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A Reconsideration of Multiple Equilibria in the Analysis of One-Period Government Bonds with Default Risk

Yanling Guo

Abstract
In this paper, the author considers sovereign debt in the form of one-period government bonds with default risk, which can be purchased by and traded among domestic and foreign investors. She shows that the "good equilibrium" is the only stable equilibrium under some quite general assumptions, while the "bad equilibrium" is an unstable one – a possible explanation for why the former is observed in practice. Given the "good equilibrium", the author further shows that the domestic debt share also determines the default risk: a higher domestic debt share means a lower default risk, ceteris paribus, which leads to a lower risk premium; while a lower domestic debt share means a higher default risk and a higher risk premium. Finally, she discusses some alternative interpretations of the domestic debt share.

JEL F34 H63 H74 H62 H6 H87
Keywords Public debt; sovereign debt; sovereign default; domestic debt; external debt; fiscal policy; government bond; government borrowing

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1 Introduction

Recently, the term sovereign default has received considerable attention in both the literature and the public discussion. Sovereign default is defined as a default on sovereign debt. In this paper, I focus on the analysis of the so-called one-period government bond, which is the basic form of sovereign debt and hence has received the most attention in the literature. The one-period government bond is defined as non-collateralized and non-committable public debt, which promises state-non-contingent repayment after one period. Here, I only consider the government bonds purchased by and traded among the private investors, although the result can, after some adaptation, also be applied to sovereign debt owned by public lenders.

Prior studies modeling this particular form of sovereign debt consider either domestic or external debt, classified according to whether the debt is owned by residents or by foreigners, respectively. The recent literature also models government bonds, which are a combination of the two forms of debt, because the globalization of financial markets makes both residents and foreigners more likely to purchase and trade government bonds with one another – a modeling strategy that is also adopted in this paper. This modeling strategy necessarily introduces a new parameter, here called α, which represents the domestic debt share, i.e., the share of public debt owned by domestic citizens. After having shown that only the "good equilibrium" is a stable equilibrium, I also study the effect of α.[1] This research subject was selected for both theoretical and practical reasons.

To date, the literature mostly concentrates on the analysis of the effect of the outstanding amount of debt on default risk. Most empirical research, e.g., Reinhart and Rogoff (2011), shows that an increasing amount of outstanding debt increases default risk, although the relationship is not linear. This empirical finding has theoretical foundations, first laid by Eaton and Gersovitz (1981), who has proven this positive correlation analytically. Another strand of literature, led by Calvo (1988), has adopted a different modeling strategy and also obtained this positive correlation but only in the so-called "good equilibrium". Calvo (1988) demonstrates that there is also a so-called "bad equilibrium" in which all effects are

[1] Because globalization has made the researchers increasingly aware of the effect of α, other researchers have – parallel to me – derived similar results; see, e.g., Engler and Steffen (2014).

reversed, i.e., the default risk can decline even with a rising amount of outstanding debt. Although this theoretical concept of the "bad equilibrium" is not supported by empirical findings, it can weaken the political will to reduce debt because governments experiencing difficulty can argue that they are simply affected by the "bad equilibrium". In this paper, I show that while the "bad equilibrium" indeed exists, it is an unstable equilibrium and hence almost certainly occurs with a probability of zero.

My finding that only the "good equilibrium" is a stable equilibrium while the "bad equilibrium" is unstable supports the call for more fiscal consolidation as a remedy for a debt crisis. However, in practice, governments facing difficulty often only recognize that they have accumulated excessive debt when it is already too late, i.e., although fiscal consolidation can reduce the default risk, it is often difficult to conduct due to a lack of support from the citizens, who have to suffer most when the government cuts expenditures or raises taxes. One alternative solution suggested in the literature is to raise expected output because rising output reduces debt as a share of GDP and increases the tax base, thereby reducing default risk and interest costs in the "good equilibrium". In Appendix B.1, I also use an example to show that more debt can even benefit the economy when it can generate a higher yield than the agreed upon cost of interest. However, governments experiencing difficulty in repaying their debt often simultaneously face a weak economy; in other words, if they were able to generate a high yield from borrowed money, be it in the form of direct investment or output-increasing fiscal policy, they would have no problem accessing the financial market, which always seeks good investment opportunities. Of course, the government ultimately has to increase the productivity of the economy to render the debt level sustainable, and hence structural reforms are inevitable if the government cannot reduce its debt. However, structural reforms are sometimes just as difficult to realize for the government as cutting debt. Hence, here I offer an alternative solution by considering the effect of α. After proving that the "good equilibrium" is the only stable equilibrium, I show that raising α can also reduce default risk and hence mitigate the sovereign debt problem. However, raising α could also be difficult, and hence what I show here is simply an additional possible solution, and the government has to choose which solution it prefers: to reduce debt, to raise the output or to raise α. Of course, it can also combine these three solutions to find a policy mix that seems optimal.

As already briefly discussed in Calvo (1988),[2] α can co-determine the default risk. Calvo's finding regarding default in the form of debt repudiation is in line with my paper, namely that in the "good equilibrium", a lower α means more *ex post* default risk and at least the same risk premium. However, because his focus is on domestic debt, he only considers the case in which α is close to 1.[3] Moreover, in his "bad equilibrium", the effects of all variables, including α, on interest costs have the opposite sign as in the "good equilibrium". Gennaioli et al. (2010) also consider an economy borrowing from both residents and foreigners, but they did not analyze the effect of α.

The remainder of the paper is organized as follows: Section 2 describes the model, proves the positive impact of α on the repayment propensity of the borrowing government for the Calvo-type model and then for the Eaton and Gersovitz-type model, and shows that the good equilibrium is the only stable equilibrium and hence that a higher repayment propensity almost certainly leads to lower default risk and better borrowing conditions for the government. Section 3 briefly discusses the application of my analysis in practice, and Section 4 concludes.

2 The model setup

2.1 The preamble

The model studies a small open economy in a *de facto* monetary union in the sense that this small open economy has only fiscal authority, while the monetary power is assigned to a union-level institution. Further I assume that this small open economy is so small that its performance has no influence on the monetary policy or economic development of the union. In particular, the union-wide reference interest rate is taken as given for this small open economy. Here I use this "monetary union" setting as an analytical device to abstract from monetary policy accommodation possibilities such as "inflating away" the debt and to, thereby, focus on the effect of α. Hence, the "monetary union" term used here should not be confused with a real-world monetary union such as the EMU (European Monetary Union), which

[2] In Calvo (1988), the α is coded as γ.

[3] In Appendix B.1 I will use the original Calvo model as a special case to illustrate the effect of α.

I will refer to in this paper as an "explicitly declared monetary union". Indeed, the small open economy under study does not necessarily need be a country at all but can be any administrative region with its own fiscal authority, such as a state in the USA or a province in Canada[4], and I only occasionally refer to this small open economy as a "country" to make the text shorter. Further, a country that has adopted the currency of another country and in this way abandoned its own monetary authority can also be viewed as a member in a *de facto* monetary union. In short: the monetary union in this paper is defined as a collection of sovereign bodies sharing the same currency and the same monetary policy authority that is not under the influence of this small open economy, while each union member maintains its own fiscal policy authority.

The government is assumed to be benevolent and attempts to maximize the welfare of its residents. The residents are modeled as the representative agent, as usual. To achieve its goal, the government can, in each period t, select its expenditure g_t and income tax τ_t. The difference between g_t and τ_t is lent to or borrowed from the financial market. The debt has to be repaid at the end of each period, and immediately thereafter, the new debt contract is signed at the beginning of the next period. Following the standard literature, I assume that the government cannot commit and determines its policy instruments anew for every period. The available policy instruments include, besides g_t and τ_t, θ_t, which is the default rate as a fraction of the outstanding amount of debt. The aim of the government is to maximize social welfare in terms of the current and (discounted) future utility of the representative agent. Its policy choice may be constrained in various ways, e.g., g_t may be held constant to reflect a pre-determined fiscal stance, or τ_t can be bounded from above by y_t if y_t constitutes the sole tax base.

Because the government cannot commit, there is a non-negative probability of default on the outstanding debt. If the investors in the financial market anticipate a positive default probability for the debt being negotiated, they will charge a higher interest rate to compensate for the possible loss due to default or refuse to lend if the expected debt repayment ratio is strictly below the market return. Denote the gross reference interest rate or the market return as R_t and the contracted gross

[4] Studies that regard the USA or Canada as a monetary union include Rockoff (2000) and Landon and Smith (2007)

interest rate for government debt as z_t; then, in an arbitrage-free world, there should be $z_t E_t(1-\theta_{t+1}) = R_t$ with $\theta_t \in [0,1]$. This market participation constraint takes the form of the risk-neutrality of investors, though I can imagine adapting it to the risk-averse case in which the investors charge a market return, which is higher than the risk-free alternative interest rate. Because this paper does not concern risk-aversion, I do not further consider R_t and treat it as a constant.

If the government chooses to default, then the economy will incur some default cost p. This default cost may or may not be of an economic nature. An example of a non-economic cost is the loss of a good reputation, and as an example of the economic cost, p can represent negotiation costs, retaliatory actions such as trade embargoes, or a reduction in trade credit or bank credit.[5] Here, I treat all types of default cost as possible and model p as the default cost, which will be incurred in the default period and, potentially, also in the following periods, and a positive p will reduce the social welfare in the corresponding period. Although the literature often regards exclusion from the financial market as a default cost, I do not model it as a part of p but as a constraint to the government's set of available policy mixes, i.e., the amount that can be borrowed from the financial market will be constrained to 0 immediately following a default decision. In addition to default cost, there may also be costs arising from taxation, referred to in the literature as deadweight loss and denoted here as $x(\tau_t)$.

The parameter α lies in the interval $[0,1]$ and represents the domestic debt share, namely the proportion of debt owned by domestic lenders. Because it can also be mathematically interpreted as the weight assigned to the private consumption covered by wealth, namely debt repayment minus new borrowing, in Section 3 I will discuss some alternative interpretations of α. However, otherwise, throughout this paper, I will retain the conventional interpretation of α as the domestic debt share. Following the conventional wording, I sometimes use phrases such as "a portion of $1-\alpha$ of the debt is held by foreign investors", although in this context the term "foreign investors" does not necessarily mean investors from a foreign country but rather refers to the lenders who are not within the boundaries of the fiscal authority under consideration.

[5] Gennaioli et al. (2010) argue that sovereign default will lead to deterioration in domestic banks' balance sheets and hence reduce credit supply in the domestic market.

2.2 The objective function

The following is the borrowing government's objective function:

$$\mathbb{V}(y_t, z_{t-1}b_{t-1}, \vec{p}_t; \alpha) = \sup_{b_t \in B_t, \theta_t \in \Theta_t, g_t \in G_t, \tau_t \in \Upsilon_t} \{\mathbb{U}(c_t, g_t) + \cdots$$

$$\cdots + \beta E_t \mathbb{V}(y_{t+1}, z_t b_t, \vec{p}_{t+1}(\theta_t); \alpha)\} \quad \text{s. t.} \tag{1}$$

$$c_t = y_t - x(\tau_t) - \tau_t + \alpha(1-\theta_t)z_{t-1}b_{t-1} - \alpha b_t \tag{2}$$

$$(1-\theta_t)z_{t-1}b_{t-1} + g_t = \tau_t + b_t - p(\theta_t) - \vec{p}_t \tag{3}$$

$$z_t = \inf[z : \ z_t E_t(1-\theta_{t+1}) = R_t] \tag{4}$$

Equation (1) describes the representative agent's value function, which the government attempts to maximize using the political instruments of new debt b_t, the default rate on old debt θ_t, government expenditure g_t and tax revenue τ_t. The political instruments can only be chosen within the eligible sets B_t, Θ_t, G_t and Υ_t, respectively. According to the specific model setting, B_t, Θ_t, G_t and Υ_t can be differently defined. For instance, many models assume government expenditure to be exogenous and hence restrict G_t to be a singleton, such that $G_t = \{\bar{g}\}$, while other papers allow g_t to be any non-negative value and hence $G_t = \mathbb{R}_+$. Typically, $\Theta_t \equiv [0,1]$, but in models interpreting inflation as an implicit default, Θ_t can also include negative values as in Calvo (1988), while in models in which θ_t needs to be flexible because other political instruments are strongly constrained, Θ_t can also include values above one.[6] Here, we have $\Theta_t \equiv [0,1]$ because there is no need to consider inflation or deflation as an implicit default for a government without own monetary authority. To make a decision on θ_t possible, I consider only Υ_t, which is constrained loosely enough such that an optimal policy mix that satisfies the government budget constraint and the constraints imposed on other eligible sets of political instruments always satisfies the constraint on Υ_t. Indeed, many papers do not place any constraint on Υ_t, and some set $\Upsilon_t \equiv (-\infty, y_t - x(\tau_t)]$ when interpreting τ_t as income tax.

