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PRICE INERTIA AND PRODUCTION LAGS

Assar Lindbeck and Dennis J Snower

HUMAN RESOURCES
ABSTRACT

Price Inertia and Production Lags*

The paper shows how prolonged price inertia can arise in a macroeconomic system in which there are temporary price rigidities as well as production lags in the use of intermediate goods. In this context, changes in product demand – generated, say, by changes in the money supply – have long-lasting price and quantity effects. Specifically, a temporary demand shift generates 'persistence' in price-quantity decisions, in the sense that the price-quantity effects of this shift persist for long after the shift has disappeared. A permanent demand shift generates 'sluggishness' in price-quantity decisions, in the sense that the full price effects of the shift take a long time to appear and that meanwhile quantity effects are present.

JEL classification: D1, D43, D57, E12, E31, E32
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This paper shows how production lags in the use of intermediate inputs can play an important role in explaining price inertia. A simple model is constructed in which labour is used to produce intermediate goods, and these intermediate goods are used to produce final goods. The production process for the final goods involves a lag. The prices of the final goods are set before the final demand is observed, i.e. these prices are precommitted before sales take place.

Within this set-up, prolonged price inertia arises as follows. An unanticipated fall in final demand – generated, for example, by an unanticipated fall in the money supply – has no immediate effect on final good prices (since they are precommitted) or on nominal wages (which depend in part on the final good prices). Since intermediate good prices depend in part on nominal wages, these prices fall less than proportionately to the drop in final demand. Given a production lag between the intermediate inputs and the final outputs, the final output prices in the next round of production will also not adjust fully to the demand shock. This, in turn, implies imperfect adjustment of nominal wages and intermediate good prices, and so on.

In short, a transient price precommitment can turn into prolonged price inertia as sticky final output prices lead to sticky nominal wages, which give rise to sticky intermediate input prices, which in turn make final output prices sticky in the following period. Thereby an initial, temporary price precommitment gets transmitted from one period to the next. The price inertia implies that the initial change in final demand can have significant quantity effects.

This particular model of price inertia – involving a relation running from final output prices to nominal wages to intermediate input prices and back to final output prices – is merely one simple way of illustrating a far more general theory, in which any price precommitment combined with any production lags may lead to prolonged price inertia. To take another variant of the many conceivable models with the same basic idea, suppose for example that goods which satisfy final demands can also be used as intermediate inputs that produce further outputs with a lag. Then an unanticipated fall in final demand that leaves intermediate input prices temporarily unchanged will lead to inertia in the output prices in the following period. But since these outputs are themselves used as intermediate inputs elsewhere, the next round of output prices will also be characterized by inertia, and so on. Along these lines, the possibilities for generating price inertia associated with production lags are virtually endless. Our choice of production lag specification is made merely on account of its analytical simplicity.
Although this model focuses on price inertia stemming solely from product demand changes that are not anticipated when intermediate input prices are determined, it is worth noting that fully anticipated demand changes are practically inconceivable when final output prices depend on intermediate input prices, which in turn depend on nominal wages, which depend on final output prices, and so on. Specifically, consider a fall in demand occurring at the beginning of time period $t$. As shown below, the output prices in period $t$ will adjust fully only if the prices of the associated intermediate inputs adjust fully as well. Due to production lags, however, these intermediate inputs must have been produced and bought previously—say in period $t-1$—and thus full adjustment of the intermediate input prices requires that the final demand increase be anticipated in period $t-1$. But these intermediate inputs are produced by labour, whose wage depends on the final output prices in period $t-1$, and the final outputs in period $t-1$ are themselves produced by intermediate inputs in period $t-2$, and so on indefinitely. Thus only a demand reduction that has been anticipated in the very distant past can generate full price adjustment. This feature is augmented when there are long chains of production, each associated with significant production lags, for then full price adjustment requires that the demand change be anticipated all the way along the input-output chain.

This also helps to explain why it may be impossible to index intermediate input prices to the associated output prices. In the presence of production lags, current input prices would have to be indexed to future output prices. But future prices may be difficult to predict, particularly if these in turn would have to be indexed to output prices lying even further in the future, and so on.

Our explanation of price inertia merely requires that prices be precommitted in the initial period, when the unanticipated change in final demand occurs. In all subsequent periods, the precommitment assumption is immaterial. It is sufficient that intermediate input prices in the initial period $t-1$ are not fully responsive to the demand change of period $t$ in order for the final output prices in period $t$ to be sluggish; and if these output prices affect nominal wages, which in turn affect intermediate good prices in period $t$, then the output prices in period $t+1$ will also be sluggish, even if the level of final demand in period $t+1$ is known with certainty in period $t+1$. Thus, even if prices in all periods subsequent to the initial demand change are set under full information about demand, the price inertia will not disappear.

Finally, our price inertia mechanism may also be reinforced by temporary nominal wage rigidities. To emphasize the role of production lags, we do not include such rigidities in our model, but the underlying idea is straightforward. Since the price of an output under imperfect competition depends on marginal cost, nominal wage rigidities get translated into price rigidities which, in turn, are prolonged as they feed through the production lags of an input-output system. Beyond that, the price inertia may in turn have an impact on the wage setting process, giving
rise to further wage inertia. While Blanchard (1986) has modelled such an interaction based on wage-price staggering, our analysis suggests that both wage and price inertia can be greatly magnified through the operation of production lags.

