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## Modelling the interaction between flooding events and economic growth

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# Modelling the interaction between flooding events and economic growth



by

**Johanna Grames**

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# Modelling the interaction between flooding events and economic growth

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## Abstract

Recently socio-hydrology models have been proposed to analyse the interplay of community risk-coping culture, flooding damage and economic growth.

These models descriptively explain the feedbacks between socio-economic development and natural disasters such as floods. Complementary to these descriptive models, we develop a dynamic optimization model, where the inter-temporal decision of an economic agent interacts with the hydrological system. We assume a standard macro-economic growth model where agents derive utility from consumption and output depends on physical capital that can be accumulated through investment. To this framework we add the occurrence of flooding events which will destroy part of the capital.

We identify two specific periodic long term solutions and denote them rich and poor economies. Whereas rich economies can afford to invest in flood defence and therefore avoid flood damage and develop high living standards, poor economies prefer consumption instead of investing in flood defence capital and end up facing flood damages every time the water level rises. Nevertheless, they manage to sustain at least a low level of physical capital. We identify optimal investment strategies and compare simulations with more frequent and more intense high water level events.

Keywords: flood socio-hydrology dynamic optimization investment strategy

# 1 Introduction

Since the beginning of time, people have settled close to rivers and this is still the case nowadays. Rivers enable ways of transport, supply water for industry and agriculture and enhance the quality of living due to lively nature and beautiful scenery. However, living close to rivers also involves the risk of flooding, one of the most devastating natural threats on Earth (Ohl and Tapsell, 2000), whose impact has increased over the past decades in many regions of the world (Dankers et al., 2014, Hall et al., 2014). In order to avoid flood damage, societies have developed projects involving structural defences (e.g., dams, levees, retention basins) and non-structural measures (e.g., land-planning, insurance, forecasting, see e.g. Kundzewicz (2002)). These investments are costly, but may avoid damage in the future. This is an interesting dynamic trade-off structure which we aim to analyse in a stylized socio-hydrological model that is embedded in a macroeconomic set up. To account for the dynamic nature of optimal investment strategies, we apply dynamic optimization methods.

Floods and their consequences have been studied with different model approaches: Recent Integrated Assessment Models (IAM) aim to understand the interaction of society and floods (Merz et al., 2014) in a broad context. Climate change leads to more and bigger floods in certain regions. Such models typically do not account for the impact of changes in the environment on economic growth (Estrada et al., 2015). The aim of Agent Based Models (ABM) such as Dawson et al. (2011), Safarzyńska et al. (2013) and Li et al. (2015) is to understand the impact of floods on individual behaviour. ABMs can provide a qualitative analysis of the consequences of floods on different levels: the individual/micro-level, the aggregated economy/macro-level and the firm level/meso-level. Complementary Input-Output-Models (Hasegawa Ryoji, Koks et al., 2014) provide a quantitative cost-benefit-analysis of case studies. Okuyama analysed these model frameworks as well as computational equilibrium models for disasters. A dynamic spatial computable general equilibrium model based on the dynamic structure of a Ramsey growth model was developed by Nakajima et al. (2014) to numerically measure flood damage costs. It displays the dynamic tradeoff between the costs today and future savings, investments and consumption. Besides simulation modelling approaches, optimization models have been developed to calculate optimal dike heights (Brekelmans et al., 2012, Chahim et al., 2012, 2013, Eijgenraam, 2006).

So far, floods have been rarely analysed in a macroeconomic model of economic growth considering not only direct and indirect damage costs, but also loss of future potential economic growth through dynamic consumption and investment decisions.

In environmental economics this approach is quite common. Economic growth models have been applied to study, e.g., the effect of climate change on long run economic growth (Xepapadeas et al., 2005). More formally, these models commonly postulate that pollution causes economic losses via a damage function that is positively related to an increasing temperature caused by pollution. (Millner and Dietz, 2015, Morisugi and Mutoh, 2012, Rezai et al., 2014). Pollution itself is commonly modeled via the flow or stock of emissions. Indeed, emissions and investment in emission abatement have strong analogies to extreme water events (floods, droughts) and investment in abatement (flood defence capital, reservoirs), respectively. It therefore seems an obvious choice to apply this modelling framework also in the context of flood modelling. Similar to the increase in the temperature that underlies the economic damage in climate change models, the water level underlies the occurrence of floodings and hence the economic damage. There is a new research line, socio-hydrology, that deals with such coupled systems. The main thrust of socio-hydrology is to add a new perspective to former models and studies in hydrology by coupling dynamics of human populations, economic growth and general resource availability

(Sivapalan et al., 2012). Socio-hydrology aims at understanding emergent patterns and paradoxes that result from long-term co-evolution of non-linearly coupled human-water systems. Elshafei et al. (2014) and Sivapalan and Blöschl (2015) developed prototype frameworks for socio-hydrology models. Di Baldassarre et al. (2013) and Viglione et al. (2014) developed a socio-hydrology model to explain the feedbacks between settlements close to rivers and flooding events. Di Baldassarre et al. (2015) use the model to capture processes such as the “levee effect” (e.g., Montz and Tobin (2008)) and the “adaptation effect” (IPCC, 2012, Mechler and Bouwer, 2014, Penning-Rowsell, 1996), which traditional flood risk models do not include. Pande et al. (2014) were one of the first who added a water related problem to a standard economic model of finitely lived agents, the so called overlapping-generations model (OLG). In this paper, we build a macro-economic model in the context of floods and use a dynamic optimization model which is a different perspective from the more common descriptive models, simulations and scenario analyses.

Our model uses the model of Di Baldassarre et al. (2013) and Viglione et al. (2014) as a starting point. Their simulations show that building high levees leads to fewer flooding events with higher impacts which may slow down economic growth. Protecting a settlement by levees can, however, increase the damage to downstream settlement due to the loss of flood retention volume. Furthermore, building levees or any other defence capital will lower flood risk and may therefore increase the willingness of citizens to build close to the river. If water levels rise higher than the crest of the levees, the physical capital next to the river is destroyed. Since there is a higher physical capital stock next to the river, the flood hits even harder on the economy. Based on their model set up we build an economic model to analyse the tradeoffs and feedbacks associated with settlements close to rivers. We assume two types of capital: physical capital and defence capital. Decision makers can invest in physical capital, such as machines, buildings and infrastructure. On the other hand, investments in defence capital can avoid the actual damage of floods and have thereby a positive influence on output. Total output of the economy consequently depends on both capital stocks. We apply a periodic non-autonomous exogenous function to represent the water level. The periodic water function is introduced in Grames et al. (2015). Even though the assumption of non-stochastic flood occurrence is a strong one, we believe that useful insights on the system can still be obtained. Assuming the periodic non-autonomous exogenous function for flood occurrence allows us to solve the dynamic optimization problem, for which we further develop the solution method of Moser et al. (2014) where a similar mathematical problem in the context of renewable energy has been solved.

In this paper, we aim to understand the mechanisms behind investment decisions in the context of flood prevention. For this purpose we choose a stylized macro-economic model to investigate the optimal investment strategy between flood protection measures and physical capital to enable economic growth.

The remainder of the paper is organized as follows. The following section provides an introduction to the feedbacks between society and floods and outlines the model framework and its equations. In a first step we present various simulations of our model and show the sensitivity of the resulting dynamics on the investment strategy chosen. To determine the optimal investment strategy between physical and defence capital taking into account the dynamic feedback between the economic and hydrological system we next apply the tools of dynamic optimization. We also show the sensitivity of the model dynamics on the initial endowment of the economy. In particular, the optimal investment strategies will be determined by the state of the economy. Furthermore, we investigate how the optimal investment strategy will change depending on the frequency and amplitude of the high level water events and whether a more efficient flood

