School of Economics and Finance Cross-Country Co-movement in Long-Term Interest Rates: A DSGE Approach

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Cross-Country Co-movement in Long-Term Interest Rates: A DSGE Approach^{*}

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Abstract

Long-term interest rates in a number of small-open inflation targeting economies co-move more strongly with US long-term rates than with short-term rates in those economies. We augment a standard small open-economy model with imperfectly substitutable government bonds and timevarying term premia that captures this phenomenon. The estimated model fits a range of US and UK data remarkably well, and produces term premium estimates that are comparable to estimates from the affine term structure model literature. We find that the strong co-movement between US and UK long-term interest rates arises primarily via correlated policy rate expectations, rather than through correlated term premia. This is due to policymakers in both economies responding to foreign productivity and discount factor shocks that cause persistent changes in inflation. We also overcome the common failure of similar models to account for the large influence of foreign disturbances on domestic economies found empirically, where in our model around 40% of the variation in UK GDP can be explained by shocks originating in the US economy.

JEL Classification: E43, E44, F30, F44, F65, G15

Keywords: DSGE Model, Small Open Economy, Yield Curve, Long-Term Interest Rates, Term Premia, Co-movement

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1 Introduction

Monetary authorities around the world affect agents' consumption and investment decisions, in part by influencing long-term nominal interest rates, in order to deliver an environment of low and stable inflation and maximum employment. This can be achieved (absent the zero lower bound (ZLB)) by adjusting the short-term policy rate (which we refer to as 'conventional' policy) and/or by explicitly targeting longer maturity interest rates (such as the 10-year government bond yield, which we refer to as 'unconventional' policy). However, a number of studies (Bernanke (2013), Kulish and Rees (2011), Wright (2011), Dahlquist and Hasseltoft (2013), Swanson and Williams (2014) and Jotikasthira et al. (2015)) have highlighted the strong co-movement over the post-war period between long-term interest rates in a number of inflation targeting small open economies (such as Australia, Canada, New Zealand and the UK) and long-term interest rates in the US. In fact, the correlation with US long-term rates appears to be stronger than the correlation between short- and long-term rates in the small-open economies themselves (see Figure 1 of Bernanke (2013) and Kulish and Rees (2011), the discussion in Dahlquist and Hasseltoft (2013) and Jotikasthira et al. (2015)). Figure 1 illustrates this phenomenon for the UK, where the correlation between UK and US 10-year government bond yields over the period from 1997 to 2013 is as high as 93%¹. The co-movement between the short-term (3-month) UK interest rate and the 10-year yield is somewhat weaker, particularly over the ZLB period.

Based on these observations, previous studies have pointed to i) the existence of significant term premia in domestic interest rates, that create a 'wedge' between expectations over the path of future policy rates and long-term interest rates and ii) term premia being correlated internationally (see Wright (2011), Dahlquist and Hasseltoft (2013) and Jotikasthira et al. (2015)). The term premium captures the 'compensation' required by investors to hold long-term bonds as opposed to rolling over a series of short-term bonds. Broadly speaking, this premium ought to reflect uncertainty around inflation and real economic variables, and internationally correlated term premia are consistent with domestic and foreign premia being subject to common shocks.

If conventional policy primarily acts to influence the expected path of short-term interest rates, it follows that the extent to which conventional policy can influence long-term interest rates materially depends on the size and dynamics of the term premium. In other words, the premise that long-term interest rate co-movement reflects correlated term premia – and not correlated policy rate expectations – generates an additional layer of complexity for monetary policymakers operating in a small-open economy. Monetary authorities may implicitly have to account for a decoupling of short- and long-term domestic interest rates, and for the possibility that long-term interest rates are influenced by factors beyond expected rates and *domestic* shocks. In order to conduct domestic monetary policy effectively, it is therefore important to understand how term premia are formed, how they respond to macroeconomic shocks, and the extent to which the strong co-movement between long-term yields arises via expectations about the policy rates or correlated term premia. Policy makers across the world are devote much attention and resource to these issues, as highlighted in a speech given by Ben Bernanke at the 2013 Annual Monetary/Macroeconomics Conference: 'The Past and Future of Monetary Policy'.²

¹The relative correlations do not simply reflect the downward trend in long-term interest rates: we obtain almost identical estimates when using first-differenced yields.

²This issue is also discussed more recently by Ben Bernanke in his blog.

In general, standard small open economy (SoE) dynamic stochastic general equilibrium (DSGE) models (such as those proposed by Gali and Monacelli (2005), Adolfson et al. (2007), Christiano et al. (2011), Burgess et al. (2013) among others) are not set up to consider these issues, as they either do not include or cannot replicate the high observed correlations across long-term interest rates.³ This is because a distinct role for long-term interest rates requires certain types of frictions which, in general, are not part of theoretical models that have been developed. In addition, it has been shown by Justiniano and Preston (2010) that estimated SoE DSGE models fail to adequately capture the large contribution of foreign disturbances to fluctuations in the domestic economies, found in empirical studies. This suggests that, even with the introduction of term premia into an open economy model, we would still not necessarily replicate the cross-country co-movement in long-term interest rates observed in the data.

To our knowledge, this paper is the first study that attempts to introduce long-term interest rates with embedded term premia into an *open* macroeconomy model. We do so by incorporating imperfectly substitutable assets (as in Andres et al. (2004), Chen et al. (2012), Harrison (2012) and Liu et al. (2014) for closed economy models) into an otherwise standard micro-founded SoE DSGE model. To summarise briefly, in each economy (domestic and foreign) we introduce a simple financial intermediary that holds both short- and long-term government debt, where the holdings are financed using oneperiod deposits received from households. The domestic financial intermediary holds long-term debt issued by the domestic government, and also a fraction of the long-term debt issued by the foreign government. Foreign intermediaries, however, are restricted to holding only foreign short- and longterm assets (in line with the small open economy assumption). Similar to Harrison (2012), portfolio adjustment costs are introduced into financial intermediaries' profit functions to capture the stylised fact that, while intermediaries would prefer to hold long-maturity assets, this weakens their ability to meet short-term demand to withdraw deposits. Intermediaries' balance sheets are subject to both short- and long-maturity 'risk premium shocks' (à la Smets and Wouters (2007)). The effective interest rate faced by domestic agents in the model depends on short-term domestic interest rates, long-term interest rates (domestic and foreign), the quantity of short- and long-term debt (domestic and foreign) and the exchange rate.

Our set-up allows us to obtain a model-implied term structure decomposition of domestic and foreign long-term interest rates into short-rate expectations and term premia. In line with past evidence, we find a significant role for term premia in accounting for variation in long-term interest rates. We also use the decomposition to guide our exploration of the key mechanisms that underlie the cross-country interest rate co-movement implied by the model. While previous studies (e.g. Jotikasthira et al. (2015)) suggest that term premia drive the co-movement, we are able to account for the strong co-movement between US and UK long-term interest rates via correlated policy rate expectations. This result stems from monetary policy responses to foreign shocks (productivity and discount factor), that cause persistent deviations of inflation in both countries away from their steady-states. Forward-looking agents adjust their expectations of the path of policy rates quickly in response to these shocks, and more quickly relative to the adjustment in actual policy rates, due to strong interest rate smoothing preferences displayed by policymakers in both countries. As foreign shocks move inflation rates across countries in the same direction, policy rate expectations account for a significant proportion of strong

 $^{^{3}}$ Kulish and Rees (2011) illustrate that standard SoE DSGE models can reproduce the strong correlation seen between domestic and foreign long-term interest rates, but this is achieved only through an equally strong correlation between the short- and long-term yields, as the expectations hypothesis holds in Kulish and Rees model.

correlation in long-term interest rates. As central banks only gradually adjust their policy instruments, we do not see as strong a correlation between short- and long-term interest rates.

The term structure decomposition of the long-term interest rate also allows us to derive term premium estimates for both US and UK. As a cross-check for our model, we compare these measures with estimates obtained using empirical time-series affine term structure models. We take comfort from the similarity between the DSGE and time-series estimates. Our model appears to deliver term premium estimates that are broadly comparable to those obtained from models designed to closely fit the whole term structure of interest rates in a no-arbitrage framework.

Overall, the estimated model fits a large range of US and UK economic and financial data remarkably well, including over recession periods occurring within the sample (including the 'Great Recession' in 2008/9). Furthermore, the model is able to replicate the correlation between the US and UK GDP. As discussed in Kollmann (1996) and Kollmann (2001) (and the references within), this has traditionally been a characteristic of the data that open economy DSGE models have struggled to reproduce. We show that this feature is a consequence of the incomplete asset market structure, and sticky prices and wages in the model (Kollmann, 2001). As mentioned, Justiniano and Preston (2010) show that 'estimated' SoE models fail to identify the importance of foreign disturbances for the domestic economy, in sharp contrast with the evidence from the open economy empirical literature (see Cushman and Zha (1997) and Kose et al. (2003) among others), which suggests that foreign shocks are able to explain about 40% of the variance in domestic GDP and 80% of the variance in domestic long-term interest rates.

More than 50% of the variance in foreign and domestic long-term rates is explained by a very persistent US productivity shock. We explain that this reflects agents' concerns about investing in long-term assets when inflation is positive and output (consumption) is low i.e. the response to a negative productivity shock. In this case long-term nominal assets become unattractive as high inflation erodes their payoff at times when payoffs are needed the most. This is consistent with the analysis of Rudebusch and Swanson (2012) and the empirical results presented by Piazzesi and Schneider (2007) who argue that long-term assets do not provide a good hedge against times of low consumption and high inflation.

The paper is organised as follows: Section 2 outlines the novel features of the model relative to standard SoE models. Section 3 provides preliminary analysis and details of the model estimation, and Sections 4 and 5 discuss the empirical results.

2 Theoretical Model

Our baseline model has been used in a large number of SoE studies, the features/predictions of which are well understood (see Gali and Monacelli (2005), Justiniano and Preston (2010), ?, Adolfson et al. (2007), and Christiano et al. (2011) among others). In this section, we describe the main differences in our otherwise 'standard' model, where we augment the model with two additional assets – domestic and foreign long-term government debt – and include financial intermediaries which operate in the

 $^{^{4}}$ This result has been confirmed in many subsequent studies. See Christiano et al. (2011) and Burgess et al. (2013) among others.

government bond markets.⁵ Tables 1 and 2 in the appendix summarise the linearised (around a non-stochastic steady-state) model's first-order conditions. The structural parameters and their prior moments are provided in Tables 3, 4 and 5.

2.1 Domestic Financial Intermediaries

In the model, simple financial intermediaries operate in between households and government bond markets. These firms issue deposits to households that pay a gross interest rate r_t^h , and the proceeds from these deposits are used to purchase a portfolio of short- and long- term government issued bonds (paying interest r_t^S and r_t^L , respectively), as well as a small fraction of long term debt issued by the foreign government (paying interest $r_t^{L,*}$). Similar to Andres et al. (2004), Chen et al. (2012), Harrison (2012) and Liu et al. (2014), we follow Woodford (2001) in our treatment of long-term bonds, where government bonds are modelled as perpetuities that cost $p_{L,t}$ at time t and pay an exponentially decaying coupon κ^s at time t + s + 1 where $0 < \kappa \leq 1$. As explained in Woodford (2001) and Chen et al. (2012) the advantage of this formulation is that the period t price of a bond issued s periods ago, $p_{L-s,t}$, can be expressed as a function of the coupon and the current price $p_{L,t}$:

$$p_{L-s,t} = \kappa^s p_{L,t} \tag{1}$$

This relation allows us to express the intermediary balance sheet equations and the government budget constraints (below) in a parsimonious form.⁶ Furthermore, for simplicity, we rule out the existence of a secondary market for long-term bonds, implying that agents who invest in long-term debt must hold it until maturity.⁷ Finally, we assume that all government bonds issued are purchased by these firms. The intermediary's balance sheet is given by

$$b_{\kappa,t}^{h} = \frac{b_{\kappa,t}^{S}}{\varepsilon_{t}^{b^{S}}} + \frac{p_{L,t}b_{\kappa,t}^{L}}{\varepsilon_{t}^{b^{L}}} + \frac{q_{t}\left(1-\varrho\right)p_{L,t}^{*}b_{\kappa,t}^{L,*}}{\varepsilon_{t}^{b^{L}}}$$

where $b_{\kappa,t}^S$, $b_{\kappa,t}^L$ and $b_{\kappa,t}^{L,*}$ denote the quantities held of short-term domestic, long-term domestic, and long-term foreign government bonds, respectively. $p_{L,t}$ and $p_{L,t}^*$ denote the prices of domestic and foreign long-term bonds ($p_{S,t}$ is set to unity), given by

$$p_{L,t} = \frac{1}{r_t^L - \kappa}$$
$$p_{L,t}^* = \frac{1}{r_t^{L,*} - \kappa^*}$$

The term $(1 - \varrho)$ reflects the fraction of the foreign long-term debt held by the domestic financial intermediary, and q_t is the real exchange rate. Motivated by the work of Smets and Wouters (2007) we assume the balance sheet equation is subject to two 'financial' shocks: a short- and a long-term risk premium shock denoted by $\varepsilon_t^{b^S}$ and $\varepsilon_t^{\bar{b}^L}$, respectively. The intermediary's profit function is given by

⁵The full model is provided in an online Appendix.