Within the above context, the analysis shows how a temporary price precommitment can generate price sluggishness and positive output responses to permanent demand shifts, as well as persistence in the price and quantity effects of temporary demand shifts.
1. Introduction

Price inertia is commonly regarded as an important reason why product demand shocks have significant short-run effects on output and employment. As we will argue, existing theories of price inertia do not tell the full story; they provide partial explanations, not all of which are convincing. This paper offers another explanation which, we suggest, may be significant and commonplace in practice. It generates predictions that are observationally distinct from those of the current theories.

The aim of the paper is to show how production lags in the use of intermediate inputs can play an important role in explaining price inertia. To highlight this role in a particularly simple way, we construct a model in which labor is used to produce intermediate goods, and these intermediate goods are used to produce final goods. The production process for the final goods involves a lag. The prices of the final goods are set before the final demand is observed, i.e. these prices are precommitted before sales take place. Within this setup, prolonged price inertia arises as follows. An unanticipated fall in final demand - generated, for example, by an unanticipated fall in the money supply - has no immediate effect on final good prices (since they are precommitted) or on nominal wages (which depend in part on the final good prices). Since intermediate good prices depend in part on nominal wages, these prices fall less than proportionately to the drop in final demand. Given a production lag between the intermediate inputs and the final outputs, the final output prices in the next round of production also will not adjust fully to the demand shock. This, in turn, implies imperfect adjustment of nominal wages and intermediate good prices, and so on.

In short, a transient price precommitment can turn into prolonged price inertia as sticky final output prices lead to sticky nominal wages, which give rise to sticky intermediate input prices, which in turn make final output prices sticky in the following period. Thereby an initial, temporary price precommitment gets transmitted from one period to the next. The underlying reason is that since the final goods prices are precommitted, the pricing decisions for final and intermediate goods are made at different times, so that price inertia arises as price changes gradually feed through

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the input-output system. This price inertia implies that the initial change in final demand can have significant quantity effects.

The assumptions underlying the model - (i) that final goods are produced by means of intermediate goods, (ii) that this production process involves lags, (iii) that nominal wages depend on final output prices, and (iv) that final output prices are precommitted when sales take place - are uncontroversial and plausible. The last assumption may, however, require some comment. Setting prices before sales are observed is a common phenomenon in practice. Department stores, supermarkets, automobile retailers, and furniture stores all attach prices to their products before customers decide whether to purchase them. The reason for this appears to be the existence of transactions costs and moral hazard problems. For example, if department stores would bargain over prices with each of their customers, there would be large negotiation costs and managers would have to relinquish control of pricing to their sales personnel, who may then be tempted to offer favorable terms to their families and friends.

It is important to emphasize that our particular model of price inertia - involving a relation running from final output prices to nominal wages to intermediate input prices and back to final output prices - is merely one simple way of illustrating a far more general theory, in which any price precommitment combined with any production lags may lead to prolonged price inertia. To take another variant of the many conceivable models with the same basic idea, suppose for example that goods which satisfy final demands can also be used as intermediate inputs that produce further outputs with a lag. Then an unanticipated fall in final demand that leaves intermediate input prices temporarily unchanged will lead to inertia in the output prices in the following period. But since these outputs are themselves used as intermediate inputs elsewhere, the next round of output prices will also be characterized by inertia, and so on. Along these lines, the possibilities for generating price inertia associated with production lags are virtually endless. Our choice of production lag specification was made merely on account of its analytical simplicity.

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The paper is organized as follows. Section 2 relates our contribution to the existing literature on nominal rigidities. Section 3 presents our model of price-quantity decisions. Sections 4 and 5 use this model to examine the price-quantity effects of temporary and permanent shifts in the product demand function. Section 6
concludes.

2. Relation to the Literature

Since this paper is concerned with nominal price inertia, it shares its domain of interest with menu costs and "near-rationality" theories\(^1\) and the wage-price staggering theories\(^2\). Unlike the menu cost and near-rationality theories, our model provides an explanation for why prices in practice are often changed frequently in the same direction, but not by sufficiently large amounts to obviate the need for significant quantity adjustments.\(^3\) Like the menu cost and near-rationality theories, our analysis seeks to explain why standard variations in aggregate demand can generate large output-employment fluctuations. Yet whereas the quantity effects of demand variations will not arise in the menu cost theory if, as appears plausible, the costs of output-employment adjustment exceed the cost of price adjustment, this is not the case for price inertia generated by production lags in an input-output system.\(^4\)

In contrast to the staggering theory, our analysis makes the degree of price inertia depend on technologically given production lags rather than on contract periods.\(^5\) As noted in the previous section, however, our analysis is complementary to the staggering theory, since nominal wage and price sluggishness arising from staggered wage-price contracts may be substantially amplified in the presence of production lags. Furthermore, these lags may provide a new rationale for price staggering: for technological reasons these lags are not perfectly synchronized across production sectors and thus the corresponding pricing decisions will not be perfectly synchronized as well.\(^6\)

Although the temporary price rigidity presupposed in our model need have nothing to do with menu costs, our analysis is also complementary to the menu cost theory, for

\(^1\) The seminal contributions are those of Akerlof and Yellen (1985) and Mankiw (1985).


\(^3\) Since the menu costs of large price changes do not appear to be significantly greater than those of small price changes, it is not clear how the menu cost theory can account for this phenomenon.

\(^4\) This is easy to show in the analytical framework below, although for brevity we do not do so.

\(^5\) The staggering theory requires that the length of price contracts is "sluggish" in response to demand changes (for otherwise changes in contract length would obviate the need for quantity adjustments); our theory requires that the length of the production lag is "sluggish" in this respect.

