

Choi, Dong Beom; Eisenbach, Thomas M.; Yorulmazer, Tanju

Working Paper

Watering a lemon tree: Heterogeneous risk taking and monetary policy transmission

Staff Report, No. 724

Provided in Cooperation with:
Federal Reserve Bank of New York

Suggested Citation: Choi, Dong Beom; Eisenbach, Thomas M.; Yorulmazer, Tanju (2015) : Watering a lemon tree: Heterogeneous risk taking and monetary policy transmission, Staff Report, No. 724, Federal Reserve Bank of New York, New York, NY

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Federal Reserve Bank of New York
Staff Reports

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Dong Beom Choi
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Watering a Lemon Tree: Heterogeneous Risk Taking and Monetary Policy Transmission

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JEL classification: E52, E58, G20

Abstract

We build a general equilibrium model with financial frictions that impede the effectiveness of monetary policy in stimulating output. Agents with heterogeneous productivity can increase investment by leveraging up, but this increases interim liquidity risk. In equilibrium, the more productive agents choose higher leverage, invest more, and take on higher liquidity risk. Therefore, these agents respond less than the agents with lower productivity to monetary policy that reduces the equilibrium interest rate. Overall quality of investment deteriorates, which can generate a negative spiral, dampening the effect of a monetary stimulus: Worse overall quality leads to lower liquidation values, increasing the cost of liquidity risk. This reduces the demand for loanable funds, further decreasing the interest rate, which then leads to further quality deterioration. When this feedback is strong, monetary policy can lose its effectiveness in stimulating aggregate output even if it leads to significant drops in the interest rate.

Key words: monetary policy transmission, financial frictions, heterogeneous agents, financial intermediaries

Choi, Eisenbach: Federal Reserve Bank of New York (e-mail: dongbeom.choi@ny.frb.org, thomas.eisenbach@ny.frb.org). Yorulmazer: University of Amsterdam (e-mail: t.yorulmazer@uva.nl). For helpful comments, the authors thank Viral Acharya, Tobias Adrian, Gara Afonso, Markus Brunnermeier, Eduardo Dávila, Mark Flannery, Douglas Gale, Itay Goldstein, Charles Kahn, Enrico Perotti, Jean-Charles Rochet, Tano Santos, Andrea Tambalotti, Jenny Tang (discussant), James Vickery, and Zhenyu Wang, as well as audiences at the Federal Reserve Bank of New York, the Federal Reserve System Committee on Macroeconomics, and the University of Amsterdam. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

1 Introduction

The experiences of the run-up to the financial crisis of 2007–09 and the following recession have raised important questions about the potential implications of loose monetary policy. The emerging concerns on the “risk-taking channel” of monetary policy (see e.g. [Borio and Zhu, 2012](#)), which focuses on the “quality” of the additional risk taking rather than the “quantity”, and the apparent paradox of a sluggish response in economic output concurrent with worrying signs of risk taking in financial markets have troubled policy makers (see e.g. [Stein, 2014](#); [Tarullo, 2014](#)). However, it is not obvious why policy makers should pay attention to the “quality” of risk-taking if the primary objective of loose monetary policy is stimulating investment to increase aggregate output. In this paper, we build a general equilibrium model with financial frictions and describe how the quality of overall investment deteriorates in response to monetary stimulus, and discuss how this can impede the potential for monetary policy to stimulate the economy. Aggregate output can become unresponsive to monetary stimulus even with significantly lowered interest rates due to a feedback between investment quality deterioration in response to the lower interest rate and decrease in aggregate loan demand further lowering the interest rate, leading to credit reallocation in the economy. Our mechanism is therefore different from both the conventional liquidity trap in which output becomes unresponsive to monetary stimulus because the interest rate becomes unresponsive as well as the balance sheet channel where monetary policy has an amplification effect.

The model features heterogeneous agents, all risk neutral, that differ in their productivity. Each agent has an initial endowment and can also borrow from other agents to invest more with an expected cost of borrowing equal to the risk-free interest rate. The risk-free rate is an equilibrium market clearing rate which equates the supply and demand of loanable funds in the economy and therefore equals the expected rate of return for lending.

In the absence of any financial friction only the most productive type invests—absorbing all the loanable funds—which maximizes aggregate output in this economy. However, as a first financial friction we assume that borrowing comes with interim liquidity risk. Investment projects are long-term but borrowing is short-term such that borrowing agents are subject to liquidity shocks at an interim date. When hit by the shock, an agent has to liquidate its assets in a secondary market at a discount. Furthermore, the liquidity shock becomes more likely with higher leverage, thus ex ante liquidity risk increases as agents lever up. This implies that the marginal cost of additional borrowing becomes higher as an

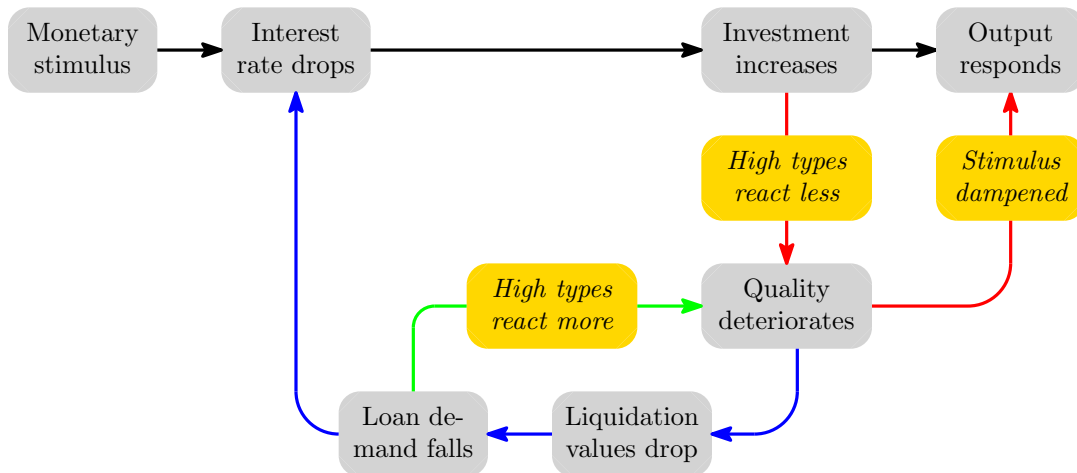


Figure 1: Negative feedback spirals dampening the effect of monetary policy

agent's leverage increases. In equilibrium, there is a marginal type, such that agents more productive than the marginal type borrow and invest, and agents less productive become lenders. Note that in equilibrium, an agent's marginal return from additional borrowing and investing, which is different across types, is equal to the marginal cost of funding, which is same across types, plus the endogenous cost of liquidity risk, which is increasing in leverage. Hence, among the borrowers, higher types can afford to take on more liquidity risk and in equilibrium they would borrow more and invest more.

Figure 1 illustrates the key mechanisms of the model. When the central bank provides monetary stimulus by injecting additional loanable funds into the economy, the market clearing interest rate drops, leading to an increase in investment which should raise output (black arrows). In our model, however, the agents' investment responds heterogeneously to the interest rate drop. Since high types are already exposed to significant liquidity risk, they are more reluctant to raise additional debt so a decrease in the interest rate raises low types' borrowing more than high types' (intensive margin). In addition, less productive types who were lenders start investing (extensive margin). Overall, a decrease in the risk-free rate shifts the distribution of investment towards lower types and the average quality of investment in the economy deteriorates. As a result, the stimulating effect of monetary policy on output is dampened (red arrows).

This opens the door for negative feedback effects between quality deterioration and lower loan demand when we introduce our second financial friction: a lemons problem in the secondary market for liquidated assets where the liquidation value depends on the

overall quality of investment, e.g. due to incomplete information, opaqueness or complexity of the underlying assets.¹ As discussed above, monetary stimulus reduces the quality of average investment which, in turn, leads to a decrease in the equilibrium liquidation value. This increases the cost of being hit by a liquidity shock and reduces borrowers' demand for funds, causing further downward pressure on the interest rate (blue arrows). Furthermore, when better quality borrowers react more to the decrease in the liquidation value, average quality deteriorates directly as well, which strengthens the lemons spiral (green arrow).

In this feedback process, the risk-free rate drops and the overall quality of investment deteriorates significantly as funds are reallocated from high types to low types. As a result, monetary stimulus can lead to a large drop in the interest rate but only a small increase (or potentially even a decrease) in aggregate output since total borrowing shifts from agents with high productivity to agents with low productivity. When the economy is trapped in this negative spiral, the effect of monetary loosening can be limited and in extreme cases, while actual investment goes up, output can go down since resources shift to low productivity types especially when leverage and liquidation risk are high.

We argue that the heterogenous responses of agents' loan demand to changes in the interest rate and the liquidation price generate and amplify the impairment of monetary stimulus transmission in general equilibrium. First, the stimulus effect becomes weaker when high types respond less to a drop in the interest rate compared to low types, leading to a larger deterioration of overall investment quality. This effect is more pronounced when liquidity risk increases steeply with leverage. The negative spiral arises when this quality deterioration leads to a decrease in the liquidation value. Second, the effect of monetary stimulus is further impaired as high types' demand becomes more responsive to changes in the liquidation price compared to low types' demand. If high types' loan demand decreases more than low types' when the liquidation price decreases, this itself reduces the liquidation value through further quality deterioration while making aggregate loan demand more inelastic. In Section 5, using a numerical example, we illustrate how

¹Morgan (2002) analyzes the pattern of disagreement between bond raters and provides evidence that banks and insurance firms are inherently more opaque than other types of firms. In particular, he shows that Moody's and S&P have split ratings more often over financial intermediaries. This uncertainty over banks stems from certain assets, loans and trading assets in particular, the risks of which are hard to observe or easy to change. Hirtle (2006) finds that characteristics related to opacity increased market reaction to CEO certification of bank financial statements, suggesting that this certification had greater value for market participants for more opaque banks. Flannery et al. (2013) examine bank equity's trading characteristics during "normal" periods and two "crisis" periods between 1993 and 2009. They find only limited (mixed) evidence that banks are unusually opaque during normal periods. However, crises raise the adverse selection costs of trading bank shares relative to those of nonbank control firms.

the dampening effect is stronger as the liquidity risk becomes more severe and more funds are injected into the economy, e.g. during the downturn as micro-founded in Appendix A, which coincides with recent empirical evidence.² This business cycle dependency implies that our credit-reallocation effect of monetary policy would not be symmetric between stimulus, which is usually implemented during the macroeconomic downturn, and tightening, which is implemented during the boom. We also analyze other policy options such as intervention in secondary markets to support liquidation values and discuss the possibility of positive feedback, which, when combined with the monetary stimulus, could help the central bank to stimulate the aggregate output by offsetting the negative spiral arising in our model.