⁶For further discussion of the benefits of approach, see Chen et al. (2012).

 $^{^{7}}$ See the discussion in Andres et al. (2004) for the advantages of this assumption.

$$\xi_{\kappa,t} = \underbrace{b_{\kappa,t}^{h} + \frac{r_{t-1}^{S}}{\pi_{t}^{c}} b_{\kappa,t-1}^{S} + \frac{r_{t}^{L}}{\pi_{t}^{c}} p_{L,t} b_{\kappa,t-1}^{L} + (1-\varrho) \frac{r_{t}^{L,*}}{\pi_{t}^{c,*}} p_{L,t}^{*} b_{\kappa,t-1}^{L,*}}_{\text{revenues}}}_{\text{revenues}} - \underbrace{\frac{b_{\kappa,t}^{S}}{\varepsilon_{t}^{b^{S}}} - \frac{p_{L,t} b_{\kappa,t}^{L}}{\varepsilon_{t}^{b^{L}}} - \frac{q_{t} (1-\varrho) p_{L,t}^{*} b_{\kappa,t}^{L,*}}{\varepsilon_{t}^{b^{L}}} - \frac{r_{t-1}^{h} b_{\kappa,t-1}^{h}}{\pi_{t}^{c}} b_{\kappa,t-1}^{h} - \frac{x}{2} \left(\delta \frac{b_{\kappa,t-1}^{S}}{b_{\kappa,t-1}^{L} + b_{\kappa,t-1}^{L,*}} - 1 \right)^{2} \frac{1}{\pi_{t}^{c}}}_{e_{t}^{$$

where the prices of domestic and foreign consumption goods are denoted by π_t^c and $\pi_t^{c,*}$, respectively, such that profits reflect real returns on government debt holdings less new bond purchases and interest payments on household deposits. An additional expenditure for the intermediary is an 'adjustment cost', $\frac{x}{2} \left(\delta \frac{b_{\kappa,t-1}^s}{b_{\kappa,t-1}^L + b_{\kappa,t-1}^{L,*}} - 1 \right)^2$, where δ is the inverse of the steady-state ratio of short- to long-term bonds, $\frac{b_{\kappa,t-1}^s}{b_{\kappa,t-1}^L + b_{\kappa,t-1}^{L,*}}$ and x is a free parameter. This implies that it is costly for intermediaries when bond holdings deviate from their steady-state values, and that excess holdings of short-term bonds or a shortage of long-term bonds is more costly. The adjustment cost captures the stylised fact that while intermediaries would like to hold long-maturity assets, this weakens their ability to meet short-term demand to withdraw deposits, and so also want to hold short-term bonds. Substituting the balance sheet equation into the (one-period ahead) profit function, and defining $\bar{b}_{\kappa,t}^L = p_{L,t} b_{\kappa,t}^L$ and $\bar{b}_{\kappa,t}^{L,*} = q_t (1-\varrho) p_{L,t}^* b_{\kappa,t}^{L,*}$, gives

$$E_{t}\xi_{\kappa,t+1} = \frac{r_{t}^{S}}{E_{t}\pi_{t+1}^{c}}b_{\kappa,t}^{S} + E_{t}\left\{\frac{r_{t+1}^{L}}{\pi_{t+1}^{c}}\frac{p_{L,t+1}}{p_{L,t}}\right\}\bar{b}_{\kappa,t}^{L} + E_{t}\left\{\frac{r_{t+1}^{L,*}}{\pi_{t+1}^{c,**}}\frac{p_{L,t+1}^{*}}{p_{L,t}^{*}}\frac{q_{t+1}}{q_{t}}\right\}\bar{b}_{\kappa,t}^{L,*}$$
$$-E_{t}\left\{\frac{r_{t}^{h}}{\pi_{t+1}^{c}}\right\}b_{\kappa,t}^{h} - \frac{x}{2}\left(\delta\frac{b_{\kappa,t}^{S}}{b_{\kappa,t}^{L} + b_{\kappa,t}^{L,*}} - 1\right)^{2}\frac{1}{E_{t}\pi_{t+1}^{c}}$$

2.2 Interest Rates and Term Premia

Profit maximisation with respect to domestic short-term debt, and domestic and foreign long-term debt, subject to the balance sheet condition delivers expressions for the effective rate faced by house-holds, the long-term interest rate and the exchange rate:

Short-Term Household Interest Rate

$$\frac{r_t^h}{\varepsilon_t^{b^S}} = r_t^S - x \left(\delta \frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + b_{\kappa,t}^{L,*}} - 1 \right) \frac{\delta}{b_{\kappa,t}^L + b_{\kappa,t}^{L,*}}$$
(2)

Domestic Long-term Interest Rate

$$E_t \left\{ r_{t+1}^L \frac{p_{L,t+1}}{p_{L,t}} \right\} = \frac{r_t^h}{\varepsilon_t^{b^L}} - x \left(\delta \frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + b_{\kappa,t}^{L,*}} - 1 \right) \frac{1}{p_{L,t} \left(b_{\kappa,t}^L + b_{\kappa,t}^{L,*} \right)}$$
(3)

Foreign Long-term Interest Rate

$$E_t \left\{ \frac{r_{t+1}^{L,*}}{\pi_{t+1}^*} \frac{p_{L,t+1}^*}{p_{L,t}^*} \frac{q_{t+1}}{q_t} \right\} = \frac{r_t^h}{\varepsilon_t^{b^{L,*}} E_t \pi_{t+1}^c} - x \left(\delta \frac{b_{\kappa,t}^S}{b_{\kappa,t}^L + b_{\kappa,t}^{L,*}} - 1 \right) \frac{1}{\frac{r_t^{L,*} p_{L,t}^*}{\pi_t^{c^{*,*}}} \left(b_{\kappa,t}^L + b_{\kappa,t}^{L,*} \right) E_t \pi_{t+1}^c}$$
(4)

As shown in the the online Appendix, the domestic and foreign long-term interest rates can be decomposed into two primary components: a component that reflects expectations of future policy rates, and a term premium.⁸ The term premium in the model can be further decomposed into risk premium and liquidity premium components. It is perhaps important to emphasise that the risk premium component is exogenous, while the liquidity premium is an endogenous variable. The domestic long-term interest rate is given by

$$\hat{r}_{t}^{L} = \underbrace{(1 - \beta\kappa) \sum_{i=0}^{\infty} (\beta\kappa)^{i} E_{t} \hat{r}_{t+i}^{S}}_{\text{Policy Rate Expectations } (\widehat{PE}_{t})} + \underbrace{(1 - \beta\kappa) \sum_{i=0}^{\infty} (\beta\kappa)^{i} \left(E_{t} \hat{\varepsilon}_{t+i}^{b^{S}} - E_{t} \hat{\varepsilon}_{t+i}^{b^{L}}\right)}_{\text{Risk Premium } (\widehat{RP}_{t})} \\ \underbrace{\frac{\tilde{x} (1 - \beta\kappa) (\delta\beta + 1 - \kappa\beta)}{\beta} \sum_{i=0}^{\infty} (\beta\kappa)^{i} \left[\frac{b^{L}}{b^{L} + (1 - \varrho) b^{L,*}} E_{t} \hat{b}_{t+i}^{L} + \frac{(1 - \varrho) b^{L,*}}{b^{L} + (1 - \varrho) b^{L,*}} E_{t} \hat{b}_{t+i}^{L,*} - E_{t} \hat{b}_{t+i}^{S}}_{\text{Liquidity Premium } (\widehat{LP}_{t})} \right] }_{\text{Liquidity Premium } (\widehat{LP}_{t})}$$

or

$$\hat{r}_t^L = \widehat{PE}_t + \underbrace{\widehat{RP}_t + \widehat{LP}_t}_{\text{Term Premium } (\widehat{TP}_t)}$$
(5)

where \hat{r}_t^L is the long-term interest expressed relative to its steady-state value, and $\beta \kappa$ is the subjective discount factor. A large part of our analysis uses these decompositions, as they allow us to *i*) understand the channels though which the strong correlation between US and UK long-term interest rates arise (through synchronised changes in either policy expectations or term premia) and *ii*) derive DSGE term premium estimates, that can be compared against estimates obtained using no-arbitrage term structure models. Similarly, the decomposition of the foreign long-term interest rate is given by

$$\hat{r}_{t}^{L,*} = \underbrace{(1 - \beta\kappa^{*})\sum_{i=0}^{\infty} (\beta\kappa^{*})^{i} E_{t}\hat{r}_{t+i}^{S,*} + (1 - \beta\kappa^{*})\sum_{i=0}^{\infty} (\beta\kappa^{*})^{i} \left(E_{t}\hat{\varepsilon}_{t+i}^{b^{S,*}} - E_{t}\hat{\varepsilon}_{t+i}^{b^{L,*}}\right)}_{\text{Policy Rate Expectations } \left(\widehat{PE}_{t}^{*}\right)} \underbrace{\text{Risk Premium } \left(\widehat{RP}_{t}^{*}\right)}_{\text{Kisk Premium } \left(\widehat{RP}_{t}^{*}\right)} + \underbrace{\frac{\tilde{x}^{*} \left(1 - \beta\kappa^{*}\right) \left(\delta^{*}\beta + 1 - \kappa^{*}\beta\right)}{\beta} \sum_{i=0}^{\infty} (\beta\kappa^{*})^{i} \left[E_{t}\hat{b}_{t+i}^{L,*} - E_{t}\hat{b}_{t+i}^{S,*}\right]}_{\text{Liquidity Premium } \left(\widehat{LP}_{t}^{*}\right)}$$

or

$$\hat{r}_{t}^{L,*} = \widehat{PE}_{t}^{*} + \underbrace{\widehat{RP}_{t}^{*} + \widehat{LP}_{t}^{*}}_{\text{Term Premium } \left(\widehat{TP}_{t}^{*}\right)}$$
(6)

 8 Using the linearised equations in Tables 1 and 2 these expressions can be obtained by substitution.

2.3 Exchange Rate

In addition to the interest rate expressions, (3) and (4) can be combined to produce an Uncovered Interest Rate Parity (UIP) condition for the exchange rate. That is, the exchange rate in the model is determined using long-term real interest rate differentials (as opposed to short-term interest rates as is more commonly the case) (see Table 1). This is due to the fact that we restrict households from investing in domestic and/or foreign assets directly, and that intermediaries, through which households invest, can only hold long-maturity foreign debt. As shown in the decomposition of the long-term interest rate (expressions (5) and (6)), the long-term interest rate is comprised of policy expectation and term premium components. Using (3) and (4), it follows that the real exchange rate is a function of the expected real short-term interest rate differential and (domestic and foreign) term premia.

It is typically the case in the SoE literature that the UIP condition is expressed purely in terms of expected policy rates (see Adolfson et al. (2007) and Burgess et al. (2013) among others), and so an additional difference in our model, relative to the literature, is that deviations from the 'conventional' UIP condition reflect relative term premia, as opposed to the standard exchange rate risk premium shocks (Adolfson et al. (2007) and Christiano et al. (2011)). We later show that the estimated model fits data for the real exchange rate remarkably well (Figure 2 shows one step ahead Kalman Filter projections (red dashed line) against the data (blue solid line)), which supports the formulation of our model.