[6] An example is Juessen et al. (2011), who model non-strategic government default in which both τ_t and g_t are predetermined and hence θ_t may sometimes be above one to meet the budget constraint.

The eligible sets of the political instruments can also be a function of another political instrument, as nearly all papers assume that new debt taking is restricted to zero if there is a default in the current period and the default is in the form of a contract violation, i.e., $B_t(\theta_t) = \{0\} \ \forall \ \theta_t > 0$.[7] For models in which exclusion from the financial market may also take place in the following periods after the initial default, the previous constraint will become $B_i(\theta_s) = \{0\} \ \forall \ \theta_s > 0$ and $i \in [s,t]$, with s and t denoting the first and last period of the default era, and t may be ∞, which means permanent exclusion from the financial market as in Eaton and Gersovitz (1981). These constraints may also appear in expectational form, i.e., one can assume that future exclusion from the financial market occurs with some positive probability as in Arellano (2008), in which $\Pr(B_i(\theta_s) = \{0\} \mid B_{i-1}(\theta_s) = \{0\}) = \text{const} > 0 \ \forall \ \theta_s > 0$ and $i \in [s+1, \infty)$.

The periodic utility of the representative agent is derived from the absorption of private consumption c_t and public goods provisioning g_t. Equation (2) describes the financing source of private consumption: the average citizen consumes his after-tax income $y_t - \tau_t - x(\tau_t)$, plus government bond repayment, which is possibly partially repudiated, $\alpha(1-\theta_t)z_{t-1}b_{t-1}$, minus purchases of new government bonds αb_t. Here, I do not consider private external borrowing or lending because such activity cannot be influenced by the government using the political instruments available here; hence, aggregate saving or dissaving appears in the form of government bond purchases, and the government can adjust b_t to smooth economy-wide consumption as long as b_t is not restricted to 0 due to a default decision. Note that here the before-tax income is $y_t - x(\tau_t)$ and not just y_t because the distortionary effect from taxation may reduce the output. Therefore, y_t should rather be interpreted as endowment or potential output, i.e., the output that could be achieved if there were no distortion arising from the income tax.

Equation (3) is the government budget constraint, which states that the repayment of old debt, $(1-\theta_t)z_{t-1}b_{t-1}$, and government expenditure, g_t, is financed by taxation, τ_t, and new debt taking, b_t, net of the default cost, which is the punishment imposed by investors on current default, $p(\theta_t)$, or on past default, $\vec{p}_t$. Typically,

[7] Of course this does not necessarily hold for models in which default takes place implicitly in the form of inflation or surprising levies on capital income, here of bond repayment receipt. In this case, it is plausible that the financial market is prepared to continue lending to the government even in the event of a *de facto* default.

$p(\theta_t)$ and $\vec{p}_t$ do not co-exist, i.e., when there is still some cost due to past default, then we say that this government is further in a default period and cannot contract new debt on which it could default, and, consequently, $p(\theta_t) = 0$. Only after the end of the default period, which implies that there is no *burden of the past*, $\vec{p}_t$, can the government again contract new debt and would incur default cost in the next period if it were to again repudiate the debt contract. This consideration regarding the non-co-existence of $\vec{p}_t$ and $p(\theta_t)$ is reasonable, but loosening this assumption does not have a considerable analytical impact because $\vec{p}_t$ is a type of *sunk cost* and will not affect the current trade-off among various political choices. Nonetheless, here I maintain the non-co-existence assumption, and hence the government can only optimize on b_t or θ_t if $\vec{p}_t = 0$, i.e., only after the last default is resolved through settlements with the investors can the government take on new debt and possibly default again.

The gross contracted interest rate z_t is non-state-contingent, while the *ex post* interest rate $z_t(1-\theta_{t+1})$ is state-contingent because the choice of θ_{t+1} will depend on the circumstances in the next period. After choosing the optimal values for b_t, θ_t, g_t and τ_t, welfare, expressed as the value function of the representative agent, will depend on the existing debt burden $z_{t-1}b_{t-1}$ and possibly on the burden of the past $\vec{p}_t$, as well as on the current endowment, y_t, which by assumption does not depend on the past debt taking and repayment decisions of the government. Further, the value function also depends on α, although α is a parameter and not a state variable.

Equation (4) differs from the usual participation constraint equation in the literature in that it assumes that among all contracted interest rates that give lenders the market return R_t in expectational form, the smallest possible interest rate will always be contracted. Thus, I assume that the government is initiating a debt contract $\{z_t, b_t\}$ and will always offer the lowest possible z_t, which makes the financial market prepared to lend the amount b_t that is desired by the government to maximize social welfare. The government will always choose the smallest possible interest rate because a lower z_t means less debt burden and less default cost due to lower default probability in the next period and is thus preferable for the

government compared with a larger z_t that sustains the same amount of b_t.[8] In the event that the debt contract is not initiated by the government but by the financial market, the lowest possible z_t will also be proposed by the financial market if investors compete, as suggested in Eaton and Gersovitz (1981). According to those authors, competition among investors will lead them to offer the most favorable borrowing condition – here the smallest possible z_t – to the government to obtain the debt contract, as they can obtain the market return in expectational form under this suggested borrowing condition. Hence, under different institutional assumptions, the lowest possible interest rate can be rationalized. Therefore, I only consider the lowest possible contracted interest rate, which by definition excludes the multiple equilibria problem that arises when the set $\{z: \ z_t E_t[1-\theta_{t+1}] = R_t\}$ contains more than one element. In Section 2.6, I will show that this smallest possible contracted interest rate conjecture also holds in much more general cases and even in the absence of the institutional assumptions just made.

To summarize: the value of the value function depends on the state variables endowment y_t, outstanding debt amount $z_{t-1}b_{t-1}$, and the possible inherited default cost, $\vec{p}_t$, as well as the parameter α. To improve legibility, in the following I will omit y_t and α as arguments for $\mathbb{V}$ as long as they are not necessary for the reader's understanding.

2.3 Deriving the FOCs

By inserting Equation (3) into (2), the objective function of the government is simplified to:

[8] Arellano (2008) has also noted that a government always prefers a higher bond price to a lower bond price.

$$\mathbb{V}(z_{t-1}b_{t-1},\vec{p}_t) = \sup_{b_t\in B_t,\theta_t\in[0,1],g_t\in G_t} \{\mathbb{U}(c_t,g_t)+\beta E_t\mathbb{V}(z_t b_t,\vec{p}_{t+1}(\theta_t))\} \quad \text{s. t.} \tag{5}$$

$$c_t = y_t - x(\tau_t) - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} + \cdots$$
$$\cdots + (1-\alpha)b_t - g_t - p(\theta_t) - \vec{p}_t \tag{6}$$
$$\tau_t = (1-\theta_t)z_{t-1}b_{t-1} + g_t - b_t + p(\theta_t) + \vec{p}_t \tag{7}$$
$$z_t = \inf[z:\ z_t E_t[1-\theta_{t+1}] = R_t]$$

In Equation (5), the government is not attempting to adjust τ_t because τ_t is determined by the budget constraint (3), now rewritten in (7). Equivalently, here one could choose another political instrument instead of τ_t that does not serve as an optimizer, a candidate for which is θ_t as is used in example B.1. Equations (6) and (7) are not truly constraints in the sense that they do not place further constraints on the optimizers but merely describe how c_t and τ_t are determined. Equivalently, one could plug them into Equation (5) to eliminate the corresponding variables, and I only write them down here separately for better legibility. Equation (6) states that private consumption c_t is equal to output, possibly reduced by the distortion due to the tax load, $y_t - x(\tau_t)$, net of debt repayment to the foreign investors, $(1-\alpha)(1-\theta_t)z_{t-1}b_{t-1}$, plus new funding received from them, $(1-\alpha)b_t$, minus government expenditure g_t and the penalty cost imposed on current and past default, $p(\theta_t)+\vec{p}_t$. Equation (7) states that the tax is used to cover total debt repayment, $(1-\theta_t)z_{t-1}b_{t-1}$, government expenditure, g_t, and default costs, $p(\theta_t)+\vec{p}_t$, net of new debt taking, b_t.

Because b_t can be constrained to zero when default occurs in the form of a contract violation, if exclusion from the financial market is one punitive instrument available to the investors, there will be a "jump" in the value function when the government switches between default and non-default, as long as the decision maker is not in the so-called last period in which $b_t = 0$ regardless of the default or non-default decision. Hence, in general, one has to distinguish between the value function given the default decision, $\mathbb{V}^d$, and the value function given the non-default decision, $\mathbb{V}^n$. Given access to the financial market, the value function is henceforth $\mathbb{V}^f = \max\left(\mathbb{V}^d,\mathbb{V}^n\right)$, which implies that $\vec{p}_t = \emptyset$.

Given the non-default decision, the government can choose g_t and b_t to optimize:

$$\begin{aligned}
\mathbb{V}^n(z_{t-1}b_{t-1}) = & \sup_{b_t \in B_t, g_t \in G_t} \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}^f(z_t b_t)\} \quad \text{s.t.} \\
& c_t = y_t - x(\tau_t) - (1-\alpha) z_{t-1} b_{t-1} + (1-\alpha) b_t - g_t \\
& \tau_t = z_{t-1} b_{t-1} + g_t - b_t \\
& z_t = \inf[z : \ z_t E_t [1 - \theta_{t+1}] = R_t]
\end{aligned} \tag{8}$$

For any function $f(x_1, x_2, \cdots)$, denote $f_i(x_1, x_2, \cdots)$ as the *i-th* first derivative, i.e., $f_i(x_1, x_2, \cdots) \equiv \frac{\partial f(x_1, x_2, \cdots)}{\partial x_i}$, and use f_t' or $f'(x_t)$ as a shortcut for $\frac{\partial f(x_t)}{\partial x_t}$; then, the first-order condition for (8) with respect to b_t reads as follows:

$$\mathbb{U}_1(c_t, g_t) * (x_1(\tau_t) + 1 - \alpha) = -\beta E_t \mathbb{V}^{f\,\prime}(z_t b_t) * (z_t + z_t' b_t) \tag{9}$$

The above equation describes the optimal decision regarding new debt taking b_t as an inter-temporal trade-off. The left-hand side captures the benefit from one additional unit of b_t; by taking one more unit of debt, the economy can obtain $(1-\alpha)$ units of additional transfer from abroad, and the total output can be raised by $x_1(\tau_t)$, as more debt financing means less tax to finance government expenditures and leads to less distortion in output, and the total increase in consumption resulting from borrowing abroad and less tax distortion will enhance social welfare, as each additional unit of consumption can increase current utility by $\mathbb{U}_1(c_t, g_t)$. This benefit from debt-taking for today's well-being has a cost for the future because each unit of new debt raises the amount of outstanding debt for tomorrow by $(z_t + z_t' b_t)$,[9] and each additional unit of debt repayment obligation will reduce the expected future social welfare by $-E_t \mathbb{V}^{f\,\prime}(z_t b_t)$, discounted by β. At the optimal amount of new debt taking, the LHS should be equated to the RHS.

[9] Note that $z_t(b_t)$ is an increasing function, as proven in Eaton and Gersovitz (1981), and hence one more unit of debt does not only raise the amount of outstanding debt for tomorrow by z_t but also by $z_t' b_t$, the increased interest rate cost due to more debt taking.

Now consider the FOC w. r. t. g_t:

$$\mathbb{U}_2(c_t, g_t) = \mathbb{U}_1(c_t, g_t) * (x_1(\tau_t) + 1) \tag{10}$$

Equation (10) describes the optimal decision concerning government expenditure, g_t, as an intra-temporal trade-off. One additional unit of public goods provisioning will directly increase current utility by $\mathbb{U}_2(c_t, g_t)$. However, government expenditure also needs to be financed by taxation and thus will reduce after-tax income in a one-to-one manner. In addition, the before-tax income will be reduced by $x_1(\tau_t)$ due to the more distortionary taxation, and each additional consumption decrease resulting from more taxation and less total income will reduce current utility by $\mathbb{U}_1(c_t, g_t)$. In the event that the government expenditure is restricted to a singleton, the above equation simply drops out.

