where  $\theta > 1$ .

Workers live for two periods: young age and old age. Firms and workers are risk-neutral and share a common discount factor  $\delta \in [0, 1)$ . At each period  $t$  ( $t = 0, 1, 2, \dots$ ), a mass  $M$  of young workers enter the labor market. Young workers are in excess supply:  $M > H + L$ . The outside option of an unemployed agent (young or old) is normalized to zero.

As above, we assume that bonding arrangements (fines, transfer fees, non-compete clauses, etc.) are not feasible. However, to focus on the role of asymmetric learning, we assume that workers are not protected by limited liability because limited liability would generate inefficiencies in the dynamic model even when learning is symmetric.

At the beginning of a period, a firm can be in one of the following states:

- i) The firm has a vacant position because its worker retired at the end of the previous period (that is, the worker was old).
- ii) The firm does not have a vacant position because its worker was young in the previous period.

Both types of firms may have incumbent workers and may also become poachers. Thus, we no longer use the term “poacher” as synonymous with a type- $h$  firm; a type- $h$  firm may behave sometimes as an incumbent firm and sometimes as a poacher. The same applies to type- $l$  firms.<sup>22</sup>

In each period  $t$ , the timing of actions is as follows:

**Timing.**

*Date 1.* Each type- $i$  firm with an incumbent (old) worker who is known to be of type  $\tau$  independently chooses wage  $w_i(\tau) \in \mathbb{R}$ .

*Date 2.* After observing all wage offers  $w_i$ , all firms that have a vacant position simultaneously make offers according to the function  $w^p(w_i)$ .

*Date 3.* A worker who holds an offer  $w$  accepts all poaching offers such that  $w^p(w) > w$  and rejects all poaching offers such that  $w^p(w) \leq w$  (as described in Assumption A1).

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<sup>22</sup>For some constellations of parameters, it is possible to have equilibria in which some or all poachers are  $l$ -firms. For the sake of brevity, we do not analyze those cases here; our goal is to present the case that is most similar to the static model.

*Date 4.* All firms that do not have a worker at this date randomly select one young agent from the outside pool and offer the wage  $w_i^y \in \mathbb{R}$  (i.e.,  $w_i^y$  could be positive or negative), for  $i \in \{l, h\}$ .<sup>23</sup>

*Date 5.* Payoffs are realized and a young worker's type is revealed to the worker's employer.

In sum, a firm that hires a young worker (whose average type is  $\mu$ ) learns about the type of its worker only at the end of the period (at Date 5) and thus begins the subsequent period with an old worker whose type is known. For simplicity, we also assume that firm performance is not observed by outsiders or (equivalently) that observed firm performance is noisy and thus not informative about a young worker's type.

A type- $h$  firm can attempt to poach a worker from a type- $l$  firm or from another type- $h$  firm. In general, we also allow type- $l$  firms to make poaching offers. However, for simplicity, we (implicitly) restrict our analysis to a set of parameters for which, in equilibrium, workers would strictly prefer poaching offers from type- $h$  firms. Thus, without loss of generality, we assume that type- $l$  firms cannot poach workers.

To understand the differences between the static model and the dynamic model, we initially focus on the symmetric learning version of the model and later solve the model with asymmetric learning. The mathematical details and the proofs are relegated to Appendix 1.4.

## 5.1. Benchmark: Symmetric Learning

Under symmetric learning, all firms have the same information about an old worker's type, i.e., they learn the employed young workers' types at Date 5 of each period. As the equilibrium will be time-invariant, for simplicity we ignore time subscripts. At Date 1 of each period, a type- $i$  firm with an incumbent worker who is of a known type  $\tau$  offers the wage:

$$w_i^S = \begin{cases} \text{any } w < 0 & \tau \leq \underline{\tau}_i \\ 0 & \tau \in [\underline{\tau}_i, \hat{\tau}_i] \\ w^{pS}(\tau) & \tau \in [\hat{\tau}_i, \tau_i^\#] \\ \text{any } w < w^{pS}(\tau) & \tau \in [\tau_i^\#, \bar{\tau}] \end{cases}, \quad (25)$$

where  $\underline{\tau}_i$ ,  $\hat{\tau}_i$ ,  $\tau_i^\#$  and function  $w^{pS}(\tau)$  are to be determined in equilibrium.

Because poachers compete à la Bertrand, their equilibrium value function,  $V_h^{pS}(\tau)$ , when poaching a worker of type  $\tau$  should be equal to the value they derive from hiring a young

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<sup>23</sup>It is always in a firm's interest to hire a young – as opposed to an old – employee from the pool of the unemployed.

worker,  $V_h^{yS}$ :

$$V_h^{pS}(\tau) - V_h^{yS} = 0. \quad (26)$$

In other words, poachers enjoy no net surplus in equilibrium. From this equation, we obtain the following poaching wage offered by a type- $h$  firm with a vacancy to a worker with talent  $\tau$  (this is shown in Appendix 1.4):

$$w^{pS}(\tau) = \theta\gamma(\tau - \mu) - \frac{\delta}{1 + \delta(1 - F(\underline{\tau}_h))} \int_{\underline{\tau}_h}^{\bar{\tau}} (\theta\tau - \theta\gamma\mu)f(\tau)d\tau. \quad (27)$$

In the dynamic model, for a given  $\tau$ , the offer made by a poacher is lower than that in the static model. In the dynamic setting, hiring a young worker has an option value: At Date 5 of the first period of employment, the firm learns the worker's type and thus has the option to retain this worker for the subsequent period. The value of this option is given by the second term on the right-hand side of (27). Thus, poaching an old worker comes at a cost, which is the value of this option. The existence of this option is the main qualitative difference between the static model and the dynamic model.

The threshold  $\hat{\tau}_i$  corresponds to the level of talent above which a poacher offers a positive wage to a worker of type  $\tau > \hat{\tau}_i$ . Because information is symmetric, the poaching wage depends only on a worker's talent; therefore, we set  $\hat{\tau}_l = \hat{\tau}_h = \hat{\tau}$ , and thus the threshold  $\hat{\tau}$  is given by  $w^{pS}(\hat{\tau}) = 0$ .

The first-period wage  $w^{yS}$  of a young worker is given by

$$w^{yS} = -\delta \int_{\hat{\tau}}^{\bar{\tau}} w^{pS}(\tau)f(\tau)d\tau. \quad (28)$$

Note that this wage is always negative and equal to the discounted expected wage received by this worker in the second period. In other words, young workers have zero expected surplus. This result is a consequence of our assumptions that the worker's outside option is zero and that there is no limited liability. We know from Terviö (2009) that, in a dynamic model with symmetric learning, limited liability creates inefficiencies: There is excessive retention of mediocre types. Because we want to isolate the effect of asymmetric learning on welfare, we choose not to impose limited liability, which also implies that, unlike Terviö (2009), the first-best allocation is obtained in our benchmark model with symmetric learning.

Threshold  $\underline{\tau}_i$  from (25) is determined by

$$V_i^{oS}(\underline{\tau}_i) - V_i^{yS} = 0, \quad (29)$$

where  $V_i^{oS}(\tau)$  is the value function a type- $i$  firm from retaining an incumbent (old) worker with talent  $\tau$ , and  $V_i^{yS}$  is the value from hiring a young worker. For a type- $h$  firm, this



threshold is given by:

$$\underline{\tau}_h = \gamma\mu + \delta \int_{\underline{\tau}_h}^{\bar{\tau}} (\tau - \underline{\tau}_h) f(\tau) d\tau. \quad (30)$$

The decision to retain a worker is given by the following trade-off. The left-hand side of (30) is the immediate gain from retaining an old worker of type  $\underline{\tau}_h$ ; the right-hand side is the benefit from hiring a young worker from the outside pool. This benefit has two components. First, a young worker from the outside pool produces (in expectation)  $\gamma\mu$  during the first year of employment. Second, hiring a young worker again gives the firm the option to retain this worker in the subsequent period. The value of this option is given by the second term on the right-hand side of (30) (See Appendix 1.4 for the derivations).

For a type- $l$  firm the retention threshold  $\underline{\tau}_l$  is given by:

$$\underline{\tau}_l = \gamma\mu + \delta \int_{\underline{\tau}_l}^{\tau_l^\#} (\tau - \underline{\tau}_l) f(\tau) d\tau + \delta \int_{\tau_l^\#}^{\bar{\tau}} w^{pS}(\tau) f(\tau) d\tau. \quad (31)$$

The first two terms on the right-hand side of (31) are analogous to those in (30). The key difference between these two conditions is the last term on the right-hand side of (31), which represents the present value of the wages paid to those workers who are poached in equilibrium in the second year of employment. A firm of type  $l$  is able to capture such surplus by offering a negative wage to young workers. Thus, these firms are compensated for being talent discoverers; even if their best workers leave to work for other firms, type- $l$  firms capture all the surplus generated by an efficient allocation of talent.

Finally, the poaching threshold  $\tau^\#$  is determined by

$$V_l^{oS}(\tau^\#) = V_l^{yS}. \quad (32)$$

This condition follows because  $V_l^{oS}(\tau)$  is decreasing; thus, for all  $\tau > \tau^\#$ , an  $l$ -firm prefers hiring a young worker (which yields value  $V_l^{yS}$ ) to retaining the incumbent worker at wage  $w^{pS}(\tau)$ .

Solving simultaneously for (27), (28), (30), (31), and (32) fully characterizes the unique equilibrium. Equilibrium existence is not an issue, provided that the number of potential poachers is sufficiently large (i.e.,  $H$  is sufficiently large; the analog of Assumption A2 must hold for every period).