Related literature: Our paper is related to various strands of the literature. Our transmission mechanism is related to both the credit channel and the risk taking channel of monetary policy. Endogenous credit reallocation is critical in our monetary transmission impairment. Our mechanism is different from the standard balance sheet channel (e.g. [Bernanke and Gertler 1989, 1995](#)) in which an external finance premium resulting from agency problems is the main driver. In that case, monetary policy has an amplification effect since it relaxes the financial constraints of borrowers, whereas in our case there are no agency problems and a *dampening* effect arises.³ Borrowers in our model can be viewed as financial intermediaries facing risks from maturity mismatch, and our model presents a novel distortion of monetary transmission through a bank lending channel ([Kashyap and Stein, 2000](#); [Peek and Rosengren, 2000](#)). [Benmelech and Bergman \(2012\)](#) also study how the real economy becomes unresponsive to monetary stimulus through financial frictions.

There is an emerging literature on the risk taking channel of monetary policy which focuses on how monetary policy affects the “quality” of lending rather than the “quantity” (for an overview, see [Borio and Zhu, 2012](#); [De Nicolò et al., 2010](#); [Adrian and Shin, 2010](#)). Empirical evidence relating monetary loosening and quality deterioration is documented by [Jiménez et al. \(2014\)](#); [Ioannidou et al. \(2009\)](#); [Peydró and Maddaloni \(2011\)](#); [Paligoro and Santos \(2012\)](#); [Altunbas and Marques-Ibanez \(2011\)](#); [Dell’Ariccia et al. \(2013\)](#). However, theoretical understanding of this channel is lagging behind, both in terms of

²[Tenreyro and Thwaites \(2015\)](#) show that monetary stimulus is less effective in recessions than in expansions and [Bech et al. \(2014\)](#) document that the stimulus effect is further impaired when associated with a financial crisis.

³Thus, agents in a standard setup are financially constrained and the shadow costs of capital are different across the agents in equilibrium. In our setup, agents are making an unconstrained decision and thus the marginal costs of capital are equalized in equilibrium.

mechanisms and potential social costs.⁴ Our model explains both the mechanism and the cost of the quality deterioration—it can make monetary stimulus ineffective through the credit channel.

Our paper is also related to the literature on fire sales and costly liquidation of assets. The idea that fire sales can occur when potential buyers are financially constrained and assets are not easily deployable were shown by [Williamson \(1988\)](#) and [Shleifer and Vishny \(1992\)](#). [Holmström and Tirole \(1998\)](#) study an ex-ante investment decision facing this interim risk, and [Allen and Gale \(1994, 1998\)](#) feature models where the price of assets are determined by the level of liquidity in the market resulting in cash-in-the-market pricing. There is strong empirical support for this idea in the corporate-finance literature, such as [Pulvino \(1998\)](#), [Acharya et al. \(2006\)](#), [Berger et al. \(1996\)](#) and [Stromberg \(2000\)](#). The evidence of such specificity for banks and financial institutions is studied by [James \(1991\)](#). [Rosenthal and Wang \(1993\)](#) use a model in which sellers may not be able to extract the fundamental value due to the informational rents earned by the privately informed bidders.⁵

Finally, our paper contributes to the broad literature on incorporating financial frictions into macroeconomic analysis. In particular, we analyze how the frictions in the secondary market generates macro effects. [Kiyotaki and Moore \(1997\)](#) study the effect of resalability of financial assets in secondary markets on aggregate investments, and [Kurlat \(2013\)](#) builds a model in which this friction comes from a lemons problem in the secondary market. [Bolton et al. \(2011\)](#) and [Malherbe \(2014\)](#) also study an economy in which incomplete information in the secondary market affects investment decisions. For a general review, see e.g. [Brunnermeier et al. \(2013\)](#).

The paper is organized as follows. Section 2 discusses the model setup. Section 3 analyzes the leverage and investment decisions of individual agents, and analyzes the effects of interest rates and asset prices on such choices. Section 4 analyzes the mechanism of monetary transmission impairment in a general equilibrium setup. Section 5 illustrates

⁴[Dell’Ariccia et al. \(2014\)](#) is the only theoretical study to our knowledge.

⁵Fire sales and increased haircuts were commonplace during the recent crisis. [Shin \(2009\)](#) documents based on data from Bloomberg, that the typical haircuts on treasuries, corporate bonds, AAA asset-backed securities, AAA residential mortgage-backed securities and AAA jumbo prime mortgages are respectively, less than 0.5%, 5%, 3%, 2% and 5%, whereas, in March 2008, these haircuts respectively rose to between 0.25% and 3%, 10%, 15%, 20% and 30%. [Gorton and Metrick \(2010, 2012\)](#) analyze haircuts in an inter-dealer market for less liquid collateral and show that during 2007-08, the repo haircuts on a variety of assets rose on average from zero in early 2007 to nearly 50 percent in late 2008. They also report that some collateralized debt obligations could not be financed at all (100% haircut) during the crisis.

the model with a numerical example. Section 6 discusses policy implications and the model's assumptions and Section 7 concludes. All proofs are in Appendix B.

2 Model setup

Primitives: Consider a model with three dates $t = 0, 1, 2$. There is a continuum of agents with measure 1 indexed by their type $\theta \in [0, 1]$, which reflects heterogeneous productivity across the agents. All agents are risk neutral and have discount factors of 1. At $t = 0$, each agent has an endowment of E and access to a type-specific investment technology with constant returns to scale which pays off a random return at $t = 2$. Specifically, one unit of investment implemented by an agent of type θ yields $R\theta$ with probability p and 0 with probability $1 - p$. Thus, higher θ implies higher individual productivity whereas p and R capture aggregate productivity variables, such as TFP, which are the same across types.⁶

Borrowing/lending: At $t = 0$, agents can increase their total investment by borrowing on top of their endowment. Let D_θ denote the amount agent θ borrows at $t = 0$, then the agent's total investment is $I_\theta = E + D_\theta$. Agents can alternatively choose to lend their initial endowment E in case they decide not to take the risky investment at $t = 0$, which corresponds to $D_\theta < 0$. Lenders are competitive so the interest rate r_θ promised by a borrower of type θ guarantees that all lenders receive the risk-free rate r in expectation. The risk-free rate r , in turn, is determined endogenously by market clearing in the market for loanable funds at $t = 0$.

Note that total investment in this economy at $t = 0$, denoted by I , can be written as:

$$I = \int_0^1 I_\theta d\theta = \int_0^1 (D_\theta + E) d\theta,$$

and taking the heterogeneous productivity into account, the average quality (productivity) of investment is given by

$$q = \frac{\int_0^1 pR\theta (D_\theta + E) d\theta}{\int_0^1 (D_\theta + E) d\theta}, \quad (1)$$

which is a function of D_θ .

⁶Only the product $p \times R$ matters in our analysis with risk neutral agents, while the separation of p and R becomes useful when we provide a micro-foundation of the run risks in Appendix A.

Liquidity risk: The key friction in our setup is that borrowers face liquidity risk in the interim period $t = 1$ which is a function of leverage. As micro-founded in Appendix A, borrowing is short-term and needs to be rolled over at $t = 1$, which creates the potential for liquidity problems.⁷ When experiencing an interim liquidity problem, a borrower of type θ is forced to liquidate the long-term assets in a secondary market at a discount, which is costly for the agent. We assume that the probability of an interim liquidity problem increases with leverage, thus debt becomes endogenously more costly as one increases leverage although its expected rate of return to the lender is given by r .

For simplicity, we focus on a particular source of interim liquidity risk—liquidity run risk—in a reduced-form setup micro-founded in Appendix A.⁸ We denote by $\alpha(D)$ the ex-ante, as of $t = 0$, probability that an agent with debt level D experiences a run at $t = 1$.⁹ When hit by this liquidity run, the agent is forced to liquidate all asset holdings at a discounted price P per unit which is identical across the agents.¹⁰ We assume the following properties of $\alpha(D)$:

1. $\alpha'(D) \geq 0$ (increasing liquidity risk driven by leverage).
2. $\alpha''(D) \geq 0$ (weakly convex liquidity risk driven by leverage).
3. $\alpha(D) = 0$ for $D \leq 0$ (no liquidity risk for the lenders)
4. $\alpha'(0) = 0$ (regularity condition), $\alpha'''(D) \approx 0$ (this condition is for simplicity)

Liquidation values: We focus on the secondary market with financial frictions. Our benchmark case, in particular, considers a liquidation market with incomplete information such that potential buyers can't distinguish the sellers of the liquidated assets, but only observe the overall quality of investment in the economy defined as q in equation (1). Formally, we assume lemons pricing due to incomplete information such that $P = f(q)$ with $f' \geq 0$ and $f(q) < 1 + r$ for all q . Since the average investment quality q depends on the distribution of D_θ across different types, P becomes higher (lower) as high (low) types invest more. We also consider different price functions with different financial frictions in

⁷This is a typical maturity-mismatch problem that financial institutions face, thus we can consider the borrowers in our model as “banks”.

⁸Any other type of interim liquidity risk that is increasing (in a weakly convex way) in leverage qualitatively generates the same results.

⁹Kashyap et al. (2014) adopt a similar reduced-form setup where interim liquidity risk is captured by ex-ante probability of a run, which is a function of balance-sheet entries.

¹⁰This implies an anonymous secondary market. We also consider a case with different liquidation prices across types in Section 6.3.

Section 6.3 such as cash-in-the-market pricing (Allen and Gale, 1994). We assume that no output is lost through the secondary market liquidation process, thus only transfers occur. This assumption implies that the dampening effect we show is *not* due to resources lost in inefficient liquidation; the effect is due purely to changes in the equilibrium distribution of borrowing levels $\{D_\theta\}$ across types.

Monetary policy: In addition to the agents’ initial endowment E , the central bank provides liquidity L to the market for loanable funds at $t = 0$. The equilibrium risk-free rate r then equates aggregate loan supply, consisting of the public supply L and the private supply from lending agents, and aggregate loan demand, consisting of productive agents’ borrowing. We identify monetary policy as changes in central bank exogenous liquidity supply L .¹¹ Note that in this setup the central bank can create resources at $t = 0$ which are then invested by borrowers and produce output at $t = 2$. As in Allen et al. (2014) and Keister (2014), we assume that monetary stimulus at $t = 0$ has costs at $t = 2$ given by $c(L)$ which is increasing in L to ensure that monetary policy is not a “free lunch”. Although not explicitly modeled in this paper, these costs can be interpreted as, e.g., welfare losses from nominal price distortions or additional taxes necessary to meet the government’s consolidated budget constraint, or less public goods provision as in Allen et al. (2014) and Keister (2014).

Definition of equilibrium: The equilibrium of our economy is characterized by the private decision variables $\{D_\theta\}$ and the price variables r and P satisfying the following conditions:

1. Agents choose optimal borrowing/lending $D_\theta(r, P)$ taking prices r and P as given.
2. The risk-free rate r clears the market for loanable funds:

$$E + L = \int_0^1 D_\theta(r, P) d\theta$$

3. The secondary market price P satisfies the pricing rule given the private decision variables $\{D_\theta\}$ such that $P = f(q)$ where q is defined by (1).