Now consider $\mathbb{V}^d$, the value function in the event of default. Given the default decision, the government can choose g_t and θ_t to maximize:

$$\begin{aligned}
\mathbb{V}^d(z_{t-1}b_{t-1}) = \sup_{\theta_t \in [0,1], g_t \in G_t} & \{\mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t \mathbb{V}^f(0) + \cdots \right. \\
& \left. \cdots + (1-\kappa) E_t \mathbb{V}^a(\vec{p}_{t+1}(\theta_t))\right]\} \quad \text{s. t.} \\
& c_t = y_t - x(\tau_t) - (1-\alpha)(1-\theta_t) z_{t-1} b_{t-1} - g_t - p(\theta_t) \\
& \tau_t = (1-\theta_t) z_{t-1} b_{t-1} + g_t + p(\theta_t)
\end{aligned} \tag{11}$$

Equation (11) states that the default decision may place the economy in autarky from the next period on, the value function of which is denoted $\mathbb{V}^a$. Being in autarky means that, in each period, the economy is excluded from the financial market, but with some predetermined non-negative probability κ, it may regain access to the financial market with no old debt in the next period, and this probability is the same at which an economy can return to the financial market with no old debt directly after a default event. $\mathbb{V}^a$ with inherited default cost $\vec{p}$ is expressed as follows:

$$\mathbb{V}^a(\vec{p}) = \sup_{g_t \in G_t} \{\mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t \mathbb{V}^f(0) + (1-\kappa) E_t \mathbb{V}^a(\vec{p})\right]\} \quad \text{s. t.}$$
$$c_t = y_t - x(\tau_t) - g_t - \vec{p}$$
$$\tau_t = g_t + \vec{p}$$

The idea of a random return to the financial market was introduced in Arellano (2008) to capture the observed differences in the lengths of default periods. Before her, and following Eaton and Gersovitz (1981), the literature considering financial market exclusion as a means of punishment for defaulting governments often sets $\kappa = 0$, i.e., once excluded from the financial market, the economy will remain in autarky forever.

The optimal policy mix (θ_t, g_t) in the event of the decision to default is given by the first-order conditions of Equation (11). The FOC w. r. t. θ_t is:

$$\begin{aligned} &\mathbb{U}_1(c_t, g_t) * (x_1(\tau_t)(z_{t-1}b_{t-1} - p_1(\theta_t)) + (1-\alpha) z_{t-1} b_{t-1} - p_1(\theta_t)) \\ =& -\beta(1-\kappa) E_t \mathbb{V}_1^a(\vec{p}_{t+1}(\theta_t)) * \vec{p}'_{t+1}(\theta_t) \end{aligned} \tag{12}$$

The above equation states that an increment in the default rate can reduce the tax by the amount of outstanding debt, $z_{t-1}b_{t-1}$, net of the resulting increase in the penalty cost $p_1(\theta_t)$, and each unit of reduction in tax can raise the output by $x_1(\tau_t)$; moreover, one additional unit of default also increases domestic wealth by $(1-\alpha)z_{t-1}b_{t-1}$, net of the increase in the penalty cost $p_1(\theta_t)$, and each unit of the resulting increase in consumption will raise current utility by $\mathbb{U}_1(c_t, g_t)$. This "benefit" from default will be traded off against its cost, namely the rise in the future penalty cost $\vec{p}'_{t+1}(\theta_t)$, which reduces the future welfare in the event of autarky by $-E_t \mathbb{V}_1^a(\vec{p}_{t+1}(\theta_t))$ for each additional unit of the penalty cost. This loss of welfare in the future will enter with probability $(1-\kappa)$ and is discounted by β. Because θ_t is constrained in the interval $[0,1]$, the above equation may not hold in equality, in which case no repudiation or full repudiation will occur. To ensure that no multiple solutions exist, it is sufficient to let $\mathbb{V}^d$ be non-convex in θ_t, which is satisfied in the majority of models in which $x_(\tau_t)$ is convex and $p_(\theta_t)$ as well as $\vec{p}_{t+1}(\theta_t)$ are non-concave.

Analogous to the case in which $\mathbb{V}^n$ applies, I also derive the optimality condition for $\mathbb{V}^d$ w. r. t. g_t, which should be considered if G_t is no singleton:

$$\mathbb{U}_2(c_t, g_t) = \mathbb{U}_1(c_t, g_t) * (x_1(\tau_t) + 1)$$

The above FOC is functionally identical to (10).

From solving the first-order conditions, we have the optimal policy mix (θ_t, g_t, τ_t) and (b_t, g_t, τ_t) conditional on the default or non-default decision, respectively. Using $\mathbb{V}^f = \max\left(\mathbb{V}^d, \mathbb{V}^n\right)$ yields the optimal policy mix $(b_t, \theta_t, g_t, \tau_t)$ either in the form of $(0, \theta_t, g_t, \tau_t)$ or in the form of $(b_t, 0, g_t, \tau_t)$, according to whether default or non-default is optimal for this small open economy. In the event that default will not cause financial market exclusion in the current period, we can simplify this procedure by directly optimizing $\mathbb{V}^f$ over $(b_t, \theta_t, g_t, \tau_t)$ using the first-order conditions (9), (12) and (10) as well as the budget constraint (7).

After setting up the general model, it is interesting to observe how the state variables y_t[10] and $z_{t-1}b_{t-1}$ as well as the model parameter α affect the debt repayment behavior of the government. By taking the inductive approach and analyzing the two main types of models, each of which is a special form of the general model I have set up here, it is easy to verify that a lower $z_{t-1}b_{t-1}$ will reduce the default risk and that a higher $z_{t-1}b_{t-1}$ will increase the default risk – a result as expected from and in line with the existing literature and hence not elucidated here. The effect of α has yet to be extensively studied in the literature, and hence the following subsections will be devoted to it. To state the result in advance: a higher/lower α will lead, *ceteris paribus*, to a lower/higher or at least not a higher/lower default risk. Note that this knowledge does not concern the equilibrium outcome and merely states that after the interest rate is contracted, then in the next period, when all state variables have been realized and when the government has to make the default or repayment decision, a government with a higher α will have less incentive to default. However, without the "smallest-possible interest rate will be contracted" assumption made above, this knowledge of a lower *ex post* default incentive may lead to an even higher interest rate cost in the so called "bad equilibrium" as defined in Calvo (1988), which may in turn raise the default risk in the equilibrium. In

[10] If output is autocorrelated as in, e.g., Aguiar and Gopinath (2006).

Section 2.6, I will explore the conditions under which we can ensure that an *ex post* lower default incentive due to factors such as a lower $z_{t-1}b_{t-1}$ or higher α will lead to *ex ante* lower default risk, i.e., a lower default probability or lower default rate in equilibrium.

Note that when I analyze the models based on Calvo (1988) and those based on Eaton and Gersovitz (1981) separately in Sections 2.4 and 2.5, not all of the elements mentioned above appear in every type of model. For instance, in the models based on Calvo (1988), there is no exclusion from the financial market following the default decision, while in the models based on Eaton and Gersovitz (1981), there is no deadweight loss. Hence, the model set up thus far is not really a unifying model for which the proofs will be conducted; it is rather a framework that unifies the notations for the two very different types of models such that the results drawn from them are more easily comparable. In the following subsection, I will first analyze the effect of α for models based on Eaton and Gersovitz (1981).

2.4 Effect of α in the model of default probability

The first type of model, primarily used to explain external default, was first introduced in Eaton and Gersovitz (1981) and then further developed, among others, by Aguiar and Gopinath (2006) and Arellano (2008). In this type of model, the default cost, including exclusion from the financial market and a potential decline in output during the default period, is assumed to be independent of the fraction of debt being repudiated. Together with the increasing "benefit" from default due to the wealth transfer effect, a typical government in this model world will always choose to default on its entire stock of outstanding debt whenever default is preferable to non-default, as I will show below. Consequently, the expected default rate is equal to the probability of default, denoted by the parameter λ: $E_t(\theta_{t+1}) = \Pr(\theta_{t+1} > 0) \equiv \lambda_t$. Therefore, I will refer to this type of model as the "model of the default probability" when analyzing the correlation between the default risk, here λ, and the parameter α. Moreover, this type of model usually does not consider deadweight loss from taxation because the wealth transfer effect is sufficient to explain the existence of default, a modeling strategy that I will maintain to simplify the analysis.

Under the above assumptions, the objective function of the government is a special form of the value function (5):

$$\begin{aligned}
\mathbb{V}(y_t,(1-\alpha)z_{t-1}b_{t-1},\vec{p}_t) = \sup_{b_t\in B_t,\theta_t\in[0,1],g_t\in G_t} & \{\mathbb{U}(c_t,g_t)+\cdots \\
& \cdots+\beta E_t\mathbb{V}(y_{t+1},(1-\alpha)z_t b_t,\vec{p}_{t+1})\} \quad \text{s. t.} \\
& c_t = y_t-(1-\alpha)(1-\theta_t)z_{t-1}b_{t-1}+\cdots \\
& \cdots+(1-\alpha)b_t-g_t-p_t-\vec{p}_t \\
& z_t = \inf[z:\ z_t E_t[1-\theta_{t+1}]=R_t]
\end{aligned}$$

In words: the value function is a function of the state variables of endowment y_t, debt service to foreigners $(1-\alpha)z_{t-1}b_{t-1}$, and inherited default cost from last default event $\vec{p}_t$, which is null for $\mathbb{V}^f$ and some constant $\vec{p}$ for $\mathbb{V}^a$, which is usually equal to the current default cost p_t.[11] Here, (7) drops out because τ_t no longer directly affects c_t due to the assumption of no deadweight loss; hence, it is unnecessary to model it explicitly.

Given default decision, the corresponding value function is:

$$\begin{aligned}
\mathbb{V}^d(y_t,(1-\alpha)z_{t-1}b_{t-1}) = \sup_{\theta_t\in[0,1],g_t\in G_t} & \{\mathbb{U}(c_t,g_t)+\beta\left[\kappa E_t\mathbb{V}^f(y_{t+1},0)+\cdots\right. \\
& \left.\cdots+(1-\kappa)E_t\mathbb{V}^a_{t+1}(y_{t+1},\vec{p})\right]\} \quad \text{s. t.} \\
& c_t = y_t-(1-\alpha)(1-\theta_t)z_{t-1}b_{t-1}-g_t-p_t
\end{aligned}$$

The value function in autarky is:

[11] To verify that the value function depends solely on the three state variables, optimize the value function over $(1-\alpha)b_t$, $1-\theta_t$ and g_t which are one-by-one mappings of the optimizers shown in the formula.

$$\begin{aligned}\mathbb{V}_t^a(y_t,\vec{p}) = \sup_{g_t\in G_t}\{&\mathbb{U}(c_t,g_t)+\beta\left[\kappa E_t\mathbb{V}^f(y_{t+1},0)+\cdots\right.\\ &\left.\cdots+(1-\kappa)E_t\mathbb{V}_{t+1}^a(y_{t+1},\vec{p})\right]\} \quad \text{s.t.}\\ c_t = y_t - g_t - \vec{p}\end{aligned}$$

By plugging in $\mathbb{V}^a$, the value function for default can be written in a more parsimonious way as:

$$\begin{aligned}\mathbb{V}^d(y_t,(1-\alpha)z_{t-1}b_{t-1}) = \sup_{\theta_t\in[0,1],g_t\in G_t}\{&\mathbb{U}(c_t,g_t)+\beta\left[\kappa E_t\mathbb{V}^f(y_{t+1},0)+\cdots\right.\\ &\left.\cdots+(1-\kappa)E_t\mathbb{V}_{t+1}^a(y_{t+1},\vec{p})\right]\} \quad \text{s.t.}\\ c_t = y_t-(1-\alpha)(1-\theta_t)z_{t-1}b_{t-1}-g_t-p_t\\ \mathbb{V}_\tau^a(y_\tau,\vec{p}) = \mathbb{U}((y_\tau-g_\tau^a-\vec{p}),g_\tau^a)+\beta&\left[\kappa E_\tau\mathbb{V}^f(y_{\tau+1},0)+\cdots\right.\\ &\left.\cdots+(1-\kappa)E_\tau\mathbb{V}_{\tau+1}^a(y_{\tau+1},\vec{p})\right]\forall\tau\geq t+1\end{aligned}$$

In the above expression, g_τ^a stands for the optimal government expenditure in autarky at time τ, and g_τ^a is a function of y_τ and $\vec{p}$ only.[12]

The first-order derivative of $\mathbb{V}^d$ over θ_t is:

$$\mathbb{U}_1(c_t,g_t)*(1-\alpha)z_{t-1}b_{t-1}$$

The marginal utility from consumption $\mathbb{U}_1(c_t,g_t)$ is always positive. Further, $z_{t-1}b_{t-1}>0$ whenever the government contemplates default. Therefore, the above term will be strictly positive whenever $\alpha<1$, i.e., if we are not confronting purely domestic debt. Hence, the optimal value of θ_t is always one given a default decision, i.e., whenever the government chooses to default, it will choose to default on the entire stock of debt.

[12] This statement also holds in cases in which the distribution of $y_{\tau+1}$ may depend on y_τ as in Aguiar and Gopinath (2006).