We now discuss two important properties of the equilibrium. First, we have the following result:

**Proposition 6**  $\underline{\tau}_l \geq \underline{\tau}_h$ .

This result indicates that  $l$ -firms are more likely to fire workers with low talent than are  $h$ -firms. The intuition is as follows: It is more efficient for  $l$ -firms to act as talent discoverers

than as producers because  $l$ -firms are as efficient as  $h$ -firms in discovering talent, but less efficient at producing output. Thus,  $l$ -firms have a comparative (but not absolute) advantage at discovering talent and should thus do more of it in an efficient allocation.

We also have that:

**Proposition 7** *The unique equilibrium under symmetric learning is efficient (in the Kaldor-Hicks sense).*

Although it is not surprising that under symmetric learning the first-best outcome is achieved, we note that, unlike the static case, a hypothetical social planner has to consider two different trade-offs. First, we require an efficient allocation of workers to firms. As discussed above, the social planner would then choose the poaching threshold  $\tau^\#$  by trading off the loss in firm-specific skills and the gain from assigning a worker to a more productive firm. Second, the social planner must find the optimal rate of talent discovery. The social planner chooses the retention threshold  $\underline{\tau}_i$  by trading off the loss in firm-specific skills and the gain from sampling a young worker and learning about the worker's type in the subsequent period.

## 5.2. Asymmetric Learning

We begin by noting that Lemmas 1 and 2 also hold for the dynamic version of the model; we can easily check that the simple proofs of these lemmas are valid in each period. Thus, these results are invariant to the number of periods in the game. Therefore, in any equilibrium, only the best workers are retained, and all retained workers are offered the same wage.

To characterize the equilibrium, in each period, we need to find three types of thresholds. As discussed above,  $\tilde{\tau}_i$ ,  $i \in \{l, h\}$ , denotes the threshold such that all types  $\tau \geq \tilde{\tau}_i$  are retained. Here, the only difference from the static case is that both types of firms can retain workers. We define  $\underline{\tau}_i$  (as in the symmetric learning case) as the threshold for which all types  $\tau \leq \underline{\tau}_i$  are fired.<sup>24</sup> Finally, we define  $\hat{\tau}_i$  as the lowest type that is poached in equilibrium.

An equilibrium is fully determined by a sequence of thresholds  $\{\tilde{\tau}_l, \tilde{\tau}_h, \hat{\tau}_l, \hat{\tau}_h, \underline{\tau}_l, \underline{\tau}_h\}_t$ ,  $t = 0, 1, \dots, \infty$ . For simplicity, we focus only on equilibria in which these thresholds are time-invariant. Thus, we can drop the time subscript from the analysis that follows.

**Proposition 8** *Any equilibrium  $\{\tilde{\tau}_l, \tilde{\tau}_h, \hat{\tau}_l, \hat{\tau}_h, \underline{\tau}_l, \underline{\tau}_h\}$  has the following properties:*

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<sup>24</sup>As above, there could be a subset  $P$  of types that are poached in equilibrium. For simplicity, we focus only on cases in which  $P$  is an interval.

1. For a given  $\{\hat{\tau}_l, \hat{\tau}_h, \underline{\tau}_l, \underline{\tau}_h\}$ , threshold  $\tilde{\tau}_i$  is either  $\bar{\tau}$  or the least element of the set of fixed points of

$$H_i(\tau) \equiv i(\tau - \gamma\mu) - \frac{w_i^* - w_i^y - \delta i \int_{\tau}^{\bar{\tau}} (x - \gamma\mu) dF(x)}{1 + \delta(1 - F(\tau))}. \quad (33)$$

(Recall that  $i \in \{1, \theta\}$ ). All retained workers are offered wage

$$w_i^* = \max \left\{ \theta\gamma \left( \int_{\tilde{\tau}_i}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_i)} - \mu \right) + w_h^y - \frac{\delta \int_{\tilde{\tau}_h}^{\bar{\tau}} (\theta\tau - w_h^* - \theta\gamma\mu + w_h^y) f(\tau) d\tau}{[1 + \delta(1 - F(\tilde{\tau}_h))]}, 0 \right\}, \quad (34)$$

where

$$w_h^* = \max \left\{ \theta\gamma \left( \int_{\tilde{\tau}_h}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_h)} - \mu \right) + w_h^y - \delta\theta \int_{\tilde{\tau}_h}^{\bar{\tau}} (1 - \gamma)\tau f(\tau) d\tau, 0 \right\}, \quad (35)$$

all workers who are poached (if any) are paid

$$w_i^{**} = \theta\gamma \left( \int_{\hat{\tau}_i}^{\tilde{\tau}_i} \frac{\tau f(\tau) d\tau}{F(\tilde{\tau}_i) - F(\hat{\tau}_i)} - \mu \right) + w_h^y - \frac{\delta \int_{\tilde{\tau}_h}^{\bar{\tau}} (\theta\tau - w_h^* - \theta\gamma\mu + w_h^y) f(\tau) d\tau}{1 + \delta(1 - F(\tilde{\tau}_h))}, \quad (36)$$

and all young workers who agree to work for a type- $i$  firm are offered wage

$$w_i^y = -\delta(1 - F(\tilde{\tau}_i))w_i^* - \delta(F(\tilde{\tau}_i) - F(\hat{\tau}_i)) \max \{w_i^{**}, 0\}. \quad (37)$$

2. All types  $\tau_i \in [0, \underline{\tau}_i]$  are fired in equilibrium.

3. If  $\underline{\tau}_i < \tilde{\tau}_i$ , then types  $[\hat{\tau}_i, \tilde{\tau}_i]$ , with  $\hat{\tau}_i \geq \underline{\tau}_i$ , are poached in equilibrium and are paid  $w_i^{**}$ .

From this proposition we conclude that the equilibrium displays the same type of talent misallocation as in the static model: The best types  $[\tilde{\tau}_i, \bar{\tau}]$  are retained and the mediocre types  $[\hat{\tau}_i, \tilde{\tau}_i]$  are poached. Thus, our main conclusions continue to hold in the dynamic model.

To understand the differences between the static and the dynamic cases, consider the poaching wage given by (34). This wage differs from (4) only because of

$$w_h^y - \frac{\delta \int_{\tilde{\tau}_h}^{\bar{\tau}} (\theta\tau - w_h^* - \theta\gamma\mu + w_h^y) f(\tau) d\tau}{[1 + \delta(1 - F(\tilde{\tau}_h))]} \quad (38)$$

This expression has two terms: the present value of payments to a young worker and the option value of hiring a young worker. The first term is a consequence of our assumption

of unlimited liability and the second term reflects the value of learning. Because  $w_h^y$  is negative, both of these terms reduce the incentives to poach and thus make it less likely that inefficient poaching occurs in equilibrium. However, these terms are often not large enough to eliminate poaching entirely. Furthermore, if we impose limited liability, we obtain  $w_h^y = 0$ , and expressions (33), (34) and (36) are simplified and remain valid.

In the dynamic model, a new result is that inefficient poaching is less likely to be sustained in equilibrium. An equilibrium without poaching is more likely if firms are more patient (i.e., if  $\delta$  is high). Conversely, if firms are very impatient (i.e., if  $\delta$  is low), inefficient poaching is more likely.<sup>25</sup> Firms might become more impatient because they experience a higher death rate, which could be a consequence of tougher competition in product markets. Thus, increasing “short-termism” makes inefficient poaching more likely.

## 6. Conclusions

Our theory is not meant to be a general theory of labor markets. We expect our analysis to be relevant to those industries in which talent is not easily observed, such that incumbent employers enjoy a natural advantage in discovering talent. In such industries, labor market competition may lead to misallocation of talent, and allocative efficiency might be improved by restricting competition. Examples that fit such descriptions include innovative industries, such as information technology, and some sectors of the financial industry, such as asset management.

Our model predicts that employee earnings are higher in sectors with greater revenue dispersion and, more strikingly, that within-job earnings growth is also higher in such sectors. Consistent with such implications, Andersson et al. (2009) show that the “rewards to loyalty” (i.e., within-job earnings growth for those employees retained by their firms) are greater in software sectors with high revenue dispersion. In contrast, between-job earnings growth is smaller in such sectors.

Another novel empirical prediction of our model is that employees retained or promoted by their employers are better than those who are successfully poached and “promoted” to positions in other firms (after controlling for observable characteristics). In a recent paper, Berk, van Binsbergen, and Liu (2014) study the labor market for mutual fund managers and find that internal “promotions” (i.e., when a manager is given control over a larger value of assets) add value to the firm, whereas external promotions (i.e., external hires with an increase of assets under management) do not. Our model provides a possible equilibrium explanation for this finding. More importantly, our model helps us better understand the welfare consequences of competition for talent in such labor markets.

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<sup>25</sup>As  $\delta \rightarrow 0$ , the equilibrium converges to an equilibrium of the static case.

