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## Radar scanning the world production frontier

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# **Darmstadt Discussion Papers in ECONOMICS**

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**Radar Scanning the World Production Frontier**

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# Radar Scanning the World Production Frontier

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by

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## Abstract

In this paper we report the results from a detailed investigation of the shifts of the world production frontier function over the period 1980-2010. Analogous to a radar we implement a novel measurement approach for these shifts using nonparametrically computed productivity measures to scan the frontier shifts across the entire input-output space. The shifts of the frontier function measured in this way are analyzed by various regression methods (including robust and nonparametric). The results point towards substantial non-neutrality of technological progress and furthermore show that technological progress is more pronounced in regions of high output per worker and in regions where physical and human capital are intensely used.

**JEL classification:** C14, E23, O11, O47

**Keywords:** non-neutral technological change, world production frontier, nonparametric frontier function

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# 1 Introduction

The concept of neutrality of technological progress is important for economic theory and empirical analyses. Neutrality implies a uniform proportional outward shifting of the world technology frontier across the whole range of input and output quantities. In reality, however, there is a strong prior that technological progress is uneven and depends on the output level at which an economy operates (e.g. the size or the development stage of a country) as well as on the mix of inputs employed (Rousseau 2008). Starting with Atkinson and Stiglitz (1969) as an early predecessor of this literature, a growing body of literature is challenging the neutrality concept from the theoretical side and from an empirical perspective (see e.g. Acemoglu and Zilibotti (2001), Acemoglu (2002), Caselli and Coleman (2006)).

More recently, nonparametric methods of efficiency analysis and in particular the Malmquist productivity index have been increasingly applied to measure total factor productivity change. This index allows to decompose productivity change into several components, most notably technological change (shifts of the frontier function) and efficiency change (catching up to or falling behind from the frontier function). The application to macroeconomic productivity research started with Färe et al. (1994) and subsequently developed with several key contributions. Kumar and Russell (2002) use nonparametric methods of efficiency analysis to decompose labor productivity growth into the components technological change, catching-up and capital accumulation (movements along the frontier function) as a third component for a sample of 57 countries observed during 1965-90. Henderson and Russell (2005) extend the basic setup of Kumar and Russell (2002) by incorporating human capital instead of raw labor. Human capital is constructed there as suggested by Hall and Jones (1999) using results from Mincerian wage equations for estimating the returns of education in different countries. Badunenko, Henderson and Zelenyuk (2008) update the study of Kumar and Russell (2002) and are particularly interested in the development during the 1990s. They also expand the country sample and focus on the analysis of transition economies within the broader sample. Henderson and Zelenyuk (2007) take account of the estimation uncertainty in the analysis of Henderson and Russell (2005) by bootstrapping the efficiency scores. Allen (2012) extends the time span back to 1820 for an analysis of a subgroup of countries along the lines of Kumar and Russell (2002). Badunenko, Henderson and Russell (2013) reconsider the study of Henderson and Russell (2005) by increasing the country sample, updating the time period to 2007 and employing more recent data on educational attainment (from Barro and Lee). In a different approach, Jerzmanowski (2007) uses conventional data envelopment analysis jointly with a Cobb-Douglas production function to assess the validity of the Cobb-Douglas functional form assumption on the one hand and the uneven shifts of the frontier function at different intensities of physical to human capital on the other hand.

All these studies provide consistent evidence against the neutrality of technological change by nonparametric methods. In this paper we take a systematic look at the non-neutrality of technological change by implementing a measurement device analogous to a radar for scanning the changes of the world production frontier at varying input intensities and output levels. We use data from the recently released version 8.0 of the Penn World Table jointly with the updated Barro-Lee data set which provide improved measures of output, physical capital and human capital inputs. Our sample comprises 110 countries over the period 1980-2010. We separately investigate also the impact of the IT and computer revolution during the 1990s as well as the impact of the financial crisis and the Great Recession.

The analysis proceeds by introducing the nonparametric measurement approach in section 2, explaining the data handling in section 3, summarizing the dependence of the frontier shifts on the direction of the efficiency measurement in section 4 and concluding in section 5.

## 2 Nonparametric Efficiency Measurement

The empirical analysis reported in this paper centers on the nonparametric approach to efficiency analysis introduced by Charnes et al. (1978) and called data envelopment analysis (DEA). In the subsequent analysis, we apply the input oriented version of DEA to scan the frontier function across an exhaustive set of directions in the input space and a grid of output levels to track the shifts of the frontier function between two points in time. The main idea is to fix an input-output point outside of the production possibility sets of both periods to be compared and then to measure the distance of this point towards the frontier functions of both periods along a set of directions in the input space and a grid of output levels. This can be viewed analogous to a radar scanning of the sky and tracking the routes of airplanes.

The axiomatic approach to efficiency measurement on which DEA is based departs from the technology set  $T = \{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^{p+q} : \mathbf{x} \geq \mathbf{0} \text{ can produce } \mathbf{y} \geq \mathbf{0}\}$  defining the set of feasible input-output combinations with the input quantities collected in the  $p$ -vector  $\mathbf{x}$  and the output quantities in the  $q$ -vector  $\mathbf{y}$ .<sup>1</sup> The boundary of the technology set is the subset  $F = \{(\mathbf{x}, \mathbf{y}) \in T : (\theta\mathbf{x}, \mathbf{y}) \notin T \forall 0 < \theta < 1 \wedge (\mathbf{x}, \rho\mathbf{y}) \notin T \forall \rho > 1\}$  which comprises all input-output combinations leaving the technology set either if inputs would be reduced ( $0 < \theta < 1$ ) or if outputs would be increased ( $\rho > 1$ ) by an arbitrary small amount, or both. This boundary defines the frontier function and the radial input-oriented distance function  $D(\mathbf{x}, \mathbf{y}) = \sup\{\delta \geq 1 : (\mathbf{x}/\delta, \mathbf{y}) \in T\}$  according to Shephard (1970) measures the distance towards this frontier function. The factor  $\delta$  is inversely related to  $\theta$ . It is equal to one for all efficient input-output points on the frontier function and larger than one if efficiency can be improved to reach the frontier function by reducing all inputs proportionately. Thus,  $\delta$  is a measure of inefficiency which is larger for a larger distance towards the frontier function.

