

Dolatabadi, Sepideh; Narayan, Paresh Kumar; Nielsen, Morten Ørregaard; Xu, Ke

## Working Paper

# Economic significance of commodity return forecasts from the fractionally cointegrated VAR model

Queen's Economics Department Working Paper, No. 1337

### Provided in Cooperation with:

Queen's University, Department of Economics (QED)

*Suggested Citation:* Dolatabadi, Sepideh; Narayan, Paresh Kumar; Nielsen, Morten Ørregaard; Xu, Ke (2015) : Economic significance of commodity return forecasts from the fractionally cointegrated VAR model, Queen's Economics Department Working Paper, No. 1337, Queen's University, Department of Economics, Kingston (Ontario)

This Version is available at:

<https://hdl.handle.net/10419/122047>

#### Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

#### Terms of use:

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



Queen's Economics Department Working Paper No. 1337

# Economic significance of commodity return forecasts from the fractionally cointegrated VAR model

Sepideh Dolatabadi  
Queen's University

Paresh Kumar Narayan  
Deakin University

Morten Årregaard Nielsen  
Queen's University and CREATES

Ke Xu  
Queen's University

Department of Economics  
Queen's University  
94 University Avenue  
Kingston, Ontario, Canada  
K7L 3N6

1-2015

# Economic significance of commodity return forecasts from the fractionally cointegrated VAR model\*

Sepideh Dolatabadi      Paresh Kumar Narayan  
Queen's University      Deakin University

Morten Ørregaard Nielsen<sup>†</sup>      Ke Xu  
Queen's University and CREATES      Queen's University

January 8, 2015

## Abstract

Based on recent evidence of fractional cointegration in commodity spot and futures markets, we investigate whether a fractionally cointegrated model can provide statistically and/or economically significant forecasts of commodity returns. Specifically, we propose to model and forecast commodity spot and futures prices using a fractionally cointegrated vector autoregressive model that generalizes the more well-known cointegrated vector autoregressive model to allow fractional integration. We derive the best linear predictor forecast for this model and perform an out-of-sample forecast comparison with forecasts from the more standard (non-fractional) model. In our empirical analysis to daily data on 15 commodity spot and futures markets, the fractional model is found to be superior in terms of in-sample fit and also out-of-sample forecasting based on statistical metrics of forecast comparison. We then analyze the economic significance of the forecasts through a dynamic trading strategy based on a portfolio with weights derived from a mean-variance utility function. This analysis leads to statistically significant and economically meaningful profits in the commodity markets, and also shows that the fractional model generates higher profits on average compared with the non-fractional model.

**Keywords:** commodity markets, economic significance, forecasting, fractional cointegration, futures markets, price discovery, vector error correction model.

**JEL Classification:** C32, G11.

---

\*This article was prepared for a keynote address at the Conference on Recent Developments in Financial Econometrics and Applications, Centre for Economics and Financial Econometrics Research, Deakin University, Australia, which was given by M.Ø. Nielsen in December, 2014. We are grateful to the participants at the conference as well as Peter Extercate and Michał Popiel for useful comments and suggestions, and to the Canada Research Chairs program, the Social Sciences and Humanities Research Council of Canada (SSHRC), and the Center for Research in Econometric Analysis of Time Series (CREATES, funded by the Danish National Research Foundation, DNRF78) for financial support.

<sup>†</sup>Corresponding author. Email: [mon@econ.queensu.ca](mailto:mon@econ.queensu.ca)

# 1 Introduction

The forecastability of commodity market returns is a very active area of research in financial economics. In particular, recent research has shown that commodity spot and futures prices are fractionally cointegrated; see, *inter alia*, Baillie and Bollerslev (1994), Lien and Tse (1999), Maynard and Phillips (2001), Coakley, Dollery, and Kellard (2011), and Dolatabadi, Nielsen, and Xu (2014). The implication is that a fractionally cointegrated model may provide a better statistical fit when modeling and forecasting commodity prices and returns. Relatedly, the understanding of how commodity market return forecasts can be used to devise trading strategies appears as of yet to be rather limited.

In this paper, we make two contributions to this literature. Our first contribution is methodological. We propose to model and forecast commodity spot and futures prices using the recently developed fractionally cointegrated vector autoregressive (FCVAR) model of Johansen (2008) and Johansen and Nielsen (2012). Specifically, we derive the best linear predictor for the FCVAR model and show that it takes a relatively simple form. We thus demonstrate how to forecast commodity spot and futures prices and returns based on the FCVAR model, and we evaluate these using statistical measures of forecast performance. Our second contribution is to investigate the economic significance of the FCVAR model forecasts through a dynamic trading strategy based on a portfolio of two assets with portfolio weights derived from a mean-variance utility function and from returns forecasts. Throughout, we compare with forecasts from the more standard (non-fractional) cointegrated vector autoregressive (CVAR) model of Johansen (1995).

We apply the FCVAR model to spot and futures prices of 15 commodities and demonstrate that it provides superior statistical in-sample fit compared with the more standard CVAR model. We also estimate price discovery from both models, see Hasbrouck (1995), Gonzalo and Granger (1995), Figuerola-Feretti and Gonzalo (2010), and Dolatabadi, Nielsen, and Xu (2015). This tells us whether price discovery is by the commodity spot or futures market, which may be important from a forecasting point of view since historical information from the dominant market could be useful in forecasting prices and returns in the non-dominant market. In any case, both the FCVAR and CVAR models are joint models of the two prices series, and as such are able to take into account the price discovery information in modeling and forecasting. With the FCVAR model we find that for 12 commodities the futures market dominates price discovery, as much theory predicts (e.g., Hasbrouck, 1995), whereas using the CVAR model price discovery is dominated by the futures market for only 10 commodities. In both cases, the empirical evidence favors price discovery in the spot market for soybean, sugar, and crude oil.

In our empirical analysis we consider both short horizon ( $h = 1$ ) and long horizon ( $h = 5$  and  $h = 21$ ) forecasting. Using a variety of out-of-sample statistical forecasting evaluation metrics, we see that the FCVAR model tends to outperform the CVAR model in terms of tests of forecast superiority, although these are not always significant. Thus, the FCVAR model has superior statistical in-sample fit, and the out-of-sample forecasting results are less strong but still suggest a slight advantage to the FCVAR model.

As an additional metric of forecast performance and comparison, we examine the economic— as opposed to purely statistical—significance of returns forecasts. We do this by investigating

whether the returns forecasts can generate significant excess returns when implemented in a dynamic portfolio trading strategy. For our main empirical analysis we find that using returns forecasts from both FCMVAR and CMVAR models in simple mean-variance trading strategies leads to statistically significant and economically meaningful profits in the commodity markets, and furthermore that profits are higher on average when based on forecasts from the FCMVAR model than when based on forecasts from the CMVAR model.

Our finding that profits from commodity markets are statistically significant and economically meaningful are consistent with a broad range of studies which show, using different approaches, that commodity markets are profitable. For example, Miffre and Rallis (2007), Szakmary, Shen and Sharma (2010), and Narayan, Ahmed, and Narayan (2014) show profitability using technical trading and momentum trading strategies. However, given the profitability of these approaches, limited focus has been on using a forecasting model-based approach to estimate profits (an exception is Narayan, Narayan, and Sharma, 2013).

In spite of this, there is a clear acceptance of the fact that a forecasting based model that draws its profitability analysis from a utility function, such as a mean-variance utility function, has a theoretical appeal, see e.g. Marquering and Verbeek (2004) and Campbell and Thompson (2008). On the basis of this evidence, commodity markets are treated as an investment class. As the focus on theoretically motivated profitability analysis gains momentum, following, for example, the work mentioned above, the emphasis on and hence demand for appropriate forecasting models will increase.

We note from the outset that, although trading strategies based on commodity spot prices are not really feasible, because it would be too expensive to take possession of the commodity, we nonetheless consider simultaneous modeling of commodity spot and futures prices. In terms of applying these as forecasting models for futures returns, it has no relevance whether spot prices can be traded on or not, and hence this point is irrelevant for all our results regarding futures markets, futures price and returns forecasting, and trading strategies involving commodity futures. For trading strategies involving commodity spot markets, these can still be considered a useful basis for comparison of forecast performance in terms of economic significance, even if the trading strategies are infeasible; a related point was also made in, e.g., Graham-Higgs, Rambaldi, and Davidson (1999), Wang (2000), and Narayan, Narayan, and Sharma (2013). Thus, even if portfolios involving commodity spot positions are infeasible, we consider such “artificial portfolios” as a means of forecast evaluation and comparison.

Finally, to demonstrate the robustness of our empirical results, we repeat our analysis with several different variations. First, in the forecasting models, we forecast returns over both short and long horizons. Second, we use more than one out-of-sample forecast evaluation technique. Third, when estimating profits using the mean-variance investor utility function, where the choice of the investor’s risk aversion coefficient influences portfolio weights, we consider both low, medium, and high risk-aversion investors. All these results confirm (i) that portfolio returns are statistically different from zero and economically meaningful and (ii) portfolio returns derived from FCMVAR model forecasts are higher on average than those obtained from CMVAR model forecasts.

The remainder of the paper is organized as follows. The econometric model is explained in the next section, where, in particular, the best linear predictor is derived and forecasting is discussed in Section 2.4. In Section 3 we discuss the commodity data and conduct some

preliminary data analysis. Section 4 contains the empirical results, and is divided into subsections on estimation, statistical forecast comparison, economic significance of forecasts, and alternative risk-aversion results. Finally, in Section 5 we provide concluding remarks.

## 2 Econometric methodology: the fractionally cointegrated VAR model

Our empirical analysis applies the FCVAR model, see Johansen (2008) and Johansen and Nielsen (2012), as well as its non-fractional counterpart. The model is a generalization of Johansen's (1995) CVAR model to allow for fractionally integrated (or just fractional) time series.

### 2.1 Fractional integration and cointegration

Fractional time series models are based on the fractional difference operator

$$\Delta^d X_t = \sum_{n=0}^{\infty} \pi_n(-d) X_{t-n}, \quad (1)$$

where the fractional coefficients  $\pi_n(u)$  are defined in terms of the binomial expansion  $(1 - z)^{-u} = \sum_{n=0}^{\infty} \pi_n(u) z^n$ , i.e.,

$$\pi_n(u) = \frac{u(u+1) \cdots (u+n-1)}{n!}.$$

For details and for many intermediate results regarding this expansion and the fractional coefficients, see, for example, Johansen and Nielsen (2014, Appendix A). Efficient calculation of fractional differences, which we apply in our estimation, is discussed in Jensen and Nielsen (2014).

With the definition of the fractional difference operator in (1), a time series  $X_t$  is said to be fractional of order  $d$ , denoted  $X_t \in I(d)$ , if  $\Delta^d X_t$  is fractional of order zero, i.e., if  $\Delta^d X_t \in I(0)$ . The latter property can be defined in the frequency domain as having spectral density that is finite and non-zero near the origin or in terms of the linear representation coefficients if the sum of these is non-zero and finite, see, for example, Johansen and Nielsen (2012). An example of a process that is fractional of order zero is the stationary and invertible ARMA model. Finally, then, a time series  $X_t \in I(d)$  for which one or more linear combinations are fractional of a lower order, i.e., for which there exists a  $p \times r$  matrix  $\beta$  such that  $\beta' X_t \in I(d-b)$  with  $b > 0$ , is said to be (fractionally) cointegrated.