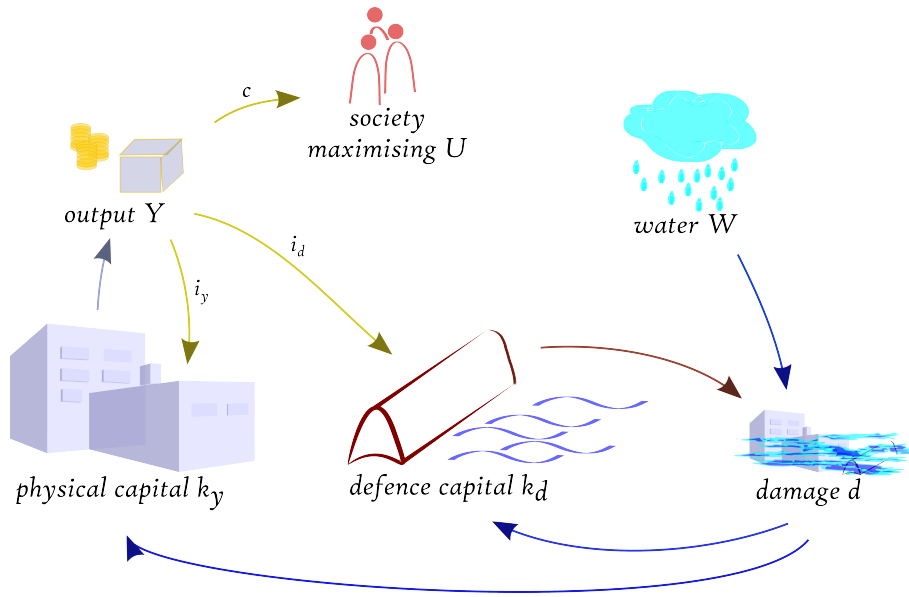


Figure 1: Overview of the dynamics within the presented model. The society chooses to consume ( $c(t)$ ) or invest ( $i_y(t)$  and  $i_d(t)$ ) the economic output  $Y(t)$  into the capital stocks.

defence capital may foster economic growth. The paper concludes by discussing our scenarios in the context of flooding in various regions of the world.

## 2 Modelling the interaction between flood events and economic growth

### 2.1 Feedbacks between society and floods

Floods affect settlements close to rivers by destroying existing capital. Societies have developed different approaches to prevent or mitigate the damage. Building dikes, levees or flood control basins may prevent flood waters entering the settlements. Warning systems to assist in evacuations and settling further away from the river (Viglione et al., 2014) may also be regarded as mitigation measures.

In our model we represent all flood prevention technologies by one variable and name it *defence capital*. Similarly we model the physical capital stock — which represents machines, buildings, infrastructure — by one variable named *physical capital*. We assume that a flood causes damage of physical capital if the water level exceeds a specific threshold of the defence capital. The society chooses how much it invests into defence capital and therefore influences the occurrence of floodings.

The physical capital stock is used to produce economic output. Aggregate output in an economy can be used for consumption and investments in either physical or defence capital stock. We assume that the decision of the optimal share of output used for consumption and investment is taken by a social planner. This means we abstract from a market framework where factor remunerations such as interest rates on capital or wages for labour input would determine the optimal allocation of output between consumption and the two types of investment.

We assume a closed economy, which implies that all of the produced output will be used, and no further trade with other communities is possible.

Fig. 1 displays the dynamics of the model. Economic output  $Y(t)$  depends on the amount

of physical capital  $k_y(t)$ . The output can be either consumed  $c(t)$  or invested in physical  $i_y(t)$  or defence capital  $i_d(t)$ . The society chooses the level of consumption and the amount of investment into physical and defence capital in order to maximize utility. The defence capital can prevent the damage  $d(W(t), k_d(t))$  caused by flooding events. The occurrence of flooding events depends on the water level  $W(t)$ . In case of flooding, both capital stocks are damaged.

## 2.2 Model equations

To model the aforementioned interaction between society and flood events we first define the utility function of the social planner and its choice variables. Next, we determine how output is produced in the economy and explain the dynamics of physical and defence capital which constitute the dynamic constraints for the optimization problem of the social planner. To model the water level we introduce an exogenous periodic function over time. Together with the level of defence capital, the water level will then determine the extent of the damage.

### 2.2.1 Utility function

The objective of the social planner is to maximize the discounted stream of aggregate utility  $U(c(t)) = \ln(c)$  which depends positively on the consumption level  $c(t)$ :

$$\max_{\{c(t), i_d(t)\}} \int_0^\infty e^{-\rho t} U(c(t)) dt \quad (1)$$

where  $\rho$  denotes the time preference and indicates to which extent the social planner prefers utility of consumption today compared to utility of consumption tomorrow. Consumption  $c(t)$  and investment in defence capital  $i_d(t)$  are control variables <sup>1</sup> to be chosen optimally to maximize equation (1), given the level of output and dynamic constraints of physical and defence capital as stated below. More specifically, the dynamic optimization of the social planner guarantees that any decision taken today also incorporates the feedback on the future evolution of the system.

Since at every time period consumption together with investment in physical and defence capital is bounded by the available output, the choice of two variables implies the optimal choice of the third variable (investment in physical capital in our case).

### 2.2.2 Economic output

Output  $Y(t)$  is given by a simple Cobb Douglas-production function

$$Y(t) = Ak_y(t)^\alpha \quad (2)$$

that depends on the physical capital stock  $k_y(t)$  and an exogenous level of technology  $A$ . The production input factor labor is normalized to one.  $\alpha \in [0, 1]$  denotes the elasticity of the production input factor capital.

Output can be used for consumption  $c(t)$  as well as for investment in physical capital  $i_y(t)$  and investment in defence capital  $i_d(t)$ . Since output is given in [\$] and the unit of the defence capital is [m] we need to transform investment in defence capital  $i_d$  given in [m] into costs  $Q(i_d(t)) = \theta_0(\theta_1 i_d(t) + \theta_2 i_d(t)^2)$  given in [\$]. The parameters  $\theta_i$  weight the linear and quadratic parts of the costs and are calculated according to Slijkhuis et al. (1997) and Bedford et al. (2008).

The overall budget constraint for the social planner is therefore given as:

$$Y(t) = c(t) + i_y(t) + Q(i_d(t)) \quad (3)$$

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<sup>1</sup>In a less technical setting we refer to the control variables as decision variables.



### 2.2.3 State dynamics

Following the standard Ramsey model we write the dynamic constraints by the following two state equations for physical and defence capital:

$$\dot{k}_y(t) = i_y(t) - d(k_d(t), W(t))k_y(t) - \delta_y k_y(t) \quad (4)$$

$$\dot{k}_d(t) = i_d(t) - \kappa_d d(k_d(t), W(t))k_d(t) - \delta_d k_d(t) \quad (5)$$

Each capital stock can be augmented by investments  $i_y$  and respectively  $i_d$  and depreciates by a constant rate  $\delta_y$ , respectively  $\delta_d$ . Moreover, flood damage  $d(k_d(t), W(t))$  decreases both capital stocks.<sup>2</sup> The flood damage rate  $d(k_d(t), W(t))$  is in the interval  $[0, 1]$ . We allow for the fact that the damage may be different for physical and defence capital by introducing the parameter  $\kappa_d$  in equation (5).

### 2.2.4 Damage function

We assume a damage function  $d(k_d(t), W(t))$  analogous to Viglione et al. (2014). The amount of damage is related to the flood intensity  $W_{eff}(W(t), k_d(t)) = W(t) + \xi_d k_d(t)$  which is a function of the water level  $W(t)$  and the additional amount of water  $\xi_d k_d(t)$ . This additional amount of water occurs due to existing defence capital  $k_d(t)$  such as levees: Levees at one place protect this area from flooding, but increase water levels further down the river due to loss of flood plain retention (Viglione et al., 2014).

If the flood intensity  $W_{eff}(W(t), k_d(t)) = W(t) + \xi_d k_d(t)$  exceeds the flood defence capital  $k_d(t)$  and the levees spill over, a damage of the overall capital stock occurs. The higher the effective water level  $W_{eff}(W(t), k_d(t))$ , the higher the direct damage of the flooding (Jonkman et al., 2008). The damage rate  $d(k_d(t), W(t)) \in [0, 1]$  gives the relative damage of the capital stocks. Beyond  $k_d(t)$ , the damage of the flood is proportional to the effective water level of the flood  $W_{eff}$  and, also, to the flood duration, which is the time intervall when  $W_{eff}(W(t), k_d(t)) > k_d(t)$  is true. This assumption reflects the common situation that structural damage is related to the water level, while damage to industry production and stocks is related to the duration of the inundation. The damage rate is then represented as follows.

$$d(k_d(t), W(t)) = \begin{cases} 1 - \exp(-W_{eff}(t)) & \text{if } W_{eff}(W(t), k_d(t)) > k_d(t) \\ 0 & \text{else} \end{cases} \quad (6)$$

For ease of obtaining a numerical solution of the optimization model, we approximate the damage function (6) with a continuous function. Still, damage ( $d(k_d(t), W(t)) > \epsilon$  with a positive  $\epsilon$  close to zero) only occurs if  $W_{eff}(W(t), k_d(t)) > k_d(t)$ . We choose the signum-approximation function and base it on the following four assumptions: First, the minimum value is 0 for the water level  $W \leq 0$ . Second, if  $W_{eff}(W(t), k_d(t)) = W + \xi_d k_d > k_d$  and  $W \rightarrow \infty$  we reach the maximum value 1. Third, the inflection point is at  $W + \xi_d k_d = k_d$ . Fourth, the gradient at the inflection point is chosen such as to approach infinity to approximate the jump between 0 and the relative damage  $d > 0$  in equation (6). Furthermore, we add a multiplicative term  $(1 - \frac{1}{1+W(t)^\eta})$  that is increasing in the water level  $W(t)$  and bounded by the interval  $[0, 1]$ . This term ensures that the damage is higher for a more intense flooding.

$$d(k_d(t), W(t)) = \frac{1}{2} \left( \tau_3 + \frac{\tau_2 + W(t) - (1 - \xi_d)k_d(t)}{\sqrt{(W(t) - (1 - \xi_d)k_d(t))^2 + \tau_1}} \right) \left( 1 - \frac{1}{1 + W(t)^\eta} \right) \quad (7)$$

<sup>2</sup>Rezai et al. (2014) model similar dynamics for pollution.



