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On the long-run equilibrium value of Tobin's average Q

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FINANCIAL DISTORTIONS AND MACROECONOMIC
PERFORMANCE: EXPECTATIONS, CONSTRAINTS AND
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On the Long-Run Equilibrium Value of Tobin's Average Q

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Abstract

This note considers Tobin's average Q in a framework where firms finance investment by equities and debt. The determination of its long-run equilibrium value Q^o is based on positing equality of the loan rate and, adjusted for a risk premium, the return on equities. Q^o can thus be characterized as a ratio of two rates representing the somewhat modified interest costs and profits of the firms. The familiar benchmark value $Q^o = 1$ obtains if another condition on the risk premium holds true, which may or may not be the case. An elementary numerical check demonstrates that possible deviations of Q^o from unity are not overly dramatic.

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1 Introduction

A central variable in macroeconomic growth models where firms finance their investment by debt and equities is the ratio of outside finance to the capital stock. This concept was first put forward by Kaldor (1966), who simply referred to it as

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the “valuation ratio”, and then brought to prominence by Tobin (1969) and a number of subsequent writings, from when on it became known as Tobin’s (average) Q .¹

For Tobin himself and much of the literature, the obvious value for a long-run equilibrium was $Q = 1$ (for example, Tobin 1969: 23). Regarding possible reactions in the economy to deviations of Q from this benchmark, probably best known is the ‘common sense’ argument “that the incentive to make new capital investments is high when the securities giving title to their future earnings can be sold for more than the investment cost, i.e., when q exceeds one” (Tobin 1996: 15), while arbitrage opportunities deter the production of new capital goods in the opposite case.² Considering the substantial delays in such a process, Tobin (1998: 107) interestingly adds later that “in a sense the stock market acts out in advance and in purely financial terms the subsequent and much slower adjustment of the physical stock of capital”.³

Under closer scrutiny these arguments seem to apply to investment at the margin or, in more technical terms, with the notion of marginal rather than average Q .⁴ This may be one of the reasons why contemporary structuralist, post-Keynesian modelling has practically no recourse to them. As a consequence, Q is here generally different from unity in a steady state; some representative examples are the stock-flow consistent models of a real-financial interaction by Godley and Lavoie (2007), Dos Santos and Zezza (2008), Le Heron and Mouakil (2008), Van Treeck (2008), or Bernardo et al. (2015). In these works, Q may actually attain any value in a steady state. It appears as the accidental result of the interplay of some components in the model that, however, bear no clear relation to the valuation ratio. That is, so far the models are not very thoroughly concerned with an explicit theory of the determination of the long-run equilibrium value of Q .

The present note sets out to provide some elementary insights that may prove useful for developing such a theory, which abstains from marginalist concepts. To this end, it characterizes Q by a ratio of certain rates of return on the real

¹We write capital ‘ Q ’ in a purely descriptive context such as presently in the Introduction, and switch to lower case ‘ q ’ when we turn to the formal arguments.

²As far as Q may affect investment decisions, this reasoning was already essentially anticipated by Keynes (1936: 151): “the daily revaluations of the stock exchange . . . inevitably exert a decisive influence on the rate of current investment. For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased . . . if it can be floated off on the Stock Exchange at an immediate profit.” (See also the footnote that Keynes adds to this quotation.)

³At the same place, Tobin also sketches other equilibrator mechanisms (pp. 106f).

⁴Marginal Q as a key determinant of investment was formally introduced in neoclassical models, which typically include adjustment costs. Marginal and average Q were shown to coincide if these costs exhibit constant returns (Hayashi, 1982).

and financial assets, and this relationship serves to work out consistency conditions for a long-run equilibrium position. In particular, one condition with a suitable risk premium entering it is identified that, together with the others, would yield $Q = 1$ in equilibrium. Because, however, it cannot be expected to hold true in general, we will also carry out a provisional numerical check to get a first feeling for possible deviations of Q from unity. It is found out in this way that typically these deviations will be of a fairly moderate order of magnitude.

