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Optimal Public Information Dissemination

Introducing Observational Learning into a Generalized Beauty Contest

Hendrik Hüning [*] *and Lukas Meub* [†]

Abstract

We develop a dynamic two-period generalized beauty contest to study the optimal level of publicity when disclosed information is subject to multiplier effects inherent to social learning. We build upon the static case, where all agents receive a private signal about an unknown fundamental state and only a fraction of all agents receive an additional public signal. However, in our model, agents no longer act simultaneously; rather, agents informed by both signals act in the first period, while those uninformed about the public signal delay their action and learn about informed agents' actions. We show that in the unique equilibrium of our dynamic game, informed agents overreact more strongly to public signals. The optimal dissemination of public information is thus considerably lower than the static case suggests. If the social learning signal is reasonably precise, aggregate welfare is higher than in the static case. Our results hold relevance for the optimal information policy design of public authorities.

Keywords: generalized beauty contest, monetary policy, optimal communication, social learning, strategic complementarities

JEL: D82, D83, E5

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Public authorities such as central banks have a profound impact on financial markets as agents pay close attention to information disclosed. Public information serves agents as a valuable signal for an economy's fundamentals and - by influencing market expectations - it affects asset prices and interest rates. However, Morris and Shin (2002) (henceforth MS) have shown in their seminal generalized beauty contest model that the dissemination of public information might also have detrimental welfare effects in markets characterized by strategic complementarities. Accordingly, it might be optimal for central banks to strategically withhold some information. Cornand and Heinemann (2008) (henceforth CH) introduce the degree of publicity as an efficient instrument to preclude overreactions to public information. Although multiplier effects fostering the dissemination of public information are acknowledged concerning their relevance to assess the optimal degree of publicity, such distortions to a straightforward management of publicity and overall market expectations have not been modeled. We consider observational learning as one multiplier effect of outstanding relevance. Financial market participants in transparent global markets are able to delay their actions to strategically outwait other participants, observe their actions and thus infer underlying private and public information. These dynamics in the penetration of public information substantially alter central banks' optimal policy design.

In this paper, we extend the analysis presented by CH to a two-period model incorporating observational learning into the generalized beauty contest with strategic complementarities. In our framework, agents who are uninformed about the public signal can learn from preceding actions of the informed agents. Similar to herding models with pay-off externalities (cp. for instance Dasgupta 2000 or Chari and Kehoe 2004), agents have the incentive to coordinate within and across periods. This extension from the static to a dynamic setting allows us to analyze the interplay of pay-off externalities induced by strategic complementarities and information externalities inherent to social learning. We show that as long as the social learning signal comprising the actions of the informed agents is not overly imprecise, aggregate welfare is higher than in the static case. However, the overreaction to disclosed public information will exceed the overreaction in the static case. This finding results from informed agents anticipating the behavior of the uninformed agents, who aim to coordinate across periods by relying on the social learning signal. In response to observational learning as a multiplier effect to disclosed information, the central bank optimally chooses a lower level of publicity ex ante and thus excludes more agents from receiving the public information. Furthermore, the central bank discloses information at maximal precision if the respective public signal is not extremely imprecise.

The relevance of public information policy has grown considerably since the mid-1990s, when central bank actions and communication became increasingly transparent. This increase in transparency was justified by two main arguments: first, it had become widely accepted that increased transparency improves the credibility of the democratic legitimacy of the central bank actions; and second, increased transparency was expected to improve the effectiveness of monetary policy by enabling central banks to more successfully manage market expectations

(Woodford 2003; Haan et al. 2007).

Following the generalized beauty contest model introduced by MS, a broad discussion has emerged, questioning the superiority of comprehensive central bank transparency. Financial markets are predominantly characterized by strategic complementarities, which lead to agents striving to match the economy's fundamentals and - at the same time - the likely actions of other market participants. Therefore, public information contains useful information about both the fundamentals and the likely actions of other market participants, merely given its public nature. This dichotomous informational value allows agents to more effectively coordinate with the actions of others by placing a disproportionally high weight on public information. Markets are subsequently characterized by an overreaction to public information and they fail to establish the socially desirable outcome of weights being assigned according to signals' relative precision. Consequently, it might be optimal for a central bank to reduce transparency to avoid such an overreaction to public information.

As Woodford (2005) shows, this undesirable outcome of public information dissemination crucially depends on the specification of the welfare function: if coordination per se is a welfare objective, MS's proposition fails to hold. Svensson (2006) argues that the result established by MS only holds for an implausibly low precision of public information relative to private information. Morris et al. (2006) replied that assuming correlated signals, the result even holds when public information is more precise than private information.

A number of theoretical papers have considered partial publicity as a policy instrument to avoid the detrimental welfare effects of public information dissemination under strategic complementarities. Morris and Shin (2007) consider one public and n-semipublic signals within a modified generalized beauty contest game. In equilibrium, the weight accorded to the semi-public signals decreases with increasing fragmentation and there is a trade-off between fragmentation and the precisions of signals. CH emphasize that the MS framework only offers the possibility to reduce the precision of public information to mitigate the potentially detrimental effects of disseminating public information. In relaxing this assumption, they introduce an additional parameter allowing the central bank to restrict the fraction of market participants receiving the public information and thus actively manage the publicity of announcements as a strategic means to withhold information. They show that it can be optimal to disseminate public information with maximum precision to a certain fraction of all agents only.[3]

Arato et al. (2014) introduce endogenous information acquisition to the framework of CH. In their model, public information is costly and agents can choose whether or not to gather the public information. As in Hellwig and Veldkamp (2009) - where agents' actions are strategic complements - information acquisition becomes a strategic complement, leading to multiple equilibria. Arato et al. show that the central bank can guide agents to an unique partial publicity equilibrium if an increasing pricing rule is introduced, rather than using constant

[3] Trabelsi (2010) emphasizes the similarity of the fragmentation approach in Morris and Shin (2007) and the publicity approach in CH. By introducing private signals in addition to the n-semipublic signals in the framework of Morris and Shin, Trabelsi shows that both approaches induce the same impact of reducing the detrimental effect of public information: one might either reduce the publicity of public information or introduce fragmented information.

prices for public information.[4]

Another recent paper related to our research objective is Molavi et al. (2014). Contrary to our game, agents do not receive an exogenous public signal but rather observe some neighboring agents' actions. By studying social learning in a dynamic generalized beauty contest game, they show that in a Markov perfect Bayesian equilibrium agents eventually reach consensus in their actions. However, Molavi et al. focus on the asymptotic properties of actions due to social learning in a generalized beauty contest framework, while we are interested in the optimal level of publicity of the public information when a coordination motive and social learning interact.

From the class of dynamic games with strategic complementarities, the two-period model developed in Angeletos and Werning (2006) is closest to our framework. Similar to the framework in CH, agents are divided into two groups: the first group merely receives a private signal - which is an unbiased signal of the underlying state - and subsequently acts in the first period, whereby average actions reflect the fundamental; and the second group acts in the second period and receives a private signal as well as a noisy public signal about the average action of the first period. This endogenous public signal serves as a focal point for agents in the second period. The authors show that such an endogenously modelled public signal might lead to a multiplicity of equilibria rather than a unique solution. However, agents have a binary action set of attacking or not-attacking the status-quo while we study the equilibrium for a continuous action set.

Our results hold relevance for the optimal design of central bank communication. As acknowledged by CH, the optimal dissemination of public information should account for multiplier effects such as observational learning. We show that intertemporal coordination motives aggravate agents' overreaction to public information if the social learning signal - i.e. the observable actions of the informed agents - contains substantial information. Observational learning subsequently transmits disclosed public information from informed to initially uninformed agents, thus reducing - ceteris paribus - the optimal degree of publicity. Nevertheless, assuming optimal publicity, social learning enhances aggregate welfare. Our two-period approach further emphasizes central banks' ability to influence the fundamentals by disseminating public information.

The remainder of the paper is organized as follows. Section 1 motivates our approach to analyze the generalized beauty contest in a dynamic two-period model, introduces our model and derives the equilibrium. Section 2 presents our welfare analysis. Section 3 discusses variations and extensions to our model, as well as implications drawn from related experimental studies, before Section 4 concludes.

1 A Dynamic Generalized Beauty Contest Model

1.1 A two-period approach

In the static model by CH, all agents receive a private signal about an unknown fundamental state of the economy. A fraction of all agents additionally receive a public signal, while all others

[4]Other models introducing costly information acquisition to the generalized beauty contest framework of MS include Colombo and Femminis (2008), Ui (2014) and Colombo et al. (2014).

remain uninformed about this signal issued by the central bank. It is assumed that the central bank can determine the fraction of agents receiving its public signal by designing their public announcements accordingly. However, informed and uninformed agents act simultaneously and thus the model constitutes a static generalized beauty contest game. There is neither a time lag in agents' decisions nor observational learning. Accordingly, the model does not account for a crucial characteristic of financial markets where participants can delay their actions to learn about first movers' information when observing their actions.

Our model can be rationalized as follows: agents strive to align their actions to the unknown fundamental state and - at the same time - follow a coordination motive. They all listen to public announcements of the central bank and face a signal-extraction problem of inferring the respective signal conveyed by the announcement. However, only a certain fraction are capable of inferring the public signal. In the words of Morris and Shin (2007): *"... as any central banker knows, it is not so easy to communicate information in such a way that it become(s) common knowledge within the private sector. If different listeners interpret an announcement differently, then the content of the announcement does not become common knowledge. If some listeners pay attention to the announcement, while others do not, then the content of the announcement does not become common knowledge. Intuitively, the more one attempts to communicate, the more likely it is that some listeners will not pay attention to all the information, and the less common knowledge."* All others wait an instant of one period to observe the actions of informed agents to gain insights into the newly announced public signal and learn about the likely common action in the market, as well as the fundamental state.

Similar considerations follow when the public signal is interpreted as a trigger to initiate an action in the first place. One might assume the fraction of uninformed agents that did not receive or failed to understand the public signal as having no intention to take an action, since no new information seems to be disclosed. Only by observing other agents' actions is a reaction induced and private information gathered regarding the new fundamental state. Hence, in this scenario, the decision is not delayed strategically to receive additional information through observational learning. The delay rather follows from the lack of a trigger that initiates an action, which – to the informed agents – has been the observation of the public signal.

In our model, sequential choice and intertemporal coordination incentives interact, i.e. the game features information externalities due to learning and payoff externalities due to strategic complementarities. As a result, informed agents are concerned about how their actions influence the successors. This anticipation aspect distinguishes our model from most herding models following the seminal contributions of Banerjee (1992) and Bikhchandani et al. (1992), where the only externality is a purely informational one. With strategic complementarities being absent in many herding models, agents do not conduct forward-looking behavior, which makes the models more difficult to apply to actual financial markets Dasgupta (2000).