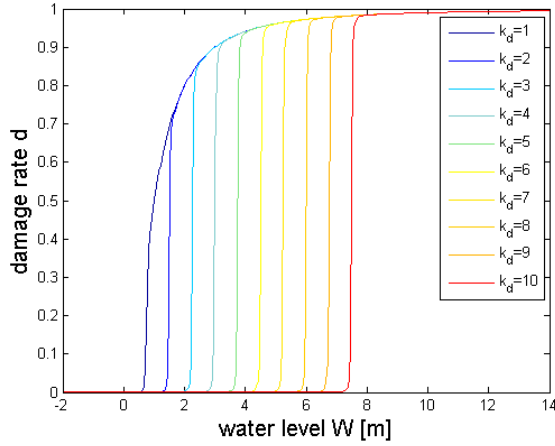


Figure 2: Form of the damage rate as a function of the water level  $W(t)$  given for various levels of the defence capital  $k_d(t)$  and for  $\xi_d = 0.5$ . Both the water level and the defence capital are given in meters.

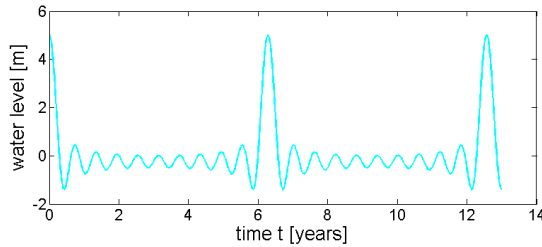


Figure 3: The periodic water level function gives quite frequent flood events. In brackets we display the units for the time and the water level itself.

The coefficients  $\tau_i$  adjust the accuracy of the approximation of (6) with (7), for the calculations we used  $\tau_1 = 0.001$ ,  $\tau_2 = 0$  and  $\tau_3 = 1$ .

Fig. 2 shows the damage rate with respect to the water level  $W(t)$  for different values of defence capital stock  $k_d(t)$ . If the defence capital is higher than the water level, the damage is closer to zero (no damage) until the inflection point  $W(t) = (1 - \xi_d)k_d(t)$  given in equation (6) and then close to one (total damage).

### 2.2.5 Water function

The water level  $W(t)$  [m] is approximated with a continuous function (Viglione et al. (2014) uses a discrete time series for flood events) to allow an analytical solution of the model. A similar function was developed by Langer (2014) and explained in Grames et al. (2015). The parameter  $\kappa_s$  determines the maximum level of water to be reached during a flood and  $\kappa_m$  controls the frequency of flood events.

$$W(t) = \frac{1}{2} \sum_{\kappa=1}^{\kappa_s} \cos(\kappa_m \kappa t) \quad (8)$$

The water function is shown in Fig.3. The water level is 0 when the river is bankfull and therefore the function (8) can be negative. Negative water levels  $W(t) < 0$  are simply treated like  $W(t) = 0$ , since the water level only affects  $d(k_d(t), W(t))$ , and  $d(k_d(t), 0) = d(k_d(t), w_-)$  holds for any  $w_- < 0$ .

### 2.2.6 Model summary

In summary, our model is represented by the following set of equations, where we have substituted  $i_y(t)$  from equation (3) into equation (4):

$$\max_{\{c(t) \in [0, Y(t)], i_d(t) \in [0, Y(t) - c(t)]\}} \int_0^\infty e^{-\rho t} U(c(t)) dt \quad (9a)$$

$$\begin{aligned} \text{s.t.} \\ \dot{k}_y(t) &= Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y(t) - \delta_y k_y(t) \end{aligned} \quad (9b)$$

$$\dot{k}_d(t) = i_d(t) - \kappa_d d(k_d(t), W(t))k_d(t) - \delta_d k_d(t) \quad (9c)$$

$$U(c(t)) = \ln(c(t)) \quad (9d)$$

$$Q(i_d(t)) = \theta_0(\theta_1 i_d(t) + \theta_2 i_d(t)^2) \quad (9e)$$

$$W(t) = \frac{1}{2} \sum_{\kappa=1}^{\kappa_s} \cos(\kappa_m \kappa t) \quad (9f)$$

$$d(k_d(t), W(t)) = \frac{1}{2} \left( \tau_3 + \frac{\tau_2 + W(t) - (1 - \xi_d)k_d(t)}{\sqrt{(W(t) - (1 - \xi_d)k_d(t))^2 + \tau_1}} \right) \left( 1 - \frac{1}{1 + W(t)^\eta} \right) \quad (9g)$$

The variables and parameters are shown in Table 1 and in Table 2. We chose them based on basic literature and to replicate the stylized facts discussed in the introduction.

Table 1: Variables of the model and their units of measurement

Decision variable		Interpretation	Unit
$c$		Consumption	$10^9 \$$
$i_y$		Investment in $k_y$	$10^9 \$$
$i_d$		Increase in $k_d$ after investment of $Q(i_d)$	m
Endogenous variable		Interpretation	Unit
$Y$		Output	$10^9 \$$
$k_y$		physical capital	$10^9 \$$
$k_d$		Defence capital	m
$d$		Damage rate	1/year
$W_{eff}$		Effective water level	m
$Q$		Costs for defence capital	\$
Exogenous variable		Interpretation	Unit
$W$		Water level	m

periodic

## 3 Results

### 3.1 Simulation

To gain a better understanding of the model dynamics we start with numerical simulations of the uncontrolled system where the dynamics of the control variables are exogenously given. As-

Table 2: Parameters of the model and their units of measurement

Parameter	Interpretation	Unit	Base case	Case study
$A$	Technology	[ ]	2.3	
$\alpha$	Output elasticity of physical capital	[ ]	0.3	
$\rho$	Time preference rate	1/year	0.07	
$\delta_y$	Depreciation rate of econ. capital	1/year	0.1	
$\delta_d$	Depreciation rate of defence capital	1/year	0.1	
$\kappa_m$	Frequency of floods	$1/(2\pi)$ /year	1	2
$\kappa_s$	Water level of floods	1/2 m	5	10
$\kappa_d$	Damage of defence capital relative to physical capital	[ ]	1	0.1
$\eta$	Increase in damage due to a higher water level	[ ]	2	
$\tau_1$	Approximation parameter in the damage function	[ ]	0.001	$\theta_0$ is calcu-
$\tau_2$	Water peak approximation parameter	[ ]	0	
$\tau_3$	Approximation parameter in the damage function	[ ]	1	
$\theta_0$	Scaling parameter for dike heightening costs <sup>3</sup>	$10^9\$/m$	0.5	
$\theta_1$	Weight for linear dike heightening costs	[ ]	0.5	
$\theta_2$	Weight for quadratic dike heightening costs	[ ]	0.5	
$\xi_d$	Additional rise of the water level due to existing defence capital	[ ]	0.5	

lated due to Slikhuis et al. (1997) and Bedford et al. (2008)

suming perfect consumption smoothing, we postulate  $c(t)$  to be constant over time. Investment into physical and defence capital,  $i_y(t)$  and  $i_d(t)$  are, therefore, functions of the exogenous consumption level and the aggregate economic output  $Y(t)$ . To determine the specific investment in either one of the capital stocks we propose two alternative settings: We may keep the defence capital constant and therefore choose the investment  $i_d(t)$  equal to the sum of the depreciation rate of the flood defence capital  $\delta k_d(t)$  and the damage  $d(W(t), k_d(t))k_d(t)$ . The investment in physical capital  $i_y(t)$  is then determined by the budget constraint (3). Alternatively, we assume that the total amount available for investments  $Y(t) - c(t) = i(t) = i_y(t) + Q(i_d(t))$  is proportionally split between both investment options, i.e. for our simulations we assume  $Q(i_d(t)) = 0.3i(t)$  and  $i_y(t) = i(t) - Q(i_d(t)) = 0.7i(t)$ .

