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Trade and Productivity: The Family Connection Redux

Klaus Prettner¹ and Holger Strulik²

Abstract

We investigate the effects of human capital accumulation on trade and productivity by integrating a micro-founded education and fertility decision of households into a model of international trade with firm heterogeneity. Our theoretical framework leads to two testable implications: i) the export share of a country increases with the education level of its population, ii) the average profitability of firms located in a country also increases with the education level of its population. We find that these implications are supported by empirical evidence for a panel of OECD countries from 1960 to 2010.

JEL: F12, F14, I20, J11
Keywords: firm heterogeneity, international competitiveness, education, fertility decline

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1. Introduction

Over the last decades, industrialized countries experienced considerable demographical changes toward lower birth rates. While crude birth rates fell in all developed countries, the extent to which this happened differed considerably between them. For example, the U.S. had a crude birth rate of 24 children per 1000 inhabitants in 1950 and ended up with 14 children per 1000 inhabitants in 2010, Japan started with the same rate in 1950 but ended up with 9 children per 1000 inhabitants in 2010. European countries like France, Italy, and the United Kingdom started from lower levels than Japan and the U.S. in 1950 and find themselves somewhere in between these two countries nowadays.

The immediate question that economists are confronted with is whether declining fertility will be a millstone around the neck of economic prosperity. This question has been analyzed extensively from different points of view. Some economists argue that support ratios have been declining and will decline even further such that fewer and fewer workers will be available for producing the goods and services that are consumed by all the individuals in an economy (see for example Gruescu, 2007; Bloom et al., 2010), others emphasize that the sustainability of social security and pension systems is threatened (see for example Gruber and Wise, 1998), and yet others analyze the changing savings behavior of individuals in the wake of changing demography (see for example Heijdra and Ligthart, 2006; Bloom et al., 2007; Krueger and Ludwig, 2007).

In this paper we focus on the effects of declining fertility on international competitiveness and productivity of domestic firms via its impact on education. To analyze this question, we integrate into the state-of-the-art trade literature with firm heterogeneity (Melitz, 2003; Helpman et al., 2004) a micro-founded fertility and education decision at the household level. We show that declining fertility has beneficial effects on productivity and on international competitiveness of domestic firms because it is accompanied by higher schooling investments. The reason is a child quantity-quality substitution at the household level (Becker and Lewis, 1973; Galor, 2005). Higher schooling investments then translate into higher average human capital and therefore into higher labor productivity, which in turn raises the probability for entrepreneurs to establish a profitable firm.

These theoretical implications are helpful to explain the finding that for OECD countries in the
period 1960 – 2010 export share and output per firm are positively correlated with education. Our results indicate that investments in human capital accumulation, especially in higher education, are an important determinant of a country’s export share, that is, its international competitiveness. Furthermore, as far as international competitiveness is concerned, falling birth rates need not be as bad as it is often argued.

So far there exists relatively little research on the role of education in the new trade literature. Yeaple (2005) proposes a model in which homogeneous firms have access to a “high-tech technology” and a “low-tech technology”. The work force is heterogeneous in their skill level and high skilled workers have a comparative advantage in high-tech production. In equilibrium, exporting firms are shown to be larger, to employ more high skilled labor, and to pay higher wages. A similar result has been derived by Manasse and Turrini (2001) for an economy in which firms are led by worker-entrepreneurs of different ability. The role of population size (or its growth rate) is not investigated in these studies. Our study, by contrast, focuses on a homogeneous workforce and the impact of its human capital endowment on firm productivity through managerial education and human capital externalities at the firm level. In conjunction with a fertility-education trade-off at the household side, it establishes a negative association between population growth and firm productivity. Finally, Prettner and Strulik (2014) analyze the differential impact of the scale of an economy (in terms of its population size) and the education of its workforce on per capita GDP relative to other countries. For their analysis, they use a trade model based on Eaton and Kortum (2001) in which they introduce endogenous education decisions of households. Consistent with empirical regularities, they show that population size has a positive effect on relative per capita GDP if and only if the countries under consideration are closed to international trade, while education always has a positive effect on relative per capita GDP, irrespective of trade-openness.

The association between international trade and the fertility and education behavior of households has also been investigated in the seminal papers of Galor and Mountford (2006, 2008). Employing a Ricardian argument for trade, these studies identify a causal impact of trade on fertility and education. They consider a two-region model in which the North is endowed with a better industrial technology and specializes on industrial goods, while the South specializes on agriculture. Because industrial production is relatively skill-intensive and agriculture is relatively labor intensive, trade increases the demand for education in the North and induces higher fertility in the
South. Empirically, Galor and Mountford find that within OECD countries the volume of trade is positively associated with education and negatively associated with fertility, while the opposite is found for non-OECD countries.

Acknowledging the important channel established by Galor and Mountford, our paper explains the complementing channel running from the fertility and education decisions of households to aggregate productivity and the volume of trade. We focus on trade between countries that produce at least some manufacturing goods and hence do not specialize in agricultural production. These countries are populated by heterogeneous firms and differ in the education level of their population. Within this framework we explain how choices at the household level lead to cross-country differences in aggregate productivity through the induced selection of firms into international trade. It is important to emphasize that the trade framework underlying our analysis generalizes the one used by Galor and Mountford (2006, 2008): while they assume that there are only two types of goods — manufactured goods and agricultural goods — the Melitz (2003) framework allows for differentiated manufactured varieties. Consequently, in our case a country that exports more of some varieties does not necessarily import more agricultural goods. Instead, our model predicts that countries populated by a better educated workforce produce with higher productivity and export more manufactured goods, a fact that allows them to import more manufactured varieties from other countries. Galor and Mountford aptly coined the catchphrase “the family connection” for their studies on trade and productivity. The present paper resumes this line of research.

Better education and the implied higher average productivity of firms does not necessarily imply more exports when fertility is endogenous. The child quantity-quality trade-off at the household level leads also, taken for itself, to lower exports through less population growth (the home market effect). The prediction from the theoretical model is thus, in principle, ambiguous. This motivates our empirical analysis. We find that the productivity channel dominates, i.e., we corroborate the hypothesis that more education promotes productivity as well as the export share of GDP.

The paper proceeds as follows. Section 2 sets up a basic model that integrates a microfounded fertility and education decision into the modern trade theory that is based on firm heterogeneity. The model is simple enough to assess analytically the impact of falling birth rates and rising education on aggregate productivity and international competitiveness of domestic firms. The simple model, however, entails the drawback that changes in family behavior have to be conceptualized as
triggered by exogenous parametric changes of the underlying model, in particular of the weight of children and education in the preferences of parents. An intellectually more appealing way to study the “family connection” is to rely on endogenous changes of family behavior and their feedback on firm productivity and income growth. In Section 3 we therefore extend our theory in this respect and solve numerically for the long-run adjustment dynamics. Section 4 provides empirical evidence on the association of trade and productivity with education investments. Section 5 discusses implications and limitations of our study.

2. THE MODEL

In this section we develop an analytically tractable model in which we rely on international differences in parent’s preferences with respect to education and fertility to illustrate the main points of our argument. In Section 3 we show that our results prevail when we relax the assumption of international differences in preferences and explain fertility and education differentials by a differentiated timing of the take-off to growth and the onset of the demographic transition. This, however, comes at the price of analytical tractability implying that we have to resort to numerical investigations.

2.1. Households. Consider a society populated by two overlapping generations: children and adults. Time is discrete and measured in terms of generations. We suppress time arguments whenever possible. Individual (household or firm) variables are denoted by lowercase letters, aggregate variables are denoted by capital letters. Adults experience utility from consuming an aggregate consumption good, from having a divisible number of children $n$ and from providing their children with education $e$. Having a child incurs a minimum time cost $\phi$ (child rearing cost) and, following Galor and Weil (2000), a unit of child education requires an additional time cost $e$. Consequently, parents forgo wage income if they invest more in quantity or quality of children. It turned out that the algebra involved in the solution for the general equilibrium is more convenient when we measure all quantities in terms of units of human capital (efficiency units). Let therefore $c$ and $w$ denote consumption and wages per unit of human capital and let $P$ denote the aggregate price level. A household’s budget constraint is then given by

$$w (1 - \phi n - en) = Pc. \quad (1)$$
The left hand side of (1) represents lifetime income after taking into account child care in terms of quality and quantity investments, while the right hand side comprises consumption expenditures.