2.4 Exports

In our model, the evolution of foreign debt is determined in the foreign economy. We derive an expression for exports that ensures consistency between the domestic economy's debt and the evolution of the foreign long-term debt given imports (in the online Appendix). In other words, exports are used to 'close' the model. This is another attractive feature of this model, as the export demand function is typically assumed in these types of models, and not derived from agents' optimising behavior (see Justiniano and Preston (2010), ?, Adolfson et al. (2007), and Christiano et al. (2011) among others). We can derive the following expression

$$c_t^{x,d} = q_t v_t^m c_t^{m,d} + \frac{\bar{b}_t^{L,*,d}}{\varepsilon_t^{\bar{b}_L}} - \frac{r_t^{L,*,d}}{\pi_t^*} \frac{p_{L,t}^*}{p_{L,t-1}^*} \frac{q_t}{q_{t-1}} \bar{b}_{\kappa,t-1}^{L,*} + \frac{x}{2} \left(\delta \frac{b_{t-1}^S}{\bar{b}_{t-1}^L + \bar{b}_{t-1}^{L,*}} - 1 \right)^2 \frac{1}{\pi_t^c}$$
(7)

where exports can be viewed as what we need to 'pay' foreign agents for imports and foreign assets after the capital returns on investing in foreign assets are subtracted. In other words, exports act as residuals in the net foreign asset position accumulation equation.

3 Preliminary Analysis

3.1 Data

The model is estimated using data for the US and UK, where we include in our dataset real GDP per capita, inflation, policy rates, the slope of the zero-coupon yield curve (the 10-year yield less the policy rate) and the bilateral real exchange rate for the period between 1976Q1 and 2013Q2. We detrend the real GDP series using a one-sided HP filter for both the US and UK, though the results are unchanged if using a two-sided HP filter.⁹ UK and US inflation are constructed using the CPI and GDP deflator series, respectively. We obtain data for the US from the Federal Reserve Economic Data source, maintained by the Federal Reserve Bank of St. Louis, and data for the UK are obtained from the Bank of England database (see Appendix A for more details).

3.2 Calibrated Parameters

Table 3 summarises the parameters that are calibrated prior to the estimation of the model. We follow the literature and set the discount factor equal to 0.99, which implies a steady-state value of the (annual) real interest rate of around 4% (for both countries). The US steady-state values of the government spending to GDP ratio, price and wage markup are 0.18, 1.20 and 1.05, respectively, in line with the values in Smets and Wouters (2007) and Christiano et al. (2005). The calibration of the same quantities for the UK economy (g = 0.17, $\lambda_y = 1.20$ and $\lambda_w = 1.05$) is based on the work of Burgess et al. (2013). The steady-state values of the import and export prices markup ($\lambda_m = \lambda_x = 1.2$) are in line with those used in Adolfson et al. (2007), and the steady-state values of US and UK total domestic debt to GDP ratios ($\frac{b^* + \bar{b}^{L,*}}{y^*} = 0.52$ and $\frac{b + \bar{b}^L}{y} = 0.41$) are based on averages of the data used in Mumtaz and Surico (2013). Finally, for both the US and UK, the steady-state values of hours, TFP, the slope of the yield curve and the spread between the policy rate and the effective interest rate faced by households are calibrated to unity. This simply relects the fact that the data is demeaned prior to the estimation of the model.

3.3 **Prior Distributions**

Tables 4 and 5 summarise the prior density probability function of the estimated parameters (which we refer to as primitive priors $\pi(\vartheta)$). Again, these values are in line with those used in the literature (see Smets and Wouters (2007), Justiniano et al. (2010), Christiano et al. (2011) and Burgess et al. (2013) among others) and we do not discuss them further here. In addition, we follow Del Negro and Schorfheide (2008), Liu et al. (2013) and Christiano et al. (2011) (among others) and form our priors 'endogenously'. This requires another set of 'priors' that reflect our beliefs regarding selected data moments, which are described in Table 6.

As explained in Del Negro and Schorfheide (2008), eliciting priors are derived by combining Bayesian techniques and calibration approaches. This intuitive approach formalises the decisions most re-

⁹The two-sided suggests that over the couple of years before the financial crisis the output gap was around 3%. Given inflation was around target levels for the US and UK, these estimates do not seem economically plausible. The one-sided filter does not produce this puzzle and suggests that the output gap was around zero during this period (as is widely believed).

searchers make when deciding the prior moments of the estimated structural parameters. We briefly outline the main idea here, though interested readers are advised to explore the preceding references.

Let $\mathcal{M}(\vartheta)$ denote a vector of DSGE model-implied data moments (expressed as function of the structural parameters vector) and $\widehat{\mathcal{M}}$ its empirical counterpart. Let us further assume that two vectors of moments are the same up to a vector of measurement errors \mathcal{V}

$$\widehat{\mathcal{M}} = \mathcal{M}(\vartheta) + \mathcal{V} \tag{8}$$

Then, as explained in Del Negro and Schorfheide (2008), a conditional distribution that reflects the beliefs about the above moment conditions can be obtained by combining the conditional density of (8), $\mathcal{L}\left(\mathcal{M}(\vartheta)|\widehat{\mathcal{M}}\right)$, Bayes theorem, and the primitive prior distribution of the structural parameter vector:

$$p\left(\vartheta|\widehat{\mathcal{M}}\right) \propto \mathcal{L}\left(\mathcal{M}(\vartheta)|\widehat{\mathcal{M}}\right)\pi(\vartheta) \tag{9}$$

There are several advantages of using this type of prior. For instance, as we can infer from (9), structural parameters are no longer treated as independent, as is typically assumed in the DSGE literature. Furthermore, shock processes are unobserved variables which makes it difficult to justify beliefs regarding the persistence and the volatility of these exogenous processes. In this setup this is not a problem, since these prior moments adjust endogenously to 'match' the selected data moments. Finally, the empirical application in Del Negro and Schorfheide (2008) suggests that these priors can be helpful when DSGE parameters are not well identified.

3.4 Posterior Estimation

The posterior distribution of the DSGE parameter vector is approximated using the steps described in An and Schorfheide (2007). Specifically, we first employ the minimisation routine *csminwel* developed by Chris Sims to obtain the posterior mode, and the inverse hessian matrix around this point. Next, we use the posterior mode to initiate a random walk Metropolis-Hastings algorithm, and the scaled inverse hessian matrix, to ensure an acceptance rate of posterior draws between 25% - 33%. Using parallel computing we approximate the posterior distribution of the structural parameter vector via a large number (32) of chains. For each chain we simulate 350000 draws, discard the first 250000, and from the remaining 100000 draws save one of every hundred. Finally, from these 32000 draws, we randomly select 1000 that are used to produce the statistics reported in our results.

4 Empirical Results

In this section, we discuss the main features of the estimated model, where we examine the posterior distribution of the structural parameter vector, and the ability of the model to match the data. We then proceed to examine the term premium estimates from the DSGE model, and draw comparisons with estimates from the term structure modelling literature.

4.1 Parameter Estimates

Tables 7 and 8 report the posterior moments of the estimated structural parameters, for US and UK respectively. The vast majority of the estimates are in line with those reported in the previous studies, and we keep the discussion of the parameter estimates brief as a result.

The steady-state domestic long-term debt to GDP ratio $\left(\frac{\bar{b}L}{y}, \frac{\bar{b}L,*}{y^*}\right)$ is approximately 20% in both countries, while the total UK debt-GDP ratio $\left(\frac{b+\bar{b}L+\bar{b}L,*,*d}{y}\right)$ is around 60%. According to these estimates, UK fiscal authorities react more aggressively ($\theta = 1.57$) to deviations in short-term debt away from its steady-state, compared to US authorities ($\theta^* = 0.19$). In addition, UK agents face a higher adjustment cost when they alter their portfolio ($100\chi = 9.81$) relative to the US ($100\chi^* = 7.25$), though the overlap between the posterior distribution of the two parameters is large, suggesting that this difference is not significant in both statistical and economic terms. These values are higher than the estimates reported in Chen et al. (2012), but lower than the value in Harrison (2012).

In line with the evidence for the UK presented in Burgess et al. (2013), we estimate that monetary authorities in both countries respond to inflation deviations from target, and smooth interest rate decisions, to a similar degree ($\phi_{\pi}^* = 1.68$, $\phi_{\pi} = 1.63$, $\phi_{R}^* = 0.86$ and $\phi_{R} = 0.82$). However, they react to changes in the output gap differently ($\phi_y^* = 0.07$ and $\phi_y = 0.31$). For the US, the estimates of the inverse inter-temporal substitution and labour supply elasticity parameters ($\sigma_C^* = 1.51$ and $\varphi^* = 1.98$) are similar to those reported in Smets and Wouters (2007). UK households have almost log-utility preferences ($\sigma_C = 1.02$), and US and UK preferences over leisure exhibit a similar degree of curvature ($\varphi = 1.88$). As discussed in Burgess et al. (2013), the degree of consumption smoothing in the UK (b = 0.39) is significantly lower than in the US $(b^* = 0.77, \text{ see Smets and Wouters (2007) and})$ Justiniano et al. (2010)). Again our estimates of the price and wage Philip curve parameters for the US are consistent with those reported in the literature (see Christiano et al. (2005), Smets and Wouters (2007) and Justiniano et al. (2010)), and for the UK (see Burgess et al. (2013), Adolfson et al. (2007) and Christiano et al. (2011)). Finally, Tables 7 and 8 suggest that US shocks are more persistent compared to UK disturbances (consistent with Kulish and Rees (2011)). However uncertainty, as summarised by the trace of the covariance matrix of the structural shocks, is larger in the domestic economy compared to the foreign.

4.2 Model Fit

Following Adolfson et al. (2007), Figure 2 shows the one-step-ahead Kalman filter (in sample) model predictions (the red dashed lines) against the observed data series (the blue solid lines). Overall, the model is able to match a range of real economy and financial data remarkably well, including over recession periods within our sample. Figure 2 potentially also highlights some interesting features of the model. Similar to Del Negro et al. (In Press), our model is able to fit US inflation during the Great Recession, and as explained in Del Negro et al., this due to the high degree of nominal price rigidity (see Table 7) that induces the marginal cost to become more persistent, more endogenous (in the sense it responds more to shocks) and its dynamics are influenced by the degree to which inflation expectations are anchored to the target. This suggests that US central bank actions to stabilise inflation expectations were effective. In our model this is further supported by the fact that the US policy rate as well as the slope of the US yield curve predicted by the model are very close to what it is

observed in the data (which by construction, this implies that the model-implied US long-term interest rate is also close to the data). Our analysis later in the paper illustrates that the policy expectations component of the long-term interest rate (see expression 6) accounts for the majority of its variation, and that these expectations are mainly driven by Central Bank's ability to stabilise inflation. In other words, according to our model the long-term interest prevailed during the Great Recession reflected the expected path of the policy rates anticipated by markets, in order that inflation returned back to its target and for the output gap to close (as implied by the Taylor Rule). With sterling depreciating more than 20% during the recent financial crisis, it is not hard to understand why CPI inflation in the UK did not fall during that period. The model seems capable of replicating the evolution of the real exchange rate and, therefore, correctly captures the import price pressures on the CPI inflation.

4.3 Moment Estimates

Table 9 shows a selection of key moments from the estimated model. Reassuringly, the model matches the correlation between the US and UK long-term interest rates, and the correlation between shortand long-term interest rates. A promising feature of Table 9 is the ability of the model to match the observed correlation between US and UK GDP. As discussed in Kollmann (1996) and Kollmann (2001) (and the references within), this correlation has been a characteristic of the data that past open economy DSGE studies have had difficulty in replicating. As shown in Kollmann (2001), sticky prices, sticky wages, and incomplete asset markets (which are all features of our model) can help these theoretical models to move closer to matching the correlation in the data. In our case, however, this moment is replicated exactly, and we interpret this as further support for the model structure proposed in this study.

4.4 Term Premium Estimates

As described in Section 2, we are able to derive term premium estimates directly from our model, and it potentially interesting to compare our structural model estimate with estimates from the macrofinance literature. There is large body of research dedicated to the estimation of term premia, where the majority of studies use reduced-form no-arbitrage term structure models (see Dai and Singleton (2000), Duffee (2002) and Kim and Wright (2005) among others). These estimates reflect the difference between the observed bond yield, and the average expected short rate over the life of the long-term bond (e.g. $\widetilde{TP}_t^* = r_t^{L,*} - \frac{1}{n} \sum_{i=0}^{n-1} E_t r_{t+i}^*$ using the notation in this paper). For our comparisons, we use term premia estimates obtained using the methodology developed in Adrian et al. (2013), for the US and UK.¹⁰ The US estimate is obtained from the Federal Reserve of New York website, and the UK estimate is taken from Malik and Meldrum (2014), who use the same approach to derive UK term premium estimates from 1997Q1 onwards. The DSGE term premia are expressed as percentage deviations from their steady-state, so there is naturally a level-difference is that, while the reducedform term premium estimates reflect the difference between bond yields and average (equally-weighted)

¹⁰The estimation of term structure models is a computationally challenging task, as it requires numerical Maximum Likelihood optimization techniques to be applied to large scale models. Recently, Adrian et al. (2013) proposed a methodology/model where the Maximum Likelihood optimization problem can be replaced by a three-step ordinary least squares estimator. The authors illustrate that their model performs equally well (if not better) relative to models that have been widely used in the literature.

policy rate expectations over the life of the bond, the DSGE term premia are based on unequally weighted expectations. That is, as shown in equation (??), longer-term expectations are more heavily discounted relative to the near-term (by a factor of $\beta\kappa$).