Both cases are shown in the following Figs.4-6 where we plot the water level  $W$  as well as the effective water level  $W_{eff}(W(t), k_d(t)) = W + \xi_d k_d$  and the dynamics of the state variables  $k_y(t)$  and  $k_d(t)$ . The dynamics are qualitatively similar for both cases: Whenever a flooding hits (the effective water level  $W_{eff}(t)$  is above the defence capital  $k_d(t)$ ) damage occurs and reduces the total capital stock  $k(t)$  and hence the growth rate of the economy.

We present results of our simulations for two different sets of initial values. Higher initial

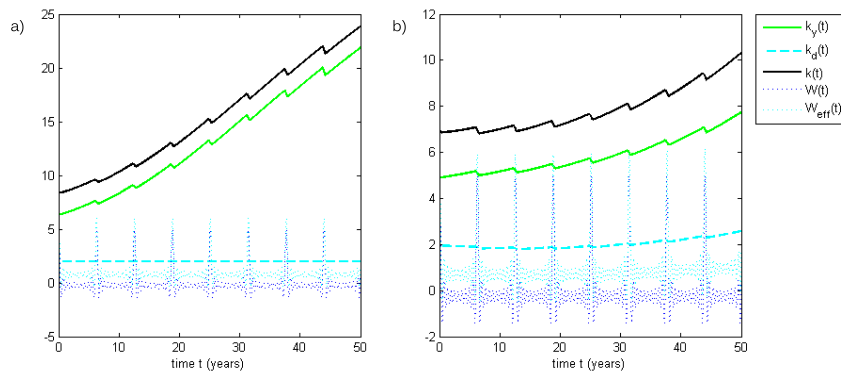


Figure 4: Simulation run of the physical capital  $k_y(t)$ , the defence capital  $k_d(t)$ , their sum  $k(t)$ , the exogenous water level  $W(t)$  and the endogenous effective water level  $W_{eff}(t)$ . a) Constant  $k_d = 2$  with  $k_y(t_0) = 6.5$  and b) proportional investments with  $k_d(t_0) = 2$  and  $k_y(t_0) = 6.5$  lead to economic growth. The unit of  $k_y$  is [\$], all the other variables are given in [m].

capital stocks ( $k_y(t_0) = 6.5$  and  $k_d(t_0) = 2$ ) enable the economy to grow (see Fig.4). Moreover, keeping the amount of defence capital constant (Fig.4 a)) allows even faster growth compared to ever increasing amounts of investment in defence capital (Fig.4 b)) .

A small change in the initial capital stocks can make a significant difference in the long term behaviour of the capital stocks and hence on economic growth. If the economy does not have enough physical capital in terms of infrastructure, machines and buildings to produce economic output, it cannot withstand floods and economic growth will decline in the long run. If the society still tries to keep the level of the defence capital constant (see Fig.5 a)) they even have to invest such a large part of their output in defence capital that their physical capital depreciates and the economy crashes. The situation is not as severe in case two (see Fig.5 b)) where an economy invests in defence capital proportional to the existing capital stock. However, also in this case, the economy will shrink in the long run. In order to avoid such a doomsday scenario when initial capital stocks are too low, an alternative is to reduce the amount of investment. For instance, if  $Q(i_d(t))$  is only 25% instead of 30% of the total investments, economic growth is sustainable even for low levels of initial capital stocks (see Fig.6).

Overall, our simulations indicate that constant levels of decision variables that do not adapt

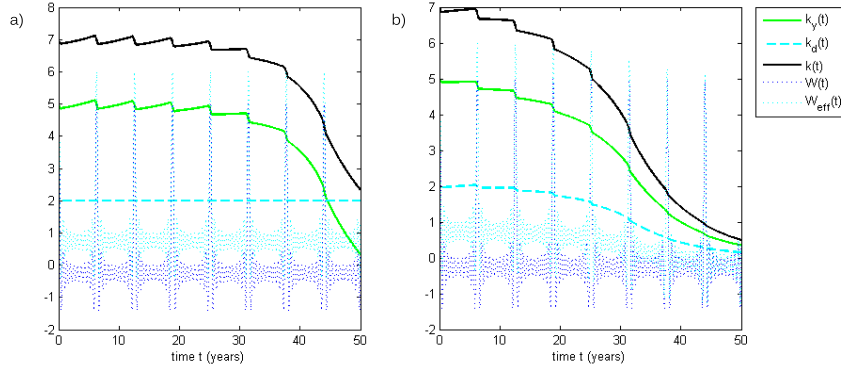


Figure 5: Simulation run of the physical capital  $k_y(t)$ , the defence capital  $k_d(t)$ , their sum  $k(t)$ , the exogenous water level  $W(t)$  and the endogenous effective water level  $W_{eff}(t)$ . a) Constant  $k_d = 2$  with  $k_y(t_0) = 5$  and b) proportional investments with  $k_d(t_0) = 2$  and  $k_y(t_0) = 5$  run into economic disaster.

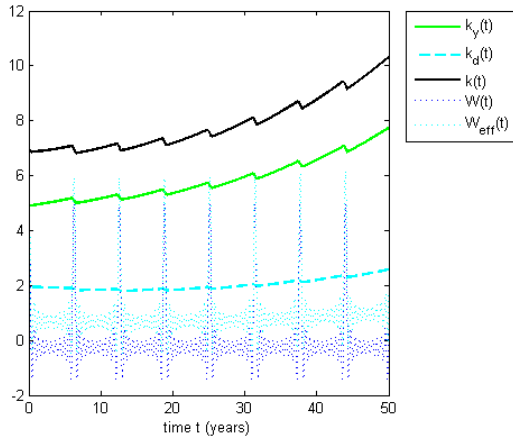


Figure 6: Simulation run where the initial values  $k_d(t_0) = 2$  and  $k_y(t_0) = 5$  are enough to enable economic growth if the investment in defence capital is only 25% of the total investment.

to the state of the economy, may in the long run lead to a collapse of the economy. We therefore need to consider dynamic decision rules that react to the state of the model. Dynamic optimization methods are the tools to implement these dynamic decision rules.

## 3.2 Dynamic Optimization

Given the dynamics of the capital stocks, the exogenous water function, and the functional forms of the damage function and aggregate economic output, the social planner maximizes the discounted flow of utility by choosing the optimal consumption and the optimal amount of investments into defence capital. Since the exogenous function of the water level is periodic, the optimal decisions on consumption and investment will also follow a periodic time path.

### 3.2.1 Optimal consumption and investment decisions

Before we present detailed analytical and numerical results of the model we give an intuitive explanation of the dynamics of the model. Total aggregate output of the economy is consumed

or reinvested into either one of the capital stocks (see equation (3)). Applying optimal control theory (A), we derive the optimal dynamics of consumption and investment decisions:

$$\dot{c}(t) = c(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho] \quad (10)$$

$$\begin{aligned} \dot{i}_d(t) = & \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2} [A\alpha k_y(t)^{\alpha-1} + (\kappa_d - 1)d(k_d(t), W(t)) \\ & + \kappa_d d'(k_d(t), W(t))k_d + \delta_d - \delta_y] \\ & + \frac{1}{2\theta_0\theta_2} [d'(k_d(t), W(t))k_y] \end{aligned} \quad (11)$$

Both, the consumption path and the investment path, depend on the exogenous periodic function  $W(t)$  and consequently, they will be periodic as well. Note that  $W(t)$  indirectly influences  $\dot{c}(t)$  because the capital stock  $k_y(t)$  is a function of  $W(t)$ .

The consumption dynamics are the same as in the standard Ramsey model with a social planner (Ramsey (1928)). A higher marginal product of physical capital (as given by the first derivative of the production function with respect to physical capital) as well as a lower rate of capital depreciation and time preference will positively affect the consumption growth rate. Damage acts like an additional depreciation on the marginal product of physical capital.