2 Some basic relationships in the business sector

Consider an economy where besides retained earnings, firms finance fixed investment from external sources by issuing equities and raising credits. Accordingly, their accumulated liabilities are given by the loans outstanding L and by the evaluation $p_e E$ of the firms on the stock market, E being the number of shares and p_e their price. The assets on the other side of the balance sheet of the firms are the stock of fixed capital K valued at the current price p .⁵ Tobin's q is the ratio of the liabilities to the assets, and λ and e are the single ratios of the loans and equities, which add up to q :

$$q = \frac{L + p_e E}{pK}, \quad \lambda = \frac{L}{pK}, \quad e = \frac{p_e E}{pK} \quad (1)$$

Let a dot above a dynamic variable x denote its derivative with respect to time, $\dot{x} = dx/dt$, and a caret its growth rate $\hat{x} = \dot{x}/x$. Furthermore, denote the growth rates of loans and equities as $g_L = \hat{L}$ and $g_E = \hat{E}$, respectively, and by g and π the growth rate of the capital stock, $g = \hat{K}$, and the rate of goods price inflation, $\pi = \hat{p}$. The latter two may well be supposed to be positive, that is, the economy is generally growing in an inflationary environment.

Since $\dot{q} = \dot{\lambda} + \dot{e} = \lambda \hat{\lambda} + e \hat{e}$ and $\hat{\lambda} = g_L - g - \pi$, $\hat{e} = \hat{p}_e + g_E - g - \pi$, the motions of q are described by the equation

$$\dot{q} = \lambda g_L + e(\hat{p}_e + g_E) - q(g + \pi) \quad (2)$$

(the last term uses $\lambda + e = q$). In our discussion of the long-run equilibrium features in this framework, the values of the variables in a steady state are indicated by a superscript 'o'. To begin with, what is invariant in this position is not the stock prices but rather the ratio q , that is, $\dot{q} = 0$ when the loan and stock markets are

⁵The short-term as well as long-term financial assets in the corporate business are thus neglected, or the loans L may thought of net of them.

growing in step. Solving this equality in (2) for stock price inflation and writing from now on $\pi_e = \hat{p}_e$, the equilibrium value π_e^o is seen to be given by

$$\pi_e^o := \pi_e \Big|_{\dot{q}=0} = \frac{1}{e} \left[q(g + \pi) - \lambda g_L - e g_E \right] \quad (3)$$

If in addition both markets themselves are on an equilibrium path, so that $\hat{\lambda} = g_L - \pi - g = 0$ (which also implies $\hat{e} = 0$), the prices on the stock market rise at a rate

$$\pi_e \Big|_{\dot{q}=\hat{\lambda}=0} = \pi + (g - g_E) \quad (4)$$

It is thus readily checked that stock prices rise at the same rate as goods prices if, and only if, the firms issue equities at the real rate of growth, $g_E = g$. Generally, stock price inflation exceeds the inflation rate for goods by the difference between g_E and g . To get an impression of a reasonable order of magnitude of such an equilibrium stock price inflation, we can refer to US data and observe that over the last three decades there was hardly ever a positive net issuance of equities. Firms rather followed a strategy of buying their shares back from the market, so that typically g_E was distinctly negative.⁶ If in the (very) long term prices on the stock market show a tendency to rise faster than prices in the real sector, this is therefore not only due to a permanent speculative pressure.⁷

The core of the model is the finance equation of the firms. It says that the net investment $g pK$ is financed by internal and external sources. Internal financing is what the firms retain from their surplus $r pK$ (r being the rate of profit net of depreciation) after paying the interest costs iL (i the loan rate), the corporate income tax (based on a proportional tax rate τ_c levied on $r pK - iL$), and the dividends $d pK$ (d defined as the ratio of the dividends to the capital stock). External financing is given by (i) issuing new shares \dot{E} at the going price p_e and (ii) raising new loans \dot{L} . Taken together, $g pK = (1 - \tau_c)(r pK - iL) - d pK + \dot{L} + p_e \dot{E}$. The finance equation in intensive form is obtained by normalizing the variables by the capital stock pK . Using $\dot{L}/pK = (\dot{L}/L)(L/pK) = \lambda g_L$, $p_e \dot{E}/pK = (p_e \dot{E}/p_e E)(p_e E/pK) = e g_E$, it reads,

$$g = (1 - \tau_c)(r - i\lambda) - d + \lambda g_L + e g_E \quad (5)$$

Given the limited purpose of this note we need not discuss the determination of the variables in this identity; especially the profit rate may be treated as in the

⁶May it suffice to refer to Figures 2 and 3 in Skott and Ryoo (2008: 831) or, without their time aggregation, to Figure 1 in Ryoo (2010: 166).