In the event of purely domestic debt, i.e., $\alpha = 1$, default does not make sense because it does not transfer any wealth to the domestic economy. With the additional assumption that the government will only choose to default more when doing so can render the representative agent better off, the government will always choose $\theta_t = 0$, given a default decision.[13] However, because default can trigger financial market exclusion and possibly also other default costs, for $\alpha = 1$, the default probability λ will be zero, i.e., the government will never default, and hence does not have to pay any risk premium on its bond issuance, and we have $z_t = R_t$.

Now again consider the more interesting case in which $\alpha < 1$ and hence $\theta_t = 1$ given a default decision. Plug $\theta_t = 1$ into the expression of $\mathbb{V}^d$ and we obtain:

$$\begin{aligned}
\mathbb{V}^d(y_t, (1-\alpha)z_{t-1}b_{t-1}) = \sup_{g_t \in G_t} \{ & \mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t \mathbb{V}^f(y_{t+1}, 0) + \cdots \right. \\
& \left. \cdots + (1-\kappa) E_t \mathbb{V}^a_{t+1}(y_{t+1}, \vec{p}) \right] \} \quad \text{s.t.} \\
c_t = y_t - g_t - p_t & \\
\mathbb{V}^a_\tau(y_\tau, \vec{p}) = \mathbb{U}((y_\tau - g^a_\tau - \vec{p}), g^a_\tau) & + \beta \left[\kappa E_\tau \mathbb{V}^f(y_{\tau+1}, 0) + \cdots \right. \\
& \left. \cdots + (1-\kappa) E_\tau \mathbb{V}^a_{\tau+1}(y_{\tau+1}, \vec{p}) \right] \forall \tau \geq t+1
\end{aligned}$$

Denote the optimal choice of g_t given default as g^d_t. By checking the above expression, we can see that g^d_t is independent of $(1-\alpha)z_{t-1}b_{t-1}$ because this term disappears after plugging in $\theta_t = 1$. Then, write the above expression in an even more compact way:

$$\begin{aligned}
\mathbb{V}^d_t = \mathbb{U}((y_t - g^d_t - p_t), g^d_t) + \beta \left[\kappa E_t \mathbb{V}^f(0) + (1-\kappa) E_t \mathbb{V}^a_{t+1}(\vec{p}) \right] \quad \text{s.t.} \\
\mathbb{V}^a_\tau(\vec{p}) = \mathbb{U}((y_\tau - g^a_\tau - \vec{p}), g^a_\tau) + \beta \left[\kappa E_\tau \mathbb{V}^f(0) + (1-\kappa) E_t \mathbb{V}^a_{\tau+1}(\vec{p}) \right] \\
\forall \tau \geq t+1
\end{aligned}$$

As is evident from above, the value from default, $\mathbb{V}^d$, does not depend on α. Note that $\mathbb{V}^d$ does depend on y_t, and I have only dropped y_t as an input argument

[13] Actually θ_t is not exactly equal to 0 but the point to the right of it, i.e., near zero, but positive, and hence it is regarded as default.

$\forall\, t$ to make the expression simpler and more legible, which is innocuous in this context because we are not interested in the exogenous variable y_t.

Now, consider the value function given non-default:

$$\begin{aligned}\mathbb{V}^n((1-\alpha)z_{t-1}b_{t-1}) = \sup_{b_t\in B_t, g_t\in G_t} &\{\mathbb{U}(c_t,g_t)+\beta E_t\mathbb{V}^f((1-\alpha)z_t b_t)\} \quad \text{s. t.}\\ c_t &= y_t-(1-\alpha)z_{t-1}b_{t-1}+(1-\alpha)b_t-g_t\\ z_t &= \inf[z:\ z_t(1-\lambda_t)=R_t]\end{aligned}$$

In the expression above, the contracted interest rate is a function of the default probability $\lambda_t \equiv \Pr(\mathbb{V}^d_{t+1} > \mathbb{V}^n_{t+1})$[14] with $\mathbb{V}^n_{t+1}$ depending on $(1-\alpha)z_t b_t$. An implication is that z_t is also a function of $(1-\alpha)z_t b_t$ or, equivalently, a function of $(1-\alpha)b_t$. Here I have replaced $E_t[1-\theta_{t+1}]$ with $(1-\lambda_t)$ because the expected default rate is interchangeable with the default probability λ_t as explained before.

By denoting the optimal values of b_t and g_t given non-default as b^n_t and g^n_t, respectively, we obtain $\mathbb{V}^n$ in a more compact way:

$$\begin{aligned}\mathbb{V}^n((1-\alpha)z_{t-1}b_{t-1}) &= \mathbb{U}((y_t-(1-\alpha)z_{t-1}b_{t-1}+(1-\alpha)b^n_t-g^n_t),g^n_t)+\cdots\\ &\cdots+\beta E_t\mathbb{V}^f((1-\alpha)z_t b^n_t) \quad \text{s. t.}\\ z_t &= \inf[z:\ z_t(1-\lambda_t)=R_t]\end{aligned}$$

Analogous to the proof used in Eaton and Gersovitz (1981), I show that $\mathbb{V}^n$ increases with α, which is summarized in **Theorem 1**:

Theorem 1: for any $0 \le \alpha_1 < \alpha_2 < 1$ and any given outstanding amount of debt $z_{t-1}b_{t-1} \ge 0$, it holds that $\mathbb{V}^n((1-\alpha_1)z_{t-1}b_{t-1}) \le \mathbb{V}^n((1-\alpha_2)z_{t-1}b_{t-1})$.

The formal proof of **Theorem 1** can be found in Appendix A.1; the following is only a sketch of the underlying idea: as α increases, i.e., the government internalizes the wealth position of lenders to a larger extent in its objective function, the debt repayment will reduce the current consumption of the representative agent by a smaller amount, and hence repayment is more worthwhile for the government.

[14] The assumption that only the smallest possible interest rate will be contracted has ensured that z_t can be expressed as a function of other terms.

Moreover, when the financial market anticipates this attitude toward repayment, then the government will face a lower interest rate for its current borrowing b_t^n, or in other words, for the same future repayment obligation $z_t b_t^n$, the government can borrow more today, which further increases current consumption. Therefore, a higher α makes the repayment decision more valuable to the government.

Because the value of repayment, $\mathbb{V}^n$, increases with α, while the value of default, $\mathbb{V}^d$, is independent of α, as a consequence, the default probability λ_t, which is the probability of $\mathbb{V}^n$ being smaller than $\mathbb{V}^d$, will decrease with α. Together with the previously gained knowledge that $\lambda_t = 0$ for $\alpha = 1$, we can say that for all $\alpha \in [0,1]$, the default probability λ_t will decrease or at least not increase when increasing α, i.e., the greater weight assigned to the lenders' wealth position in the objective function makes the government less prone to default, given the same outstanding amount of debt $z_{t-1}b_{t-1}$ and other state variables.

However, due to the interaction between z_t and λ_t, i.e., z_t will increase as λ_t increases due to the market participation constraint while λ_t will increase as z_t increases because $\mathbb{V}^n$ decreases, *ceteris paribus*, as z_t increases, there may be a "good equilibrium" characterized by low z_t and low λ_t and a "bad equilibrium " characterized by high z_t and high λ_t given the same conditions. Moreover, in the "bad equilibrium", the contracted interest rate z_t may increase as α increases. Using the smallest-possible-interest-rate-will-be-contracted assumption, I have already excluded the "bad equilibrium" and thereby ensured that the lower default propensity due to a higher α shown above indeed leads to more favorable borrowing conditions for the government in the form of a lower z_t. In Section 2.6, I will address the multiple equilibria problem in a more general way and show that only the "good equilibrium" is a stable equilibrium that can exist in a world afflicted with shocks. Before doing so, I will first prove in the next section that a higher α will also lower the default propensity in a Calvo-type model in which the default rate is a continuous function of the state variables.

2.5 Effect of α in the model of the default rate

The second type of model that is primarily used to explain domestic default was first presented in Calvo (1988) as a two-period model that focuses on the default rate decision as a trade-off between deadweight loss due to taxation and the default

cost from debt repudiation. The default cost may be linear or convex and can stand for dispute or renegotiation or some other penalty cost; it can also be the inflation cost, etc. In either case, the default decision will not trigger exclusion from the financial market. In other words, this type of model is characterized by the independence of the maximal available amount of new debt from the current default decision, and hence there will be no "jump" in the value function of the representative agent when switching between default and non-default decisions. Accordingly, the default rate can be determined as a continuous function of the underlying variables. Consequently, I will refer to this type of model as "model of the default rate" when analyzing the correlation between α and the default risk θ_t. The recent literature on non-strategic default, represented by, e.g., Uribe (2006), Schabert (2010) and Juessen et al. (2011), also falls into this category. However, because they have set both g_t and τ_t to be exogenous to model a given fiscal stance, the default rate θ_t will rather be derived from the exogenous fiscal and monetary stance and hence be independent of α because θ_t is not the result of an optimizing process. However, if we interpret the fiscal stance itself to be the result of an optimization process, then α may still have an impact on the equilibrium default rate and hence on the government borrowing condition. Here, I only consider the Calvo-type model in which τ_t is constrained loosely enough such that the government can optimize over θ_t, and I do so first in a two-period setting; however, the result can also be extended to an infinite-horizon model as is shown in Appendix A.3.

In the two-period model without an inherited penalty cost, debt is contracted in period 0, while the repayment decision is made in period 1. Thus, the value function in period 1 is again a special form of (5):

$$\begin{aligned}
\mathbb{V}(z_0 b_0) = & \sup_{\theta_1 \in [0,1], g_1 \in G_1} \{\mathbb{U}(c_1, g_1)\} \quad \text{s.t.} \\
& c_1 = y_1 - x(\tau_1) - (1-\alpha)(1-\theta_1) z_0 b_0 - g_1 - p(\theta_1) \\
& \tau_1 = (1-\theta_1) z_0 b_0 + g_1 + p(\theta_1)
\end{aligned}$$

Note that here we cannot optimize over b_1 not because a possible default decision has triggered exclusion from the financial market but because this is the

last period, and hence no new debt can be contracted regardless of whether the government fully repays or defaults on (part of) its debt.

The contracted interest rate in period 0, z_0, must satisfy the following market participation constraint, which will be taken as given by the government in period 1, the period of debt repayment:

$$z_0 = \inf[z: \ z_0 E_0[1-\theta_1] = R_0]$$

The first-order condition to determine the optimal default rate θ_1 reads as follows:

$$\mathbb{U}_1(c_1, g_1) * (x_1(\tau_1)(z_0 b_0 - p_1(\theta_1)) + (1-\alpha) z_0 b_0 - p_1(\theta_1)) = 0 \tag{13}$$

The LHS of the above expression is a decreasing function of α for any positive debt stock $z_0 b_0 > 0$. It is also a decreasing function of θ_1 around the optimum due to the assumption made concerning the curvature of the deadweight loss function and the penalty cost function. Consequently, the optimal default rate θ^* will decrease or at least not increase with α. This can be formally expressed as:

Theorem 2: In a two-period model, given $\mathbb{U}_1(c,g) > 0$, $x'(\tau) > 0$, $x''(\tau) > 0$ and $p''(\theta) \geq 0$ in the vicinity of the equilibrium, it holds that $\frac{d\theta^*}{d\alpha} \leq 0$.

The formal proof of **Theorem 2** can be found in Appendix A.2; here, I only provide a brief depiction of the underlying idea: Each additional unit of default rate θ will increase consumption by reducing the taxation needed to finance the debt service by $z_0 b_0 - p_1(\theta_1)$, which is the outstanding debt obligation net of the increase in the default cost. Each unit of reduced taxation to service debt in turn reduces deadweight loss by $x_1(\tau_1)$. This marginal "benefit" of default will be smaller the larger θ is because $x(\tau)$ and $p(\theta)$ are convex and non-concave, respectively, by assumption, and in the optimum $x_1(\tau_1)$ is non-negative. The marginal cost of one additional unit of default is $p_1(\theta_1) - (1-\alpha) z_0 b_0$, the marginal change in the default cost net of the wealth transfer from abroad. This effect is larger the larger θ is because $p(\theta)$ is assumed to be non-concave. Thus, when θ rises, the marginal change in consumption, which is the marginal "benefit" minus the marginal cost, will fall and with it the marginal utility because the marginal

utility from consumption is positive. When α rises/falls, then the marginal utility will fall/rise because a higher/lower α reduces/enhances the wealth transfer effect. To let the marginal utility again rise/fall to zero, the optimal default rate θ^* then needs to fall/rise, and hence a higher α will lower the optimal default rate θ^* until it reaches the lower bound, and a lower α will raise θ^* until it reaches the upper bound. In other words, a greater weight assigned to the lenders' wealth position in the objective function will lower the government's propensity to default; inversely, a lower α will increase the default propensity.