The dynamics of the investment in flood defence capital are more complex. The marginal product of physical capital and a lower rate of depreciation of physical capital positively influence the investment rate  $\dot{i}_d(t)$ , whereas a low rate of depreciation of the defence capital will reduce the optimal investment rate in flood defence capital because less investment is necessary to sustain the defence capital. Moreover, since the factor  $(\kappa_d - 1)$  is nonpositive, when damage occurs, investments in defence capital decreases. The latter effect can be explained by the assumption that, in case of  $\kappa_d > 1$ , the damage to defence capital is more severe than the damage to physical capital. Consequently, investment in defence capital will be reduced. In case the damage rate for both types of capital is the same ( $\kappa_d = 1$ ), damage does not directly influence the investment behaviour. However, the first derivative of damage with respect to the defence capital is zero or close to zero, so neither of the terms affect the investment dynamics. In general, all investment decisions are scaled by the cost parameters  $\theta_0$ ,  $\theta_1$  and  $\theta_2$ . Lower costs enable higher investments.

### 3.2.2 Optimal long term capital stocks

Our results indicate that any optimal path of consumption and investment that the social planner decides on will end up in one of two possible long run solutions/limit sets (see B). Note, that mathematical limit sets are different from an economic equilibrium which denotes a situation where all markets clear. We name the inner equilibrium which has high capital stocks and therefore high economic output the *rich economy* and the boundary equilibrium which only sustains a comparatively small physical capital stock and no defence capital *poor economy*. This notation will become apparent when we consider the long run economic state of the economy in each case.

To identify both equilibria we solved the optimization problem first analytically using the Pontryagin maximum principle (Pontryagin, 1962) and then numerically using the specific MATLAB® -Toolbox *OCMat* from Grass and Seidl (2013) and the parameter values given in Table 2.

The *rich economy* (Fig.7 a)) invests just enough to avoid floodings and consequently flood damage. Even though the social planner never stops investing into flood prevention measures ( $i_d(t) > 0$ ) in the long term, they lower the investments when they are not urgent and rather invest in physical capital  $k_y(t)$  to increase the economic output  $Y(t)$ . In such an economy,

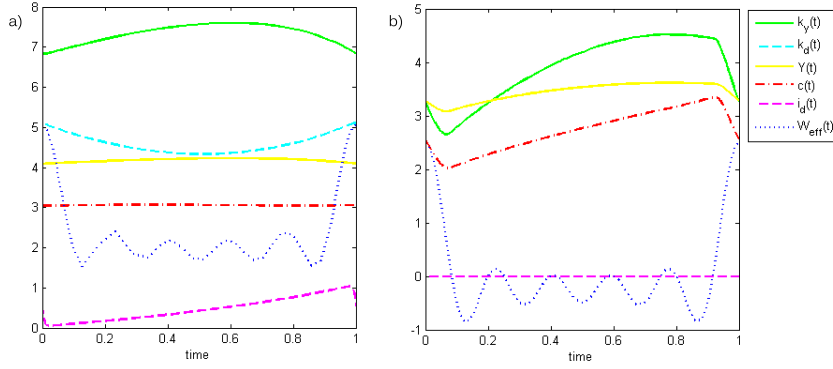


Figure 7: One limit cycle (in normalized time) of the long-term behaviour of a a) rich economy, b) poor economy showing the time series of the physical capital  $k_y(t)$ , the defence capital  $k_d(t)$ , the economic output  $Y(t)$ , the consumption  $c(t)$ , the investment in defence capital  $i_d(t)$  and the exogenous effective water level  $W(t)$ . Note the different scales for a) and b).

the aggregate output is quite high and therefore a constant consumption path is sustainable. These so called smooth consumption paths are characteristic of developed economies and are also commonly shown to be consistent with economic growth.

In contrast, *poor economies* (Fig.7 b)) do not invest at all in defence capital. Mathematically they move to a boundary periodic solution with  $i_d(t) = 0$ . Without any investments  $i_d(t)$  the defence capital  $k_d(t)$  remains zero (and so the effective water  $W_{eff}$  level equals the exogenous water level  $W$ ). Consequently, they are vulnerable and every time a high water level occurs, flooding hits the economy. The physical capital stock  $k_y(t)$  decreases and less economic output  $Y(t)$  is produced. Interestingly, the social planner already anticipates the damage shortly before a flood hits and prefers to distribute the output to consumption rather than investment in physical capital. Therefore consumption  $c(t)$  strictly increases until a flood hits and less consumption is possible during a flooding event. It takes time to recover and to reach the old consumption level again.

It is useful to highlight the optimal investment strategy for the rich economy: The investments in flood defence capital are always positive and increase before a flood hits. In reality, societies tend to invest in flood defence infrastructure only after big flooding events have occurred. An example is the Danube flood of 1954 which resulted in construction of a flood relief channel in Vienna. Decision processes to invest in flood defence management are mostly based on political decisions and financial considerations and only effective if stakeholders have an immediate memory of past flooding. However, the optimization model shows that investing in flood defence capital before floods would be economically more advisable.

The long-term state dynamics of the capital stocks  $k_y(t)$  and  $k_d(t)$  clearly identify the limit cycle. Note that the cycling is counterclockwise. For the rich economy (Fig.8 a)) we see a negative correlation of the capital stocks: Since the social planner wants to keep consumption smooth, increasing investments in one capital stock lowers the investments in the other capital stock. Moreover, a lower physical capital stock yields less output. This allows less investments and therefore a lower total capital stock. This is always the case after high water levels, when the priority is to build up defence capital. This means that floods do not only affect the economy directly via damage, but also indirectly through a lower level of output and therefore lower capital stocks.

The limit cycle for the poor economy (Fig.8 b)) is trivial. Since there is no defence capital  $k_d(t)$ , the physical capital basically increases after a flooding, reaches its maximum slightly before a



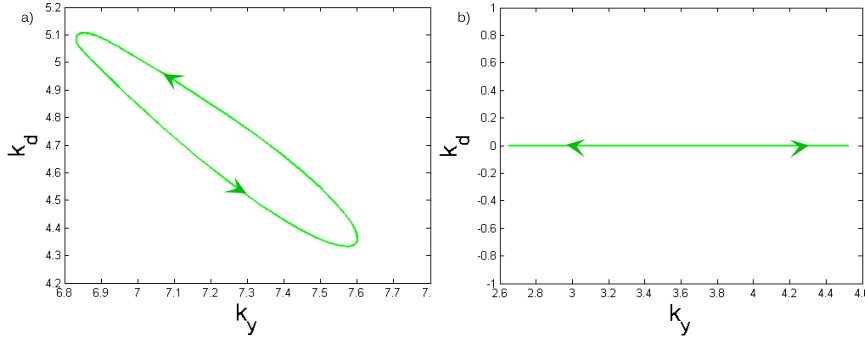


Figure 8: The state dynamics of the a) rich economy, b) poor economy.

flooding due to the anticipation effect and decreases quickly when a flood hits the economy.

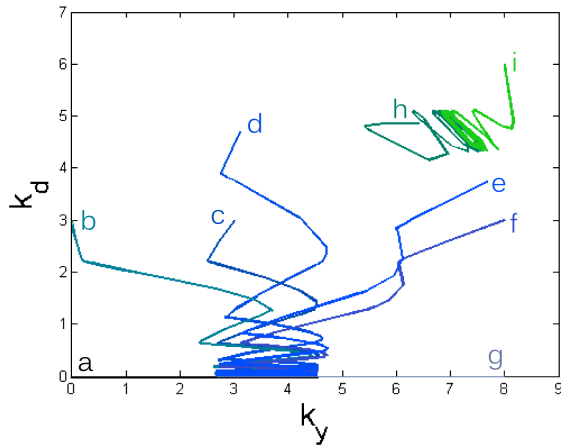


Figure 9: Different initial conditions (points a – g) in several economies converge to different long-term behaviour.  $k_y$  and  $k_d$  are the physical capital and the defense capital, respectively.

So far we have studied the long-term behaviour along the limit cycles. It is also important to understand the path towards one of the two equilibria. Depending on the initial values of the capital stocks  $k_y$  and  $k_d$  the economy follows a path to one of the equilibria. For the base case where we set the parameters according to Table 2 we choose the set of the starting points (a)-(i) and show the different paths in Fig.9. Both initial capital stocks have to be close or higher than the long-term values of a rich economy in order for the economy to stay rich. This is only the case for the initial points h and i out of those chosen.