\(^6\) This is not the way price staggering has been rationalized in the current literature. Ball and Cecchetti (1988) do so via firms' incentive to gain information about their rivals' prices before setting their own prices. Ball and Romer (1989) rationalize it by supposing that there are firm-specific shocks falling on different firms at different times.
it is easy to show that the effects of menu costs become more pronounced when they interact across long chains of production with lags. Furthermore, menu costs within our analytical framework could be used to explain how even perfectly anticipated final demand become associated with price inertia and significant quantity effects.

Our analysis also has common objectives with the contributions that rationalize price inertia by showing how demand shocks are cushioned by movements of inventories and order backlogs. These theories should be observationally distinct, however, since movements of inventories and order backlogs do not depend only on the length of production lags.

The main idea underlying this paper is related to an insightful contribution by Gordon (1990), in which he argues that "with thousands of firms buying thousands of components, containing ingredients from many other firms, the typical firm has no idea of the identity of its full set of suppliers when all the indirect links within the input-output table are considered. Because the informational problem of trying to anticipate the effect of a currently perceived nominal demand change on the weighted average costs of all these suppliers is difficult to formulate and probably impossible to solve ... the sensible firm just waits by the mailbox for news of cost increases and then, Okun-like, passes them on as price increases" (pp.1150-1). In other words, Gordon suggests that firms' inability to predict cost variations within a complex input-output system causes them systematically to underestimate the magnitude of all nominal demand changes, so that price inertia results. While our analysis shares Gordon's concern with the economy's input-output structure, it differs from his in that it emphasizes the length of production lags as a crucial determinant of price inertia whereas Gordon emphasizes systematic expectational errors in an intersectoral setting.

3. The Model

3a. Underlying Assumptions

Our model economy produces one intermediate good and several final (consumption)
goods. The intermediate good \( I \) is produced by means of labor \( L \) with a Cobb-Douglas technology:

\[
I = L^\mu
\]  

(1)

where \( \mu \) is a constant \( (0 < \mu < 1) \). There are \( F \) intermediate-good producers, each of whom is a price-taker, maximizing profit \( P_i \cdot I_i - W_i \cdot L_i \), while taking the intermediate good price \( P_i \) and the nominal wage \( W_i \) as exogenous to the output \( (I_i) \) and employment \( (L_i) \) decisions. The resulting labor demand \( (L_d^I) \) and intermediate good supply \( (I) \) are

\[
L_d^I = \left( \frac{W_i}{P_i \cdot \mu} \right)^{-\frac{1}{1-\mu}} \quad I = \left( \frac{W_i}{P_i \cdot \mu} \right)^{-\frac{\mu}{1-\mu}}
\]  

(2)

where \( \mu \) is a constant \( (0 < \mu < 1) \).

There are \( F \) differentiated final goods, each produced by a different firm. Each firm faces symmetric demand and cost conditions. For analytical simplicity, let the representative firm produce the final good \( q_f \) by means of only the intermediate good \( I \). The technology is again Cobb-Douglas, but now there is a one-period production lag:

\[
q_f^t = I_{t-1}^{\gamma}
\]  

(3)

where \( \gamma \) is a constant \( (0 < \gamma < 1) \).

Each final good producer is a Cournot oligopolist,\( ^{10} \) taking other firms' quantities as given when making its own price-quantity decisions \( (P_i \text{ and } q_i) \). Letting \( \pi_i \) be the aggregate price index of the final good and \( M \) be the nominal money supply, the demand for the firm's product has the following constant-elasticity form:

\[
q_i^t = \alpha \cdot \left( \frac{M}{\pi_i} \right)^\epsilon \cdot \left( \frac{P_i}{\pi_i} \right)^{-\eta}
\]  

(4)

where \( \alpha, \epsilon, \) and \( \eta \) are positive constants. Since all final goods producers face symmetric demand and cost conditions, they all charge the same price \( P_i \) in equilibrium and thus

\( ^9 \) We scale \( L_i, I_i, \) and \( q_i \) so that the multiplicative constants in the production functions (1) and (3) are unity.

\( ^{10} \) As explained in footnote 14, qualitative conclusions remain intact under other types of oligopolistic interactions.
the aggregate price index is $\Pi_t = p_t$.

The nominal wage $W$ and the intermediate input price $P$ are predetermined when each final good producer makes his production $(q_t)$, employment $(n_t)$, and pricing $(p_t)$ decisions. Let $\Pi_t^e$ be the period-$t$ aggregate price level of final goods that the final goods producer expects in period $t-1$, and $\delta$ be the real discount factor. Then, in period $t=0$, each final good producer seeks to maximize the real present value of profit:

$$z = \sum_{t=1}^{\infty} \frac{\delta^t \cdot q_t \cdot \Pi_t - \delta^{t-1} \cdot P_{t-1} \cdot I_{t-1}}{\Pi_t^e}$$  \hfill (5)

subject to the production function (3) and the final demand function (4).

The objective function (5) is standard, but it may nevertheless be useful to clarify how labor and the intermediate good is valued, since it may appear at first glance that the input is valued at historical cost rather than opportunity cost. Following Malinvaud (1972, p.253) and others, the general expression for the present value of a firm's nominal profit is

$$z = \sum_{t=1}^{\infty} \delta^t \cdot [p_t \cdot q_t - p_t \cdot a_t]$$  \hfill (5')

where $p_t$ is a vector of prices of all the commodities going through the firm's production process, and $q_t$ and $a_t$ are the corresponding vectors of all the firm's outputs and inputs. With respect to our analysis, $p_t = (p_t', P_t')$, $q_t = (q_t, 0)$, and $a_t = (0, I)$. Since the production process lasts one period, $q_i = 0$. Making these substitutions into (4') yields the nominal counterpart of our objective function (5): $\sum (\delta^t \cdot p_t \cdot q_t - \delta^{t-1} \cdot P_{t-1} \cdot I_{t-1})$.