For explaining the measurement let the input-output data for  $n$  country observations of  $p$  inputs be collected in the  $p \times n$  matrix  $\mathbf{X}$  and for  $q$  outputs in the  $q \times n$  matrix  $\mathbf{Y}$ . Let  $\mathbf{x}_i$  and  $\mathbf{y}_i$  denote the  $i$ th column of  $\mathbf{X}$  and  $\mathbf{Y}$ , respectively, which contain the input and output quantities of country  $i$ . Then the linear programming problem to compute the Farrell (1957) distance function via the efficiency measure  $\theta$  can be stated as

$$\max_{\theta, \boldsymbol{\lambda}} \left\{ \theta : \theta \mathbf{x}_i \geq \sum_{j=1}^n \lambda_j \mathbf{x}_j, \mathbf{y}_i \leq \sum_{j=1}^n \lambda_j \mathbf{y}_j, \sum_{j=1}^n \lambda_j = 1, \lambda_1, \dots, \lambda_n \geq 0 \right\} \quad (1)$$

or more compactly in matrix notation

$$\max_{\theta, \boldsymbol{\lambda}} \{ \theta : \theta \mathbf{x}_i \geq \mathbf{X}\boldsymbol{\lambda}, \mathbf{y}_i \leq \mathbf{Y}\boldsymbol{\lambda}, \mathbf{1}'\boldsymbol{\lambda} = 1, \boldsymbol{\lambda} \geq \mathbf{0} \}. \quad (2)$$

The frontier function is a piece-wise linear function enveloping the sample of input-output combinations. The solution value for a country  $i$ ,  $\theta_i$ , can then be transformed to the solution value of the associated Shephard efficiency measure  $\delta_i = 1/\theta_i$  for country  $i$ . A further part of the solution is the  $n \times 1$  vector  $\boldsymbol{\lambda}_i = (\lambda_{1i}, \dots, \lambda_{ni})' \geq \mathbf{0}$  containing the weight factors by which the input and output quantities of (potentially) all countries are combined to generate the efficient input-output combination of country  $i$  on the frontier function. The restriction  $\sum_{j=1}^n \lambda_{ji} = \mathbf{1}'\boldsymbol{\lambda}_i = 1$  (with  $\mathbf{1}$  as a  $n \times 1$  vector of ones) allows for variable returns to scale<sup>2</sup> (see Banker et al. (1984)).

To implement the input-oriented radial efficiency measure analogous to a radar we proceed along the following steps:

**Step 1.** We build a grid of points for the input directions and the output levels. For the input directions we first determine the efficient countries for both periods under investigation<sup>3</sup> and then determine a grid of  $k = K/L$  and  $h = H/L$  (where  $H$  denotes human capital input,  $K$  physical capital input and  $L$  raw labor input) such that grid points are at the center of all frontier facets of both periods. For the output levels we determine a grid of the values of the single output  $Y$  of the efficient countries of both periods starting with 90 percent of the smallest output level, proceeding with the means of each two consecutively larger output values and ending with the smaller of the largest output levels of both periods. Combining, this leads to a grid with a total of 30938 different points spanning the whole relevant input-output space where the frontier function actually shifts when we consider the period 1980-2005. This grid is denoted as the set  $G$ .

**Step 2.** We fix the position of the radar at the origin of the input space and control the direction by the point  $(\mathbf{x}_0, y_0)$  where  $y_0$  denotes one of the grid values for the output level and  $\mathbf{x}_0$  denotes the input coordinates. The latter are determined by projecting the grid values for the input vector  $(K, H, L) = L \cdot (k, h, 1)$  on a circle with the radius of the observed combination of minimum input values of the efficient firms in each of both periods, denoted by  $(K_{\min}, H_{\min}, L_{\min})$ . Thus, with  $r = \sqrt{K_{\min}^2 + H_{\min}^2 + L_{\min}^2}$  and  $d = \sqrt{k^2 + h^2 + 1}$  we compute  $\mathbf{x}_0 = \frac{r}{d} \cdot (k, h, 1)$ . In this way, the efficiency towards the frontier functions

<sup>1</sup>See Färe and Primont (1995) and Hackman (2008) for detailed expositions of the axiomatic approach.

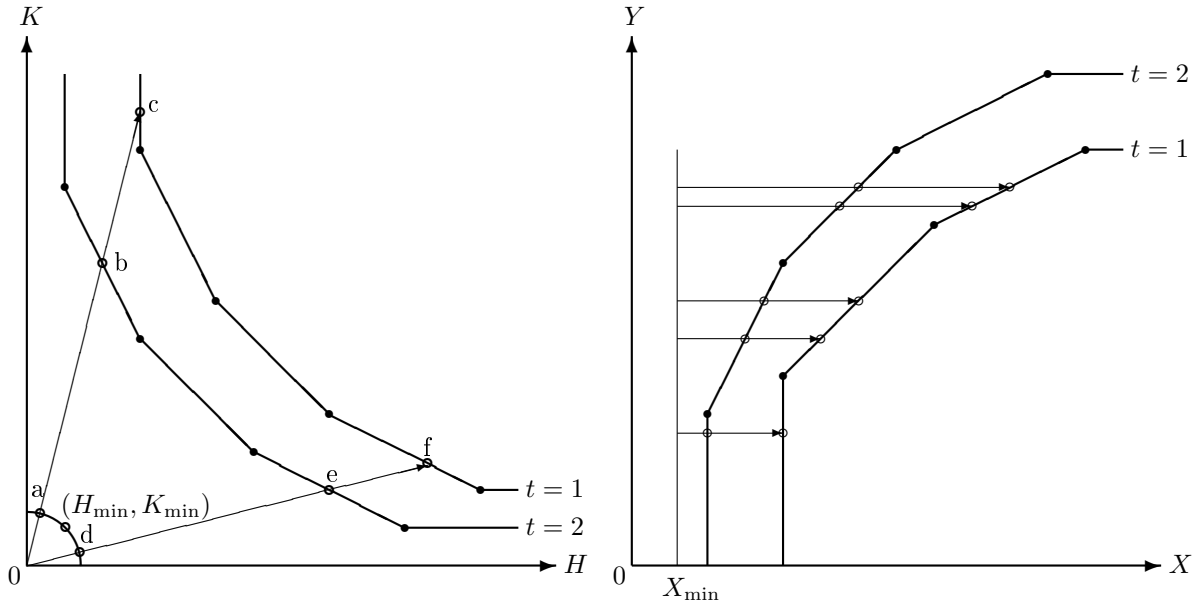
<sup>2</sup>This lets the analysis be less restrictive than in the case of studies assessing non-neutrality from a plot of  $Y/L$  against  $K/L$  where constant returns to scale are implicitly imposed.

<sup>3</sup>This is done by computing (1) for all countries and both periods and selecting the observations with  $\theta = 1$ .

of both periods is measured along a ray through the origin and a point well outside both frontier functions with properly determined intensities of physical and human capital.

In order to see this more clearly, figure 1 shows the situation for the setting to which we stick later in the empirical implementation with a single output  $Y$  and physical capital  $K$  and human capital  $H$  as two inputs, while ignoring raw labor  $L$  in the figure. The thick solid lines depict the frontier isoquants in the input space (left panel) for the periods  $t = 1$  and  $t = 2$  and the same output level as well as the production frontiers in the input-output space (right panel) for a prototypical input  $X$  and the periods  $t = 1$  and  $t = 2$ . The radar is positioned at the origin and rotates along the circle through the point  $(H_{\min}, K_{\min})$  in the input space and along the vertical line at  $X_{\min}$  in the input-output space. The arrows indicate the direction vectors for the efficiency measurement.