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## A. Appendix

### 1.1. Proofs for Subsection 3.3

#### Proposition 1.

**Proof.** *Part 1.* Lemma 2 implies that an equilibrium with retention must have a threshold  $\tilde{\tau}$  as in Part 1 of the proposition above (If no type is retained in equilibrium, we set  $\tilde{\tau} = \bar{\tau}$  and this part is trivially proved). Lemma 1 implies that all types in  $[\tilde{\tau}, \bar{\tau}]$  are paid the same wage. To prevent poaching, this wage must be such that  $w^* \geq w^p(w^*)$ . Because poachers know that all types in  $[\tilde{\tau}, \bar{\tau}]$  are offered  $w^*$ , their beliefs must be given by  $F(\tau | \tau \geq \tilde{\tau})$  upon observing  $w^*$ , which implies

$$w^p(w^*) = \theta\gamma \left( \int_{\tilde{\tau}}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tilde{\tau}) - \mu \right). \quad (39)$$

Because  $w^* \geq w^p(w^*)$ , a necessary condition for an incumbent with type  $\tau \in [\tilde{\tau}, \bar{\tau}]$  not to deviate and fire the worker is

$$\tau - w^p(w^*) \geq \gamma\mu, \quad (40)$$

a condition that is equivalent to

$$\tilde{\tau} = w^p(w^*) + \gamma\mu. \quad (41)$$

We now show that  $w^* = w^p(w^*)$  if the equilibrium threshold is  $\tilde{\tau}$ . Suppose first that  $w^* > w^p(w^*)$  and consider a deviation from an incumbent with type  $\tau > \tilde{\tau}$  who chooses to



offer  $w^p(w^*)$  instead. For this not to constitute a profitable deviation, we must have that

$$w^p(w^p(w^*)) > w^p(w^*) = \theta\gamma \left( \int_{\tilde{\tau}}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tilde{\tau}) - \mu \right), \quad (42)$$

which can only happen if

$$\int_0^{\bar{\tau}} \tau dF^W(\tau | w^p(w^*)) > \int_{\tilde{\tau}}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tilde{\tau}).$$

This condition requires the existence of at least one  $\tau'' > \tilde{\tau} \geq w^p(w^*) + \gamma\mu$  such that its probability of deviation is strictly greater than that of some type  $\tau' \in (\tilde{\tau}, \tau'')$ . But this is ruled out by Assumption E2. Thus,  $w^* = w^p(w^*)$ .

Next, we show that  $\tilde{\tau}$  is unique. Define the function

$$H(\tau) = \tau - \theta\gamma \left( \int_{\tau}^{\bar{\tau}} x dF(x | x \geq \tau) - \mu \right) - \gamma\mu. \quad (43)$$

Clearly, the existence of an equilibrium with retention requires this function to be non-negative for some  $\tau$  (to see this, insert (42) in (41)). Because  $H(\tau)$  is continuous and  $H(0) = -\gamma\mu < 0$ , at least one fixed point exists if and only if

$$\max_{\tau \in [0, \bar{\tau}]} H(\tau) \geq 0. \quad (44)$$

(If (44) does not hold, the unique equilibrium displays no retention). Assuming that (44) holds, we define the threshold  $\tilde{\tau}$  as the least element of the set of fixed points of  $H(\tau)$ :

$$\tilde{\tau} = \min_{\{\tau: H(\tau)=0\}} \tau. \quad (45)$$

Clearly,  $\tilde{\tau} \geq \gamma\mu$ . At  $\tilde{\tau}$ , the incumbent is just indifferent between retaining the worker for  $w^*$  or firing the worker:

$$\tilde{\tau} = \theta\gamma \left( \int_{\tilde{\tau}}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tilde{\tau}) - \mu \right) + \gamma\mu. \quad (46)$$

To show that this threshold is part of an equilibrium, notice first that because  $H(0) < 0$ ,  $H(\tau)$  crosses zero from below at  $\tilde{\tau}$ , which is also a necessary condition for an equilibrium. We only need to show that no other  $\tau > \tilde{\tau}$  can be an equilibrium. To see this, suppose that there is  $\tau' > \tilde{\tau}$  such that only types  $\tau > \tau'$  are retained at wage

$$w' = \theta\gamma \left( \int_{\tau'}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tau') - \mu \right). \quad (47)$$

Then, an incumbent with type  $\tilde{\tau} + \varepsilon$ , with  $\varepsilon > 0$  arbitrarily small, could deviate and offer  $w^* < w'$ , with

$$w^* = \theta\gamma \left( \int_{\tilde{\tau}}^{\bar{\tau}} \tau dF(\tau \mid \tau \geq \tilde{\tau}) - \mu \right). \quad (48)$$

If type  $\tilde{\tau} + \varepsilon$  is successfully retained after this deviation, then the incumbent is strictly better off. For such a deviation not to be profitable, poachers' beliefs must be such that  $w^p(w^*) > w^*$ . But again this could only be true if there is some type  $\tau'' > \tilde{\tau}$  whose probability of deviation is strictly greater than that of a type  $\tau \in (\tilde{\tau}, \tau'')$ . This is ruled out by Assumption E2. Thus,  $\tilde{\tau}$  is uniquely determined as the least fixed point of  $H(\tau)$  and the retention wage is given by  $w^*$  as in (48).

*Part 2.* It follows trivially from Assumption E1.

*Part 3.* Suppose that there is some type  $\tau'$  in  $[\gamma\mu, \tilde{\tau}]$  that is retained in equilibrium. Lemma 2 implies that all types in  $[\tau', \tilde{\tau}]$  are also retained and Lemma 1 implies that all types in  $[\tau', \bar{\tau}]$  must be paid the same wage. But, because  $\tau' \leq \tilde{\tau}$ , then by the definition of  $\tilde{\tau}$  in (45), we have  $H(\tau') \leq 0$ . Thus, type  $\tau'$  cannot be profitably retained. Thus, all types in  $[\gamma\mu, \tilde{\tau}]$  must be either poached (and thus included in set  $P$ ) or fired (and thus included in set  $S$ ). Thus, if an equilibrium exists, Part 3 must hold.

To complete the proof, we only need to show that at least one equilibrium exists. Suppose first that  $\max_{\tau \in [0, \bar{\tau}]} H(\tau) \geq 0$ . In this case, we know that there exists a unique  $\tilde{\tau} < \bar{\tau}$ . The following fully characterizes one possible equilibrium:

Consider the retention wages

$$w(\tau) = \begin{cases} w^* & \text{if } \tau \in [\tilde{\tau}, \bar{\tau}] \\ 0 & \text{if } \tau \in [\mu, \tilde{\tau}] \\ -1 & \text{if } \tau \in [0, \mu] \end{cases}, \quad (49)$$

the poaching wages on the equilibrium path

$$w^p(w) = \begin{cases} w^* & \text{if } w = w^* \\ \theta\gamma \left( \int_{\mu}^{\tilde{\tau}} \tau dF(\tau \mid \tau \in [\mu, \tilde{\tau}]) - \mu \right) & \text{if } w = 0 \\ -1 & \text{if } w = -1 \end{cases}, \quad (50)$$

and beliefs such that  $F(\tau \mid \tau \geq w + \gamma\mu)$  for any  $w$  that is off the equilibrium path. In this equilibrium,  $P = [\mu, \tilde{\tau}]$  and  $S = [\gamma\mu, \mu]$ .

If instead we have  $\max_{\tau \in [0, \bar{\tau}]} H(\tau) < 0$ , then no type is retained, and an equilibrium in which all types  $\tau \geq \mu$  are offered  $w = 0$ , and types below  $\mu$  are fired, exists and is sustained by beliefs such that  $F(\tau \mid \tau \geq w + \gamma\mu)$  for any  $w$  that is off the equilibrium path. This equilibrium implies  $P = [\mu, \bar{\tau}]$  and  $S = [\gamma\mu, \mu]$ . ■

**Proposition 2.**

**Proof.** We proved the existence of this equilibrium in the proof of Proposition 1. This equilibrium is the most efficient one because any other equilibrium must have at least one of the following:

1. Some  $\tau' < \mu$  being poached, and/or
2. some  $\tau' > \mu$  being fired.

In Case 1, allocational efficiency can be improved by firing  $\tau'$ . In Case 2, allocational efficiency can be improved by letting poachers hire  $\tau'$ . ■

**Lemma 3.**

**Proof.** Suppose that there is an equilibrium in which  $w' = w^p(w') > w = w^p(w)$ . In such an equilibrium,

$$E[\tau | w'] \equiv \int_0^{\bar{\tau}} \tau dF(\tau | w') > \int_0^{\bar{\tau}} \tau dF(\tau | w) \equiv E[\tau | w], \quad (51)$$

(because of (4) and Bayesian rationality on the equilibrium path). Suppose now that  $p(w') < p(w)$ . Then an incumbent firm facing a worker with type  $\tau' \geq E[\tau | w']$  could deviate from the equilibrium and offer this worker  $w$ . The worker has now a strictly lower probability of being poached and receives a strictly lower wage if retained. The incumbent firm is strictly better off after this deviation. Thus,  $p(w)$  must be non-decreasing in equilibrium. ■

**1.2. Mixed-strategy Equilibria**

Here we allow workers to randomize between accepting the incumbent's or the poacher's offer, whenever they are indifferent between the two offers.

For brevity, we only consider the case in which  $1 \geq \theta\gamma$ . From (7) we have that  $\tau^\# = \bar{\tau}$ , thus poaching is always inefficient. Because equilibria in which workers play strictly-mixed strategies must involve some poaching, it follows trivially that such equilibria will also be inefficient. Furthermore, the source of inefficiency is the same as in the pure-strategy equilibria: there is too much poaching. Thus, the policy implications are also unchanged.

Although the equilibrium still involves excessive poaching, mixed strategies may improve allocational efficiency by allowing for the retention of some types in  $[\gamma\mu, \tilde{\tau}]$  with some positive probability (but not with probability 1).