¹¹This setup is for expositional purposes, and targeting L is equivalent to having the central bank target r since there is a one-to-one relation between L and r in equilibrium. If we adopt the conventional bank lending channel interpretation, L could be referred to as aggregate reserves divided by the reserve requirement that the central bank can adjust, and r as the interbank lending rate.

3 Individual agent behavior

We first analyze the optimizing behavior of individual agents taking prices r and P as given. Our analysis specifically focuses on how different types change their $t = 0$ leverage—and therefore investment levels—differently in response to changes in these prices. We first show that more productive types react less elastically to changes in the interest rate r . We then discuss how changes in the liquidation value P affect different types differently.

We now describe and solve for the individual agent’s optimization problem at $t = 0$. Given the initial endowment E , each agent chooses either to be a “lender” or a “borrower”, and how much to lend or borrow taking the prices P and r as given. Formally, agent θ chooses an optimal $D_\theta \geq -E$ to maximize his expected payoff $\Pi(D; r, P)$ which is defined as

$$\Pi(D; r, P) = \alpha(D) (D + E) P + (1 - \alpha(D)) (D + E) pR\theta - (1 + r) D \quad (2)$$

where $D > 0$ when borrowing, and $D < 0$ when lending with $\alpha(D) = 0$ for $D \leq 0$. Note that the equilibrium market clearing rate $1 + r$ is the expected rate of return for lending and the expected cost of borrowing, common across all agents in this economy. Therefore a borrower’s expected payoff can be simply written as the total expected payoff from the investment (the first two terms in (2)) minus the expected cost of debt (the last term) since all agents are risk neutral and the loanable funds market is competitive with no agency problem. When an agent θ borrows D in addition to his endowment to invest $D + E$, he ex-ante anticipates that the liquidity run occurs at $t = 1$ with probability $\alpha(D)$ leaving only P per unit of investment whereas he expects to collect $pR\theta$ when he does not experience a run. When lending, an agent compares the expected return of lending $1 + r$ and that of investing $pR\theta$. This immediately implies that the risk-free rate determines the marginal type separating agents into borrowers and lenders.

Lemma 1. *Given r , there exists θ^* such that types $\theta > \theta^*$ become borrowers and types $\theta < \theta^*$ become lenders where θ^* satisfies $1 + r = pR\theta^*$.*

Given the expected borrowing cost $1 + r$, agents compare their marginal product from the investment to this marginal cost of funding. Agents with high productivity choose to produce and borrow, while those with low productivity choose not to produce and lend their endowment instead. The marginal producer’s marginal product is equal to the marginal cost of funding when there is no liquidity risk. It is obvious that less productive agents lend all their endowment to the borrowers ($D_\theta = -E$ for $\theta < \theta^*$). Figure 2 illustrates the expected payoff as a function of D for different types.

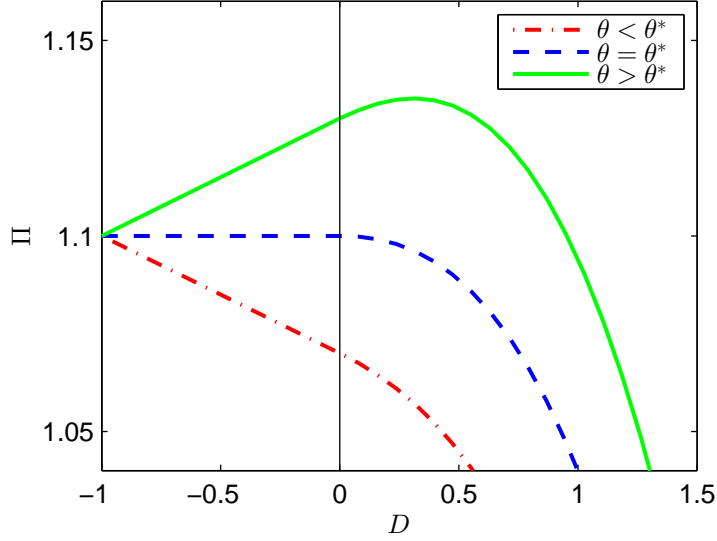


Figure 2: Expected payoff Π as a function of borrowing D for different types θ . The functional forms and parameter values used are the same as in Section 5 with $a = 0.1$, $r = 0.1$ and $P = 0.8$.

Borrower θ 's optimal loan demand D_θ can be derived from the first order condition¹²

$$\underbrace{pR\theta}_{\text{marginal product}} - \underbrace{(\alpha'(D_\theta)(D_\theta + E) + \alpha(D_\theta))}_{\text{type-specific liquidity risk premium}} (pR\theta - P) = \underbrace{1 + r}_{\text{marginal funding cost}}, \quad (4)$$

which characterizes how borrowers build up liquidity risk endogenously as they increase borrowing. Without the liquidity risk, a productive agent should keep on increasing its investment as long as the funding cost is lower than its marginal product. However, liquidity risk also increases as leverage goes up making the additional borrowing more costly. In equilibrium, the wedge between the marginal product of investment and the marginal funding cost, $pR\theta - (1 + r)$, is filled with the type-specific liquidity risk premium (Figure 3). Note that this wedge becomes larger when the interest rate is lower, so that agents have “room” to take additional liquidity risk when the funding cost is lower. Also note that this wedge is larger for more productive agents, thus they can take more liquidity risk by building up higher leverage. In equilibrium, more productive agents hold larger

¹²The second order condition is satisfied with weakly convex and non-decreasing α :

$$-(\alpha''(D_\theta)(D_\theta + E) + 2\alpha'(D_\theta))(pR\theta - P) < 0 \quad (3)$$

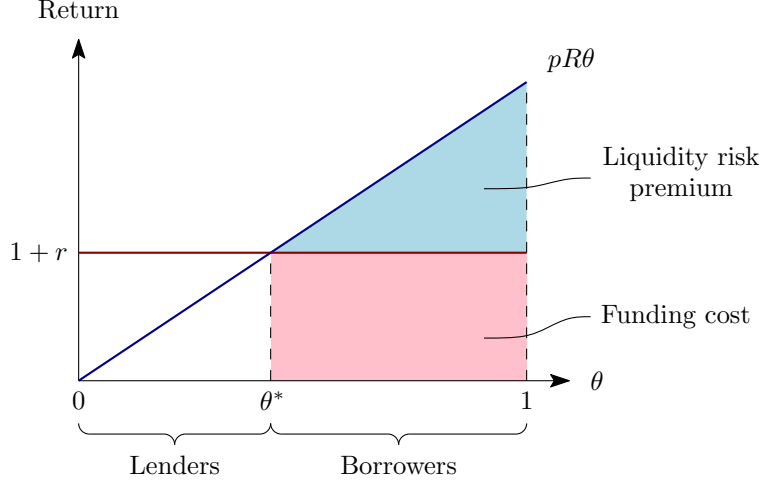


Figure 3: Optimal liquidity risk exposure

debt outstanding, filling the wedge with larger liquidity risk premium.¹³

Proposition 1. *For given r and P , more productive agents borrow more than less productive agents, i.e. D_θ is strictly increasing in θ for all $\theta > \theta^*$.*

We now analyze how different types respond differently to the changes in the interest rate. A decrease in the interest rate r leads all borrowers to lever up. Note that we have the following from the implicit function theorem:

$$\frac{\partial D_\theta}{\partial r} = \frac{-1}{(\alpha''(D_\theta)(D_\theta + E) + 2\alpha'(D_\theta))(pR\theta - P)} < 0$$

In addition, it is clear that $\partial D_\theta / \partial r$ is increasing in θ under our assumption of $\alpha''(D) \geq 0$ and $\alpha'''(D) \approx 0$ since D_θ is increasing in θ . We therefore know that the productive borrowers respond less elastically to the changes in the interest rate than the less productive borrowers. In summary, we have the following proposition.

Proposition 2. *For a reduction in r , all borrowers increase their debt, i.e. $\partial D_\theta / \partial r < 0$ for all $\theta > \theta^*$. Agents with high productivity respond less than agents with low productivity, i.e. $|\partial D_\theta / \partial r|$ is decreasing in θ for all $\theta > \theta^*$.*

¹³Note that there is no borrowing constraint or agency problem, thus there is no external finance premium that could be different across types, unlike in the conventional credit channel. The marginal funding cost is equal to $1 + r$ for all agents with the binding first order condition, thus agents are not financially constrained in our setup.

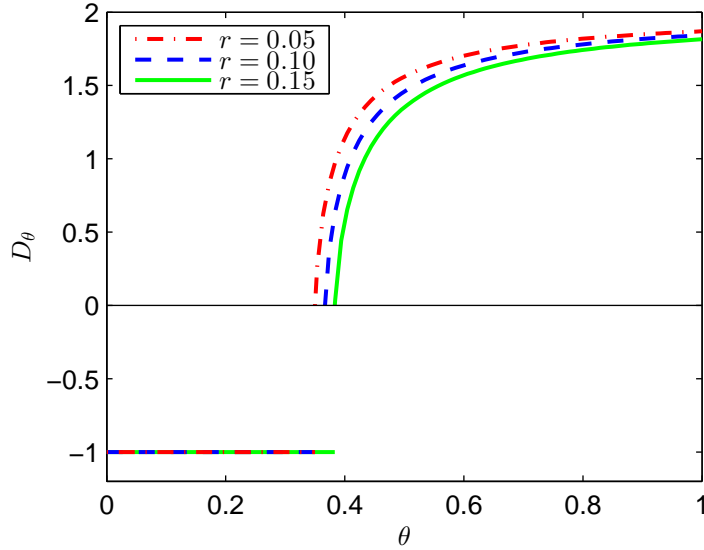


Figure 4: Optimal borrowing D_θ as a function of type θ for different interest rates r . The functional forms and parameter values used are the same as in Section 5 with $a = 0.1$ and $P = 0.8$.

Figure 4 illustrates the optimal borrowing D_θ as a function of type θ for different levels of r . Economically, changes in the interest rate affect the difference between the marginal product and the marginal funding cost by the same amount for all agents, and they adjust their leverage accordingly to fill the wedge. High- θ borrowers adjust less than low- θ borrowers since their liquidity risk premium responds more sensitively because of two factors: (i) convex liquidity risk α and (ii) different per-unit financial loss $pR\theta - P$ given the forced liquidation which is increasing in θ .

As an example, consider a decrease in r , which decreases the marginal funding cost on the RHS of equation (4). Borrowers increase their debt levels until the liquidity risk premium on the LHS of equation (4) rises sufficiently to fill the wedge as illustrated in Figure 5. For a unit increase in debt holdings, the liquidity risk premium of higher types rises faster than that of the lower types. In other words, the high types are more reluctant to increase their debt since they are already taking higher leverage and the additional increase in leverage is more costly for them. Thus, a lower type's increase in borrowing is larger than that of a higher type for the same decrease in the interest rate.