We have a closer look at the paths to a rich economy: Starting at (h), where the defence capital is almost high enough to completely avoid floodings, we see a drop in the physical capital first, before it converges to the limit cycle. In contrast, if we start with a much higher defence capital at point (i), which does not bring any extra benefit compared to the long-term level, investments in defence capital are stopped immediately and the defence capital stock depreciates, while investments in economic capital are slightly positive. The main part of the output is consumed directly, unless the defence capital stock has reached the level where it may be too small to prevent damage from floods. So, even if the community could afford more capital, they prefer to only invest as much as necessary to avoid floodings and rather consume the output right away.

The paths to the long run equilibrium of the poor economy can be similarly diverse. Starting

without any capital (point a) can never enable an economy to build up defence capital and become rich. Yet, the economy will manage to build up some physical capital significantly different from zero. If there is defence capital, but it is not effective enough, not even a wealthy economy (e,f,g) will manage to stay rich in the long term. Even if the defence capital is high enough to avoid floodings (d), a too small economy in the beginning will limit the economy to sustain its defence capital and it will use all its physical capital to produce output and for consumption. Again, the economies b and c show characteristics of a and d. If neither the economic output nor the flood management is close to the standards of a rich economy, it will always remain poor.

### 3.2.3 Higher frequency and higher intensity of floods changes the investment behaviour

So far we have studied the dynamics of the model under one specific set of parameters. We next investigate how the optimal decisions of the social planner will change when she faces a different environment, e.g., a different occurrence of high level water events. We study two cases: First, we assume a higher frequency of floods, and secondly we assume higher water levels which can lead to stronger floodings.

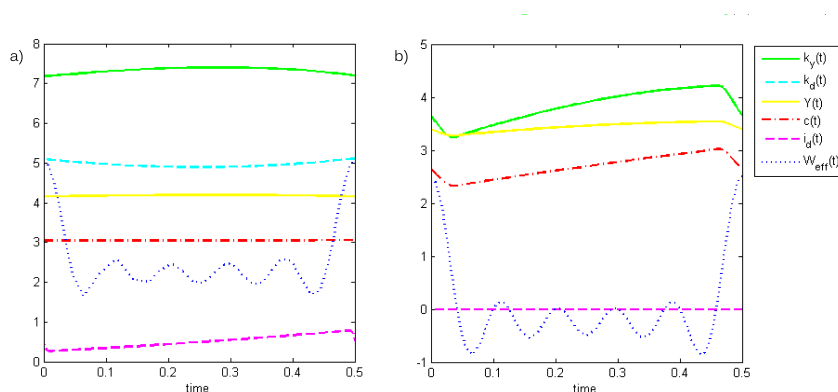


Figure 10: One limit cycle in case of a higher frequency of floods and therefore time period  $[0, 0.5]^4$  for a) a rich economy, b) a poor economy. Parameters as in base case of Table 2, but  $\kappa_m = 2$ .

Doubling the frequency of high level water events ( $\kappa_m = 2$ ) naturally leads to a smaller time period of the limit cycle. Fig.10 displays less variation in the dynamics of the state and control variables than in the base case. Intuitively, we would expect that a doubling of the flood frequency would translate into a 50%-reduction in the variations of the levels of the state and control variables since the time to accumulate capital without being hit by a flood is only half. However, this is only true for poor economies. For rich economies, the difference between the highest and lowest level of the capital stock along the limit cycle is not even a third in case of double flood frequency. Even more counterintuitive is the finding that a rich economy facing a higher frequency of high water levels manages to have the same consumption rate and even higher capital stocks on average as compared to the case with lower frequencies of high water levels. Both the defence and the physical capital stock are higher on average than in the base case. So only very rich economies manage to stay rich when they are facing higher flood frequencies.

Poor economies suffer from higher flood frequencies. Since more floods lead to shorter flood

durations, the damage is not as high, but occurs more often. Not only is the range of the values of the capital stocks smaller than in the base case, also the range of the consumption level is halved. Moreover, on average poor economies facing more floodings consume less and have less economic output.

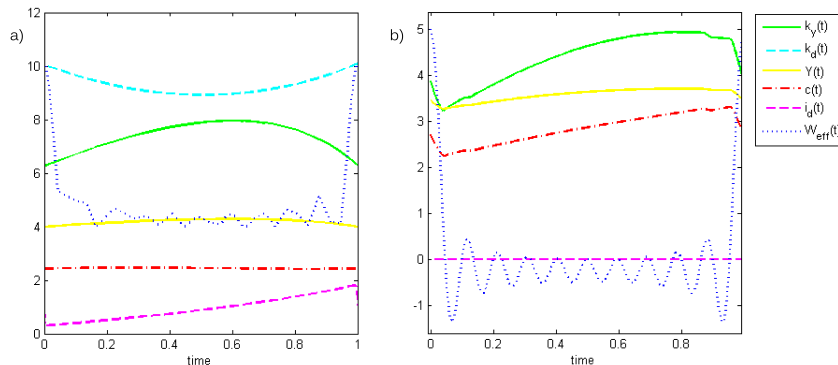


Figure 11: One limit cycle in case of bigger floods for a) a rich economy, b) a poor economy. Parameters as in base case of Table 2, but  $\kappa_s = 10$ .

For the second case we vary the amplitude of the floods ( $\kappa_s = 10$ ) and show the results in Fig.11. In order to protect against higher water levels, rich economies will start to invest in defence capital earlier and to a larger extent. Consequently, less economic output is left to invest in physical capital or for consumption. Rich economies can consume 20% less than rich economies in the base case scenario. This is the only chance they can keep the physical capital almost at the same level and therefore produce a critical amount of economic output.

Surprisingly, poor economies converge in the case of stronger floods to an economic state with higher capital stocks and higher consumption levels compared to the base case scenario. Although floodings hit harder, each flood is shorter which results in a wealthier economy.

When we compare rich and poor economies in case of bigger floods, the capital stocks are much higher for rich economies, so they seem to be wealthier. However, consumption and therefore the average utility in one limit cycle of the society is 17% higher for poor economies. This means that poor communities in heavily flooded areas should actually not invest in defence capital but rather invest in physical capital, thereby increasing output and allowing for higher consumption levels, even though they have to give up a smooth consumption path.

The results depend on the parameters and the characteristics of the damage function.

### 3.2.4 Less damage in the defence capital stock influences the dynamics of the capital stocks

Fig.12 shows a case where the defence capital is not as vulnerable as the physical capital ( $\kappa_d = 0.1$ ). For this case we only need to analyze rich economies, since poor economies do not even have defence capital and therefore defence capital cannot be damaged. Fig.12 shows very similar patterns to the base case. It appears that the floods do not destroy defence capital as heavily as physical capital. Assuming an equilibrium without any damage would simply look like the base case scenario. Since the social planner knows that damage does not affect the defence capital very much, she chooses a lower investment in defence capital than in the base case and therefore allows small flooding events for a very short time, where both capital stocks are damaged. As a consequence, the economic output is slightly lower, but the consumption

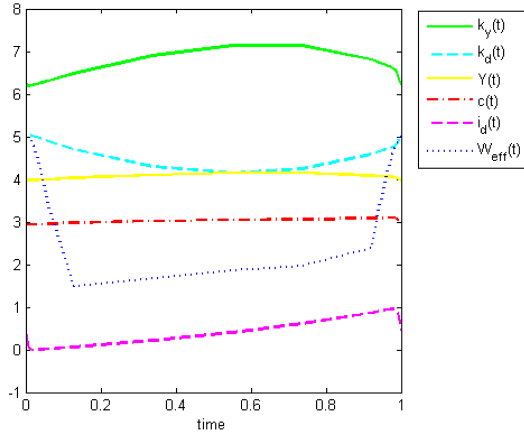


Figure 12: One limit cycle in case of a more robust flood defence capital. Parameters as in base case of Table 2, but  $\kappa_d = 0.1$ . Note, due to numerical discretization of the solution  $W_{eff}$  is displayed different, but has the same oscillating behaviour as in the other figures.

increases in the time in-between the flooding events. This suggests that, in this model, people do care more about the defence capital, if it is more vulnerable.

## 4 Discussion

In this paper we studied a socio-hydrological model of high water level events potentially causing floodings in an economic decision framework. In the model, a social planner, representing the society and knowing occurrence and magnitude of high water levels in advance, decides how to optimally distribute the economic output between consumption, investment in flood defence capital and investment into physical capital. Investments in flood defence capital do not only avoid direct damage in the future, but also save opportunity costs for reconstruction. This allows investments in physical capital and consequently more economic growth in the future Hochrainer-Stigler et al. (2013).