Figures 3 and 4 show the term premium estimates, with the reduced-form estimates in the red dashed lines and the DSGE estimates in the solid blue lines. With the above caveats in mind, we interpret the similarity between the two estimates for both countries as a very positive signal. The two figures suggest that the DSGE model can produce relatively meaningful term premium estimates, and gives us confidence in proceeding to decompose term premia into the effects of structural shocks and/or channels. The correlation between the time-series and DSGE estimate is around 0.7 for US and 0.5 for UK. For the US and UK, the term premium estimates rise prior to recessions, and stay elevated for a few years after the economy has returned to positive GDP growth. This is consistent with the conventional theory and empirical evidence suggesting that term premia are countercyclical (Rudebusch and Swanson, 2012).

5 Cross-Country Co-movement

In this section, we explore the co-movement of long-term interest rates and GDP growth. Figures 5 to 10 show forecast variance decompositions for GDP, long-term interest rates, term premia and policy rates. The figures show the contribution of demand, supply, policy and 'financial' shocks, originating in the domestic and foreign economies over horizons up to 10 years.

5.1 Interest Rate Variation and Co-movement

Figure 6 shows the forecast variance decomposition for the UK long-term interest rate. The left-handside of the figure highlights the importance of foreign disturbances for the domestic economy that we find in our model. Foreign shocks explain 60 - 70% of the variation in the long-term rate at short horizons, and around 80% at longer horizons. The right-hand-side of the figure shows the breakdown into the various types of shocks. According to this figure, domestic financial shocks (that is $\varepsilon_t^{b^S}$ and $\varepsilon_t^{b^L}$, the 'risk premium' components of term premia) account for a substantial part of the variation in the long-term rate, in particular at shorter horizons. We see a similar story for the US long-term interest rate variance decomposition (shown in Figure 7), where the foreign financial shocks account for almost 50% of the variation at short horizons. In line with previous evidence, therefore, term premia account for a substantial proportion of the variation in long-term interest rates in our model.

Turning to the co-movement between US and UK long-term interest rates, we can ask through which component the foreign disturbances impact UK long-term rates, policy rate expectations or term premia. Figures 8 and 9 show the contribution of foreign (and domestic) shocks to variation in the UK term premium and policy rate expectations, respectively (expression (5)). The figures show that the foreign contributions largely transmit through the policy rate expectations component, as opposed to the term premium (which is largely driven by domestic financial shocks). This is a slightly different result to the conclusions in Jotikasthira et al. (2015), who emphasize the role of correlated term premia in the co-movement between US and UK long-term interest rates. While it is not easy to compare our structural model (and different term premium definition) with the reduced-form framework used in their study, it is interesting that our model can almost fully account for the co-movement of interest rates through the policy rate expectations component of bond yields.

Since the risk premium component of the term premium is a function of two domestic exogenous processes, the only channel through which foreign shocks can impact the domestic term premium is via the (endogenous) liquidity premium (equation (5)). Figure 8 suggests that, aside from the domestic risk premium shocks, US demand (discount factor) and supply (TFP) shocks account for a large part of the variation in the domestic term premium (through the liquidity component). We explore these mechanisms further in Section 5.3.

5.2 Co-movement of Foreign and Domestic GDP

As outlined earlier, Justiniano and Preston (2010) have shown that estimated SoE DSGE models have difficulty in accounting for the substantial influence of foreign disturbances identified in numerous empirical studies (see Cushman and Zha (1997) and Kose et al. (2003) among others). Subsequent studies that use more elaborate SoE models (for instance Adolfson et al. (2007), Christiano et al. (2011) and Burgess et al. (2013)) have confirmed these findings and, as a result, question the usefulness of SoE models for understanding economic fluctuations across countries. As shown in Figure 5, the estimated model assigns a much larger role of foreign shocks to fluctuations in the domestic economy, and shows that our model overcomes the common limitation of SoE DSGE models. Foreign disturbances explain around 20% of the variation in GDP at forecast horizons up to 12 quarters ahead, and this contribution increases at longer horizons, exceeding 40% at business cycle frequencies.

5.3 Impulse Response Analysis

As outlined above, our analysis suggests that the main shocks underlying the strong cross-country GDP and long-term interest rate correlations seen in Table 9 are foreign supply and demand shocks. We explore the channels that generate this co-movement over the remainder of the paper, where we set out the mechanisms through which these shocks impact the domestic and foreign economies and their implications for interest rates. To shorten our analysis, we concentrate on the case of a foreign supply shock, where the set of impulse responses following a foreign TFP shock are shown in Figure 11, but the intuition that underlies the co-movement of interest rates is the same in the case of discount factor shocks.

5.3.1 Macroeconomic Effects

The effects of a supply shock on the foreign economy are as follows: following a positive TFP shock, the supply side of the economy expands, and the improvement in the technology employed to produce the final output generates a decline in firms' marginal cost, causing inflation to fall (although wages rise due to the wealth effect). Due to declining inflation, policymakers act to simulate demand by lowering the policy rate. As can be seen in Figure 11 and Table 7, the shock is very persistent and leads to a protracted period of low policy rates, due to persistantly below-target inflation. In this environment, long-term bond prices rise, as agents in the model anticipate the period of low shortterm rates. The immediate response of long-term bond yields, despite it taking around a year for the policy rate to reach its trough, is a natural consequence of the forward looking nature of asset prices. In this scenario, the long-term interest rate falls by more than the path of expected policy rates, reflecting a declining liquidity component of the US term premium (the risk premium component is zero in this case).

In the domestic economy, there is an initial decline in output, as agents substitute away from domestic to imported goods. This is a consequence of the appreciation of the exchange rate (due to the fall in foreign long-term rates and the UIP condition). While domestic imports are more expensive following the exchange rate movements, domestic exports increase in response to the increase in foreign demand, and this offsets some of the fall in domestic consumption. Shortly following these initial responses, higher exports become the dominant effect, and the GDP impact becomes positive and remains above its steady-state for some time. In equilibrium, imports and holdings of the foreign long-term bond (before any capital gains) are financed through the domestic exports (Table 1), and so exports increase following increases in imports and foreign bond holdings. While the exchange rate appreciation makes the foreign long-term bond less attractive, the effect of higher foreign asset prices dominates, and the domestic value of the foreign long-term asset increases. As the exchange rate appreciates, import price inflation falls, but the effect of this on inflation is offset as domestically generated inflation rises due to higher wages (a wealth effect). At the same time, however, higher domestic relative prices offset a large part of the wage increase through the terms of trade effect on firms. Eventually, the effect from lower import prices dominates and lowers CPI inflation persistently below its steady-state. Overall, therefore, the shock causes output to rise (with a minor delay) and CPI inflation to fall, similar to the foreign economy. The monetary policymaker again sets policy based on a (Taylor-type) rule, and act to boost demand to bring inflation back to target by lowering the policy rate. Again, a lower path of expected policy rates is associated with a decrease in the long-term interest rate, and similar to the US economy the effect on the domestic term premium is negative and of a similar magnitude.

5.3.2 Bond Market Implications

The preceding analysis implies strong persistent effects of supply shocks, which generate a corresponding monetary policy reaction. The persistence of the effects is important: inflation is below its steady-state even ten years after the appearance of the shock, and this subsequently has a sizeable impact the pricing of long-term interest rates. In the opposite case, a negative supply shock is particularly detrimental to long-term bond investments, where payoffs from these assets are eroded due to high inflation at a time when output (or consumption) is lower. The more difficult it is for the US policymakers is to restore inflation back to its target and close the output gap, the more concerned the agents are going to be about their long-term investments. The high contribution (more than 60%) of supply shocks to variation in US long-term interest rates reflects this concern (Figure 7). This is consistent with the analysis of Rudebusch and Swanson (2012) and the empirical results presented by Piazzesi and Schneider (2007) who argue that when output (consumption) and inflation move in opposite directions, long-term bonds are undesirable, since high inflation erodes their payoff in 'bad times'.¹¹ As Piazzesi and Schneider (2007) and Rudebusch and Swanson (2012) use nonlinear DSGE models these concerns (high inflation/low consumption) are transmitted to the real economy

¹¹Although Piazzesi and Schneider (2007) talk about consumption and inflation surprises and not explicitly about structural shocks explicitly, the inflation surprise in their model moves consumption/output and inflation in the opposite direction and this is consistent with the TFP shock in our model.

via the risk premium, which in our linearised framework is exogenous (expression (6)). Despite the fact that our model does not capture higher order effects, the first-order impact of these shocks similarly captures potential concerns to long-term bond investors.

We noted earlier that discount factor shocks also play a significant role in long-term interest rate variation. While the mechanisms through which the co-movement arises differs relative to supply shocks, discount factor shocks generate similarly persistent deviations of inflation away from steadystate in the US and UK and the subsequent policymaker response to these deviations (Figure 12).

5.3.3 The 08/09 Financial Crisis

As explained above, in the model, following a positive TFP shock, agents' policy rate expectations adjust immediately, and this drives up long-term bond prices almost instantaneously. This is because agents observe the path of policy rates in response to the shock, and bond yields fall in anticipation of this (and prior to the trough in policy rates due to policymakers' preferences about smooth instrument changes). Following the positive TFP shock and the associated bond market activity, agents are particularly exposed to negative shocks. This exposure manifests not only through the possibility of a reversal in policy rates relative to the expected path, but also due to the high liquidity premium that results from the prior accumulation of long-term bonds. This additional propagation implies that, from this position, a negative shock has dramatic consequences for the economy over and above the reduction in output caused by lower productivity. The 'Great Moderation' was a period of robust growth, low (below the steady-state) inflation and high asset prices. This would be consistent with a positive TFP shock, an interpretation which is supported by the US GDP historical decomposition shown in Figure 13. Similarly, the 'Great Recession' is interpreted by the model as a negative TFP shock. The severity of the crisis is consistent with the mechanisms just described, and a similar explanation for the 'boom' and the 'bust' in asset prices prior to and during the financial crisis has been proposed by Christiano et al. (2008) and Christiano et al. (2010). While the analysis of these studies is based on an anticipated productivity shock, the 'mechanism' that generates these effects is very similar. During the 'boom' phase, asset prices increased as policy rates were expected to fall, due to the fact that expected marginal cost and consequently inflation decreased (under the 'Inflation Targeting' regime). While the 'bust' was caused by the fact that the lower expected path of inflation and positive output were not realised and asset prices became inconsistent with the path of expected policy rates after the adverse shock.

6 Conclusion

We develop a small open economy DSGE model in order to understand the stylised fact that long-term interest rates in a number of small-open inflation targeting economies co-move more strongly with US long-term rates, than with short-term rates in those economies. The core of our model is very similar to existing models in the small open-economy literature, and our main innovation is to augment this otherwise 'standard' model with two more assets: domestic and foreign long-term government bonds. We further assume that short- and long-term assets are imperfectly substitutable in both domestic and foreign economies, which allows us to relax the expectations hypothesis of the yield curve, and allows for a time-varying term premium in our model. The estimated model fits real-economy and financial US and UK data remarkably well overall, and displays good fit even during the recessions that occur within our sample (including the 'Great Recession' in 2008/9). Furthermore, the model-implied US and UK term premium estimates demonstrate a great degree of similarity with measures obtained using reduced-form affine term-structure models (Adrian et al., 2013). Our analysis shows that the strong correlation between US and UK long-term interest rates arises via the policy rate expectations component of bond yields, as opposed to via term premia, as previously suggested (Jotikasthira et al., 2015). This is due to monetary authorities in both economies responding to foreign (productivity and discount factor) shocks that cause persistent deviations of inflation in both countries away from their steady-states. Expectations about the path of policy rates adjust much faster to these foreign shocks compared to actual policy rates, since asset prices are forward-looking and policymakers in both countries adjust their policy instruments gradually. Our analysis suggests that foreign shocks are able to explain about 40% of the variance in domestic GDP and 80% of the variance in domestic long-term interest rates. More than 50% of the variance in foreign and domestic long-term rates is explained by a very persistent (almost non-stationary) US productivity shock. This is consistent with the empirical evidence and the analysis presented by Piazzesi and Schneider (2007) and Rudebusch and Swanson (2012), who argue that when output and inflation tend to move in opposite directions long-term assets are unattractive as they have low real payoff in 'bad times'.