We next analyze how agents respond to the changes in the secondary market price P .

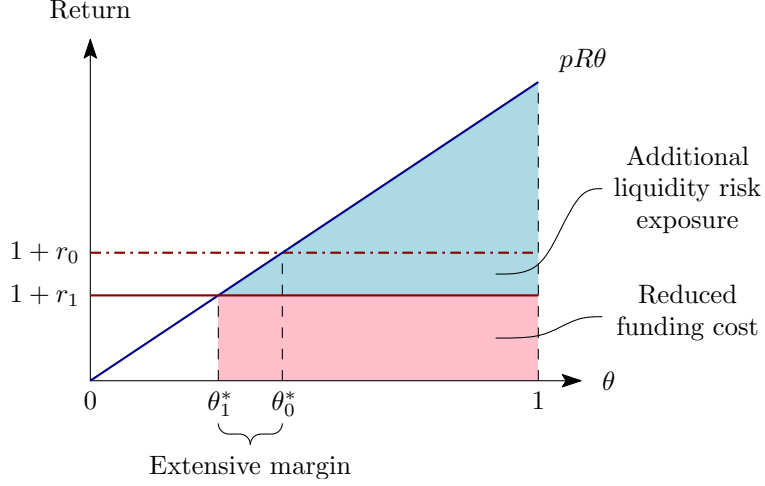


Figure 5: Effect of lower interest rate on equilibrium liquidity risk exposure

From the implicit function theorem, we have

$$\frac{\partial D_\theta}{\partial P} = \frac{\alpha'(D_\theta)(D_\theta + E) + \alpha(D_\theta)}{(\alpha''(D_\theta)(D_\theta + E) + 2\alpha'(D_\theta))(pR\theta - P)}$$

and the following proposition holds.

Proposition 3. *For a reduction in P , all borrowers decrease their debt, i.e. $\partial D_\theta / \partial P > 0$ for all $\theta > \theta^*$. However, the heterogeneity in response is ambiguous, i.e. $\partial D_\theta / \partial P$ can be increasing or decreasing in θ .*

Figure 6 illustrates the optimal borrowing D_θ as a function of type θ for different liquidation prices P . Changes in P affect the loss in case of forced liquidation and thus the liquidity risk premium. Higher P implies lower cost of liquidation and agents lever up to satisfy (4). If P decreases, on the contrary, agents delever to reduce their liquidity risk exposure.

Unlike the interest rate response, it is not obvious which type responds more strongly to changes in P . However, we show the following corollary comparing the relative cross-section effects of the two variables r and P .

Corollary 1. $\left| \frac{\partial D_{\theta_H} / \partial r}{\partial D_{\theta_L} / \partial r} \right| < \left| \frac{\partial D_{\theta_H} / \partial P}{\partial D_{\theta_L} / \partial P} \right|$ holds for any $\theta_H > \theta_L > \theta^*$.

As discussed before, a change in r generates the same amount of slack in the RHS of (4) across different types. A change in P , however, generates different amounts of slack across different types. Indeed, it generates larger slack for higher types since it directly affects liquidity risk premium whereas interest rate changes only affects the cost of funding.

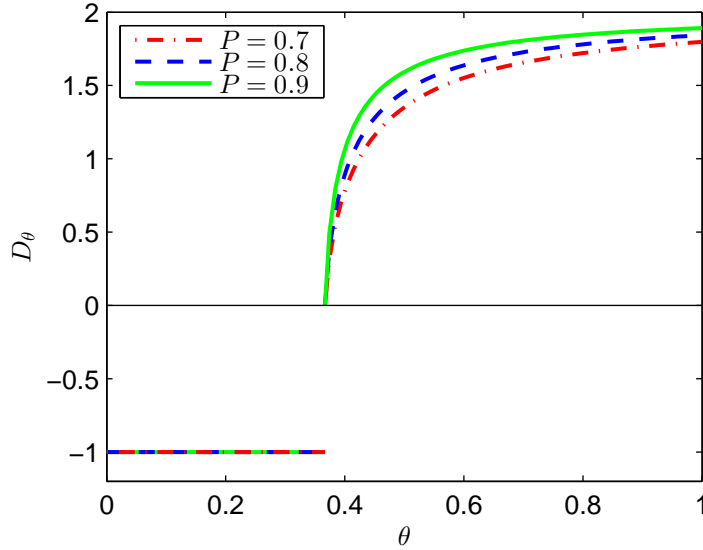


Figure 6: Optimal borrowing D_θ as a function of type θ for different liquidation values P . The functional forms and parameter values used are the same as in Section 5 with $a = 0.1$ and $r = 0.1$.

4 Monetary policy with heterogeneous risk taking

We are interested in the effects of monetary policy in the initial period $t = 0$ on aggregate output in the final period $t = 2$. When additional liquidity ΔL is injected, aggregate investment increases by the same amount, such that $\Delta I = \Delta L$. In the first-best case—if there were no frictions—the most productive agent with $\theta = 1$ would absorb all additional funds and aggregate output would increase by $pR\Delta L$ (ignoring the policy cost $c(L)$).

In our analysis, however, financial frictions can critically distort the transmission mechanism through resource reallocation. Since agents in our model are heterogeneous in their investment productivity, changes in aggregate output also depend on how the distribution of initial investment across different types changes. Therefore we have two channels of monetary policy: Monetary policy—a change in L —affects aggregate output (i) through its effect on aggregate investment—a change in I —and (ii) through its effect on the average quality of investment—a change in q .

As shown in Lemma 1, agents are split endogenously into borrowers and lenders around a marginal type θ^* such that total investment at $t = 0$ can be written as:

$$I = \int_{\theta^*}^1 (D_\theta + E) d\theta$$

and the average quality of investment can be written as:

$$q = \frac{\int_{\theta^*}^1 pR\theta (D_\theta + E) d\theta}{\int_{\theta^*}^1 (D_\theta + E) d\theta} \quad (5)$$

Remember that we assume no output is lost through the secondary market liquidation process in the interim period $t = 1$. Aggregate output in the final period $t = 2$ can therefore be written as the average quality of investment times the aggregate amount invested:

$$\begin{aligned} Y &= \int_{\theta^*}^1 pR\theta (D_\theta + E) d\theta \\ &= q \times I \end{aligned}$$

Denoting output net of the costs of monetary policy by $\bar{Y} = Y - c(L)$, the effect of monetary policy in the form of changes in central bank liquidity L can then be decomposed into three parts:

$$\frac{d\bar{Y}}{dL} = \underbrace{q \times \frac{dI}{dL}}_{\text{new investment}} + \underbrace{\frac{dq}{dL} \times I}_{\text{change in quality}} - \underbrace{c'(L)}_{\text{marginal cost}}$$

The first and third part are straightforward and standard. In fact, in our model total investment equals total loanable funds, $I = L + E$, so investment changes one-for-one with monetary policy, $dI/dL = 1$.¹⁴ Our focus is therefore the second part, how monetary policy affects the average quality of investment. While the effect on aggregate investment is always positive, the effect on average quality can be negative, dampening the effectiveness of monetary policy. If quality deteriorates sufficiently, it may even reverse the effect of monetary stimulus on output.

We can decompose the quality effect as follows:

$$\frac{dq}{dL} = \underbrace{\frac{dq}{dr}}_{\text{“quality elasticity”}} \times \underbrace{\frac{dr}{dL}}_{\text{“stimulus pass-through”}} \quad (6)$$

Monetary policy affects the average quality of investment through its effect on the equilibrium risk-free rate which, in turn, affects average quality. If the first factor in the decomposition (6) – which we refer to as “quality elasticity” – is positive and the second

¹⁴We don’t have any hoarding of liquidity which would reduce investment, e.g. as in [Diamond and Rajan \(2011\)](#) or [Gale and Yorulmazer \(2013\)](#).

factor – which we refer to as “stimulus pass-through” – is negative, monetary stimulus decreases the interest rate but at the same time lowers the quality of investment. Digging deeper into these two parts highlights the effects of our model and the mechanism of negative feedback between the two factors, (i) investment quality deterioration in response to the lower interest rate, and (ii) decrease in aggregate loan demand in response to the quality deterioration, leading to a further decrease of the interest rate.

4.1 Quality elasticity

First, consider the quality elasticity, i.e. the effect of the risk-free rate r on the average quality of investment q . Our analysis focuses on when this factor becomes positive and large, implying that a decrease in the interest rate leads to a large deterioration in investment quality.

As illustrated in the expression for q in (5), average quality is determined by the distribution of borrowing D_θ across types θ . The optimal borrowing, in turn, depends on the risk-free rate r as well as the secondary-market price P . When the secondary market price is an endogenous variable, we can further decompose the quality elasticity into a direct and an indirect effect:

$$\frac{dq}{dr} = \underbrace{\frac{\partial q}{\partial r}}_{\text{direct effect}} + \underbrace{\frac{\partial q}{\partial P} \times \frac{dP}{dr}}_{\text{indirect effect}} \quad (7)$$

The direct effect characterizes how overall quality changes in response to the change in r , holding P fixed. The answer depends on how loan demand changes differently across types. Using the expression for average quality in (5), we see that the direct effect of r on q consists of an extensive margin and an intensive margin:

$$\frac{\partial q}{\partial r} = \underbrace{\frac{(q - pR\theta^*) (D_{\theta^*} + E) \frac{d\theta^*}{dr}}{\int_{\theta^*}^1 (D_\theta + E) d\theta}}_{\text{extensive margin}} - \underbrace{\frac{\int_{\theta^*}^1 (q - pR\theta) \frac{\partial D_\theta}{\partial r} d\theta}{\int_{\theta^*}^1 (D_\theta + E) d\theta}}_{\text{intensive margin}} \quad (8)$$

The first term reflects a change of the marginal type θ^* in response to changes in r . Since the quality of the average borrower’s investment is always higher than that of the marginal borrower, $q > pR\theta^*$, the sign of the effect of r on the extensive margin of quality is the same as the sign of the effect on the marginal type $d\theta^*/dr$, which is positive.

The interesting part is the sign of the intensive-margin effect, that is how different

types respond to changes in r , which is determined by the integral in the numerator of the respective expression. Intuitively, for a lower interest rate, average quality should decrease (increase) if D_θ increases more for low (high) θ types. Formally, note the two factors integrated over: The first factor, $q - pR\theta$, is linear in θ , positive at the lower bound of the integral and negative at the upper bound; since q is biased upward, integrating only over $q - pR\theta$ would yield a positive result. The second factor, $\partial D_\theta/\partial r$, the direct effect of the risk-free rate r on the borrowing D_θ of type θ is negative; this factor plays the role of a weighting of different types, determining whether the positive or the negative part of $q - pR\theta$ dominates the integral. The weighting and ultimately the sign of the intensive-margin effect therefore depends on differences in sensitivity across types. Since Proposition 2 shows that $|\partial D_\theta/\partial r|$ is decreasing in θ , i.e. high types are less sensitive to interest rate changes, we have that $\partial q/\partial r$ is positive. Therefore, overall investment quality deteriorates through both extensive and intensive margins when the interest rate decreases.