An equilibrium is characterized in the same way as in the pure-strategy case, except that we now need to describe the equilibrium behavior of a worker who faces two equivalent offers. Whenever an equilibrium with strictly-mixed strategies exists, there exists a function

$p(w)$  that maps incumbent wage offers into probabilities of acceptance. In this appendix, we describe the equilibrium properties of this function.

Define  $w(\tau)$  as the equilibrium wage offer that an incumbent makes to a worker of type  $\tau$  and let  $p(\tau) \equiv p(w(\tau))$ . Lemma 3 shows that  $p(w)$  is nondecreasing in  $w$ , which trivially implies that  $p(\tau)$  is also non-decreasing in  $\tau$ . Another equilibrium property of  $p(\tau)$  is as follows:

**Lemma 4** *Function  $p(\tau)$  is continuous for all  $\tau$  such that  $p(\tau) > 0$ .*

**Proof.** Consider  $\tau'$  and let  $\lim_{\varepsilon \rightarrow 0} p(\tau') - p(\tau' - \varepsilon) \equiv \delta$ . For a deviation not to be profitable, we need

$$p(\tau')(\tau' - \varepsilon - \gamma\mu - w(\tau')) \leq p(\tau' - \varepsilon)(\tau' - \varepsilon - \gamma\mu - w(\tau' - \varepsilon)) \quad (52)$$

and

$$p(\tau')(\tau' - \gamma\mu - w(\tau')) \geq p(\tau' - \varepsilon)(\tau' - \gamma\mu - w(\tau' - \varepsilon)) \quad (53)$$

We take the limit as  $\varepsilon \rightarrow 0$  and let  $\tilde{w}(\tau') \equiv \lim_{\varepsilon \rightarrow 0} w(\tau' - \varepsilon)$ . Then

$$p(\tau')(\tau' - \gamma\mu - w(\tau')) \leq (p(\tau') - \delta)(\tau' - \gamma\mu - \tilde{w}(\tau')) \quad (54)$$

and

$$p(\tau')(\tau' - \gamma\mu - w(\tau')) \geq (p(\tau') - \delta)(\tau' - \gamma\mu - \tilde{w}(\tau')), \quad (55)$$

which implies that  $\delta = 0$ , i.e.,  $p(\tau)$  must be continuous. ■

The next result follows directly from Lemmas 3 and 4:

**Corollary 2** *For  $\tau \in [\tau', \bar{\tau}]$  such that  $p(\tau') > 0$ , we can find sets  $A_1, A_2, \dots$  such that  $\bigcup_i A_i = [\tau', \bar{\tau}]$  and that, for each  $A_i$ , either  $p(\tau)$  is constant for  $\tau \in A_i$  or  $p(\tau)$  is strictly increasing for  $\tau \in A_i$ .*

In other words,  $p(\tau)$  is defined over regions of *pooling* (i.e.,  $p(\tau)$  is constant over an interval) and *fully-revealing separation* (i.e.,  $p(\tau)$  is strictly increasing over an interval, so that types in this interval are fully revealed in equilibrium).

Suppose that the interval  $[a, b]$  is an equilibrium pooling region with  $p(\tau) \in (0, 1)$  for  $\tau \in [a, b]$ , and assume that this interval is not contained in any other pooling interval. The equilibrium wage must be

$$w(\tau) = w^p = \theta\gamma \left( \int_a^b \frac{\tau f(\tau)}{F(b) - F(a)} d\tau - \mu \right) \text{ for } \tau \in [a, b]. \quad (56)$$

To find  $p(\tau)$  for  $\tau \in [a, b]$  notice there must exist at least one separating interval to the right or to the left of  $[a, b]$ . From continuity,

$$\lim_{\tau \rightarrow a} p(\tau) = \lim_{\tau \rightarrow b} p(\tau), \quad (57)$$

which implies that we can characterize  $p(\tau)$  for  $\tau \in [a, b]$  by the limit of  $p(\tau)$  over any fully-revealing separation region in the neighborhood of  $[a, b]$ . This implies that it suffices to characterize  $p(\tau)$  over separation regions.

Let  $[c, d]$  denote a fully-revealing separation interval, so that type  $\tau \in [c, d]$  is fully revealed in equilibrium. Due to competition among poachers,  $w^p(w(\tau)) = \theta\gamma(\tau - \mu)$ . In order to obtain separation, the probability schedule must be such that it prevents an incumbent employer with a worker of type  $\tau$  from pretending that the worker is of type  $\hat{\tau} \in [c, d]$  and  $\hat{\tau} \neq \tau$ . Thus, the following incentive compatibility constraint must hold for any such  $\hat{\tau}$ :

$$p(\tau) [\tau - \gamma\mu - \theta\gamma(\tau - \mu)] \geq p(\hat{\tau}) ([\tau - \gamma\mu - \theta\gamma(\hat{\tau} - \mu)]). \quad (58)$$

Define

$$U(\tau) = \max_{x \in [c, d]} p(x) [\tau - \gamma\mu - \theta\gamma(x - \mu)] + \gamma\mu. \quad (59)$$

By the envelope theorem we obtain:

$$\frac{\partial U(\tau)}{\partial \tau} = p(x^*) = p(\tau), \quad (60)$$

where the second equality follows from the IC condition in (58): If  $\tau$  is fully revealed in equilibrium, then  $x^* = \tau$ .

Integrating (60) yields

$$U(\tau) = U(d) - \int_{\tau}^d p(x) dx. \quad (61)$$

For simplicity we assume that the function  $p(\tau)$  is twice differentiable over the interval  $[c, d]$ . Then the next lemma allows us to solve for  $p(\tau)$ .

**Lemma 5** *All incentive constraints are satisfied if and only if the following two sets of constraints hold:*

(i) *Local incentive compatibility:*

$$p'(\tau) [\tau - \gamma\mu - \theta\gamma(\tau - \mu)] - \theta\gamma p(\tau) = 0 \quad (62)$$

(ii) *Monotonicity:*

$$p'(\tau) \geq 0. \quad (63)$$

**Proof.** Assume first that all incentive compatibility constraints are satisfied, then it must be that the following first and second order conditions are satisfied at  $x^* = \tau$

$$FOC : p'(x^*) [\tau - \gamma\mu - \theta\gamma(x^* - \mu)] - \theta\gamma p(x^*) = 0 \quad (64)$$

$$SOC : p''(x^*) [\tau - \gamma\mu - \theta\gamma(x^* - \mu)] - 2\theta\gamma p'(x^*) \leq 0 \quad (65)$$

Replacing  $x^*$  with  $\tau$  and totally differentiating the local incentive compatibility constraint with respect to  $\tau$ , we obtain:

$$p''(\tau) [\tau - \gamma\mu - \theta\gamma(\tau - \mu)] - 2\theta\gamma p'(\tau) + p'(\tau) = 0. \quad (66)$$

From the second order condition, this equation implies that  $p'(\tau) \geq 0$ .

Now, suppose that both the monotonicity and local incentive compatibility conditions hold. This must imply that all incentive compatibility constraints are satisfied:

$$p(\tau) [\tau - \gamma\mu - \theta\gamma(\tau - \mu)] \geq p(\hat{\tau}) [\hat{\tau} - \gamma\mu - \theta\gamma(\hat{\tau} - \mu)] \text{ for any } \tau \neq \hat{\tau}. \quad (67)$$

This equation can be rewritten as:

$$p(\tau) [\tau - \gamma\mu - \theta\gamma(\tau - \mu)] \geq p(\hat{\tau}) [\hat{\tau} - \gamma\mu - \theta\gamma(\hat{\tau} - \mu)] - (\hat{\tau} - \tau)p(\hat{\tau})$$

or

$$(\hat{\tau} - \tau)p(\hat{\tau}) \geq p(\tau) [\tau - \gamma\mu - \theta\gamma(\tau - \mu)] - p(\hat{\tau}) [\hat{\tau} - \gamma\mu - \theta\gamma(\hat{\tau} - \mu)], \quad (68)$$

which implies

$$\begin{aligned} \int_{\tau}^{\hat{\tau}} p(\hat{\tau}) dx &\geq \int_{\tau}^{\hat{\tau}} \{p(x) + p'(x) [x - \gamma\mu - \theta\gamma(x - \mu)] - \theta\gamma p(x)\} dx \\ &\quad - \int_{\hat{\tau}}^{\tau} \{p(x) + p'(x) [x - \gamma\mu - \theta\gamma(x - \mu)] - \theta\gamma p(x)\} dx. \end{aligned} \quad (69)$$

If the local incentive compatibility constraint holds and  $\hat{\tau} \geq \tau$ , this condition becomes:

$$\int_{\tau}^{\hat{\tau}} p(\hat{\tau}) dx \geq \int_{\tau}^{\hat{\tau}} p(x) dx, \quad (70)$$

which always holds for  $p'(\tau) \geq 0$ . If  $\hat{\tau} < \tau$ , the condition becomes:

$$\int_{\hat{\tau}}^{\tau} p(x) dx \geq \int_{\hat{\tau}}^{\tau} p(\hat{\tau}) dx, \quad (71)$$

which always holds for  $p'(\tau) \geq 0$ . ■

This lemma allows us to characterize  $p(\tau)$  by solving the differential equation in (62):

**Corollary 3** *In any mixed-strategy equilibrium, the probability that type  $\tau$  is retained is*

$$p(\tau) = K [(1 - \theta\gamma)\tau + \gamma\mu(\theta - 1)]^{\frac{\theta\gamma}{1-\theta\gamma}}, \quad (72)$$

where  $K \geq 0$  is a constant.