To show that this objective function contains the appropriate valuation of inputs in terms of their opportunity costs, it is crucial to be clear about the timing of decisions in our model. In any period $t$ the final good producer decides to purchase $I_t$ intermediate goods in order to produce $q_{t+1}$ of the output at price $p_{t+1}$ in the following period, given the intermediate good price $P_t$. In short, the decisions $(I_t, q_{t+1}, p_{t+1})$ are all made in period $t$. Consequently, the inputs should be valued in terms of their opportunity costs at date $t$. The opportunity cost of the intermediate input $I_t$

\[\text{Footnote}\] Note that expectations formulated prior to period $t-1$ are not relevant to the producer's maximization problem, since it is only in period $t-1$ that the output and pricing decisions for period $t$ are made.
at date \( t \) is \( P_t \)\(^{12}\) (It is clearly not appropriate to value the intermediate input \( I_t \) in terms of its opportunity cost at time \( t+1 \), when the revenues generated by that input accrue. That opportunity cost is generally around zero, since at date \( t+1 \) the intermediate good is already embodied in the output and can no longer be extracted in its original form to be used as intermediate input elsewhere.\(^{13} \)\(^{14}\)

To express the nominal present value \( \sum (\delta^t \cdot P_t \cdot q_t - \delta^{t+1} \cdot P_{t+1} \cdot I_{t+1}) \) in real terms involves deflating both the period \( t-1 \) cost \( (P_{t-1} \cdot I_{t-1}) \) and the period \( t \) revenue \( (P_t \cdot q_t) \) by the period \( t-1 \) expectation of the aggregate price level in period \( t \): \( E_{t-1}(\Pi_t) \). The simplest way to see this is to suppose that the firm lives for just two periods, 0 and 1, so that its nominal profit becomes \( -P_0 \cdot I_0 + \delta \cdot P_1 \cdot q_1 \). Since the input, output, and pricing decisions are all made at time \( t=0 \) while the profit accrues at time \( t=1 \), nominal profit must be deflated by \( E_0(\Pi_1) \) in order to obtain real profit. Thus profits are evaluated in real terms with respect to the prices that are expected to prevail in the period when these profits accrue. Extending this reasoning to an arbitrary number of periods yields the objective function (5).

Defining \( e = 1 - 1/\eta \), where \( 1/\eta \) is Lerner's index of monopoly power, the intermediate good producer's first-order condition for profit maximization is\(^{15}\)

\[
\delta \cdot e \cdot \gamma \cdot I_{t-1} = \frac{P_{t-1}}{P_t} \quad \text{for } t > 1
\]

The left-hand expression is the discounted marginal product of the intermediate

\(^{12}\)This is true regardless of whether the intermediate goods are purchased at the time when they enter the production process (as in our model) or whether they have been purchased previously.

\(^{13}\)For example, the opportunity cost of chrome on car fenders and cinnamon in cookies is nil since these intermediate goods can no longer be separated from the goods they are in.

\(^{14}\)As for inventories carried forward from period \( t \) to period \( t+1 \), such as gasoline inventories at gas stations, they should clearly be valued in terms of their opportunity costs at time \( t+1 \), as implied by (4'). Letting \( k_t \) and \( k_{t+1} \) be the stocks of inventories of the produced good at dates \( t \) and \( t+1 \), respectively, the relevant valuation is \( \sum \delta^t \cdot P_t \cdot k_t - \delta^{t+1} \cdot P_{t+1} \cdot k_{t+1} \). Thus our analysis is quite consistent with the view that the retail price of gasoline jumps immediately after a disturbance in oil production, even though the crude oil used to produce the gasoline that was being sold had been purchased at a lower price.

\(^{15}\)Other forms of imperfect competition among firms (rather than Cournot competition) yield a similar first-order condition. Suppose, for example, that the oligopolistic interactions among firms may be summarized by \( (\partial \tilde{q}_i / \partial q_i) = \xi \), where \( \tilde{q}_i \) is aggregate output produced by each of the other final good producers and \( \xi \) is a positive constant. Clearly, \( \xi = 1 \) under cartel behavior, \( \xi = 1/F \) under Cournot behavior, and \( \xi = 0 \) under Bertrand (perfectly competitive) behavior. Then all that changes in our analysis is our definition of the market power parameter \( e \), which now becomes \( e = 1 - \xi / \eta \).
input times the market power parameter \( (e) \); the right-hand side expression is the real factor cost\(^{16}\).

To close the model, the nominal wage \( (W) \) need to be determined. To focus attention on how price inertia can arise from production lags in an input-output system, rather than from nominal wage rigidities, we assume such nominal rigidities away by supposing quite simply that the real wage \( (w) \) is constant through time:\(^{17}\)

\[
w_t = \frac{W_t}{\pi_t} = w
\]

On this basis we now proceed to derive the price-quantity dynamics for our model economy.

3b. Price-Quantity Dynamics

The price-quantity dynamics in our model may be derived from the market-clearing conditions for the intermediate good and the final goods. Using "\( \ln \)" to denote logarithms of variables, and recalling that \( p_t = \pi_t \) in equilibrium, the market-clearing condition for each final good \( (q_f^t = q_f^t) \) is

\[
\hat{l}_{t-1} = \frac{1}{\tau} \left( \hat{x} + e \cdot \left( \hat{M} - \hat{n}_t \right) \right)
\]

by the production function (3) and the demand function (4).