Observational learning causes a dynamic penetration of public information and might raise publicity of disclosed information above the degree initially intended by the central bank. Since the actions of informed agents are observed by the agents who did not receive the public signal in the first place, the focal role of the public signal continues to the second period. Moreover, as

mentioned above, informed agents in our game account for their actions' influence on subsequent play. This anticipation effect amplifies the focal role of the public signal, which alters the optimal level of publicity in comparison to the static CH case. Put simply, optimal central bank behavior has to account for decisions taken non-simultaneously, as well as the dynamics of the process by which public information is disseminated and influenced by observational learning.

1.2 The model

In the following, we first introduce agents' utility and the signal structure of the game, before subsequently formulating the optimal actions conditional upon available signals. Finally, we identify the unique linear equilibrium of the game.

Utility

There is a continuum of agents indexed by $i \in [0,1]$. There are two periods indexed by $t = 0, 1$. Each agent i acts in either $t = 0$ or $t = 1$ but never in both periods and chooses an action $a_i \in \mathbb{R}$, which maximizes his utility given by

$$U_i(a, \theta) = -(1-r)(a_i - \theta_t)^2 - r(L_i - \bar{L}) \tag{1}$$

with

$$L_i \equiv \int_0^1 (a_i - a_j)^2 dj, \;\; \bar{L} \equiv \int_0^1 L_j dj,$$

where $\theta_t = \{\theta_0, \theta_1\}$ is the fundamental state of period t and r is a constant with $0 < r < 1$.[5] According to (1), agents minimize the loss from the squared distance to the fundamental state of the current period (first term) and minimize the individual loss compared to the average loss from discoordination (second term).

Signal structure and evolution of the fundamental state

All agents receive a private signal that is identically distributed around $\theta_t : x_{it} = \theta_t + \varepsilon_i \; with \; \varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. The precision of private signals is defined as $\beta \equiv \frac{1}{\sigma_\varepsilon^2}$. All private signals are uncorrelated.

A fraction P of all agents i observe the realization of a public signal $y = \theta_0 + \eta \; with \; \eta \sim N(0, \sigma_\eta^2)$. We define $P \equiv \frac{informed \; agents}{all \; agents}$ such that $0 < P \leq 1$. Public signals are perfectly correlated, i.e. every informed agent observes exactly the same public signal with precision $\alpha \equiv \frac{1}{\sigma_\eta^2}$. For simplicity, we assume an exogenous decision order. Agents who are informed about the public signal y take an action in the first period $t = 0$, whereas uninformed agents take an action in the second period $t = 1$.[6] We assume the fundamental state to be drawn from the real line in

[5] We abstain from considering the case $r = 1$, in which we would face a pure coordination game with multiple equilibria.

[6] We might well construct a scenario in which agents face (constant) waiting costs when choosing to delay their actions or act immediately. However, agents being informed about the public signal can be assumed to have weaker incentives to wait in order to learn from observable actions than uninformed agents (cp. Frisell 2003). Depending on the significance of waiting costs, there should be an equilibrium where all informed agents act in

$t = 0$. For $t = 1$, the fundamental state evolves in response to the average action in $t = 0$ such that

$$\theta_1 = \bar{a}_0.^{7}$$

Agents uninformed about the public signal y receive a noisy exogenous public signal in $t = 1$, defined as

$$s \sim N(\bar{a}_0, \sigma_s^2) \text{ with } \delta \equiv \tfrac{1}{\sigma_s^2}.^{8}$$

To rationalize this social learning signal and the evolution of the fundamental state, we could think of an immediate price reaction after public announcements by authorities like the central bank. These assumptions about the evolution of the fundamental state and the social learning signal allow us to model the social learning signal as an exogenous signal where all agents have common priors over the precision of all signals.[9]

Note that for $P = 1$, the game collapses to the static game introduced by MS as all agents are informed, act in the same period $t = 0$ and there is no observational learning. On the contrary, the fewer agents who receive the public signal y, the more agents who act in $t = 1$ and learn from the preceding agents' actions.[10] Agents of both periods account for the focal role of the social learning signal, since it serves as a focal point for coordination in $t = 1$ and across periods by transmitting information on the average action from $t = 0$. However, the focal role vanishes for an extremely imprecise social learning signal and our model approaches the CH framework. Accordingly, our richer setting represents a more general framework comprising MS and CH as special cases.

Finally, it is assumed that all agents process the signals in a linear fashion - which is common knowledge - such that

$$a_{i0}(\Omega) = \gamma x_{i0} + (1 - \gamma) y \tag{2}$$

and

$$a_{i1}(\Omega) = \gamma x_{i1} + (1 - \gamma) s \tag{3}$$

with $\gamma \in [0, 1]$ being the weight on the private signal. We thus restrict our attention to linear equilibria. Following the related literature, we assume that $r, P, \alpha, \beta, \delta$ are common knowledge.

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the first period and all uninformed agents delay their action to the second period. This case would be equivalent to our scenario featuring an exogenous decision sequence. Please note that such an endogenously derived decision sequence would substantially complicate our analysis. Since our focus lies on optimal publicity given multiplier effects inherent to social learning and not the distinct timing of actions, we assume the decision order to be exogenous and abstain from modeling waiting costs.

[7] For games in which the history of play influences the fundamental state, see for instance Curtat (1996). However, note that - unlike in Curtat's game - our agents' actions not only depend on the current fundamental state because we assume inter-temporal coordination incentives. Therefore, we do not establish a Markov perfect equilibrium (cp. Maskin and Tirole 2001).

[8] Instead of modeling social learning as a noisy public signal, we could introduce perfect learning. Furthermore, one might consider heterogeneous private signal precision. Both aspects are discussed in section 3.

[9] We discuss the difficulties of modeling the social learning signal endogenously in further detail in the context of (13) and formally in appendix E.

[10] For $P = 0$, all agents act in $t = 1$ and there is no average action and no social learning signal. Agents would merely rely on their private information for optimal play. Since we chose to analyze the interplay between coordination and social learning, we abstract from this trivial case and set $P > 0$.

Optimal action

For both informed agents and uninformed agents, the FOC of (1) is the utility maximizing condition

$$a_{it}^{*} = (1-r)E_i(\theta_t|\Omega) + rE_i(\bar{a}|\Omega). \tag{4}$$

However, the information set Ω to form the expectations $E_i(\theta_t)$ and $E_i(\bar{a})$ differs between the two groups. The fraction P of informed agents receive both a private signal x_{i0} and the public signal y. The fraction $(1-P)$ of uninformed agents receive x_{i1} and a signal s on the average action of the informed from observational learning. We can formulate optimal individual actions conditional upon available signals. For informed agents, (4) becomes

$$a_{i0}^{*} = (1-r)E_i(\theta_0|x_{i0}, y) + rE_i(\bar{a}|x_{i0}, y), \tag{5}$$

where

$$E_i\left(\theta_0|x_{i0}, y\right) = \frac{\beta}{\alpha+\beta}x_{i0} + \left(1 - \frac{\beta}{\alpha+\beta}\right)y$$

and

$$E_i(\bar{a}|x_{i0}, y) = PE_i(\bar{a}_0|x_{i0}, y) + (1-P)E_i(\bar{a}_1|x_{i0}, y).$$

For uninformed agents, we can write

$$a_{i1}^{*} = (1-r)E_i(\theta_1|x_{i1}, s) + rE_i(\bar{a}|x_{i1}, s), \tag{6}$$

where

$$E_i(\theta_1|x_{i1}, s) = \frac{\beta}{\delta+\beta}x_{i1} + \left(1 - \frac{\beta}{\delta+\beta}\right)s$$

and

$$E_i(\bar{a}|x_{i1}, s) = PE_i(\bar{a}_0|x_{i1}, s) + (1-P)E_i(\bar{a}_1|x_{i1}, s).$$

While the expectations about the fundamental are equivalent to the static games of MS and CH - where private and public signals are weighted by their relative precision - the expectation about the average action differs, as it comprises expectations concerning the average action in both $t = 0$ and $t = 1$. These expectations about average actions are weighted by the fraction of agents acting in the respective period. It becomes clear that besides the signal structure, the main difference of our framework compared with previous models lies in the inter-temporal coordination incentives.

Linear Equilibrium: Optimal weights on private and public signals in $t = 1$

In order to solve for the optimal actions in both periods, we start by deriving the optimal weights on private and public signals for uninformed agents and identify the optimal and the average action. In a second step, we insert this average optimal action into the optimality conditions of informed agents and solve for their optimal weights and action.

By drawing upon equation (6), we write the optimal action for uninformed agents as

$$a_{i1}^{*} = r\left\{P\left[\frac{\beta}{\delta+\beta}x_{i1} + \left(1-\frac{\beta}{\delta+\beta}\right)s\right] + (1-P)\left[\frac{\gamma_1\beta}{\delta+\beta}x_{i1} + \left(1-\frac{\gamma_1\beta}{\delta+\beta}\right)s\right]\right\} + (1-r)\left[\frac{\beta}{\delta+\beta}x_{i1} + \left(1-\frac{\beta}{\delta+\beta}\right)s\right] \quad (7)$$

The last term depicts the Bayesian considerations of the two pieces of information to form the expectation on θ_1. The first term contains the expectation about the average action in the preceding period and the expectation about the average action among the uninformed within the same period. It is important to note that since we assume $\theta_1 = \bar{a}_0$, both the private and social learning signal serve as valuable information to derive the likely actions of informed agents, where we can write $E_i(\bar{a}_0|x_{i1}, s) = E_i(\theta_1|x_{i1}, s) = \left[\frac{\beta}{\delta+\beta}x_{i1} + \left(1-\frac{\beta}{\delta+\beta}\right)s\right]$.[11] The expectation on the average action in the same period - in this case the expression right after $(1-P)$ - is the same as in the static case. Derived in detail in appendix A, solving for the optimal weight on the private signal for uninformed agents yields

$$\gamma_1^{*} = \frac{\beta[1-r(1-P)]}{\delta+\beta[1-r(1-P)]}. \quad (8)$$

By symmetry, the optimal weight on the public signal is given by $(1-\gamma_1^{*})$, respectively. In our framework, uninformed agents play the same game as the informed agents in CH, except that rather than receiving the public signal y, they receive the public social learning signal s. Furthermore, the optimal weight on the respective public signal increase in $(1-P)$ and not in P. For $P \to 0$, the optimal weight on the private signal approaches the static MS case. For $P \to 1$, the optimal weight on the private signal approaches the efficient Bayesian weight on θ_1 as the social learning signal s loses its focal role. Uninformed agents thereby acknowledge that for higher P (lower $1-P$), there are fewer agents with whom they could profitably coordinate on s.