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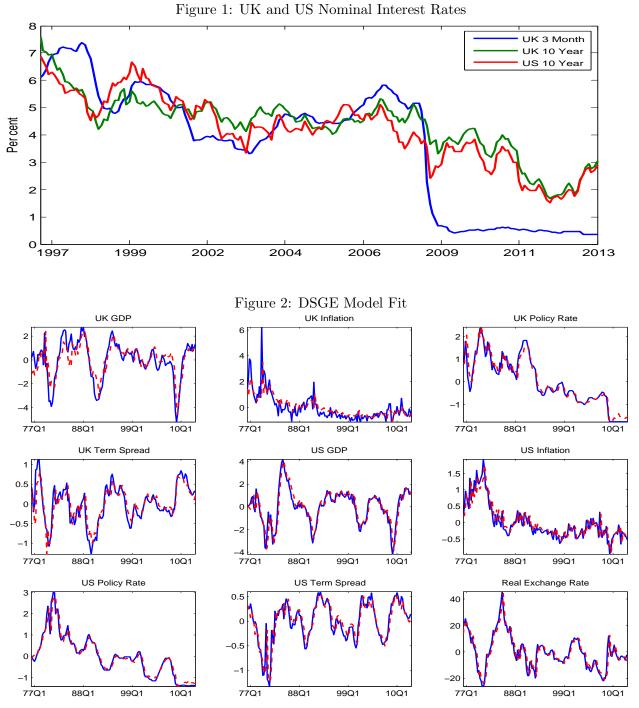
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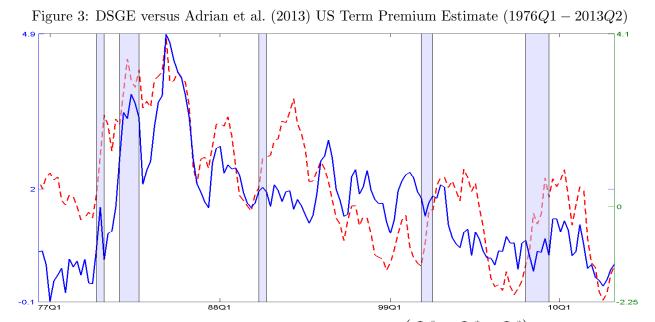
A Data

The list below shows ONS and FRED codes for the series used to construct our dataset.

- UK Real GDP: ONS Code ABMI.Q
- UK Population: ONS Code MGSL.Q
- UK CPI: Bank of England Database
- UK Policy Rate: ONS Code ABEDR.Q
- UK 10 Year Yield: Bank of England Database (quarterly average)
- UK Nominal Wages: ONS Code KAB7.M (quarterly average)
- US Nominal GDP: FRED Code GDPC96
- US Population: FRED Code POP
- US GDP Deflator: FRED Code GDPDEF
- US Policy Rate: FRED Code FEDFUNDS
- US 10 Year Yield: Bank of England Database (quarterly average)
- US Nominal Wages: FRED Code COMPRNFB
- US/UK Nominal Exchange Rate: Bank of England Database



Notes: Observed data shown in the blue lines, model Kalman Filter one step ahead predictions $(E_{t-1}x_t)$. shown in red-dashed lines.



Notes: The DSGE Model US Kalman Smoothed Term Premium $\left(\widehat{TP}_t^* = \widehat{RP}_t^* + \widehat{LP}_t^*\right)$ estimate (blue solid line, right hand axis) against the Adrian et al. (2013) Affine Term-Structure Model Term Premium estimate (red dashed line, left hand axis). The DSGE estimate is expressed as percentage deviations from the steady-state, while the time series estimate is in levels. Shaded areas denote NBER US recession periods.

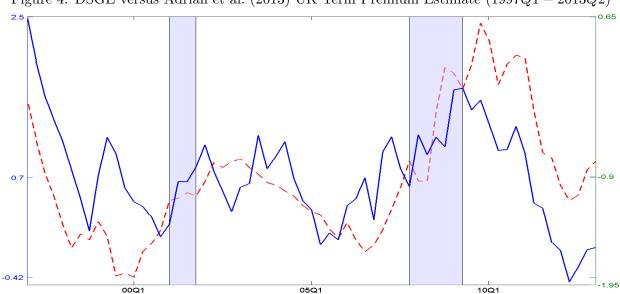


Figure 4: DSGE versus Adrian et al. (2013) UK Term Premium Estimate (1997Q1 - 2013Q2)

Notes: The DSGE Model US Kalman Smoothed Term Premium $(\widehat{TP}_t = \widehat{RP}_t + \widehat{LP}_t)$ estimate (blue solid line, right hand axis) against the Adrian et al. (2013) Affine Term-Structure Model Term Premium estimate (red dashed line, left hand axis). The DSGE estimate is expressed as percentage deviations from the steady-state, while the time series estimate is in levels. Shaded areas denote NBER US recession periods.

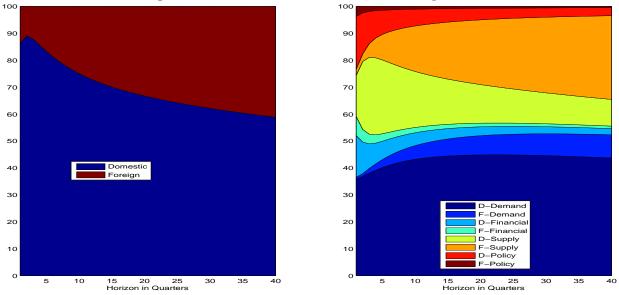


Figure 5: UK GDP Forecast Variance Decomposition

Notes: F-Supply includes: US TFP shock, F-Demand includes US discount factor shock and government spending shocks, F-Policy includes: US monetary policy shock, F-Financial includes: US short-term risk premium and US long-term risk premium. D-Supply includes: UK TFP shock, D-Demand includes UK discount factor and government shocks, D-Policy includes: UK monetary policy shock, D-Financial includes: UK short-term risk premium, UK long-term risk premium and exchange rate risk premium shocks.

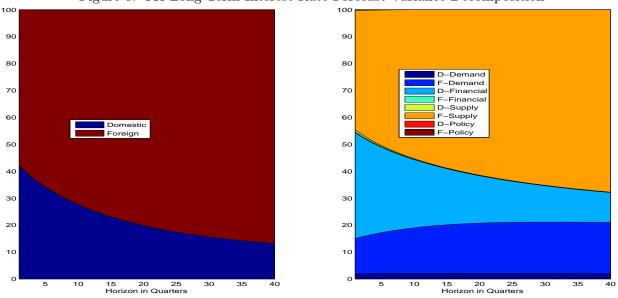


Figure 6: UK Long-Term Interest Rate Forecast Variance Decomposition

Notes: F-Supply includes: US TFP shock, F-Demand includes US discount factor shock and government spending shocks, F-Policy includes: US monetary policy shock, F-Financial includes: US short-term risk premium and US long-term risk premium. D-Supply includes: UK TFP shock, D-Demand includes UK discount factor and government shocks, D-Policy includes: UK monetary policy shock, D-Financial includes: UK short-term risk premium, UK long-term risk premium and exchange rate risk premium shocks.

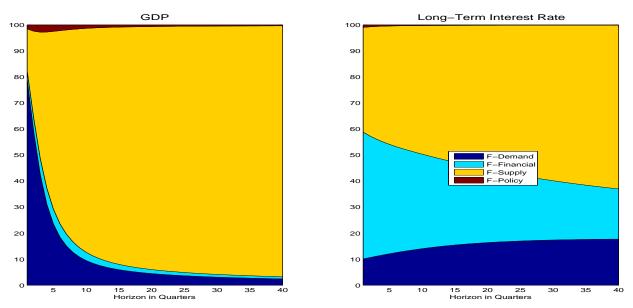


Figure 7: US GDP & Long-Term Interest Rate Forecast Variance Decomposition

Notes: F-Supply includes: US TFP shock, F-Demand includes US discount factor shock and government spending shocks, F-Policy includes: US-monetary policy shock, F-Financial includes: US short-term risk premium and US long-term risk premium.

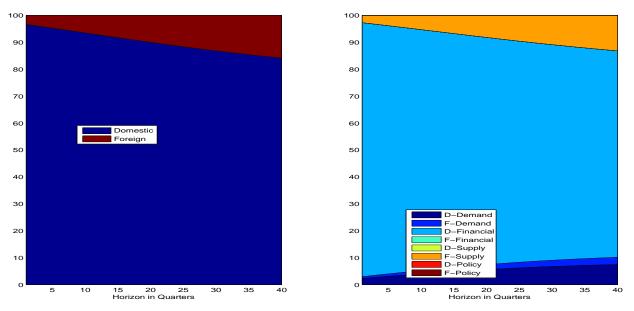


Figure 8: UK Term Premium Forecast Variance Decomposition

Notes: F-Supply includes: US TFP shock, F-Demand is the US discount factor shock and government spending shocks, F-Policy contains: US monetary policy shock, F-Financial includes: US short-term risk premium and US long-term risk premium. D-Supply includes: UK TFP shock, D-Demand includes UK discount factor and government shocks, D-Policy includes: UK monetary policy shock, D-Financial includes: UK short-term risk premium, UK long-term risk premium and exchange rate risk premium shocks.

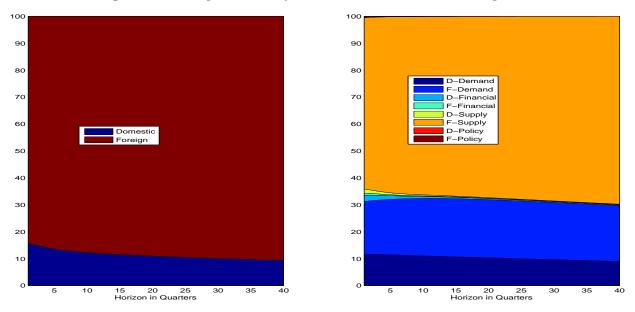


Figure 9: UK Expected Policy Rate Forecast Variance Decomposition

Notes: F-Supply includes: US TFP shock, F-Demand is the US discount factor shock and government spending shocks, F-Policy contains: US monetary policy shock, F-Financial includes: US short-term risk premium and US long-term risk premium. D-Supply includes: UK TFP shock, D-Demand includes UK discount factor and government shocks, D-Policy includes: UK monetary policy shock, D-Financial includes: UK short-term risk premium, UK long-term risk premium and exchange rate risk premium shocks.

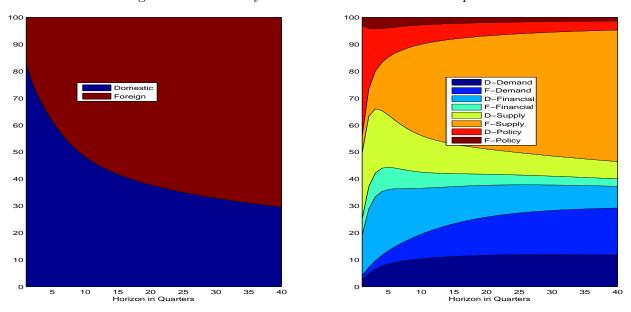
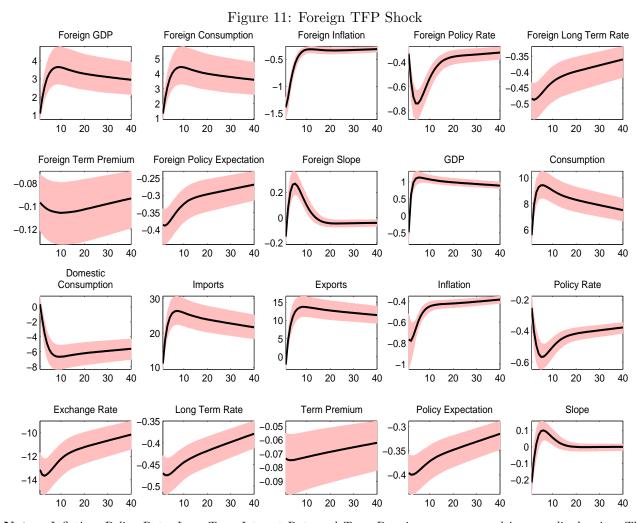
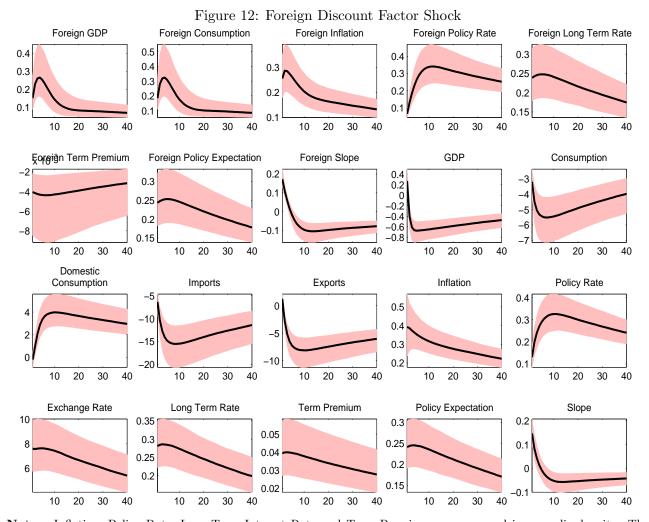