Indeed, if the penalty cost is assumed to be a constant fraction of the repudiated debt, then $p(\theta_1) = \omega\theta_1 z_0 b_0$, and hence $p_1(\theta_1) = \omega z_0 b_0$ is a constant. In this case, $\alpha = 0$ will make the government always choose to fully default on its debt because (13) will become $\mathbb{U}_1(c_1, g_1) * (x_1(\tau_1) + 1)(1 - \omega) z_0 b_0$, which is positive for any $\theta_1 \in [0, 1]$. A more detailed analysis of this case of linear default cost can be found in the example in Appendix B.1.

Although I have shown the negative correlation between α and θ^* first for the two-period model, this result also holds for the corresponding infinite-horizon model, the proof of which can be found in Appendix A.3:

Theorem 3: In an infinite-horizon model, given $\mathbb{U}_1(c, g) > 0$, $x'(\tau) > 0$, $x''(\tau) > 0$ and $p''(\theta) \geq 0$ in the vicinity of the equilibrium, it holds that $\frac{d\theta^*}{d\alpha} \leq 0$.

As in the last section, the negative correlation between α and the default rate θ_t is based on a predetermined contracted interest rate z_{t-1}. Moreover, because θ_t itself and its expectation will have influence on z_{t-1} through the market participation constraint, there may exist multiple equilibria, and in the so-called "bad equilibrium" a higher α may even raise z_{t-1}. Although the institutional assumption made in Section 2 can already exclude this bad equilibrium and ensure that a higher α will indeed lower the interest cost for the government, I will show in Section 2.6 that the negative correlation between α and z also holds under much more general assumptions.

2.6 The multiple equilibria

The term "multiple equilibria" can have various meanings depending on the context. Here, I use this term as in Calvo (1988). More precisely: the set of contracted interest rates $\{z : \ z_t E_t[1 - \theta_{t+1}] = R_t\}$ contains two elements. The lower equilibrium

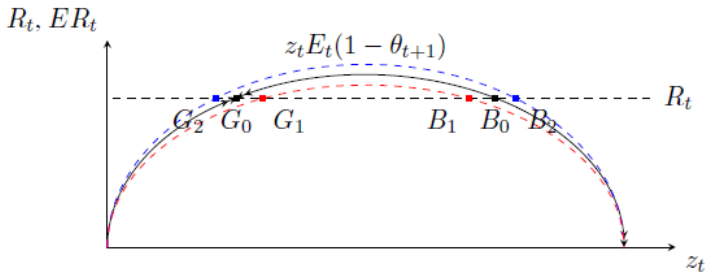

Figure 1: The multiple equilibria

interest rate is referred to as the "good equilibrium" and the other one is called the "bad equilibrium". As most authors have noted, the higher "bad equilibrium" has several counter-intuitive features such as an interest rate decreasing with an increasing outstanding amount of debt, which is rarely observed in reality. Indeed, examining the properties of the equilibria in greater detail, I found that the "bad equilibrium" is an unstable equilibrium while the "good equilibrium" is a stable equilibrium as is illustrated in Figure 1.

From the previous sections, we know that, in general, the default rate θ_{t+1} will rise or at least not fall with a rising contracted interest rate z_t because a higher z_t means a higher debt obligation for tomorrow, given the contracted debt amount b_t. Hence, the expected return $ER_t \equiv z_t E_t(1-\theta_{t+1})$ will first rise with rising z_t but at an increasingly slower rate because the expected default rate $E_t\theta_{t+1}$ also rises with the rising interest cost, and after some point, the increase in expected default may outweigh the increase in the contracted interest rate and the expected return will fall with further increasing z_t, as illustrated in Figure 1. Note that the exact form of the expected return as a function of the contracted interest rate z_t may not appear exactly identical, and in example B.1, we will see this function as a kinked straight line rather than a smooth curve as in Figure 1, but the qualitative effect of the contracted interest rate on the expected return remains the same.

In Figure 1, the expected return $z_t E_t(1-\theta_{t+1})$ is represented by the black solid curve, which first rises and then falls as z_t increases. At some point, the expected default $E_t\theta_{t+1}$ will reach its maximum of one, and hence the expected

return will become zero and lie on the x-axis. This part is not of interest and hence not plotted here, and we consider only the part above the x-axis. In equilibrium, the expected return should be equated to the market return R_t, which is represented by the horizontal black dashed line, and the intersections of R_t with the black curve $z_t E_t(1-\theta_{t+1})$ constitute the two equilibrium points G_0 and B_0. The point G_0 is the so-called "good equilibrium" because it maintains the same market return with a lower, and hence more favorable, interest cost for the government, compared with the "bad equilibrium" represented by the point B_0.

The "bad equilibrium" has some odd features, e.g., if some shock increases the exogenous market return R_t and thus shifts the horizontal line upward, then B_0 will move to the left, i.e., tighter monetary policy would reduce the interest cost z_t even in the bad equilibrium, which is counterintuitive and rarely observed. It also predicts that a rise in the amount of outstanding debt can even reduce the interest cost – a prediction apparently not shared by most researchers who recommend fiscal consolidation to reduce the public debt.[15] Regarding the correlation between α and the borrowing cost: in Sections 2.4 and 2.5, I have already shown that a higher α will lower the default propensity of the government and hence shift the expected return curve upward to the blue dashed curve while a lower α will increase the default propensity and therefore shift the expected return curve downward to the red dashed curve. If the economy is in the good equilibrium, then a higher α will lower the contracted interest rate because G_2 is to the left of G_0; a lower α will raise the contracted interest rate because G_1 is to the right of G_0. However, if the economy were in a bad equilibrium, then a higher α would actually raise the contracted interest rate because B_2 is to the right of B_0; a lower α would lower the contracted interest rate because B_1 is to the left of B_0.

The explanation for the opposite behavior of the two equilibria is the following: In the vicinity of the good equilibrium, the change in expected return is primarily determined by the change in the contracted interest rate; hence, after a positive shock such as a higher α or lower $z_{t-1}b_{t-1}$, which drives the expected return above the market return, the contracted interest rate will decline to bring the expected return down to the market return; after a negative shock such as a higher R_t, the

[15] Cogan et al. (2013) have shown using a DSGE model how to implement such a fiscal consolidation that not only increases long-run output but also has a short-run stimulating impact.

contracted interest rate will rise to increase the expected return to the market return. In the vicinity of the bad equilibrium, the expected return is dominated by the default risk $E_t\theta_{t+1}$, which is positively correlated with z_t. Hence, a positive shock such as a higher α will require a higher $E_t\theta_{t+1}$, which means a higher z_t, and a negative shock will lower the interest cost z_t. With the "smallest-possible-interest-rate-is-always-contracted" assumption, we can ensure that this economy is always in the good equilibrium and hence that a higher α will lower the contracted interest rate and a lower α will raise the contracted interest rate – a conclusion that is in line with our intuition.

However, why should the economy ever be in a "bad equilibrium"? By examining the expected return curve in greater detail, we observe that the good equilibrium G_0 is a stable equilibrium while the bad equilibrium B_0 is unstable. For example, if some shock were to drive the contracted interest rate to a value between G_0 and B_0, then there would be excess demand for government bonds because the expected return is higher than the market return, which would lead to a continually lower z_t until it converges to G_0.[16] Moreover, if the initial interest cost z_t were to lie below G_0, then there would be no demand because the expected return is below the market return, and z_t would continue to rise until it converges to G_0. Hence the good equilibrium G_0 is also a stable equilibrium, and after any small deviation from it, possibly due to external shocks, it will soon converge back to G_0 as long as the price adjustment mechanism works in the usual way, namely, that excess demand will raise the price or lower the yield of the government bond while excessive supply will work in the opposite way. If the shock were to drive the interest cost above B_0, then either the lack of demand would drive z_t increasingly higher such that no debt can be contracted in the equilibrium, or, in the event that the market participants are aware of the existence of the multiple equilibria problem, they may realize that the reason that no debt can be contracted is that they are bargaining at a "too high" interest rate, and hence they will switch to a low z_t, and the contracted interest rate again converges to G_0. In any case, the economy will not return to B_0, and hence B_0 is an unstable equilibrium: if any force were to drive the economy away from this point, then no return to B_0 would follow. Because B_0 is just one

[16] The arrows along the curve represent the direction of the movement of z_t.

point among a continuum of points on the curve, the probability that the economy would ever be in the bad equilibrium is actually almost zero.[17]

Now consider again the correlation between α and the equilibrium borrowing cost. Initially, the economy is very probably at G_0, as I have explained above. A rising α, which shifts the expected return curve upward to the blue curve would cause excess demand and thus lower the equilibrium borrowing cost to G_2; a falling α, which shifts the expected return curve downward to the red curve would cause insufficient demand and thus raise the equilibrium borrowing cost to G_1 – meaning a negative correlation between α and the borrowing cost, as expected. In the rather unlikely case in which the economy was initially at B_0: a rising α will cause an excess demand because the blue curve is above the black curve, and the excess demand will continue to reduce z_t until G_2 has been reached. A falling α will cause insufficient demand, which will either continually increase z_t such that no debt can be contracted or lead the economy to "jump" to some fairly low z_t and from there converge to G_1. Hence, the negative correlation between α and the borrowing cost will be maintained in the case of a rising α; whereas, a falling α will either lead to no equilibrium, i.e., a complete cessation of lending, which means the worst possible borrowing condition for the government, or a one-time positive correlation between α and the borrowing cost as the economy has now switched to the good equilibrium after repeatedly failed negotiations that made the economy aware that it was in the vicinity of a bad equilibrium. However, because the economy is now in a good equilibrium, at least from now on, the negative correlation between α and the borrowing cost will hold. Hence, the negative correlation between α and the borrowing cost almost certainly holds in a fairly general setting, even without the "smallest-possible-interest-rate-is-always-contracted" assumption. In other words, greater consideration of the lenders' wealth position will generally reduce the equilibrium borrowing cost of the government, while a lower α will instead increase its borrowing cost.

[17] Note that the validity of the conclusion that the "bad equilibrium" is unstable hinges on the model assumption that the government cannot commit. If this assumption is violated, there may be a stable "bad equilibrium". One example is Corsetti and Dedola (2014), in which the government can commit with a not too low probability, and hence there is a stable "bad equilibrium".

3 Discussion

3.1 The measurement of α

One reason that researchers have devoted little attention to α is the general belief that whenever default becomes likely, foreigners will simply sell their debt claims to domestic investors, potentially even at par.[18] In such a case, there would be no need to consider α explicitly because this parameter is expected to be 1 if default is imminent. Indeed, in the case of tradable assets such as government bonds, the parameter α should be measured at the time of debt repayment. When issuing bonds, investors can only estimate α. Because in the imminent default case, domestic investors are most likely to buy, their wealth relative to the outstanding amount of debt and the concentration of their wealth[19] would largely determine the value of α. Following this logic, in the case of small economies with large-scale government debt, it is questionable whether domestic investors have sufficient wealth to take on all public debt at par. Thus, as government bonds are freely tradable, a capital-intensive economy can generally enjoy more favorable borrowing conditions than an assimilable economy endowed with less capital. The reason is that when default looms, the debt claim holders of the former economy can more easily find buyers to take over their debt claims in the belief that their wealth position will be better regarded by the borrowing government.

The parameter α has thus far been interpreted as the fraction of public debt held by domestic residents – a standard interpretation in the literature. The underlying assumption for interpreting α in this way is that the government acts as an agent of the principals, its voters or citizens, and hence its task is to maximize the aggregate welfare of the voters, who are usually the residents. Therefore, domestic investors, who are often also voters, dare to buy bonds from foreign investors when default looms because their government has more incentive to repay them. However, due to the process of globalization and integration in financial markets, it is common for a fiscal authority to borrow from other fiscal areas to overcome capital shortages in its own economy. If α were strictly interpreted as the fraction

[18] See, e.g., Broner et al. (2006).

[19] According to Broner et al. (2006), a low concentration of wealth is necessary to exclude the possibility that domestic investors collude and thus lower the repurchase price of the bonds.

of debt held by residents in the borrowing economy, as is traditionally the case, then the corresponding default risk would be high for most of the smaller economies in the world, due to the low α value. Though small economies with less capital are indeed often viewed as riskier borrowers, it seems unrealistic to assume that they all have a higher default propensity. This is especially the case when we consider that this small open economy may also be a local government with its own fiscal authority. The reason that investors often deem a borrowing county that is part of a wealthy country more reliable than a borrowing smaller country might be that they intuitively include investors outside the boundary of the borrowing county but within the same country when gauging α. This intuition appears reasonable because α actually represents the weight that the borrowing government assigns to investors. There is no reason that a borrowing local government should inevitably discriminate against investors from other counties or states because they may well have the same importance for the government as the local investors. There are many possible reasons for non-local and local investors being of equal importance to the local government. For instance, capital can flow freely within a country, and hence local and non-local investors actually make the same contribution to local finances. Alternatively, firms can freely choose the locations of their businesses and hence contribute the same to the local labor market. These reasons could also be applied to investors from other countries, albeit possibly to a lesser extent. For example, countries in an explicitly declared union may also be so integrated that investors from other countries are not particularly discriminated against by the borrowing government, and hence the borrowing government no longer regards the default on the foreign investors' claims as a welcome wealth transfer to domestic citizens. Of course, what really matters is the actual consideration that the borrowing government accords to investors' wealth position and not mere membership in a country or union. In a model with different voter groups, as in Alesina and Tabellini (1990), α would be re-interpreted as the part of the public debt that is held by the relevant voter group if there is no interconnection or interaction as described above among the different voter groups.