Figure 1 illustrates the aforementioned results. It can be seen that the equilibrium weight on the private signal increases in P and approaches the Bayesian weight at the limit where both signals are weighted by their relative precision. Since we assume equal signal precision for this illustration, signals are assigned equal Bayesian weights of 0.5 for $P \to 1$.

Conclusion 1: *The more agents are informed by the public signal, the weaker the social learning signal's focal role. In the limit ($P \to 1$), weights on private and public signals are accorded relative to their precision (Bayesian weights).*

[11] At first, it might seem odd that private signals hold information on the actions of first movers. Nonetheless, in games where preceding actions influence the fundamental state and subsequent agents receive signals about the resulting new state, all signals - both private and public - contain information about the likely actions of others.

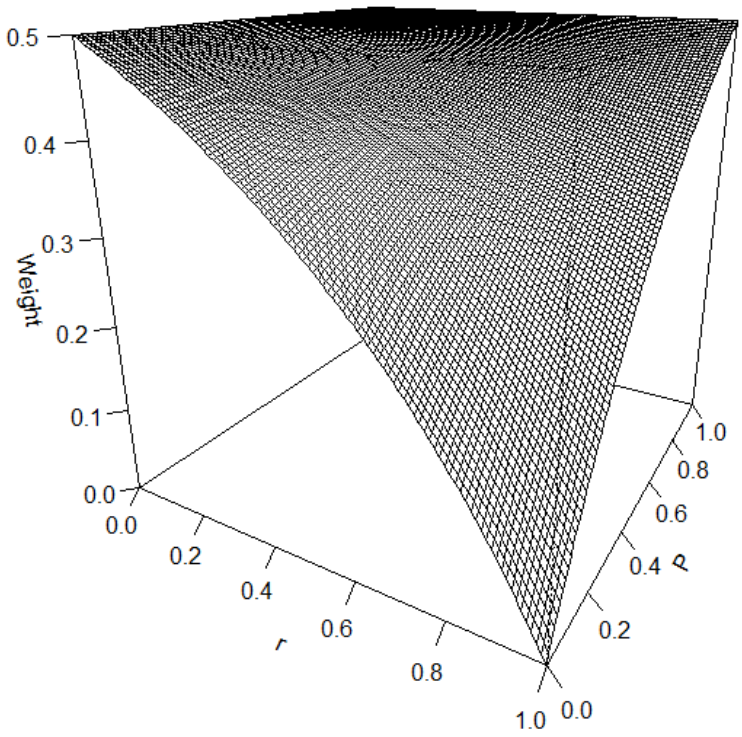


Figure 1: Optimal weight on private signal in $t = 1$

Note: the illustration assumes equal precision of the private and the social learning signal $\beta = \delta = 0.01$

Now we can state the optimal actions for the subgame of uninformed agents as

$$a_{i1}^* = \frac{\beta[1 - r(1 - P)]}{\delta + \beta[1 - r(1 - P)]} x_{i1} + \left[1 - \frac{\beta[1 - r(1 - P)]}{\delta + \beta[1 - r(1 - P)]}\right] s \tag{9}$$

and the average action becomes

$$\begin{aligned} \bar{a}_1^* &= \int_P^1 a_{i1}^* \\ &= \frac{\beta[1 - r(1 - P)]}{\delta + \beta[1 - r(1 - P)]} \theta_1 + \left[1 - \frac{\beta[1 - r(1 - P)]}{\delta + \beta[1 - r(1 - P)]}\right] s \\ &= \frac{\delta s + \beta[1 - r(1 - P)]\theta_1}{\delta + \beta[1 - r(1 - P)]}. \end{aligned} \tag{10}$$

In appendix C, we show that that the linear equilibrium established here is the unique equilibrium of the subgame in $t = 1$.

Linear Equilibrium: Optimal weights on private and public signals in $t = 0$

Following the procedure of backward induction, we insert the optimal average action in $t = 1$ of (10) into the optimality condition (4) of the informed agents. Since r, P, β and δ are common knowledge, informed agents can derive the optimal weight on the private signal γ_1^* of the uninformed agents. However, informed agents neither observe the fundamental state θ_1 nor the average action signaled by s. Therefore, they need to build expectations about these random variables. First, since

$$E(\theta_1) = E(\bar{a}_0) = \theta_0$$

we obtain

$$E_i(\theta_1|x_{i0}, y) = \tfrac{\beta}{\alpha+\beta}x_{i0} + \left(1 - \tfrac{\beta}{\alpha+\beta}\right) y.^{12}$$

In other words, since the expected fundamental state remains constant over periods, informed agents rely on their signals (x_{i0} and y) in a Bayesian fashion to form an expectation on θ_1. Second, since

$$E(s) = \bar{a}_0$$

we obtain

$$E_i(s|x_{i0}, y) = \frac{\gamma_0\beta}{\alpha+\beta}x_{i0} + \left(1 - \frac{\gamma_0\beta}{\alpha+\beta}\right) y.$$

In other words, since s is an unbiased signal of $\bar{a}_0$, informed agents can form the same expectation for s as they do for the average action of all informed agents. Summarizing the optimal individual action of informed agents based on (5) gives

$$\begin{aligned} a_{i0}^* = {} & (1-r)\left[\frac{\beta}{\alpha+\beta}x_{i0} + \left(1 - \frac{\beta}{\alpha+\beta}\right) y\right] + rP\left[\frac{\gamma_0\beta}{\alpha+\beta}x_{i0} + \left(1 - \frac{\gamma_0\beta}{\alpha+\beta}\right) y\right] \\ & + r(1-P)\left\{\gamma_1^*\left[\frac{\beta}{\alpha+\beta}x_{i0} + \left(1 - \frac{\beta}{\alpha+\beta}\right) y\right] + (1-\gamma_1^*)\left[\frac{\gamma_0\beta}{\alpha+\beta}x_{i0} + \left(1 - \frac{\gamma_0\beta}{\alpha+\beta}\right) y\right]\right\} \end{aligned} \quad (11)$$

The first term of this expression replicates the static MS case, whereby agents form Bayesian expectation about θ_0. The second term gives informed agents' incentive to account for average play in the same period, which is weighted by the coordination incentive parameterized by r and the corresponding fraction P of informed agents. The last term formulates the anticipation of the average action of the $(1 - P)$ uninformed in the second period. Solving for the optimal weight on the private signal for informed agents - detailed in appendix B - yields

$$\gamma_0^* = \frac{\beta[1 - r(1 - \gamma_1^*(1-P))]}{\alpha + \beta[1 - r(1 - \gamma_1^*(1-P))]}. \quad (12)$$

By symmetry, the optimal weight on the public signal can be written as $(1 - \gamma_0^*)$. Optimal

[12] One might wonder why x_{i0} and y serve as valuable signals for θ_1 as they are signals for θ_0. Recall that $\theta_1 = \bar{a}_0$ and $\bar{a}_0$ is a linear combination of x_{i0} and y, which can be written as $\gamma_0 x_{i0} + (1 - \gamma_0)y$ with $\gamma_0 \in [0, 1]$, where x_{i0} and y are normally distributed random variables with $E(x_{i0}) = E(y) = \theta_0$. Therefore, $E(\theta_1|x_{i0}, y) = \theta_0$.

actions of the subgame in $t = 0$ are then given by

$$a_{i0}^* = \frac{\beta[1 - r(1 - \gamma_1^*(1 - P))]}{\alpha + \beta[1 - r(1 - \gamma_1^*(1 - P))]} x_{i0} + \left[1 - \frac{\beta[1 - r(1 - \gamma_1^*(1 - P))]}{\alpha + \beta[1 - r(1 - \gamma_1^*(1 - P))]}\right] y$$

and the average action becomes

$$\begin{aligned} \bar{a}_0^* &= \int_P^1 a_{i0}^* \\ &= \frac{\beta[1 - r(1 - \gamma_1^*(1 - P))]}{\alpha + \beta[1 - r(1 - \gamma_1^*(1 - P))]} \theta_0 + \left[1 - \frac{\beta[1 - r(1 - \gamma_1^*(1 - P))]}{\alpha + \beta[1 - r(1 - \gamma_1^*(1 - P))]}\right] y \\ &= \frac{\alpha y + \beta[1 - r(1 - \gamma_1^*(1 - P))]\theta_0}{\alpha + \beta[1 - r(1 - \gamma_1^*(1 - P))]}. \end{aligned} \tag{13}$$

In appendix D, we show that the linear equilibrium established here is the unique equilibrium of the subgame in $t = 0$.[13] Expressing $\bar{a}_0^*$ in terms of the random variables θ_0, η and ϵ_i yields

$$\bar{a}_0^* = \theta_0 + \frac{\alpha\eta}{\alpha + \beta[1 - r(1 - \gamma_1^*(1 - P))]}.$$

In equilibrium, the average action of the informed agents is distorted away from the fundamental state according to the noise in the public signal η and the corresponding weight that placed on the public signal y. This average action is transmitted to the uninformed agents in $t = 1$ through a noisy public signal, which is

$$s = \theta_0 + \tfrac{\alpha\phi}{\alpha + \beta[1 - r(1 - \gamma_1^*(1 - P))]} \text{ with } \phi \sim N(\eta, \sigma_\phi^2).$$

Since $E(\phi) = \eta$, $E(s) = \bar{a}_0$. The optimal weights for informed agents show some interesting properties when compared to the static cases in MS and CH. First, it can be seen that for $P = 1$ - where all agents become informed and act in the first period - the optimal weight on the private signal collapses to the static MS case. Second, for any $0 < P < 1$, the optimal weight on the private signal depends on the optimal weight on the private signal in the second period (γ_1^*). For $\gamma_1^* \to 0$ - which is the case for $\delta \to \infty$ or $\beta \to 0$ - the equilibrium weight on the private signal again collapses to the MS case, since $1 - \gamma_1^*(1 - P) = 1$. Put simply, for an extremely precise social learning signal or an extremely imprecise private signal, informed agents assume that uninformed agents follow their average action signaled by s. Third, for $\gamma_1^* \to 1$ - which is the case for $\delta \to 0$ or $\beta \to \infty$ - the equilibrium weight on the private signal corresponds to that in CH, since $1 - \gamma_1^*(1 - P) = P$. Put simply, for the case that the social learning signal is extremely imprecise or the private signal is extremely precise, the dynamic game analyzed here replicates the static case of CH, given that social learning adds nothing to the game.