Suppose utility has the log-form such that

$$u = \log (c \cdot h) + \alpha \log (n) + \gamma \log (e),$$

in which $h$ is human capital of the household, $c \cdot h$ is household consumption in units of goods, and $\alpha \in (0, 1)$ and $\gamma \in (0, \alpha)$ are the weights of child quantity and quality, respectively. Our parameter restrictions ensure $\alpha > \gamma$, which is needed for a positive population size in period $t + 1$ because otherwise parents would choose to have no children at all. Note that the household side of standard trade models is captured by the special case in which parents live indefinitely and do not have children implying that the human capital adjusted lifetime wage income of parents would be $w$. Maximizing (2) subject to (1) provides the household’s demand for children and their education:

$$n = \frac{\alpha - \gamma}{\phi (1 + \alpha)}, \quad e = \frac{\gamma \phi}{\alpha - \gamma}. \quad (3)$$

The implied time spent on working is given by $1 - (\phi + e)n = 1/(1 + \alpha)$. The quantity-quality trade-off can be easily established: If parents desire more children, they reduce education and vice versa (Becker and Lewis, 1973; Galor and Weil, 2000). Formally, the trade-off is obtained from (3) as $n = \gamma/(1 + \alpha) \cdot (1/e)$.

For simplicity we assume that education of a child is transformed one-to-one into human capital of an adult. This means that human capital of a member of the next generation is given by $h_{t+1} = e_t = \gamma \phi/\alpha - \gamma)$. Let $L_t$ denote the size of the adult generation at time $t$. Aggregate human capital of the next generation is then given by $H_{t+1} = n_t h_{t+1} L_t = \gamma L_t/(1 + \alpha)$. Notice that aggregate human capital of the next generation is increasing with the desire for education $\gamma$ and decreasing with the desire for children $\alpha$. This result is straightforwardly explained. A substitution of child quantity $n$ by education $e$ keeps total child expenditure $e \cdot n$ constant and sets free $\phi \Delta n$ units of parental time, which can be used to earn extra income. The additional income is partly spent on education such that overall education expenditure rises more strongly than child quantity falls. At the macro side of the economy this trade-off implies that human capital per person increases more strongly than the number of persons falls such that total available human capital increases.
Aggregate consumption consists of a homogeneous consumption good \( Z \), which can be traded without costs, and of a continuum of costly tradeable CES consumption goods (as in Helpman et al., 2004; and Demidova, 2008). The sub-utility function of the representative individual with respect to consumption is iso-elastic in homogeneous good consumption and CES good consumption, that is, \( c = Z^n Q^{1-\eta} \). This implies that households optimally spend a share \( \eta \) of their consumption budget on \( Z \) and a share \( 1-\eta \) on \( Q \). The CES part of the sub-utility function is given by

\[
Q = \left[ \int_{\omega \in \Omega} q(\omega)^\rho d\omega \right]^{1/\rho}.
\]

(4)

In this expression the measure \( \Omega \) denotes the mass of available goods, \( q(\omega) \) denotes the quantity of each good \( \omega \), and \( \rho \) determines the elasticity of substitution between the CES goods as \( \sigma = 1/(1-\rho) > 1 \). Total expenditure on CES consumption goods is then given by

\[
\frac{1-\eta}{1+\alpha} wL = \int_{\omega \in \Omega} p(\omega) q(\omega) d\omega = R,
\]

(5)
in which \( p(\omega) \) is the price of variety \( \omega \) and \( R \) refers to the aggregate revenues of all firms that produce heterogeneous goods. The numerator on the left hand side of (5) adjusts for consumption of the homogeneous good and the denominator adjusts for the effect that parents with children do not supply their whole available time on the labor market implying that their lifetime wages are lower than \( w \). Since all households are symmetric, their consumption allocation problem is solved by maximizing (4) subject to (5). This leads to the demand function \( q(\omega) = (p(\omega)/P)^{-\sigma} Q \), with \( P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \). Optimal expenditure for each variety \( \omega \) can then be written as

\[
r(\omega) = p(\omega) q(\omega) = R \left[ \frac{p(\omega)}{P} \right]^{1-\sigma},
\]

(6)

where we note that \( R = PQ \).

2.2. Firms. Human capital is the only factor of production. The homogeneous consumption good is produced with a constant input coefficient, which is normalized to one, implying a unit wage in efficiency units (measured in terms of human capital). In line with Helpman (2004) and Demidova (2008), costless trade of the homogeneous good ensures factor price equalization at the human capital adjusted wage rate \( w = 1 \) for all countries. Note that the notation in efficiency units implies that better educated workers receive higher labor income and that firms pay higher wages
in countries populated by a better educated workforce. The modeling of the production sector for CES goods follows the literature of international trade with firm heterogeneity (Eaton and Kortum, 2001, 2002; Bernard et al., 2003; Melitz, 2003; Helpman, 2004; Demidova, 2008) with the last three being especially relevant in our context.

There exists a continuum of firms, each producing a variety $\omega$. Each firm has therefore access to a production technology $A(\omega)$ that follows from the standard profit function in the literature (see, for example, Melitz, 2003; p. 1699). Production requires a fixed cost $f$, which is measured in terms of human capital and which is the same for all firms, while productivity of human capital, $A(\omega)$, differs between them. Human capital demand $\tilde{h}(\omega)$ is given by $\tilde{h}(\omega) = f + q(\omega)/A(\omega)$. The intuition is that if firms want to produce a desired quantity $q(\omega)$ given the firm-specific productivity $A(\omega)$, they first have to incur the fixed cost of production in terms of human capital, $f$, and then they have to employ $q(\omega)/A(\omega)$ units of human capital to be able to fulfill their manufacturing schedule.

Profits are given by revenues of sales net of the wage bill and amount to $\pi = p(q) \cdot q - w \left[ (q/A) + f \right]$. Maximizing profits, taking the demand function of households into account, leads to the optimal pricing rule $p(A) = w/\rho A$, implying a markup $1/\rho$ over marginal costs $1/A$.

Using the demand function $q(\omega)$, the expression for profits can be reformulated as $\pi = r(\omega)/\sigma - f$, in which $r(\omega) = R \left[ p(\omega)/P \right]^{1-\sigma}$ are firm-specific revenues and $r/\sigma$ denotes operating profits. Inserting $p(\omega) = 1/(\rho A)$ into $r$ provides an expression of firm revenues as a function of macroeconomic aggregates and firm-specific productivity:

$$
 r = R \left( \frac{1}{\rho AP} \right)^{1-\sigma} = R(\rho AP)^{\sigma-1},
$$

such that profits become $\pi = (R/\sigma)(\rho AP)^{\sigma-1} - f$. Using $q(\omega) = [p(\omega)/P]^{-\sigma} Q$, the following ratios between the size and revenue of two firms indexed by 1 and 2 can be calculated

$$
\frac{q(A_1)}{q(A_2)} = \left( \frac{A_1}{A_2} \right)^{\sigma}, \quad \frac{r(A_1)}{r(A_2)} = \left( \frac{A_1}{A_2} \right)^{\sigma-1}.
$$

This implies that relative firm size in terms of the produced quantity as well as relative revenue crucially depend on firm-specific productivity, replicating a central result of Melitz (2003). Note that this result cannot be derived by relying on a model with homogeneous firms.
2.3. **Aggregation over firms.** For the sake of tractability and in line with the theoretical and empirical literature (Axtell, 2001; Helpman et al., 2004; Chaney, 2008; Eaton et al., 2011), we assume that firm productivity is derived from a Pareto distribution \( G(A) = 1 - \left( \frac{A_m}{A} \right)^a \), where minimum productivity \( A_m \) is equal to one. In deviation from the earlier literature, however, we postulate that the shape parameter \( a \) is decreasing in the human capital endowment per person, capturing the phenomenon that in an economy where people are better educated, it is easier for an entrepreneur to establish a productive enterprise.

There exist at least two complementing channels through which education of the workforce exerts a positive effect on firm productivity. The first channel acknowledges positive external effects of human capital in (team) production, which are observationally equivalent to an increase in productivity (Lucas, 1988; Battu et al., 2003; Munch and Skaksen, 2008). The second channel takes into account that the education of entrepreneurs has a positive impact on firm productivity, i.e., that better educated entrepreneurs are more likely to draw a favorable firm productivity. Empirically there exists supporting evidence for both channels. Firms of high productivity are on average run by better educated managers and employ a better educated work force (see Grossmann, 2007; La Porta and Shleifer, 2008; Bloom and Van Reenen, 2007, 2010; Syverson, 2011; and, in particular, Gennaioli et al., 2013). Taking human capital externalities and education of entrepreneurs into account is one particularly reasonable way to endogenize differences across countries in the productivity distribution of firms, which has been investigated within the Melitz framework by Demidova (2008).

A parsimonious and analytically convenient way to represent the positive effect of human capital on firm productivity is to set the shape parameter of the Pareto distribution to \( a = 1/h \). Since \( 1/h = (\alpha - \gamma)/(\gamma \phi) \), the cumulative Pareto distribution function reads

\[
G(A) = 1 - \left( \frac{1}{A} \right)^{\frac{\alpha - \gamma}{\gamma \phi}}
\]

and has positive support over \([1, \infty)\). This treatment formalizes the notion that the probability to draw a high productivity firm is not exogenous — as in Melitz (2003) — but influenced positively by the level of education in an economy. As explained above, this is so because of external knowledge spillovers between employers and/or because higher education increases the probability to draw a capable manager of the firm.
The density function of firm productivity \( g(A) \) is given by

\[
 g(A) = G'(A) = \frac{\alpha - \gamma}{\gamma \phi} A^{-\left(1 + \frac{\alpha - \gamma}{\gamma \phi}\right)}.
\] (9)

The existence of the expected value of \( A \) and its variance are ensured by the parameter restriction \((\alpha - \gamma)/\gamma \phi > 2\), a sufficient condition for which is \( n \geq 1 \), implying non-negative population growth. Expected productivity is then determined as \( E(A) = \int_1^\infty Ag(A)dA = (\alpha - \gamma)/(\alpha - \gamma - \gamma \phi) \).