Because the truly important factor is how the borrowing government regards investors' interests, it is also possible for an economy to have a high α value without being a member of a wealthy country or union. One such alternative is a legal system that protects investors' interests. As noted in the literature, a

government with debt problems may be tempted to eliminate or manipulate legal restrictions to facilitate default. Whether this is feasible certainly depends on the specific construction of the legal system with regard to factors such as how easily it can be manipulated or how investor-friendly it is. In Appendix B.1, I show how domestic welfare can increase when the government internalizes the lenders' interests more than the residents do – possibly through a rigorous lender-protecting law system that facilitates the acquisition of capital in the economy, which is necessary for growth. In that example, I model a growth-improving program as a direct investment with a higher yield than the interest cost, but it could also be in the form of a reduction in a distortionary tax that increases output, as modeled in Cogan et al. (2013). There may also be other reasons that a small open economy could have a high α value, but this is not subject of this paper. This paper only shows that, first, a higher α value is associated with a lower default risk and lower borrowing cost, second, only the α value at the time of debt repayment matters, and third, the α value corresponds to the weight assigned to lenders by the borrowing government in its objective function. These three points should also hold even if the government debt is not tradable or is difficult to trade, with the only difference being that in that case, domestic wealth would matter less in determining α.

3.2 The applicability of α to real-world issues

The finding that a higher α can reduce default propensity appears to be empirically confirmed, as Reinhart and Rogoff (2011) have observed that domestic default occurs less frequently and often occurs under much more difficult economic conditions than does external default. As their data cover several forms of public debt and are not limited to one-period government bonds, which are analyzed in this paper, it would be interesting for future research to investigate whether the analytical analysis of other forms of sovereign debt yields qualitatively identical results as the findings shown here. Reinhart and Rogoff (2011) distinguish between domestic and external debt according to the law under which the debt is issued. This may suggest that institutional features such as the legal system are the predominant factor and not the country's wealth or union membership, because a sovereign body can only borrow from abroad under domestic law if its legal system is considered trustworthy by global investors. However, as those authors noted, the status of

domestic or external debt remains largely the same when one shifts the classifying criterion from juridical governance to other criteria such as the citizenship of the lenders or currency denomination. In any event, their empirical result is compatible with the hypothesis that a higher α reduces default risk while that higher α may result from a good legal system or from a large proportion of domestic lenders.

The model applied here assumes rational expectations – which is standard in the literature but has been questioned in recent discussions concerning modeling strategy. It would be interesting to determine whether the model outcome would change if this assumption were violated. For instance, when arguing that non-local investors may contribute identically to the domestic economy as local investors and hence receive the same weight, it is actually assumed that people can correctly identify the indirect ownership, i.e., when residents work for or own a stake in a non-local investing institution such as a bank, then they can correctly anticipate that the default of that bank has the same wealth effect as a default of residents. If a non-local investor is from the same country, residents will be more likely to have sympathies and to recognize such indirect links because they identify themselves with people from their own country, even if they are from other regions. However, if the non-local investor is from another country, then it is questionable whether the residents will have such sentiments. Thus, it is possible that at the time of debt repayment, people may underestimate α and hence default too readily, and only in the aftermath when the wealth loss becomes increasingly evident will they realize that the default was not sensible, and a defaulting politician might have to resign, as documented in Borensztein and Panizza (2009). However, in this example, the possible violation of the rational expectation assumption only affects the factor determining α and not α itself. Thus, the statement that a higher α can reduce the default risk still holds, but the economic links between lender and borrower countries may no longer correlate positively with α.

Another assumption underlying this model is that all public debt is pooled together and hence not distinguishable. The fact that the purchase and possession of government bonds is often regarded as anonymous by the financial market justifies this assumption. This anonymity assumption also allows some investors to doubt whether their interests will be honored by the government because the government has no means of determining whether its bonds are purchased by voters. However, the government can trace the sentiment of the average voter by,

e.g., considering poll results,[20] and polling, if representative enough, has already incorporated the congruence between the public debt and the wealth position of voters.[21] The anonymity assumption is stronger than the non-distinguishability assumption, and hence the result also holds in situations in which public debt holding is not anonymous but nonetheless non-distinguishable, i.e., the government cannot discriminate against lenders even if it knows their identity, possibly because such discriminatory behavior in the form of selective default is prohibited by law. Regarding the real world, this non-distinguishability assumption seems to be plausible and the norm because a selective default could destroy the trust of the foreign investors and hence they would not invest in such an economy. Consequently, a developed economy often establishes a juridical framework that makes such discrimination difficult. A less obvious form of a selective default is to default on all debt but only rescue the domestic industry, as described in Gennaioli et al. (2010). However, if the economic integration in the union is symmetric, i.e., domestic citizens are stakeholders not only of domestic firms but also of foreign firms, then such a *de facto* selective default will be ineffective because rescuing domestic industry and rescuing foreign industry have the same welfare effect for domestic citizens.

3.3 A selection of related literature

Admittedly, the analysis here is rather simplified and stylized to focus on the effect of α, which has not been extensively studied before. Factors such as inertia in debt distress as shown in Binder et al. (2015) are not considered. Thus, the conclusions drawn here apply in the first place to the type of sovereign debt modeled in this paper, namely, the one-period government bond. Whether the same relationship holds for other types of public debt remains to be shown by further research. I consider the study of the effect of α in alternative model settings worthwhile, as the model outcome here suggests α to be a potentially new determinant of the

[20] The likely significance of the poll results for the default or repayment decision of the government is, for instance, documented by Tomz (2002).

[21] Of course, if the voters are unable to recognize such indirect ownership as in the above example, then only their direct government bond ownership would enter α.

default risk of sovereign debt.[22] Furthermore, the model considers only borrowing from the financial market, and hence the market return consideration is a key factor in determining the debt contract. However, many developing countries, especially during financial distress, heavily rely on borrowing from international organizations such as the IMF or the World Bank. Whether the findings in this paper concerning the effect of α still hold for public lending, which does not necessarily require the maintenance of the market return but nonetheless requires a minimum return, remains to be shown. For further reading on public lending, the interested reader may consult, e.g., Binder and Bluhm (2010) regarding IMF program participation.

To focus on the effect of α, I have adopted a partial equilibrium (PE) analysis. Although conducting a general equilibrium (GE) analysis is not the purpose of this paper, I constructed the model in a way to facilitate its implementation in a GE analysis. Hence, instead of specifying a particular functional form, I have used a general functional form with some specifications to its first and second derivatives, such that it can be easily integrated into another model with specific functional forms. For instance, the deadweight loss from taxation is taken as exogenous in this PE analysis; when implemented in a GE analysis, the deadweight loss can well be an endogenous outcome such as reduced labor supply due to a higher income tax as in Schabert (2010), and the model outcome will be qualitatively the same as long as the deadweight loss function in the reduced form is convex, as specified in this paper.[23] Consequently, the result here is rather complementary to existing work and can be used as a foundation for the analysis of further relevant factors in a more full-fledged framework such as a DSGE or an agent-based model to quantitatively assess the risk and value of government borrowing. The model can in general also be implemented in a model comparison approach as in Taylor and Wieland (2009).

[22] Certainly, the α value is not the sole factor affecting the default risk of government bonds. Other well-known factors include, a.o., output, the debt burden or the term structure as in, e.g., Borgy et al. (2011). For a survey of the different aspects studied in the existing sovereign default literature, see, e.g., Stähler (2011).

[23] Whether the model result still holds when one or more of the specifications of the functional forms are violated has not been checked here. Hence, the specifications are sufficient rather than necessary conditions.

4 Conclusion

In this paper, I have investigated the relationship between α, a parameter representing how well the average lender's wealth position is regarded by the borrowing government, and the default risk as well as the associated borrowing cost for the government. The main finding is that a higher α can reduce the propensity of government default. By showing that the good equilibrium is the only stable equilibrium, I demonstrate that a lower default propensity also leads to a lower default risk and hence more favorable borrowing conditions, which in turn enhance the government's ability to repay. This theoretical finding is in line with empirical results from the literature, although further research is still needed to extend the model to a broader scope of application. In the case that α can change over time, e.g., when government debt is freely tradable, only the α value at the time of debt repayment is relevant and thus needs to be assessed when debt is issued.

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[24] The names are sorted alphabetically according to surname.

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A Appendix

A.1 Proof of Theorem 1

Consider two different values of α with $0 \leq \alpha_1 < \alpha_2 < 1$. Given the outstanding amount of debt $z_{t-1}b_{t-1} \geq 0$ and any endowment y_t, denote the corresponding value function given non-default decision for α_1 and α_2 as $\mathbb{V}^n((1-\alpha_1)z_{t-1}b_{t-1})$ and $\mathbb{V}^n((1-\alpha_2)z_{t-1}b_{t-1})$, respectively. Further denote the associated optimal policy choice for new debt taking and government expenditure as (b^n_{1t}, g^n_{1t}) and (b^n_{2t}, g^n_{2t}), respectively. The contracted interest rate z_t is a function of $(1-\alpha)b_t$: $z_t = z((1-\alpha)b_t)$. Denote the equilibrium contracted interest rates for α_1 and α_2 as z_{1t} and z_{2t} with $z_{1t} \equiv z((1-\alpha_1)b^n_{1t})$ and $z_{2t} \equiv z((1-\alpha_2)b^n_{2t})$. Then we have:

$$
\begin{aligned}
&\mathbb{V}^n((1-\alpha_2)z_{t-1}b_{t-1}) \\
=&\mathbb{U}((y_t-(1-\alpha_2)z_{t-1}b_{t-1}+(1-\alpha_2)b^n_{2t}-g^n_{2t}), g^n_{2t}) + \beta E_t \mathbb{V}^f((1-\alpha_2)z_{2t}b^n_{2t}) \\
\geq&\mathbb{U}((y_t-(1-\alpha_2)z_{t-1}b_{t-1}+(1-\alpha_2)\frac{1-\alpha_1}{1-\alpha_2}b^n_{1t}-g^n_{1t}), g^n_{1t}) \\
&\quad + \beta E_t \mathbb{V}^f((1-\alpha_2)z\left((1-\alpha_2)\frac{1-\alpha_1}{1-\alpha_2}b^n_{1t}\right)\frac{1-\alpha_1}{1-\alpha_2}b^n_{1t}) \\
\geq&U(y_t-(1-\alpha_1)z_{t-1}b_{t-1}+(1-\alpha_1)b^n_{1t}-g^n_{1t}), g^n_{1t}) + \beta E_t \mathbb{V}^f((1-\alpha_1)z_{1t}b^n_{1t}) \\
=&\mathbb{V}^n((1-\alpha_1)z_{t-1}b_{t-1})
\end{aligned}
$$

A.2 Proof of Theorem 2

According to Equation (13) it holds that:

$$x'(\tau_1)(z_0b_0 - p'(\theta_1)) + (1-\alpha)z_0b_0 - p'(\theta_1) = 0$$

First consider the first term, $x'(\tau_1)(z_0b_0 - p'(\theta_1))$: From the above equation it follows that $x'(\tau_1)(z_0b_0 - p'(\theta_1))$ and $(1-\alpha)z_0b_0 - p'(\theta_1)$ should have different signs. At the optimum we have $x'(\tau_1) > 0$, because otherwise welfare can be increased by, e.g., increasing tax financed government expenditure. Hence, the sign of $x'(\tau_1)(z_0b_0 - p'(\theta_1))$ is determined by the sign of $z_0b_0 - p'(\theta_1)$, which means that $z_0b_0 - p'(\theta_1)$ and $(1-\alpha)z_0b_0 - p'(\theta_1)$ should have different signs; because

$z_0b_0 - p'(\theta_1) > (1-\alpha)z_0b_0 - p'(\theta_1)\ \forall\ z_0b_0 > 0$, it holds that $z_0b_0 - p'(\theta_1) > 0$. Furthermore, following the budget constraint it holds that the first derivative of τ_1 over θ_1 is $-z_0b_0 + p'(\theta_1) < 0$; therefore, around the optimum, an increasing θ_1 always lowers τ_1 and hence also lowers $x'(\tau_1)$ because $x''(\tau) > 0$. In addition, when θ_1 increases, then $-p'(\theta_1)$ will decrease or at least not increase because the penalty cost function $p(\theta_1)$ is assumed to be non-concave. Thus, $z_0b_0 - p'(\theta_1)$ also decreases when increasing θ_1. Consequently, the entire first term will decrease when θ_1 increases. Now consider the second term $(1-\alpha)z_0b_0 - p'(\theta_1)$: We see that the second term also moves in the opposite direction as θ does because by assumption, $p''(\theta_1) \geq 0$. Hence the entire LHS decreases with θ. On the other hand, a higher α will lower the LHS at the rate z_0b_0, which needs to be offset by a lower θ_1 to let the equation hold further until θ_1 reaches its lower bound; likewise, a lower α leads to a higher θ_1 until θ_1 reaches its upper bound. Consequently, it holds that $\frac{d\theta^*}{d\alpha} \leq 0$, and the adjustment of θ continues until the upper or lower bound is reached. When denoting the LHS as a function f, then we can express the proof in a more formal way:

$$\frac{\partial f}{\partial \theta} = -x''(\tau_1)(z_0b_0 - p'(\theta_1))^2 - x'(\tau_1)p''(\theta_1) - p''(\theta_1) < 0$$
$$\frac{\partial f}{\partial \alpha} = -z_0b_0 < 0 \Rightarrow$$
$$\frac{d\theta^*}{d\alpha} = -\frac{\partial f/\partial \alpha}{\partial f/\partial \theta} < 0$$

Thus, when the optimal default rate θ^* lies in the interval $[0,1]$, then we have $\frac{d\theta^*}{d\alpha} < 0$; and when the upper or lower bound has been reached, we must have $\frac{d\theta^*}{d\alpha} = 0$. These results combined, we obtain $\frac{d\theta^*}{d\alpha} \leq 0$.