Figure 10: UK Policy Rate Forecast Variance Decomposition

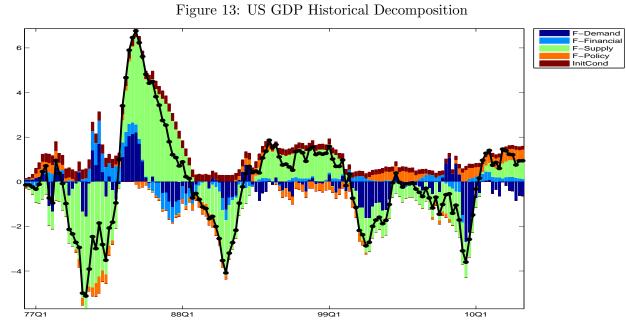
Notes: F-Supply includes: US TFP shock, F-Demand is the US discount factor shock and government spending shocks, F-Policy contains: US monetary policy shock, F-Financial includes: US short-term risk premium and US long-term risk premium. D-Supply includes: UK TFP shock, D-Demand includes UK discount factor and government shocks, D-Policy includes: UK monetary policy shock, D-Financial includes: UK short-term risk premium, UK long-term risk premium and exchange rate risk premium shocks.



Notes: Inflation, Policy Rate, Long-Term Interest Rate and Term Premium are expressed in annualised units. The solid line represents the impulse response function produced using the posterior mode, and the shaded area is the corresponding 5^{th} and 95^{th} percentile of the posterior distribution centered around the posterior mode impulse response function. The term 'Foreign' refers to US variables. The first eight subplots show the responses of the foreign variables to the foreign disturbance, while the remaining subplots plot the responses of the domestic economy variables.



Notes: Inflation, Policy Rate, Long-Term Interest Rate and Term Premium are expressed in annualised units. The solid line represents the impulse response function produced using the posterior mode, and the shaded area is the corresponding 5^{th} and 95^{th} percentile of the posterior distribution centered around the posterior mode impulse response function. The term 'Foreign' refers to US variables. The first eight subplots show the responses of the foreign variables to the foreign disturbance, while the remaining subplots plot the responses of the domestic economy variables.



Notes: F-Supply includes: US TFP shock, F-Demand includes US discount factor shock and government spending shocks, F-Policy includes: US-monetary policy shock, F-Financial includes: US short-term risk premium and US long-term risk premium.

C Tables

Description	Mnemonics
Domestic Inflation	$\hat{\pi}_t = \frac{\beta}{1+\beta\kappa_y}\hat{\pi}_{t+1} + \frac{\kappa_y}{1+\beta\kappa_y}\hat{\pi}_{t-1} + \frac{(1-\xi_y)(1-\beta\xi_y)}{\xi_y(1+\beta\kappa_y)}\widehat{mc}_t$
Domestic Marginal Cost	$\hat{mc}_t = \hat{w}_t - \hat{ ilde{p}}_t - \hat{z}_t$
Import Price Inflation	$\hat{\pi}_t^m = \frac{\beta}{1+\beta\kappa_m} \hat{\pi}_{t+1}^m + \frac{\kappa_m}{1+\beta\kappa_m} \hat{\pi}_{t-1}^m + \frac{(1-\xi_m)(1-\beta\xi_m)}{\xi_m(1+\beta\kappa_m)} \widehat{mc}_t^m$
Importers Marginal Cost	$\widehat{mc}_t^m = \hat{q}_t - \hat{\widetilde{p}}_t^m$
Marginal Utility of Consumption	$\hat{\lambda}_t = \frac{1}{1-b\beta} \left\{ \left[\hat{d}_t - b\beta \hat{d}_{t+1} \right] - \frac{\sigma_C}{1-b} \left[\left(1 + \beta b^2 \right) \hat{c}_t - b\beta \hat{c}_{t+1} - b\hat{c}_{t-1} \right] \right\}$
Consumption Euler Equation	$\hat{\lambda}_t = \hat{\lambda}_{t+1} + E_t \Delta \hat{d}_{t+1} + r_t^h - \hat{\pi}_{t+1}^c$
Wage Phillips Curve Equation 1	$\hat{f}_t = (1 - \beta \xi_w) \left[-\frac{1}{\lambda_y - 1} \hat{w}_t^{new} + \hat{\lambda}_t + \frac{\lambda_y}{\lambda_y - 1} \hat{w}_t + \hat{h}_t \right]$
	$+\beta\xi_w \left[\frac{1}{\lambda_y - 1} \left(E_t \hat{\pi}_{t+1}^c - \kappa_w \hat{\pi}_t^c + E_t \Delta \hat{w}_{t+1}^{new}\right) + E_t \hat{f}_{t+1}\right]$
Wage Phillips Curve Equation 2	$\hat{f}_t = (1 - \beta \xi_w) \left[\hat{d}_t + (1 + \varphi) \frac{\lambda_y}{\lambda_y - 1} (\hat{w}_t - \hat{w}_t^{new}) + (1 + \varphi) \hat{\tilde{h}}_t \right]$
	$+\beta\xi_w\left[(1+\varphi)\frac{\lambda_y}{\lambda_y-1}\left(E_t\hat{\pi}_{t+1}^c-\kappa_w\hat{\pi}_t^c+E_t\Delta\hat{w}_{t+1}^{new}\right)+E_t\hat{f}_{t+1}\right]$
Wage Phillips Curve Equation 3	$\hat{w}_{t}^{new} - \hat{w}_{t} = \frac{\xi_{w}}{1 - \xi_{w}} \left(\hat{\pi}_{t}^{c} - \kappa_{w} \hat{\pi}_{t-1}^{c} + \hat{w}_{t} - \hat{w}_{t-1} \right)$
CPI Inflation	$\hat{\pi}_{t}^{c} - \hat{\tilde{p}}_{t-1}^{c} = (1 - \alpha)\hat{\pi}_{t} + \alpha \tilde{p}^{m}(1 - \eta) \left(\hat{\pi}_{t}^{m} + \hat{\tilde{p}}_{t-1}^{m} - \hat{\tilde{p}}_{t-1}\right)$
Production Function	$\hat{y}_t = \hat{h}_t + \hat{z}_t$
Goods Market Clearing Condition	$\hat{y}_t = \frac{c^d}{y}\hat{c}_t^d + \frac{x}{y}\hat{x}_t + g\left(\hat{g}_t + \hat{y}_t\right)$
Import Relative Prices	$\hat{\hat{p}}_t^m = \hat{\hat{p}}_{t-1}^m + \hat{\pi}_t^m - \hat{\pi}_t^c$
Domestic Relative Prices	$\begin{bmatrix} \tilde{p}_{t} = \tilde{p}_{t-1} + \hat{\pi}_{t} - \hat{\pi}_{t}^{c} \\ \tilde{p}_{t-1} = \tilde{p}_{t-1} + \hat{\pi}_{t} - \hat{\pi}_{t}^{c} \end{bmatrix}$
Household Effective Rate	$\hat{r}_{t}^{h} = \hat{r}_{t}^{S} + \hat{\varepsilon}_{t}^{b^{S}} + \tilde{x}\delta \left[\frac{b^{L}}{b^{L} + (1-\varrho)b^{L,*}} \hat{b}_{t}^{L} + \frac{(1-\varrho)b^{L,*}}{b^{L} + (1-\varrho)b^{L,*}} \hat{b}_{t}^{L,*} - \hat{b}_{t}^{S} \right]$
Long-Term Debt Value	$\hat{b}_t^L = \hat{p}_{L,t} + \hat{b}_t^L$
Long-Term Interest Rate	$\hat{r}_{t+1}^{L} + \hat{p}_{L,t+1} - \hat{p}_{L,t} = \hat{r}_{t}^{h} - \hat{\varepsilon}_{t}^{b^{L}}$
	$\left +\tilde{x} \left(r^{L} - \kappa \right) \left[\frac{b^{L}}{b^{L} + (1 - \varrho)b^{L,*}} \hat{b}_{t}^{L} + \frac{(1 - \varrho)b^{L,*}}{b^{L} + (1 - \varrho)b^{L,*}} \hat{b}_{t}^{L,*} - \hat{b}_{t}^{S} \right] \right.$
UIP Condition	$\hat{r}_{t+1}^{L,*} - \hat{\pi}_{t+1}^{c,*} + \hat{p}_{L,t+1}^* - \hat{p}_{L,t}^* + \hat{q}_{t+1} - \hat{q}_t = \hat{r}_t^h - \hat{\varepsilon}_t^{b^{L,*}}$
	$+\frac{\tilde{x}(r^{L,*}-\kappa^{*})}{r^{L,*}}\left[\frac{b^{L}}{b^{L}+(1-\varrho)b^{L,*}}\hat{b}_{t}^{L}+\frac{(1-\varrho)b^{L,*}}{b^{L}+(1-\varrho)b^{L,*}}\hat{b}_{t}^{L,*}-\hat{b}_{t}^{S}\right]$
Foreign Long-Term Debt Value	$\hat{b}_{t}^{L,*,d} = \hat{q}_{t} + \hat{p}_{L,t}^{*} + \hat{b}_{t}^{L,*}$
Long-Term Asset Price	$\hat{p}_{L,t} = -\frac{1}{1-eta\kappa}\hat{r}_t^L$
Short-Term Debt	$ \frac{b^{S}}{y} \left(\hat{b}_{t}^{S} - \hat{\varepsilon}_{t}^{b^{S}} \right) + \frac{b^{L}}{y} \left(\hat{\bar{b}}_{t}^{L} - \hat{\varepsilon}_{t}^{\bar{b}^{L}} \right) + \frac{b^{S}}{y} \theta \hat{b}^{S}{}_{t-1}^{S} = \frac{1}{\beta} \frac{b^{S}}{y} \left(\hat{b}_{t-1}^{S} + \hat{r}_{t-1} - \hat{\pi}_{t}^{c} \right) $
	$+rac{1}{eta}rac{b^L}{y}\left(\hat{ar{b}}_{t-1}^L+\hat{r}_t^L+\Delta\hat{p}_{L,t}-\hat{\pi}_t^c ight)+ ilde{p}g\left(\hat{ar{p}}_t+\hat{g}_t+\hat{y}_t ight)$
Taylor Rule	$\hat{r}_t = \phi_R \hat{r}_{t-1} + (1 - \phi_R) \left(\dot{\phi}_{\pi^c} \hat{\pi}_t^c + \phi_y \hat{y}_t \right) + \sigma_R \omega_t^R $
Exports	$\frac{x}{y}\hat{x}_t = \frac{m}{y}\left(\hat{q}_t + \hat{m}_t\right) + \frac{b^{L,*}}{y}\left(\hat{\bar{b}}_t^{L,*,d} - \hat{\varepsilon}_t^{\bar{b}^{L,*}}\right)$
	$-\frac{1}{\beta}\frac{b^{L,*}}{y}\left(\hat{\bar{b}}_{t-1}^{L,*,d}+\hat{r}_{t}^{L,*}-\hat{\pi}_{t}^{c,*}+\Delta\hat{p}_{L,t}^{*}+\Delta\hat{q}_{t}\right)$
Imports	$\hat{m}_t = -\eta \hat{p}_t^m + \hat{c}_t$
Domestic Consumption	$\hat{c}_t^d = -\eta \hat{\tilde{p}}_t + \hat{c}_t$
TFP	$\hat{z}_t = \rho_z \hat{z}_{t-1} + \sigma_z \omega_t^z$
Long-Term Debt Risk Premium	$\hat{b}_{\hat{t}}^L = \rho_{\bar{b}L} \hat{b}_{t-1}^L + \sigma_{\bar{b}L} \omega_t^{\bar{b}L}$
Discount Factor	$\vec{d}_t = \rho_d \vec{d}_{t-1} + \sigma_d \omega_t^d$
Short-Term Debt Risk Premium	$\hat{b}_t^S = \rho_b s \hat{b}_{t-1}^S + \sigma_b s \omega_t^{b^S}$
Government Spending	$\hat{g}_t = \rho_g \hat{g}_{t-1} + \sigma_g \omega_t^g$

Table 1: Domestic Economy Block Model Equations

Notes: Δ denotes the difference operator $(\Delta y_t = y_t - y_{t-1})$. We assume that the supply of long-term assets is fixed in both economies, namely $\hat{b}_t^L = \hat{b}_t^{L,*} = 0$ for all t.