Now turn to the market-clearing condition for the intermediate good. Given the intermediate good supply by an individual firm (equation (2)\(^{16}\)) and the constancy of the real wage (equation (7)\(^{17}\)), the aggregate supply of the intermediate good in period \( t-1 \) is

\[
\hat{r}_{t-1} = \phi + \frac{\mu}{1-\mu} \left( \hat{w} - \hat{n}_{t-1} + \hat{P}_{t-1} \right)
\]

Given the first-order condition (6), the aggregate demand for the intermediate good becomes

\(^{16}\)We implicitly assume that, in any time period \( t \), the firm has an incentive to use its intermediate inputs to produce output rather than to hold them as inventories, i.e. the profit from using the intermediate good to produce the output \( (P_t \cdot q_t - P_{t-1} \cdot l_{t-1}) \) is greater than the capital gain from holding the intermediate good \( (P_t \cdot l_{t-1} - P_{t-1} \cdot l_{t-1}) \). This condition reduces to \( (P_t/P_{t-1}) < (1/\delta) \cdot [1 + (1 - e \cdot \gamma)/(e \cdot \gamma)] \), where \( (1/\delta) \cdot [1 + (1 - e \cdot \gamma)/(e \cdot \gamma)] > 1. \)

\(^{17}\)This real wage may, for example, be viewed as the workers' reservation wage.
Thus the market clearing condition for the intermediate good ($\tilde{I}_t^m$) may be expressed as

$$\hat{\tilde{I}}_{t+1} = \hat{F} + \frac{1}{1-\gamma} \left( \hat{\delta} + \hat{e} + \hat{\gamma} - \hat{\tilde{I}}_{t+1} + \tilde{n}_t \right)$$  \hspace{1cm} (6')

Substituting the market-clearing condition for the final good (9) into that for the intermediate good (11), we obtain the following price dynamics function:

$$\hat{\tilde{n}}_t = a \cdot \hat{\tilde{n}}_{t-1} + (1-a) \cdot \hat{M} + b$$  \hspace{1cm} (12)

where the money supply $M$ is taken as a time-invariant parameter, and

$$a = \frac{\mu \cdot \gamma}{\mu \cdot \gamma + e \cdot (1-\mu \cdot \gamma)}$$  \hspace{1cm} (13a)

$$b = a \cdot \left( \hat{\delta} + \hat{e} + \hat{\gamma} + \hat{\mu} \cdot \hat{\tilde{w}} \right) + (1-\gamma) \cdot \hat{F} + \left( \frac{1-\mu}{\mu} \right) \cdot \hat{\phi} - \frac{\alpha}{\gamma}$$  \hspace{1cm} (13b)

The coefficient $a$, which we call the "price inertia coefficient", measures the degree to which last period's prices affect current prices. Recalling that $0 < \mu, \gamma < 1$, it follows that $0 < a < 1.18$

The time path of prices, given by the solution to the difference equation (12), is

$$\hat{\tilde{n}}_t = \left( \hat{M} + \frac{b}{1-a} \right) \cdot [1 - a^t] + \hat{\tilde{n}}_0 \cdot a^t$$  \hspace{1cm} (14)

assuming that $a \neq 1$. The corresponding output trajectory may be found by substituting equation (14) into the aggregate final demand function and letting $\hat{p}_t = \hat{\tilde{n}}_t$.

18The condition that $a < 1$ can be shown to be equivalent to the condition that $E \cdot [1/(1-\gamma)] \cdot [1 + (1/(1-\gamma)) \cdot (1-\mu)/\mu] > 0$. 

10
In this context, we now examine the price-quantity effects of permanent and temporary shifts in product demand.

4. Price Sluggishness: The Effects of a Permanent Demand Shift

We define "price sluggishness" as occurring when a permanent demand shift has delayed effects on the price level. To fix ideas, let the permanent demand shift take the form of a permanent fall in the money supply $M$ in period 0. Then we say that price sluggishness occurs when the price level $\hat{\pi}_t$ falls less than proportionately to the money supply in subsequent periods, so that it takes some time for the level of real money balances $(\hat{M}t^{\hat{\pi}} / \hat{\pi}_t)$ to be restored to its long-run equilibrium level.

Specifically, consider the following sequence of events. Initially, at the beginning of period $t=0$, the money supply is $\hat{M}_0$, the price level is at its corresponding stationary equilibrium level ($\hat{\pi}_0 = \hat{M}_0 + [b/(1-a)]$, and similarly for the output and price of the intermediate good ($\hat{I}_0 = \hat{I}_0$ and $\hat{P}_0$, respectively). Then, in the course of period $t=0$ - while the intermediate good $\hat{I}_0$ is being used to produce the final good $\hat{q}_1$ - the money supply falls to $\hat{M}_1 (< \hat{M}_0)$ and remains at this new level thereafter. This permanent monetary shock is unforeseen at the beginning of period $t=0$ and thus does not affect the demand and supply, and hence the price and quantity, of the intermediate good in period $t=0$. Moreover, since the final output supply decision $\hat{q}_1$ is made at the beginning of period $t=0$ (when the intermediate demand decision $\hat{I}_0$ is made), the monetary shock does not affect final output in period $t=1$. And since the final output price $\hat{P}_1$ is precommitted, the monetary shock does not affect it either. From period 2 onwards, however, the price-quantity decisions concerning the intermediate and final goods take the monetary shock into account.