A.3 Proof of Theorem 3

Consider the value function for the infinite-horizon model:

$$\begin{aligned}
\mathbb{V}(y_t, z_{t-1}b_{t-1}; \alpha) = & \sup_{b_t \in B_t, \theta_t \in [0,1], g_t \in G_t} \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}(y_{t+1}, z_t b_t; \alpha)\} \quad \text{s. t.} \\
& c_t = y_t - x(\tau_t) - (1-\alpha)(1-\theta_t) z_{t-1} b_{t-1} + \cdots \\
& \quad \cdots + (1-\alpha) b_t - g_t - p(\theta_t) \\
& \tau_t = (1-\theta_t) z_{t-1} b_{t-1} + g_t - b_t + p(\theta_t) \\
& z_t = \inf[z : \ z_t E_t [1 - \theta_{t+1}] = R_t]
\end{aligned}$$

Taking the FOC over θ_t, we get the same equation as (13). Then following the same steps as in the proof of **Theorem 2**, we obtain the same result, namely $\frac{d\theta^*}{d\alpha} \leq 0$.

B Examples

I illustrate the previous analysis in Section 2 by re-considering two well-known models from the earlier literature and extending them with the parameter α. Both models can be interpreted as special cases of the rather general framework constructed before, and each represents one type of sovereign default model. The first example comes from the seminal work in Calvo (1988) and serves as a representative of the model of the default rate as treated in Section 2.5.

B.1 The two-period model by Calvo

In Calvo's model about default in the form of contract violation, there are two periods: the debt contract is signed in the first period and the repayment or repudiation decision is made in the second period. A default decision will incur default cost as a fraction of the repudiated amount. Since there is no third period, no new loan will be contracted in the second period, and the value function of the second period (the period of repayment decision) collapses to the utility function:

$$\mathbb{V}(zb) = \sup_{\theta\in[0,1],g\in\{\bar{g}\},\tau\in\mathbb{R}} \{\mathbb{U}(y-\tau-x(\tau)+\alpha(1-\theta)zb,g)\} \tag{14}$$

$$\equiv \sup_{\theta\in[0,1],\tau\in\mathbb{R}} \{y-\tau-x(\tau)+\alpha(1-\theta)zb\} \quad \text{s.t.} \tag{15}$$

$$(1-\theta)zb+g = \tau - p(\theta) = \tau - \omega\theta zb \tag{16}$$

For notational simplicity, I have dropped out all time subscripts as Calvo did in his paper, which is viable as this is a two-period model and each relevant variable is determined only one time. We should keep in mind that the contracted gross interest rate z and the debt amount b are both determined in period 0 while all other variables are determined in period 1, the period of debt repayment.

Equation (14) is the utility function of the representative agent in the repayment period, which is assumed to be a function of after tax final wealth, which will be all consumed, as well as a function of government expenditure. Because government expenditure is constant, this maximization problem collapses to maximize the after tax final wealth, which is the sum of total income y (including labor income and capital income) and wealth receipt $\alpha(1-\theta)zb$, minus taxation τ and deadweight loss $x(\tau)$, as stated in Equation (15). Since Calvo's model is about domestic debt, he has set $\alpha = 1$; however, to investigate the influence of α on the repayment decision, I will further allow α to vary within the interval $[0,1]$

The contracted gross interest rate from the last period must give the investors the market return R:

$$zE_0[1-\theta] = (1-\theta)z = R$$

Here the expectation symbol can be dropped out since we are dealing with a deterministic case.

And as explained before, I will only consider the minimum interest rate which satisfies the participation constraint of the financial investors, i.e., I will only

consider the "good equilibrium" in the Calvo model.[25] Hence the above equation becomes:

$$z = \inf[z : (1-\theta)z = R] \tag{17}$$

The deadweight loss function due to taxation satisfies the following:

$$\begin{aligned} &x(0) = x'(0) = 0 \\ &x''(\tau) > 0 \quad \forall \tau \\ &\lim_{\tau \to \infty} x''(\tau) = \infty = -\lim_{\tau \to -\infty} x''(\tau) \end{aligned} \tag{18}$$

Rearranging Equation (16) yields:

$$\theta = \frac{zb + g - \tau}{(1-\omega)zb} \tag{19}$$

Equation (19) imparts that the default rate θ is decreasing when increasing tax load τ, which is intuitive: with rising taxation at a given government expenditure, more debt can be repaid; further, less default cost will be incurred, which in turn increases the repay ability of the government.

Plugging the government budget constraint (16) into the government maximization calculus (15) to eliminate θ,[26] and focusing on the part which can be influenced by the government policy, it can be shown that the maximization problem of the government is equivalent to the following minimization calculus:

$$\inf_{\tau \in \mathbb{R}} \{x(\tau) - \frac{\omega - (1-\alpha)}{1-\omega}\tau\}$$

[25] Indeed, as Calvo has noted, the equilibrium interest rate in the "bad equilibrium" will "paradoxically" decrease with increasing amount of outstanding debt. The reason that a "bad equilibrium" with this characteristic is rarely observed has been detailed in Section 2.6.

[26] Here I do not optimize over θ as I did in the main analysis, but optimize over τ instead, to facilitate the comparison with the result from Calvo (1988), also to show that the conclusion concerning the effect of α holds irrespective of the way of solving the model.

Hence, the desired taxation τ^* is characterized by $x'(\tau^*) = \frac{\omega-(1-\alpha)}{1-\omega}$. In the case of domestic debt, i.e., $\alpha = 1$, the result is the same as in Calvo (1988): $x'(\tau^*) = \frac{\omega}{1-\omega}$. As α decreases, the policy decision will put relatively more weight on the interest of the domestic tax payers than on the interest of the lenders; as a consequence, $x'(\tau^*)$ will decrease, which means less desired tax load τ due to the assumed property of the deadweight loss function as stated in (18), which in turn means that more will be repudiated as θ is a decreasing function of τ (Equation (19)), and θ will increase until it reaches its upper bound. Hence, in the case of a higher α and given all other factors constant, a government will repudiate less or equal amounts, as I have already shown in Section 2.5. Note that the θ here is not necessarily on the equilibrium path, but the institutional assumption made in Section 2 assures that the equilibrium repudiation ratio never increases with increasing α; and as I have shown in 2.6, the negative correlation between θ and α also almost certainly holds under much more general assumptions. We will see next that the maximal available loan amount for this small open economy strictly increases with α, too. In other words, a government with a higher α is less credit constrained than a comparative government with a lower α.

The impact of α on the repayment behavior and the associated borrowing condition for this small open economy is illustrated in Figure 2, which is comparable to the Figure 2 in Calvo (1988) but was modified to highlight the different repayment decisions and borrowing conditions due to different α values.

The two dashed lines radiating in north east direction represent the boundary of τ^*, the tax level chosen by the government of this small open economy: since θ is constrained in the interval $[0,1]$, τ^* is also constrained, namely, in the interval $[\bar{g}+\omega bz, \bar{g}+bz]$. Hence, for α below some threshold value, the desired taxation τ^* will be so low that full repudiation will occur for any b. Since the lenders anticipate that, the government of this small open economy will be completely constrained from the financial market and cannot borrow at all, i.e., the low α value cannot maintain enough trust for the lenders to invest in this economy. Indeed, suppose $\alpha = 0$, i.e., the government does not consider the welfare of the lenders at all, then more repudiation is the dominant strategy because it means more wealth transfer to the borrowers. This wealth transfer effect exceeds the repudiation cost, since the repudiation cost is assumed to be a fraction of the repudiated amount of debt.

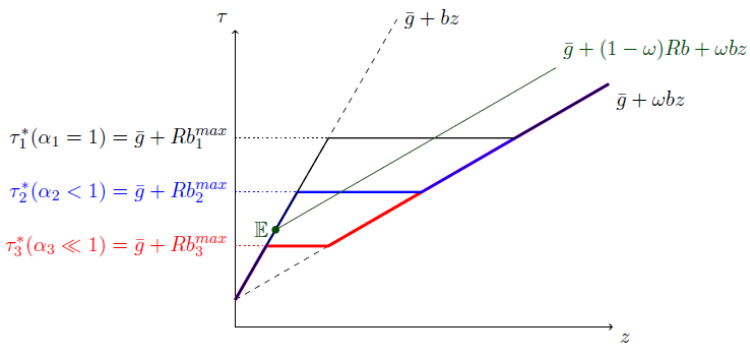

Figure 2: The equilibria under different α

Further, more default also means a reduction of deadweight loss due to less tax load which enables this small open economy to consume more. Hence, in the last period, the government will always choose full repudiation; and when the financial market participants anticipate this, they will not lend to the government of this small open economy at all.

When $\alpha = 1$, Figure 2 will be the same as in Calvo (1988): the wealth loss of the lenders resulting from a default is fully taken into account in the government's decision making. Since a default is nothing but a wealth transfer from lenders to tax payers which does not increase the welfare of the representative agent, the only reason for the government to default is to reduce deadweight loss resulting from too high taxation. Therefore, the government will only default if the gain from reducing deadweight loss exceeds the default cost, while wealth transfer from lenders to tax payers plays no role here. As a consequence, the government will have a higher desired tax level, represented by τ_1 (the black line), and is less inclined to default and hence the financial market participants are more willing to grant loans to the government since they anticipate a higher repay propensity.

As α declines, the horizontal part in the solid line, which presents the desired tax level τ^*, will also decline. All other lines remain in the same position because they are derived from the government budget constraint and the financial market participation constraint which are independent of α. Therefore, as α becomes small enough, the solid line will lie completely below the line representing the

"consistency condition", $\bar{g}+(1-\omega)Rb+\omega bz$, i.e., no contract $\{z,b\}$ can fulfill the market participation constraint for the financial investors, and hence the government is fully debt constrained. One such unsustainable desired tax level is depicted as the red line τ_3 and the tax level taking account of the boundary of θ corresponds to the red path ⌐.

For all values of α which support an equilibrium contract,[27] the contracted interest rate z always equals the market return R, since it lies on the full-repayment path $\bar{g}+bz$ and hence no repudiation will be expected under this condition.[28] However, the maximal available debt amount will increase when increasing α. To see this, note that $\bar{g}+zb \leq \tau^*$ is the necessary condition for an equilibrium contract to exist, hence the maximal available debt amount b^{max} is equal to $\frac{\tau^*-\bar{g}}{z} = \frac{\tau^*-\bar{g}}{R}$. Because τ^* increases when increasing α, so does b^{max}. In other words: a higher α increases the investors' trust in the government's ability and willingness to repay its debt, hence they are willing to lend more to the government.