[13] From (13), it becomes clear why modeling the social learning signal as an endogenous signal is impossible in our case. Recall that we assume inter-temporal coordination incentives, whereby agents try to match average actions of both their own period and the respective other period. Intuitively, we face the problem that the precision of an endogenous social learning signal is not identifiable because the average action in $t = 0$ derived by the optimal behavior of agents depends on the optimal weights on the signals in $t = 1$, which in turn depend on the precision of the social learning signal constituted by the average action in $t = 0$. In E, we provide a formal argument regarding this problem.

Figure 2 illustrates the optimal weight on the private signal depending on the coordination motive r and the fraction P of agents being informed about the public signal y. It can be seen that for an increasing r, the optimal weight on the private signal decreases for all P. However, for $P = 1$ and $P \to 0$, the optimal weight on the private signal reaches zero for maximum r. For intermediate levels of publicity P, the optimal weight on the private signal remains significantly above zero. Even when coordination becomes the predominant motive in $t = 0$, informed agents do not assign full weight to the public signal due to a critical mass of uninformed agents relying on the social learning signal, which imperfectly transmits the public signal y. For any

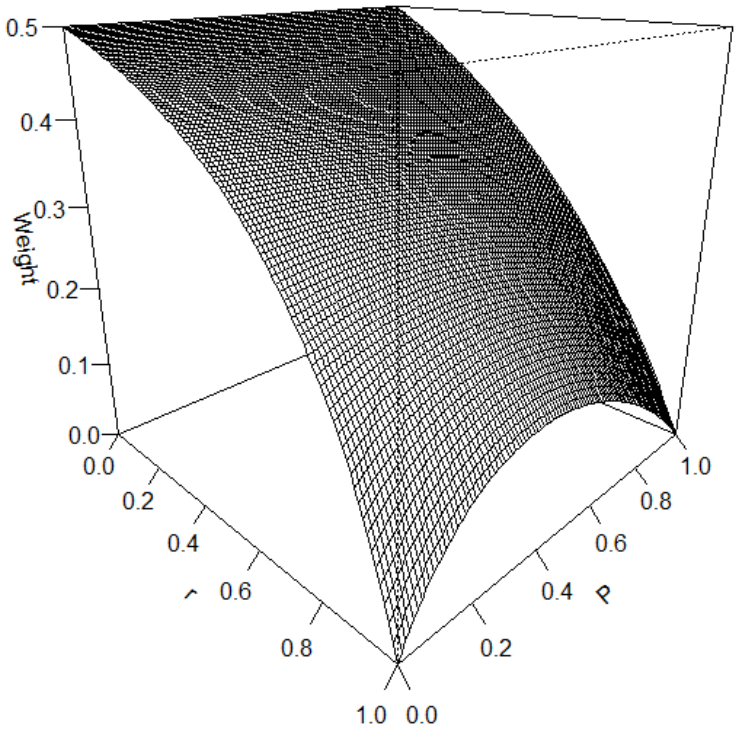


Figure 2: Optimal weight on private signal in $t = 0$

Note: the illustration assumes equal precision of the private, the public and the social learning signal $\alpha = \beta = \delta = 0.01$

intermediate weight of γ_1^* - i.e. for a reasonable precision of the social learning signal somewhere in the region of α and β - and for $0 < P < 1$, the optimal weight for informed agents on the private signal is higher than in the MS case, although - more importantly – it is lower than in the CH case. Figure 3 illustrates a straightforward comparison of the CH, the MS and our own case with respect to the optimal weight on the private signal of informed agents.
This constitutes one of the central arguments of our paper, which can be summarized as follows:

if informed agents expect uninformed agents to delay their action and learn from prior actions, the focal role of the public signal y is preserved by the social learning signal s. Informed agents account for uninformed agents' social learning and thus place a higher weight on the public signal and less on their private signal for any P. This holds true for social learning signals that are not extremely imprecise.

Conclusion 2: *For reasonable precisions of the social learning signal, the optimal weight on the private signal is higher than in the static MS case but lower than in the CH case for any* $0 < P < 1$. *The partially public signal has a stronger focal role when introducing social learning.*

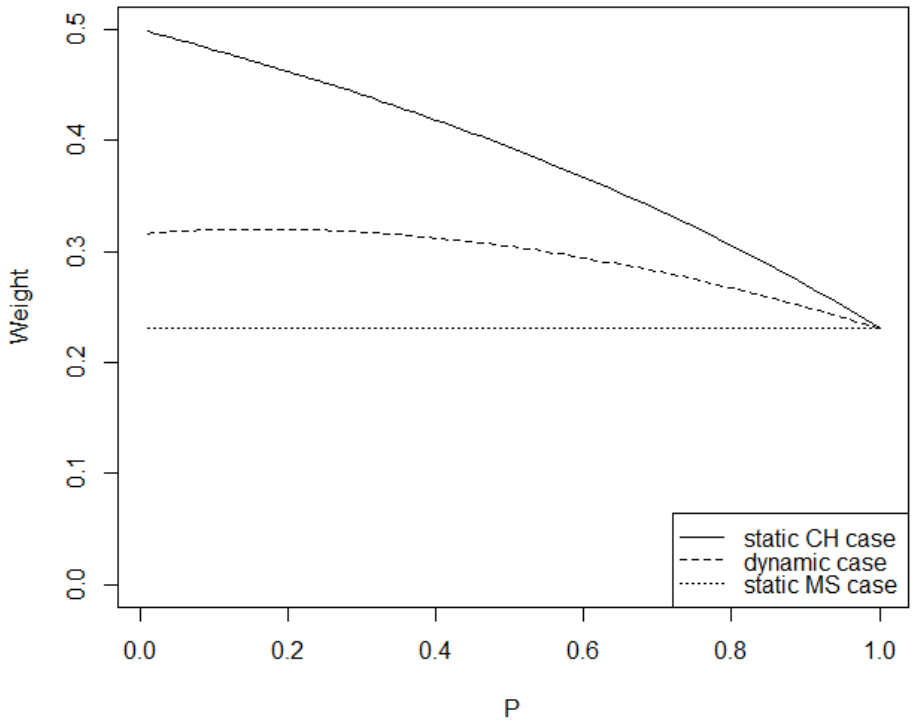


Figure 3: Optimal weight on private signal in comparison to CH and MS

Note: the illustration assumes equal precision of the private, the public and the social learning signal $\alpha = \beta = \delta = 0.01$ and $r = 0.7$

2 Welfare: The optimal degree of publicity

Our welfare specification follows the same formalization as in MS and CH, whereby coordination adds to utility on the individual level but not to aggregate welfare. Our model is thus prone to the criticism expressed by Woodford (2005): if coordination was a welfare objective per se,

public information should always be released at maximum precision and publicity. Nonetheless, we follow the argument by CH: while agents individually benefit from accurately predicting expectations of other agents, efficiency requires actions to reflect the fundamental state and any form of overreaction to the public signal is socially undesirable.

Consider a pre-stage of our game where the central bank chooses a degree of publicity P to maximize expected welfare. As the central bank chooses P such that actions are close to the fundamentals, we can write

$$\begin{aligned} E[W(a,\theta)] &= -E[\int_{i=0}^{P}(a_{i0}-\theta_0)^2+\int_{P}^{1}(a_{i1}-\theta_1)^2] \\ &= -\int_{i=0}^{P}E\{[\gamma_0^* x_{i0}+(1-\gamma_0^*)y-\theta_0]^2\}di-\int_{P}^{1}E\{[\gamma_1^* x_{i1}+(1-\gamma_1^*)s-\theta_1]^2\}di \\ &= -P[\gamma_0^{*2}E(\epsilon_i^2)+(1-\gamma_0^*)^2E(\eta^2)]-(1-P)[\gamma_1^{*2}E(\epsilon_i^2)+(1-\gamma_1^*)^2E(\phi^2)]. \end{aligned}$$

Now inserting optimal weights for both periods - γ_0^* from (12), γ_1^* from (8) - yields

$$\begin{aligned} = -P\left\{\left[\frac{\beta[1-r(1-\gamma_1^*(1-P))]}{\alpha+\beta[1-r(1-\gamma_1^*(1-P))]}\right]^2\frac{1}{\beta}+\left[1-\frac{\beta[1-r(1-\gamma_1^*(1-P))]}{\alpha+\beta[1-r(1-\gamma_1^*(1-P))]}\right]^2\frac{1}{\alpha}\right\} \\ -(1-P)\left\{\left[\frac{\beta[1-r(1-P))]}{\delta+\beta[1-r(1-P))]}\right]^2\frac{1}{\beta}+\left[1-\frac{\beta[1-r(1-P))]}{\delta+\beta[1-r(1-P))]}\right]^2\frac{1}{\delta}\right\}, \end{aligned}$$

which can be rearranged to obtain

$$= -P\frac{\beta[1-r(1-\gamma_1^*(1-P))]^2+\alpha}{[\alpha+\beta(1-r(1-\gamma_1^*(1-P)))]^2}-(1-P)\frac{\beta[1-r(1-P))]^2+\delta}{[\delta+\beta(1-r(1-P))]^2}.$$

It can be seen that for $\delta\to 0$, $\gamma_1^*\to 1$ and the limit of the above expression becomes

$$= -P\frac{\beta(1-rP)^2+\alpha}{[\alpha+\beta(1-rP)]^2}-(1-P)\frac{1}{\beta}.$$

This expression replicates the expected welfare of the static CH case. Although agents act in different periods, the game collapses to the static case for a very imprecise social learning signal compared to the private signal. The intuition of this result is straightforward: if the social learning signal holds no value for the uninformed in $t=1$, they fully rely on their private signal, which in turn is anticipated by the first movers. The information set for informed and uninformed agents becomes equivalent to the static case. We analyze the expression above in further detail by carrying out various graphical illustrations with distinct parametrizations. We illustrate our findings graphically to enable an intuitive understanding since first derivatives of the expected welfare function are quite complex (see appendix F).