Figure 1: Radar Scanning in Input Space (left) and in Input-Output Space (right)



**Step 3.** Denoting the fixed position of the radar by  $\mathbf{x}_0 = (K_0, H_0, L_0)$  and  $\mathbf{y}_0 = (Y_0)$  we solve the linear programs

$$\max_{\theta_1, \lambda} \{ \theta_1 : \theta_1 \mathbf{x}_0 \geq \mathbf{X}_1 \lambda, \mathbf{y}_0 \leq \mathbf{Y}_1 \lambda, \mathbf{1}' \lambda = 1, \lambda \geq 0 \} \quad (3)$$

for the first period (with the corresponding input and output data in  $\mathbf{X}_1$  and  $\mathbf{Y}_1$ ) and

$$\max_{\theta_2, \lambda} \{ \theta_2 : \theta_2 \mathbf{x}_0 \geq \mathbf{X}_2 \lambda, \mathbf{y}_0 \leq \mathbf{Y}_2 \lambda, \mathbf{1}' \lambda = 1, \lambda \geq 0 \} \quad (4)$$

for the second period (with the corresponding input and output data in  $\mathbf{X}_2$  and  $\mathbf{Y}_2$ ). The computations of the efficiency measures are performed using the package *FEAR* for R, being documented in Wilson (2008).

From these calculations we obtain measures of the distance towards the frontier function of period 1 as the solution value  $\hat{\theta}_1$  for all grid points in the set  $G$  of program (3) and towards the frontier function of period 2 as the solution value  $\hat{\theta}_2$  of program (4) for each specified direction. In figure 1 along the steeper ray through the points a, b and c we get  $\hat{\theta}_1 = \frac{0c}{0a}$  and  $\hat{\theta}_2 = \frac{0b}{0a}$  (with  $0a$  denoting the length of the line from the origin 0 to the point a, etc.). Along the flatter ray through the points d, e and f we get  $\hat{\theta}_1 = \frac{0f}{0d}$  and  $\hat{\theta}_2 = \frac{0e}{0d}$ . Note that the efficiency measures depend only on the shifts of the frontier function at different regions of the input space along the particular ray chosen. The Farrell efficiency measures  $\hat{\theta}_1$  and  $\hat{\theta}_2$  are then transformed to the Shephard measures  $\hat{\delta}_1 = 1/\hat{\theta}_1$  and  $\hat{\delta}_2 = 1/\hat{\theta}_2$ . For measuring the extent and the direction (forward or backward) of the movement of the frontier function in the specified direction

we compute the average annual (log) percentage growth rate  $\Delta \ln \hat{\delta} = \frac{100}{\Delta t} \cdot (\ln \hat{\delta}_2 - \ln \hat{\delta}_1)$  over the time span from period 1 to period 2 (with  $\Delta t$  as the length of this time span). Positive values of this change measure indicate productivity improvements by the a forward shifting frontier part along the chosen ray while negative values indicate productivity deteriorations by a backward shifting frontier part.<sup>4</sup> The right part of figure 1 shows that the extent of the frontier function shifts may also be differential at different output levels.

**Step 4.** We summarize the dependence of  $\Delta \ln \hat{\delta}$  on the chosen directions by regressing  $\Delta \ln \hat{\delta}$  on the two ratios  $k = K/L$ ,  $h = H/L$  and the output level  $Y$  without and with considering interactions. As methods for estimating these response surface regressions we use OLS as well as a robust linear regression method known as the MM estimator. In addition, we also use nonparametric estimation methods such as the generalized additive model (GAM) without interactions up to a fully nonparametric estimation which also accounts for interactions. These estimation methods are briefly reviewed in the subsequent section 4 together with the results. Before we turn to issues of data handling in the next section.

### 3 Data Handling

The data for the empirical analysis are taken from the latest release of the Penn World Table (PWT, version 8.0), which now provides GDP data constructed from the output side rather than from the expenditure side. In addition, the data base also contains improved measures of the labor force and capital stocks. This lets the PWT 8.0 be much better suited for the purpose of macroeconomic productivity analysis. Feenstra et al. (2013) document the data base in detail and Feenstra et al. (2009) provide a conceptual comparison of output-side and expenditure-side real GDP measures. Also available now is a new version of the Barro-Lee data set on educational attainment data and information about average schooling years at primary, secondary and higher levels. This data set has also been updated and expanded and now covers the period 1950 to 2010 in 5-year intervals for 146 countries. Barro and Lee (2013) provide a documentation of the primary data sources, procedures and comparisons with other human capital measures.

As output variable for country  $i$  and year  $t$  we use real output per worker computed as the output-side real GDP at chained PPPs (series `rgdpo` in the PWT), denoted  $Y_{it}$ . Raw labor input  $L_{it}$  is measured by the number of workers in the economy (number of persons engaged, `emp`). Capital stocks  $K_{it}$  are used as the physical capital input variable. The PWT 8.0 unfortunately contains no direct real capital measure at chained PPPs analogous to the output measure. Instead, the series `ck` contains the capital stock at current PPPs (that is in PPPs but not real) whereas the series `rkna` contains the capital stock at constant 2005 national prices (that is real but not in PPPs). To solve this dilemma, we take the series `cgdpo`, containing output-side GDP at current PPPs, and multiply the ratio of the capital stock `ck` to this output-side GDP at current PPPs with the output-side real GDP at chained PPPs (`rgdpo`) to reach a real capital measure at chained PPPs (i.e. we compute  $\mathbf{rk} = (\mathbf{ck}/\mathbf{cgdpo}) \cdot \mathbf{rgdpo}$ ).<sup>5</sup>

Human capital per worker is constructed according to the suggestion of Hall and Jones (1999) based on previous work on Mincerian wage equations of Psacharopoulos (1994). Accordingly, the log of human capital per worker is a piece-wise linear function with returns to education of 0.134 up to the fourth year of education, 0.101 up to the eighth year and 0.068 beyond the eighth year. Formally, the human capital measure  $H_{it}$  is thus

$$H_{it} = h_{it} \cdot L_{it} = \exp(\phi(E_{it})) \cdot L_{it} \quad \text{with } \phi(E_{it}) = \begin{cases} 0.134 \cdot E_{it} & \text{for } 0 \leq E_{it} \leq 4 \\ 0.536 + 0.101 \cdot E_{it} & \text{for } 4 < E_{it} \leq 8 \\ 0.940 + 0.068 \cdot E_{it} & \text{for } E_{it} > 8 \end{cases}$$

where  $E_{it}$  denotes the average years of schooling in the population aged at least 25 years from Barro and Lee (2013), labeled there as the series `yr_sch`. This measure of human capital is also used by Henderson and Russell (2005) and by Badunenko et al. (2013).

<sup>4</sup>By a forward shifting frontier part we mean a frontier function part shifting towards the origin. Analogously, backward shifting means shifting away from the origin.

<sup>5</sup>The alternative way to compute  $\mathbf{rk} = (\mathbf{rkna}/\mathbf{rgdpna}) \cdot \mathbf{rgdpo}$  with the series `rgdpna` as the real GDP at constant 2005 national prices leads to a real capital stock series which is very highly correlated (correlation coefficient  $\approx 0.99$ ) with the variant we opt for here.