Inspecting the derivatives

\[
 \frac{\partial E(A)}{\partial \alpha} = -\frac{\gamma \phi}{(\alpha - \gamma - \gamma \phi)^2} < 0, \quad \frac{\partial E(A)}{\partial \gamma} = \frac{\alpha \phi}{(\alpha - \gamma - \gamma \phi)^2} > 0,
\]

we conclude that expected firm productivity is higher in an economy in which people prefer to have fewer children and/or prefer to educate their children better. This is the central consequence of the child quality-quantity tradeoff, which implies that parents with fewer children invest more in education of each child. The expansion of education investments per child leads to a higher average human capital stock which in turn leads to a counterclockwise rotation of the density function of the distribution from which firms draw their productivity levels. Consequently, the probability that firms draw a high productivity level rises and the probability that they draw a low productivity level falls.

2.4. Firm entry and exit. Firms are initially identical and face a fixed investment cost to enter the market, which is expressed in units of human capital and denoted by \( f_e \). After entering, firms draw a productivity level \( A \). Firms drawing a low \( A \) that would not allow their operating profits to cover their fixed costs choose to exit immediately. The other firms start to produce and henceforth face a constant probability \( \delta \) of a bad shock in every period that would force them to exit. In equilibrium, on which we focus from now on, entry is equal to exit. The value function of a firm — assuming that there is no discounting of the future on top of the risk of exit — is

\[
v(A) = \max \left( 0, \sum_{t=0}^{\infty} (1 - \delta)^t \pi(A) \right) = \max \left( 0, \frac{\pi(A)}{\delta} \right).
\]

Analogous to Melitz (2003), we define the cut-off level of productivity as \( A^* = \inf(A : v(A) > 0) \). Firms with lower productivity, that is, firms facing \( A < A^* \), exit immediately and do not produce. At the cut-off level of productivity, firms do not make any profits such that \( \pi(A^*) = 0 \). Together
with the density function of the Pareto distribution, the probability of successful entry, \(1 - G(A^*)\), determines the distribution of productivity conditional on entry denoted by \(\mu(A)\) as

\[
\mu(A) = \begin{cases} 
\frac{g(A)}{1 - G(A^*)} & \text{if } A > A^*, \\
0 & \text{otherwise,}
\end{cases}
\]

where

\[
g(A) = \frac{\alpha - \gamma}{\gamma \phi} A^{\frac{\alpha - \gamma}{\gamma \phi}} (A)^{-\left(1 + \frac{\alpha - \gamma}{\gamma \phi}\right)}.
\]

Consequently, productivity \(A\) is a function of the cut-off productivity for successful entry, \(A^*\). Defining average productivity as \(\bar{A} = \left[\int_{A^*}^{\infty} A^{\sigma - 1} \mu(A) dA\right]^{\frac{1}{\sigma - 1}}\), we arrive at

\[
\bar{A} = \frac{\alpha - \gamma}{(1 - \sigma) \gamma \phi + \alpha - \gamma} A^*,
\]

which is a linear function of the cut-off productivity level. Non-negative average productivity requires \((1 - \sigma) \gamma \phi + \alpha - \gamma > 0\), implying \((\alpha - \gamma)/(\gamma \phi) > \sigma - 1 > 0\), where the latter inequality ensures that average productivity is finite. It always holds because \(\sigma > 1\).

We define average revenues by \(\bar{r} = r(\bar{A})\), average profits by \(\bar{\pi} = \pi(\bar{A}) = \bar{r}/\sigma - f\) and note that a firm does not make profits at the cut-off level of productivity implying \(\pi(A^*) = r(A^*)/\sigma - f = 0 \Rightarrow r(A^*) = \sigma f\). Using \(\bar{r} = r(A^*) \left[\bar{A}(A^*)/A^*\right]^{\frac{1}{\sigma - 1}} = r(A^*)(\alpha - \gamma)/\left[(1 - \sigma) \gamma \phi + \alpha - \gamma\right]\), average profits at the cut-off are given by

\[
\bar{\pi} = f \cdot \left[\frac{\alpha - \gamma}{(1 - \sigma) \gamma \phi + \alpha - \gamma} - 1\right] \equiv f \cdot \kappa.
\]

Melitz (2003) refers to this expression as the zero cut-off profit condition. In our case, where productivity levels are drawn from a Pareto distribution, the term in square brackets is independent of \(A^*\) and given by the compound constant \(\kappa\).

The present value of average profit flows, \(\bar{\pi}\), is determined by the probability of a bad shock driving the firms under consideration out of business: \(\bar{\pi} = \pi/\delta = \int_{A^*}^{\infty} v(A) \mu(A) dA\). Since the probability of successful entry is \(1 - G(A^*)\) and the expected value of entry net of fixed costs has to be zero, we can solve for average profits as

\[
\bar{\pi} = \delta f e^{(A^*)^{\frac{\alpha - \gamma}{\gamma \phi}}}.
\]
Equations (11) and (12) can be solved for average profits and the cut-off level of productivity:

\[
\pi = \frac{f \gamma (\sigma - 1) \phi}{\alpha - \gamma - (\sigma - 1) \phi},
\]
\[
A^* = \exp \left\{ \gamma \left[ \ln(\delta) - \ln \left( \frac{f \gamma (\sigma - 1) \phi}{\alpha - \gamma - (\sigma - 1) \phi} \right) + \ln(f_e) \right] + \ln \left( f_e \right) \right\}.
\]

This leads to the following proposition.

**Proposition 1.** In a closed economy, an increase in the desire for fertility (\(\alpha\)) decreases average productivity and average profits, whereas an increase in the desire for education (\(\gamma\)) increases average productivity and average profits.

**Proof.** The derivatives of \(\pi\) with respect to \(\alpha\) and \(\gamma\) are, respectively,

\[
\frac{\partial \pi}{\partial \alpha} = -\frac{f \gamma (\sigma - 1) \phi}{[\alpha - \gamma - (\sigma - 1) \phi]^2} < 0,
\]
\[
\frac{\partial \pi}{\partial \gamma} = \frac{f \alpha (\sigma - 1) \phi}{[\alpha - \gamma - (\sigma - 1) \phi]^2} > 0,
\]

implying that average profits decrease with the desire for fertility and increase with the desire for education. Furthermore, recall that Equation (12) implies that \(\frac{\pi}{\delta f_e} = A^*\). Since \(\alpha - \gamma > 0\), the base and the exponent of this expression are both increasing with \(\gamma\) and decreasing with \(\alpha\). Consequently, the cut-off productivity also decreases with the desire for fertility and increases with the desire for education. This establishes that \(A^*\) is an increasing function of \(\gamma\) and a decreasing function of \(\alpha\).

Furthermore, when using Equation (10), we can show that average productivity increases with \(\gamma\) and decreases with \(\eta\) for given \(A^*\)

\[
\frac{\partial A}{\partial \alpha} = -A^* \alpha \phi \left\{ \frac{[\alpha - \gamma]/[(1 - \sigma) \gamma \phi + \alpha - \gamma]}{[\gamma - \alpha + \gamma (1 - \sigma) \phi]^2} \right\}^{\frac{1}{\gamma - \alpha + \gamma (1 - \sigma) \phi}} < 0,
\]
\[
\frac{\partial A}{\partial \gamma} = A^* \alpha \phi \left\{ \frac{[\alpha - \gamma]/[(1 - \sigma) \gamma \phi + \alpha - \gamma]}{[\gamma - \alpha + \gamma (1 - \sigma) \phi]^2} \right\}^{\frac{1}{\gamma - \alpha + \gamma (1 - \sigma) \phi}} > 0,
\]

which establishes the proof.