### 2.2 The FCVAR model and interpretation

For a time series  $Y_t$  of dimension  $p$ , the well-known CVAR model is given in error correction form as

$$\Delta Y_t = \alpha \beta' Y_{t-1} + \sum_{i=1}^k \Gamma_i \Delta Y_{t-i} + \varepsilon_t = \alpha \beta' L Y_t + \sum_{i=1}^k \Gamma_i L^i \Delta Y_t + \varepsilon_t, \quad (2)$$

where, as usual,  $\varepsilon_t$  is  $p$ -dimensional independent and identically distributed with mean zero and covariance matrix  $\Omega$ . The simplest way to derive the FCVAR model from the CVAR is to replace the difference and lag operators,  $\Delta$  and  $L = 1 - \Delta$ , in (2) by their fractional counterparts,  $\Delta^b$  and  $L_b = 1 - \Delta^b$ , respectively, and apply the resulting model to  $Y_t = \Delta^{d-b}X_t$ . We then obtain

$$\Delta^d X_t = \alpha\beta' \Delta^{d-b} L_b X_t + \sum_{i=1}^k \Gamma_i \Delta^d L_b^i X_t + \varepsilon_t, \quad (3)$$

where  $\Delta^d$  is the fractional difference operator, and  $L_b = 1 - \Delta^b$  is the fractional lag operator.<sup>1</sup>

Model (3) nests Johansen's (1995) CVAR model in (2) as the special case  $d = b = 1$ . Some of the parameters are well-known from the CVAR model and these have the usual interpretations also in the FCVAR model. The most important of these are the long-run parameters  $\alpha$  and  $\beta$ , which are  $p \times r$  matrices with  $0 \leq r \leq p$ . The rank  $r$  is termed the cointegration, or cofractional, rank. The columns of  $\beta$  constitute the  $r$  cointegration (cofractional) vectors such that  $\beta' X_t$  are the cointegrating combinations of the variables in the system, i.e. the long-run equilibrium relations. The parameters in  $\alpha$  are the adjustment or loading coefficients which represent the speed of adjustment towards equilibrium for each of the variables. The short-run dynamics of the variables are governed by the parameters  $(\Gamma_1, \dots, \Gamma_k)$  in the autoregressive augmentation.

The FCVAR model has two additional parameters compared with the CVAR model, namely the fractional parameters  $d$  and  $b$ . Here,  $d$  denotes the fractional integration order of the observable time series. As would presumably be the case for most—if not all—financial asset prices, we assume in our study that these are integrated of order  $d = 1$ . That is, we consider  $d = 1$  to be fixed and known, and therefore not estimated. On the other hand, the parameter  $b$  is estimated jointly with the remaining parameters, and determines the degree of fractional cointegration, i.e. the reduction in fractional integration order of  $\beta' X_t$  compared to  $X_t$  itself.

The FCVAR model (3) thus has the same main structure as the standard CVAR model (2), in that it allows for modeling of both cointegration and adjustment towards equilibrium, but is more general since it accommodates fractional integration and cointegration.

We note that the fractional difference as defined in (1) is an infinite series, but any observed sample will include only a finite number of observations. This makes calculation of the fractional differences as defined in (1) impossible. In practice, therefore, the summation in (1) would need to be truncated at  $n = t - 1$ , and the bias introduced by application of such a truncation is analyzed by Johansen and Nielsen (2014) using higher-order expansions in a simpler model. They show that, albeit in a simpler model, this bias can be avoided by including a level parameter,  $\mu$ , that shifts each of the series by a constant. We follow this suggestion (and set  $d = 1$ ), and consider the unobserved components formulation

$$X_t = \mu + X_t^0, \quad \Delta X_t^0 = L_b \alpha \beta' X_t^0 + \sum_{i=1}^k \Gamma_i \Delta L_b^i X_t^0 + \varepsilon_t \quad (4)$$

---

<sup>1</sup>Both the fractional difference and fractional lag operators are defined in terms of their binomial expansion in the lag operator,  $L$ , as in (1). Note that the expansion of  $L_b$  has no term in  $L^0$  and thus only lagged disequilibrium errors appear in (3).

from which we easily derive the model

$$\Delta(X_t - \mu) = \alpha\beta' \Delta^{1-b} L_b(X_t - \mu) + \sum_{i=1}^k \Gamma_i \Delta L_b^i(X_t - \mu) + \varepsilon_t. \quad (5)$$

The formulation (5) includes the so-called restricted constant, which may be obtained as  $\rho' = -\beta'\mu$  and is interpreted as the mean level of the long-run equilibria when these are stationary, i.e.  $E\beta'X_t + \rho' = 0$ . In the CVAR model this parameter enters as

$$\Delta Y_t = \alpha(\beta'Y_{t-1} + \rho') + \sum_{i=1}^k \Gamma_i \Delta Y_{t-i} + \varepsilon_t. \quad (6)$$

More generally, the level parameter  $\mu$  in (5) is meant to accommodate a non-zero starting point for the first observation on the process, i.e., for  $X_1$ . Our empirical work applies the version of the FCVAR model given in (5) and we provide comparisons with the CVAR model in (6).

The asymptotic analysis of the FCVAR model is provided in Johansen and Nielsen (2010, 2012), where it is shown that the maximum likelihood estimator of  $(b, \alpha, \Gamma_1, \dots, \Gamma_k)$  is asymptotically normal, while the maximum likelihood estimator of  $(\beta, \rho)$  is asymptotically mixed normal when  $b > 1/2$  and asymptotically normal when  $b < 1/2$ . The important implication is that standard asymptotic inference can be applied using (quasi) likelihood ratio tests and the  $\chi^2$ -distribution.

Likelihood ratio (trace-type) tests for cointegration rank can be calculated as well, and hypotheses on the cointegration rank can be tested in the same way as in the CVAR model. In the FCVAR model, the asymptotic distribution of the tests for cointegration rank depends on the unknown (true value of the) scalar parameter  $b$ , which complicates empirical analysis compared to the CVAR model. However, the distribution can be simulated on a case-by-case basis. The calculation of maximum likelihood estimators and test statistics is discussed in detail in Johansen and Nielsen (2012) and Nielsen and Popiel (2014), with the latter providing Matlab computer programs that we apply in our empirical analysis.

## 2.3 Economic model and price discovery

Fractional cointegration between spot and futures log-prices can be derived from an economic model. Specifically, following Figuerola-Ferretti and Gonzalo (2010), Dolatabadi, Nielsen, and Xu (2014, 2015) consider an economic model within which spot log-prices are assumed to be  $I(1)$  and no-arbitrage conditions imply that futures log-prices are then also  $I(1)$  and the two log-prices are fractionally cointegrated. It is shown how the FCVAR model provides a natural methodology for the analysis of the relation between spot and futures log-prices, and how the model can be used to analyze price discovery as well. In the notation of the previous subsections, we then let  $X_t = (s_t, f_t)'$ , where  $s_t$  and  $f_t$  denote the log-spot and log-futures prices at time  $t$ , respectively.

Specifically,  $X_t$  may be decomposed into a transitory (stationary) part,  $\beta'X_t$ , and a permanent part,  $W_t = \alpha'_\perp X_t$ , using the identity

$$\begin{aligned} X_t &= (\beta_\perp (\alpha'_\perp \beta_\perp)^{-1} \alpha'_\perp + \alpha (\beta' \alpha)^{-1} \beta') X_t \\ &= A_1 W_t + A_2 \beta' X_t. \end{aligned}$$



Here,  $W_t$  is the common permanent component of  $X_t$ ; that is, in the case of spot and futures log-prices, it is the long-run dominant (fundamental or efficient) price. Thus, the proportions of price discovery attributable to each market may be inferred from the elements of  $\alpha_\perp$ , after being normalized so that the elements sum to unity. For further details, we refer the reader to Gonzalo and Granger (1995) and Dolatabadi, Nielsen, and Xu (2015).

## 2.4 Forecasting from the FCVAR model

We now discuss how to forecast log-prices, that is  $X_t$ , as well as returns,  $\Delta X_t$ , from the FCVAR model. Because the model is autoregressive, the best linear predictor takes a simple form and is relatively straightforward to calculate. We note that  $\Delta(X_{t+1} - \mu) = X_{t+1} - X_t$  for  $t \geq 1$  and rearrange (5) as

$$X_{t+1} = X_t + \alpha\beta' \Delta^{1-b} L_b(X_{t+1} - \mu) + \sum_{i=1}^k \Gamma_i \Delta L_b^i(X_{t+1} - \mu) + \varepsilon_{t+1}. \quad (7)$$

Since  $L_b = 1 - \Delta^b$  is a lag operator, so that  $L_b^i X_{t+1}$  is known at time  $t$ , this equation can be used to calculate forecasts from the model.

We let conditional expectation given the information set at time  $t$  be denoted  $E_t(\cdot)$ , and the best linear predictor forecast of any variable  $Z_{t+1}$  given information available at time  $t$  be denoted  $\hat{Z}_{t+1|t} = E_t(Z_{t+1})$ . Clearly, we then have that the forecast of the innovation for period  $t + 1$  at time  $t$  is  $\hat{\varepsilon}_{t+1|t} = E_t(\varepsilon_{t+1}) = 0$ , and  $\hat{X}_{t+1|t}$  is then easily found from (7). Inserting also coefficient estimates based on data available up to time  $t$ , denoted<sup>2</sup>  $(\hat{b}, \hat{\mu}, \hat{\alpha}, \hat{\beta}, \hat{\Gamma}_1, \dots, \hat{\Gamma}_k)$ , we have that

$$\hat{X}_{t+1|t} = X_t + \hat{\alpha}\hat{\beta}' \Delta^{1-\hat{b}} L_{\hat{b}}(X_{t+1} - \hat{\mu}) + \sum_{i=1}^k \hat{\Gamma}_i \Delta L_{\hat{b}}^i(X_{t+1} - \hat{\mu}). \quad (8)$$

This defines the forecast of log-prices for period  $t + 1$  given information at time  $t$ . From (8) we can derive the forecast of returns as

$$\hat{r}_{t+1|t} = 100(\hat{X}_{t+1|t} - X_t). \quad (9)$$

We note that, after constructing a series of one-step ahead log-price forecasts,  $\hat{X}_{t+1|t}$  for a range of  $t$ , the return forecast (9) is different from  $100\Delta\hat{X}_{t+1|t}$ , which may seem the obvious forecast of returns based on forecasts of log-prices, given the definition of returns as 100 times the first difference of log-prices. However, since  $X_t$  is known at time  $t$ , clearly (9) is the appropriate forecast of returns.

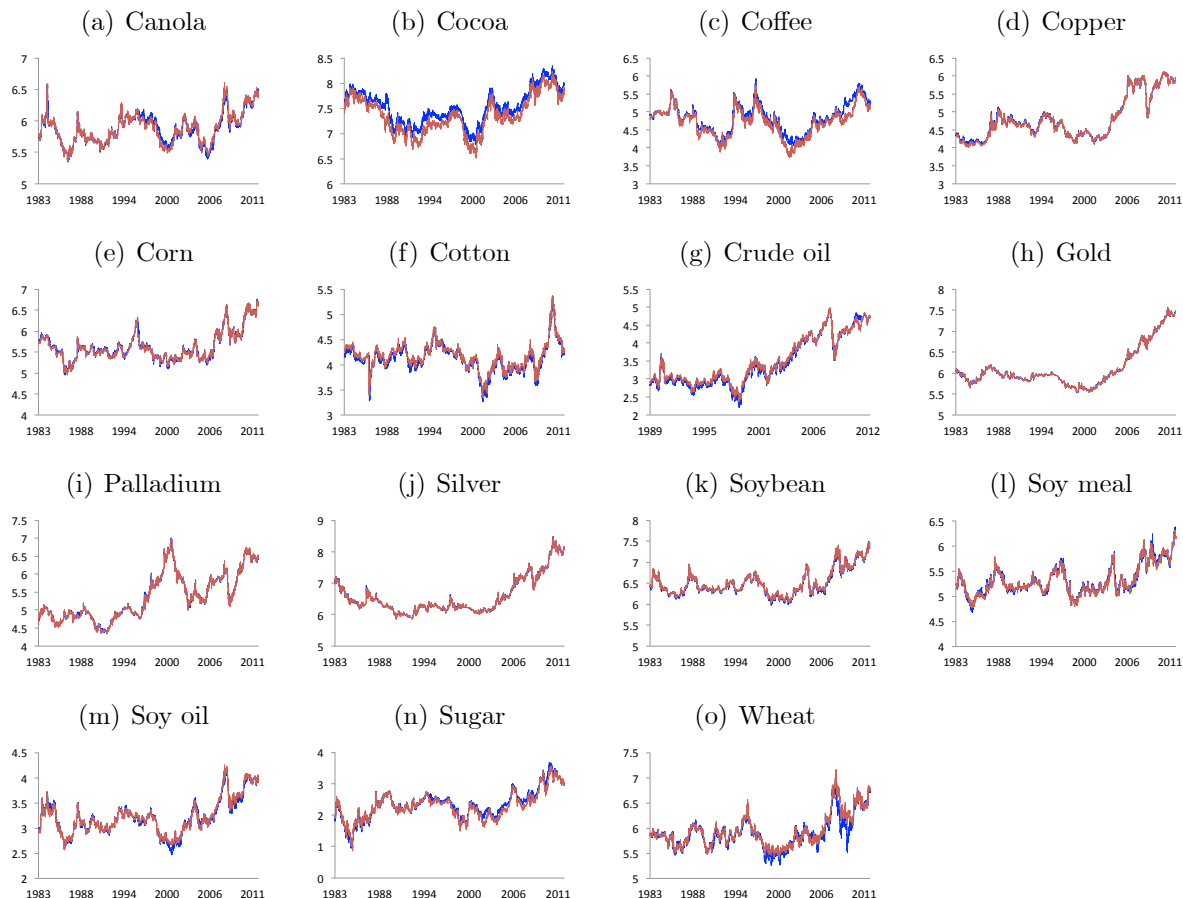
Multi-period ahead forecasts can be generated recursively. That is, to calculate the  $h$ -step ahead forecast, we first generalize (8) as