The constant  $K$  is pinned down by the boundaries of  $[c, d]$ . The indeterminacy of  $K$  reflects the potential multiplicity of equilibria. Once a boundary condition is chosen,  $K$  is uniquely determined. For example, if  $d = \bar{\tau}$  and type  $\bar{\tau}$  is retained with probability 1, then

$$K = [(1 - \theta\gamma)\bar{\tau} + \theta\mu(\theta - 1)]^{\frac{\theta\gamma}{\theta\gamma-1}}. \quad (73)$$

### 1.3. Proofs for Subsection 4.3

**Proposition 4.**

**Proof.** As before, we assume that E1 and E2 hold.

To find the equilibrium, we work backwards. At Date  $2\frac{1}{2}$ , the incumbent observes a poaching wage  $w^p$ . The incumbent pays the poaching wage and retains type  $\tau$  if and only if  $\tau - w^p \geq \gamma\mu$ .

At Date 2, a worker with a wage offer  $w_l$  receives a poaching offer equal to

$$\theta\gamma \left( \int_0^{\bar{\tau}} \tau dF(\tau | w_l) - \mu \right). \quad (74)$$

The beliefs represented by  $F(\tau | w_l)$  must be Bayesian on the equilibrium path and consistent with E2.

At Date 1, the incumbent chooses  $w_l$ . We argue that an incumbent offers a unique wage  $w_l = 0$  to any retained employee, i.e., an employee with talent  $\tau \geq \gamma\mu$ . The argument is similar to the one used to prove Lemma 1. Suppose that there are two types  $\tau' > \tau''$  and that the incumbent wants to retain both of them. Suppose the incumbent offers two different wages  $w'_l > w''_l$  and suppose the poacher's offers are  $w^p(w'_l) > w^p(w''_l)$ . Then, there is a profitable deviation for the incumbent, which is to offer  $w''_l$  to both types. Now, suppose that  $w_l > 0$ . Then, the incumbent could deviate and offer  $w'_l = 0$ ; Assumption E2 implies that  $w^p(0) < w^p(w_l)$ . Thus,  $w_l = 0$ . E1 implies that all  $\tau < \gamma\mu$  receive negative offers. Maximum retention implies that the incumbent offers  $w_l = 0$  to all  $\tau \geq \gamma\mu$ . This proves Part 2 of the proposition and that there is a unique  $\tilde{\tau}' \in [\gamma\mu, \bar{\tau}]$  such that all types  $\tau > \tilde{\tau}'$

are retained. Then, it follows that the equilibrium poaching wage is given by

$$w^p = \theta\gamma \left( \frac{\int_{\gamma\mu}^{\tilde{\tau}'} \tau dF(\tau)}{F(\tilde{\tau}') - F(\gamma\mu)} - \mu \right), \quad (75)$$

and thus all retained workers are offered wage

$$w^{*'} = \max \left\{ \theta\gamma \left( \frac{\int_{\gamma\mu}^{\tilde{\tau}'} \tau dF(\tau)}{F(\tilde{\tau}') - F(\gamma\mu)} - \mu \right), 0 \right\}, \quad (76)$$

because the incumbent only needs to offer  $w_l^c = \max\{w^p, 0\}$ . If  $w^p$  is strictly positive, then clearly all types  $\tau \in (\gamma\mu, \tilde{\tau}')$  are poached in equilibrium. If  $w^p \leq 0$ , then no one is poached and thus  $\tilde{\tau}' = \gamma\mu$ . This proves Part 3.

To prove Part 1, suppose first that  $G(\bar{\tau}) < 0$ . Then, the incumbent does not wish to retain any type, implying that  $\tilde{\tau}' = \bar{\tau}$ .

Suppose now that  $G(\bar{\tau}) \geq 0$ . If  $G(\tau) \geq 0$  for all  $\tau$ , then the incumbent can retain any type for a given equilibrium  $w^p$  and still make a net profit. Thus, all types higher than  $\gamma\mu$  are retained. Finally, if  $G(\tau) < 0$  for some  $\tau$ , then the set  $Y$  is non-empty and the equilibrium threshold must be in  $Y$  (which has at least two elements because  $G(0) > 0$ ). Consider a candidate equilibrium threshold  $\tau^* \in Y$ , with respective equilibrium poaching wage  $w^{p*}$ , and assume that  $\tau^*$  is not the largest element of  $Y$ . Then, a single poacher may deviate and offer an alternative poaching wage equal to

$$w^{p'} = \tilde{\tau}' - \alpha - \gamma\mu, \quad (77)$$

where  $\tilde{\tau}'$  is the largest element in  $Y$  and  $\alpha > 0$  is sufficiently small so that  $w^{p'} < w^{p*}$ . This poacher would be successful at poaching all types  $[\gamma\mu, \tilde{\tau}' - \alpha)$  at a wage that is strictly lower than the one implied by the zero net profit condition. Thus, this deviation is profitable. Thus, the equilibrium threshold must be  $\tilde{\tau}'$ , i.e., the largest element of  $Y$ . ■

**Proposition 5.**

**Proof.** Threshold  $\tilde{\tau}$  is defined by the lowest value that solves

$$\tilde{\tau} - \gamma\mu = \theta\gamma \left( \frac{\int_{\tilde{\tau}}^{\bar{\tau}} \tau dF(\tau)}{1 - F(\tilde{\tau})} - \mu \right). \quad (78)$$

(We assume an interior solution for simplicity; if the solution is not interior, then there is no retention, and the proposition is trivially proven).



Threshold  $\tilde{\tau}'$  is defined by the largest value that solves

$$\tilde{\tau}' - \gamma\mu = \theta\gamma \left( \frac{\int_{\gamma\mu}^{\tilde{\tau}'} \tau dF(\tau)}{F(\tilde{\tau}') - F(\gamma\mu)} - \mu \right). \quad (79)$$

Suppose that  $\tilde{\tau}' > \tilde{\tau}$ . Then, it must be that

$$\frac{\int_{\gamma\mu}^{\tilde{\tau}'} \tau dF(\tau)}{F(\tilde{\tau}') - F(\gamma\mu)} > \frac{\int_{\tilde{\tau}}^{\tilde{\tau}} \tau dF(\tau)}{1 - F(\tilde{\tau})}, \quad (80)$$

which cannot be true. ■

## 1.4. Dynamic model

### Symmetric learning - Derivation of expressions in Subsection 5.1

The poaching wage offered to a worker with talent  $\tau$  is implicitly determined by the equilibrium condition:

$$V_h^{pS}(\tau) - V_h^{yS} = 0, \quad (81)$$

where

$$V_h^{pS}(\tau) = \theta\gamma\tau - w^{pS}(\tau) + \delta \max \left\{ V_h^{yS}, V_h^{pS}(\tau) \right\}, \quad (82)$$

$$V_h^{yS} = \theta\gamma\mu - w^{yS} + \delta V_h^{oS}, \quad (83)$$

and

$$V_h^{oS} = F(\underline{\tau}_h) \max \left\{ V_h^{yS}, V_h^{pS}(\tau) \right\} + \theta \int_{\underline{\tau}_h}^{\tilde{\tau}} \tau f(\tau) d\tau - \int_{\hat{\tau}_h}^{\tilde{\tau}} w^{pS}(\tau) f(\tau) d\tau + \delta(1 - F(\underline{\tau}_h)) V_h^{yS}. \quad (84)$$

By replacing (82) and (83) into (81), we obtain the following expression for the poaching wage (recall that this is only defined for non-negative wages):

$$w^{pS}(\tau) = \theta\gamma(\tau - \mu) + w^{yS} - \delta(V_h^{oS} - V_h^{yS}). \quad (85)$$

The threshold  $\hat{\tau}_i$  corresponds to the level of talent above which a poacher offers a positive wage to a worker of type  $\tau > \hat{\tau}_i$ . Because information is symmetric, the poaching wage depends only on a worker's talent, therefore we set  $\hat{\tau}_l = \hat{\tau}_h = \hat{\tau}$ , and thus threshold  $\hat{\tau}$  is given by  $w^{pS}(\hat{\tau}) = 0$ .