To summarize, productivity is higher in an economy in which parents prefer to have fewer and better educated children because the average human capital stock is higher in such an economy and the corresponding distribution from which firms draw their productivity level has a fatter upper
2.5. Open Economy. Similar to Melitz (2003), the transition to trade does not affect individual firm level variables but the decision of firms to enter the home and foreign markets. Assume that there are \( m + 1 \) countries and iceberg transport costs apply for the CES consumption good such that \( \tau > 1 \) units have to be shipped in order for one unit to arrive at the destination (see e.g. Samuelson, 1954; Krugman, 1980; Baldwin et al., 2003). Furthermore, there are fixed costs for exploring the export market and for adjusting the product to foreign standards. These are denoted by \( f_{ex} > 0 \) and measured in terms of human capital. The domestic price charged by a firm is \( p_d(A) = 1/\left(\rho A\right) \) and the corresponding export price is \( p_x(A) = \tau p_d(A) = \tau/(\rho A) \). The revenues from domestic sales are \( r_d(A) = R(P\rho A)^{\sigma-1} \). Following Helpman et al. (2004) in assuming that countries do not differ too much in size such that all countries face the same wages, demand, and cut-off levels, the corresponding revenues from export sales amount to

\[
r_x(A) = R\left(\frac{P \rho A}{\tau}\right)^{\sigma-1} = \tau^{1-\sigma} r_d(A). \tag{15}\]

The revenue of a firm depends on its export status and is given by

\[
r(A) = \begin{cases} 
  r_d(A) & \text{non exporting firms,} \\
  r_d(A) + r_x(A) = (1 + m \tau^{1-\sigma}) r_d(A) & \text{exporting firms.}
\end{cases}
\]

Following Melitz (2003), we assume that the export fixed cost \( f_{ex} \) is paid every period, which is tantamount to paying a discounted stream of these costs \( (f_x = f_{ex}/\delta) \) only once at the beginning of the export business. The profits of domestic sales and of foreign sales are then given by

\[
\pi_d = \frac{r_d(A)}{\sigma} - f, \quad \pi_x = \frac{r_x(A)}{\sigma} - f_x, \tag{16}
\]

and total profits amount to \( \pi(A) = \pi_d(A) + \max \left[ 0, \pi_x(A) \right] \). The domestic and export cut-off levels of productivity are given by \( A^* = \inf \left[ A : v(A) > 0 \right] \) and \( A_x^* = \inf \left[ A : A \geq A^* \right. \text{ and } \pi_x(A) > 0 \] , respectively, where it is apparent that \( A_x^* \geq A^* \). The cut-off levels of productivity can be determined via the conditions \( \pi_d(A^*) = 0 \) and \( \pi_x(A_x^*) = 0 \). The central underlying assumption is that \( \tau^{\sigma-1} f_x > f \) or \( A_x^* > A^* \) which ensures that no firms exist that only export and do not supply to domestic consumers. This means that export costs are sufficiently large such that the following partitioning
of firms occurs (Melitz, 2003; Helpman, 2006):

- $A < A^*$: firms exit,
- $A^* < A \leq A_x$: firms produce for the home market only,
- $A > A_x^*$: firms produce for home and export markets.

The probability of exporting conditional on successful entry, which is tantamount to the export share of a country, is a unique function of $A^*$:

$$
\Phi_x = \frac{1 - G(A^*_x)}{1 - G(A^*)} \left[ \frac{A^*}{A^*_x} \right]^{\frac{\alpha - \gamma}{\gamma \phi}}.
$$

(17)

Define $\tilde{A}_x$ as the average productivity of exporting firms and let $\tilde{A}$ be an index of average productivity of all firms competing in a country (domestic and foreign). Then average revenues consist of revenues due to domestic sales and revenues due to foreign sales $\overline{\pi} = r_d(\tilde{A}) + \Phi_x r_x(\tilde{A})$ such that average profits amount to $\overline{\pi} = \pi_d[\tilde{A}(A^*)] + \Phi_x \pi_x[\tilde{A}(A^*_x)]$. The cut-off productivity condition for firms requiring indifference between exporting and non-exporting is $\pi_x(A^*_x) = 0 \iff \pi_x(\tilde{A}_x) = f_x \kappa$. Using Equations (8), (15), and (16) we get

$$
A_x^* = \left( \frac{f_x}{\kappa} \right)^{\frac{1}{\sigma-1}} \tau A^*.
$$

(18)

implying that $A_x^*$ is a linear function of $A^*$. From the zero cut-off profit conditions of domestic producers and of exporters we get average profits as

$$
\overline{\pi} = \pi_d(\tilde{A}) + \Phi_x \pi_x(\tilde{A}_x) = f \kappa + \Phi_x m f_x \kappa.
$$

(19)

Again we have that the expected net present value of a successful firm must be zero due to free entry, that is, $v_e = [1 - G(A^*)] \bar{\pi}/\delta - f_e \frac{1}{\delta} = 0$, which is the case for $\bar{\pi} = \delta f_e/[1 - G(A^*)]$. Consequently, the free entry condition remains unchanged as compared to the closed economy case. Regardless of the profit differences across firms due to the export status, the expected value of future profits in equilibrium must be equal to fixed investment costs. The two Equations (18) and (19) determine $\bar{\pi}$ and $A_x^*$. Once $A^*$ is known from the closed economy solution, we get $A_x^*$, $\tilde{A}$, $\tilde{A}_x$, $1 - G(A^*)$, and $\Phi_x$. Recalling that $\kappa = (\alpha - \gamma)/[(1 - \sigma) \gamma \phi + \alpha - \gamma - 1]$ in case of our particular Pareto distribution,
we arrive at

\[ A_x^* = B \tau e^{\left(\gamma - \alpha\right)\sigma} \]  

(20)

\[ \bar{\pi} = \left[ \frac{(\sigma - 1)\gamma \phi}{(1 - \sigma)\gamma \phi + \alpha - \gamma} \right] f + mf_x \left( \frac{1}{B \tau} \right)^{\frac{\alpha - \gamma}{\sigma}} \]  

(21)

with \( B \equiv (f_x/f)^{\frac{1}{\sigma - 1}} \) and

\[ C \equiv \gamma \left[ \ln(\delta) - \ln \left( \frac{f \gamma (\sigma - 1) \phi}{\alpha - \gamma - \gamma (\sigma - 1) \phi} \right) + \ln (f_e) \right]. \]

At this stage we can state the two central results of our paper.

**Proposition 2.** For the open economy, an increase in the desire for fertility (\( \alpha \)) decreases average productivity and average profits, while an increase in the desire for education (\( \gamma \)) increases average productivity and average profits.

**Proof.** It can be shown that, as in autarky, \( \partial \bar{\pi}/\partial \alpha < 0 \) and \( \partial \bar{\pi}/\partial \gamma > 0 \). From Equations (14), (20), the expression \( (\bar{\pi}/f_e)^{\frac{\gamma \phi}{\alpha - \gamma}} = A^* \), and the fact that in this expression the base and the exponent both decrease with \( \alpha \) and increase with \( \gamma \) due to \( \alpha - \gamma > 0 \), it follows that the export cut-off productivity increases with the desire for education and decreases with the desire for having many children. Together with Proposition 1, this establishes that the same holds true for average productivity in an economy. \( \square \)

Let international competitiveness be defined by the export share, that is, the probability of the firms in a country to export (\( \Phi_x \)). We then observe the following result.

**Proposition 3.** An increase in the desire for fertility (\( \alpha \)) decreases international competitiveness, while an increase in the desire for education (\( \gamma \)) increases international competitiveness.

**Proof.** Substituting for \( A_x^* \) in the probability of exporting conditional on successful entry yields

\[ \Phi_x = \left[ \left( \frac{f_x}{f} \right)^{\frac{1}{\pi - \tau}} \right]^{\frac{\gamma \phi}{\alpha - \gamma}}. \]  

(22)

The derivative of this expression with respect to \( \alpha \) is negative, whereas the derivative with respect to \( \gamma \) is positive. \( \square \)
Recalling from Proposition 2 that firms are larger in terms of profits in economies in which parents prefer fewer and better educated children, we conclude from Proposition 3 that, ceteris paribus, the export share of GDP is increasing with education and declining with fertility.

The intuition for the results described in Propositions 2 and 3 is similar to the closed economy case: The average human capital stock is higher in an economy in which parents invest more in education of their children. This affects the distribution of productivity levels to the extent that it becomes more likely for a domestic firm to draw a high productivity level. Consequently, domestic firms will also be more competitive abroad.