Both the PWT 8.0 and the new Barro-Lee dataset have been extended to the year 2010. To increase country coverage we restrict the empirical analysis to the period 1980-2010. We exclude countries which are either exceptional since they are merely large cities than countries (Hong Kong, Luxembourg and Singapore) or which are relatively small major oil producing countries (Bahrain, Brunei, Kuwait, Qatar, Saudia Arabia). Since the analysis in this paper focuses exclusively on the shifts of the frontier function those countries can be very influential and potentially can heavily bias the results. In addition, we exclude 16 mainly former Soviet Republics and Eastern European countries under Soviet hegemony with no data for the subperiod 1980-1990. These countries experienced their rather special transformation phase from a central planning economy during the 1990s. This leaves us with  $n = 110$  countries with complete observations for the period 1980-2010 as well as all five year averages.

## 4 Response Surface Estimates

In our framework we are working in a four-dimensional input-output space spanned by physical capital, human capital, raw labor and the output. Our direction vector is also three-dimensional with the output level as the fourth dimension. Even if we reduce the dimension by one through the application of the ratios  $k$  and  $h$  there remains the task to systematically determine in which way  $\Delta \ln \delta$  jointly depends on  $k$ ,  $h$  and  $Y$ . This is done by means of regression analysis which is the device to estimate so-called response surfaces defined as “a regression model in which each observation corresponds to one experiment, the dependent variable is some quantity that was estimated in the experiments, and the independent variables are functions of the various parameter values, chosen by the experimenter, which characterize each experiment” (Davidson and MacKinnon 1993, pp. 755f.). In this application, the experiment consists of the choice of a particular combination of direction vector and output level from the set  $G$  and the outcome of the experiment is the frontier shift measured along this direction at the indicated output level. The response surfaces serve to summarize the dependence of the frontier shifts on the entire set of directions.

The response surfaces are estimated by different regression methods. The first method is ordinary linear least squares regression (OLS) with heteroskedasticity and autocorrelation consistent standard errors computed according to the proposal of Lumley and Heagerty (1999). Autocorrelation may be caused by the systematic scan of the entire frontier and may bias the standard errors. On the other hand, the exogeneity of the regressors is guaranteed by the design of the experiment so that inconsistency of regression parameter estimates is not a problem. Moreover, the large sample available lets the efficiency loss due to heteroskedasticity and autocorrelation not be particularly important (this fact will be seen later from the large  $t$ -statistics we obtain).

Since outliers may influence the estimation of the frontier function and therefore the assessment of its shifts and we are also not quite sure about the functional form of the response surfaces, we apply a robust regression estimators and nonparametric regression methods. The MM-estimator of Yohai (1987) is a robust regression estimator specifically designed to combine the advantages of a high breakdown point (the fraction of contaminated observations in the sample that can lead to an arbitrarily large deviation of the estimator) and high estimation efficiency.<sup>6</sup>

Nonparametric regressions are computed as generalized additive models (GAM) when no interactions of the angles are supposed. For the computation we use the penalized likelihood approach implemented by Wood in the R-package “mgcv”. The details of the functionalities available in the package are described at length in Wood (2006) with brief overview given in Wood (2001). The fit is computed using thin plate regression splines which avoid the choice of knot locations (see Wood (2003)). When interactions are allowed for, two-dimensional and three-dimensional spline bases are used. The number of knots is generally not critical. Smoothing parameters are chosen by the restricted maximum likelihood estimation (REML) approach.

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<sup>6</sup>This goal is achieved by using an initial M-estimator searching for the regression parameters associated with the smallest robust measure of scale of the residuals (actually an S-estimator), followed by a second M-estimation which can be computed by the iteratively reweighted least squares (IRWLS) algorithm. Maronna et al. (2006, pp. 124ff.) provide a formal exposition. The implementation used in this paper is that of Yohai et al. (1991) in the R-package “robust”.



## 4.1 Results Without Interactions Included

In the remainder of this section we analyze different specifications of the response surface regressions for the period 1980-2005. To limit the effects of single years we compare the frontier functions of the five year averages 1976-1980 as the first period and 2001-2005 as the second period of the inputs and the output. The subperiod 1995-2005 will be analyzed later on to explore the impact of the IT and computer revolution separately. Likewise, the period 2005-2010 covering the effects of the financial crisis and the Great Recession is also analyzed separately below.

The response surfaces are first estimated without including interactions. This means that we simply regress  $\Delta \ln \delta_i$  on the output levels  $Y_i$  and the directions indicated by  $k_i$  and  $h_i$  (all in natural logs) as the basic linear specification

$$\Delta \ln \delta_i = \beta_0 + \beta_1 \ln Y_i + \beta_2 \ln k_i + \beta_3 \ln h_i + u_i$$

or on three additive sets of one-dimensional spline functions in the case of the basic GAM specification

$$\Delta \ln \delta_i = \beta_0 + s_1(\ln Y_i) + s_2(\ln k_i) + s_3(\ln h_i) + u_i$$

where the functions  $s_1(\cdot)$ ,  $s_2(\cdot)$  and  $s_3(\cdot)$  are represented by the splines. Note that in this section  $i$  indexes an element of the set  $G$  of directions and thus  $i \in \{1, \dots, 30938\}$ .

Table 1 reports the response surface regression estimates with all three methods. We find positive effects of  $Y$  and  $h$  which are highly significant with large  $t$ -statistics. The effect of  $k$  is negative but insignificant in the OLS regression while it gains significance in the robust MM regression.<sup>7</sup> Goodness of fit is reasonably large with  $R^2$ s of about 43% for the OLS and 37% for the MM regressions. The results imply that shifts of the frontier function depend on the output level and also on the direction as chosen by  $k$  and  $h$ , which provides strong evidence of non-neutrality.

The GAM regression results show that all three spline terms are highly significant with quite large effective degrees of freedom (edf). This points to a pronounced degree of nonlinearity in the relationship which simultaneously is also evidence in favor of non-neutrality of technological change. This is supported by the increase of the  $R^2$  measure to about 62%. To analyze this nonlinear relationship in more detail figure 2 shows the response surface estimated by the GAM in the form of contour plots. These plots show the levels of the fitted values of the regression evaluated at various combinations of  $Y$ ,  $k$  and  $h$  in different shadings. The shading is dark gray for negative values of the response surface and becomes lighter for increasing values of the response. Note that the axes are denominated in natural logs. Since three variables ( $Y$ ,  $k$ ,  $h$ ) influence the response and contour plots are only suited to depict two-dimensional relationships, the figure shows the results as the response on the  $k$ - $h$  space for different quantiles of  $Y$  (i.e. 0.1, 0.25, 0.5, 0.75, 0.9).