Corollary 2. *The direct effect of a decline in the interest rate r is a deterioration in average investment quality q , i.e., $\partial q/\partial r > 0$.*

The indirect effect through P in equation (7) combines the response of quality to the liquidation value, $\partial q/\partial P$, with the change in the equilibrium liquidation value in response to a change in the risk-free rate, dP/dr . Since, as we show later in Section 4.3, P and r co-move in equilibrium, i.e. $dP/dr > 0$, the sign of the indirect effect depends on $\partial q/\partial P$. The marginal type θ^* doesn't depend on P , so the effect works purely through the intensive margin, i.e. how different types respond differently to changes in P :

$$\frac{\partial q}{\partial P} = - \frac{\int_{\theta^*}^1 (q - pR\theta) \frac{\partial D_\theta}{\partial P} d\theta}{\underbrace{\int_{\theta^*}^1 (D_\theta + E) d\theta}_{\text{intensive margin}}}$$

As in the case of the direct effect of the risk-free rate on quality, $\partial q/\partial r$ in (8), the sensitivity of each type θ to the liquidation value, $\partial D_\theta/\partial P$, plays the role of a weighting of the different types, determining whether the integral in the numerator is positive or negative. Again, the difference in sensitivity across types, how $\partial D_\theta/\partial P$ changes with θ , is a key element; average quality decreases if high- θ types reduce their borrowing more than low- θ types in response to a lower liquidation price. The heterogenous response of different types to changes in P is not a necessary element of our dampening mechanism but can make it stronger when combined with the direct effect $\partial q/\partial r$.

Corollary 3. *If high types respond sufficiently strongly to changes in the liquidation value P , the indirect effect through P amplifies the quality deterioration after a decline in the interest rate.*

In sum, the quality elasticity is positive as low-productivity agents respond more to interest rate changes than high-productivity agents (red arrow in Figure 1), and this effect becomes stronger when the liquidation price P drops with monetary stimulus and high productivity types are *more* sensitive to changes in P (green arrow in Figure 1). We now analyze the second factor in the main decomposition (6), showing that quality deterioration may be amplified by depressing aggregate loan demand and lowering the market clearing interest rate further (blue arrow in Figure 1).

4.2 Stimulus pass-through

In this section, we analyze the stimulus pass-through, i.e. the effect of a liquidity injection L on the interest rate r , to complete our analysis of the effect of monetary policy on quality deterioration. Note that the market clearing condition equating supply and demand of loanable funds is given by:

$$L + E = \int_{\theta^*}^1 (D_\theta + E) d\theta$$

Implicit differentiation yields the equilibrium stimulus pass-through as the inverse of the effect of r on the aggregate demand for loanable funds which is composed of a change in the extensive margin and the intensive margin:

$$\begin{aligned} \frac{dr}{dL} &= \left(\frac{d}{dr} \int_{\theta^*}^1 (D_\theta + E) d\theta \right)^{-1} \\ &= \left(\underbrace{-(D_{\theta^*} + E) \frac{d\theta^*}{dr}}_{\text{extensive margin}} + \underbrace{\int_{\theta^*}^1 \frac{dD_\theta}{dr} d\theta}_{\text{intensive margin}} \right)^{-1} = \left(-\frac{E}{pR} + \int_{\theta^*}^1 \frac{dD_\theta}{dr} d\theta \right)^{-1} \end{aligned} \quad (9)$$

since $D_{\theta^*} = 0$ for the marginal type and $d\theta^*/dr = -E/pR$ from Lemma 1. This implies that when additional funds are injected, the market clearing interest rate drops more if aggregate loan demand is less elastic. The extensive margin effect is standard and negative: fewer agents want to borrow if the interest rate increases. The interesting effect is the intensive margin, which captures the leverage changes of all infra-marginal borrowers. Given the dependence of optimal borrowing D_θ on the risk-free rate r and the price P ,

the change in leverage goes through two channels:

$$\frac{dD_\theta}{dr} = \underbrace{\frac{\partial D_\theta}{\partial r}}_{\text{direct effect}} + \underbrace{\frac{\partial D_\theta}{\partial P} \times \frac{dP}{dr}}_{\text{indirect effect}} \quad (10)$$

The direct effect is the standard price effect where each borrower borrows more for a lower interest rate, $\partial D_\theta / \partial r < 0$. However, the indirect effect can work *against* the direct effect when it has a positive sign. Since the liquidation value P captures how costly a liquidity shock is, agents borrow less for a lower liquidation value, that is, $\partial D_\theta / \partial P > 0$, as shown in Proposition 3. The indirect effect through P reduces the responsiveness of borrower demand to changes in the interest rate since P and r co-move in equilibrium, i.e. $dP/dr > 0$, as we show in Section 4.3. Hence, agents' loan demand becomes less elastic, and the equilibrium market clearing interest rate becomes more sensitive in response to loanable funds injection; the interest rate drops more for the same amount of fund injections when the indirect effect dampens the aggregate loan demand.

Corollary 4. *The indirect effect through the liquidation value P amplifies the stimulus pass-through by making loan demand less elastic.*

Note that even with P exogenously fixed, our model yields similar effects of monetary policy through changes in the average borrower quality. With fixed P , we have $dP/dr = 0$ in equations (7) and (10) and therefore equation (6) simplifies to

$$\begin{aligned} \frac{dq}{dL} &= \frac{dq}{dr} \times \frac{dr}{dL} \\ &= \frac{\partial q}{\partial r} \times \left(-\frac{E}{pR} + \int_{\theta^*}^1 \frac{\partial D_\theta}{\partial r} d\theta \right)^{-1}, \end{aligned}$$

with no second round effect through changes in P . We already showed that $\partial q / \partial r > 0$ (Corollary 2) and the second term is obviously negative, resulting in the negative effect of loosening monetary policy on asset quality and output, even though the effect is stronger when price changes depress loan demand further. In the next Section, we show that the indirect effect of the risk-free rate r on quality q and therefore output Y that works through the price P strengthens the impairment of monetary policy transmission.

4.3 Feedback through liquidation values

We didn't assume any specific secondary market pricing rule in the analysis of Section 4 so far, and thus our dampening mechanism can arise with various alternative frictions in the secondary market. We now examine the case of incomplete information in the secondary market in which underlying assets could be considered as "opaque" or "complex". Under such lemons pricing a negative spiral arises between average investment quality and the elasticity of aggregate loan demand with the potential to severely strengthen the dampening mechanism.

As described in Section 2, we assume that buyers in the secondary market in $t = 1$ cannot observe the sellers' types but know the average quality of investment in the economy q . The secondary market price therefore reflects this average quality such that $P = f(q)$ with $f'(q) \geq 0$.

Proposition 2 and Corollary 2 imply that heterogeneous responses to monetary stimulus lower the aggregate quality of investment q in a partial equilibrium sense, and thus the price P in the secondary market. This leads to a further general equilibrium effect; aggregate loan demand is squeezed due to increased liquidation costs (Proposition 3), deleveraging pressure arises that further lowers the market clearing interest rate, and this lower interest rate again leads to the heterogeneous responses lowering the quality and the liquidation price further generating a feedback loop. We now revisit the analysis of Section 4 and discuss what makes this dampening effect stronger in general equilibrium (characterized by equation (6)).

Note that average quality q is a function of each type's optimal debt level D_θ , thus depends on the risk-free rate r as well as the liquidation value P . The equilibrium liquidation value is therefore implicitly defined by the fixed-point condition

$$P = f(q(r, P)) \tag{11}$$

We focus on the case with a stable fixed point satisfying $f'(q) \partial q / \partial P < 1$. Given this implicit definition of P in (11), the equilibrium effect of r on P is given by

$$\frac{dP}{dr} = \frac{f'(q) \partial q / \partial r}{1 - f'(q) \partial q / \partial P} \tag{12}$$

Corollary 5. *In equilibrium, the liquidation value P and the interest rate r are positively related, i.e. $dP/dr > 0$.*

The equilibrium co-movement between P and r is necessary for the amplification effects in both the quality elasticity and the stimulus pass-through. Note that given the expression for dP/dr , these amplification effects are stronger when $\partial q/\partial r$, $\partial q/\partial P$ and/or $f'(q)$ is larger.

In sum, combining the effects of r and P on average quality q we see that our model can generate a strong spiral. Injections of liquidity increase the supply of loanable funds which puts downward pressure on the interest rate. Any reduction in the interest rate leads worse borrowers to lever up relatively more than higher quality borrowers leading to a deterioration in the expected quality of assets sold in the secondary market at $t = 1$. This worse quality leads to a decrease in the liquidation value which reduces borrowers' demand for funds, causing further downward pressure on the the interest rate. If, in addition, better borrowers react more to the decrease in the liquidation value, average quality deteriorates directly (through r) as well as indirectly (through P) which strengthens the spiral further. The overall effect of the liquidity injection is then a large drop in the interest rate but only a small increase or potentially even a decrease in total output since total borrowing shifts from agents with high productivity to agents with low productivity.

Suppose that the central bank wishes to induce further risk taking of productive high- θ types without affecting less productive low- θ types. Corollary 1 implies that this goal can be achieved more effectively by raising P than lowering r as liquidity provision in the secondary market directly affects the liquidity risk, our primary source of the financial friction. When the liquidity risk is critical, the productive types don't respond to the interest rate decrease since they are already highly levered and any additional borrowing is very costly. Raising P generates larger room for additional risk taking for the productive types by directly reducing the liquidity risk premium. We come back to these issues when we discuss more policy implications of our model in Section 6.