Using (83) in (84), we obtain:

$$\begin{aligned} V_h^{oS} &= F(\underline{\tau}_h) [\theta\gamma\mu - w^{yS} + \delta V_h^{oS}] + \theta \int_{\underline{\tau}_h}^{\bar{\tau}} \tau f(\tau) d\tau \\ &\quad - \int_{\hat{\tau}_h}^{\bar{\tau}} w^{pS}(\tau) f(\tau) d\tau + \delta(1 - F(\underline{\tau}_h)) V_h^{yS}. \end{aligned} \quad (86)$$

Subtracting  $V_h^{yS}$  from both sides yields

$$\begin{aligned} V_h^{oS} - V_h^{yS} &= -[1 - F(\underline{\tau}_h)] [\theta\gamma\mu - w^{yS} + \delta V_h^{oS}] + \theta \int_{\underline{\tau}_h}^{\bar{\tau}} \tau f(\tau) d\tau \\ &\quad - \int_{\hat{\tau}_h}^{\bar{\tau}} w^{pS}(\tau) f(\tau) d\tau + \delta(1 - F(\underline{\tau}_h)) V_h^{yS}, \end{aligned} \quad (87)$$

or

$$V_h^{oS} - V_h^{yS} = \frac{\int_{\underline{\tau}_h}^{\bar{\tau}} (\theta\tau - \theta\gamma\mu) f(\tau) d\tau - \int_{\hat{\tau}_h}^{\bar{\tau}} w^{pS}(\tau) f(\tau) d\tau + (1 - F(\underline{\tau}_h)) w^{yS}}{1 + \delta(1 - F(\underline{\tau}_h))}. \quad (88)$$

The first-period wage of a young worker is given by the first-period participation constraint:<sup>26</sup>

$$w^{yS} = -\delta \int_{\hat{\tau}_h}^{\bar{\tau}} w^{pS}(\tau) f(\tau) d\tau. \quad (89)$$

Therefore, we can replace  $\int_{\hat{\tau}_h}^{\bar{\tau}} w^{pS}(\tau) f(\tau) d\tau$  by  $-w^{yS}/\delta$  in (88) to obtain:

$$V_h^{oS} - V_h^{yS} = \frac{\int_{\underline{\tau}_h}^{\bar{\tau}} (\theta\tau - \theta\gamma\mu) f(\tau) d\tau}{1 + \delta(1 - F(\underline{\tau}_h))} + \frac{w^{yS}}{\delta}. \quad (90)$$

Now, plug (90) into (85) to find the poaching wage offered to a worker with talent  $\tau$  (this function is defined only for values of  $\tau$  such that  $w^{pS}(\tau) \geq 0$ ):

$$w^{pS}(\tau) = \theta\gamma\tau - \theta\gamma\mu - \frac{\delta}{1 + \delta(1 - F(\underline{\tau}_h))} \int_{\underline{\tau}_h}^{\bar{\tau}} (\theta\tau - \theta\gamma\mu) f(\tau) d\tau. \quad (91)$$

We now derive the expressions for  $\underline{\tau}_h$  and  $\underline{\tau}_l$ . An  $h$ -firm is willing to retain a worker of type  $\underline{\tau}_h$  for a wage of zero if the following condition holds:

$$V_h^{oS}(\underline{\tau}_h) - V_h^{yS} = 0, \quad (92)$$

where

$$V_h^{oS}(\underline{\tau}_h) = \theta\underline{\tau}_h + \delta V_h^{yS}, \quad (93)$$

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<sup>26</sup>Note that the wage of a young worker is independent of the type of the firm.

and  $V_h^{yS}$  is given by equation (83) (Recall that in equilibrium  $V_h^{yS} = V^{pS}(\tau)$  for any  $\tau \geq \hat{\tau}$ ). We can rewrite (92) as

$$\begin{aligned} & \theta_{\underline{\tau}_h} - \theta\gamma\mu + w^{yS} - \delta(V_h^{oS} - V_h^{yS}) = 0 \\ \Leftrightarrow & \theta_{\underline{\tau}_h} - \theta\gamma\mu - \frac{\delta}{1 + \delta(1 - F(\underline{\tau}_h))} \int_{\underline{\tau}_h}^{\bar{\tau}} (\theta\tau - \theta\gamma\mu) f(\tau) d\tau = 0 \\ \Leftrightarrow & \underline{\tau}_h - \gamma\mu - \delta \int_{\underline{\tau}_h}^{\bar{\tau}} (\tau - \underline{\tau}_h) f(\tau) d\tau = 0. \end{aligned} \quad (94)$$

The equilibrium threshold  $\underline{\tau}_h$  is given by the unique solution to (94) (note that the left-hand side of (94) is increasing in  $\underline{\tau}_h$  and is negative for  $\underline{\tau}_h = 0$  and positive for  $\underline{\tau}_h = \bar{\tau}$ ). Then, we have a closed form solution for the poaching wage in (91). By setting  $w^{pS}(\hat{\tau}) = 0$  in (91), we then obtain a unique equilibrium value for  $\hat{\tau}$ .

We now need to find threshold  $\underline{\tau}_l$ . An  $l$ -firm is willing to retain a worker of type  $\underline{\tau}_l$  for a wage of zero if the following condition holds:

$$V_l^{oS}(\underline{\tau}_l) - V_l^{yS} = 0,$$

where

$$V_l^{oS}(\underline{\tau}_l) = \underline{\tau}_l + \delta V_l^{yS}, \quad (95)$$

$$V_l^{yS} = \gamma\mu - w^{yS} + \delta V_l^{oS}, \quad (96)$$

and

$$\begin{aligned} V_l^{oS} &= (F(\underline{\tau}_l) + 1 - F(\tau_l^\#))V_l^{yS} + \int_{\underline{\tau}_l}^{\tau_l^\#} \tau f(\tau) d\tau \\ &\quad - \int_{\hat{\tau}_l}^{\tau_l^\#} w^{pS}(\tau) f(\tau) d\tau + \delta(F(\tau_l^\#) - F(\underline{\tau}_l))V_l^{yS}. \end{aligned} \quad (97)$$

We use (95), (96), and (97) to obtain:

$$V_l^{oS}(\underline{\tau}_l) - V_l^{yS} = 0 \Leftrightarrow \underline{\tau}_l - \gamma\mu - \frac{\delta \left( \int_{\underline{\tau}_l}^{\tau_l^\#} (\tau - \gamma\mu) f(\tau) d\tau + \int_{\tau_l^\#}^{\bar{\tau}} w^p(\tau) f(\tau) d\tau \right)}{1 + \delta(F(\tau_l^\#) - F(\underline{\tau}_l))} = 0 \quad (98)$$

$$\Leftrightarrow \underline{\tau}_l - \gamma\mu - \delta \int_{\underline{\tau}_l}^{\tau_l^\#} (\tau - \underline{\tau}_l) f(\tau) d\tau - \delta \int_{\tau_l^\#}^{\bar{\tau}} w^p(\tau) f(\tau) d\tau = 0, \quad (99)$$

which again determines a unique  $\underline{\tau}_l$  for a given  $\tau_l^\#$ .

Now, we only need to find  $\tau_l^\#$ . Poaching exists only if the incremental surplus to the

poacher is larger than the net loss to the incumbent firm:

$$V_l^{oS}(\tau_l) - V_l^{yS} \leq V_h^{pS}(\tau_l) - V_h^{yS}. \quad (100)$$

To see that this must hold in any equilibrium with poaching, note that if it did not hold, the incumbent could offer a slightly larger wage and profitably prevent poaching. Thus, if an interior  $\tau_l^\#$  exists, it is determined by one of the solutions to (100) with equality, which yields:

$$\tau_l^\# - \gamma\mu - \frac{\delta \int_{\underline{\tau}_l}^{\tau_l^\#} (\tau - \gamma\mu) f(\tau) d\tau + \delta \int_{\tau_l^\#}^{\bar{\tau}} w^p(\tau) f(\tau) d\tau}{1 + \delta(F(\tau_l^\#) - F(\underline{\tau}_l))} = \theta\gamma(\tau_l^\# - \mu) - \frac{\delta \int_{\underline{\tau}_h}^{\bar{\tau}} (\theta\tau - \theta\gamma\mu) f(\tau) d\tau}{1 + \delta(1 - F(\underline{\tau}_h))}. \quad (101)$$

(If there is no interior solution, the equilibrium is such that no one is poached). If there is more than one solution, only one of such solutions is an equilibrium. To see this, note that if  $\tau_l$  is poached in any equilibrium, then  $\tau_l' > \tau_l$  will also be poached because  $\tau_l - \gamma\mu - \theta\gamma(\tau_l - \mu)$  is strictly decreasing in  $\tau_l$  (note that the value of future options do not change with  $\tau_l$ ). Thus, there is a unique set of values  $(\underline{\tau}_l, \underline{\tau}_h, \tau_l^\#, \hat{\tau}, w^{pS})$  and function  $w^{pS}(\tau)$  that characterize the equilibrium.

### Proposition 6

**Proof.** Begin by rewriting (99) as

$$\underline{\tau}_l - \gamma\mu = \delta \int_{\underline{\tau}_l}^{\bar{\tau}} (\tau - \underline{\tau}_l) f(\tau) d\tau + \delta \int_{\tau_l^\#}^{\bar{\tau}} (w^p(\tau) - \tau + \underline{\tau}_l) f(\tau) d\tau. \quad (102)$$

The left-hand side of equation (102) increases with  $\underline{\tau}_l$  and the right-hand side (RHS) decreases with  $\underline{\tau}_l$ . If  $\tau_l^\# = \bar{\tau}$ , then the conditions defined by equations (102) and (94) are the same and  $\underline{\tau}_l = \underline{\tau}_h$ . If  $\tau_l^\# < \bar{\tau}$ , then  $\delta \int_{\tau_l^\#}^{\bar{\tau}} (w^p(\tau) - \tau + \underline{\tau}_l) f(\tau) d\tau > 0$ , which increases the RHS and thus increases the value for  $\underline{\tau}_l$ . ■

### Proposition 7

The intuition for this proposition is straightforward. As there are no labor market frictions, perfect competition for talent implies that the allocation of workers to firms is efficient. Thus, the only potential source of inefficiency is the choice between the retention of an old worker and the hiring of a young worker. Hiring a young worker is a potential source of externalities, as everyone learns about the talent of a young worker, which increases the number of options available to all players. However, because a firm that hires a young worker can extract all of the worker's surplus by charging a negative wage, and because Bertrand competition implies that poachers obtain zero net surplus from their poaching activity, a firm extracts all of the expected surplus from its decision to hire a young worker. Thus, the firm

internalizes all of the potential costs and benefits of such a decision, and thus the firm's optimal private decision is also socially optimal.