Figure 3 shows how the expected return $(1-\theta)z$ varies with an increasing contracted interest rate z under different debt amounts b:

From Equation (19) it follows that θ is an increasing function of z. However, because θ is lower-bounded by zero, for z less than or equal to $\frac{\tau^*-\bar{g}}{b}$, the expected return $(1-\theta)z$ is equal to the contracted interest rate z, which means that the first part of the expected return curve is a 45° straight line. After this peak point has been reached, the expected return curve will become a downward sloping line because $(1-\theta)z = \frac{-\omega zb-\bar{g}+\tau^*}{(1-\omega)b}$ is a linear decreasing function of z. When z becomes as high as $\frac{\tau^*-\bar{g}}{\omega b}$, θ will reach its upper bound of one, from here on the expected return curve will overlap with the x-axis, which I do not plot here. Although Figure 3 does not look identical as Figure 1 in Section 2.6, it maintains the feature of having a part above the x-axis which first increases and then decreases, hence the analysis in 2.6 with regard to the multiple equilibria problem also applies here.

In Figure 3 I have set $\tau^* = 2$, $\bar{g} = 1$, $R = 1.11$ and $\omega = 0.5$. Then I vary b between $\frac{2}{3}$, $\frac{1}{R}$ and 1. For $b = \frac{2}{3}$, the expected return curve cuts the market return

[27] Here I have only depicted two values of α: the full internalization of lender's interest, $\alpha_1 = 1$, and a smaller α_2 which also allows the maintenance of expected market return

[28] The other intersection of the consistency condition line with the tax path is the so called "bad equilibrium" and can be excluded as shown in 2.6.

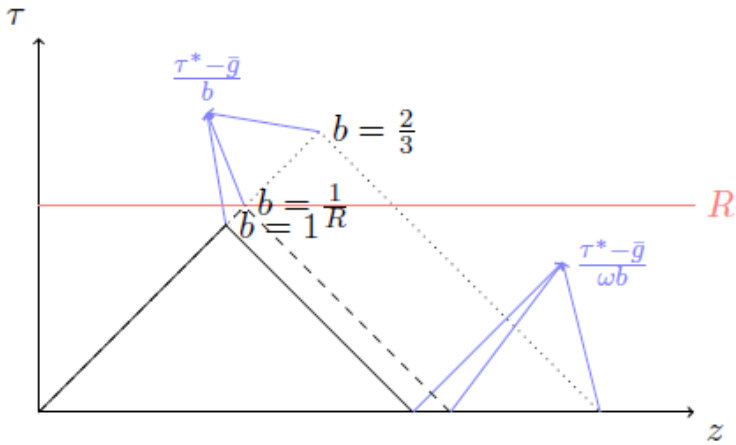

Figure 3: The equilibria under different b

R at two points, and I have shown in 2.6 that only the left intersection for which $\theta = 0$ and $z = R$ can be a stable equilibrium. As b increases, the expected return curve moves downward or inward. When $b = \frac{1}{R}$, which equals the maximal debt amount $b^{max} = \frac{\tau^*-\bar{g}}{R}$, the expected return curve has only one tangent point with the red line representing the market return, which is also the only equilibrium in this case. Furthermore, before b^{max} has been reached, the stable equilibrium always lies at the point in which $\theta = 0$ and $z = R$. When b further increases, e.g., to $b = 1$, the expected return curve will lie below the market return line, and there exists no equilibrium. Since τ^* increases with increasing α, a higher α will shift all the curves upward or outward, so that a curve associated with $b > \frac{1}{R}$ will eventually become tangent to R, which shows that a higher α will enable a higher b^{max}.

Usually, a more favorable borrowing condition (here in the form of less credit constraint, i.e., a higher b^{max}) also means a welfare increase (or at least not a decrease) for the respective economy. However, in the original Calvo model, this welfare effect cannot be seen directly since only the last period is considered, in which more debt rather means more duty to repay, hence a higher b value will rather reduce the average consumption, here a proxy for social welfare. To find the

reason for government borrowing, we need to go one period back and try to figure out why this government wants to borrow at all.

In period 0, the government chooses to borrow b because the desired debt amount can help improve the economy. If the government is constrained in borrowing, then it will simply borrow as much as it can. The welfare enhancement attained from borrowing can arise for many reasons: from consumption smoothing to inter-temporal allocation of tax load to some profitable investment opportunity which needs to be financed by debt, etc. For simplicity here I only consider the case of favorable investment opportunity, i.e., with one unit of borrowed money, which is contracted under the market return R, the government can conduct some investment, which will increase its next period's revenue by R' with $R' > R$. Hence the last period's consumption reads as follows:

$$\begin{aligned} c &= y - \tau - x(\tau) + \alpha Rb \\ &= y + R'b - g - (1-\alpha)Rb - x(g + Rb - R'b) \end{aligned}$$

Taking the first derivative yields:

$$\frac{\partial c}{\partial b} = R' - (1-\alpha)R + x'(g + Rb - R'b)(R' - R)$$

By assumption, we have that $R' - R > 0$ and hence $R' - (1-\alpha)R > 0$. Further $x'(\tau) > 0 \; \forall \; \tau > 0$, hence $\frac{\partial c}{\partial b}$ is strictly positive, i.e., the consumption in the last period increases with increasing debt taken in the previous period and hence the government will always take as much debt as possible. Therefore, a looser borrowing constraint, i.e., a higher b^{max}, always leads to a better economy which is now able to consume more.

Another interesting finding here is that even if we assume that the domestic economy (not the government) cannot internalize the foreign lenders' interest as their own and only considers the domestic consumption in the last period $c^d = y + R'b - g - Rb - x(g + Rb - R'b)$,[29] then also this term strictly increases

[29] This could be the case if the lenders' interest is internalized by the government through an investor-friendly legal system.

with increasing b. Hence more consideration of the lenders' interest, i.e., a higher α, can improve the domestic economy and increase its ability to consume, although the government now has (partially) incorporated the foreign lender's interest and no longer focuses solely on the domestic citizen's welfare. This is because here I assume rationality of the financial market, which implies that the foreign lenders can always correctly anticipate the government's repay behavior in the next period. If they feel that their interest will not be adequately considered in the next period and hence there is a higher default risk, then they will raise the contracted interest rate and/or, as in this case, reduce the maximal available loan amount for the government. As a result, the domestic economy will always pay the expected market return for the contracted debt amount, if it can get any; however, if the lenders' interest is considered more, then it can borrow more due to the perceived lower default risk, and hence a higher α will make the domestic citizens better off. To be precise: the final domestic consumption is $c^d = y + (R' - R)b^{max} - g - x(g - (R' - R)b^{max})$ which is increasing in b^{max}. As mentioned above, a higher α will increase (and at least not decrease) b^{max}, hence more consideration of lenders' interest will also make the borrowing economy better off, whose welfare increases due to more favorable borrowing conditions.

B.2 The model by Eaton and Gersovitz

In Eaton and Gersovitz (1981), purely external debt has been considered. As mentioned, I extend this model by the parameter α such that purely external debt can be viewed as an extreme case in which $\alpha = 0$. But the feature that there is either full repayment or full repudiation remains, and hence the expected default rate is equal to the default probability as elaborated in 2.4.

To the model: In each period, a random output y_t has been drawn, from which the old debt b_{t-1} has to be repaid (possibly with repudiation on the fraction of θ_t). In the case of full repayment, new debt $b_t \in B_t$ can be taken. In the case of default, $B_t = 0 \; \forall \; t \geq \tau$ if $\theta_\tau > 0$, i.e., a one-time (possibly partial) default will exclude the government from financial market participation forever. In Eaton and Gersovitz (1981), the absorption c_t is $y_t - (1 - \theta_t)z_{t-1}b_{t-1} + b_t$, since any repayment of outstanding debt will reduce the economy's absorption one-to-one.

I want to show, however, how α affects the repay propensity of the government, hence in the extended model, the economy's absorption reads as follows:

$$c_t = y_t - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} + (1-\alpha)b_t \tag{20}$$

The government's objective function is the maximization of the current and the discounted future utility. The utility in period t constitutes of the absorption c_t minus some penalty cost P_t which is imposed in the case of default in addition to the financial market exclusion:

$$\max_{\theta_t \in [0,1]} E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_\tau - P_\tau) \right]$$

One straightforward conclusion from Equation (20) is that, in the case of purely domestic debt, i.e., $\alpha = 1$, the government of this small open economy will have no incentive to repudiate in any state, since repudiation is just the wealth transfer from one citizen to another citizen and will not affect the economy-wide absorption at all. Anticipating this, the financial market is always willing to lend to the government, regardless of the contracted debt amount as well as of the expected level and volatility of the future output. However, in this case, the optimal debt amount for the government, b_t, is also undetermined, because this model assumes that there is no storage technology, nor is there any deadweight loss due to taxation, thus, all debt is immediately absorbed, and hence borrowing money from domestic lenders just leads to the same absorption and consequently has no impact on the welfare of the domestic citizens.

For $\alpha < 1$, the government will in some states prefer default, namely whenever this decision can enhance the welfare of the representative agent. And whenever the government chooses to default, it will default on the entire stock of outstanding debt, i.e., set $\theta_t = 1$, since both exclusion from the financial market and the possible penalty cost P_t will occur for any $\theta_t > 0$, independent of the amount being repudiated. Denote the probability of default in the next period as λ_t, then in equilibrium:

$$z_t(1-\lambda_t) = R_t \tag{21}$$

In the following I will show that the default probability λ_t is a decreasing function of α, i.e., the more the financial investors expect their interests to be considered by the government, the lower the anticipated default probability will be.

Denote the value function in the case of default as $\mathbb{V}_t^D$, then $\mathbb{V}_t^D$ for a government with outstanding amount of debt $z_{t-1}b_{t-1}$ and the parameter α is given by:

$$\mathbb{V}_t^D = E_t\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(y_\tau - P_\tau)\right] \tag{22}$$

Since the above expression is independent of α, the value of default, $\mathbb{V}_t^D$, is also independent of α.

Denote the value function in the case of full repayment as $\mathbb{V}_t^R$, then the value function for period t with access to the financial market is given by $\mathbb{V}_t^f = \max(\mathbb{V}_t^D, \mathbb{V}_t^R)$. Further, $\mathbb{V}_t^R$ for a government with an outstanding amount of debt $z_{t-1}b_{t-1}$ and the parameter α can be expressed as:

$$\begin{aligned}\mathbb{V}_t^R((1-\alpha)z_{t-1}b_{t-1}) = \sup_{b_t} U(y_t - (1-\alpha)z_{t-1}b_{t-1} + (1-\alpha)b_t) + \cdots \\ \cdots + \beta E_t[\mathbb{V}_{t+1}^f((1-\alpha)z_t b_t)]\end{aligned}$$

Now consider two governments with different α values, assume $\alpha_1 > \alpha_2$, and all other variables being equal, it can be shown that the government with the higher α value will also have a higher $\mathbb{V}_t^R$:

$$\begin{aligned}
\mathbb{V}_t^R((1-\alpha_1)z_{t-1}b_{t-1}) &= U(y_t - (1-\alpha_1)z_{t-1}b_{t-1} + (1-\alpha_1)b_{1t}^*) + \cdots \\
&\cdots + \beta E_t[\mathbb{V}_{t+1}((1-\alpha_1)z_{1t}^* b_{1t}^*)] \\
&\geq U(y_t - (1-\alpha_1)z_{t-1}b_{t-1} + (1-\alpha_1)\frac{1-\alpha_2}{1-\alpha_1}b_{2t}^*) + \cdots \\
&\cdots + \beta E_t[\mathbb{V}_{t+1}((1-\alpha_1)z_{2t}^* \frac{1-\alpha_2}{1-\alpha_1}b_{2t}^*)] \\
&\geq U(y_t - (1-\alpha_2)z_{t-1}b_{t-1} + (1-\alpha_2)b_{2t}^*) + \cdots \\
&\cdots + \beta E_t[\mathbb{V}_{t+1}((1-\alpha_2)z_{2t}^* b_{2t}^*)] \\
&= \mathbb{V}_t^R((1-\alpha_2)z_{t-1}b_{t-1})
\end{aligned} \tag{23}$$

In the above expression, b_{it}^* with $i \in \{1,2\}$ is the optimal new debt amount chosen by the respective government, and z_{it}^* with $i \in \{1,2\}$ is the contracted interest rate associated with the respective α_i and b_{it}^*. In the case of positive outstanding amount of debt, i.e., $z_{t-1}b_{t-1} > 0$, the second $\geq$ will hold with strict inequality.

The inequality (23) implies that a government with a higher α will always value the full repayment more (at least not less) than a government with a lower α, while the value from full default remains the same according to Equation (22). Therefore, in any state, a government with a higher α will always pay back its debt whenever the comparative government with a lower α chooses to repay its debt; and the government with a higher α will only choose to default if the comparative government with a lower α also decides to default. Note that $\lambda_t \equiv \Pr(\mathbb{V}_{t+1}^D > \mathbb{V}_{t+1}^R)$, hence λ_t is a decreasing function in α. From 2.6 we know that a government with a lower propensity to default can enjoy more favorable borrowing conditions, which means that the borrowing costs for the government, here the contracted interest rate z_t, will also decrease when increasing α.

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The Editor

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