5 Numerical example

In this section we present a numerical example to illustrate the impaired transmission of monetary policy in our framework. We choose quadratic functions for the run probability at $t = 1$ and the cost of monetary policy at $t = 2$:

$$\alpha(D) = aD^2, \quad c(L) = bL^2$$

Table 1: Parameters of numerical example

Parameter	Description
$E = 1$	Aggregate endowment
$R = 4$	Maximum project payoff
$p = 0.75$	Project success probability
$a \in \{0, 0.05, 0.1\}$	Liquidity risk parameter
$b = 2$	Cost parameter
$\delta = 1$	Liquidation cost

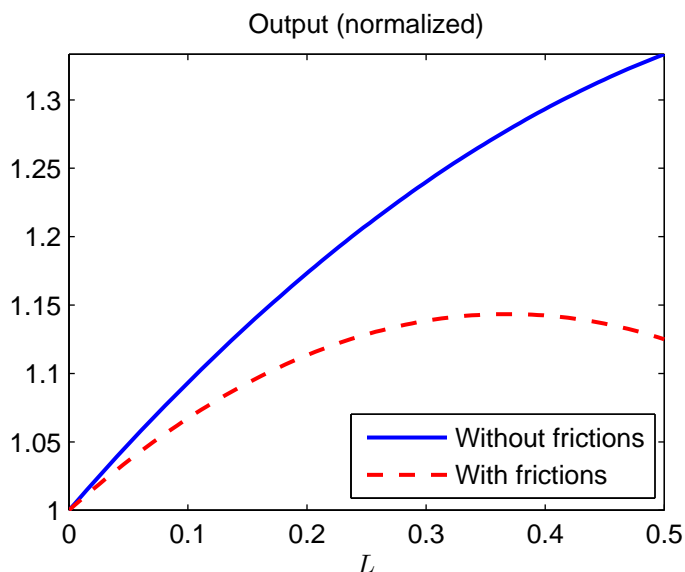


Figure 7: Effect of monetary policy on output with and without frictions

For the lemons pricing we assume $f(q) = q - \delta$ where $\delta > 0$ is a liquidation cost that doesn't affect final output, e.g. a transfer to bankruptcy lawyers, or a risk premium for the buyers. For the parameters of the model we use the values in Table 1.

Figure 7 compares the effect of monetary policy in the first-best economy ($a = 0$) and in a second-best economy characterized by our frictions ($a = 0.1$). In the first-best economy without frictions, only the highest type $\theta = 1$ invests and any liquidity injected is allocated to the highest type. Starting from $L = 0$, monetary stimulus at $t = 0$ increases output at $t = 2$ at a rate equal to the highest type's expected return, $pR = 3$. Since we assume that monetary policy at $t = 0$ has costs $c(L)$ at $t = 2$, the stimulus effect is concave even in the first-best economy. In contrast, in the second-best economy with agents facing

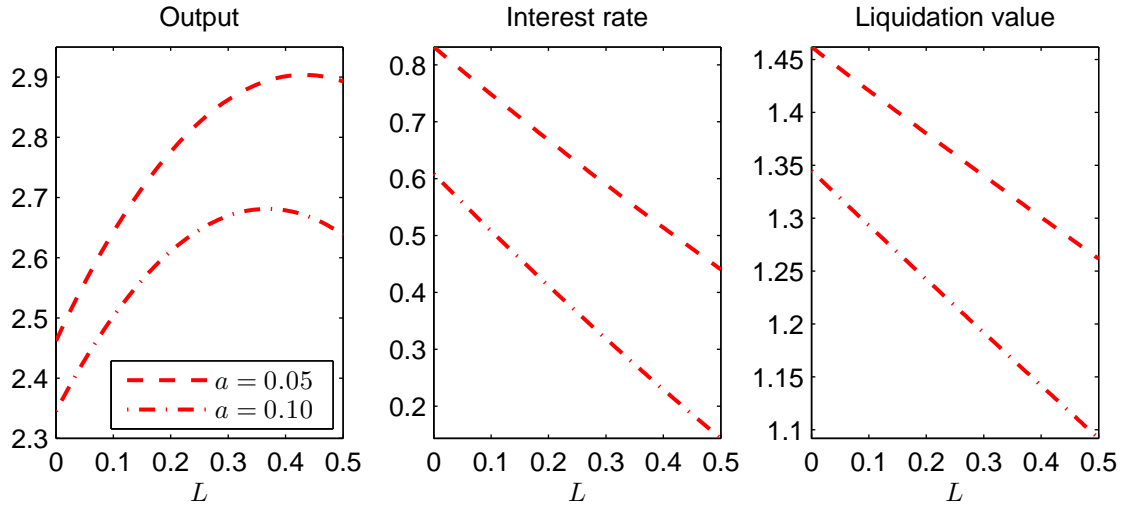


Figure 8: Effect of monetary policy under different severities of liquidity risk

liquidity risk and lemons pricing, the effect of monetary policy is considerably impaired. Since it is no longer the case that only the highest type invests, any stimulus is distributed across a range of types, resulting in a lower slope starting at $L = 0$. As stimulus increases, the quality deterioration effect kicks in and final output is strongly concave and eventually decreasing in L .

Figure 8 shows the effect of monetary policy under two scenarios varying in the severity of the liquidity risk, $a \in \{0.05, 0.10\}$. The left panel shows final output \bar{Y} , the middle panel the equilibrium interest rate r and the right panel the lemons price P . The first thing to note is that more severe liquidity risk reduces the level of output overall. This is significant, since aggregate investment is always $I = E + L$ so the difference in output for a given level of L is due only to the endogenous distribution of borrowing across types. This is reflected in the different levels of the interest rate r and the liquidation value P as well. With higher liquidity risk, overall borrowing demand is lower so the equilibrium interest rate is lower. This, however, leads worse types to invest which is reflected in the lower liquidation value (remember that average quality satisfies $q = P + \delta$). Furthermore, we see that the effect of monetary policy is weaker in the scenario with more severe liquidity risk: output responds less and flattens earlier while the interest rate and liquidation value drop faster as stimulus increases.

6 Discussion

In this section, we discuss policy implications of our model as well as the critical assumptions.

6.1 Monetary stimulus vs. tightening

Our model is static and we don't specifically distinguish monetary stimulus and tightening in the analysis. We argue, however, that the quality effect in the transmission mechanism (characterized by dq/dL) should not be thought of as symmetric for stimulus and tightening since the macroeconomic contexts—captured by the exogenous parameters in our model—for the two scenarios are different.

Note that our mechanism critically depends on financial frictions, in particular endogenously increasing cost of leverage and secondary market frictions. Both of these frictions should not be assumed constant over the business cycle but rather more severe in downturns than in upturns. In our specific setup, the concern about liquidity risk underlying the heterogeneous responses is more relevant—implying higher and faster increasing α —during downturns with low aggregate productivity p .¹⁵

The cyclical nature of our frictions combined with the cyclical nature of monetary policy implies that our effect through heterogeneous responses is not symmetric for tightening and loosening policy. When monetary policy is tightened during booms our financial frictions are less of a concern and we don't expect heterogeneous responses and the feedbacks to play an important role. Rather, the channel we describe applies mainly to situations when aggregate productivity is low and the policy maker wants to stimulate the economy, but individual agents are facing more severe financial frictions. Our theoretical predictions coincide with recent empirical findings of [Bech et al. \(2014\)](#) and [Tenreyro and Thwaites \(2015\)](#).

6.2 Secondary market intervention

Our policy analysis so far has focused on monetary loosening at $t = 0$, which can be strongly impaired by the feedback between liquidation values at $t = 1$ and loan demand at $t = 0$. Naturally, this feedback effect could be alleviated through an intervention in secondary markets to support liquidation values. If such a program for $t = 1$ were

¹⁵See [Appendix A](#) for more discussion of the link between α and aggregate conditions.

announced or anticipated in $t = 0$, it could counteract the credit misallocation at $t = 0$.

One such program would be to announce a floor for the secondary market price, which would correspond to the case of exogenously fixed P . However, this could be costly since the policy maker has to credibly commit to purchasing any amount of assets at that price. An alternative program would be to support private buyers with subsidies or loss-sharing arrangements. In the example of Section 5, this would correspond to a reduction in the wedge δ between average quality and liquidation value if it reflects a buyer risk premium.

One benefit of this intervention is that it could generate a positive spiral that partially offsets the negative spiral discussed in the paper; higher P with lower δ increases aggregate loan demand and raises the interest rate, which leads to an improvement in overall investment quality through the heterogeneous responses and thus a further increase in P . If the policy maker implements monetary stimulus and simultaneously announces these programs, the impairment effect could be alleviated.

6.3 Cash-in-the-market pricing

We now analyze an alternative case to illustrate how the dampening mechanism due to heterogeneous responses can arise even without the lemons pricing assumption. In particular, we drop our assumption of an anonymous secondary market, examining a case where buyers can distinguish seller types, but total market liquidity is limited leading to cash-in-the-market pricing in the secondary market. In this case, the increase in aggregate investment due to monetary stimulus leads to a larger cash-in-the-market discount for every forced seller. This strengthens the dampening effect on output by making aggregate loan demand less elastic, leading to a further decrease in the interest rate, which again reduces aggregate investment quality. However, there is no further feedback through an effect of lower quality on liquidation prices.

We consider an economy with limited liquidity in the secondary market at $t = 1$, where more initial investment and thus more early liquidation lowers the liquidation value for every seller. There is no incomplete information in the secondary market, such that potential buyers can perfectly identify the quality θ of the assets. However, the amount of cash available to purchase assets is limited and equal to an amount C , e.g. due to limited participation as in [Allen and Gale \(1994\)](#). As a result, when sufficiently large amounts are being sold in the secondary market, the assets may suffer from a discount to fundamental value and this discount increases in the aggregate amount of assets liquidated.

Let V denote the average value of the assets being sold in the secondary market such

that

$$V = \int_{\theta^*}^1 \alpha(D_\theta) pR\theta (D_\theta + E) d\theta.$$

When the cash available in the market is less than V , assets are sold at a discount. Note that the buyers in the secondary market can perfectly identify each asset and can acquire them using their cash. As a result, each asset should offer the same rate of return to the buyers, i.e. suffers the same proportional discount $1 - \Delta$, where $\Delta = C/V$. Hence, the price in the secondary market for an asset sold by type θ is given as $P_\theta = \Delta pR\theta$, where

$$\Delta = \begin{cases} 1 & \text{for } C \geq V \\ \frac{C}{V} & \text{for } C < V \end{cases}.$$

Differentiating the cash-in-the-market price for the case where the liquidity constraint binds, we get:

$$\begin{aligned} \frac{dP_\theta}{dr} &= -pR\theta \frac{C}{V^2} \\ &\quad \times \left(-\alpha(D_{\theta^*}) pR\theta^* (D_{\theta^*} + E) \frac{d\theta^*}{dr} + \int_{\theta^*}^1 (\alpha'(D_\theta) (D_\theta + E) + \alpha(D_\theta)) \frac{dD_\theta}{dr} pR\theta d\theta \right) \\ &= pR\theta \frac{C}{V^2} \int_{\theta^*}^1 (\alpha'(D_\theta) (D_\theta + E) + \alpha(D_\theta)) \left(-\frac{dD_\theta}{dr} \right) pR\theta d\theta < 0 \end{aligned}$$

since we have $D_{\theta^*} = 0$ so that $\alpha(D_{\theta^*}) = 0$ for the marginal type, $\alpha'(D_\theta) (D_\theta + E) + \alpha(D_\theta) > 0$ from the first-order condition (4), and $dD_\theta/dr < 0$ from Proposition 2.