⁷Equation (4) is put forward for making this elementary observation but it is not necessarily needed in the following analysis; working with eq. (3) will then already do.

post-Keynesian or Kaleckian models of the real sector studying growth and income distribution. We do not even need to make an assumption about the causality in this relationship. That is, it can be left open which of the variables is predetermined in some sense, by a function or a dynamic adjustment equation, and which is then residually determined. For example, one may follow Hein (2013) and assume a constant proportion \bar{d}_e of the dividend payments to the stock market value of the firms (rather than to their capital stock), so that $d = e\bar{d}_e$; or, what is more common in the literature, one may assume a constant after-tax retention rate s_f , so that the dividends normalized by the capital stock are determined as $d = (1 - s_f)(1 - \tau_c)(r - i\lambda)$ and therefore vary with r, i as well as λ .

3 The return on equities

To study the financial sector of the economy insofar as Tobin's q plays a role for its long-run equilibrium position, we begin with the rate of return r_e that the rentiers earn by holding equities. It is given by the dividends in relation to this investment $p_e E$, plus the capital gains π_e . Solving the finance equation (5) for d , the equilibrium value r_e^o when $\dot{q} = 0$ (and thus (3) holds true) is computed as

$$r_e^o := \frac{d}{e} + \pi_e^o = \frac{1}{e} \left[(1 - \tau_c)(r - i\lambda) - g + q(\pi + g) \right] \quad (6)$$

We next introduce a risk premium ξ_e of the equities. Given that loans and equities are not perfect substitutes, it is that premium that compensates the rentiers for the greater risk on the stock market and makes them indifferent between the returns from their investment in equities and the interest that they could earn from lending their money to the firms (directly or indirectly, intermediated by commercial banks). In such a situation, and only in such a situation, will the rentiers no longer reshuffle their portfolio and the two ratios e and λ can remain invariant. Let us thus formulate:⁸

Equilibrium Condition 1

With respect to a given risk premium ξ_e of the rentiers regarding their investment on the stock market,

$$r_e^o - \xi_e = i$$

prevails in a long-run equilibrium position.

⁸While the premium ξ_e is here treated as given, in a full-fledged dynamic model it may interact with q and other key variables. Also, for simplicity, we do not distinguish between the loan rate of the firms and a possibly different interest rate relevant to the rentiers.

To make it perfectly clear, Condition 1 is a necessary but by no means sufficient condition for a long-run equilibrium to come about.

Actually, eq. (6) indicates that r_e is inversely related to the equity ratio e (which also holds when $\dot{q} \neq 0$). Hence a relatively low e can be one reason for a situation where $r_e - \xi_e$ exceeds the interest rate i . The latter feature, however, will induce a larger group of rentiers to increase their demand on the stock market, which should then speed up the increase in stock prices and thus have a positive effect on the entire ratio $e = p_e E / pK$. This, in turn, introduces a negative feedback on the return rate r_e , so that the gap between $r_e - \xi_e$ and i is reduced. In this way we have sketched an elementary mechanism that may tend to restore Condition 1 in case it were violated.⁹

4 Tobin's q in the steady state: Model version A

The next two sections are directed at a characterization of the long-run equilibrium value of Tobin's q . Since, the ratios q , λ , e and the ratios in the desired portfolio of the rentiers are not independent of one another, their equilibrium values may generally be determined in a simultaneous manner. Before setting up a full-fledged model design it is, however, useful to treat some of them as parametrically given and examine the resulting implications.