Figure 2 shows that for low quantiles of  $Y$  productivity growth at the frontier is positive only in regions of the  $k$ - $h$  space where  $h$  is large jointly with  $k$  being either large or small (see the upper left and right region in the figure). Productivity growth at large  $h$  and small  $k$  may be due to small (in terms of output) and presumably less manufacturing oriented countries. The region where productivity is positive and the frontier function shifts forward becomes much larger when higher quantiles of  $Y$  are considered. Here, the magnitude of the productivity change also increases. In case of the 0.9-quantile of  $Y$  backward shifts of the frontier function associated with negative productivity growth at the frontier are observable only for small  $h$  values and medium values of  $k$ . Thus the frontier shifts forward at high levels of human capital per worker, physical capital per worker and output pointing to the force of complementarities. The forward shift at high levels of human capital per worker and low levels of physical capital per worker is puzzling, however. The entire set of results suggests the presence of interaction effects to which we turn now.

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<sup>7</sup>This gain of significance may be viewed with caution, however, since the standard errors of the OLS regression are corrected for autocorrelation and heteroskedasticity (see above), but the standard errors of the MM estimation are not corrected.

Table 1: Pure Linear Regression Results

	OLS	MM		GAM
$c$	-4.776 (53.046)	-2.808 (61.573)	$c$	-0.097 (23.400)
$\ln Y$	0.346 (52.841)	0.178 (49.789)	$s_1(\ln Y)$	8.878 (0.000)
$\ln k$	-0.028 (1.329)	-0.039 (11.187)	$s_2(\ln k)$	8.668 (0.000)
$\ln h$	0.760 (10.958)	0.873 (87.137)	$s_3(\ln h)$	6.445 (0.000)
$R^2$	0.429	0.372	$R^2$	0.619
$n$	30938	30938	$n$	30938

Note: Reported in parentheses below the regression coefficients are  $t$ -statistics. For the spline terms  $s(\cdot)$  the effective degrees of freedom are reported with the  $p$ -values of the associated  $F$ -statistics for the joint significance in parentheses.

Figure 2: GAM Regressions without Interactions (Period 1980-2005)

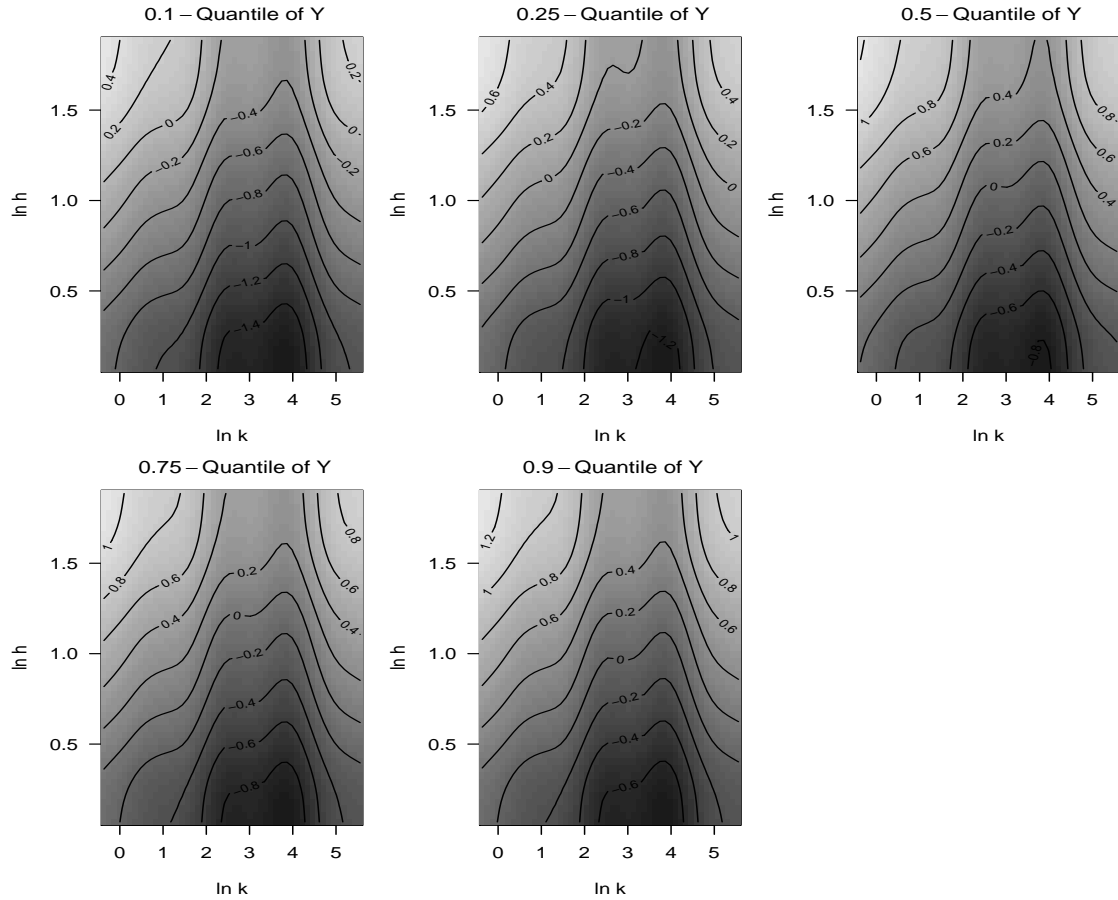


Table 2: Regression Results with Two-Way Interactions

	OLS	MM		GAM
$c$	-4.231 (20.000)	-1.443 (22.854)	$c$	-0.097 (59.983)
$\ln Y$	0.366 (22.828)	0.136 (26.123)	$s_{12}(\ln Y, \ln k)$	28.303 (0.000)
$\ln k$	-0.209 (5.058)	-1.261 (93.513)	$s_{13}(\ln Y, \ln h)$	27.385 (0.000)
$\ln h$	0.071 (0.455)	3.507 (89.395)	$s_{23}(\ln k, \ln h)$	26.545 (0.000)
$\ln Y \cdot \ln k$	-0.004 (1.220)	0.082 (74.324)		
$\ln Y \cdot \ln h$	-0.006 (0.625)	-0.295 (93.148)		
$\ln k \cdot \ln h$	0.224 (13.678)	0.271 (97.372)		
$R^2$	0.467	0.502	$R^2$	0.942
$n$	30938	30938	$n$	30938

Note: Reported in parentheses below the regression coefficients are  $t$ -statistics. For the spline terms  $s(\cdot)$  the effective degrees of freedom are reported with the  $p$ -values of the associated  $F$ -statistics for the joint significance in parentheses.

## 4.2 Results With Two-Way Interactions Included

In the following we make the specification more flexible by including interactions. We start with two-way interactions where the response surface regressions are for OLS and MM

$$\Delta \ln \delta_i = \beta_0 + \beta_1 \ln Y_i + \beta_2 \ln k_i + \beta_3 \ln h_i + \beta_4 \ln Y_i \ln k_i + \beta_5 \ln Y_i \ln h_i + \beta_6 \ln k_i \ln h_i + u_i.$$

In the case of the basic GAM specification we have

$$\Delta \ln \delta_i = \beta_0 + s_{12}(\ln Y_i, \ln k_i) + s_{13}(\ln Y_i, \ln h_i) + s_{23}(\ln k_i, \ln h_i) + u_i$$

where the functions  $s_{12}(\cdot, \cdot)$ ,  $s_{13}(\cdot, \cdot)$  and  $s_{23}(\cdot, \cdot)$  are now representing two-dimensional spline bases.