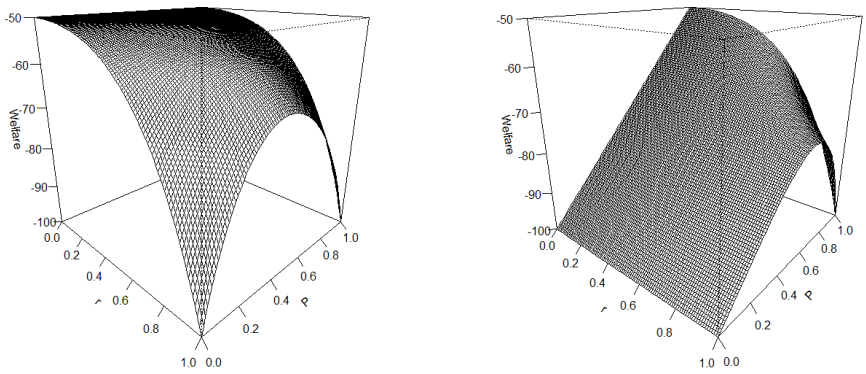
Comparing the static and dynamic case

Figure 4a depicts the expected welfare with respect to P and r in our framework, while figure 4b depicts expected welfare as formulated in CH, which we identified as being equivalent to our case for $\delta \to 0$. For both illustrations, we assume equal precision of the underlying signals ($\alpha = \beta = \delta$). In the static CH case, the optimal level of publicity is lower than 1 if, and only if, $\frac{\alpha}{\beta} < 3r - 1$, or for $\alpha = \beta$ if, and only if, $r > \frac{2}{3}$. In our dynamic case, the optimal level of publicity not only depends on the coordination motive r and the relative precision of private and public signals; rather, optimal publicity crucially depends on the precision of the social learning signal relative to the precision of both the private and public signal. For the case illustrated by figure 4a, the optimal publicity P is smaller than 1 and - more importantly - smaller than in the static CH case for any $r > 0$. The stronger the coordination motive, the higher the loss from a non-optimal level of publicity. Non-optimality thereby results from either too low or too high publicity .

For $r = 0$, any $P > 0$ is welfare maximizing. Put simply, if there are no coordination incentives, all signals hold the same information value to estimate the underlying fundamental. Since there is no desire to coordinate, there is no overreaction to the public signal and uninformed agents receive the same information by observing the average action of the informed agents. For $r \to 1$, $P = 0.44$ is welfare maximizing, which is lower than in the static CH case as observational learning transmits disclosed public information from informed to initially uninformed agents. This transmission is anticipated and accounted for by the informed agents, the focal role of the central bank's signal grows stronger and – ceteris paribus – the optimal degree of publicity decreases.

Conclusion 3: *For equivalent signal precisions, the welfare maximizing degree of publicity is lower in the dynamic case than in the static CH case.*

If the social learning signal is less precise than the public and private signal (figure 5a), the optimal level of publicity P is higher than under equal signal precision (figure 4a). Intuitively, the welfare maximizing central bank benefits from informing more agents due to the higher precision of its own public signal compared to the social learning signal. As before, the stronger the coordination motive, the higher the loss from a non-optimal level of publicity. However, in this case, losses are asymmetrically distributed, i.e. publicity higher than the optimum is less welfare damaging compared to publicity lower than the optimum, since the public signal y is more precise than the social learning signal. If the social learning signal is more precise than the public and private signal, the optimal level of publicity P substantially decreases. In our illustration of figure 5b, the social learning signal is twice as precise and thus the optimal P becomes zero. Effective social learning and the imprecise public signal evoke an optimal zero public disclosure policy, where P is chosen infinitesimal.

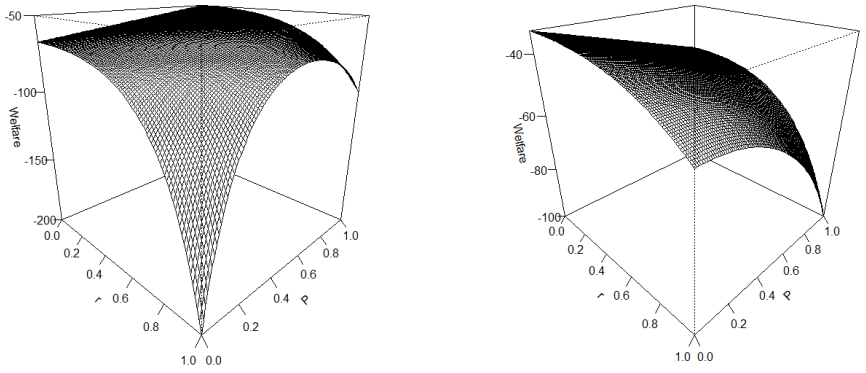


(a) dynamic case (b) static CH case

Figure 4: Welfare depending on r and P for the static and the dynamic case

Note: the illustrations assumes equal precision of the private, the public and the social learning signal $\alpha = \beta = \delta = 0.01$

Conclusion 4: *For a weak precision of the social learning signal, optimal publicity is high, whereas for strong precision the optimal publicity is low. Therefore, the effectiveness of social learning determines optimal central bank policy with respect to publicity.*



(a) low precision: $\delta = 0.005$ (b) high precision: $\delta = 0.02$

Figure 5: Welfare depending on r and P for high and low precision of s

Note: the illustration assumes equal precision of the private and the public signal $\alpha = \beta = 0.01$

Optimal publicity and the precision of central bank's public signal

As CH note, if the public signal y can be provided with infinite precision $\alpha \to \infty$, the expected welfare loss is zero and the first best solution is established. However, if there is an upper limit $\bar{\alpha}$

for the provision of the public signal, the central bank has to choose the optimal level of publicity and precision simultaneously. CH show that if the central bank can set $P = P^*$, the optimal precision is always maximal in the static case. The main theorem follows: public information should always be provided with maximum precision, but - under certain conditions - not to all agents. In our case, where the effectiveness of social learning plays a crucial role for the optimal level of publicity, this theorem no longer holds. Figure 6a evaluates expected welfare at P^* with respect to α and r.

For any $r < \frac{2}{3}$, overall welfare is strictly increasing in α. Optimal precision is given by the feasible maximum as stated by the main theorem of CH. For instance, for $r = 0$, welfare strictly increases in α whereby such an increase in the precision of the public signal substantially contributes to welfare only if $\alpha > \beta = \delta$. For the case of a weaker precision in the public signal than the private and social learning signal, the central bank optimally chooses a small P to prevent actions from being distorted away from the fundamental state. Conversely, for $\alpha > \beta = \delta$, optimal P is high as more agents should be informed to benefit from the precise public signal.

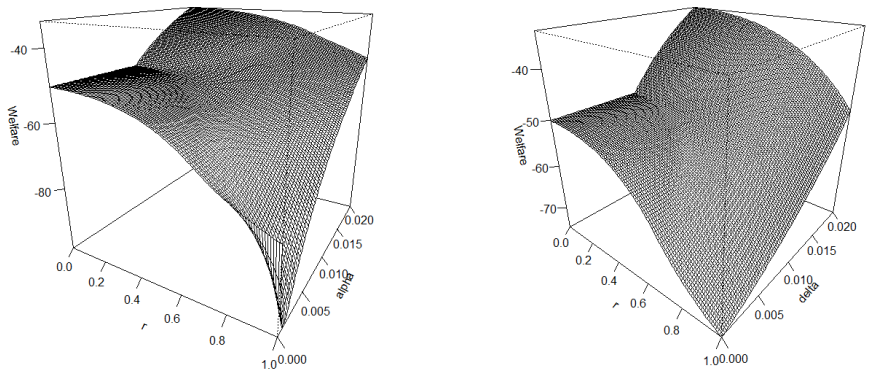
For any $r > \frac{2}{3}$, the optimal precision is not necessarily the maximal feasible one. Only if the central bank can guarantee a minimum precision of their issued public signal, optimal precision equals the the maximal feasible. Following our illustration, this minimum constitutes at $\alpha > 0.001$. Accordingly, the public signal must be at least one tenth as precise as the private signal and the social learning signal. For precisions lower than this threshold, it becomes optimal for the central bank to completely withhold information and provide a signal carrying no information at all. However, Svensson (2006) argues - based upon empirical findings on the precision of private versus public sector forecasts, e.g. by Romer and Romer (2000) - that it seems unlikely to assume the relative precision of public information being sufficiently weak for this finding to become relevant.

Conclusion 5: *Given optimal publicity P^*, the optimal precision of the public signal is at the maximum only if $r < \frac{2}{3}$. For $r > \frac{2}{3}$, it can be optimal to withhold very imprecise public information and issue a signal carrying no information at all.*

Desirability of social learning

On the one hand, social learning leads to the uninformed agents in our model being better informed than in the static CH case, whereby uninformed agents might more accurately match the fundamental state by their actions. On the other hand, social learning serves a focal role that fosters coordination and aligns the actions of all agents. Naturally, the question arises concerning whether social learning is generally desirable from a welfare perspective.

In our dynamic beauty contest model, social learning only adds to overall welfare if the central bank chooses the optimal level of publicity ($P = P^*$) and the social learning signal carries some information and is not extremely imprecise. Consider the public signal y to be very precise (high α) and at the same time the social learning signal s to be very imprecise (low δ): in this case, if the central bank chooses a very low level of publicity ($P < P^*$), more agents act upon the social



(a) public signal precision: α (b) social learning signal precision: δ

Figure 6: Welfare depending on r evaluated at $P = P^*$

Note: the illustration assumes equal precision for a) the private and the social learning signal $\beta = \delta = 0.01$ and b) the private and the public signal $\beta = \alpha = 0.01$

learning signal, which potentially distorts actions further away from the fundamental than the public signal. Social learning subsequently reduces welfare and becomes socially undesirable. By contrast, if the social learning signal is very precise (high δ) and the public signal is very imprecise (low α) and the central bank chooses a very high level of publicity ($P > P^*$), the contrary holds true: too many agents act upon the public signal, which potentially distorts actions further away from the fundamental than the social learning signal. Social learning subsequently becomes desirable as the positive impact on welfare potentially becomes strong. However, if the central bank chooses publicity optimally, welfare at worse reaches the level of the static CH case (cp. figure 6b). As aforementioned, expected welfare in our model corresponds to expected welfare in CH in case of an extremely imprecise social learning signal $\delta \to 0$. Contrastingly, if the social learning signal carries at least some information, welfare is strictly higher for all r.

Conclusion 6: *Social learning adds to overall welfare if the central bank follows the welfare optimizing level of publicity ($P = P^*$) and the respective social learning signal is not extremely imprecise.*

3 Discussion

In this section, we bring up variations and extensions of our main framework. Firstly, we discuss a variation of our model in which the media distributes public announcements to a wider audience than initially intended by the central bank. Secondly, we consider heterogeneous private signal precision between informed and uninformed agents. Thirdly, we introduce an alternative

specification of the social learning signal and the evolution of the fundamental state. Fourthly, we connect our model to the related literature concerning the central bank's influence on an economy's fundamentals. Finally, we discuss our theoretical findings in the light of experimental evidence on generalized beauty contests and social learning.