$$\hat{X}_{t+j|t} = \hat{X}_{t+j-1|t} + \hat{\alpha}\hat{\beta}' \Delta^{1-\hat{b}} L_{\hat{b}}(\hat{X}_{t+j|t} - \hat{\mu}) + \sum_{i=1}^k \hat{\Gamma}_i \Delta L_{\hat{b}}^i(\hat{X}_{t+j|t} - \hat{\mu}), \quad (10)$$

---

<sup>2</sup>To emphasize that these estimates are based on data available at time  $t$ , they could be denoted by a subscript  $t$ . However, to avoid cluttering the notation we omit this subscript and let it be understood in the sequel.

Figure 1: Daily commodity log-prices



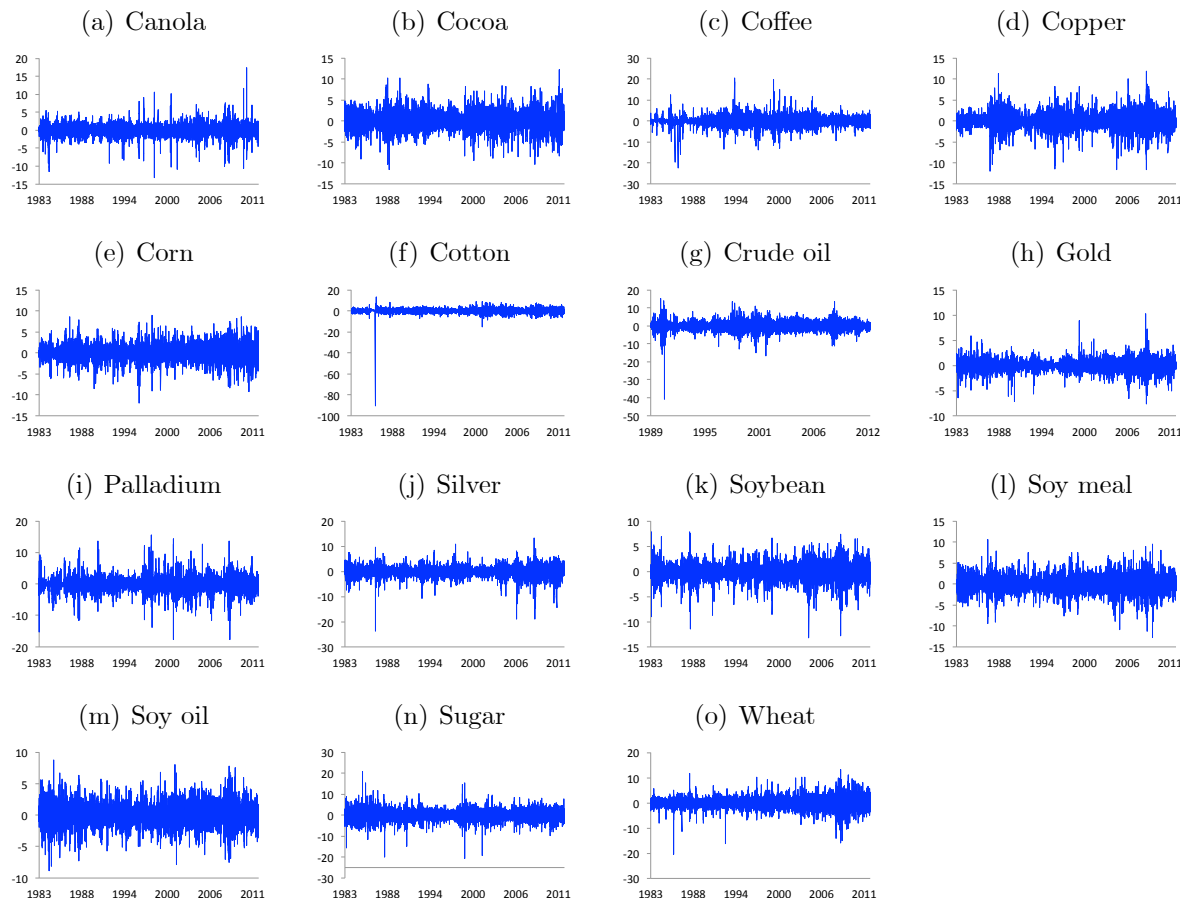
Notes: Each plot shows daily commodity log-prices. The blue lines are spot prices and the red lines are futures prices. The sample is 3/30/1983–10/12/12, except for crude oil where it is 7/24/89–10/12/12.

where  $\hat{X}_{s|t} = X_s$  for  $s \leq t$ . Then forecasts are calculated recursively from (10) for  $j = 1, 2, \dots, h$  to generate  $h$ -step ahead forecasts of log-prices,  $\hat{X}_{t+h|t}$ . Given these,  $h$ -step ahead forecasts of returns are calculated as in (9) using the recursively generated log-price forecasts on the right-hand side. We will apply the returns forecast (9) in our empirical analysis below for several forecast horizons,  $h$ , and we will compare with the similarly obtained returns forecast based on the CVAR model (6).

### 3 Preliminary analysis of the data

In our empirical analysis we have data on 15 commodity spot and futures markets. These are canola, cocoa, coffee, copper, corn, cotton, crude oil, gold, palladium, silver, soybean, soy meal, soy oil, sugar, and wheat. The data set is obtained from Narayan, Ahmed and Narayan (2014), and we refer the reader to this paper for details on the data such as the precise construction of the futures price series. The data are daily (5-day) time-series on spot and futures prices for each commodity starting on March 30, 1983, and ending October

Figure 2: Daily commodity spot returns



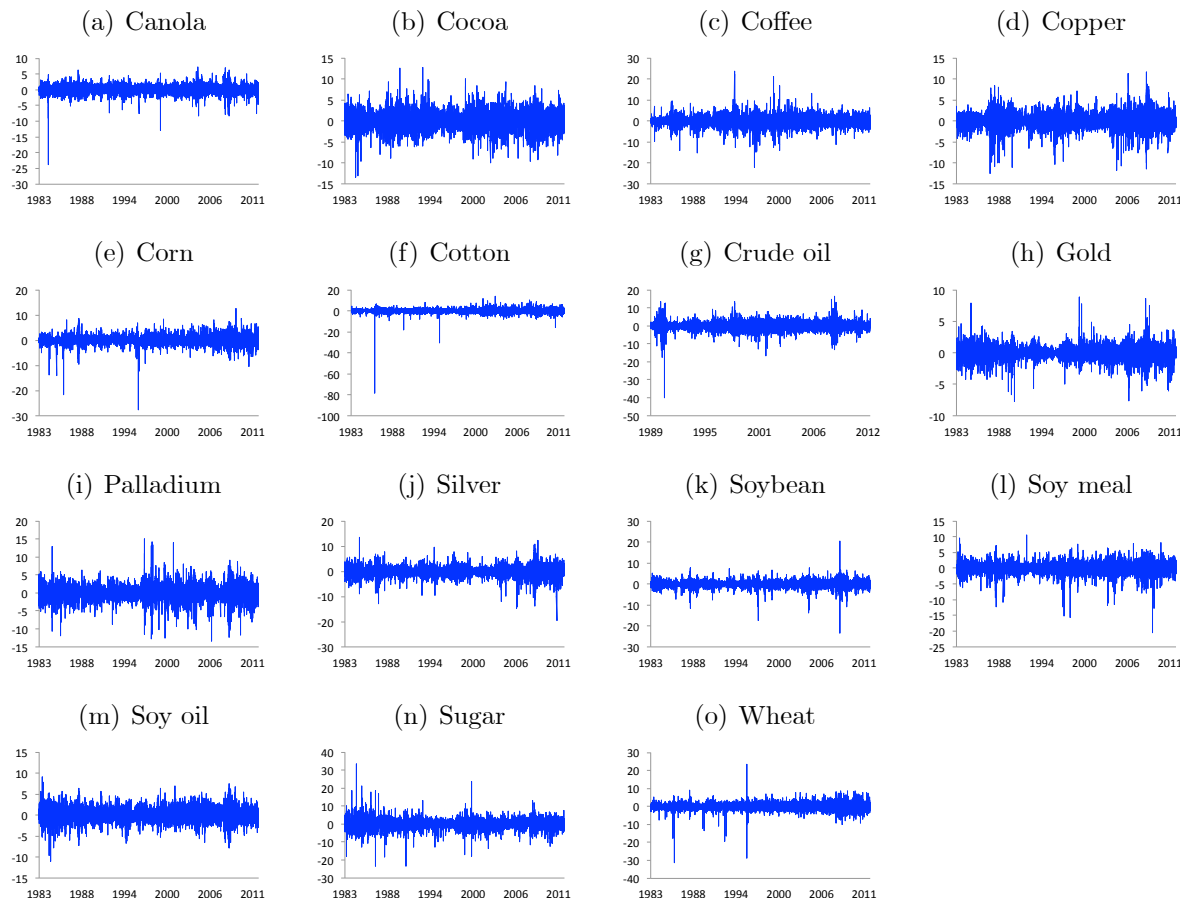
Notes: Each plot shows daily commodity spot returns. The sample is 3/30/1983–10/12/12, except for crude oil where it is 7/24/89–10/12/12.

12, 2012, for a total of 7708 observations. For crude oil the sample period is slightly shorter, starting on July 24, 1989, and ending October 12, 2012, yielding 6060 observations.

The commodity spot and futures log-price series and corresponding spot and futures returns are plotted in Figures 1–3. Returns are computed as the first difference of the log-price series, multiplied by 100 to yield a (continuously compounded) percentage return. Three tendencies emerge from the figures. First, the log-price series do not appear to have obvious time trends in Figure 1. This finding was supported by statistical tests in the estimation of our models below, where any trend included was statistically insignificant and therefore removed. Second, the spot and futures log-price series appear to move together in the long-run, supporting the notion that they are cointegrated. Third, there is clearly heterogeneity among the commodities. For example, the variance of returns in Figures 2 and 3 varies substantially across commodities.

In Table 1 we present some descriptive statistics for each of the commodity spot and futures returns series. These statistics confirm the tendencies observed in the figures. The sample mean returns for the spot market vary from 0.004% to 0.031% per day and in the futures market from –0.001% to 0.030% per day. A similar disparity in sample standard

Figure 3: Daily commodity futures returns



Notes: Each plot shows daily commodity futures returns. The sample is 3/30/1983–10/12/12, except for crude oil where it is 7/24/89–10/12/12.

deviation, skewness, and kurtosis is found. The implication here is that these specific commodities can potentially offer investors quite different risk-return trade-offs when considered from an investment portfolio point of view. The last column in the table reports the first-order autocorrelation coefficient for each series. These are all quite small, ranging from  $-0.085$  to  $0.095$ , but several are in fact statistically significant due to the large sample size. Nonetheless, the small autocorrelation coefficients suggest that all returns are clearly stationary  $I(0)$  processes, thus confirming our modeling choice of fixing  $d = 1$  in the FCVAR analysis, but also suggest that past returns alone will likely not be very good predictors of returns in the future.