Then, by (14), the price effects of the permanent monetary shock are

$$\frac{d\hat{\pi}_t}{dM} = 1 - a^t$$

for $t>1$, where $d\hat{\pi}_t$ is the difference between the price level in period $t$ in the presence and absence of the monetary shock. Equation (16) implies that $(d\hat{\pi}_t/dM)$ is positive, less than unity, but asymptotically approaches unity. This means that the permanent monetary shock leads to a sequence of price decreases that eventually restore real money balances
(M/π) to their initial stationary equilibrium level.

By (15), the quantity effects of the permanent monetary shock are

$$\frac{dq_t}{dM} = \varepsilon \cdot \left[ 1 - \frac{d\hat{p}_t}{dM} \right] = \varepsilon \cdot \alpha t^{t-1}$$

for $t > 1$. Observe that $(dq_t/dM) > 0$ and $(dq_t/dM) \to 0$ as $t \to \infty$, so that when the money supply falls, output first declines and then returns asymptotically to its initial level.

The underlying mechanism illustrated in Figure 1, which pictures the price dynamics function (12) as a line mapping $\hat{p}_t^{\pi}$ into $\hat{p}_t^{\pi}$. In period $t=0$ this function is at $PC_0$, and the economy is at the corresponding long-run equilibrium point $A$. The permanent contractionary monetary shock shifts the price dynamics line downwards to $PC_1$ in period $t=1$ and keeps it there for all subsequent periods. Given that the final good prices are precommitted in period $t=1$, the period $1$ price level remains at $\hat{p}_0^{\pi}$. In the following period ($t=2$), the final good price falls by the vertical distance from $A$ to $B_1$. This leads to a corresponding fall in the nominal wage and the intermediate good price in period $t=2$, and thus the final good price in period $t=3$ falls by the vertical distance from $B_1$ to $B_2$, and so on. This chain reaction of price reductions continues until the economy eventually reaches its new long-run equilibrium point $D$. In this way, a temporary rigidity in the pricing of intermediate goods turns into prolonged price sluggishness.

As Figure 1 implies, all that the model requires for this to happen is that the price inertia coefficient $a$ be positive, which (by (13a)) is the case when the production functions (1) and (3) display diminishing returns ($0 < \mu, \gamma < 1$) and the elasticity of product demand ($\varepsilon$) is finite. The greater is the price inertia coefficient, the steeper is the slope of the price dynamics line and the greater is the degree of price sluggishness. Specifically, the greater is the price inertia coefficient, the longer it takes for a fraction $\kappa$ of the full price effect to appear (where $\kappa$ is in an $\varepsilon$-neighborhood of the new equilibrium).

Figures 2 add to the intuitive understanding of these results by describing the sequence of price-quantity equilibria explicitly in terms of the interaction between the final good and intermediate good markets. The market-clearing condition for the final good market is

$$\sum_{i=1}^{n} q_{t}^{i} = P_{t}^{F}$$

Clearly, if $a=0$, full adjustment takes place within a period of the monetary shock. Observe that the degree of monopoly power ($\varepsilon$) of the final good producers affects the intercept $b$, but not the slope $a$, of the price dynamics line (12). Thus the greater the degree of monopoly power (ceteris paribus), the greater will be the price level, while the degree of price sluggishness remains unchanged.
good (equation (8)) is illustrated by the QE locus in Figure 2a. This locus is downward-sloping in \( \hat{I}_{t+1} - \hat{\pi}_t \) space, since the supply of the final good \( \hat{q}_t \) depends positively on the intermediate input \( \hat{I}_{t+1} \) and the final demand depends negatively on the price level \( \hat{\pi}_t \). The market-clearing condition for the intermediate good (equation (11)) is pictured by the IE locus. This locus is upward-sloping in \( \hat{I}_{t+1} - \hat{\pi}_t \) space, since an increase in the final good price level \( \hat{\pi}_t \) (ceteris paribus) raises the demand relative to the supply of intermediate goods at any given intermediate good price, thereby leading to a rise in the quantity (and price) of the intermediate good.

The initial equilibrium (at the beginning of period 0) is described by the intersection between the \( QE_0 \) and \( IE_0 \) loci, point \( A \) in Figure 2a. Then (in the course of period 0) the permanent monetary shock occurs. This shock however affects neither the intermediate good price-quantity combination in period 0, nor the final good price-quantity combination in period 1, since these decisions were made at the beginning of period 0, before the monetary shock became known. Thus market activity is pictured by points \( A \) in Figures 2a-c, so that the intermediate good price-quantity combination in period 0 is \((P_0, I_0)\) and the final good price-quantity combination in period 1 is \((\hat{\pi}_0, \hat{q}_0) = (\hat{\pi}_1, \hat{q}_1)\).

Now turn to the next round of price-quantity decisions: the period-1 decisions for the intermediate good \((P_1, I_1)\) and the period-2 decisions for the final goods \((\hat{\pi}_2, \hat{q}_2)\). By equation (8), the permanent monetary shock causes the QE locus to shift downward to \( QE_1 \) in period 1 and to remain there for all subsequent periods. By equation (11), the IE locus in period 1 remains at its original position \((IE_0 = IE_1 \text{ in Figure 2a})\), since the final good price is precommitted in period 1 \((\hat{\pi}_1 = \hat{\pi}_0)\). The downward shift of the QE locus to \( QE_1 \) along this unchanged IE locus means that the general equilibrium moves from point \( A \) to point \( B_1 \) in the Figure 2a.