To prove this result formally, we proceed as follows. We first state the necessary and sufficient conditions for an allocation, here fully characterized by thresholds  $(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*)$ , to be (Kaldor-Hicks) efficient (i.e., to maximize a social welfare function with equal weights to all players). We then show that we can construct a set of prices (wages) that sustains such an allocation as a decentralized equilibrium of our game. Thus, an efficient allocation is also a decentralized equilibrium. Because the decentralized equilibrium is unique, it is thus always efficient.

**Proof.** For simplicity, without loss of generality we consider only symmetric allocations in which all firms and workers of the same type and in identical situations are assigned the same surplus by a hypothetical social planner. Under this assumption, to derive the efficiency conditions we can work with an alternative interpretation of the model in which there is only one firm of each type.

Consider an allocation associated with the thresholds  $(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*)$ . Let  $S^*(\tau_l, \tau_h)$  denote the total surplus generated by this allocation, conditional on knowing the incumbent workers' types  $(\tau_l, \tau_h)$  (if one or both firms do not have incumbent workers, define the surplus accordingly as being conditional only on the type of the existing incumbent worker, if any). This allocation is efficient if and only if, for any other allocation with conditional surplus  $S'(\tau_l, \tau_h)$ ,

$$S^*(\tau_l, \tau_h) \geq S'(\tau_l, \tau_h) \text{ for all } (\tau_l, \tau_h). \quad (103)$$

We can focus on conditional surplus because, under the current interpretation, there are only two firms and at most two incumbent workers.

To maximize (conditional) surplus, we list three necessary conditions:<sup>27</sup>

(1) For any given  $\tau_l$ , firm  $l$  retains this type instead of hiring a young worker if and only if:

$$U_l^o(\tau_l) + u(\tau_l) + U_h^{ol} + u(\tau_h, \tau_l) \geq U_l^y + u_f(\tau_l) + u^{yl} + U_h^{yl} + u_f(\tau_h, \tau_l), \quad (104)$$

where  $U_i^o(\tau_i)$  is the expected payoff to  $i$  of retaining  $\tau_i$  under the allocation,  $u(\tau_i)$  is the expected payoff to worker  $\tau_i$  of being retained by  $i$ ,  $U_h^{ol}$  is the expected payoff to  $h$  of  $l$  retaining  $\tau_l$ ,  $u(\tau_h, \tau_j)$  is the expected payoff to worker  $\tau_h$ , who currently works for firm  $h$ , if worker  $\tau_l$  is retained by  $l$  (if  $h$  has no incumbent worker, we set this value to zero),  $U_i^y$  is the expected payoff to  $i$  of hiring a young worker,  $u_f(\tau_l)$  is the expected payoff to a worker of type  $\tau_l$  of being fired by  $l$ ,  $u^{yi}$  is the expected payoff to a young worker of being hired by

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<sup>27</sup>In what follows, for simplicity we assume that all workers who remain unemployed are assigned zero net surplus by the social planner. This is without loss of generality. In addition, in line with the previous assumption that only  $h$  firms can be poachers, we focus on the cases where there are no job transitions from  $h$  to  $l$ .

$i$ ,  $U_h^{yl}$  is the expected payoff to  $h$  of  $l$  hiring a young worker, and  $u_f(\tau_h, \tau_l)$  is the expected payoff to worker  $\tau_h$  if worker  $\tau_l$  is fired by firm  $l$  (if firm  $h$  has no incumbent worker, we set this value to zero).

(2) If firm  $h$  has a vacancy,  $h$  poaches a worker of type  $\tau_l$  instead of hiring a young worker if and only if:

$$U_h^p(\tau_l) + u^h(\tau_l) + U_l^y + u^{yl} \geq U_h^y + u^{yh} + \max\{U_l^o(\tau_l) + u(\tau_l), U_l^y + u^{yl}\}. \quad (105)$$

where  $U_h^p(\tau_l)$  is the expected payoff to  $h$  of poaching  $\tau_l$  and  $u^h(\tau_l)$  is the expected payoff to worker  $\tau_l$  of being hired by  $h$ .

(3) For any given  $\tau_h$  and  $\tau_l$ , firm  $h$  retains this type if and only if:

$$U_h^o(\tau_h) + u(\tau_h) + \max\{U_l^o(\tau_l) + u(\tau_l), U_l^y + u^{yl}\} \geq \max\{U_h^y + u^{yh} + \max\{U_l^o(\tau_l) + u(\tau_l), U_l^y + u^{yl}\}, U_h^p(\tau_l) + u^h(\tau_l) + U_l^y + u^{yl}\}. \quad (106)$$

Now, consider the efficient allocation, which is determined by the thresholds  $(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*)$ . Note first that these thresholds fully determine the following wages:

$$w^{p*}(\tau) = \theta\gamma\tau - \theta\gamma\mu - \frac{\delta}{1 + \delta(1 - F(\underline{\tau}_h^*))} \int_{\underline{\tau}_h^*}^{\tau} (\theta\tau - \theta\gamma\mu) f(\tau) d\tau, \quad (107)$$

$$w^{y*} = -\delta \int_{\hat{\tau}^*}^{\tau} w^{pS}(\tau) f(\tau) d\tau, \quad (108)$$

where  $\hat{\tau}^*$  is the threshold for which  $w^{p*}(\hat{\tau}^*) = 0$ . Given these wages, then we can easily verify that we can uniquely define  $V_h^{p*}(\tau)$ ,  $V_h^{o*}(\tau)$ ,  $V_h^{y*}$ ,  $V_h^{o*}$ ,  $V_l^{o*}(\tau)$ ,  $V_l^{y*}$ , and  $V_l^{o*}$  as the value functions as before, but taking the thresholds  $(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*)$  as given.

We now need to show that such wages can sustain a decentralized equilibrium such that Conditions (1)-(3) hold. Start with (104). First, if  $u^{yl} \neq 0$ , then use (positive or negative) lump-sum transfers from the worker to firm  $l$  to create a new allocation on the right-hand side of (104), without changing its total surplus, so that  $U_l^y$  under this new allocation is equal to the old  $U_l^y$  plus the old  $u^{yl}$ , and thus the new  $u^{yl}$  becomes zero:

$$U_l^o(\tau_l) + u(\tau_l) + U_h^{ol} + u(\tau_h, \tau_l) \geq U_l^y + u_f(\tau_l) + U_h^{yl} + u_f(\tau_h, \tau_l). \quad (109)$$

Second, consider  $U_h^{ol}$ . Suppose that  $h$  has a vacancy. If  $U_h^{ol} \neq V_h^{y*}$ , make transfers to or from all the other players until  $U_h^{ol} = V_h^{y*}$  and the surplus on left-hand side is unchanged.<sup>28</sup>

<sup>28</sup>Notice that such transfers can always be made because the initial allocation is assumed to be efficient and thus has the maximum possible conditional surplus. If, counterfactually,  $V_h^{y*}$  was higher than the maximum surplus, an allocation that delivered  $V_h^{y*}$  (which is possible by construction) would be superior to the efficient

Make similar transfers in the analogous case in which  $h$  has a worker of type  $\tau_h$  until  $U_h^{ol} = \max\{V_h^{y*}, V_h^{o*}(\tau_h)\}$ . Make similar transfers on the right-hand side until  $U_h^{yl} = V_h^{y*}$  or  $U_h^{yl} = \max\{V_h^{y*}, V_h^{o*}(\tau_h)\}$ , depending on which case is relevant. Then, we can rewrite the condition above as

$$U_l^o(\tau_l) + u(\tau_l) + u(\tau_h, \tau_l) \geq U_l^y + u_f(\tau_l) + u_f(\tau_h, \tau_l). \quad (110)$$

Third, consider  $u(\tau_h, \tau_l)$ . This term is zero if  $h$  has a vacancy. If instead  $h$  has an incumbent who is retained (i.e., if  $\tau_h \geq \tilde{\tau}_h^*$ ), then  $u(\tau_h, \tau_l) = u(\tau_h)$ . Suppose in this case that  $\tau_h \leq \hat{\tau}^*$ . If  $u(\tau_h) \neq 0$ , make transfers so that  $u(\tau_h) = 0$ . If instead  $\tau_h > \hat{\tau}^*$ , if  $u(\tau_h) \neq V_h^{p*}(\tau_h) - V_h^{y*}$ , make transfers until  $u(\tau_h) = V_h^{p*}(\tau_h) - V_h^{y*}$ . Similarly, we have that  $u_f(\tau_h, \tau_l) = u(\tau_h)$  if  $\tau_h \geq \tilde{\tau}_h^*$  and  $u_f(\tau_h, \tau_l) = 0$  otherwise. Make transfers on the right-hand side so that  $u(\tau_h) = 0$  or  $u(\tau_h) = V_h^{p*}(\tau_h) - V_h^{y*}$ , depending on which case is relevant. Then, we can rewrite the condition above as

$$U_l^o(\tau_l) + u(\tau_l) \geq U_l^y + u_f(\tau_l). \quad (111)$$

Finally, suppose first that  $\tau_l \leq \hat{\tau}^*$ . If  $u(\tau_l) \neq 0$ , make transfers to or from  $l$  so that  $u^l(\tau_l) = 0$ . Suppose now that  $\tau_l > \hat{\tau}^*$ . If  $u(\tau_l) \neq V_h^{p*}(\tau_l) - V_h^{y*}$ , make transfers to or from  $l$  so that  $u(\tau_l) = V_h^{p*}(\tau_l) - V_h^{y*}$ . Similarly, make transfers on the right-hand side so that  $u_f(\tau_l) = 0$  or  $u_f(\tau_l) = V_h^{p*}(\tau_l) - V_h^{y*}$ , depending on which case is relevant. Then, we can rewrite the condition above as

$$U_l^o(\tau_l) \geq U_l^y, \quad (112)$$

which by construction is equivalent to  $V_l^{o*}(\tau_l) \geq V_l^{y*}$ . But this is also a necessary condition for the retention of type  $\tau_l$  in a competitive equilibrium given thresholds  $(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*)$ . Thus, condition (104) is compatible with a decentralized equilibrium with thresholds  $(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*)$ .