## 4 Empirical results and economic significance

The section has three parts. In the first part, we present estimation results for the FCVAR and CVAR models based on the first 75% of the sample. We use a relatively large fraction of the total sample for estimation because the fractional models tend to require large sample

Table 1: Selected descriptive statistics of commodity spot and futures returns

Commodity	Spot market returns					Futures market returns				
	Mean	S.d.	Skew.	Kurt.	AC	Mean	S.d.	Skew.	Kurt.	AC
Canola	0.009	1.437	-0.282	13.968	0.002	0.009	1.295	-1.169	21.957	0.095
Cocoa	0.005	1.784	-0.018	6.625	-0.085	0.004	1.929	-0.007	5.962	0.004
Coffee	0.004	1.931	-0.042	17.243	0.003	0.003	2.279	0.012	11.817	0.000
Copper	0.020	1.656	-0.294	8.725	-0.039	0.021	1.722	-0.426	8.035	-0.032
Corn	0.011	1.664	-0.139	6.546	0.034	0.012	1.659	-1.003	21.212	0.053
Cotton	0.000	1.979	-12.412	570.578	0.014	-0.001	1.956	-8.933	347.753	0.065
Crude oil	0.031	2.341	-0.979	22.202	0.014	0.030	2.412	-0.891	20.238	-0.004
Gold	0.019	0.995	-0.131	10.097	-0.029	0.019	1.003	-0.103	10.153	-0.006
Palladium	0.022	1.958	-0.106	12.200	0.019	0.024	1.961	-0.234	8.802	0.082
Silver	0.015	1.794	-1.063	15.324	-0.016	0.015	1.802	-0.697	10.896	-0.009
Soybean	0.012	1.503	-0.568	7.950	-0.009	0.011	1.539	-0.934	20.509	0.018
Soy meal	0.012	1.699	-0.200	6.556	0.030	0.012	1.678	-0.904	12.272	0.056
Soy oil	0.013	1.570	0.020	5.137	0.020	0.013	1.509	-0.047	5.661	0.058
Sugar	0.015	2.231	-0.133	9.567	-0.039	0.014	2.514	0.218	15.249	-0.044
Wheat	0.011	2.049	-0.273	9.052	-0.019	0.011	1.857	-1.217	29.765	-0.021

Notes: This table reports selected descriptive statistics for the 15 commodity spot and futures return series. Specifically, the table reports the sample mean, standard deviation, skewness, kurtosis, and the first-order sample autocorrelation of returns (in percentage).

sizes for reliable estimation. In the second part, we present and discuss results for out-of-sample forecasting for the remaining part of the sample, based on the FCVAR and CVAR models. The third part of the results is about the economic significance of return forecasts. In other words, this is where we evaluate the forecasting models by asking: how beneficial are these forecasting models to investors?

## 4.1 Estimation results

Before we can estimate the FCVAR model and apply it in forecasting, we have to make some model selection choices. First, because we include the level parameter  $\mu$  in the FCVAR model, we apply estimation conditional on  $N = 0$  initial values for all our results, following Johansen and Nielsen (2014). Experimentation with different values of  $N$  showed little effect. For the CVAR model we used the formulation with a restricted constant term and applied estimation conditional on  $k + 1$  initial values, such that maximum likelihood estimation is reduced rank regression (Johansen, 1995). Second, we have to specify the lag length,  $k$ , in the vector error correction model (5). We apply several different statistics to carefully select the lag length, namely the Bayesian Information Criterion (BIC), the LR test statistic for significance of  $\Gamma_k$ , and univariate Ljung-Box Q tests (with  $h = 10$  lags) for each of the two residual series, in each case based on the model that has full rank  $r = p$ , where  $p$  is the dimension of the system. In addition, we examined the unrestricted estimates of  $b$  and  $\beta_2$  which, when the lag length is misspecified, will sometimes be very far from what should

Table 2: Estimation results for CVAR and FCVAR models of commodity prices

Commodity	CVAR model					FCVAR model						
	$k$	$-\hat{\beta}_2$	$-\hat{\rho}$	$\hat{\alpha}_{\perp,1}$	$\hat{\alpha}_{\perp,2}$	$k$	$\hat{b}$	$se(\hat{b})$	$-\hat{\beta}_2$	$\hat{\beta}'\hat{\mu}$	$\hat{\alpha}_{\perp,1}$	$\hat{\alpha}_{\perp,2}$
Canola	4	1.044	-0.251	0.048	0.952	3	0.424	0.068	1.291	-1.694	0.439	0.561
Cocoa	7	0.915	0.784	0.954	0.046	7	0.217	0.008	0.882	0.999	0.109	0.891
Copper	3	0.991	0.068	0.124	0.876	0	0.337	0.032	1.054	-0.177	0.056	0.944
Coffee	2	0.956	0.303	0.171	0.829	1	0.535	0.056	1.034	-0.077	0.437	0.563
Corn	1	1.057	-0.298	0.305	0.695	1	0.922	0.044	1.058	-0.302	0.327	0.673
Cotton	0	1.073	-0.349	0.108	0.891	0	1.025	0.033	1.063	-0.312	0.101	0.898
Crude oil	1	1.052	-0.236	0.729	0.270	1	0.699	0.052	1.078	-0.314	0.984	0.016
Gold	2	1.000	-0.003	0.226	0.773	0	0.758	0.018	1.003	-0.019	0.136	0.863
Palladium	3	1.002	0.000	0.333	0.666	3	0.987	0.034	1.000	0.012	0.323	0.676
Silver	3	0.993	0.042	0.231	0.768	2	0.632	0.048	0.995	0.027	0.190	0.810
Soybean	1	1.029	-0.201	0.628	0.371	1	0.868	0.035	1.027	-0.196	0.659	0.340
Soy meal	3	1.042	-0.211	0.254	0.745	4	0.614	0.052	0.994	0.021	0.005	0.994
Soy oil	3	1.169	-0.519	0.881	0.118	2	0.433	0.052	1.432	-1.230	0.045	0.954
Sugar	3	1.119	-0.219	0.861	0.138	2	0.471	0.081	1.021	-0.126	0.956	0.043
Wheat	1	1.110	-0.652	0.470	0.529	1	0.992	0.050	1.110	-0.653	0.471	0.528

Notes: This table reports estimation results for CVAR and FCVAR models applied to the first 75% of the sample of commodity spot and futures log-prices. The columns include lag-order ( $k$ ), estimates of the cointegration coefficient,  $-\hat{\beta}_2$ , and the restricted constant term,  $-\hat{\rho}$  or  $\hat{\beta}'\hat{\mu}$ , as well as the price discovery coefficients  $\hat{\alpha}_{\perp,1}$  and  $\hat{\alpha}_{\perp,2}$ , normalized to add to unity. For the FCVAR, we also report the estimated fractional parameter,  $\hat{b}$ , and its standard error,  $se(\hat{b})$ .

be expected. In particular, due to a non-identification issue in the FCVAR model with misspecified lag length, it is sometimes found that, e.g.,  $\hat{b} = 0.05$  or similar, see Johansen and Nielsen (2010, Section 2.3) for a theoretical discussion of this phenomenon. For each commodity, we first use the BIC as a rough guide to choose the lag length, and starting from there we find the nearest lag length which satisfies the criteria (i)  $\Gamma_k$  is significant based on the LR test, (ii) the unrestricted estimates of  $b$  and  $\beta_2$  are reasonable (very widely defined), and (iii) the Ljung-Box Q tests for serial correlation in the two residual series do not show signs of misspecification. Third, we select the cointegrating rank,  $r$ , by sequentially testing (using the LR trace statistic) the hypotheses  $r = 0, 1, 2$  until rejection. The critical values for the rank test are simulated case-by-case for the FCVAR model, and for the CVAR model we used Johansen (1995, Table 15.2). The selected cointegration rank is then the last non-rejected hypothesis.

Table 2 reports results<sup>3</sup> from estimation of CVAR and FCVAR models for commodity spot and futures log-prices, i.e. with  $X_t = (s_t, f_t)'$  in the notation of Section 2. For the estimation we use only the first 75% of the sample, and reserve the remainder for out-of-sample forecasting. This leaves  $T = 5781$  observations in the estimation sample, except for crude oil where  $T = 4545$ . Several conclusions emerge from Table 2. First of all, fewer

<sup>3</sup>Full estimation results are available from the authors upon request.

lags are usually needed in the lag-augmentation for the FCVAR model compared with the CVAR model. This is expected since the FCVAR model includes the additional parameter  $b$  to model dependence. However, although the FCVAR model in this way includes one additional parameter, for each additional lag included in the CVAR model, four additional parameters need to be estimated, and hence the CVAR model in most cases includes a larger number of parameters than the FCVAR model.

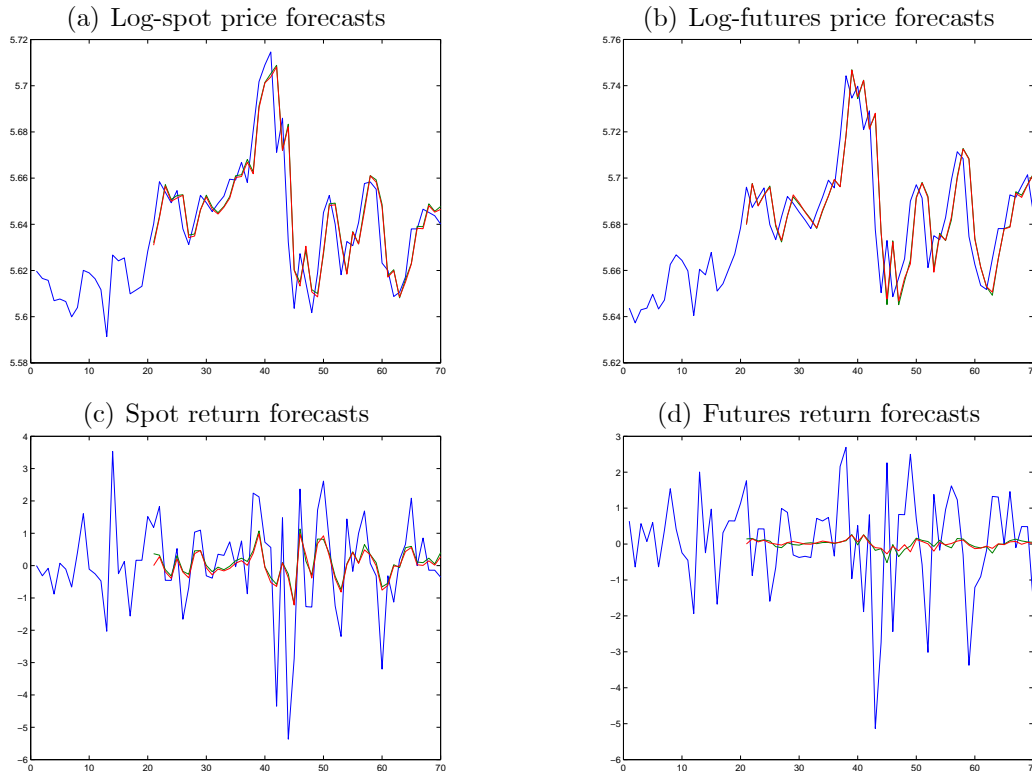
Secondly, in spite of the larger number of parameters in the CVAR model, the FCVAR model provides a much better statistical in-sample fit in most cases. This is seen through the extremely strong statistical significance of the difference of the fractional parameter,  $b$ , from unity, but also from the maximized log-likelihood and BIC values (not reported). The point estimates  $\hat{b}$  range from just over 0.2 to about 1.0, showing a wide variety of fractional cointegration properties across the different commodities. The estimates  $-\hat{\beta}_2$  of the cointegration coefficients are close to unity, as expected from an efficient markets hypothesis point of view, although they may deviate from unity still. The latter reflects a market in long-run backwardation or contango, see Figuerola-Ferretti and Gonzalo (2010) or Dolatabadi, Nielsen, and Xu (2014).