The underlying activity in the intermediate good market is pictured in Figure 2c. The precommitment of the final good price \((\hat{\pi}_1 = \hat{\pi}_0)\) means that the nominal wage remains unchanged \((\hat{W}_1 = \hat{W}_0)\), and consequently the intermediate good supply curve does not shift: \( I_0^s = I_1^s \) in Figure 2c. But the intermediate good demand curve drops to \( I_1^d \), in response to a drop in the final good price level (explained below). Thus the intermediate good price falls to \( \hat{P}_1 \) and the associated quantity falls to \( \hat{I}_1 \). Observe that even if the intermediate good demand curve were to fall proportionately to the monetary shock, the intermediate good price would fall less than proportionately to the final good price.

The final goods market is pictured in Figure 2b. Here the supply curve drops in response to the fall in the intermediate good price, while demand curve drops in response to the monetary shock. But since the intermediate good price falls less than proportionately to the monetary shock, the supply curve drops by less than the demand curve (i.e. the supply curve shifts only from \( q_1^s \) to \( q_2^s \), while the corresponding demand
curve shifts from \( q_1^d \) to \( q_2^d \). Thus the final good price falls less than proportionately to the monetary shock (from \( \hat{\pi}_1 \) to \( \hat{\pi}_2 \)) and final output falls (from \( \hat{q}_1 \) to \( \hat{q}_2 \)).

In the following round of price-quantity decisions are the period-2 decisions for the intermediate good (\( P_2, I_2 \)) and the period-3 decisions for the final goods (\( \hat{\pi}_3, \hat{q}_3 \)). By equation (11), the previous drop in the final good price level to \( \hat{\pi}_2 \) leads to a drop in the \( IE \) locus (from \( IE_1 \) to \( IE_2 \) in Figure 2a). As noted, the \( QE \) locus remains unchanged at \( QE_1 \). The downward shift of the \( IE \) locus along the \( QE_1 \) locus makes the general equilibrium move to point \( B_2 \) in Figure 2a.

Figure 2c shows the corresponding changes in the intermediate good market. The fall in the final good price to \( \hat{\pi}_2 \) leads to a proportional fall in the nominal wage to \( \hat{W}_2 \) (since the real wage remains constant). Thus the intermediate good supply curve shifts down to \( I_2^s \). Since the period-3 final good price falls by less (to \( \hat{\pi}_3 \)), as explained below, the intermediate good demand curve shifts down to \( I_2^d \). Thus the new equilibrium in the intermediate good market is characterized by a lower price (at \( \hat{P}_2 \)) and a higher quantity (at \( \hat{I}_2 \)).

The fall in the intermediate good price to \( \hat{P}_2 \) leads to an outward shift of the final good supply curve to \( q_3^s \) in Figure 2b. Since the final good demand curve remains at \( q_3^d \), the new equilibrium in the final good market features a lower price (at \( \hat{\pi}_3 \)) and a higher quantity (at \( \hat{q}_3 \)). The final good price \( \hat{\pi}_3 \) falls by less than the intermediate good price \( \hat{P}_2 \), since the final good supply curve shifts along an unchanged demand curve.

In this way, the final good price level gradually falls towards its now long-term equilibrium value \( \hat{\pi}_\infty \) and the outputs of the intermediate and final goods rise gradually towards their original levels.

Put differently the impact effect of the monetary shock is to shift each final good producer’s marginal revenue curve downwards in proportion to the monetary shock, while the associated marginal cost curve shifts downwards by less (since the intermediate good price is sluggish). Thus the price and quantity of the final good both fall. Thereafter, the marginal revenue curve remains unchanged, while the marginal cost curve falls gradually (as the intermediate good price continues to fall). Consequently the price of the final good continues to fall, while the output gradually rises back to its original level.

To summarize,

**Proposition 1:** When the production functions of the final and intermediate good

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\[2^{1} \text{Yet since the final good price falls less than proportionately to the monetary shock, the intermediate good demand curve must also shift less than proportionately, which makes the intermediate good price even more sluggish.} \]
producers are characterized by positive, but diminishing, returns \((0 < \mu, \gamma < 1)\) and the wealth elasticity of product demand \((\epsilon)\) is finite, a permanent drop in the money supply, occurring in period 0, has the following price-quantity effects, starting from an initial stationary equilibrium:

(i) the price level falls asymptotically to its new stationary equilibrium level, and
(ii) the quantity transacted falls beneath its initial level in period 1 and then rises asymptotically back to that level.

5. Price Persistence: The Effects of a Temporary Demand Shift

We define "persistence" in price setting as occurring when a temporary demand shift has prolonged effects on the price level. Specifically, let the temporary demand shift take the form of a drop in the money supply in period 0, reversing itself in period 2, when it returns to its original level and remains there. Then there is persistence in price setting if, starting from an initial stationary equilibrium price level \(\hat{\pi}_0\) in period 0, the price level remains below \(\hat{\pi}_0\) for more than two periods.

As above, the drop in the money supply occurs in the course of period 0 and is not anticipated by agents making their decisions before then. Thus in period 0 the price and quantity of the intermediate good is not affected; nor is the period-1 price of the final good (since it is precommitted) or the period-1 final output (since the output supply decision was made in period 0, when the intermediate inputs were purchased).

Then, in the context of our model, it can indeed be shown that, if the production functions of the final and intermediate good producers are characterized by positive, but diminishing returns \((0 < \mu, \gamma < 1)\) and the wealth elasticity of product demand \((\epsilon)\) is finite - so that the price inertia coefficient \(a\) is less than unity - the temporary monetary shock \((dM_1 = M_1 - M_0)\) causes the price level in period 2 to fall less than proportionately to the money supply and then to return gradually to its long-run stationary level. By (14), the price effects of the temporary monetary shock are

\[
\frac{d\hat{\pi}_t}{dM_1} = (1-a)^{t-1}
\]

for \(t>1\), where \(d\hat{\pi}_t\) is again the difference between the final good price level in period \(t\) in the presence and absence of the monetary shock. Observe that \((d\hat{\pi}_t/dM_1)\) is positive, less than unity, and asymptotically approaches zero. This means that the temporary monetary shock generates persistent price effects that gradually die out.