This implies that insufficient market liquidity leads to a drop in the equilibrium liquidation value if more funds are injected, $dP_\theta/dL < 0$ for all θ . This affects dr/dL through the indirect effect of (10); monetary stimulus increases aggregate investment but at the same time lowers the interim liquidation prices with limited liquidity in the secondary market, thus making the loan demand less elastic. This induces a further decline in the market clearing interest rate r , which leads to further deterioration in aggregate investment quality q on top of the direct effect discussed in the previous subsection. While we assumed that the liquidity in the secondary market is fixed at C , this is not necessary for our results. As long as capital is slow-moving to the secondary market, there is cash-in-the-market pricing in the secondary market and our results go through qualitatively (Mitchell et al., 2007; Duffie, 2010; Acharya et al., 2013).

6.4 Source of heterogeneous responses

Proposition 2, which implies heterogeneous responses to changes in interest rates, is critical for the dampening mechanism. As discussed in Section 3, this relies on liquidity risk increasing in leverage in a weakly convex way. However, any other cost that is similarly increasing in balance sheet size can generate the heterogeneous responses of Proposition 2. If we consider the model’s agents as financial intermediaries, these could be from regulatory burden or the cost of deviating from a target leverage ratio. After the prolonged loosening, some financial intermediaries could become more reluctant to grow further due to the increased balance sheet costs, which induces heterogeneous responses across types.

If we interpret the high types as the traditional banking sector and the low types as the shadow banking sector, our mechanism predicts a rapid growth of the shadow banking sector in response to the prolonged monetary loosening.¹⁶ When the interest rate decreases, traditional banks facing regulatory costs respond less compared to shadow banks with less regulatory burden. More resources are allocated to “opaque” shadow banks and as a result secondary market could become more illiquid. This then generates a negative feedback described in our paper, which could end up shifting resources from the regulated banks to the shadow banks.

Note that this type of heterogeneity arises only when there is excessive loanable funds at $t = 0$, i.e. with excessive loosening compared to the loan demand. In our setup, the lenders are willing to lend but it is the borrowers who are reluctant to borrow more. In the conventional balance sheet channel, the borrowers are willing to borrow more but it is the lenders who are reluctant to lend (due to financial frictions such as agency problems). Therefore, our model describes how excess liquidity in the loanable funds market distorts resource allocation, whereas in models of the conventional credit channel liquidity demand is higher than supply (implying a positive external finance premium).

7 Conclusion

We build a general equilibrium model with heterogeneous agents facing financial frictions and show that monetary policy can become less effective than desired in stimulating output due to a feedback between the deterioration of asset quality and the reduction of

¹⁶It has been widely argued that the shadow banking sector could emerge due to regulatory arbitrage. However, it is not well understood in what circumstances this sector could grow faster than the traditional banking sector.

loan demand elasticity. More productive agents choose to invest more by borrowing, but at the same time they become exposed to higher liquidity risk due to high leverage. Agents increase their debt when the interest rate is lowered, but this additional risk taking is greater for less productive agents because high productivity agents are reluctant to lever up due to the existing high liquidity risk. The overall quality of the assets thus drops due to this heterogeneous responsiveness to the interest rate cut, which can further decrease asset liquidation values and increase liquidity risk. The elevated liquidity risk then depresses aggregate loan demand, which lowers the interest rate further. This again affects the agents differently and further decreases the investment quality. When the economy is trapped in this negative spiral, the aggregate output becomes less sensitive to monetary policy (or can even decrease) even with a significant decrease in the interest rate.

We discuss two types of heterogeneous responses that could generate and further strengthen the dampening effect on monetary policy transmission. First, we show that the less productive agents increase their borrowing more than the agents with high productivity in response to the same amount of interest rate drop, implying the potential quality deterioration following the monetary stimulus. This alone can generate the negative spiral if it induces a decrease in the liquidation value. Second, the dampening effect becomes stronger as the agents with higher productivity become relatively more sensitive to the changes in the liquidation value than the less productive agents—this not only makes loan demand more inelastic, but the quality deterioration itself could also lead to further deterioration. We argue that certain combinations of financial frictions (we consider liquidity risk and asset opaqueness in our setup) critically strengthens the dampening effect, and this could be alleviated with other policy measures such as an intervention in the secondary market to support liquidation values reversing the negative spiral.

References

- Acharya, V., S. T. Bharath, and A. Srinivasan (2006). Does industry-wide distress affect defaulted firms? Evidence from creditor recoveries. *Journal of Financial Economics* 85(3), 787–821.
- Acharya, V. V., H. S. Shin, and T. Yorulmazer (2013). A theory of arbitrage capital. *Review of Corporate Finance Studies* 2(1), 62–97.
- Adrian, T. and H. S. Shin (2010). Financial intermediaries and monetary economics. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*. Elsevier.
- Allen, F., E. Carletti, I. Goldstein, and A. Leonello (2014). Government guarantees and financial stability. Working Paper.
- Allen, F. and D. Gale (1994). Limited market participation and volatility of asset prices. *American Economic Review* 84(4), 933–955.
- Allen, F. and D. Gale (1998). Optimal financial crises. *Journal of Finance* 53(4), 1245–1284.
- Altunbas, Y., L. G. and D. Marques-Ibanez (2011). Does monetary policy affect bank risk-taking? Working Paper.
- Bech, M., L. Gambacorta, and E. Kharroubi (2014). Monetary policy in a downturn: Are financial crises special? *International Finance* 17(1), 99–119.
- Benmelech, E. and N. K. Bergman (2012). Credit traps. *American Economic Review* 102(6), 3004–3032.
- Berger, P., E. Ofek, and I. Swary (1996). Investor valuation of the abandonment option. *Journal of Financial Economics* 42, 257–287.
- Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctuations. *American Economic Review* 79(1), 14 – 31.
- Bernanke, B. and M. Gertler (1995). Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic perspectives* 9(4), 27 – 48.

- Bolton, P., T. Santos, and J. Scheinkman (2011). Inside and outside liquidity. *Quarterly Journal of Economics* 126(1), 259–321.
- Borio, C. and H. Zhu (2012). Capital regulation, risk-taking and monetary policy: A missing link in the transmission mechanism? *Journal of Financial Stability* 8(4), 236–251.
- Brunnermeier, M. K., T. M. Eisenbach, and Y. Sannikov (2013). Macroeconomics with financial frictions: A survey. In D. Acemoglu, M. Arellano, and E. Dekel (Eds.), *Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society*. Cambridge University Press.
- De Nicolò, G., G. Dell’Ariccia, L. Laeven, and F. Valencia (2010). Monetary policy and bank risk-taking. IMF Working Paper.
- Dell’Ariccia, G., L. Laeven, and R. Marquez (2014). Monetary policy, leverage and bank risk-taking. *Journal of Economic Theory* (forthcoming).
- Dell’Ariccia, G., L. Laeven, and G. Suarez (2013). Bank leverage and monetary policy’s risk-taking channel: Evidence from the United States. IMF Working Paper.
- Diamond, D. W. and R. G. Rajan (2011, March). Fear of fire sales, illiquidity seeking, and credit freezes. *Quarterly Journal of Economics* 126(2), 557–591.
- Duffie, D. (2010). Presidential address: Asset price dynamics with slow-moving capital. *Journal of Finance* 65(4), 1237–1267.
- Eisenbach, T. M. (2014). Rollover risk as market discipline: A two-sided inefficiency. Working Paper.
- Eisenbach, T. M., T. Keister, J. McAndrews, and T. Yorulmazer (2014). Stability of funding models: An analytical framework. *Federal Reserve Bank of New York Economic Policy Review* 20(1), 29–49.
- Flannery, M., S. Kwan, and M. Nimalendran (2013). The 2007–2009 financial crisis and bank opaqueness. *Journal of Financial Intermediation* 22, 55–84.
- Gale, D. and T. Yorulmazer (2013). Liquidity hoarding. *Theoretical Economics* 8(2), 291–324.

- Goldstein, I. and A. Pauzner (2005). Demand-deposit contracts and the probability of bank runs. *Journal of Finance* 60(3), 1293–1327.
- Gorton, G. and A. Metrick (2010). Haircuts. *Federal Reserve Bank of St. Louis Review* 92(6), 507–519.
- Gorton, G. and A. Metrick (2012). Securitized banking and the run on repo. *Journal of Financial Economics* 104(3), 425–451.
- Hirtle, B. (2006). Stock market reaction to financial statement certification by bank holding company CEOs. *Journal of Money, Credit and Banking* 38, 1263–1291.
- Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy* 106(1), 1–40.
- Ioannidou, V., S. Ongena, and J. L. Peydró (2009). Monetary policy, risk-taking and pricing: Evidence from a quasi-natural experiment. CentER Discussion Paper Series No. 2009-31S.
- James, C. (1991). The losses realized in bank failures. *Journal of Finance* 46(4), 1223–1242.
- Jiménez, G., S. Ongena, J. L. Peydró, and J. Saurina (2014). Hazardous times for monetary policy: What do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking? *Econometrica* (forthcoming).
- Kashyap, A. K. and J. C. Stein (2000). What do a million observations on banks say about the transmission of monetary policy? *American Economic Review* 90(3), 407–428.
- Kashyap, A. K., D. P. Tsomocos, and A. P. Vardoulakis (2014). How does macroprudential regulation change bank credit supply? NBER working paper.
- Keister, T. (2014). Bailouts and financial fragility. Working Paper.
- Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy* 105(2), 211–248.
- Kurlat, P. (2013). Lemons markets and the transmission of aggregate shocks. *American Economic Review* 103(4), 1463–1489.
- Malherbe, F. (2014). Self-fulfilling liquidity dry-ups. *Journal of Finance* 69(2), 947–970.

- Mitchell, M., L. H. Pedersen, and T. Pulvino (2007). Slow moving capital. *American Economic Review* 97(2), 215–220.
- Morgan, D. (2002). Rating banks: Risk and uncertainty in an opaque industry. *Rating Banks: Risk and Uncertainty in an Opaque Industry* 92(4), 874–888.
- Morris, S. and H. S. Shin (2003). Global games: Theory and applications. In M. Dewatripont, L. P. Hansen, and S. J. Turnovsky (Eds.), *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society)*. Cambridge University Press.
- Morris, S. and H. S. Shin (2010). Illiquidity component of credit risk. Working Paper.
- Paligorova, T. and J. A. C. Santos (2012). Monetary policy and bank risk-taking: Evidence from the corporate loan market. Working Paper.
- Peek, J. and E. S. Rosengren (2000). Collateral damage: Effects of the Japanese bank crisis on real activity in the United States. *American Economic Review* 90(1), 30–45.
- Peydró, J. L. and A. Maddaloni (2011). Bank risk-taking, securitization, supervision, and low interest rates: Evidence from the Euro area and U.S. lending standards. *Review of Financial Studies* 24, 2121–2165.
- Pulvino, T. C. (1998). Do asset fire sales exist: An empirical investigation of commercial aircraft sale transactions. *Journal of Finance* 53, 939–978.
- Rosenthal, R. W. and R. Wang (1993). An explanation of inefficiency in markets and a justification for buy-and-hold strategies. *Canadian Journal of Economics*, 609–624.
- Shin, H. S. (2009). Reflections on Northern Rock: The bank run that heralded the global financial crisis. *Journal of Economic Perspectives* 23(1), 101–19.
- Shleifer, A. and R. W. Vishny (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance* 47(4), 1343–1366.
- Stein, J. (2014). Incorporating financial stability considerations into a monetary policy framework. Speech at the International Monetary Fund Spring Meetings on April 13, 2014.