The final two columns for each model shows the price discovery coefficients,  $\hat{\alpha}_{\perp,1}$  and  $\hat{\alpha}_{\perp,2}$ , normalized to add to unity. The first is the proportion of price discovery in the spot market, and the second is the proportion of price discovery in the futures market. It is seen that, according to both the CVAR and FCVAR models, the futures market dominates price discovery for most commodities, as expected from theory (e.g., Hasbrouck, 1995). In particular, according to the CVAR model, the spot market is dominant in the price discovery process for five commodities (cocoa, crude oil, soybean, soy meal, and sugar), and according to the FCVAR for only three commodities (crude oil, soybean, and sugar). Thus, for the latter three commodities, there is strong statistical evidence of price discovery in the spot market.

Our findings on price discovery connects with the literature on price discovery in commodity markets; see, e.g., Figuerola-Ferretti and Gonzalo (2010), Dolatabadi, Nielsen, and Xu (2015), and the papers cited therein. In this literature there are several studies which show that price discovery is dominated by the futures market. Our study confirms this broad view, but at the same time points to three commodities where price discovery is not dominated by the futures market. Thus, while our results are consistent in spirit with the literature, suggesting that for most commodities futures market dictates price discovery, this evidence is not completely general—a finding consistent with Dolatabadi, Nielsen, and Xu (2015).

These price discovery results are not trivial outcomes because the dominance of one market over another indicates the market which has the highest information content. This has implications for investors because the market which has most information can then be used to forecast the market which has less information. In univariate regression-style forecasting models, one would then consider using past information from the dominant market to forecast prices or returns in the non-dominant market. However, our FCVAR (and CVAR) models are joint models for spot and futures price series, and will therefore forecast both series simultaneously and automatically take the price discovery information into account. In our forecasting analysis, we will explicitly consider the forecasting performance (in terms of statistical or economic significance) of the dominant market *vis-à-vis* the non-dominant

Figure 4: Daily canola forecasts



Notes: The four plots show forecasts as well as subsequently realized values for (a) log-spot prices, (b) log-futures prices, (c) spot returns, and (d) futures returns. Each plot shows the last 20 observations in the estimation sample together with the first 50 out-of-sample one-step ahead forecasts and the subsequently realized values. In each subplot there are three lines: The blue denotes the data observations, the red line are the recursive FCVAR forecasts, and the green line the recursive CVAR forecasts.

market to analyze their relative forecastability.

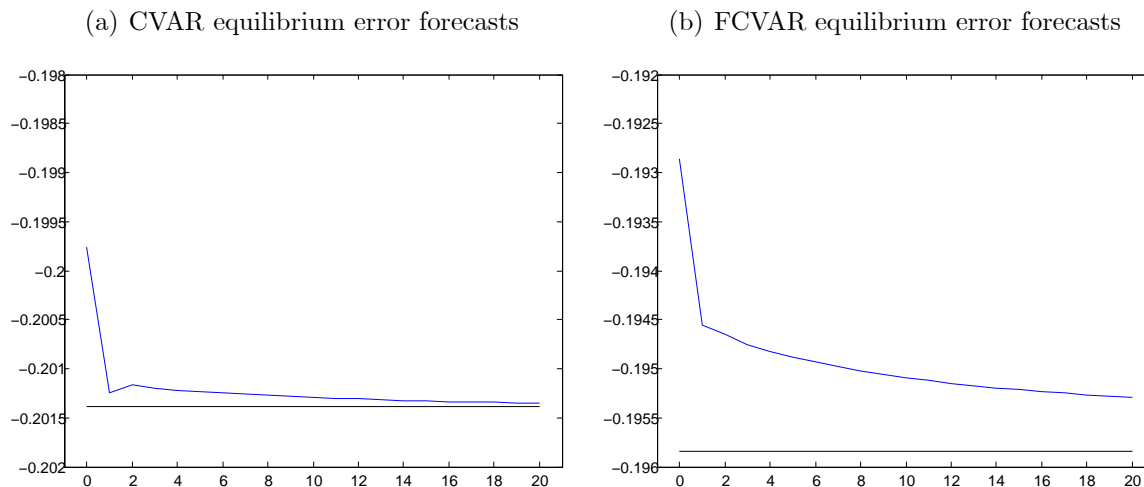
## 4.2 Statistical out-of-sample forecast comparison

In this subsection we move on to out-of-sample forecasting. Specifically, starting from the estimation results in the previous subsection, we recursively generate one-step ahead (daily) returns forecasts, re-estimating the model each period. We generate a total of 1927 out-of-sample returns forecasts in this manner (1515 for crude oil), to match the remaining 25% of our observations. This allows us to compare our forecasts with the actually observed out-of-sample returns series.

In Figure 4 we show forecasts as well as subsequently realized values for daily forecasting ( $h = 1$ ) of canola (a) log-spot prices, (b) log-futures prices, (c) spot returns, and (d) futures returns. Each subplot shows the last 20 observations in the estimation sample (i.e.  $X_t$  for  $t = 5762, \dots, 5781$ ) together with the first 50 out-of-sample one-step ahead forecasts ( $\hat{X}_{t+1|t}$  for  $t = 5781, \dots, 5830$ ) and the corresponding realized values ( $X_t$  for  $t = 5782, \dots, 5831$ ). In each subplot there are three lines: The blue denotes the data observations, the red line are the recursive FCVAR forecasts, and the green line the recursive CVAR forecasts. It is



Figure 5: Daily soybean equilibrium error forecasts



Notes: The two plots show the last observation of the model equilibrium error, i.e.  $\hat{\beta}'X_t$ , together with the recursive  $h$ -period ahead forecasts,  $\hat{\beta}'\hat{X}_{t+h|t}$  for  $h = 1, \dots, 20$ , generated from (a) the CVAR model and (b) the FCVAR model. In each panel, the mean is indicated by a horizontal line given by  $-\hat{\rho}'$  in the CVAR case and  $\hat{\beta}'\hat{\mu}$  in the FCVAR case.

noted that the log-price forecasts track the subsequently realized observations quite well, whereas the returns are clearly predicted much less accurately. This is, of course, expected from no-arbitrage theory of efficient markets. However, we do note a substantially better forecasting performance of spot returns compared with futures returns, which could be due to the fact that the latter is the dominant price discovery market for canola (see Table 2) and hence should be more difficult to predict.

The difference between FCVAR model forecasts and CVAR model forecasts shows most clearly in the forecasts of the equilibrium error series,  $\beta'X_t$ , which is depicted in Figure 5. Here we show the last (daily) observation on the model equilibrium error (i.e.  $\hat{\beta}'X_t$  for  $t = 5781$ ), together with the recursive  $h$ -period ahead forecasts of these,  $\hat{\beta}'\hat{X}_{t+h|t}$  for  $t = 5781$  and  $h = 1, \dots, 20$ , generated from (a) the CVAR model and (b) the FCVAR model. In each panel, the horizontal line indicates the mean of the equilibrium relation, given by  $-\hat{\rho}$  in the CVAR case and by  $\hat{\beta}'\hat{\mu}$  in the FCVAR case. This time, the forecasts are depicted for soybean instead of canola. This is because the difference shows more clearly for soybean since  $\hat{b}$  is quite different from one in the FCVAR model and the lag orders are smaller (see Table 2). The latter implies that the short-run dynamics do not influence the forecasts as much.

It is clear from Panel (a) of Figure 5 that the CVAR model equilibrium error forecasts return to their mean value very quickly, within just a couple of days. This reflects the  $I(0)$  nature of  $\beta'X_t$  in the CVAR model. On the other hand, the FCVAR model equilibrium error forecasts in Panel (b) of Figure 5 revert to their mean value only very slowly, reflecting the fractional integration nature of  $\beta'X_t$  in the FCVAR model. This feature of the multi-step ahead forecasts generated by the FCVAR model is likely to impact the forecasting performance of the FCVAR model relative to the CVAR model at longer horizons, which we will investigate below.

In Table 3 we report some out-of-sample forecast comparison statistics for the one-step ahead (daily) forecasts calculated from either the FCVAR model or the CVAR model. In

Table 3: Statistical out-of-sample forecast comparison at daily ( $h = 1$ ) horizon

Commodity	CW statistic		Relative RMSE		$R^2_{OOS}$ CVAR		$R^2_{OOS}$ FCVAR	
	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures
Canola	1.3125*	2.0496**	0.0358	-0.2574	0.4061	-0.0021	0.4057	0.0031
Cocoa	1.9748**	2.6484***	-0.1174	-0.1888	0.0137	-0.0047	0.0160	-0.0009
Copper	1.4817*	0.4015	-0.0665	0.0140	0.0018	0.0000	0.0031	-0.0002
Coffee	10.4002***	0.5068	-2.4600	0.0439	0.4386	0.0003	0.4659	-0.0005
Corn	0.6461	0.6700	-0.0141	-0.0147	0.0009	0.0030	0.0012	0.0033
Cotton	0.9068	-0.3591	-0.0155	0.0015	-0.0047	0.0020	-0.0044	0.0019
Crude oil	2.7330***	-0.1680	-0.2371	0.3108	-0.0067	-0.0005	-0.0019	-0.0067
Gold	1.4257*	-1.0901	-0.0743	0.1981	-0.0399	0.0426	-0.0383	0.0388
Palladium	2.5913***	0.5101	-0.0670	-0.0050	0.1140	0.0063	0.1152	0.0064
Silver	0.6733	1.3597*	0.0333	-0.0921	-0.0740	0.0204	-0.0747	0.0222
Soybean	-0.2719	1.8780**	0.0587	-0.3402	0.0103	-0.0080	0.0092	-0.0012
Soy meal	1.3338*	1.5361*	0.0178	-0.0749	0.0055	0.0055	0.0052	0.0070
Soy oil	0.4221	1.2459	-0.0659	-0.0670	0.0049	-0.0056	0.0062	-0.0043
Sugar	0.4645	1.9495**	0.1252	-0.1104	0.0511	-0.0015	0.0487	0.0008
Wheat	-0.5548	-0.4826	0.0090	0.0061	-0.0030	-0.0035	-0.0032	-0.0036

Notes: This table reports out-of-sample forecast comparison statistics for one-step ahead ( $h = 1$ ) returns forecasts. The statistics reported are the Clark and West (2007) test statistic, the relative RMSE, and the out-of-sample  $R^2$ . The CW statistic is asymptotically standard normally distributed and positive values favors the FCVAR model. Statistical significance (one-sided) at the 10%, 5%, and 1% level is denoted by \*, \*\*, and \*\*\*, respectively. The relative RMSE is calculated such that it favors FCVAR model when it is negative.

particular, we first report the Clark and West (2007, Section 2) test statistic for equal predictive ability, which is a modification of the Diebold and Mariano (1995) test statistic to account for the fact that the CVAR model is nested within the FCVAR model class<sup>4</sup> (see also Giacomini and White, 2006, for the point about nested model classes). The CW statistic is asymptotically standard normally distributed and favors the FCVAR model forecasts when it is positive; usually this is tested as a one-sided test, and we report one-sided significance using asterisks in the table. The next statistic is the relative root mean squared error (RMSE) of the two forecasts (from the FCVAR and CVAR models, respectively), and this is calculated such that negative values favor the FCVAR model forecasts. Finally, we report the out-of-sample  $R^2$  for both sets of forecasts.