We applied dynamic optimization methods to determine the long run optimal solution of our system. Depending on the initial capital stocks of the economy, our system either converges to a rich or a poor economy in the long term. In order to compare the model results to real world data we use macro-economic data for countries, whereas we are aware that usually only parts of a country are under flood risk. So whenever we discuss rich or poor economies, we refer to broader regions or countries that are (partly) affected by floods.

The rich economy manages to build up defence capital to avoid damage and therefore follows a smooth consumption path. The consumption rate of 70% (Fig.7 a)) equals e.g. the rate in the US.<sup>5</sup> Poor economies, characterized by low levels of initial economic output or initial defence capital, optimally decide not to invest into defence capital and end up with lower capital stocks and lower consumption rates. Every time a flooding hits, physical capital is damaged and consumption decreases strongly. The average consumption rate of poor economies is higher than 80% of their total output, which is around the rate of third world countries such as Cambodia and Kenya.<sup>6</sup>

If defence capital such as levees is built, the water level may increase due to the loss of retention

<sup>5</sup><http://data.worldbank.org/indicator/NE.CON.PETC.ZS> assessed on June 3rd, 2015

<sup>6</sup><http://data.worldbank.org/indicator/NE.CON.PETC.ZS> assessed on June 3rd, 2015

volume (Di Baldassarre et al., 2009, Heine and Pinter, 2012, Remo et al., 2012). Also vulnerability may increase because of the levee effect (Ludy and Kondolf, 2012, Montz and Tobin, 2008). However, economic output and consequently consumption and capital stocks are higher since flood damage can be prevented. If the severity of floods is very high we showed that a rich economy investing in defence capital may end up with consuming less out of the total output compared to a poor economy which does not invest in defence capital. Our results are in line with actual observations. For example, the Netherlands are facing severe floods and invest a lot in their flood management systems (Eijgenraam et al., 2014, Silva et al., 2004). The consumption rate of around 50% in this scenario in our model fits the low consumption rate of the Netherlands.<sup>7</sup> The Netherlands have a higher output and the total per capita consumption is higher than in the mentioned third world countries.

Whether an economy is rich or poor depends very much on its economic capabilities including physical capital of firms and governments, infrastructure and technology, but also on existing flood defence capital. If any one of these components is too small, the economy will never have the strength to become a rich economy. It will stop investing in defence capital because it is not worth the opportunity costs of missed consumption. We see this scenario in many poor countries: Without any external help, regions such as the Mekong floodplains are flooded regularly and the locals are used to the damage (<http://www.mrcmekong.org/>). Kahn (2005) also found that rich nations suffer less from natural disasters than poor countries. Higher developed economies invest more in prevention of natural disasters and the total losses after a disaster are smaller (Schumacher and Strobl, 2011).

How is it possible to escape the trap into a poor economy? Since environmental conditions cannot be changed easily, only different economic environments can induce a difference. It is essential to invest into physical capital to bring the economy on a path to the equilibrium of the rich economy. If the country cannot afford this by itself, external help is necessary.

As soon as the economy is on the path towards the long term state of a rich economy, our model predicts that it will never revert to a poor economy given the same environmental and economic conditions. Staying rich when the economy is already there does not require any help from outside anymore. This is the case if no surprise will occur (see e.g., Merz et al. (2015)).

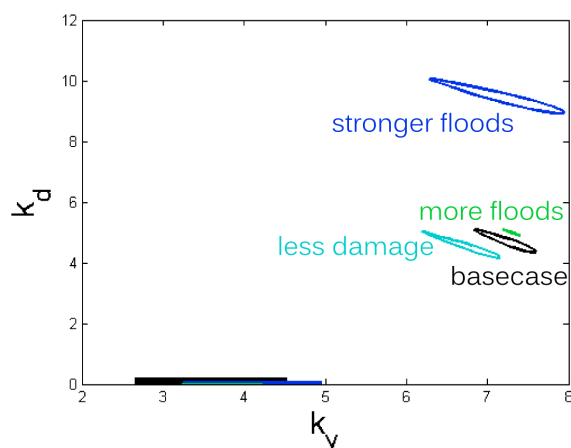


Figure 13: Long-term state dynamics for the cases of Figs. 7-12.

Fig.13 summarizes the scenarios of this paper. Each scenario is represented in a different

<sup>7</sup><http://data.worldbank.org/indicator/NE.CON.PETC.ZS> assessed on June 3rd, 2015

colour and we plot the case of a rich and a poor economy for each scenario. The amount of physical capital of the rich economies is quite similar in every scenario. Naturally, the range differs from scenario to scenario: In case of more floods we observe a lower variation of physical capital while the level of both capital stocks is higher compared to the base case.

In the scenario where we increase the severity of floods, the defence capital has to be very high in order for the economy to remain rich. So it is very hard to obtain such a rich economy and the willingness to invest in flood defence capital has to be very high, too. We only encounter this case in first world countries that are highly affected by floods such as the Netherlands. This is very much confronted with floods, can afford defence capital, and is willing to invest in it (Vis et al., 2003).

In the scenario of less damage people are minimalists and only invest in their capital stocks as much as necessary to overcome floods. As a consequence, their capital stocks are lower than in any other scenario. Their consumption is just as high as in the base case, but not as smooth since it decreases during flooding events. The consumption cycle in this scenario has similar dynamics as the poor equilibria of the other cases.

In case of poor economies, flood intensity and frequency directly impact the wealth of the economy. More floods more often cause damage of existing physical capital, but the economies have experience with floodings and rebuild the infrastructure quickly. In contrast, if bigger floods happen less frequently, the damage is much higher and the poor economies need longer and also have to invest more into physical capital to regenerate. In total, the consumption is higher than in the scenario with fewer floods. So even if floods hit harder, as long as they do not appear too often, the living standard can be relatively high in between floods.

Overall, the economic output is almost equal for all rich economies independently of the frequency and intensity of floods. Only the amount of defence capital and the variations of physical capital along the long run economic state differs. Furthermore, the economic output in poor economies is much lower than for rich economies, but it is about the same level for any poor economy in various cases.

Besides the higher economic output and the mostly higher consumption for rich economies, they do have the capacities and resources to anticipate damages before a flood hits. On the other hand, poor economies do not have the economic potential and are therefore not flexible to adjust to floods beforehand. The only anticipation is to stop investing into physical capital shortly before a flooding, but basically poor economies are affected by floods every time they occur and have to start over again rebuilding capital stocks and increasing consumption.

Optimization is important to use the resources efficiently. The simulation in Section 3.1 shows the dynamics of the model. Even in case of positive economic growth, damage occurs during every high water level event, whereas in the optimization model rich economies can avoid damage in the long run, even though they are investing less, but at the right time. Moreover, in the scenarios with declining economic growth the economies even converge to zero capital stocks. In the optimization case it will never happen that people invest in flood defence capital if they cannot even afford their basic needs for living. They therefore always manage to sustain some physical capital and to have enough resources to consume and invest again in production after a flooding event.

Comparing the results in our paper with the simulation model of Di Baldassarre et al. (2013) and Viglione et al. (2014), on which our model set up is based, we may highlight further important differences: First, they found that, in certain circumstances, investing in flood defence

capital may lead to less economic growth than facing frequent small floodings. This is because rare floodings may be catastrophic since societies erroneously consider floodplains more secure after building levees and invest in building and living there. In our optimization model, in which the social planner has the knowledge of flood occurrence and magnitude, rich economies can manage floods and therefore avoid catastrophic floodings.

Second, a lower decay of levees leads to higher growth rates in Viglione et al. (2014). In contrast, in our model the social planner decides to invest just a minimum into flood management and physical capital and, nevertheless, consumes only as much as in the scenario with a higher depreciation rate.

Our approach is to conceptualize the interaction of human decision making and flood risk management within a macro-economic framework. Our aim is to understand the mechanisms rather than matching specific cases or predicting the future development of societies. As models cannot and should not capture all details of the reality, we do not claim that this is the only true representation of communities in flood risk areas. However, it enables us to discuss certain dynamics and policies in the field of socio-hydrology.

Starting from the results in this paper, future work will focus on the sensitivity of the model results to the assumptions made, and on the assumption of perfect knowledge of future water levels by the social planner. We expect that, even though uncertainty/stochasticity of natural events will result in more complex dynamics, the results of this work will provide the fundamental baseline over which other mechanism will show up.

## 5 Acknowledgements

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# Appendix

## A Dynamics of the optimal controls

We are analyzing the model analogous to Barro and Sala-i Martin (2004) and Millner and Dietz (2015).