2.6. Heterogeneous Countries. In order to prepare for the empirical analysis of, naturally, heterogeneous countries, we now give up the symmetry assumption and allow for differing home and foreign demand levels. Following Helpman et al. (2004), we denote by $R_x$ and $P_x$ revenues and prices in the rest of the world as being obtained by appropriate aggregation over all foreign countries. This means that export revenues of a domestic firm are given by

$$r_x(A) = R_x \left( \frac{P_x \rho A}{\tau} \right)^{\sigma-1} = \tau^{1-\sigma} \frac{R_x}{P} \left( \frac{P_x}{P} \right)^{\sigma-1} r_d(A).$$

(23)

Following the same steps of analysis as before we obtain

$$A_x^* = D \tau e^{\frac{C}{((\gamma - \alpha)\phi}},$$

(24)

$$\bar{\pi} = \left[ \frac{\alpha - \gamma}{\gamma \phi + \alpha - \gamma} \right] f + f_x \left( \frac{1}{D \tau} \right)^{\frac{\alpha - \gamma}{\gamma \phi}}$$

(25)

with $D \equiv (f_x R_x / f R)^{\frac{1}{\gamma - \alpha}} P_x / P$ and $C$ given as before. Aggregate revenues at home, $R$, decline in response to an increase in $\alpha$ and do not respond to changes in $\gamma$, such that, if anything, the previously obtained results for homogenous countries are reinforced for heterogeneous countries. However, the response of the domestic price level (and consequently also the responses of $A_x^*$ and $\bar{\pi}$) with respect to variation of $\alpha$ and $\gamma$ become, in general, indeterminate. If the price level decreases very strongly in response to increasing $\alpha$ or increases very strongly in response to increasing $\gamma$, the previously outlined mechanism could be reversed. It has been shown by Demidova (2008) that unilateral increases in productivity lead unambiguously to more exports when productivity is exogenous. In our case, when productivity changes are endogenously driven by education, there is also a potentially counterbalancing effect at work. The child quantity-quality trade-off implies that
rising education is associated with declining population growth. A smaller population, taken for itself, leads to the production of fewer varieties and to higher prices due to the well-known home-market effect. The general ambiguity implies that empirical analysis has to establish whether the following hypotheses are corroborated by the data: i) countries with a better educated population are more successful in exporting, ii) firms in countries with a better educated population are more profitable. Before we proceed to the empirical analysis, we show that the central reasoning also applies in an environment where countries do not differ in the preference parameters of their population.

3. Long-Run Demographic Change and International Competitiveness

The basic model relies on parametric changes in the desire for children and education in order to motivate the connection between fertility, education, and competitiveness. This is analytically convenient but may appear to be intellectually not fully convincing. It seems to be more desirable to elicit “the family connection” as an outcome of endogenous demographic and behavioral change based on stable preferences. In this section we extend the basic model in this direction. This allows, in the spirit of Unified Growth Theory (Galor, 2005), for an explanation of observable cross-country differences as outcome of a differentiated take-off to sustained growth. Countries displaying high fertility and low investments in education are conceptualized as being at an early stage of the demographic transition, whereas countries displaying low fertility and high education have already reached a later stage of the transition. Ceteris paribus, we will not only observe that forerunners of the demographic transition produce higher income per capita than latecomers — as e.g. in Galor and Weil (2000) — but also that forerunners are populated by on average more productive firms — as observed by e.g. Gollin (2008) — and that firms of forerunners are more likely to export.

In principle there are numerous possibilities to generate transitional dynamics of a demographic-economic model (see Galor, 2005 for an overview). Here we follow a “minimal-intensive” approach by mildly extending the basic model with informal education ($\bar{e}$), which could be thought of as skills acquired by children through observation of their parents and peers at work. Let $e_t$ continue to denote costly formal acquired education. In contrast to the basic model, we assume that formal education is costly in terms of income rather than in terms of parental time such that the budget constraint reads $wh_t(1 - \delta n_t) = Pch_t + e_t n_t$. The expenditure share of education per child is
assumed to translate one-to-one into human capital per member of the next adult generation. Taking into account the normalization \( w = 1 \), the modified utility function and the modified equation of motion for human capital read:

\[
\begin{align*}
    u &= \log (c_t \cdot h_t) + \alpha \log (n_t) + \gamma \log (e_t + \bar{e}) \\
    h_{t+1} &= \frac{e_t}{h_t} + \bar{e}.
\end{align*}
\]

Henceforth we focus on the interior solution for optimal consumption, education, and fertility which is given by

\[
\begin{align*}
    c_t &= \frac{1}{(1 + \alpha)\bar{P}}, \quad e_t = \frac{h_t \gamma \phi - \bar{e} \alpha}{\alpha - \gamma}, \quad n_t = \frac{(\alpha - \gamma)h_t}{(1 + \alpha)(h_t \phi - \bar{e})}
\end{align*}
\]

and preserves the quantity-quality trade-off from the simple model. Declining fertility is observed along with increasing education,

\[
    n_t = \frac{(h_t \gamma \phi - \bar{e} \alpha)h_t}{(h_t \phi - \bar{e})(1 + \alpha)} \cdot \frac{1}{e_t}.
\]

Additionally, however, human capital of the parent, \( h_t \), is now negatively associated with fertility and it is positively associated with the education of children. Notice that for \( \bar{e} = 0 \), or for \( h_t \to \infty \), the solutions for \( n_t \) and \( h_{t+1} \) coincide with the solutions of the simple model. Using again \( a = 1/h_t \), the cumulative Pareto distribution \( G(A) \) modifies to

\[
    G(A_{t+1}) = 1 - \left( \frac{1}{A_{t+1}} \right)^{\frac{\alpha - \gamma}{\gamma \phi - \alpha \bar{e} + (\alpha - \gamma)\bar{e}}}
\]

We see that a growing level of human capital gradually increases the probability for an entrepreneur to draw a high productivity.

We continue with the analysis analogous to Section 2. The detailed calculations are summarized in the Appendix. Unfortunately, the model can no longer be assessed analytically. The resulting system of equations that we solve numerically is given by Equations (A.11) to (A.15). For the benchmark run of the model we set parameter values and initial endowments as summarized in Table 1. After solving the model, we convert generations into years assuming that a generation lasts 25 years. We set the initial time \( t_0 \) to 1750.

The results of our numerical example are displayed in Figure 1 where we see the evolution of
Figure 1. Evolution of Human Capital and International Competitiveness

Solution trajectories of the extended model for 3 different specifications of countries. Solid lines: $h(0) = 0.38$; dashed lines: $h(0) = 0.37$; dash-dotted lines: $h(0) = 0.38$ and $\gamma = 0.399$. All other parameters values as specified in Table 1.

Central variables over a time span of 250 years: human capital in panel (a), average productivity in panel (b), profits per firm in panel (c), and the probability of exporting conditional on entry in panel (d). We have normalized the trajectories (a)-(c) by dividing through their respective initial values. Results for the benchmark specification are reflected by solid lines. Human capital is predicted to

Table 1. Parameter values for the numerical example

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>1</td>
<td>$f_e$</td>
<td>1.5</td>
</tr>
<tr>
<td>$f_e$</td>
<td>2</td>
<td>$\sigma$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\bar{e}$</td>
<td>0.05</td>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.4</td>
<td>$\phi$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.2</td>
<td>$\delta$</td>
<td>0.1</td>
</tr>
<tr>
<td>$m$</td>
<td>190</td>
<td>$h_0$</td>
<td>0.38</td>
</tr>
</tbody>
</table>
increase by a factor of 1.8 from 1750 to 2000. Using a Mincer equation \( h_t = \exp(\theta s_t) \), the predicted increase is associated with an increase from 3 to 12 years of schooling when the return on schooling \((\theta)\) is 0.07. This prediction is roughly in line with the historical schooling trends observed for the Western countries (Baier et al., 2006).

Altogether the dynamic behavior of the economy is consistent with the comparative static behavior obtained for the basic economy of Section 2. This means that the association between the level of human capital and fertility on the one hand, and firm’s profits and average productivity on the other hand, are preserved: In economies richer in human capital, the fertility rate is lower and average profits per firm and average productivity are higher.

Notice that there are now two ways to explain cross-country differences in education, firm productivity, and international competitiveness. Countries may share the same fundamentals but differ in their initial endowments and thus in the timing of the take-off to growth, or they may differ in their fundamentals. This is demonstrated in Figure 1 by comparing the benchmark economy with two alternative economies. One economy shares all the fundamentals and starts with lower endowment of human capital (dashed lines). The other economy shares the same initial conditions but puts less weight on education (dash-dotted lines). Asymptotically, human capital, productivity, and international competitiveness of economies sharing the same fundamentals will converge (solid and dashed lines), while both economies outperform the fundamentally different economy (solid and dashed vs. dash-dotted lines). In any case, irrespective of the cause of contemporaneous cross-country differences in human capital, productivity and competitiveness are higher in the country that displays higher human capital and lower fertility rates.\(^1\)

4. Empirical Analysis

Our analyses in Sections 2 and 3 have elaborated why the export share and the profitability of firms are positively associated with the level of education. A first glance at the validity of these implications across countries is provided by Figure 2. The two panels of the figure show the evolution of the export share and average productivity as measured by GDP per worker together with the evolution of tertiary education for a population-weighted OECD average. We see that the \(^1\)Notice that — as a side effect — the model captures also the stylized fact that the number of firms in an economy declines with economic development. The mechanism is based on imperfect competition, international specialization, and firm productivity enhanced by education. It complements existing explanations based on factor supply and norm enforcement (Lucas, 1978; Gollin, 2008; Lindner and Strulik, 2011).
export share and firm productivity increase with rising education. This pattern is consistent with Propositions 2 and 3 and the dynamic behavior investigated in Section 3. Furthermore, Figure 3 shows the scatterplots of the logarithm of a country’s export share against the logarithm of its average years of tertiary education (left picture) and the logarithm of GDP per worker against the logarithm of average years of tertiary education (right picture). The simple correlation also indicates support of predictions i) and ii), that is, there appears to be a positive relationship between higher education and competitiveness as well as productivity.