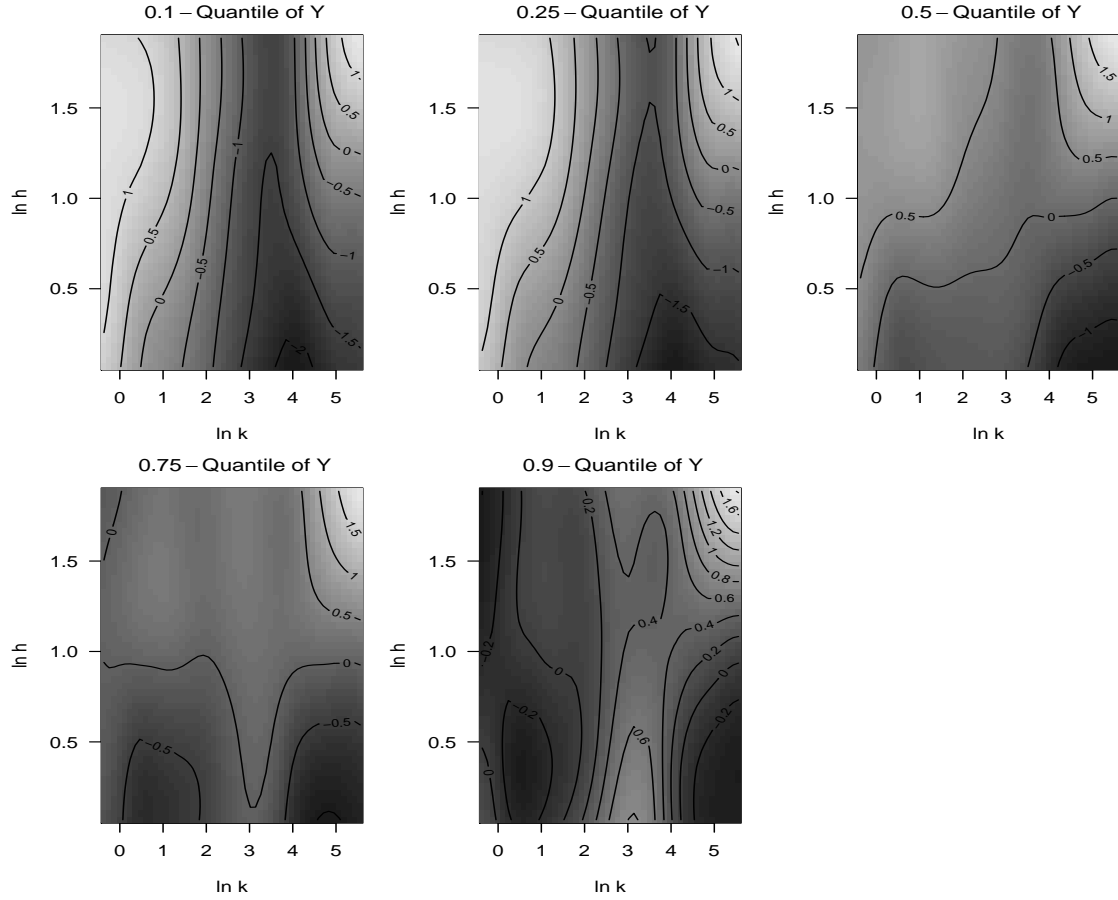
Looking at the OLS regression results in table 2 we find a significantly positive linear effect of  $Y$ , a significantly negative effect of  $k$  and an insignificantly positive effect of  $h$ . This pattern is also found in the MM regressions with much larger  $t$ -statistics there. From the interaction terms the most important seems to be the interaction of  $k$  and  $h$  reassuring that productivity growth is substantially and significantly larger in regions of the input space with large intensities of both physical and human capital. Here OLS and MM regressions agree to a large degree. Goodness of fit is improved by only a narrow margin in the case of the OLS regression but much more in the case of the MM regression.

The results of the GAM model show a much improved goodness of fit with an  $R^2$  increasing to 94%. The spline terms are highly significant altogether. The estimated response surfaces are depicted in figure 3 where we can observe that productivity growth tends to be always largest for largest values of both  $k$  and  $h$ . For values of  $Y$  below the median there is also an extended region of positive productivity growth and forward shifting frontier function parts for small values of  $k$  and medium to large values of  $h$ .

## 4.3 Results With Three-Way Interactions Included

From the above results we get the overall impression that forward shifts of the frontier function are most pronounced when the intensities of both physical and human capital are large and the output level is large as well. This is tested by introducing an additional three-way interaction term  $\ln Y_i \cdot \ln k_i \cdot \ln h_i$  in the OLS and MM regressions. The response surfaces are depicted by estimating a GAM regression with a three-dimensional spline term, leading to the specification  $\Delta \ln \delta_i = \beta_0 + s(\ln Y_i, \ln k_i, \ln h_i) + u_i$ .

Figure 3: GAM Regressions with Two-Way Interactions (Period 1980-2005)



The linear regression estimates with the OLS and MM estimators reported in table 3 show that the three-way interaction effect is positive and highly significant in both cases. Its explanatory power is not overly important since the  $R^2$  measures increase only marginally compared to the previous case with just the two-way interactions. In the case of the GAM (which here actually is a fully nonparametric three-dimensional spline fit) the increase in the  $R^2$  measure is somewhat larger. The contour plots of the response surface in figure 4 shows the same features as the previously shown plot for the two-way interactions.

#### 4.4 Frontier Shifts during the 'Productive Decade' 1995-2005

In the following we consider the subperiod 1995-2005. During this period the impact of the computer and IT revolution also materialized in productivity statistics where Robert Solow<sup>8</sup> could not find these effects previously (see Oliner et al. (2007) for a review). Here we focus on the fully nonparametric estimates with input-output data from the five-year averages over 1991-1995 to the five-year averages 2001-2005 as the first and second subperiods, respectively. The fully nonparametric estimates are now based on 30442 observations<sup>9</sup> and are associated with an  $R^2$  of about 94%. The results are depicted in figure 5 in the same way as above.

Productivity growth is indeed much larger during this decade for all output levels. It is positive for the lowest output quantiles (0.1, 0.25) except for the smallest  $h$  and  $k$ . For the other output quantiles (0.5, 0.75, 0.9) we find a quite sharp divide between forward shifts of the frontier function at large  $k$  and

<sup>8</sup>Recall the famous quote from Solow (1987): "You can see the computer age everywhere but in the productivity statistics."

<sup>9</sup>Of course, the number of observations changes when the period changes since the number of observations depends on the number of countries on the frontier function in both periods and the number of combinations of  $k$ ,  $h$  and  $Y$  resulting from this. Recall step 1 in the outline of the measurement procedure above.