In the present section we concentrate on a fixed value λ^o of the debt-asset ratio. It may even be argued that the firms borrow from commercial banks and λ^o comes about in an interaction of just these two parties, supposing that the banks do not particularly care about the firms' equity ratio as long as it remains within certain bounds.

The characterizations of q^o in which we are interested do not refer to the stock variables of the original definition of the valuation ratio. Loosely speaking, they are rather concerned with flow magnitudes, which include rates of return, of growth, and inflation. Proposition 1 presents two formulations, where the present setting with a given λ^o may be indicated by the letter 'A'.

Proposition 1

Suppose Condition 1 holds true. Then the long-run equilibrium values of q and λ are interrelated as follows,

$$q^o = \frac{r - \tau_c(r - i\lambda^o) + \lambda^o \xi_e - g}{i - \pi + \xi_e - g} \quad (7)$$

⁹Of course, this does not rule out that in a speculation dynamics there may also be other mechanisms at work, some of which could well be (locally) destabilizing.

Equivalently, putting

$$\xi_f^A := r - \tau_c(r - i\lambda^o) - (i - \pi) \quad (8)$$

and taking $e^o = q^o - \lambda^o$ into account,

$$q^o = \frac{r - \tau_c(r - i\lambda^o) - e^o\xi_e - g}{i - \pi - g} = 1 + \frac{\xi_f^A - e^o\xi_e}{i - \pi - g} \quad (9)$$

The proof is straightforward. Substitute (6) in Condition 1, multiply it by e , put $ei = qi - i\lambda$ and $e\xi_e = q\xi_e - \lambda\xi_e$, and solve the resulting equation for $q = q^o$. This yields eq. (7). The second part of the proposition is derived in the same manner, only the step $e\xi_e = q\xi_e - \lambda\xi_e$ is omitted.

We take it, of course, for granted that the denominator in (7) is positive. This equation sets up a pure functional relationship between the equilibrium values of λ and q . It is interesting to note that under a *ceteris paribus* condition an increase in the debt-asset ratio λ^o results in a higher q^o . The reaction is also economically plausible if we refer to the net worth of the firms, which is reduced by higher loans.¹⁰ A possible counter-effect could be that therefore the stock market devalues the equities of the firms, but intuitively it should not be strong enough to offset the increase in loans. Equation (7) enables us to confirm this reasoning mathematically if $(1 - \tau_c i)i > \pi + g$ is assumed. In fact, $\partial e^o / \partial \lambda^o < 0$ because $e^o = q^o - \lambda^o$ and $\partial q^o / \partial \lambda^o = (\tau_c i + \xi_e) / (i - \pi - g + \xi_e)$ is then less than one.¹¹

Equation (9) in the second part of the proposition has q^o also appearing on the right-hand side, via $e^o = q^o - \lambda^o$, but on the other hand this relationship seems to make better economic sense. The fraction in the first equation in (9) relates two quasi-rates of return, we may say, which (given $\lambda = \lambda^o$) could be of some relevance to the managers of the firms. The numerator displays the (after-tax) profit rate adjusted for growth and also discounted by the risk premium. As the latter only applies to equities, ξ_e is multiplied by the equity ratio $e = p_e E / pK$. The denominator is the real interest rate, likewise adjusted for growth.

An anonymous referee made us aware that the approach taken in Proposition 1 can be understood as an extension of a formula for q that Richard Kahn (1972) put forward already more than 40 years ago. In a framework without taxes and debt

¹⁰The net worth is here defined as the difference between assets and total liabilities. The former are given by the capital stock valued at its replacement cost, the latter by the outstanding loans and equities. In the present notation, net worth = $(1 - \lambda - e)pK = (1 - q)pK$, which means that an increase in q is tantamount to a decrease in the firms' net worth.

¹¹Recalling that i is not a riskless rate of interest but the firms' loan rate, which is higher, we will argue later that the inequality $(1 - \tau_c i)i > \pi + g$ can indeed be considered to be empirically satisfied. Otherwise we would have $\partial e^o / \partial \lambda^o > 0$, the economic reason for which would not be so clear.

financing (i.e., $\tau_c = 0$, $\lambda^o = 0$), he expressed the valuation ratio as the difference between the profit rate and the growth rate divided by the difference between the rate of return on equity and the growth rate.