Description	Mnemonics
Inflation	$\hat{\pi}_{t}^{c,*} = \frac{\beta}{1+\beta\kappa_{y}^{*}} \hat{\pi}_{t+1}^{c,*} + \frac{\kappa_{y}^{*}}{1+\beta\kappa_{y}^{*}} \hat{\pi}_{t-1}^{c,*} + \frac{(1-\xi_{y}^{*})(1-\beta\xi_{y}^{*})}{\xi_{y}^{*}(1+\beta\kappa_{y}^{*})} \widehat{mc}_{t}^{*}$
Marginal Cost	$\widehat{mc}_t^* = \hat{w}_t^* - \hat{z}_t^*$
Marginal Utility of Consumption	$\left \hat{\lambda}_{t}^{*} = \frac{1}{1 - b^{*}\beta} \left\{ \left[\hat{d}_{t}^{*} - b^{*}\beta \hat{d}_{t+1}^{*} \right] - \frac{\sigma_{C}^{*}}{1 - b^{*}} \left[\left(1 + \beta(b^{*})^{2} \right) \hat{c}_{t}^{*} - b^{*}\beta \hat{c}_{t+1}^{*} - b^{*}\hat{c}_{t-1}^{*} \right] \right\}$
Consumption Euler Equation	$\hat{\lambda}_{t}^{*} = \hat{\lambda}_{t+1}^{*} + E_{t} \Delta \hat{d}_{t+1}^{*} + r_{t}^{h,*} - \hat{\pi}_{t+1}^{c,*}$
Wage Phillips Curve Equation 1	$\hat{f}_{t}^{*} = (1 - \beta \xi_{w}^{*}) \left[-\frac{1}{\lambda_{y}^{*} - 1} \hat{w}_{t}^{new,*} + \hat{\lambda}_{t}^{*} + \frac{\lambda_{y}^{*}}{\lambda_{y}^{*} - 1} \hat{w}_{t}^{*} + \hat{h}_{t}^{*} \right]$
	$+\beta\xi_{w}^{*}\left[\frac{1}{\lambda_{w}^{1-1}}\left(E_{t}\hat{\pi}_{t+1}^{c,*}-\kappa_{w}^{*}\hat{\pi}_{t}^{c,*}+E_{t}\Delta\hat{w}_{t+1}^{new,*}\right)+E_{t}\hat{f}_{t+1}^{*}\right]$
Wage Phillips Curve Equation 2	$\hat{f}_t^* = (1 - \beta \xi_w^*) \left[\hat{d}_t^* + (1 + \varphi^*) \frac{\lambda_y^*}{\lambda_y^* - 1} (\hat{w}_t^* - \hat{w}_t^{new,*}) + (1 + \varphi^*) \hat{h}_t^* \right]$
	$+\beta\xi_{w}^{*}\left[(1+\varphi^{*})\frac{\lambda_{y}^{*}}{\lambda_{w}^{*}-1}\left(E_{t}\hat{\pi}_{t+1}^{c,*}-\kappa_{w}^{*}\hat{\pi}_{t}^{c,*}+E_{t}\Delta\hat{w}_{t+1}^{new,*}\right)+E_{t}\hat{f}_{t+1}^{*}\right]$
Wage Phillips Curve Equation 3	$\hat{w}_{t}^{new,*} - \hat{w}_{t}^{*} = \frac{\xi_{w}^{*}}{1 - \xi_{w}^{*}} \left(\hat{\pi}_{t}^{c,*} - \kappa_{w} \hat{\pi}_{t-1}^{c,*} + \hat{w}_{t}^{*} - \hat{w}_{t-1}^{*} \right)$
Production Function	$\hat{y}_t^* = \hat{h}_t^* + \hat{z}_t^*$
Goods Market Clearing Condition	$\hat{y}_{t}^{*} = rac{c^{*}}{y^{*}}\hat{c}_{t}^{*} + g^{*}\left(\hat{g}_{t}^{*} + \hat{y}_{t}^{*} ight)$
Household Effective Rate	$\left \hat{r}_{t}^{h,*} = \hat{r}_{t}^{*} + \hat{\varepsilon}_{t}^{b^{S,*}} + \tilde{\chi}^{*} \delta^{*} \left[\hat{b}_{t}^{L,*} - \hat{b}_{t}^{S,*} \right] \right $
Long-Term Debt Value	$\hat{b}_{t}^{L,*} = \hat{p}_{L,t}^{*} + \hat{b}_{t}^{L,*}$
Long-Term Interest Rate	$\left[\hat{r}_{t+1}^{L,*} + E_t \Delta \hat{p}_{L,t+1}^* = \hat{r}_t^{h,*} - \hat{\varepsilon}_t^{\bar{b}^{L,*}} + \tilde{x}^* \left(r^{L,*} - \kappa^* \right) \left[\hat{b}_t^{L,*} - \hat{b}_t^{S,*} \right] \right]$
Long-Term Asset Price	$\hat{p}_{L,t}^* = -\frac{1}{1-\beta\kappa^*}\hat{r}_t^{L,*}$
Short-Term Debt	$\frac{b^{S,*}}{y^*} \left(\hat{b}_t^{S,*} - \hat{\varepsilon}_t^{b^{S,*}} \right) + \frac{b^{L,*}}{y^*} \left(\hat{\bar{b}}_t^{L,*} - \hat{\varepsilon}_t^{\bar{b}^{L,*}} \right) + \frac{b^{S,*}}{y^*} \theta^* \hat{b}_{t-1}^{S,*} = g^* \left(\hat{g}_t^* + \hat{y}_t^* \right)$
	$\frac{1}{\beta} \frac{b^{S,*}}{y^*} \left(\hat{b}^{S,*}_{t-1} + \hat{r}^*_{t-1} - \hat{\pi}^{c,*}_t \right) + \frac{1}{\beta} \frac{b^{L,*}}{y^*} \left(\hat{\bar{b}}^{L,*}_{t-1} + \hat{r}^{L,*}_t + \Delta \hat{p}^*_{L,t} - \hat{\pi}^{c,*}_t \right)$
Taylor Rule	$\hat{r}_{t}^{*} = \phi_{R}^{*} \hat{r}_{t-1}^{*} + (1 - \phi_{R}^{*}) \left(\phi_{\pi^{c}}^{*} \hat{\pi}_{t}^{c,*} + \phi_{y}^{*} \hat{y}_{t}^{*} \right) + \sigma_{R^{*}} \omega_{t}^{R,*}$
TFP	$\hat{z}_{t}^{*} = \rho_{z^{*}} \hat{z}_{t-1}^{*} + \sigma_{z^{*}} \omega_{t}^{z^{*}}$
Long-Term Debt Risk Premium	$\hat{b}_{t}^{L,*} = ho_{ar{b}L,*} \hat{b}_{t-1}^{L,*} + \sigma_{ar{b}L,*} \omega_{t}^{ar{b}L,*}$
Discount Factor	$\hat{d}_t^* = \rho_{d^*} \hat{d}_{t-1}^* + \sigma_{d^*} \omega_t^{d^*}$
Short-Term Debt Risk Premium	$\hat{b}_{t}^{S,*} = \rho_{b^{S,*}} \hat{b}_{t-1}^{S,*} + \sigma_{b^{S,*}} \omega_{t}^{b^{S,*}}$
Government Spending	$\hat{g}_{t}^{*} = \rho_{g^{*}} \hat{g}_{t-1}^{*} + \sigma_{g^{*}} \omega_{t}^{g^{*}}$

 Table 2: Foreign Economy Block Model Equations

Notes: Δ denotes the difference operator $(\Delta y_t = y_t - y_{t-1})$. We assume that the supply of long-term assets is fixed in both economies, namely $\hat{b}_t^L = \hat{b}_t^{L,*} = 0$ for all t.

Mnemonic	Description	Value
$100(\beta^{-1}-1)$	Time Discount Factor	1.00
$rac{r^{h,*}}{r^{*}}$ $rac{r^{L,*}}{r^{*}}$ h^{*}	Steady-State Spread Between the Foreign Long-Term and Consumer Rates	1.00
$\frac{r^{L,*}}{r^*}$	Steady-State Slope of the Foreign Yield Curve	1.00
h^*	Steady-State Foreign Hours	1.00
z^*	Steady-State Foreign TFP	1.00
g^*	Steady-State Foreign Government Spending to GDP Ratio	0.18
λ_{y^*}	Steady-State Foreign Prices Markup	1.20
λ_{y^*}	Steady-State Foreign Wages Markup	1.05
$rac{b^*+ar b^{L,*}}{u^*}$	Steady-State Total Foreing Debt to GDP Ratio	0.52
$\lambda_{y^*}^{\lambda_{y^*}}$ $rac{b^*+ar b^{L,*}}{y^*}$ $rac{r^h}{r}$ $rac{r^L}{r}$ h	Steady-State Spread Between the Domestic Long-Term and Consumer Rates	1.00
$\frac{r^L}{r}$	Steady-State Slope of the Domestic Yield Curve	1.00
\dot{h}	Steady-State Domestic Hours	1.00
z	Steady-State Domestic TFP	1.00
g	Steady-State Domestic Government Spending to GDP Ratio	0.17
λ_m	Steady-State Import Prices Markup	1.20
λ_x	Steady-State Export Prices Markup	1.20
λ_y	Steady-State Domestic Prices Markup	1.20
λ_w	Steady-State Domestic Wages Markup	1.05
$\frac{b+\bar{b}^L}{y}$	Steady-State Total Domestic Debt to GDP Ratio	0.41

Table 3: Calibrated Parameters

Table 4: Description of the Foreign Economy Estimated Parameter & Prior Moments

Mnemonic	Description	Density	Mean	STD
$\frac{\overline{b}^{L,*}}{u^*}$	Steady-State Foreign Long-Term Debt to GDP Ratio	Normal	0.20	0.01
$rac{\partial y^*}{\partial t^*}$	Foreign Transfer Policy Reaction to Government Debt	Normal	0.75	0.25
$100\chi^*$	Foreign Liquidity Adjustment Cost	Normal	10.00	1.00
ϕ_{π^*}	Foreign Policy Reaction to Inflation	Normal	1.50	0.10
ϕ_{y^*}	Foreign Policy Reaction to Output	Normal	0.13	0.05
ϕ_{R^*}	Foreign Taylor Rule Smoothing	Beta	0.75	0.10
σ_{C^*}	Foreign Inverse Intertemporal Substitution	Normal	1.50	0.10
$arphi^*$	Foreign Inverse Labour Supply Elasticity	Normal	2.00	0.10
b^*	Foreign Consumption Habit	Beta	0.70	0.10
κ_{y^*}	Foreign Prices Indexation	Beta	0.50	0.15
ξ_{y^*}	Foreign Prices Reset Probability	Beta	0.50	0.10
κ_{w^*}	Foreign Wages Indexation	Beta	0.50	0.15
ξ_{w^*}	Foreign Wages Reset Probability	Beta	0.50	0.10
$ ho_{z^*}$	Persistence of Foreign Productivity Shock	Beta	0.75	0.10
$ ho_{ar b^{L,*}}$	Persistence of Foreign Long-Term Debt Risk Premium Shock	Beta	0.75	0.10
$ ho_{d^*}$	Persistence of Foreign Discount Factor Shock	Beta	0.75	0.10
$ ho_{b^{S,*}}$	Persistence of Foreign Short-Term Debt Risk Premium Shock	Beta	0.75	0.10
$ ho_{g^*}$	Persistence of Foreign Government Spending Shock	Beta	0.75	0.10
$100\sigma_{z^*}$	Uncertainty of Foreign Productivity Shock	Inv-Gamma	0.50	0.20
$100\sigma_{\bar{b}^{L,*}}$	Uncertainty of Foreign Long-Term Debt Risk Premium Shock	Inv-Gamma	0.50	0.20
$100\sigma_{d^*}$	Uncertainty of Foreign Discount Factor Shock	Inv-Gamma	0.50	0.20
$100\sigma_{b^{S,*}}$	Uncertainty of Foreign Short-Term Debt Risk Premium Shock	Inv-Gamma	0.50	0.20
$100\sigma_{R^*}$	Uncertainty of Foreign Policy Shock	Inv-Gamma	0.50	0.20
$100\sigma_{g^*}$	Uncertainty of Foreign Government Spending Shock	Inv-Gamma	0.50	0.20

Notes: STD denotes the prior standard deviation moment of the estimated parameter.