Media as a multiplier

Besides social learning, there are various channels through which public information might spread to a wider audience than initially intended by the central bank. In particular, the role of the media should be considered as highly relevant, since its discussion of central bank actions and communication enables uninformed agents to receive - at least partly - the public signal. An effective way to model the role of the media in our framework would allow uninformed agents – rather than receiving s – to receive a signal $\tilde{y} = y + \psi$ with $\psi \sim N(0, \sigma_\psi^2)$, which is an unbiased signal of the issued public signal y.[14] The signal represents the media's interpretation of the central bank announcement. Assuming that informed agents anticipate the spreading of the information, the desirability of this second order public information crucially depends on how accurate the media processes the initial announcement. In the most extreme case, the media issues its information on the public signal y without any noise and all agents eventually become informed. This amplifies the focal role of public information and the central bank optimally reduces the level of publicity. Again, it is the informed agents' beliefs concerning the information set of uninformed agents and the intertemporal coordination motive that prompts the optimal level of publicity to be lower than in the static case. This replicates the general argument of our paper, namely that accounting for multiplier effects is crucial when designing central bank communication.

Heterogeneous private signal precision

An interesting variation of our model results from relaxing the assumption of equal private signal precision across periods ($\beta_0 \neq \beta_1$). Consider that private signal precision is very low for uninformed agents, e.g. because private investors are not spending as many resources to gather information compared to institutional investors. It might plausibly be assumed that agents with more precise private information are more capable of receiving and processing public information and thus act prior to the uninformed agents with imprecise private information. Uninformed agents are subsequently prompted to closely follow the social learning signal, where feedback trading might be considered a related phenomenon. Conversely, informed agents account for this reaction by assigning a higher weight on public information than under the assumption of uninformed agents acting upon private information of equal precision. Central bank communication has to account for the aggravated overreaction to public information in such an environment. The contrary logic applies if the private information of uninformed agents is more precise.

[14] An advantage of this variation of the model is that the fundamental can be modeled as constant since the impossibility of exogenously deriving the precision of the social learning signal - formulated in appendix E - no longer holds true.

Perfect and private learning

We modeled social learning by a noisy public signal. However, we could introduce perfect learning, which involves uninformed agents observing $\bar{a_0}$ without any noise. The evolution of the fundamental state becomes $\theta_1 = \bar{a_0} + \varphi$ with $\varphi \sim N(0, \sigma_\varphi^2)$ to preclude agents from observing θ_1 with full precision, given that this would result in a trivial situation. In this case of perfect learning, uninformed agents in period $t = 1$ do not form expectations about the average action of informed agents in $t = 0$, since $\bar{a_0}$ is already part of their information set. The importance of the social learning signal for uninformed and informed agents - through anticipation - is substantially higher. This amplifies the focal role of public announcements. Overall, under perfect learning, our finding of optimal publicity being sensitive to the relevance of coordination motives and social learning proves robust. Furthermore, instead of modeling social learning by a public signal received by all uninformed agents, we could assume private learning. For instance, each uninformed agent might observe an action of one neighboring agent privately, which leaves the social learning signal without a focal role. However, as shown in section 2, informed agents account for (private) learning of uninformed agents and thus place higher weight on the public signal than in the static case. This anticipation effect reduces ex ante the optimal publicity of public information. Regardless of social learning running privately or publicly, public information is subject to a multiplier effect that decreases optimal publicity. Overall, our main result proves robust against the specific process of social learning.

Central bank power

In our framework, the central bank is capable of manipulating the fundamental state, which represents a distinct expansion of power compared to the static models of CH and MS. The central bank influences informed agents' actions through its public announcements, whereby subsequently the average action of the first period determines θ_1. Therefore, every public announcement y that might bring actions closer to the fundamental state θ_0 also manipulates the fundamental θ_1. This indirect influence on the fundamental is not the same as the "activist policy" formulated in the extension of the MS framework by James and Lawler (2011), who introduced a direct influence of the central bank on fundamentals through stabilization policy. Policy interventions g are modeled as being capable of completely resolving welfare distortions due to variations in θ. In their model, policy interventions that manipulate the fundamental state are an additional means of central bank policy. In our model, the fundamental state can only be manipulated through the intermediary of informed agents' actions. Public announcements of the central bank work as self-fulfilling prophecies. This more complex process might cause the central bank's influence to be more susceptible to potential distortions, e.g. unreasonable reactions by market participants. Furthermore, to establish an optimal design of communication and fully assess the consequences of central bank manipulations in such richer settings, one would have to account for central bank reputation and the capability of informed agents to anticipate biased announcements. The state of the economy might play a crucial role as it affects which type of information is perceived as most relevant. For instance, within a recession

or a crisis, uncertainty about private information might be high, causing overreactions to public information such as central bank announcements or social learning signals to grow stronger. This would strengthen the power of central banks as agents' expectations are more easily manipulated in such environments. However, this is not the focus of our paper and we thus abstain from formally analyzing these considerations in our model. Our welfare function only draws upon deviations from the fundamental state and not the desirability of manipulating the fundamental state itself.

Experimental evidence

Dale and Morgan (2012) and Cornand and Heinemann (2014) transfer the static game introduced by MS to a laboratory setting. They present experimental evidence on the focal role of public information, finding that agents significantly overweight public information compared to private information. However, the overreaction to public information is weaker than predicted by theory. Empirical weights are best predicted by a level-2 reasoning within a cognitive hierarchy model, which precludes public information from being detrimental to welfare Cornand and Heinemann (2013). Overall, the detrimental effect of public information is diminished when agents fail to form higher order beliefs. Baeriswyl and Cornand (2014) experimentally test the partial publicity argument introduced by CH. Comparing partial publicity - where only a fraction of all agents become informed - with partial transparency - where the public signal is disclosed to all agents, albeit with some ambiguity - they show that the two approaches are theoretically equivalent in their effectiveness of controlling overreaction. This equivalency resembles the results acquired in the lab. However, the authors argue that partial transparency is easier to implement and less discriminating as there are no uninformed agents. Experiments on social learning indicate that subjects are incapable of using Bayes rule optimally (Weizsäcker 2010). Agents cling to private information and underweight the value of social learning information. Social learning is insufficient, which results in informational inefficiencies. Transferring this core finding to our framework, we might similarly expect social learning to be weaker than predicted by theory. Agents might systematically underweight the information revealed by observable actions of the informed agents and cling to private information. The same might hold true for public information received by the informed agents. These considerations connect to the finding of a weaker overreaction to public information in the lab than predicted by theory Cornand and Heinemann (2014). In sum, the overreaction to publicly observable signals such as those issued by the central bank or derived by observable actions might be weaker than predicted by theory. Cognitive limitations or distrust in observable actions reduce the relevance of social learning yet increase the relevance of private information. Consequently, the actual optimal level of publicity might exceed the level derived theoretically by our model. However, since the interactions in our two-period framework are quite complex and anticipation effects might enter into play, future empirical research should explore the effectiveness of social learning and potential distortions.

4 Conclusion

We introduce observational learning in a two-period generalized beauty contest to study the optimal level of publicity for central bank information that is prone to multiplier effects. We show that as long as the social learning signal is not extremely imprecise, the optimal degree of publicity is lower in our dynamic setting than in the static case. This finding is driven by the focal role of the central bank signal for both the informed and uninformed agents.

Our results hold relevance for the optimal design of central bank communication. In choosing the degree of publicity, the central bank merely decides on the fraction of first-hand informed agents, while the uninformed agents will imperfectly learn about the public signal by observation. Social learning by the uninformed agents is anticipated and accounted for by the informed agents, prompting them to assign a higher weight to the central bank's signal. Accordingly, overreaction is substantially aggravated by the social learning dimension as it leads to initially uninformed agents being involved in the coordination game. For a reasonably precise social learning signal, the central bank has to reduce the publicity of its signal compared to the static case. Nonetheless, social learning – as a prominent multiplier of central bank communication – enhances aggregate welfare given an optimal policy design.

References

Angeletos, George-Marios and Iván Werning. Crises and prices: Information aggregation, multiplicity, and volatility. *American Economic Review*, 96(5):1720–1736, 2006.

Arato, Hiroki, Takeo Hori, and Tomoya Nakamura. Endogenous information acquisition and partial announcement policy, 2014. URL http://www.iser.osaka-u.ac.jp/library/dp/2014/DP0892R.pdf.

Baeriswyl, Romain and Camille Cornand. Reducing overreaction to central banks' disclosures: Theory and experiment. *Journal of the European Economic Association*, 12(4):1087–1126, 2014.

Banerjee, Abhijit V. A simple model of herd behavior. *The Quarterly Journal of Economics*, 107(3):797–817, 1992.

Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch. A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy*, 100(5):992–1026, 1992.

Chari, V.V and Patrick J. Kehoe. Financial crises as herds: overturning the critiques. *Journal of Economic Theory*, 119(1):128–150, 2004.

Colombo, Luca and Gianluca Femminis. The social value of public information with costly information acquisition. *Economics Letters*, 100(2):196–199, 2008.

Colombo, Luca, Gianluca Femminis, and Alessandro Pavan. Information acquisition and welfare, 2014. URL http://faculty.wcas.northwestern.edu/~apa522/IAW-March-25-2014.pdf.

Cornand, Camille and Frank Heinemann. Optimal degree of public information dissemination. *The Economic Journal*, 118(528):718–742, 2008.

Cornand, Camille and Frank Heinemann. Limited higher-order beliefs and the social value of public information, 2013. URL http://edoc.hu-berlin.de/series/sfb-649-papers/2013-39/PDF/39.pdf.

Cornand, Camille and Frank Heinemann. Measuring agents' reaction to private and public information in games with strategic complementarities. *Experimental Economics*, 17(1):61–77, 2014.

Curtat, L. O. Markov equilibria of stochastic games with complementarities. *Games and economic behavior*, 17:177–199, 1996.

Dale, Donald J. and John Morgan. Experiments on the social value of public information, 2012. URL http://faculty.haas.berkeley.edu/rjmorgan/CNBC.pdf.

Dasgupta, Amil. Social learning with payoff complementarities, 2000. URL https://ideas.repec.org/p/ecm/wc2000/0322.html.

Frisell, Lars. On the interplay of informational spillovers and payoff externalities. *The RAND Journal of Economics*, 34(3):582–592, 2003.

Haan, Jakob de, Sylvester C.W Eijffinger, and Krzysztof Rybiński. Central bank transparency and central bank communication: Editorial introduction. *European Journal of Political Economy*, 23(1):1–8, 2007.

Hellwig, Christian and Laura Veldkamp. Knowing what others know: Coordination motives in information acquisition. *Review of Economic Studies*, 76(1):223–251, 2009.

James, Jonathan G. and Phillip Lawler. Optimal policy intervention and the social value of public information. *American Economic Review*, 101(4):1561–1574, 2011.

Maskin, E. and Jean Tirole. Markov perfect equilibrium: i. observable actions. *Journal of Economic Theory*, 100(2):191–219, 2001.

Molavi, Pooya, Ceyhun Eksin, Alejandro Ribeiro, and Ali Jadbabaie. Learning to coordinate in social networks, 2014. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2292124.