To derive the corresponding quantity effects, observe that when the temporary
monetary shock occurs in period 0 and remains at its new level until period 2 (when it
reverses itself), the quantity demanded in period 2 is affected directly by \( dM_1 \) and
indirectly by the influence of \( d\pi_1 \) on \( d\pi_2 \):\(^{22}\)

\[
\frac{dq_2}{dM_1} = -\epsilon \left( 1 - \frac{d\pi_1}{dM_1} \right) = -\epsilon \left( \frac{1 - (1-a) \cdot a}{1 - (1-a) \cdot a} \right) \text{ for } t=2 \quad (20a)
\]

In all subsequent periods, when the monetary shock has disappeared, the quantity effects
can occur only via the delayed price responses to the shock:

\[
\frac{dq_i}{dM_1} = -\epsilon \cdot \frac{d\pi_i}{dM_1} = -\epsilon \cdot (1-a) \cdot a^{t-1} \text{ for } t>2 \quad (20b)
\]

Since \( \frac{dq_2}{dM_1} > 0 \) and \( \frac{dq_i}{dM_1} < 0 \) for \( t>2 \), a temporary fall in the money supply
initially reduces final product demand and subsequently overshoots its long-run level.

Figure 1 illustrates the chain reaction of price effects. The economy is initially
at the long-run equilibrium point \( A \), whereupon the price dynamics line shifts downwards
from \( PC_0 \) to \( PC_1 \) in period 1 and then returns it to \( PC_0 \) for all subsequent periods. Due
to precommitment, the period 1 price level remains at \( \pi_0 \), and period 2 the final good
price falls by the vertical distance from \( A \) to \( B_1 \). This results in a fall in the nominal wage and the intermediate good price in period 2. At the same time, the price dynamics line shifts back from \( PC_1 \) to \( PC_0 \). Since the upward shift of the price dynamics line is
proportional to the rise in the money supply whereas the fall in the intermediate good
price is less than proportional to the previous fall in the money supply, the demand
curve and marginal revenue curve of each final good producer rises by more than the
marginal cost curve, and thus the price of the final output in period 3 rises. The
magnitude of the price rise by the vertical distance from \( B_1 \) to \( C_1 \). This, in turn, leads
to a rise in the nominal wage and the intermediate good price in period 3, and
consequently the final good price in period 4 rises by the vertical distance from \( C_1 \) to
\( C_2 \). This process continues until the economy eventually returns to its initial
stationary equilibrium point \( A \). As the figure implies, the steeper is the slope of the
price dynamics line (i.e. the greater is the price inertia coefficient \( a \)), the more
persistent will these price effects be.

In terms of Figure 2a, the temporary monetary shock shifts the \( QE \) locus from \( QE_0 \) to

\(^{22}\)Recall that the quantity demand in period 1 is precommitted.
QE in period 1 and then back to QE₀ thereafter. Since the final good price is precommitted in period 1, the IE locus remains at its original level in that period. Thus the general equilibrium point moves from point A to point B₁ in period 2, corresponding to a fall in the output and price of the final good. The reason is that since the final good price in period 1 is unchanged, the intermediate good price does not adjust fully to the shocks and thus the final good producers' marginal cost curve falls by less than the marginal revenue curve.

The drop in the period-2 final good price leads to a drop in the IE locus from IE₁ to IE₂. That, together with the return of the QE locus to QE₀ means that the general equilibrium moves from point B₁ to point C₁ in Figure 2a, corresponding to a rise in the price and quantity of the final good in period 3. The reason is that since the final good price in period 2 fell, but less than proportionately to the temporary shock, the period-2 intermediate good price does so, too, and thus the final good producers' marginal cost curve falls while the marginal revenue curve remains at its previous level.

Along these lines, the price and quantity of the final good gradually rise to their original levels and the economy returns to point A in Figure 2a.

In sum,

Proposition 2: When the production function of the final and intermediate good producers are characterized by positive, but diminishing returns (0 < μ, γ < 1) and the elasticity of product demand (ε) is finite, a temporary drop in the money supply, occurring in period 0, has the following price-quantity effects, starting from an initial stationary equilibrium price-quantity combination (̂π₀, ̂q₀):

(i) the price level falls beneath ̂π₀ in period 2, but by less than the drop in the money supply, and thereafter remains beneath ̂π₀ while rising asymptotically to ̂π₀;

(ii) the quantity transacted falls beneath ̂q₀ in period 2, rises above ̂q₀ in period 3, and thereafter remains above ̂q₀ while falling asymptotically towards ̂q₀.

6. Concluding Remarks

This paper outlines an alternative theory of price inertia, complementary to existing theories. The underlying thesis is that when the economy's productive structure is described in terms of an input-output system with production lags, a temporary price precommitment can turn into prolonged price inertia. Thereby the degree of price inertia is tied to the intertemporal production structure of the economy, rather than being exclusively associated with nominal rigidities (and the interaction of nominal rigidities among different economic agents), as in the menu cost and price staggering.
theories. Our analysis indicates that a temporary price precommitment can generate price sluggishness and positive output responses to permanent demand shifts, as well as persistence in the price and quantity effects of temporary demand shifts.
References


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Figure 4: Price dynamics following a permanent and temporary demand shift
Figures 2:
The Interaction between the Final and Intermediate Good Markets.