The results in Table 3 tend to favor the FCVAR model, although the CW statistic is not always significant. Specifically, the CW statistic favors the FCVAR model forecasts in 13/15 spot markets and 12/15 futures markets, but it is significant at the 10% level or better in only 8/15 and 6/15 cases, respectively. The relative RMSE prefers the FCVAR model forecasts in 9/15 commodities for both the spot and futures markets. In our discussion of price discovery, we mentioned the possibility that it may be advantageous to an investor to

<sup>4</sup>We are grateful to Peter Extercate for bringing this point to our attention.

Table 4: Statistical out-of-sample forecast comparison at weekly ( $h = 5$ ) horizon

Commodity	CW statistic		Relative RMSE		$R^2_{OOS}$ CVAR		$R^2_{OOS}$ FCVAR	
	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures
Canola	2.6438***	0.9552	-1.2802	-0.1717	0.0873	0.0037	0.1106	0.0071
Cocoa	1.8787**	1.7968**	-0.5961	-0.5541	0.0749	-0.0004	0.0859	0.0107
Copper	2.0594**	-0.7140	-0.4752	0.0883	0.0057	0.0003	0.0151	-0.0015
Coffee	1.3259*	0.7738	-0.2470	0.0706	0.0604	0.0039	0.0650	0.0025
Corn	0.1980	0.6900	0.2187	-0.0623	-0.0007	-0.0020	-0.0051	-0.0007
Cotton	-1.3679	1.9168**	1.5911	-0.3662	-0.0032	0.0032	-0.0354	0.0105
Crude oil	0.5499	0.1949	-0.0779	1.5238	0.0077	0.0075	0.0093	-0.0230
Gold	-0.2532	-0.1753	0.0834	0.0278	-0.0116	0.0070	-0.0133	0.0064
Palladium	1.8570**	0.3850	-0.3942	0.0233	0.0309	0.0060	0.0385	0.0056
Silver	-0.2228	1.0889	0.0578	-0.2185	-0.0224	0.0067	-0.0236	0.0111
Soybean	-0.2149	0.4864	0.0437	0.1782	0.0068	-0.0034	0.0059	-0.0070
Soy meal	1.0070	-0.5697	-0.6652	-0.0150	0.0022	-0.0017	0.0154	-0.0014
Soy oil	1.4286*	0.6064	-0.1786	0.2042	0.0063	0.0022	0.0098	-0.0018
Sugar	0.2205	2.5928***	0.0251	-1.0260	-0.0066	0.0141	-0.0071	0.0342
Wheat	-0.2891	-0.6578	0.5382	0.2557	-0.0014	-0.0016	-0.0122	-0.0068

Notes: This table reports out-of-sample forecast comparison statistics for one-week ahead ( $h = 5$ ) non-overlapping returns forecasts. The statistics reported are the Clark and West (2007) test statistic, the relative RMSE, and the out-of-sample  $R^2$ . The CW statistic is asymptotically standard normally distributed and positive values favors the FCVAR model. Statistical significance (one-sided) at the 10%, 5%, and 1% level is denoted by \*, \*\*, and \*\*\*, respectively. The relative RMSE is calculated such that it favors FCVAR model when it is negative.

invest in the non-dominant price discovery market because returns may be easier to forecast in that market. Therefore, the best forecast model for the non-dominant market for each commodity is of particular importance. When considering only the non-dominant market, the CW statistics show that the FCVAR model forecasts are preferred in 13/15 cases, with 9 of these being significant at the 10% level or better.

Where the CW statistic and relative RMSE are both statistical measures of forecast comparison, the final columns in Table 3 report the out-of-sample  $R^2$  for both sets of return forecasts. It is seen from these columns that the forecastability of returns vary greatly across commodities, and also between spot and futures markets for the same commodity. Comparing the FCVAR and CVAR forecasts, the out-of-sample  $R^2$  values are quite similar, which is perhaps not surprising given the forecast comparison statistics in the previous columns. In most cases the out-of-sample  $R^2$  is higher for the spot market than for the futures market, confirming earlier results on the relative forecastability of returns from the two markets.

Before moving on to analyzing economic significance, we investigate the robustness of the Table 3 results by considering forecasting at longer horizons. Specifically, we consider forecasting at the weekly and monthly horizons based on daily data, i.e., horizons of  $h = 5$  and  $h = 21$  periods ahead. The motivation is that these horizons could correspond to an investor

Table 5: Statistical out-of-sample forecast comparison at monthly ( $h = 21$ ) horizon

Commodity	CW statistic		Relative RMSE		$R^2_{OOS}$ CVAR		$R^2_{OOS}$ FCVAR	
	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures
Canola	0.1632	0.6326	0.8606	-0.2895	-0.0022	-0.0154	-0.0195	-0.0095
Cocoa	-0.1954	1.2195	1.1389	-0.5767	0.0155	0.0148	-0.0071	0.0262
Copper	-0.1883	0.3597	0.1550	-0.0369	0.0021	-0.0021	-0.0010	-0.0014
Coffee	2.5300***	0.0716	-5.9748	0.5427	-0.1412	0.0201	-0.0089	0.0094
Corn	0.6703	1.4352*	-0.3193	-0.2064	0.0074	-0.0145	0.0137	-0.0103
Cotton	0.2154	-0.1786	-0.0357	0.0091	-0.0544	0.0158	-0.0536	0.0156
Crude oil	2.2915**	0.0788	-1.2836	0.9617	-0.0185	0.0178	0.0074	-0.0011
Gold	1.8804**	0.8343	-0.2065	-0.0943	-0.0072	-0.0128	-0.0030	-0.0109
Palladium	0.9367	1.6715**	-0.0712	-0.0954	0.0306	-0.0213	0.0320	-0.0194
Silver	1.1347	0.0810	-0.1563	-0.0018	0.0216	-0.0066	0.0247	-0.0066
Soybean	-1.1808	1.3633*	1.1201	-0.7527	0.0288	-0.0794	0.0070	-0.0632
Soy meal	0.3658	1.6460**	-0.9170	-1.2246	0.0654	-0.0448	0.0825	-0.0193
Soy oil	0.7456	1.0300	-0.0854	-0.8347	0.0085	-0.0238	0.0102	-0.0068
Sugar	-0.6585	-1.1273	0.3880	2.0149	0.0083	0.0609	0.0006	0.0227
Wheat	-0.3110	-1.5188	0.1094	0.2589	-0.0340	-0.0199	-0.0362	-0.0251

Notes: This table reports out-of-sample forecast comparison statistics for one-month ahead ( $h = 21$ ) non-overlapping returns forecasts. The statistics reported are the Clark and West (2007) test statistic, the relative RMSE, and the out-of-sample  $R^2$ . The CW statistic is asymptotically standard normally distributed and positive values favors the FCVAR model. Statistical significance (one-sided) at the 10%, 5%, and 1% level is denoted by \*, \*\*, and \*\*\*, respectively. The relative RMSE is calculated such that it favors FCVAR model when it is negative.

that rebalances the portfolio weekly or monthly and hence needs only forecasts at those horizons. With the same motivation, therefore, we consider only non-overlapping forecasts. That is, the forecast is calculated every  $h$  periods (days) for  $h$  steps ahead. This yields a total of 385 one-week ahead ( $h = 5$ ) forecasts and 91 one-month ahead ( $h = 21$ ) forecasts of  $h$ -day returns, except for crude oil where we have 303 and 72 forecasts, respectively.

The out-of-sample forecasting results for these longer horizons are presented in Tables 4 (weekly,  $h = 5$ ) and 5 (monthly,  $h = 21$ ), which are both laid out exactly like Table 3. The results for out-of-sample forecast comparisons using statistical measures are similar for these horizons as for the daily horizon presented in Table 3. For the weekly horizon, the FCVAR model is preferred to the CVAR model by the CW statistic in 10/15 (spot market) and 11/15 (futures market) cases, and for the monthly horizon it is preferred in 10/15 (spot) and 12/15 (futures) cases. Focusing on the non-dominant price discovery market, that is, the spot market for all commodities except crude oil, soybean, and sugar, the FCVAR model is preferred to the CVAR model in 11/15 (one-week ahead forecasts) and 11/15 (one-month ahead forecasts) cases. For the out-of-sample  $R^2$ , the general tendency is, not surprisingly, that it is smaller for the longer horizon forecasts, but it remains quite similar for FCVAR and CVAR forecasts.

### 4.3 Mean-variance utility function based profits

Another possible metric of comparison and evaluation of forecasting performance is economic—rather than purely statistical—significance. That is, whether the forecasts can generate significant returns when incorporated into a dynamic trading strategy. In doing so, we also investigate whether the improved statistical in-sample fit and forecast performance of the FCVAR model relative to the CVAR model translate into economically significant profits, and whether the relatively strong forecastability of commodity returns in some markets compared with others translate into different economic significance across markets.

The economic significance question is important because the statistical superiority of a model over its competitors is just a first step in informing investors. An equally important question is: how can investors benefit from a statistically superior model? This question is directly based on the ability to forecast returns, that is, whether an investor can use forecasts from the model to devise successful trading strategies and make superior profits compared with forecasts from alternative models. In other words, these trading strategies should deliver statistically significant and meaningful profits.

In the stock return forecasting literature, a mean-variance utility function is typically utilized to derive a dynamic trading strategy for investors. We assume that the investor rebalances the portfolio every  $h$  days, where we analyze in particular  $h = 1$ ,  $h = 5$ , and  $h = 21$ , corresponding to daily, weekly, and monthly rebalancing, respectively. The investor can invest in two assets: the risk-free asset with return from period  $t$  to period  $t + h$  given by  $r_{f,t+h}$  and the risky asset with return  $r_{t+h}$ . In our setting, the risky asset is a commodity (either a spot or a futures position). The investor then forms a portfolio with weight  $w_{t+h}$  on the risky asset and this portfolio yields a return of

$$r_{p,t+h} = w_{t+h}r_{t+h} + (1 - w_{t+h})r_{f,t+h} = w_{t+h}(r_{t+h} - r_{f,t+h}) + r_{f,t+h}. \quad (11)$$

Following Marquering and Verbeek (2004) and Campbell and Thompson (2008), among others, the weight on the risky asset is determined by maximizing the investor's mean-variance utility function,

$$U(r_{p,t+h}) = E_t(r_{p,t+h}) - \frac{1}{2}\gamma Var_t(r_{p,t+h}), \quad (12)$$

where  $E_t(\cdot)$  and  $Var_t(\cdot)$  denote conditional mean and variance given information at time  $t$  and  $\gamma$  is the investor's coefficient of relative risk aversion. Maximizing  $U(r_{p,t+h})$  with respect to  $w_{t+h}$  yields the optimal weight

$$w_{t+h}^* = \frac{E_t(r_{t+h}) - r_{f,t+h}}{\gamma Var_t(r_{t+h})}, \quad (13)$$

noting that the risk-free rate carries no risk and hence does not contribute to the variance of the portfolio. Following the literature, we further constrain the optimal weight, and specifically we impose  $w_{t+h}^* \geq 0$  (no short-selling) and  $w_{t+h}^* \leq 1$  (no borrowing/leverage).

To summarize the calculation of profitability of the returns forecasts, that is their economic significance, three steps are performed: (i) forecast returns, (ii) compute portfolio weights, and (iii) calculate portfolio returns. The first step involves calculating (9) to forecast commodity (spot or futures) returns at each time period, as explained in Section 2.4.

In the second step we calculate portfolio weights from (13) given the returns forecasts  $E_t(r_{t+h}) = \hat{r}_{t+h|t}$ . For the risk-free return  $r_{f,t+h}$  we use the return on the US three-month Treasury bill, which is assumed known at time  $t$  (since it is risk-free). The risk aversion coefficient is set at six, corresponding to an investor that takes a medium level of risk, and for robustness we also consider a high risk aversion investor ( $\gamma = 12$ ) and a low risk aversion investor ( $\gamma = 3$ ). Finally, following the literature, we estimate the time-varying variance of the risky asset by a 30-period moving window sample variance of its returns.

Third, given the portfolio weights, portfolio returns are computed from (11) for each period. These are then aggregated across time and reported as an annualized portfolio return to facilitate comparison across different values of the rebalancing horizon,  $h$ .