Table 3: Regression Results with Three-Way Interactions

	OLS	MM		GAM
$c$	-5.535 (44.161)	-2.086 (21.712)	$c$	-0.097 (101.908)
$\ln Y$	0.479 (55.229)	0.189 (23.599)	$s(\ln Y, \ln k, \ln h)$	88.912 (0.000)
$\ln k$	0.172 (4.318)	-1.087 (44.223)		
$\ln h$	1.359 (9.047)	4.302 (49.023)		
$\ln Y \cdot \ln k$	-0.037 (12.157)	0.067 (32.737)		
$\ln Y \cdot \ln h$	-0.118 (12.155)	-0.362 (49.342)		
$\ln k \cdot \ln h$	-0.154 (2.711)	0.067 (3.020)		
$\ln Y \cdot \ln k \cdot \ln h$	0.033 (8.584)	0.017 (9.244)		
$R^2$	0.470	0.502	$R^2$	0.980
$n$	30938	30938	$n$	30938

Note: Reported in parentheses below the regression coefficients are  $t$ -statistics. For the spline terms  $s(\cdot)$  the effective degrees of freedom are reported with the  $p$ -values of the associated  $F$ -statistics for the joint significance in parentheses.

Figure 4: GAM Regressions with Three-Way Interactions (Period 1980-2005)

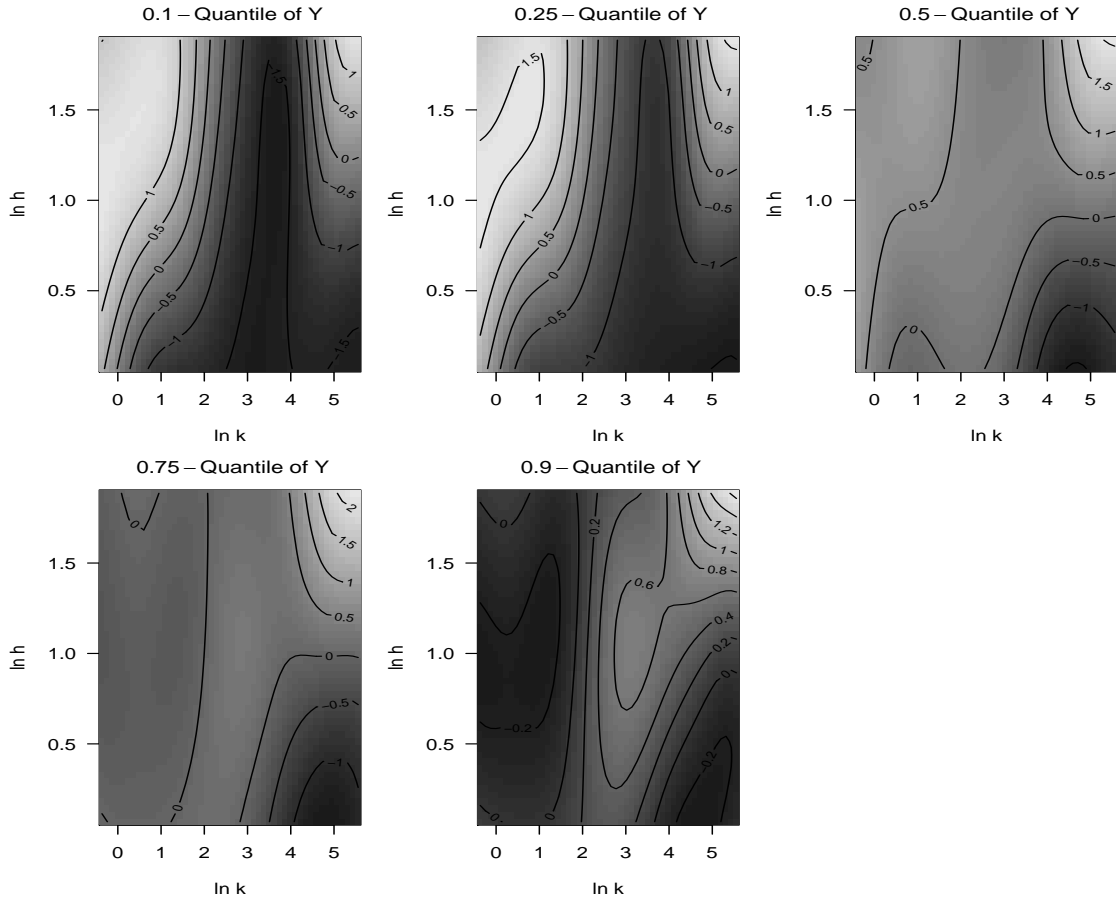
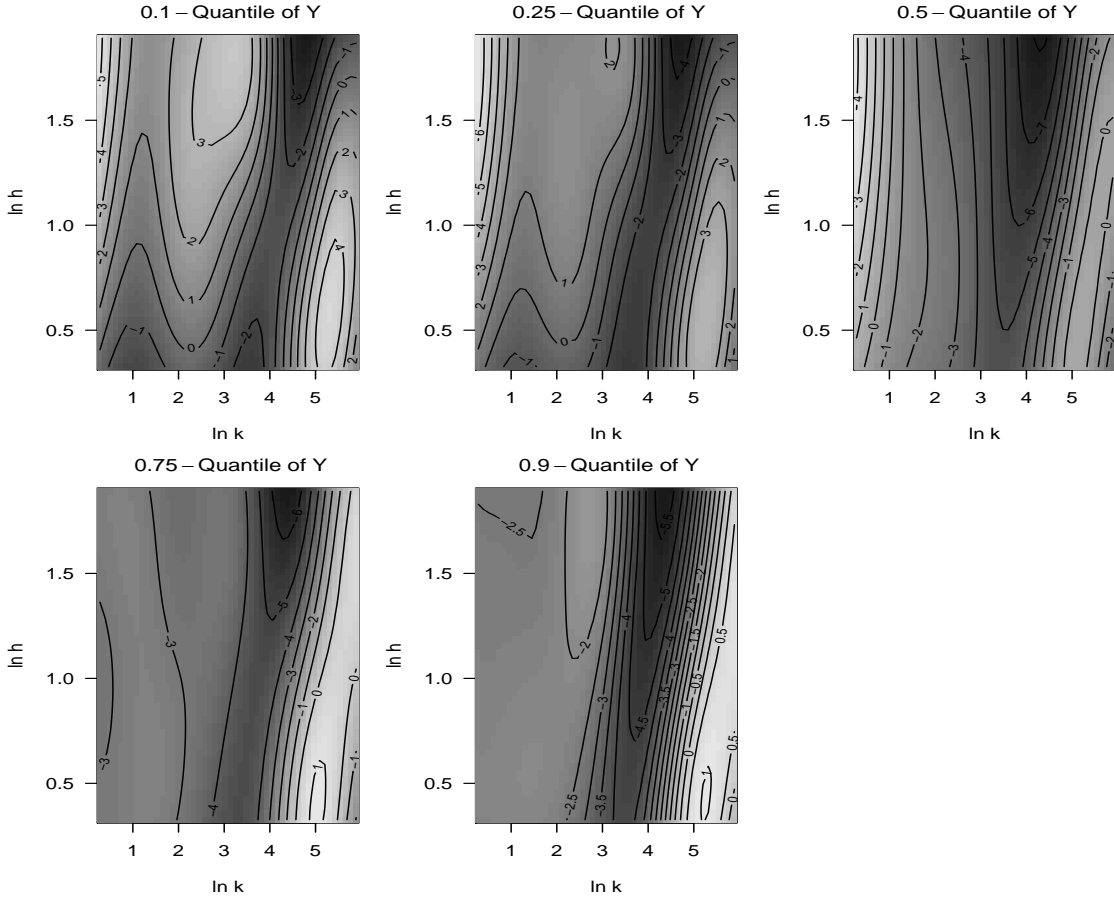