To use Tobin's expression in his famous article from 1969, the numerator in (9) might also be viewed as (a simple version of) the firms' marginal efficiency of capital (MEC), 'adjusted' for growth and risk. The benchmark $q^o = 1$ would be obtained if it were equated to the real interest cost, an equality which in these words is an old and venerable argument. The role that $e^o \xi_e$ should play for the managers in this specification of MEC is, however, not entirely clear, because it involves the risk premium of the share holders. In particular, this ξ_e also accounts for the risk from the price fluctuations on the stock market, which are of no direct concern to the operative business of the firms.

For a better distinction in this respect, the second equation in (9) introduces the specification of ξ_f^A . This difference between profits and real interest may be interpreted as risk premium for the irreversibility of fixed investment. ξ_f^A can furthermore serve as an evaluation of how successfully the managers are running the firms (therefore the index 'f'). Thus, it may be employed by the managers themselves or by the shareholders who assess their work.

A priori, ξ_f^A may or may not be equal to the other 'weighted' risk premium $e^o \xi_e$. The second part of eq. (9) reveals that $q^o = 1$ arises if and only if the two are equal, $\xi_f^A = e^o \xi_e$, while $q^o < 1$ prevails if $e^o \xi_e$ exceeds the irreversibility risk premium ξ_f^A (presupposing that $i - \pi > g$; cf. footnote 11). Because of the relatively small denominator ($i - \pi - g$), the quantitative effects of these deviations are, however, not easy to assess; a little numerical check will therefore be provided below.

In the case that ξ_f^A and $e^o \xi_e$ are equal and also not affected by a *ceteris paribus* increase in the growth rate g , eq. (9) tells us that this change has no impact on q^o , either. This is in contrast to Kaldor's conclusion in his model (1966: 317) that higher accumulation rates yield lower valuation ratios, which is a result that has also recently been obtained in a modern stock-flow consistent model with many more endogenous feedbacks (but still an exogenous growth rate); see Bernardo et al. (2015: 10).¹² In the simple setting underlying (9), the effect would come about if and only if $\xi_f^A < e^o \xi_e$, that is, if and only if the original q^o happens to be less than one.

¹²On p. 6 the authors argue that there is also some empirical evidence for this result.

5 Tobin's q in the steady state: Model version B

In this section we turn to a simultaneous determination of q^o , λ^o and e^o . To ease the discussion in a baseline version, suppose that the firms borrow directly from the rentiers by issuing corporate bonds (L), say, paying the interest rate i .¹³ This setting implies that in a steady state the rentiers will also accept the debt-asset ratio λ^o of the firms. In addition to Condition 1, we put forward a second criterion for the portfolio decisions of the rentiers that is necessary for an invariance of e and λ . Besides comparing the returns from their alternative investment in loans and equities, the rentiers are explicitly supposed to entertain the notion of a balanced portfolio. That is, defining

$$\beta = \frac{L}{L + p_e E}, \quad \varepsilon = \frac{p_e E}{L + p_e E} \quad (10)$$

the rentiers have target values β^* , ε^* for the proportions of the corporate bonds and equities in which they wish to hold their wealth—in a situation where the two assets appear equally attractive in the sense of Condition 1. In other words, β^* and ε^* serve the role of an explicit anchor for the portfolio shares. Clearly, $\beta + \varepsilon = 1$ and $e = \varepsilon q$, $\lambda = \beta q = (1 - \varepsilon)q$. Accordingly, even if Condition 1 is fulfilled but in a dynamic framework currently $\beta > \beta^*$ and therefore $\varepsilon < \varepsilon^*$, for example, the rentiers seek to decrease the share β of loans in their portfolio and to increase the share ε of equities. These adjustments should also introduce a tendency for the firms' debt-asset ratio λ to decline and for their equity ratio e to increase. Thus, the idea of the target proportions amounts to a second consistency requirement.