In order to analyze these patterns in more detail, we constructed a panel data-set for all OECD countries from 1960 to 2010 evaluated in five year steps using the World Development Indicators, the Penn World Tables of Heston et al. (2011), and the Education Statistics and the Enterprise Survey of the World Bank (2011). The data-set comprises export shares, GDP per worker and GDP per firm as productivity measures, birth rates as indicators for demographic developments, and mean years of tertiary education of the population above the age of 15 as an indicator for human capital. The focus on tertiary education is motivated by the notion that the productivity of firms is most heavily influenced by the presence of well educated managers, scientists, and engineers.

We use population size, per capita GDP, and real consumption per capita to control for country
size and income differences, and the investment shares of GDP to control for differences in capital intensity. Due to the panel structure of our data set we are also able to control for country-specific effects that remain constant over time, like geographical and cultural characteristics, as well as for time-varying effects that impact upon all countries in a similar vein, like changes in technology, in particular, transport technology\(^2\). This is in contrast to Galor and Mountford (2006, 2008) who employ a cross-country regression and therefore cannot control for country-specific characteristics.

Ideally, we would like to test the theory using data for GDP per firm as an indicator of firm profitability. But we do not have data for this variable prior to the year 2000, a drawback that reduces the size of our sample considerably. We thus use GDP per worker as an alternative proxy for firm profitability. This approach seems to be justified by the observation of a high correlation between GDP per firm and GDP per worker — a simple regression provides an adjusted \(R^2\) of more than 0.6 and a highly significant positive coefficient estimate.

To examine the validity of our theoretical implications in more detail, we estimated specifications

\(^2\)The most important geographical predictors of trade are distance to trading partners and whether a country is landlocked. Note that we analyze OECD countries between which barriers to technology adoption and diffusion are presumably less severe than between developed and developing countries.
of the form

\[
\text{Export}_{i,t} = \beta_1 ayts_{i,t-1} + \sum_{k=2}^{K} \beta_k \text{cont}_{i,t-1,k} + \epsilon_i + \psi_t + u_{i,t}, \tag{29}
\]

\[
\text{pfGDP}_{i,t} = \beta_1 ayts_{i,t-1} + \sum_{k=2}^{K} \beta_k \text{cont}_{i,t-1,k} + \epsilon_i + \psi_t + u_{i,t}, \tag{30}
\]

\[
\text{pwGDP}_{i,t} = \beta_1 ayts_{i,t-1} + \sum_{k=2}^{K} \beta_k \text{cont}_{i,t-1,k} + \epsilon_i + \psi_t + u_{i,t}, \tag{31}
\]

where the dependent variable is either the logarithm of the export share (Export), the logarithm of GDP per firm (pfGDP), or the logarithm of GDP per worker (pwGDP) in country \(i\) at time \(t\), \(ayts_{i,t-1}\) refers to the logarithm of mean years of tertiary education of the population above the age of 15 in country \(i\) at time \(t - 1\), \(\text{cont}_{i,t-1,k}\) refers to the control variable \(k \in [2, K]\) for country \(i\) at time \(t - 1\), these are, the logarithm of per capita GDP (pcGDP), the logarithm of the investment share (invest), and the logarithm of the population size (popsize). In an attempt to reduce endogeneity problems, the variables on the right hand side are lagged by one period (five years). The coefficient estimates we are interested in are the \(\beta_1\)'s, while the coefficient estimates for \(\beta_k\) refer to the marginal effects of the control variables. Each equation contains country fixed effects, \(\epsilon_i\), and time fixed effects, \(\psi_t\), while the error term is denoted by \(u_{i,t}\) and is assumed to have mean zero and a diagonal variance-covariance matrix.

Table 2 contains the results of the regressions specified in Equations (29)-(31). The positive association between the export share and average years of tertiary education is significant at the 5\% level. If we use GDP per firm as dependent variable, we estimate a positive association with average years of tertiary education but the coefficient is not statistically significant. This result is not surprising considering the small sample of 93 observations available for the estimation of 46 coefficients. When we use alternatively GDP per worker we have 223 observations for our estimation. The positive association between productivity per worker and average years of tertiary education is then significant at the 10\% level. Altogether, these results are consistent with Propositions 2 and 3 and the two predictions i)-ii).

Of course, OLS regressions of this type do not establish causation. It could very well be the case that richer countries are more productive and more successful in exporting and therefore have more resources available to invest in schooling. However, the quality-quantity trade-off apparent
Table 2. OLS estimation results for the regressions specified in Equations (29)-(31)

<table>
<thead>
<tr>
<th></th>
<th>Exports (OLS)</th>
<th>pfGDP (OLS)</th>
<th>pwGDP (OLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ayt</td>
<td>0.198</td>
<td>0.012</td>
<td>0.134</td>
</tr>
<tr>
<td></td>
<td>(0.073)**</td>
<td>(0.189)</td>
<td>(0.068)*</td>
</tr>
<tr>
<td>popsize</td>
<td>0.492</td>
<td>0.739</td>
<td>-0.322</td>
</tr>
<tr>
<td></td>
<td>(0.297)</td>
<td>(0.961)</td>
<td>(0.249)</td>
</tr>
<tr>
<td>pcGDP</td>
<td>-0.279</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>invest</td>
<td>0.267</td>
<td>0.018</td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td>(0.105)**</td>
<td>(0.287)</td>
<td>(0.083)</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.26</td>
<td>0.77</td>
</tr>
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<td>OBS</td>
<td>289</td>
<td>93</td>
<td>223</td>
</tr>
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<td>country fe</td>
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<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>time fe</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Regressors are lagged by one time period. Standard errors are reported below the coefficient estimates in parentheses. One asterisk indicates significance at the 10% level, two asterisks indicate significance at the 5% level and three asterisks indicate significance at the 1% level. $R^2$ denotes the fraction of within variation explained by the corresponding model, and OBS stands for the number of observations.

from Equation (3) and Propositions 1-3 implies that we have a powerful instrument to address the potential endogeneity of education, namely fertility. If we want to use fertility as an instrument, it has to fulfill two central requirements: i) being strongly correlated to the variable that we aim to instrument, which is suggested by our theory and substantiated by the tests for weak identification as discussed below, ii) fulfilling the exclusion restriction, i.e., it must not affect the dependent variable through channels that we do not control for in our specification. Potentially there are two other channels by which fertility could have affected the export share and the productivity of a country: i) its effect on the population size and therefore on the scale of the economy which could lead to more R&D and thereby faster productivity growth (see Romer, 1990), ii) its effect on savings decisions and therefore the capital intensity of an economy which also affects productivity levels (see Solow, 1956). Consequently, when using fertility as an instrument for education, we control for the size of the population and for the investment share in an attempt to control for the two channels that we mentioned.

A potential source of reverse causality is implied by the fact that fertility decreases as the economy develops because of the increases in household income. We therefore also control for
real per capita GDP in the specification for the export share and for real per capita consumption (\textit{cons}) in all specifications. The rationale for the latter is given by the expression for consumption in Equation (28), which establishes a linear relationship between real income and real consumption expenditures. To see this recall that consumption \( c \) is measured in units of human capital and that wages per unit of human capital were normalized to unity. Furthermore, we lag the instrument by another time period such that fertility altogether appears in the first-stage regression as lagged by two time periods (ten years).

The results are displayed in Table 3, where the first three columns contain the IV estimation results, while the other columns contain the corresponding first-stage regressions. In these first-stage regressions we see that the birth rate is negatively related to education, as predicted by our theory. The Kleibergen-Paap test statistic (\textit{Widstat}) is relatively large suggesting that we are not facing a serious weak instrument problem when using fertility as an instrument for education. Furthermore, the Kleibergen-Paap test statistic on underidentification rejects underidentification in these two specifications on the 1\% level. Since our endogenous variable is exactly identified by following the outlined procedure, we do not report the test statistic for overidentification. Altogether we see that the coefficient estimates for education become highly significant and that they increased substantially in size. Our central results are therefore reinforced by the IV regressions, which suggest that the channel we identified in Propositions 2 and 3 and the two predictions i)-ii) is indeed causal.