It is possible to replicate this argument for the other two conditions (i.e., (105) and (106)), and similarly show that none of these conditions impose restrictions on the equilibrium. The steps are tedious but simple; we omit them here for brevity.

We then conclude that, for any given efficient allocation  $(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*)$ , it is possible to construct prices (i.e., wages) that support this allocation as a decentralized equilibrium. Because we showed earlier that the decentralized equilibrium is unique, then this equilibrium must be efficient. ■

### Proposition 8

**Proof.** In what follows, we take  $\{\hat{\tau}_l, \hat{\tau}_h, \underline{\tau}_l, \underline{\tau}_h\}$  as given. Our goal is to find the unique  $\{\tilde{\tau}_l, \tilde{\tau}_h\}$  conditional on the other thresholds. Because many of the steps are similar to those in the proof of Proposition 1, we refer the reader to that proof in some instances.

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allocation, which is a contradiction.

Lemma 2 implies that an equilibrium with retention must have a threshold  $\tilde{\tau}_i$ . Lemma 1 implies that all types in  $[\tilde{\tau}_i, \bar{\tau}]$  are paid the same wage. To prevent poaching, this wage must be such that  $w_i^* \geq w^p(w_i^*)$  ( $w^p(\cdot)$  will be derived below). Because poachers know that all types in  $[\tilde{\tau}_i, \bar{\tau}]$  are offered  $w_i^*$ , their beliefs must be given by  $F(\tau \mid \tau \geq \tilde{\tau}_i)$  upon observing  $w_i^*$ . The poaching wage offered by a type- $H$  company with a vacant position is implicitly determined by the following condition:

$$V_h^p(\tau \geq \tilde{\tau}_i) - V_h^y = 0, \quad (113)$$

where

$$V_h^p(\tau \geq \tilde{\tau}_i) = \theta\gamma \int_{\tilde{\tau}_i}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_i)} - w^p(w_i) + \delta V_h^y, \quad (114)$$

$$V_i^y = i\gamma\mu - w_i^y + \delta V_i^o, \quad (115)$$

and

$$V_i^o = F(\tilde{\tau}_i)V_i^y + (1 - F(\tilde{\tau}_i)) \left( \int_{\tilde{\tau}_i}^{\bar{\tau}} \frac{i\tau f(\tau)}{(1 - F(\tilde{\tau}_i))} d\tau - w^p(w_i) + \delta V_i^y \right). \quad (116)$$

From equations (115) and (116), we obtain:

$$V_i^o - V_i^y = \frac{1}{1 + \delta(1 - F(\tilde{\tau}_i))} \left( \int_{\tilde{\tau}_i}^{\bar{\tau}} (i\tau - w^p(w_i) - i\gamma\mu) f(\tau) d\tau + (1 - F(\tilde{\tau}_i)) w_i^y \right). \quad (117)$$

The poaching wage offered by a type- $H$  firm upon observing  $w_i^*$  is

$$w^p(w_i^*) = \theta\gamma \left( \int_{\tilde{\tau}_i}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_i)} - \mu \right) + w_h^y - \frac{\delta \int_{\tilde{\tau}_h}^{\bar{\tau}} (\theta\tau - w_h^* - \theta\gamma\mu + w_h^y) f(\tau) d\tau}{[1 + \delta(1 - F(\tilde{\tau}_h))]} \quad (118)$$

Using this poaching wage, we can now proceed exactly as in the proof of Proposition 1 to show that  $w_i^* = \max\{w^p(w_i^*), 0\}$  if the equilibrium threshold is  $\tilde{\tau}_i$  for  $i \in \{l, h\}$ .<sup>29</sup> Solving it for  $w_h^*$ , we obtain

$$w_h^* = \max \left\{ \theta\gamma \left( \int_{\tilde{\tau}_h}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_h)} - \mu \right) + w_h^y - \delta\theta \int_{\tilde{\tau}_h}^{\bar{\tau}} (1 - \gamma)\tau f(\tau) d\tau, 0 \right\}, \quad (119)$$

which can be plugged into (118) to find  $w_l^*$ .

Because  $w_i^* = \max\{w^p(w_i^*), 0\}$ , a necessary condition for an incumbent with type  $\tau \in$

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<sup>29</sup>Formally, we need to modify Assumption E2 slightly to fit the dynamic setup: After observing an off-the-equilibrium-path wage  $w_i'$ , poachers believe that the probability that type  $\tau' \geq w_i' + i\gamma\mu - w_i^y + \delta(V_i^o - V_i^y)$  deviates is no less than the probability that type  $\tau'' > \tau'$  deviates. The application of this equilibrium refinement thus depends on some other equilibrium values ( $w_i^y$ ,  $V_i^o$ , and  $V_i^y$ ); this creates no difficulties as the condition can always be checked for each candidate equilibrium.



$[\tilde{\tau}_i, \bar{\tau}]$  not to deviate and fire the worker is:

$$V_i^o(\tilde{\tau}_i) \geq V_i^y, \quad (120)$$

where

$$V_i^o(\tilde{\tau}_i) = i\tilde{\tau}_i - w_i^* + \delta V_i^y. \quad (121)$$

Hence, after some rearranging, condition (120) becomes:

$$i(\tilde{\tau}_i - \gamma\mu) - \frac{w_i^* - w_i^y + \delta i \int_{\tilde{\tau}_i}^{\bar{\tau}} (\tau - \gamma\mu) f(\tau) d\tau}{1 + \delta(1 - F(\tilde{\tau}_i))} \geq 0. \quad (122)$$

The wage offered by type- $H$  firms to workers with talent  $\tau \in [\hat{\tau}_i, \tilde{\tau}_i]$  is determined by the following condition (from Bertrand competition):

$$V_h^p(\tau \in [\hat{\tau}_i, \tilde{\tau}_i]) = V_h^y, \quad (123)$$

where

$$V_h^p(\tau \in [\hat{\tau}_i, \tilde{\tau}_i]) = \theta\gamma \int_{\hat{\tau}_i}^{\tilde{\tau}_i} \frac{\tau f(\tau) d\tau}{F(\tilde{\tau}_i) - F(\hat{\tau}_i)} - w_i^{**} + \delta V_h^y. \quad (124)$$

We use equations (115), (116), and (124) to derive the wage for those workers who are poached in equilibrium:

$$w_i^{**} = \theta\gamma \left( \int_{\hat{\tau}_i}^{\tilde{\tau}_i} \frac{\tau f(\tau) d\tau}{F(\tilde{\tau}_i) - F(\hat{\tau}_i)} - \mu \right) + w_h^y - \frac{\delta \int_{\tilde{\tau}_h}^{\bar{\tau}} (\theta\tau - w_h^* - \theta\gamma\mu + w_h^y) f(\tau) d\tau}{1 + \delta(1 - F(\tilde{\tau}_h))}. \quad (125)$$

The participation constraint of a young worker is given by

$$w_i^y = -\delta(1 - F(\tilde{\tau}_i))w_i^* - \delta(F(\tilde{\tau}_i) - F(\hat{\tau}_i)) \max\{w_i^{**}, 0\}. \quad (126)$$

We now discuss the existence and uniqueness of the threshold  $\tilde{\tau}_i$ . Define the function:

$$H_i(\tau) = i(\tau - \gamma\mu) - \frac{w_i^* - w_i^y + \delta i \int_{\tau}^{\bar{\tau}} (x - \gamma\mu) f(x) dx}{1 + \delta(1 - F(\tau))}. \quad (127)$$

The existence of an equilibrium with retention requires this function to be non-negative for some  $\tilde{\tau}_i$ . Because,  $H_i(\tau)$  is continuous and  $H_i(0) < 0$ , at least one fixed point exists if and only if  $\max_{\tau \in [0, \bar{\tau}]} H_i(\tau) \geq 0$ . As before, if this latter condition does not hold, then no type

is retained by firm  $i$  in equilibrium, i.e.,  $\tilde{\tau}_i = \bar{\tau}$ . If  $\max_{\tau \in [0, \bar{\tau}]} H_i(\tau) \geq 0$ , this proves the existence of at least one threshold  $\tau'$  such that  $H_i(\tau') = 0$ .

Among all such  $\tau'$ , we define  $\tilde{\tau}_i$  as the lowest one. To show that this threshold is part of an equilibrium, notice that because  $H(0) < 0$ ,  $H(\tau)$  crosses zero from below at  $\tilde{\tau}_i$ , which is also a necessary condition for an equilibrium. To show that no other  $\tau' > \tilde{\tau}_i$  can be an equilibrium, we use the same argument as in the the proof of Proposition 1. Thus,  $\tilde{\tau}_i$  is uniquely determined given  $\{\hat{\tau}_l, \hat{\tau}_h, \underline{\tau}_l, \underline{\tau}_h\}$ . ■

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