Equilibrium Condition 2

The actual portfolio of the rentiers is balanced in a long-run equilibrium position, that is, with respect to their given target proportions β^* , ε^* ,

$$\beta^o = \beta^*, \quad \varepsilon^o = \varepsilon^*$$

In this way the equilibrium values of q , λ and e are linked by the target proportions of the rentiers, $e^o = \varepsilon^* q^o$, $\lambda^o = \beta^* q^o$. Adding the second assumption to the first one allows us to determine the steady state value of Tobin's q directly by β^* and ε^* , where the modified setting in the present section may be identified by the letter 'B'.

¹³Possible price variations from their trading on an extra market should have only weak effects, so that they can be safely ignored.

Proposition 2

Suppose Conditions 1 and 2 hold true. Then the long-run equilibrium value of Tobin's q is determined from the rentiers' target proportions β^* and ε^* as

$$q^o = \frac{(1 - \tau_c)r - g}{(1 - \beta^*\tau_c)i - \pi + \varepsilon^*\xi_e - g} \quad (11)$$

Equivalently, putting

$$\xi_f^B := r - \tau_c(r - i\beta^*) - (i - \pi) \quad (12)$$

q^o can be expressed as

$$q^o = 1 + \frac{\xi_f^B - \varepsilon^*\xi_e}{(1 - \beta^*\tau_c)i - \pi + \varepsilon^*\xi_e - g} \quad (13)$$

To prove the first part of the proposition, consider the equation $q(i - \pi + \xi_e - g) = (1 - \tau_c)r - g + (\tau_c i + \xi_e)\lambda$, which with the operations indicated in the remark on Proposition 1 is equivalent to Condition 1. Replacing λ with $(1 - \varepsilon^*)q$ and solving this equation for $q = q^o$ yields $q^o = [(1 - \tau_c)r - g] / [(1 - \tau_c)i + \varepsilon^*(\tau_c i + \xi_e) - (\pi + g)]$ and thus, with $\varepsilon^*\tau_c i = (1 - \beta^*)\tau_c i$, eq. (11). Equation (13) adds and subtracts the denominator in the numerator and rearranges these terms.

The expression in the numerator of (11) displays the growth-adjusted profit rate of the firms, where (somewhat strangely) the corporate tax rate is imputed on the entire gross profits. The denominator can be interpreted as a real interest rate pertaining to the rentiers, which is modified in three ways: (i) just like the profit rate, it is adjusted for growth; (ii) the risk premium of holding equities is added to the interest rate term, now multiplied by the weight $\varepsilon^* = p_e E / (L + p_e E)$ of the equities in the total wealth of the rentiers; (iii) the corporate tax rate carries over to the nominal interest rate that the rentiers receive from the firms (although this is a mechanism that might perhaps seem somewhat artificial, too).

While $q^o = 1$ results if the two rates above and below the fraction line in (11) are equal, it is not clear what economic principle might equate them. Similar doubts hold for the representation of q^o in eq. (13), which compares the term ξ_f^B with the weighted risk premium $\varepsilon^*\xi_e$. Regarding the former, in evaluating the performance of the firms it makes sense for the rentiers to relate the profit rate to the real interest rate, after the corporate taxes are accounted for. However, this is more appropriately done in the specification of ξ_f^A in Proposition 1, as $\tau_c\beta^*i$ in the definition of ξ_f^B is of no direct significance for the after-tax profits earned by the firms. The distortions of ξ_f^B vis-à-vis ξ_f^A , or of the criterion in (13) vis-à-vis eq. (9), which both derive from

the different denominators in Proposition 1 and 2, can be made more explicit by the relationships

$$\begin{aligned}\xi_f^B &= \xi_f^A - (q^o - 1)\tau_c i \beta^* \\ q^o &= 1 + \frac{[\xi_f - e^o \xi_e] + (q^o - 1)[\varepsilon^* \xi_e - \beta^* \tau_c i]}{(1 - \beta^* \tau_c)i - \pi + \varepsilon^* \xi_e - g}\end{aligned}$$

(which are easily verified). In short, it may be said that eq. (9) in Proposition 1 is economically more meaningful than eq. (13) in Proposition 2, but at the price that its right-hand side is not independent of the value q^o on the left-hand side.