In Table 6 we report the annualized average excess portfolio return for commodity spot markets. The returns are reported as excess returns above and beyond the risk-free rate (the return on which was 1.753% per annum over the out-of-sample forecasting period). The results are presented for a medium risk investor; that is, with risk aversion coefficient  $\gamma = 6$ . We report results for daily ( $h = 1$ ), weekly ( $h = 5$ ), and monthly ( $h = 21$ ) rebalancing and for forecasts based on both the CVAR and FCVAR models. For each commodity and each rebalancing horizon, we conduct a statistical test of the null hypothesis that excess portfolio return is zero against the two-sided alternative that excess portfolio return is different from zero and report the  $t$ -statistic for this test in parentheses. Statistical significance of the  $t$ -test at the 10%, 5%, and 1% level is denoted by one, two, and three asterisks, respectively. We interpret this as a (statistical) test of economic significance of the return forecasts.

The results in Table 6 show several clear tendencies. First of all, excess portfolio returns are positive for all commodities with daily rebalancing and significantly different from zero for 11 of 15 commodities. With longer-horizon rebalancing, excess returns are positive for all but a few commodities, and the negative returns are not significantly different from zero. The excess returns range from nearly zero and insignificant to over 8% per annum and very strongly statistically significant (with  $t$ -statistics of over 14). Compared to the very small average returns on the commodities reported in Table 1 and the average annual return of 1.753% on the risk-free asset, some of the excess returns in Table 6 are impressively large. Secondly, as expected, the highest returns are found with daily rebalancing mainly because there are more opportunities to rebalance, and returns for longer-horizon rebalancing are smaller, although often significantly different from zero. We also note that, if returns are approximately multiplied by a factor  $h$ , then variances will approximately be multiplied by a factor  $h^2$ , and from the optimal weight equation (13), the weight on the risky asset will then be multiplied by a factor  $h^{-1}$ . Thus, with longer horizons, weights will tend to be shifted towards the risk-free asset compared with the daily horizon, even for a fixed value of the risk coefficient, and this produces lower excess portfolio returns on average.

Comparing portfolio returns using weights calculated from CVAR and FCVAR based forecasts show that each model produces higher returns in roughly half of all cases, with a slight edge to the FCVAR model. However, in this comparison it makes sense to make note of the margin by which one outperforms the other, which is simple in this framework by considering, for example, an equal-weighted portfolio of all 15 commodities. In that case, the FCVAR produces an average excess portfolio return of 1.608%, 0.129%, and 0.015% per annum for daily, weekly, and monthly rebalancing, respectively. In comparison, the CVAR model produces returns of 1.525%, 0.076%, and 0.023%, respectively. Quite surprisingly,

Table 6: Annualized excess portfolio returns for commodity spot markets,  $\gamma = 6$ 

Commodity	Daily		Weekly		Monthly	
	CVAR	FCVAR	CVAR	FCVAR	CVAR	FCVAR
Canola	7.809*** (14.144)	7.343*** (13.783)	0.351*** (5.096)	0.639*** (5.470)	0.050 (0.743)	0.033 (0.793)
Coffee	7.311*** (12.880)	8.196*** (13.621)	0.195*** (3.988)	0.326*** (4.504)	0.012 (0.637)	0.013 (0.667)
Cocoa	1.378*** (4.391)	1.326*** (4.396)	0.233*** (3.811)	0.273*** (3.011)	-0.003 (-0.358)	-0.015 (-1.079)
Copper	0.171** (2.162)	0.208** (2.143)	0.027 (1.466)	0.114** (2.343)	0.020* (1.919)	0.018 (0.919)
Corn	0.279*** (2.817)	0.244*** (2.769)	0.015 (0.842)	0.065 (1.074)	0.091* (1.906)	0.071** (1.999)
Cotton	0.032 (0.430)	0.027 (0.391)	0.004 (0.235)	0.017 (0.253)	-0.026 (-0.575)	-0.024 (-0.559)
Crude oil	0.322*** (3.964)	0.336*** (4.119)	0.039 (1.260)	0.046 (1.558)	-0.008 (-0.798)	0.001 (0.333)
Gold	0.351 (1.255)	0.387 (1.306)	-0.012 (-0.247)	-0.023 (-0.395)	0.006 (0.756)	0.008 (0.872)
Palladium	2.390*** (8.029)	3.331*** (7.999)	0.179** (2.565)	0.212*** (2.623)	0.015 (0.984)	0.016 (1.089)
Silver	0.247 (0.994)	0.250 (0.945)	-0.006 (-0.232)	-0.002 (-0.056)	0.018** (2.085)	0.020** (2.181)
Soybean	0.303*** (2.816)	0.246*** (2.674)	0.026 (1.455)	0.029 (1.051)	0.038 (1.163)	-0.001 (-0.067)
Soy meal	0.201** (1.980)	0.166* (1.791)	0.012 (0.833)	0.061 (1.405)	0.058* (1.793)	0.021 (1.429)
Soy oil	0.374*** (2.978)	0.384*** (3.089)	0.022 (1.042)	0.076 (1.409)	0.020 (0.525)	0.023 (0.681)
Sugar	1.684*** (6.593)	1.643*** (6.504)	0.051 (0.963)	0.056 (1.100)	0.006 (0.891)	0.005 (0.430)
Wheat	0.028 (0.510)	0.029 (0.519)	-0.001 (-0.054)	0.042 (0.754)	0.043 (0.661)	0.040 (0.587)
Average	1.525	1.608	0.076	0.129	0.023	0.015

Notes: This table reports annualized average excess portfolio percentage returns for commodity spot markets for a medium risk investor, i.e., an investor with risk coefficient  $\gamma = 6$ . The results are reported for daily ( $h = 1$ ), weekly ( $h = 5$ ), and monthly ( $h = 21$ ) rebalancing and for forecasts based on both the CVAR and FCVAR models. In parentheses we report  $t$ -statistics for the null of zero excess returns, and statistical significance (two-sided) at the 10%, 5%, and 1% level is denoted by \*, \*\*, and \*\*\*, respectively. The final row is the average return across all commodities.

given our findings in the statistical comparisons, the CVAR model outperforms the FCVAR model with monthly rebalancing, although the difference is very small. On the other hand, the FCVAR model outperforms the CVAR model with daily and weekly rebalancing, and substantially so in the latter case.

The results for the futures markets are presented in Table 7, which is laid out as in Table 6. These results show much smaller excess returns, which reflects the earlier finding from Figure 4 and Tables 3–5 that futures returns are much more difficult to forecast than spot returns. It is also not too surprising given the findings from Table 2 that price discovery is primarily in the futures market for all commodities except crude oil, soybean, and sugar. For these three commodities, Tables 6 and 7 suggest that portfolio returns from the trading strategy may be higher in the futures market at the weekly and monthly rebalancing horizons. For all other commodities, except gold, silver, and wheat, returns are higher in the spot markets than the futures markets (with daily rebalancing).

More generally, the results in Table 7 are very similar to those found in Table 6. We still find many highly significant and positive excess returns for the futures markets, and the only negative returns are insignificant. The comparison results for an equal-weighted portfolio of all 15 commodities show the same conclusions as for the spot market. In this case, for the futures market, the FCVAR model generates average portfolio returns of 0.152%, 0.046%, and 0.008% compared with 0.152%, 0.015%, and 0.011% for the CVAR model for daily, weekly, and monthly rebalancing, respectively.

#### 4.4 Alternative risk aversion coefficients

In this section, we investigate the robustness of our findings to the choice of risk aversion coefficient,  $\gamma$ . So far we have considered an investor who takes a medium level of risk, and specifically has  $\gamma = 6$ . The choice of  $\gamma$  has obvious implications for the portfolio returns via the calculation of weights in (13), from which it is noticed that a low risk aversion investor ( $\gamma = 3$ ) will place a higher weight on the risky asset and vice versa for the high risk aversion investor ( $\gamma = 12$ ). A natural question is whether portfolio returns constructed as above are still statistically and economically significant when the investor is more or less risk averse.

In our final set of results, presented in Table 8, we report annualized excess portfolio returns for commodity spot and futures markets with daily rebalancing ( $h = 1$ ) for both a low risk investor ( $\gamma = 12$ ) and a high risk investor ( $\gamma = 3$ ). The results presented here support our earlier findings. First, excess portfolio returns are positive in all cases for the spot market and are negative (but insignificant) in only 4/30 cases for the futures market, and those four cases are all based on forecasts from the CVAR model. Excess returns from the trading strategy based on the FCVAR model forecasts are positive in all cases in Table 8.

Second, and not surprisingly, average portfolio returns are higher for the high risk investor than for the low risk investor. In fact, the high risk investor reaches average excess portfolio returns of over 16% per annum in some cases. Comparing again with the average (daily) returns for the commodity markets shown in Table 1, these returns are quite impressive.

Third, returns from the spot market are once more found to be much higher on average than returns from the futures market. As above, we ascribe this to the higher forecastability of spot returns compared with futures returns and to the price discovery in the futures



Table 7: Annualized excess portfolio returns for commodity futures markets,  $\gamma = 6$ 

Commodity	Daily		Weekly		Monthly	
	CVAR	FCVAR	CVAR	FCVAR	CVAR	FCVAR
Canola	0.349*** (4.044)	0.374*** (3.955)	0.031** (2.074)	0.098** (2.427)	-0.015 (-1.154)	0.002 (0.093)
Coffee	0.035 (0.745)	0.059 (1.104)	0.014 (1.235)	0.084* (1.796)	0.031 (0.847)	0.025 (0.784)
Cocoa	-0.030 (-0.709)	0.001 (0.027)	-0.008 (-0.960)	0.022 (0.495)	0.008 (0.827)	0.004 (0.200)
Copper	-0.004 (-0.127)	0.002 (0.961)	-0.002 (-0.354)	0.001 (0.952)	0.000 (-0.647)	0.000 (-0.106)
Corn	0.210*** (3.259)	0.228*** (3.339)	0.010 (1.042)	0.025* (1.719)	0.007 (0.993)	0.010 (1.141)
Cotton	0.025*** (3.078)	0.025*** (3.159)	0.001** (2.402)	0.015*** (2.652)	0.004** (2.342)	0.004** (2.333)
Crude oil	0.170 (0.813)	0.148 (0.760)	0.051 (1.185)	0.183 (1.272)	0.108 (1.133)	0.062 (1.140)
Gold	0.554*** (5.237)	0.456*** (4.803)	0.023** (1.989)	0.027** (2.469)	-0.002 (-0.177)	-0.001 (-0.142)
Palladium	0.260*** (3.286)	0.265*** (3.345)	0.014 (1.211)	0.020 (1.311)	-0.001 (-0.155)	0.000 (-0.061)
Silver	0.351*** (5.166)	0.285*** (5.279)	0.016 (0.975)	0.028 (0.943)	-0.007 (-0.543)	-0.006 (-0.787)
Soybean	0.003 (0.073)	0.028 (0.590)	0.011 (1.238)	0.029 (1.064)	-0.008 (-0.586)	-0.010 (-0.592)
Soy meal	0.127 (1.386)	0.140 (1.422)	0.003 (0.188)	-0.011 (-0.374)	-0.026 (-1.467)	-0.005 (-0.611)
Soy oil	0.038 (0.517)	0.072 (0.942)	0.024* (1.851)	0.033 (1.212)	-0.006 (-0.435)	-0.012 (-0.529)
Sugar	0.117 (1.106)	0.127 (1.358)	0.037** (2.562)	0.133*** (3.005)	0.062*** (2.675)	0.037* (1.894)
Wheat	0.077** (1.967)	0.075** (1.978)	-0.003 (-0.341)	0.003 (0.404)	0.004 (0.745)	0.004 (0.728)
Average	0.152	0.152	0.015	0.046	0.011	0.008

Notes: This table reports annualized average excess portfolio percentage returns for commodity futures markets for a medium risk investor, i.e., an investor with risk coefficient  $\gamma = 6$ . The results are reported for daily ( $h = 1$ ), weekly ( $h = 5$ ), and monthly ( $h = 21$ ) rebalancing and for forecasts based on both the CVAR and FCVAR models. In parentheses we report  $t$ -statistics for the null of zero excess returns, and statistical significance (two-sided) at the 10%, 5%, and 1% level is denoted by \*, \*\*, and \*\*\*, respectively. The final row is the average return across all commodities.