Figure 5: GAM Regressions with Three-Way Interactions (Period 1995-2005)



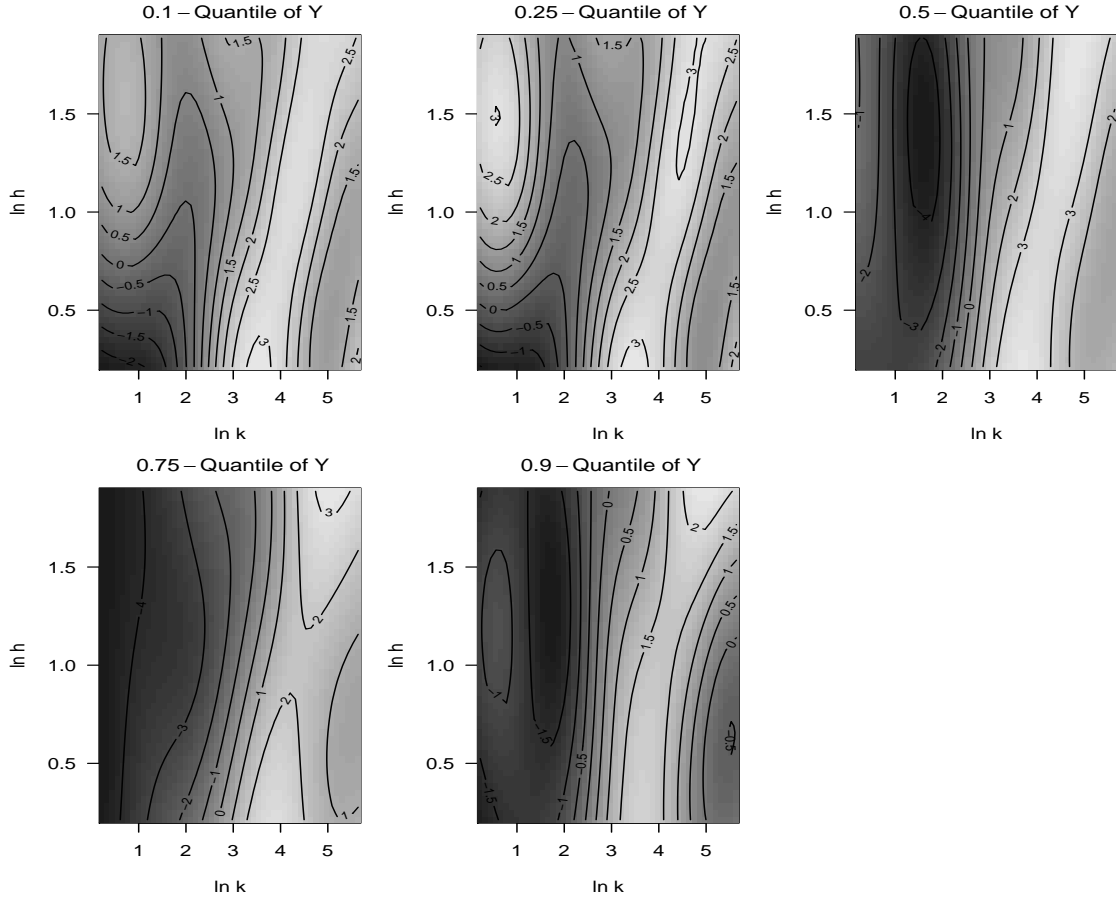
backward shifts at low  $k$ . Thus, productivity growth is driven by physical capital and the importance of human capital seems to be reduced during this period. The role of capital (esp. IT-related capital) is also stressed by Jorgenson and Vu (2005) and by Jorgenson (2001) for the US economy. See also the analysis of Czernich et al. (2011) for the growth effects of broadband access in OECD countries during this period.

#### 4.5 Frontier Shifts during the Great Recession

Now we turn to the subperiod 2005-2010, covering the years of the financial crises which led to the breakdown of Lehmann Brothers and triggered the so-called Great Recession. We focus again on the fully nonparametric estimates with input-output data now from the five-year averages over 2001-2005 to the five-year averages 2006-10 as the first and second subperiods, respectively.<sup>10</sup> Figure 6 shows the nonparametric estimates which are based on 23324 observations and are associated with an  $R^2$  of about 94%. It appears that we observe predominantly positive productivity growth for low output quantiles and are faced with marked backward shifts of the frontier function for the larger output quantiles. For larger output quantiles, forward shifts of the frontier function can only be detected in the regions with a high intensity of physical capital. Thus it appears that less developed countries were also less affected by the Great Recession.

<sup>10</sup>The Great Recession ended officially in 2009 but the recovery was still incomplete in the following years, see e.g. Ng and Wright (2013).

Figure 6: GAM Regressions with Three-Way Interactions (Period 2005-2010)



## 5 Conclusion

In the above analysis we implement a measurement device analogous to a radar for tracking the shifts of the world production frontier along a set of specified directions for the whole input-output space spanned by raw labor input, physical capital input, human capital input and the output variable. The bottom line of our findings is that we establish vivid evidence against the neutrality of technological progress for our sample of 110 countries over the period 1980-2010. This is also in accord with the other nonparametric studies reviewed in the introduction, although these studies focus on other aspects of macroeconomic productivity change and take different methodological approaches.

Using both parametric and nonparametric response surface estimates we find that the forward shifts of the frontier function induced by technological progress appear to be more pronounced in regions of the frontier function where human capital is more intensely used per worker. Simultaneously, physical capital per worker is either particularly high or particularly low in these regions. Moreover, forward shifts of the frontier function are more pronounced when the output level is higher and in that case largest when both intensities of human and physical capital are large. All this is strong evidence against the neutrality of technological progress and points to the importance of scale effects. The analysis of subperiods shows as the impact of the computer and IT revolution during the 1990s forward shifts of the frontier function are more widespread and we observe again the mutual enforcement of  $h$ ,  $k$  and  $Y$ . The years of the Great Recession are characterized by dominating backward shifts of the frontier function at high output levels except when the intensity of physical capital is large and largely independent of the intensity of human capital.

Future work will consider a more detailed evaluation of the nonparametric results and of the specific countries driving the frontier shifts. A more in-depth investigation of the impact of IT would also be a promising line of research within the radar framework.

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