6 A numerical check

If reference is made to the differences between ξ_f^A and $e^o \xi_e$ or ξ_f^B and $\varepsilon^* \xi_e$, respectively, it must be recognized that, irrespective of a more detailed story explaining them, the premia ξ_e and ξ_f^A , ξ_f^B are likely to be independent. Their equality in eqs (9) or (13) can therefore not be taken for granted and the long-run equilibrium value q^o may well be different from unity. In this section we want to get a feeling for a typical order of magnitude of these deviations, where we consider model versions A and B together.

Note that there are two reasons why our results cannot be directly related to the empirical time series for Tobin's q that can be found at various places. First, our ratios are the long-run equilibrium values, which might not necessarily be well captured by the empirical medium-term time averages. Second, the empirical specifications of what is called Tobin's q may be somewhat different from the present definition. Thus, our purpose is not so much an empirical assessment of Tobin's q but rather some hints that future calibrations of elaborate stock-flow consistent models may take into account.

g	π	i	r	τ_c
2.50	2.50	8.00	14.64	25.00

Table 1: Benchmark values (in %) characterizing the real sector.

We begin our little numerical study by putting forward reasonable values for the variables in the real sector, where we confine ourselves to US data.¹⁴ With

¹⁴Given our limited purpose and the problems of international data comparability, we abstain from considering other OECD countries. In particular, different tax conventions and the measurement of

respect to an underlying time unit of one year, Table 1 presents the benchmark values we are working with. The 2.50% for the capital growth rate and goods price inflation are the approximate time averages for the US nonfinancial business sector over the years 1983–2007, a period that is often referred to as the Great Moderation. The value for the interest rate is close to the average of the prime rate over the same period; corporate AAA (BAA) bond rates are a little lower (somewhat higher, respectively). The profit rate $r = (1 - \tau_v)hu - \delta$ is obtained from the following estimates: $\tau_v = 0.0875$ (the rate at which production is taxed), $h = 0.30$ (the profit share), $u = 0.90$ (the output-capital ratio), $\delta = 0.10$ (the capital depreciation rate); thus $r = 0.146375$. Lastly, the value for the corporate tax rate τ_c is another (grossly rounded) time average.¹⁵

The debt-asset ratio, which constitutes model A, is the statistic surrounded by the greatest uncertainty. Apart from data issues, one first has to clarify the conceptual problem whether in the specification of “net debt” only the firms’ short-term financial assets are netted out or more. Because of these ambiguities we consider three alternative values of a given λ^o , namely 0.20, 0.30, 0.40.¹⁶ The corresponding equity target proportions ε^* in model B are $\varepsilon^* = 1 - \beta^* = 1 - \lambda^o/q^o$, hence with respect to a benchmark $q^o = 1$ we consider $\varepsilon^* = 0.80, 0.70, 0.60$.

Our benchmark risk premia are likewise based on $q^o = 1$. Together with the profit and real interest rates from above and taking $\lambda^o = 0.30$ as an example, the risk premium ξ_f results as $\xi_f^A = \xi_f^B = 0.060781250$ in (8) and (12). Putting $e^o = \varepsilon^*$ and $\xi_f^A - e^o \xi_e = \xi_f^B - \varepsilon^* \xi_e = 0$, the value of ξ_e that brings about $q^o = 1$ is computed as $\xi_e = 0.086830357$. It may be claimed that both premia do not seem very implausible. In particular, it makes sense that the stock market risk of the rentiers from the volatile prices exceeds the risk of the firms from the irreversibility of their fixed capital, and that in quantitative terms we have $\xi_f < \xi_e = \xi_f + 2.60\%$.

The middle block of Table 2 shows the same risk premia if we suppose $\lambda^o = 0.20$ and $\lambda^o = 0.40$. Both premia are seen to increase with the debt asset ratio, where the variation in ξ_f is rather minor and the one in ξ_e is somewhat larger,

capital (to determine the output-capital ratio for the profit rate) would make things difficult. Furthermore, in the end these countries should not yield orders of magnitude that are drastically different from our benchmark values.