### A.1 The Hamiltonian

To analytically optimize the model given in equations (9) we formulate the Hamiltonian function.

$$\begin{aligned}\mathcal{H}(c(t), i_d(t), \mu_y(t), \mu_d(t)) &= U(c(t)) + \mu_y(t)[Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y - \delta_y k_y(t)] \\ &+ \mu_d[i_d(t) - \kappa_d d(k_d(t), W(t))k_d - \delta_d k_d(t)]\end{aligned}\quad (12)$$

The Pontryagin conditions are

$$\frac{\partial \mathcal{H}}{\partial c(t)} = U'(c(t)) + \mu_y(t)[-1] = 0 \quad (13a)$$

$$\frac{\partial \mathcal{H}}{\partial i_d(t)} = \mu_y(t)[-Q'(i_d(t))] + \mu_d = 0 \quad (13b)$$

$$\frac{\partial \mathcal{H}}{\partial k_y(t)} = \mu_y(t)[A(t)\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y] = \rho\mu_y(t) - \dot{\mu}_y(t) \quad (13c)$$

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial k_d(t)} &= \mu_y(t)[-d'(k_d(t), W(t))k_y] + \mu_d[-\kappa_d d'(k_d(t), W(t))k_d - \kappa_d d(k_d(t), W(t)) - \delta_d] \\ &= \rho\mu_d(t) - \dot{\mu}_d(t)\end{aligned}\quad (13d)$$

$$\frac{\partial \mathcal{H}}{\partial \mu_y(t)} = Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y - \delta_y k_y(t) = \dot{k}_y(t) \quad (13e)$$

$$\frac{\partial \mathcal{H}}{\partial \mu_d(t)} = i_d(t) - \kappa_d d(k_d(t), W(t))k_d - \delta_d k_d(t) = \dot{k}_d(t). \quad (13f)$$

### A.2 The canonical system

We rewrite the first order condition (13a), use the ln and take the total time derivative.

$$\mu_y(t) = U'(c(t)) = \frac{1}{c(t)} \quad (14)$$

$$\ln(\mu_y(t)) = \ln\left(\frac{1}{c(t)}\right) \quad (15)$$

$$\frac{\dot{\mu}_y(t)}{\mu_y(t)} = -\frac{\dot{c}(t)}{c(t)} \quad (16)$$

Analogous we can use the first order condition (13b).

$$\mu_d(t) = \mu_y(t)[Q'(i_d(t))] = \mu_y(t)\theta_0[\theta_1 + 2\theta_2 i_d(t)] \quad (17)$$

$$\begin{aligned}\ln(\mu_d(t)) &= \ln(\mu_y(t)\theta_0[\theta_1 + 2\theta_2 i_d(t)]) \\ &= \ln(\mu_y(t)) + \ln(\theta_0) + \ln(\theta_1 + 2\theta_2 i_d(t))\end{aligned}\quad (18)$$

$$\frac{\dot{\mu}_d(t)}{\mu_d(t)} = \frac{\dot{\mu}_y(t)}{\mu_y(t)} + \frac{2\theta_2 \dot{i}_d(t)}{\theta_1 + 2\theta_2 i_d(t)} \quad (19)$$

So we use (16),(19),(13c), (13d),(13e), and (13f) to write the canonical system.

$$\dot{c}(t) = -c(t) \frac{\dot{\mu}_y(t)}{\mu_y(t)} \quad (20a)$$

$$\dot{i}_d(t) = \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2} \left[ \frac{\dot{\mu}_d(t)}{\mu_d(t)} - \frac{\dot{\mu}_y(t)}{\mu_y(t)} \right] \quad (20b)$$

$$\dot{\mu}_y(t) = -\mu_y(t) [A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho] \quad (20c)$$

$$\begin{aligned} \dot{\mu}_d(t) = & \mu_y(t) [d'(k_d(t), W(t)) k_y] \\ & + \mu_d [\kappa_d d'(k_d(t), W(t)) k_d + \kappa_d d(k_d(t), W(t)) + \delta_d + \rho] \end{aligned} \quad (20d)$$

$$\dot{k}_y(t) = A k_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t)) k_y - \delta_y k_y(t) \quad (20e)$$

$$\dot{k}_d(t) = i_d(t) - \kappa_d d(k_d(t), W(t)) k_d - \delta_d k_d(t) \quad (20f)$$

### A.3 Euler equations for optimal controls

The dynamics of the optimal controls are given by the Euler equations. Applying the Pontryagin conditions to this control problem we yield the Euler equations for the optimal controls. We substitute (20c) into (20a) to describe the optimal consumption and additional (20d) and (17) into (20b) to see the optimal investments in defence capital.

$$\begin{aligned} \dot{c}(t) &= -c(t) \frac{-\mu_y(t) [A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho]}{\mu_y(t)} \\ &= c(t) [A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho] \quad (21) \\ \dot{i}_d(t) &= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2} \left[ \frac{\frac{\mu_d(t)}{\theta_0 [\theta_1 + 2\theta_2 i_d(t)]} [d'(k_d(t), W(t)) k_y]}{\mu_d(t)} \right. \\ &\quad + \frac{\mu_d [\kappa_d d'(k_d(t), W(t)) k_d + \kappa_d d(k_d(t), W(t)) + \delta_d + \rho]}{\mu_d(t)} \\ &\quad \left. - \frac{-\mu_y(t) [A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho]}{\mu_y(t)} \right] \\ &= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2} \left[ \frac{1}{\theta_0 [\theta_1 + 2\theta_2 i_d(t)]} [d'(k_d(t), W(t)) k_y] \right. \\ &\quad + [\kappa_d d'(k_d(t), W(t)) k_d + \kappa_d d(k_d(t), W(t)) + \delta_d + \rho] \\ &\quad \left. + [A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho] \right] \\ &= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2} [A\alpha k_y(t)^{\alpha-1} + (\kappa_d - 1) d(k_d(t), W(t)) + \kappa_d d'(k_d(t), W(t)) k_d + \delta_d - \delta_y] \\ &\quad + \frac{1}{2\theta_0 \theta_2} [d'(k_d(t), W(t)) k_y] \quad (22) \end{aligned}$$

## B Two solutions of the model

To solve the model given in Eqs.9 we proceed as follows. First, to find an initial solution, we redefine the periodic water function  $W(\gamma, \bar{W}, t) := \bar{W} + \gamma \Omega(t)$ , where  $\Omega(t)$  refers to the water function Eq.9f.

For the continuation of the function with a periodic solution we consider the more general



boundary value problem (BVP)

$$\dot{x}(t) = f(x(t), W(\gamma, \bar{W}, t)), \quad x(t) \in R^n, \quad t \in [0, 1] \quad (23a)$$

$$x(0) = x(1) \quad (23b)$$

with

$$W(\gamma, \bar{W}, t) = \bar{W} + \gamma\Omega(t), \quad \Omega(0) = \Omega(1). \quad (23c)$$

For  $\gamma = 0$  and  $\bar{W} = 1$  we found two feasible and optimal solutions  $\hat{x}_1$  and  $\hat{x}_2$ , each corresponding to a different constraint constellation (i.e.  $i_d(t) > 0$  and  $i_d(t) = 0$ ). For these two cases the following continuation steps were used: Since  $x(\cdot) \equiv \hat{x}$  is an isolated solution and  $f_x(\hat{x}, \bar{W})$  is non-singular,  $f_\gamma(\hat{x}, \bar{W}) \neq 0$  and the minimal period of  $\Omega(t)$  is one. For an isolated solution there exists  $\varepsilon > 0$  such that for every  $\gamma \in B_\varepsilon(0)$  a unique solution  $x(\cdot, \gamma)$  for 23 exists. Numerically these solutions can be found e.g. by the pseudo-arclength or Moore-Penrose continuation. As long as  $x(\cdot, \gamma)$  itself is an isolated solution and the linearization of of Eq.23 is non-singular the continuation proceeds.

For the actual computation the Moore-Penrose continuation in the implementation of the specific MATLAB<sup>®</sup> -Toolbox *OCMat* from Grass and Seidl (2013) was used, whereas it was shown that in the cases of  $\hat{x}_1$  and  $\hat{x}_2$  the linearization was always non-singular. This was done in two steps:

1. Continuation along  $\gamma$  from 0 to 1.
2. Continuation along  $\bar{W}$  from 1 to 0.

So we derived the two solutions for the model given in Eqs.9, whereas the periodic water function is  $W(\gamma, \bar{W}, t) = \bar{W} + \gamma\Omega(t) = \Omega(t)$  and therefore equals Eq.9f.

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