As a robustness check we used firm level data for a sample of developing countries collected by Gennaioli et al. (2013) and analyzed whether the education level of managers has a positive association with firm size, productivity, and success in exporting. Table 4 contains the results, where the dependent variables are the export share, the logarithm of value added, and the logarithm of employment, each belonging to firm \( i \) in region \( j \). We control for the inverse distance of a firm to the coast (\textit{invdist}), population size (\textit{popsize}), whether a firm is owned by foreigners (\textit{fgn}), and geographical influences by means of the temperature (\textit{temp}). The treatment variable is years of schooling of the manager of a firm. We see that it is positively associated with all three dependent variables at the 1\% significance level. This result further substantiates the claim that education is an important explanatory variable with regards to international competitiveness and productivity.
Table 3. IV estimation results for the regressions specified in Equations (29)-(31)

<table>
<thead>
<tr>
<th></th>
<th>Exports (IV)</th>
<th>pwgdp (IV)</th>
<th>aytst (1st-stage)</th>
<th>aytst (1st-stage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>aytst</td>
<td>0.587</td>
<td>0.453</td>
<td>-0.471</td>
<td>-0.524</td>
</tr>
<tr>
<td>(0.228)**</td>
<td>(0.187)**</td>
<td></td>
<td>(0.133)**</td>
<td>(0.132)**</td>
</tr>
<tr>
<td>birth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>birth</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(0.137)**</td>
<td>(0.132)***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>popsize</td>
<td>0.349</td>
<td>-0.318</td>
<td>0.236</td>
<td>0.204</td>
</tr>
<tr>
<td>(0.157)**</td>
<td>(0.170)*</td>
<td></td>
<td>(0.397)</td>
<td>(0.384)</td>
</tr>
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<td>pcGDP</td>
<td>-0.429</td>
<td>0.403</td>
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<tr>
<td>(0.172)**</td>
<td>(0.173)**</td>
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<td>invest</td>
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<td>0.115</td>
<td>-0.097</td>
<td>0.072</td>
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<td>(0.113)</td>
<td>(0.083)</td>
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<td>(0.139)</td>
<td>(0.097)</td>
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<td>cons</td>
<td>-0.161</td>
<td>-0.002</td>
<td>0.073</td>
<td>0.114</td>
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<tr>
<td>(0.085)*</td>
<td>(0.076)</td>
<td></td>
<td>(0.136)</td>
<td>(0.147)</td>
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<tr>
<td>Widstat</td>
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<td>19.25</td>
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<tr>
<td>Idstat</td>
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<td>15.09</td>
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<tr>
<td>F</td>
<td>64.31</td>
<td>53.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBS</td>
<td>264</td>
<td>211</td>
<td>264</td>
<td>264</td>
</tr>
<tr>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>time fe</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Regressors are lagged by one time period, while the instrument, the birth rate, is lagged by two time periods. Standard errors are reported below the coefficient estimates in parentheses. One asterisk indicates significance at the 10% level, two asterisks indicate significance at the 5% level and three asterisks indicate significance at the 1% level. Widstat denotes the Kleibergen-Paap F-statistic for the test on weak identification, Idstat is the Kleibergen-Paap F-statistic on underidentification, F refers to the F-statistic of the first-stage regression, and OBS stands for the number of observations.

5. Conclusions

In this paper we analyzed the implications of human capital accumulation for productivity and international competitiveness of domestic firms by augmenting a state-of-the-art international trade model with an explicit fertility and education decision of households. In light of the ongoing debate on losing competitiveness due to demographic change (World Economic Forum, 2011), our results indicate that some of the fears may be exaggerated. Taking the quantity-quality trade-off into account, our theory suggests that countries with lower fertility will also have a better educated work force. This in turn fosters individual and firm productivity, which increases the likelihood that a firm is internationally competitive. As a consequence, such a firm will be more successful in export markets. Empirical evidence for the OECD countries over the time span 1960 to 2010
Table 4. Firm Exports, Productivity and Manager Education

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>export share</th>
<th>(log) value added</th>
<th>(log) employment</th>
</tr>
</thead>
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<tr>
<td>education of manager</td>
<td>2.153</td>
<td>0.269</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>(0.548)***</td>
<td>(0.044)***</td>
<td>(0.027)***</td>
</tr>
<tr>
<td>invdist</td>
<td>71.295</td>
<td>-7.421</td>
<td>1.937</td>
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<tr>
<td></td>
<td>(19.172)***</td>
<td>(3.974)*</td>
<td>(0.669)***</td>
</tr>
<tr>
<td>popsize</td>
<td>9.745</td>
<td>-1.225</td>
<td>0.355</td>
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<tr>
<td></td>
<td>(2.735)***</td>
<td>(0.471)***</td>
<td>(0.113)***</td>
</tr>
<tr>
<td>fgn</td>
<td>0.229</td>
<td>0.016</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(0.044)***</td>
<td>(0.003)***</td>
<td>(0.001)***</td>
</tr>
<tr>
<td>temp</td>
<td>-1.459</td>
<td>0.278</td>
<td>-0.137</td>
</tr>
<tr>
<td></td>
<td>(1.034)</td>
<td>(0.200)</td>
<td>(0.027)***</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.20</td>
<td>0.19</td>
<td>0.30</td>
</tr>
<tr>
<td>$OBS$</td>
<td>6.312</td>
<td>6.314</td>
<td>6.314</td>
</tr>
</tbody>
</table>

Firm level data from Gennaioli et al. (2013). Standard errors in parentheses. One asterisk indicates significance at the 10% level, two asterisks indicate significance at the 5% level and three asterisks indicate significance at the 1% level.

supports the theoretical findings by indicating a positive association between the export share of a country and the average years of tertiary education of its population. This association is even stronger when instrumenting education with fertility.

The focus of our analysis has been on one particular channel by which demographic change influences productivity and competitiveness, namely, on the child quantity-quality trade-off. This does, of course, not mean that we deny other important and potentially unfavorable implications of demographic change that have already been analyzed extensively in the literature like, for example, the pressures on social security and retirement systems or the increase of the dependency ratio. Here we wanted to stress that there is also a bright side of demographic change. For that purpose we focused on productivity and competitiveness and made no claims regarding welfare. As shown in the Appendix, welfare effects resulting from fertility change are indeed ambiguous. This is so because the well-known market size effect (Krugman, 1980; Melitz, 2003) operates in the opposite direction and may outweigh the positive effects of increasing firm productivity.

In a related study (Strulik et al., 2013), we integrated the child quantity-quality trade-off into an R&D-based growth model and obtained a similarly optimistic outlook for a future of declining
birth rates. There, the driving factor was the human capital allocated for R&D and the associated knowledge spillovers in this activity. Combining this approach with the theory presented in the present paper is a challenging task for future research which may help to further identify the determinants of firm productivity in modern trade theory.

Acknowledgments

We would like to thank Volker Grossmann, Bernhard Hammer, and Astrid Krenz for valuable comments and suggestions.

Appendix

5.1. Aggregate Revenues and Profits. Aggregate revenues and aggregate profits can be written as, respectively,

\[ R = \int_0^\infty r(A) M \mu(A) dA = \int_0^\infty r(\tilde{A}) \left( \frac{r(A)}{r(\tilde{A})} \right) M \mu(A) dA \]

\[ = r(\tilde{A}) M, \quad (A.1) \]

\[ \Pi = \int_0^\infty \pi(A) M \mu(A) dA = M \left( \frac{r(\tilde{A})}{\sigma} - f \right), \quad (A.2) \]

by using \( r(A)/r(\tilde{A}) = \left( A/\tilde{A} \right)^{\sigma-1} \) and \( \int_0^\infty A^{\sigma-1} \mu(A) dA = \tilde{A}^{\sigma-1}. \) In equilibrium the number of entrants (denoted with a subscript \( e \)) is equal to the number of exiting firms such that

\[ [1 - G(A^*)] M_e = \delta M \quad (A.3) \]

\[ \left( \frac{1}{A^*} \right)^{\frac{\alpha-1}{\sigma}} M_e = \delta M, \quad (A.4) \]

where \( M_e \) denotes the mass of entrants. Labor market clearing requires

\[ L - L_z = L_p + L_e, \]

where \( L_z \) is the labor cost of production of the homogeneous good, \( L_p \) is the labor cost of CES production and \( L_e \) the labor cost of entry into CES production. Note that the same allocation between sectors holds for human capital because it is embodied and individual human capital does not depend on the sector in which a worker supplies her skills. The aggregate wage bill in the CES sector must be equal to revenue minus profits

\[ L_p = R - \pi \quad (A.5) \]

and the following holds for the labor cost of entry

\[ L_e = M_e f_e = \frac{\delta M}{1 - G(A^*)} f_e = M \pi = \Pi. \quad (A.6) \]
Thus, for aggregate revenues we have
\[ R = L_p + \Pi = L_p + L_e = \frac{1 - \eta}{1 + \alpha} L \tag{A.7} \]
where \(1 - \eta\) corrects for the budget (and the labor) allocated to the production of the homogeneous good.

5.2. **Aggregate Price Index.** Denoting the probability of successful entry by \(\mu(A)\), the price index \(P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/1-\sigma}\) can be rewritten as
\[ P = \left[ \int_0^\infty \left[ \frac{1}{\rho A} \right]^{1-\sigma} M\mu(A) dA \right]^{\frac{1}{1-\sigma}} = M^{\frac{1}{1-\sigma}} \frac{1}{\rho A} \]
with
\[ \tilde{A} = \left[ \int_0^\infty A^{\sigma-1} \mu(A) dA \right]^{\frac{1}{\sigma-1}} \]
being a weighted geometric mean of productivities and therefore tantamount to average productivity. The price index becomes \(P = M^{\frac{1}{1-\sigma}} p(\tilde{A})\) after applying \(p(A) = 1/(\rho A)\).