Morris, Stephen and Hyun Song Shin. Social value of public information. *American Economic Review*, 92(5):1521–1534, 2002.

Morris, Stephen and Hyun Song Shin. Optimal communication. *Journal of the European Economic Association*, 5(2-3):594–602, 2007.

Morris, Stephen, Hyun Song Shin, and Hui Tong. Response to social value of information: Morris and shin (2002) is actually pro transparency, not con. *American Economic Review*, 96 (1):453–455, 2006.

Romer, Christina and David Romer. Federal reserve information and the behavior of interest rates. *American Economic Review*, 90(3):429–457, 2000.

Svensson, Lars E. O. Social value of public information: Comment: Morris and shin (2002) is actually pro-transparency, not con. *American Economic Review*, 96(1):448–452, 2006.

Trabelsi, Emna. Central bank communication fragmentation as an engine for limiting, 2010. URL http://mpra.ub.uni-muenchen.de/26647/.

Ui, Takashi. The social value of public information with convex costs of information acquisition. *Economics Letters*, 125(2):249–252, 2014.

Weizsäcker, Georg. Do we follow others when we should? a simple test of rational expectations. *American Economic Review*, 100(5):2340–2360, 2010.

Woodford, Michael. *Interest & Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, 2003.

Woodford, Michael. Central bank communication and policy effectiveness, 2005. URL http://www.nber.org/papers/w11898.

Appendix A

Derivation of optimal weights on the private and the public signal in t=1

We start the derivation of optimal weights for uninformed agents by considering equation (7)

$$\begin{aligned} a_{i1}^{*} = \; & (1-r)\frac{\beta}{\beta+\delta}x_{i1} + \left(1-\frac{\beta}{\beta-\delta}\right)s \\ & + r\left\{P\left[\frac{\beta}{\beta+\delta}x_{i1} + \left(1-\frac{\beta}{\beta+\delta}\right)s\right] + (1-P)\left[\frac{\gamma_1\beta}{\beta+\delta}x_{i1} + \left(1-\frac{\gamma_1\beta}{\beta+\delta}\right)s\right]\right\}, \end{aligned}$$

which can be rearranged such that

$$a_{i1}^{*} = \left(\frac{\beta - r\beta + rP\beta + r\gamma_1\beta - r\gamma_1\beta P}{\beta+\delta}\right)x_{i1} + \left(1 - \frac{\beta - r\beta + rP\beta + r\gamma_1\beta - r\gamma_1\beta P}{\beta+\delta}\right)s.$$

It can be seen that the two multipliers of x_{i1} and s are perfectly symmetric. Since we assumed that all agents use their available information in a linear fashion (see equations (2) and (3)), we obtain

$$\gamma_1 = \frac{\beta - r\beta + rP\beta + r\gamma_1\beta - r\gamma_1\beta P}{\beta+\delta},$$

from which we can solve for the optimal weight on the private signal

$$\gamma_1^{*} = \frac{\beta[1-r(1-P)]}{\delta + \beta[1-r(1-P)]}.$$

By symmetry, the optimal weight on the public social learning signal s is then given by

$$(1-\gamma_1^{*}) = 1 - \frac{\beta[1-r(1-P)]}{\delta + \beta[1-r(1-P)]}.$$

Appendix B

Derivation of the optimal weight on the private and public signal in period t=0

We start the derivation of optimal weights for informed agents by considering equation (11)

$$\begin{aligned} a_{i0}^{*} = \; & (1-r)\left[\frac{\beta}{\alpha+\delta}x_{i0} + \left(1-\frac{\beta}{\alpha+\beta}\right)y\right] + r\left\{P\left[\frac{\gamma_0\beta}{\alpha+\beta}x_{i0} + \left(1-\frac{\gamma_0\beta}{\alpha+\beta}\right)y\right]\right. \\ & \left. +(1-P)\left[(1-\gamma_{i1}^{*})\left[\frac{\gamma_0\beta}{\alpha+\beta}x_{i0} + \left(1-\frac{\gamma_0\beta}{\alpha+\beta}\right)y\right] + \gamma_1^{*}\left[\frac{\beta}{\alpha+\beta}x_{i0} + \left(1-\frac{\beta}{\alpha+\beta}\right)y\right]\right]\right\}, \end{aligned}$$

which can be rearranged such that

$$a_{i0}^* = \left(\frac{\beta}{\alpha+\beta} - \frac{r\beta}{\alpha+\beta} + \frac{r\gamma_0\beta}{\alpha+\beta} - \frac{r\gamma_1^*\gamma_0\beta}{\alpha+\beta} + \frac{r\gamma_1^*\beta}{\alpha+\beta} + \frac{r\gamma_1^*P\gamma_0\beta}{\alpha+\beta} - \frac{r\gamma_1^*P\beta}{\alpha+\beta}\right)x_{i0}$$
$$+\left(1 - \frac{\beta}{\alpha+\beta} - \frac{r\beta}{\alpha+\beta} + \frac{r\gamma_0\beta}{\alpha+\beta} - \frac{r\gamma_1^*\gamma_0\beta}{\alpha+\beta} + \frac{r\gamma_1^*\beta}{\alpha+\beta} + \frac{r\gamma_1^*P\gamma_0\beta}{\alpha+\beta} - \frac{r\gamma_1^*P\beta}{\alpha+\beta}\right)y.$$

As in the case for uninformed agents, there is perfect symmetry in the multipliers of x_{i0} and y. Since it is assumed that information is used in a linear fashion (see equations (2) and (3)), we obtain

$$\gamma_0 = \frac{\beta}{\alpha+\beta} - \frac{r\beta}{\alpha+\beta} + \frac{r\gamma_0\beta}{\alpha+\beta} - \frac{r\gamma_1^*\gamma_0\beta}{\alpha+\beta} + \frac{r\gamma_1^*\beta}{\alpha+\beta} + \frac{r\gamma_1^*P\gamma_0\beta}{\alpha+\beta} - \frac{r\gamma_1^*P\beta}{\alpha+\beta},$$

from which we can solve for the optimal weight on the private signal

$$\gamma_0^* = \frac{\beta[1 - r(1 - \gamma_1^*(1-P))]}{\alpha + \beta[1 - r(1 - \gamma_1^*(1-P))]}.$$

By symmetry, the optimal weight on the public signal y is then given by

$$(1 - \gamma_0^*) = 1 - \frac{\beta[1 - r(1 - \gamma_1^*(1-P))]}{\alpha + \beta[1 - r(1 - \gamma_1^*(1-P))]}.$$

Appendix C

Proof of Uniqueness for t=1

Here we prove that the equilibrium strategy identified for t=1 is a linear combination of available signals x_{i1} and s. The proof follows CH (p. 738). Intuitively, it has to be proven that the infinite sum of average expectations of the average expectations...(k times) is bounded and converges to the linear equilibrium identified in section 1.2. The optimal action of uninformed agents is given by the first-order condition of the utility function

$$a_{i1} = (1 - r)E_i(\theta_1) + rE_i(\overline{a}).$$

The average action of all agents is given by

$$\overline{a} = P\int_0^1 a_{i0}(x_{i0}, y)di + (1 - P)\int_0^1 a_{i1}(x_{i1}, s)di$$

Since we assumed $\theta_1 = \overline{a}_0$, we obtain

$$\overline{a} = P\theta_1 + (1 - P)\overline{a}_1(\theta_1, s),$$

which can be plugged into the FOC such that

$$\begin{aligned}
a_{i1} &= (1-r)E_i(\theta_1) + rE_i[P\theta_1 + (1-P)\overline{a}_1(\theta_1, s)] \\
&= [1-r(1-P)]E_i(\theta_1) + r(1-P)E_i[\overline{a}_1(\theta_1, s)] \\
&= [1-r(1-P)]E_i(\theta_1) + [r(1-P)][1-r(1-P)]E_i[\overline{E}_1(\theta_1)] + [r(1-P)]^2 E_i[\overline{E}_1(\overline{a}_1|\theta_1, s)] \\
&= [1-r(1-P)]\sum_{k=0}^{\infty}[r(1-P)]^k E_i[\overline{E}_1^k(\theta_1)] + \lim_{k\to\infty}[r(1-P)]^k E_i[\overline{E}_1^k(\overline{a}_1|\theta_1, s)],
\end{aligned}$$

where $\overline{E}_1^k(\theta_1)$ denotes the average expectation of the average expectation... (k times) of θ_1 and $\overline{E}_1^k(\overline{a}_1)$ is the average expectation of the average expectation... (k times) of the average action in $t = 1$. Since we assume $0 < r < 1, r(1-P) < 1$ and thus the limes term in the above expression is zero. Thus, the equation reduces to

$$= [1-r(1-P)]\sum_{k=0}^{\infty}[r(1-P)]^k E_i[\overline{E}_1^k(\theta_1)].$$

Following Lemma 1 from MS (p. 1527), we can substitute $E_i[\overline{E}_1^k(\theta_1)] = (1-\mu^{k+1})s + \mu^{k+1}x_{i1}$ with $\mu = \frac{\beta}{\delta+\beta}$ which yields

$$\begin{aligned}
&= [1-r(1-P)]\sum_{k=0}^{\infty}[r(1-P)]^k(1-\mu^{k+1})s + \mu^{k+1}x_{i1} \\
&= [1-r(1-P)]\left[s\sum_{k=0}^{\infty}[r(1-P)]^k - s\mu\sum_{k=0}^{\infty}[r(1-P)\mu]^k + x_{i1}\mu\sum_{k=0}^{\infty}[r(1-P)\mu]^k\right].
\end{aligned}$$

Now, since $r(1-P)\mu < 1$, the infinite sums in the expression above $\sum_{k=0}^{\infty}[r(1-P)\mu]^k$ converge to $\frac{1}{1-r(1-P)\mu}$. Similarly, the expressen $\sum_{k=0}^{\infty}[r(1-P)]^k$ converges to $\frac{1}{1-r(1-P)}$.We can thus write

$$\begin{aligned}
&= [1-r(1-P)]\left[\frac{s}{1-r(1-P)} - \frac{s\mu}{1-r(1-P)\mu} + \frac{x_{i1}\mu}{1-r(1-P)\mu}\right] \\
&= s - \frac{s\mu[1-r(1-P)]}{1-r(1-p)\mu} + \frac{x_{i1}\mu[1-r(1-P)]}{1-r(1-p)\mu}.
\end{aligned}$$

Inserting $\mu = \frac{\beta}{\delta+\beta}$ and rearranging terms yields

$$= \left[1 - \frac{\beta[1-r(1-P)]}{\delta+\beta[1-r(1-P)]}\right]s + \frac{\beta[1-r(1-P)]}{\delta+\beta[1-r(1-P)]}x_{i1},$$

which replicates the linear equilibrium established in section 1.2 and completes the proof.