Table 8: Annualized daily excess portfolio returns for alternative risk coefficients

Commodity	Risk coefficient $\gamma = 3$				Risk coefficient $\gamma = 12$			
	Spot market		Futures market		Spot market		Futures market	
	CVAR	FCVAR	CVAR	FCVAR	CVAR	FCVAR	CVAR	FCVAR
Canola	15.618*** (14.144)	14.687*** (13.783)	0.699*** (4.044)	0.749*** (3.955)	3.904*** (14.144)	3.672*** (13.783)	0.175*** (4.044)	0.187*** (3.955)
Coffee	14.621*** (12.880)	16.393*** (13.621)	0.069 (0.745)	0.118 (1.104)	3.655*** (12.880)	4.098*** (13.621)	0.017 (0.745)	0.029 (1.104)
Cocoa	2.756*** (4.391)	2.652*** (4.396)	-0.060 (-0.709)	0.002 (0.027)	0.689*** (4.391)	0.663*** (4.396)	-0.015 (-0.709)	0.001 (0.027)
Copper	0.343** (2.162)	0.417** (2.143)	-0.007 (-0.127)	0.004 (0.961)	0.086** (2.162)	0.104** (2.143)	-0.002 (-0.127)	0.001 (0.961)
Corn	0.557*** (2.817)	0.488*** (2.769)	0.420*** (3.259)	0.457*** (3.339)	0.139*** (2.817)	0.122*** (2.769)	0.105*** (3.259)	0.114*** (3.339)
Cotton	0.064 (0.430)	0.053 (0.391)	0.050*** (3.078)	0.049*** (3.159)	0.016 (0.430)	0.013 (0.391)	0.013*** (3.078)	0.012*** (3.159)
Crude oil	0.643*** (3.964)	0.672*** (4.119)	0.340 (0.813)	0.297 (0.760)	0.161*** (3.964)	0.168*** (4.119)	0.085 (0.813)	0.074 (0.760)
Gold	0.701 (1.255)	0.774 (1.306)	1.109*** (5.237)	0.912*** (4.803)	0.175 (1.255)	0.193 (1.306)	0.277*** (5.237)	0.228*** (4.803)
Palladium	6.580*** (8.029)	6.662*** (7.999)	0.520*** (3.286)	0.531*** (3.345)	1.645*** (8.029)	1.666*** (7.999)	0.130*** (3.286)	0.133*** (3.345)
Silver	0.495 (0.994)	0.500 (0.945)	0.702*** (5.166)	0.570*** (5.279)	0.124 (0.994)	0.125 (0.945)	0.176*** (5.166)	0.143*** (5.279)
Soybean	0.606*** (2.816)	0.492*** (2.674)	0.005 (0.073)	0.057 (0.590)	0.152*** (2.816)	0.123*** (2.674)	0.001 (0.073)	0.014 (0.590)
Soy meal	0.403** (1.980)	0.332* (1.791)	0.253 (1.386)	0.281 (1.422)	0.101** (1.980)	0.083* (1.791)	0.063 (1.386)	0.070 (1.422)
Soy oil	0.748*** (2.978)	0.768*** (3.089)	0.076 (0.517)	0.144 (0.942)	0.187*** (2.978)	0.192*** (3.089)	0.019 (0.517)	0.036 (0.942)
Sugar	3.369*** (6.593)	3.285*** (6.504)	0.234 (1.106)	0.255 (1.358)	0.842*** (6.593)	0.821*** (6.504)	0.059 (1.106)	0.064 (1.358)
Wheat	0.057 (0.510)	0.058 (0.519)	0.154** (1.967)	0.151** (1.978)	0.014 (0.510)	0.015 (0.519)	0.039** (1.967)	0.038** (1.978)
Average	3.171	3.216	0.304	0.305	0.793	0.804	0.076	0.076

Notes: This table reports annualized average excess portfolio percentage returns for commodity spot and futures markets for daily ( $h = 1$ ) rebalancing. The results are reported for a low risk investor ( $\gamma = 12$ ) and a high risk investor ( $\gamma = 3$ ) and for forecasts based on both the CVAR and FCVAR models. In parentheses we report  $t$ -statistics for the null of zero excess returns, and statistical significance (two-sided) at the 10%, 5%, and 1% level is denoted by \*, \*\*, and \*\*\*, respectively. The final row is the average return across all commodities.

market making returns from the futures market harder to forecast. In particular, we notice from Table 1 that there is no substantial difference between average returns in the spot and futures markets, so the different profits in the two markets cannot be attributed simply to differences in the unconditional average return in the two markets.

Finally, comparing excess returns based on CVAR and FCVAR forecasts using an equal-weighted portfolio of all 15 commodities shows that the FCVAR model (CVAR model) produces average excess returns of 3.216% (3.171%) and 0.305% (0.304%) in the spot and futures market, respectively, for the low risk aversion investor. For the high risk aversion investor, excess returns for the trading strategy based on forecasts from the FCVAR model (CVAR model) are 0.804% (0.793%) in the spot market and 0.076% (0.076%) in the futures market. Thus, the FCVAR model generates the highest average return, in terms of an equal-weighted portfolio of all 15 commodities. Taken together, therefore, our result that FCVAR-based forecasts generate higher profits than CVAR-based forecasts on average across the commodities holds regardless of the risk aversion factor.

The overall implication is that our evidence that the FCVAR is a statistically superior model also extends to its economic importance. Therefore, both statistically and economically, the FCVAR model offers investors a better guide to undertaking investment portfolio decisions.

## 5 Concluding remarks

This paper is about understanding the link between statistical models of forecasting for commodity returns and their implications for investors. Identifying suitable forecasting models for asset returns is at the forefront of research in asset pricing. This is so because the accuracy of forecasts have direct implications for investors' decision making, particularly with regard to portfolio choice. In this paper we take a step in this direction by proposing an FCVAR model for forecasting commodity spot and futures returns, based on recent empirical evidence of fractional cointegration in commodity spot and futures markets. We compare the FCVAR model with the more widely used CVAR model. We derive the best linear predictor forecast for the FCVAR model and perform an out-of-sample forecast comparison with forecasts from the more standard model. In our empirical analysis to 15 commodity spot and futures markets, the fractional model is found to be superior in terms of statistical in-sample fit and also out-of-sample forecasting, although the out-of-sample forecast comparison test statistics are not always significant.

In terms of economic significance of the forecasts, we analyze this through a dynamic trading strategy based on a portfolio with weights derived from a mean-variance utility function. This analysis leads to statistically significant and economically meaningful profits in the commodity markets, and reveals the superiority of the fractional model compared with the non-fractional model in terms of economic significance; that is in terms of average portfolio return. Our results are robust on several fronts. First, our out-of-sample forecasting evaluation exercise applies a number of statistical metrics. Second, we show that forecasts from FCVAR model are significant for many commodities regardless of the forecasting horizon, i.e. regardless of how often the investor rebalances the portfolio. Third, we show that our results on profitability are robust to an investor's level of risk aversion as measured by

the coefficient of relative risk aversion that enters in the utility function and hence in the calculation of portfolio weights.

## References

- [1] Baillie, R.T., and Bollerslev, T. (1994) The long memory of the forward premium, *Journal of International Money and Finance*, 13, 565–571.
- [2] Campbell, J.Y., and Thompson, S.B., (2008) Predicting the equity premium out of sample: can anything beat the historical average?, *Review of Financial Studies*, 21, 1509–1531.
- [3] Clark, T.E., and West, K.D., (2007) Approximately normal tests for equal predictive accuracy in nested models, *Journal of Econometrics*, 138, 291–311.
- [4] Coakley, J., Dollery, J., and Kellard, N., (2011) Long memory and structural breaks in commodity futures markets, *Journal of Futures Markets*, 31, 1076–1113.
- [5] Diebold, F.X., and Mariano, R.S., (1995) Comparing predictive accuracy, *Journal of Business and Economic Statistics*, 13, 253–263.
- [6] Dolatabadi, S., Nielsen, M.Ø., and Xu, K., (2014) A fractionally cointegrated VAR model with deterministic trends and application to commodity futures markets, QED working paper 1327, Queen’s University.
- [7] Dolatabadi, S., Nielsen, M.Ø., and Xu, K., (2015) A fractionally cointegrated VAR analysis of price discovery in commodity futures markets, *Journal of Futures Markets*, forthcoming.
- [8] Figuerola-Ferretti, I., and Gonzalo, J., (2010) Modelling and measuring price discovery in commodity markets, *Journal of Econometrics*, 158, 95–107.
- [9] Giacomini, R., and White, H., (2006) Tests of conditional predictive ability, *Econometrica*, 74, 1545–1578.
- [10] Gonzalo, J., and Granger, C.W.J., (1995) Estimation of common long-memory components in cointegrated systems, *Journal of Business and Economic Statistics*, 13, 27–36.
- [11] Graham-Higgs, J., Rambaldi, A., and Davidson, B., (1999) Is the Australian wool futures market efficient as a predictor of spot prices, *Journal of Futures Markets*, 19, 565–582
- [12] Hasbrouck, J., (1995) One security, many markets: determining the contribution to price discovery, *Journal of Finance*, 50, 1175–1199.
- [13] Jensen, A.N., and Nielsen, M.Ø., (2014) A fast fractional difference algorithm, *Journal of Time Series Analysis*, 35, 428–436.

- [14] Johansen, S., (1995) *Likelihood-based inference in cointegrated vector autoregressive models*. New York: Oxford University Press.
- [15] Johansen, S., (2008) A representation theory for a class of vector autoregressive models for fractional processes, *Econometric Theory*, 24, 651–676.
- [16] Johansen, S., and Nielsen, M.Ø., (2010) Likelihood inference for a nonstationary fractional autoregressive model, *Journal of Econometrics*, 158, 51–66.
- [17] Johansen, S., and Nielsen, M.Ø., (2012) Likelihood inference for a fractionally cointegrated vector autoregressive model, *Econometrica*, 80, 2667–2732.
- [18] Johansen, S., and Nielsen, M.Ø., (2014) The role of initial values in conditional sum-of-squares estimation of nonstationary fractional time series models, QED working paper 1300, Queen’s University.
- [19] Lien, D., and Tse, Y.K., (1999) Fractional cointegration and futures hedging, *Journal of Futures Markets*, 19, 457–474.
- [20] Marquering, W., and Verbeek, M., (2004) The economic value of predicting stock index returns and volatility, *Journal of Financial and Quantitative Analysis*, 39, 407–429.
- [21] Maynard, A., and Phillips, P.C.B., (2001) Rethinking an old empirical puzzle: econometric evidence on the forward discount anomaly, *Journal of Applied Econometrics*, 16, 671–708.
- [22] Miffre, J., and Rallis, G., (2007) Momentum strategies in commodity futures markets, *Journal of Banking and Finance*, 31, 1863–1886.
- [23] Narayan, P.K., Narayan, S., and Sharma, S.S., (2013) An analysis of commodity markets: what gain for investors?, *Journal of Banking and Finance*, 37, 3878–3889.
- [24] Narayan, P.K., Ahmed, H.A., and Narayan, S., (2014) Do momentum-based trading strategies work in the commodity futures markets? *Journal of Futures Markets*, forthcoming.
- [25] Nielsen, M.Ø., and Popiel, M.K., (2014) A Matlab program and user’s guide for the fractionally cointegrated VAR model, QED working paper 1330, Queen’s University.
- [26] Szakmary, A.C., Shen, Q., and Sharma, S.C., (2010) Trend-following trading strategies in commodity futures: a re-examination, *Journal of Banking and Finance*, 34, 409–426.
- [27] Wang, J., (2000) Trading and hedging in S&P500 spot and futures markets using genetic programming, *Journal of Futures Markets*, 20, 911–942.