¹⁵For details on the data sources and the way in which especially the tax rates and the output-capital ratio Y/K were derived, the reader may be referred to Franke (2015, Section 5 and the Appendix). Regarding the perhaps somewhat unfamiliar value for Y/K it should be clarified that since we are concerned with the firm sector, the capital stock does not include housing or residential investment. Also, it can be misleading to relate nominal Y and K because their relative price underwent some systematic variation. Besides, lower values for Y/K might bring r too close to i or even below it.

¹⁶At first sight $\lambda^o = 0.20$ might appear very low. Hein and Schoder (2011: 702, 722), however, calculate ratios that are even five percentage points lower.

given λ^o	$\xi_f^A = \xi_f^B$	ξ_e	version A	version B
			based on λ^o	$\varepsilon^* = 1 - \lambda^o$
			$q^o - 1$	
0.20 :		5.35	0.192	0.233
0.30 :		6.68	0.145	0.198
0.40 :		8.46	0.105	0.165
0.20 :	5.88	7.35	0.000	0.000
0.30 :	6.08	8.68	0.000	0.000
0.40 :	6.28	10.46	0.000	0.000
0.20 :		9.35	-0.130	-0.159
0.30 :		10.68	-0.102	-0.142
0.40 :		12.46	-0.078	-0.124

Table 2: q^o from (7) and (11), respectively, for alternative values of the equity risk premium ξ_e (in %).

because according to Propositions 1 and 2 the benchmark $q^o = 1$ requires that $\xi_e = \xi_f^A/e^o = \xi_f^B/\varepsilon^* = \xi_f/(1-\lambda^o)$.

On this basis, the remainder of Table 2 gives some examples of the impact on q^o when ξ_e differs from ξ_f^A/e^o or ξ_f^B/ε^* , respectively. We fix $\xi_f^A = \xi_f^B$ and decrease/increase the stock market risk premium by two percentage points; see the upper/lower part of the table. We see that the strongest changes in q^o are obtained for the lowest debt-asset ratio, and the changes in the framework of model B are somewhat more pronounced than in model A. It is also worth noting that the positive and negative reactions are not symmetric; depending on the given indebtedness λ^o the positive reactions of q^o are 1.33 or 1.48 times stronger than the negative reactions. Generally, however, the effects can be said to be rather limited, which is the main message from our numerical exploration.

7 Conclusion

The valuation ratio of Tobin's (average) Q is a venerable concept to describe the financial situation in the business sector. As a ratio that relates the stock of liabilities

(equities and debt) of the firms to their fixed capital, a value $Q = 1$ is equivalent to a zero net worth (if the capital stock is valued at its replacement cost). Thus, $Q = 1$ is often viewed as being indicative of a financial equilibrium and one can find various arguments in the older literature why the economy may be driven in the direction of such a position. By contrast, several of the more recent and elaborated stock-flow consistent models of a post-Keynesian variety have no more recourse to them whatsoever.

As a consequence, the determination of a long-run equilibrium value $Q = Q^o$ is a side effect of the interplay of other model mechanisms and it is usually not carefully checked if the resulting Q^o and its implications are sufficiently meaningful. Still at a very general level, the present note may prove fruitful in this context because instead of a stock ratio, it characterizes Q^o as a ratio of several rates of return which, in particular, include two expressions that can be interpreted as risk premia pertaining to the firms and rentiers, respectively. These concepts can widen the horizon of contemporary models and may also be incorporated into some of their dynamic adjustment principles.

The special case $Q^o = 1$ can come about under certain precise conditions that may or may not be fulfilled. Typically, however, possible distortions from unity will be fairly moderate. Actually, an elementary numerical check yields values of Q^o between, say, 0.85 and 1.20 as a reasonable guideline for an ambitious numerical analysis. On the other hand, our approach can serve for a better understanding should larger deviations be obtained. It may then be discussed if such outcomes can still be accepted or if some modifications in the model or its numerical calibration may be necessary.

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