5.3. **Aggregate Output.** Aggregate output can be written as
\[ Q = \left[ \int_0^\infty q(A)^\rho M\mu(A) dA \right]^{\frac{1}{\rho}} = M^{\frac{1}{\rho}} \left[ \int_0^\infty q(\tilde{A})^{\rho} \left( \frac{q(A)}{q(\tilde{A})} \right)^{\rho} \mu(A) dA \right]^{\frac{1}{\rho}} = M^{\frac{1}{\rho}} q(\tilde{A}) \]
after making use of \(\left[ \int_0^\infty A^{\sigma-1} \mu(A) dA \right]^{\frac{1}{\sigma-1}} = \tilde{A}^{\sigma}\).

5.4. **The Mass of Producing Firms and the Price Index in Equilibrium.** The mass of producing firms in equilibrium (normalized by individual human capital) follows from \(\pi = \bar{\pi}/\sigma - f\) as
\[ M = \frac{R}{\bar{\pi}} = \frac{(1 - \eta)L}{(1 + \alpha)(\bar{\pi} + f)\sigma} = \frac{[\alpha - \gamma - \gamma(\sigma - 1)\psi](1 - \eta)L}{(1 + \alpha)(\alpha - \gamma)\sigma f}. \]
Multiplying both sides by individual human capital, updating for one time period and using \(H_{t+1} = h_{t+1} b_t L_t = \gamma/(1 + \alpha) L_t\) we can rewrite the equilibrium mass of producing firms in the next period (non-normalized) as
\[ h_{t+1} M_{t+1} = \frac{[\alpha - \gamma - \gamma(\sigma - 1)\psi](1 - \eta)\gamma L_t}{(\alpha - \gamma)(1 + \alpha)^2 \sigma f}. \]
The derivatives of this expression with respect to the desire of parents for the number of children and their education are, respectively,
\[ \frac{\partial h_{t+1} M_{t+1}}{\partial \alpha} = \frac{\gamma(\eta - 1) \left[ 2(\alpha - \gamma)^2 + \gamma(2\gamma - 3\alpha - 1)(\sigma - 1)\psi \right] L_t}{\sigma f(1 + \alpha)^3(\alpha - \gamma)^2 \sigma}, \]
\[ \frac{\partial h_{t+1} M_{t+1}}{\partial \gamma} = -\frac{(\eta - 1) \left[ 2(\alpha - \gamma)^2 + \gamma(\gamma - 2\alpha)(\sigma - 1)\psi \right] L_t}{\sigma f(1 + \alpha)^2(\alpha - \gamma)^2 \sigma}. \]
Both of these expressions have an ambiguous sign.
It increases in human capital of generation $t$ and the distribution $G(A)$ is obtained as

$$G(A_{t+1}) = 1 - \left( \frac{1}{A_{t+1}} \right)^{\frac{\alpha - \gamma}{\gamma \phi - \bar{c} + (\alpha-\gamma)e}},$$

$$g(A_{t+1}) = G'(A_{t+1}) = \frac{\alpha - \gamma}{\gamma \phi - \bar{c} + (\alpha-\gamma)e} A_{t+1} - \left( 1 + \frac{\alpha - \gamma}{\gamma \phi - \bar{c} + (\alpha-\gamma)e} \right).$$

The expected productivity level that a firm draws at time $t+1$ is

$$E(A_{t+1}) = \int_1^{\infty} A_t g(A_t) dA_t = \frac{\alpha - \gamma}{\alpha - \gamma - \phi - \bar{c} + (\alpha-\gamma)e}.$$ 

It increases in human capital of generation $t$. The reason is that parents with higher wages and higher human capital are more likely to invest in education of their offspring, implying that the mean of the Pareto distribution shifts outward.

The distribution of productivity conditional on entry is

$$\mu(A_{t+1}) = g(A_{t+1}) / G(A_{t+1}) = \frac{\alpha - \gamma}{\gamma \phi - \bar{c} + (\alpha-\gamma)e} \left( A^*_t \right)^{\frac{\alpha - \gamma}{\gamma \phi - \bar{c} + (\alpha-\gamma)e} - 1} A_{t+1} - \left( 1 + \frac{\alpha - \gamma}{\gamma \phi - \bar{c} + (\alpha-\gamma)e} \right),$$

and the probability of exporting conditional on entry is

$$\Phi_{x,t+1} = \frac{1 - G(A^*_t)}{1 - G(A^*)} = \left[ \frac{A^*_t}{A^*} \right]^{\frac{\alpha - \gamma}{\gamma \phi - \bar{c} + (\alpha-\gamma)e}}.$$

Defining average productivity as

$$\tilde{A}_{t+1} = \left[ \int_{A^*_t}^\infty A_{t+1}^{\sigma-1} \mu(A_{t+1}) dA_{t+1} \right]^{\frac{1}{\sigma-1}}$$

provides:

$$\tilde{A}_{t+1} = \left\{ \frac{\alpha - \gamma}{(1 - \sigma) \left[ \gamma \phi - \bar{c} + (\alpha-\gamma)e \right] + \alpha - \gamma} \right\}^{\frac{1}{\sigma-1}} A_{t+1}^{\sigma-1}.$$ (A.8)

Following the steps of the analysis for the basic model we obtain for the closed economy

$$\pi_{t+1} = \frac{f(\sigma - 1) \left( h_t (\gamma \phi + \bar{c} (\alpha-\gamma)) - \alpha \bar{c} \right)}{\alpha \bar{c} (\sigma - 1) + h_t (-\gamma + \alpha (\bar{c} (-\sigma) + \bar{c} + 1) + \gamma (\sigma - 1) (\bar{c} - \phi))}$$

$$A^*_t = \left\{ \frac{f(\sigma - 1) \left( h_t (\gamma \phi + \bar{c} (\alpha-\gamma)) - \alpha \bar{c} \right)}{\delta f_h \left[ \alpha \bar{c} (\sigma - 1) + h_t (-\gamma + \alpha (\bar{c} (-\sigma) + \bar{c} + 1) + \gamma (\sigma - 1) (\bar{c} - \phi)) \right]} \right\}^{\frac{\gamma \phi - \bar{c} + (\alpha-\gamma)e}{(\alpha-\gamma)e + \bar{c} + 1}}.$$ (A.9)

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For the derivation of the dynamic system of the open economy first note that human capital evolves according to

$$h_{t+1} = \frac{\gamma \phi - \bar{e} \alpha / h_t + (\alpha - \gamma) \bar{e}}{\alpha - \gamma}.$$  \hfill (A.11)

By making use of equations (18) and (19), we can derive the following expressions from the solutions (A.9) and (A.10) of the closed economy

$$\pi_{t+1} = \frac{(\sigma - 1)(\alpha \bar{e} - h_t(\gamma \phi + \bar{e}(\alpha - \gamma)))}{h_t(\gamma - 1)(\phi + (\alpha - \gamma)(\bar{e}(\sigma - 1) - 1)) - \alpha \bar{e}(\sigma - 1)} \left\{ m f_x \left[ \frac{(f_x)^{\frac{1}{\sigma - 1}}}{\bar{f}} \frac{(\alpha - \gamma) h_t}{h_t(\gamma \phi + \bar{e}(\alpha - \gamma))} \right] f \right\} + f,$$

$$A^*_{x,t+1} = \tau \left( \frac{f_x}{f} \right)^{\frac{1}{\alpha - \gamma}} \times$$

$$\left\{ \frac{f(\sigma - 1)(h_t(\gamma \phi + \bar{e}(\alpha - \gamma)) - \alpha \bar{e})}{\delta f_x (\alpha \bar{e}(\sigma - 1) + h_t(-\gamma + \alpha(\bar{e}(\sigma) + \bar{e} + 1) + \gamma(\sigma - 1)(\bar{e} - \phi)))} \right\} \frac{\gamma \phi - \bar{e} \alpha / h_t + (\alpha - \gamma) \bar{e}}{\alpha - \gamma}. \hfill (A.13)$$

These expressions refer to the evolution of profits per firm and cut-off productivity in the open economy. Using the equations (A.8) and (18), we can then calculate average productivity in the open economy as

$$\bar{A}_{t+1} = \frac{A^*_{x,t+1}}{\tau} \left\{ \frac{f}{f_x (1 - \sigma)} \left[ \gamma \phi - \bar{e} \alpha / h_t + (\alpha - \gamma) \bar{e} + (\alpha - \gamma) \right] \right\}^{\frac{1}{\sigma - 1}}.$$

Finally, the probability of exporting conditional on entry follows immediately from equation (17) as

$$\Phi_{x,t+1} = \left[ \frac{(f_x)^{\frac{1}{\sigma - 1}}}{\gamma \phi - \bar{e} \alpha / h_t + (\alpha - \gamma) \bar{e}} \right]. \hfill (A.15)$$

References


Becker, G. S. and Lewis, H. G. (1973). On the interaction between the quantity and quality of