Mnemonic	Description of the Domestic Economy Estimated Paran Description	Density	Mean	STD
$\frac{\overline{b+\bar{b}^L+\bar{b}^{L,*}}}{b+\bar{b}^L+\bar{b}^{L,*}}$	*	U		
11	Steady-State Domestic Total Debt to GDP Ratio	Normal	0.40	0.10
$rac{ar{b}^L}{y} = rac{g}{ heta}$	Steady-State Domestic Long-Term Debt to GDP Ratio	Normal	0.20	0.01
$ec{ heta}$	Domestic Transfer Policy Reaction to Government Debt	Normal	0.75	0.25
100χ	Domestic Liquidity Adjustment Cost	Normal	10.00	1.00
η	Substitution Elasticity Consumption	Normal	1.50	0.10
ϕ_{π}	Domestic Policy Reaction to Inflation	Normal	1.50	0.10
ϕ_y	Domestic Policy Reaction to Output	Normal	0.13	0.05
$\check{\phi_R}$	Domestic Taylor Rule Smoothing	Beta	0.75	0.05
σ_C	Domestic Inverse Intertemporal Substitution	Normal	1.50	0.10
arphi	Domestic Inverse Labour Supply Elasticity	Normal	2.00	0.10
b	Domestic Consumption Habit	Beta	0.70	0.10
κ_y	Domestic Prices Indexation	Beta	0.50	0.15
ξ_y	Domestic Prices Reset Probability	Beta	0.50	0.10
$\tilde{\kappa_m}$	Domestic Import Prices Indexation	Beta	0.50	0.15
ξ_m	Domestic Import Prices Reset Probability	Beta	0.50	0.05
κ_w	Domestic Wages Indexation	Beta	0.50	0.15
ξ_w	Domestic Wages Reset Probability	Beta	0.50	0.10
$ ho_z$	Persistence of Domestic Productivity Shock	Beta	0.75	0.10
$ ho_{ar b^L}$	Persistence of Domestic Long-Term Debt Risk Premium Shock	Beta	0.75	0.10
$ ho_d$	Persistence of Domestic Discount Factor Shock	Beta	0.75	0.10
$ ho_{b^S}$	Persistence of Domestic Short-Term Debt Risk Premium Shock	Beta	0.75	0.10
$ ho_g$	Persistence of Domestic Government Spending Shock	Beta	0.75	0.10
$100\sigma_z$	Uncertainty of Domestic Productivity Shock	Inv-Gamma	0.50	0.20
$100\sigma_{\bar{b}^L}$	Uncertainty of Domestic Long-Term Debt Risk Premium Shock	Inv-Gamma	0.50	0.20
$100\sigma_d$	Uncertainty of Domestic Discount Factor Shock	Inv-Gamma	0.50	0.20
$100\sigma_{b^S}$	Uncertainty of Domestic Short-Term Debt Risk Premium Shock	Inv-Gamma	0.50	0.20
$100\sigma_g$	Uncertainty of Domestic Government Spending Shock	Inv-Gamma	0.50	0.20
$100\sigma_R$	Uncertainty of Domestic Policy Shock	Inv-Gamma	0.50	0.20

Table 5: Description of the Domestic Economy Estimated Parameter & Prior Moments

Notes: STD denotes the prior standard deviation moment of the estimated parameter.

Mnemonics	Description	Density	Mean	STD
$Corr(r_t^L, r_t^{L,*})$	Correlation between US and UK Long-Term Rates	Normal	0.93	0.01
$Corr(r_t^L, r_t)$	Correlation between Short- and Long-Term UK Rates	Normal	0.90	0.01
$Corr(r_t, r_t^*)$	Correlation between US and UK Short-Term Rates	Normal	0.84	0.01
$Corr(y_t, y_t^*)$	Correlation between US and UK GDP	Normal	0.73	0.01
$Corr(\pi_t, \pi_t^*)$	Correlation between US and UK Inflation	Normal	0.74	0.01
$Corr(y_t, y_{t-1})$	1^{st} Order UK GDP Autocorrelation	Normal	0.92	0.01
$Corr(\pi_t, \pi_{t-1})$	1^{st} Order UK Inflation Autocorrelation	Normal	0.75	0.01
$Corr(y_t^*, y_{t-1}^*)$	1^{st} Order US GDP Autocorrelation	Normal	0.95	0.01
$Corr(\pi_t^*, \pi_{t-1}^*)$	1^{st} Order US Inflation Autocorrelation	Normal	0.90	0.01

 Table 6: Prior Distribution of Selected Moments

Notes: STD denotes the prior standard deviation of the moment.

Mnemonic	Description	Mode	Mean	5^{th}	95^{th}
$\frac{\overline{b}^{L,*}}{u^*}$	Steady-State Foreign Long-Term Debt to GDP Ratio	0.20	0.20	0.18	0.21
$rac{\partial y^*}{\partial t^*}$	Foreign Transfer Policy Reaction to Government Debt	0.19	0.20	0.13	0.31
$100\chi^*$	Foreign Liquidity Adjustment Cost	7.25	7.17	5.24	9.13
ϕ_{π^*}	Foreign Policy Reaction to Inflation	1.68	1.73	1.60	1.88
ϕ_{y^*}	Foreign Policy Reaction to Output	0.07	0.08	0.05	0.13
$\check{\phi_{R^*}}$	Foreign Taylor Rule Smoothing	0.86	0.86	0.83	0.88
σ_{C^*}	Foreign Inverse Intertemporal Substitution	1.51	1.54	1.39	1.70
$arphi^*$	Foreign Inverse Labour Supply Elasticity	1.98	1.98	1.82	2.15
b^*	Foreign Consumption Habit	0.77	0.75	0.68	0.81
κ_{y^*}	Foreign Prices Indexation	0.08	0.11	0.06	0.17
ξ_{y^*}	Foreign Prices Reset Probability	0.74	0.72	0.64	0.78
κ_{w^*}	Foreign Wages Indexation	0.43	0.46	0.17	0.71
ξ_{w^*}	Foreign Wages Reset Probability	0.39	0.39	0.24	0.54
$ ho_{z^*}$	Persistence of Foreign Productivity Shock	0.99	0.99	0.99	0.99
$ ho_{ar b^{L,*}}$	Persistence of Foreign Long-Term Debt Risk Premium Shock	0.93	0.93	0.90	0.95
$ ho_{d^*}$	Persistence of Foreign Discount Factor Shock	0.99	0.99	0.99	0.99
$ ho_{b^{S,*}}$	Persistence of Foreign Short-Term Debt Risk Premium Shock	0.89	0.88	0.85	0.91
$ ho_{g^*}$	Persistence of Foreign Government Spending Shock	0.74	0.74	0.63	0.83
$100\sigma_{z^*}$	Uncertainty of Foreign Productivity Shock	1.13	1.20	0.88	1.57
$100\sigma_{\bar{b}^{L,*}}$	Uncertainty of Foreign Long-Term Debt Risk Premium Shock	0.45	0.46	0.34	0.61
$100\sigma_{d^*}$	Uncertainty of Foreign Discount Factor Shock	1.91	2.17	1.64	2.78
$100\sigma_{b^{S,*}}$	Uncertainty of Foreign Short-Term Debt Risk Premium Shock	0.28	0.30	0.22	0.38
$100\sigma_{R^*}$	Uncertainty of Foreign Policy Shock	0.22	0.23	0.20	0.25
$100\sigma_{g^*}$	Uncertainty of Foreign Government Spending Shock	3.02	3.00	2.60	3.43

 Table 7: Foreign Parameter Estimates

Notes: The 5^{th} and 95^{th} columns refer to 0.05 and 0.95 quantiles of the posterior distribution of the structural parameter vector.

Mnemonic	Description	Mode	Mean	5^{th}	95^{th}
$\frac{b+\overline{b}^L+\overline{b}^{L,*}}{y}$	Steady-State Domestic Total Debt to GDP Ratio	0.58	0.54	0.46	0.64
$rac{ar b^L}{y}$	Steady-State Domestic Long-Term Debt to GDP Ratio	0.19	0.19	0.17	0.20
$\overset{g}{ heta}$	Domestic Transfer Policy Reaction to Government Debt	1.57	1.55	1.23	1.87
100χ	Domestic Liquidity Adjustment Cost	9.81	9.97	8.39	11.51
η	Substitution Elasticity Consumption	1.44	1.40	1.26	1.53
ϕ_{π}	Domestic Policy Reaction to Inflation	1.63	1.65	1.54	1.76
ϕ_y	Domestic Policy Reaction to Output	0.31	0.30	0.24	0.35
$\phi_R^{"}$	Domestic Taylor Rule Smoothing	0.82	0.82	0.79	0.85
σ_C	Domestic Inverse Intertemporal Substitution	1.02	1.03	1.00	1.08
arphi	Domestic Inverse Labour Supply Elasticity	1.88	1.88	1.71	2.04
b	Domestic Consumption Habit	0.39	0.37	0.30	0.44
κ_y	Domestic Prices Indexation	0.24	0.26	0.11	0.42
ξ_y	Domestic Prices Reset Probability	0.24	0.25	0.16	0.34
κ_m	Domestic Import Prices Indexation	0.17	0.22	0.08	0.35
ξ_m	Domestic Import Prices Reset Probability	0.64	0.66	0.60	0.72
κ_w	Domestic Wages Indexation	0.29	0.34	0.15	0.54
ξ_w	Domestic Wages Reset Probability	0.06	0.07	0.04	0.11
$ ho_z$	Persistence of Domestic Productivity Shock	0.75	0.74	0.67	0.81
$ ho_{ar{b}^L}$	Persistence of Domestic Long-Term Debt Risk Premium Shock	0.91	0.88	0.79	0.93
$ ho_d$	Persistence of Domestic Discount Factor Shock	0.98	0.98	0.98	0.99
$ ho_{b^S}$	Persistence of Domestic Short-Term Debt Risk Premium Shock	0.75	0.75	0.66	0.82
$ ho_g$	Persistence of Domestic Government Spending Shock	0.81	0.76	0.60	0.88
$100\sigma_z$	Uncertainty of Domestic Productivity Shock	0.87	0.99	0.78	1.23
$100\sigma_{\bar{b}^L}$	Uncertainty of Domestic Long-Term Debt Risk Premium Shock	0.48	0.60	0.38	0.94
$100\sigma_d$	Uncertainty of Domestic Discount Factor Shock	3.71	3.66	3.29	4.06
$100\sigma_{b^S}$	Uncertainty of Domestic Short-Term Debt Risk Premium Shock	0.31	0.35	0.25	0.45
$100\sigma_g$	Uncertainty of Domestic Government Spending Shock	4.85	3.53	0.50	5.65
$100\sigma_R$	Uncertainty of Domestic Policy Shock	0.30	0.30	0.27	0.34

Table 8: Domestic Parameter Estimates

Notes: The 5^{th} and 95^{th} columns refer to 0.05 and 0.95 quantiles of the posterior distribution of the structural parameter vector.

Moments	Data	Model			
		Mean	5^{th}	95^{th}	
$Corr(r_t^L, r_t^{L,*})$	0.93	0.94	0.92	0.95	
$Corr(r_t^L, r_t)$	0.90	0.91	0.89	0.92	
$Corr(r_t, r_t^*)$	0.84	0.82	0.81	0.84	
$Corr(y_t, y_t^*)$	0.72	0.71	0.69	0.72	
$Corr(c_t, c_t^*)$	0.69	0.78	0.76	0.80	
$Corr(\pi_t, \pi_t^*)$	0.74	0.76	0.74	0.77	
$Corr(y_t, y_{t-1})$	0.92	0.91	0.90	0.92	
$Corr(\pi_t, \pi_{t-1})$	0.75	0.73	0.72	0.75	
$Corr(y_t^*, y_{t-1}^*)$	0.95	0.99	0.99	0.99	
$Corr(\pi_t^*,\pi_{t-1}^*)$	0.90	0.89	0.87	0.90	

Table 9: Posterior Distribution of Selected Moments

Notes: The 5^{th} and 95^{th} columns refer to 0.05 and 0.95 quantiles of the posterior distribution centered around the posterior mode moments.



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