Appendix D

Proof of uniqueness for $t = 0$

The proof of uniqueness of the equilibrium strategy in t=0 follows the same logic as for t=1. The optimal action of informed agents is given by the first-order condition of the utility function

$$a_{i0} = (1-r)E_i(\theta_0) + rE_i(\overline{a}).$$

The average action is given by

$$\begin{aligned}\overline{a} &= P\int_0^1 a_{i0}(x_{i0}, y)di + (1-P)\int_0^1 a_{i1}(x_{i1}, s)di \\ &= P\overline{a}_0(\theta_0, y) + (1-P)\overline{a}_1(\theta_1, s),\end{aligned}$$

which can be plugged into the FOC such that

$$a_{i0} = (1-r)E_{i0}(\theta_0) + rE_{i0}(P\overline{a}_0(\theta_0, y) + (1-P)\overline{a}_1(\theta_1, s)).$$

Since $\overline{a}_1 = \gamma_1^*\theta_1 + (1-\gamma_1^*)s$, we can write

$$= (1-r)E_{i0}(\theta_0) + rPE_{i0}[\overline{a}_0(\theta_0, y)] + r(1-P)\gamma_1^* E_{i0}(\theta_1) + r(1-P)(1-\gamma_1^*)E_{i0}(s).$$

Using the fact that $E(\theta_1) = E(\theta_0)$ and $E(s) = \overline{a}_0$ and summarizing terms yields

$$\begin{aligned}&= [1 - r(1-\gamma_1^*(1-P))]\, E_{i0}(\theta_0) + r[1-\gamma_1^*(1-P)]E_{i0}[\overline{a}_0(\theta_0, y)] \\ &= [1 - r(1-\gamma_1^*(1-P))]\, E_{i0}(\theta_0) + r[1-\gamma_1^*(1-P)]\,[1 - r(1-\gamma_1^*(1-P))]\, E_{i0}[\overline{E}_0(\theta_0)] \\ &\quad + [(r(1-\gamma_1^*(1-P))]^2\, E_{i0}[\overline{E}_0(\overline{a}_0|\theta_0, y)] \\ &= [1 - r(1-\gamma_1^*(1-P))]\sum_{k=0}^{\infty}[r(1-\gamma_1^*(1-P))]^k\, E_{i0}[\overline{E}_0^k(\theta_0)] \\ &\quad + \lim_{k\to\infty}[r(1-\gamma_1^*(1-P))]^k\, E_{i0}[\overline{E}_0^k(\overline{a}_0|\theta_0, y)],\end{aligned}$$

where $\overline{E}_0^k(\theta)$ denotes the average expectation of the average expectation... (k times) of θ_0 and $\overline{E}_0^k(\overline{a}_0)$ is the average expectation of the average expectation... (k times) of the average action in t=0. Since we assume $0 < r < 1, r[1-\gamma_1^*(1-P)] < 1$ and thus the limes term in the above expression is zero. Thus, the equation reduces to

$$= [1 - r(1-\gamma_1^*(1-P))]\sum_{k=0}^{\infty}[r(1-\gamma_1^*(1-P))]^k\, E_{i0}[\overline{E}_0^k(\theta_0)].$$

Following Lemma 1 from MS (p.1527), we can substitute $E_{i0}[\overline{E}_0^k] = (1-\mu^{k+1})y + \mu^{k+1}x_{io}$ with $\mu = \frac{\beta}{\alpha+\beta}$ which yields

$$
\begin{aligned}
&= [1 - r(1-\gamma_1^*(1-P))] \sum_{k=0}^{\infty} [r(1-\gamma_1^*(1-P))]^k (1-\mu^{k+1})y + \mu^{k+1}x_{i0} \\
&= [1 - r(1-\gamma_1^*(1-P))] \left\{ y \sum_{k=0}^{\infty} [r(1-\gamma_1^*(1-P))]^k - y\mu \sum_{k=0}^{\infty} [r(1-\gamma_1^*(1-P))\mu]^k \right. \\
&\quad \left. + x_{io}\mu \sum_{k=0}^{\infty} [r(1-\gamma_1^*(1-P))\mu]^k \right\}.
\end{aligned}
$$

Now, since $r[1-\gamma_1^*(1-P)]\mu < 1$, the infinite sums in the last expression above $\sum_{k=0}^{\infty}[r(1-\gamma_1^*(1-P))\mu]^k$ converge to $\frac{1}{1-r[1-\gamma_1^*(1-P)]}$ and we obtain

$$
\begin{aligned}
&= [1 - r(1-\gamma_1^*(1-P))] \\
&\quad \left\{ \frac{y}{1-r[1-\gamma_1^*(1-P)]} - \frac{y\mu}{1-r[1-\gamma_1^*(1-P)]\mu} + \frac{x_{i0}\mu}{1-r[1-\gamma_1^*(1-P)]\mu} \right\} \\
&= y - \frac{y\mu[1-r(1-\gamma_1^*(1-P))]}{1-r[1-\gamma_1^*(1-P)]\mu} + \frac{x_{i0}\mu[1-r(1-\gamma_1^*(1-P))]}{1-r[1-\gamma_1^*(1-P)]\mu}
\end{aligned}
$$

Inserting $\mu = \frac{\beta}{\alpha+\beta}$ and rearranging terms yields

$$
= \left[1 - \frac{\beta[1-r(1-\gamma_1^*(1-P))]}{\alpha+\beta[1-r(1-\gamma_1^*(1-P))]}\right] y + \frac{\beta[1-r(1-\gamma_1^*(1-P))]}{\alpha+\beta[1-r(1-\gamma_1^*(1-P))]} x_{i0},
$$

which replicates the linear equilibrium established in section 3 and completes the proof.

Appendix E

In section 1.2, we introduced the assumption that $\theta_1 = \overline{a}_0$. Accordingly, the average action of the informed agents determines the new fundamental state of the second period, which has to be matched by the uninformed agents. In the following, we outline a formal argument concerning why it is impossible to assume a constant fundamental over periods in our framework. Consider θ drawn from the real line and constant over periods. Agents from the second period receive $\overline{a}_0$ without any noise (perfect learning). Subsequently, agents need to know how $\overline{a}_0$ is distributed around θ to derive the optimal weight relative to their private signal x_{i1}. Since we assume $a_{i0} = \gamma x_{i0} + (1-\gamma)y$ and $E(x_i) = E(y) = \theta$, $E(a_{i0}) = \theta$ and $E(\overline{a}_0) = \theta$ and $\overline{a}_0$ is an unbiased signal of the fundamental state. This holds as $\gamma + (1-\gamma) = 1$. However, the variance of $\overline{a}_0$ has to be considered relevant as it depends on the equilibrium weights on x_{i0} and y. Formally, we can write

$$
Var(\overline{a}_0) = \gamma^2\theta_0 + (1-\gamma)^2 y,
$$

where γ is $\gamma_0^* = \frac{\beta[1-r(1-\gamma_1^*(1-P))]}{\alpha+\beta[1-r(1-\gamma_1^*(1-P))]}$. This optimal weight γ_0^* relies on the optimal weight in t=1, which is $\gamma_1^* = \frac{\beta[1-r(1-P)]}{\delta+\beta[1-r(1-P)]}$. As γ_1^* is based upon the precision of the social learning signal δ, we would need to know δ to solve for $Var(\overline{a}_0)$ or its precision $\frac{1}{Var(\overline{a}_0)} = \delta$. As a matter of logic, this is not possible. The underlying intuition stems from agents having inter-temporal coordination incentives. Informed agents account for the optimal play in the subsequent second period, which makes it impossible to exogenously derive the precision of the endogenous signal $\overline{a}_{t=0}$. Therefore, we model a fundamental state that evolves according to the average action of the informed agents. This enables us to introduce an exogenous signal s with an exogenously determined precision whereby all agents form common priors over the precision of all signals.

Appendix F

Here, we present the first derivative of the welfare function

$$E[W(a,\theta)] = -P\frac{\beta[1-r(1-\gamma_1^*(1-P))]^2+\alpha}{[\alpha+\beta(1-r(1-\gamma_1^*(1-P))]^2} - (1-P)\frac{\beta[1-r(1-P)]^2+\delta}{[\delta+\beta(1-r(1-P))]^2},$$

where $\gamma_1^* = \frac{\beta[1-r(1-P)]}{\delta+\beta[1-r(1-P)]}$. Maximizing expected welfare with respect to $0 < P \leq 1$ yields for

$$\begin{aligned}
\frac{\delta E(W)}{\delta P} = & \frac{2\beta r\Big[1-r\Big(1-\frac{\beta[1-r(1-P)](1-P)}{\beta[1-r(1-P)]+\delta}\Big)\Big]\Big[\frac{\beta^2 r[1-r(1-P)](1-P)}{[\beta(1-r(1-P))+\delta]^2}-\frac{\beta r(1-P)}{\beta[1-r(1-P)]+\delta}+\frac{\beta[1-r(1-P)]}{\beta[1-r(1-P)]+\delta}\Big]P-\beta\Big[1-r\Big(1-\frac{\beta[1-r(1-P)](1-P)}{\beta[1-r(1-P)]+\delta}\Big)\Big]^2-\alpha}{\Big[\beta\Big(1-r\Big(1-\frac{\beta[1-r(1-P)](1-P)}{\beta[1-r(1-P)]+\delta}\Big)\Big)+\alpha\Big]^2} \\
& + \frac{2\beta r\Big[\frac{\beta^2 r[1-r(1-P)](1-P)}{[\beta[1-r(1-P)]+\delta]^2}-\frac{\beta r(1-P)}{\beta[1-r(1-P)]+\delta}+\frac{\beta[1-r(1-P)]}{\beta[1-r(1-P)]+\delta}\Big]\Big[-\beta\Big(1-r\Big(1-\frac{\beta[1-r(1-P)](1-P)}{\beta[1-r(1-P)]+\delta}\Big)\Big)^2P-\alpha P\Big]}{\Big[\beta\Big(1-r\Big(1-\frac{\beta[1-r(1-P)](1-P)}{\beta[1-r(1-P)]+\delta}\Big)\Big)+\alpha\Big]^3} \\
& + \frac{-2\beta r[1-r(1-P)](1-P)+\beta[1-r(1-P)]^2+\delta}{[\beta(1-r(1-P))+\delta]^2} - \frac{2\beta r[\beta[1-r(1-P)]^2(1-P)-\delta(1-P)]}{[\beta[1-r(1-P)]+\delta]^3}.
\end{aligned}$$

The first two lines of the derivative correspond to the first part of the welfare function (loss in t=0), while the last line corresponds to the second part (loss in t=1).

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