- Stromberg, P. (2000). Conflicts of interest and market illiquidity in bankruptcy auctions: Theory and tests. *Journal of Finance* 55, 2641–2692.
- Tarullo, D. (2014). Monetary policy and financial stability. Speech at the 30th Annual National Association for Business Economics Economic Policy Conference on February 25, 2014.
- Tenreyro, S. and G. Thwaites (2015). Pushing on a string: US monetary policy is less powerful in recessions. Working Paper.
- Williamson, O. E. (1988). Corporate finance and corporate governance. *Journal of Finance* 43(3), 567–591.

Appendix

A Microfoundation of run likelihood $\alpha(D)$

In this section, we derive an ex-ante likelihood of interim creditor runs on borrowers using global game techniques analogous to the analysis in Eisenbach (2014).¹⁷ Importantly, we show how run risk depends on the borrower type θ only indirectly through the leverage level D_θ , rationalizing our θ -independent functional assumption $\alpha(D)$.

We can interpret θ as the idiosyncratic productivity of type θ and the success probability p as a macroeconomic variable common across all types as of $t = 0$. Now suppose that at $t = 1$ each type receives an idiosyncratic shock to p , updating it to $p_\theta = p + u_\theta$ where u_θ is i.i.d. across types with mean 0 and cumulative distribution F_u on $[\underline{u}, \bar{u}]$.

We normalize each borrower's creditors to a continuum of measure 1. Each creditor $k \in [0, 1]$ can choose at $t = 1$ whether to withdraw or roll over until $t = 2$. Denote by $\lambda \in [0, 1]$ the fraction of creditors who choose to withdraw and by r_s the one-period interest rate promised to creditors.¹⁸ The borrower fails at $t = 1$ if total withdrawals are larger than the total liquidation value of the borrower's assets:

$$\lambda > \hat{\lambda} \equiv \frac{P(D + E)}{D(1 + r_s)}$$

The threshold $\hat{\lambda}$ is decreasing in D so for higher absolute debt, a smaller fraction of withdrawals can cause failure. A creditor who withdraws at $t = 1$ receives $(1 + r_s)D$ if the bank doesn't fail and the liquidation value $P(D + E)$ if it does fail. A creditor who rolls over to $t = 2$ expects to receive $p_\theta(1 + r_s)^2 D$ if the bank fails and 0 otherwise.

We now introduce a global game setup so that a unique failure threshold p_θ^* can be derived where a borrower θ fails if $p_\theta \leq p_\theta^*$ and survives otherwise. Suppose that p_θ is not common knowledge but creditor $k \in [0, 1]$ of borrower θ receives an i.i.d. noisy signal $s_\theta^k = p_\theta + \varepsilon_\theta^k$ instead, where $\varepsilon_\theta^k \sim \mathcal{U}[-\varepsilon, \varepsilon]$. Each borrower then chooses whether to roll over or withdraw after observing this private signal. We focus on the threshold strategy equilibrium for $\varepsilon \rightarrow 0$ such that a creditor chooses to withdraw if and only if $s_\theta^k < p_\theta^*$ for some threshold p_θ^* .

¹⁷See Morris and Shin (2010) for a similar approach. Eisenbach et al. (2014) provide a model where banks can fail due to poor fundamentals and/or a loss of significant short-term funding as well as the interaction between the two.

¹⁸Note that r_s is endogenous and is set by an ex-ante break-even condition as shown below.

A creditor exactly at the switching point, $s_\theta^k = p_\theta^*$, has to be indifferent between the two actions which requires that

$$\begin{aligned} \Pr\left[\lambda \leq \hat{\lambda} \mid s_\theta^k = p_\theta^*\right] \times (1 + r_s) D + \Pr\left[\lambda > \hat{\lambda} \mid s_\theta^k = p_\theta^*\right] \times P(D + E) \\ = \Pr\left[\lambda \leq \hat{\lambda} \mid s_\theta^k = p_\theta^*\right] \times p_\theta^* (1 + r_s)^2 D + \Pr\left[\lambda > \hat{\lambda} \mid s_\theta^k = p_\theta^*\right] \times 0 \end{aligned}$$

For $\varepsilon \rightarrow 0$, the distribution of $\lambda \mid s_\theta^k = p_\theta^*$ becomes uniform on $[0, 1]$ (Morris and Shin, 2003; Goldstein and Pauzner, 2005) and the indifference condition simplifies to

$$\hat{\lambda} (1 + r_s) D + (1 - \hat{\lambda}) P(D + E) = \hat{\lambda} p_\theta^* (1 + r_s)^2 D$$

Substituting in for $\hat{\lambda}$ and solving for p_θ^* we get

$$p_\theta^* = \frac{2(1 + r_s) D - P(D + E)}{(1 + r_s)^2 D} \quad (13)$$

For given r_s the run threshold and therefore run risk is increasing in D . Note, however, that r_s is an endogenous variable that depends on p_θ^* and D . The interest rate r_s is determined by a $t = 0$ break-even conditions for creditors:

$$F_u(p_\theta^* - p) P(D + E) + \int_{p_\theta^* - p}^{\bar{u}} (p + u_\theta) dF_u(u_\theta) (1 + r_s)^2 D = (1 + r) D \quad (14)$$

The $t = 1$ indifference condition (13) and the $t = 0$ break-even condition (14) implicitly define the interim run threshold p_θ^* as a function of the ex-ante leverage D . Lemma 1 of Eisenbach (2014) shows that the mapping $p_\theta^*(D)$ is one-to-one and satisfies $dp_\theta^*/dD > 0$.

Note that p_θ^* here depends on D but is independent of θ . Therefore, the ex-ante run risk $\alpha(D) = \Pr(p_\theta \leq p_\theta^* \mid p)$ depends only on D . We can thus write α as a function of D but not θ , as we did in main part of the paper. Note also that given D , $\alpha(D)$ becomes larger when the fundamental p is lower. Thus, the run risk is higher when the aggregate productivity is lower.

B Proofs

Proof of Proposition 1. From the implicit function theorem, the first order condition (4) implies

$$\frac{dD_\theta}{d\theta} = \frac{(1 - \alpha'(D_\theta)(D_\theta + E) - \alpha(D_\theta))pR}{(\alpha''(D_\theta)(D_\theta + E) + 2\alpha'(D_\theta))(pR\theta - P)}$$

Note that the denominator is positive from (3). The numerator is also positive since the first order condition implies

$$\begin{aligned} \alpha'(D_\theta)(D_\theta + E) + \alpha(D_\theta) &= \frac{pR\theta - (1+r)}{pR\theta - P} \\ &< 1 \quad \text{for } P < 1+r \end{aligned}$$

Therefore $dD_\theta/d\theta$ is positive. □

Proof of Proposition 2. Note that

$$\frac{\partial D_\theta}{\partial r} = \frac{-1}{(\alpha''(D_\theta)(D_\theta + E) + 2\alpha'(D_\theta))(pR\theta - P)} < 0$$

The denominator is increasing in θ since D_θ is increasing in θ and $\alpha'' \geq 0$, and the third derivative of α is very small. □

Proof of Proposition 3. From the implicit function theorem, the first order condition implies

$$\frac{\partial D_\theta}{\partial P} = \frac{\alpha'(D_\theta)(D_\theta + E) + \alpha(D_\theta)}{(\alpha''(D_\theta)(D_\theta + E) + 2\alpha'(D_\theta))(pR\theta - P)}$$

Note that the denominator is positive from (3). The numerator is also positive since all terms are positive. Thus $\partial D/\partial P$ is positive. □

Proof of Corollary 1. Note that

$$\frac{\left| \frac{\partial D_{\theta_H}/\partial P}{\partial D_{\theta_L}/\partial P} \right|}{\left| \frac{\partial D_{\theta_H}/\partial r}{\partial D_{\theta_L}/\partial r} \right|} = \frac{\alpha'(D_{\theta_H})(D_{\theta_H} + E) + \alpha(D_{\theta_H})}{\alpha'(D_{\theta_L})(D_{\theta_L} + E) + \alpha(D_{\theta_L})}$$

This is greater than 1 since $\alpha(D)$ is increasing and weakly convex in D , and D_θ is increasing in θ . □

Proof of Corollary 2. We first show that $\int_{\theta^*}^1 (q - pR\theta) d\theta > 0$. We have

$$\begin{aligned} \int_{\theta^*}^1 (q - pR\theta) d\theta &= (1 - \theta^*) \left(q - \frac{1}{1 - \theta^*} \int_{\theta^*}^1 pR\theta d\theta \right) \\ &= (1 - \theta^*) \int_{\theta^*}^1 pR\theta \left(\frac{D_\theta + E}{\int_{\theta^*}^1 (D_\vartheta + E) d\vartheta} - \frac{1}{1 - \theta^*} \right) d\theta. \end{aligned}$$

Since D_θ is increasing in θ we have

$$\begin{aligned} \int_{\theta^*}^1 pR\theta \left(\frac{D_\theta + E}{\int_{\theta^*}^1 (D_\vartheta + E) d\vartheta} - \frac{1}{1 - \theta^*} \right) d\theta &> \int_{\theta^*}^1 pR\theta d\theta \times \int_{\theta^*}^1 \left(\frac{D_\theta + E}{\int_{\theta^*}^1 (D_\vartheta + E) d\vartheta} - \frac{1}{1 - \theta^*} \right) d\theta \\ &= 0, \end{aligned}$$

as desired.

Now we show that $\int_{\theta^*}^1 (q - pR\theta) (\partial D_\theta / \partial r) d\theta < 0$. Since $\partial D_\theta / \partial r$ is negative and increasing in θ we have

$$\begin{aligned} \int_{\theta^*}^1 (q - pR\theta) \frac{\partial D_\theta}{\partial r} d\theta &< \int_{\theta^*}^1 (q - pR\theta) d\theta \times \int_{\theta^*}^1 \frac{\partial D_\theta}{\partial r} d\theta \\ &< 0, \end{aligned}$$

as desired. □

Proof of Corollary 3. We know that $q \in (pR\theta^*, pR)$ so there exists a $\hat{\theta} \in (\theta^*, 1)$ such that

$$q - pR\theta \begin{cases} > 0 & \text{for } \theta < \hat{\theta} \\ = 0 & \text{for } \theta = \hat{\theta} \\ < 0 & \text{for } \theta > \hat{\theta} \end{cases}$$

Since $\partial D_\theta / \partial P > 0$, we have that if $\partial D_\theta / \partial P$ is sufficiently increasing in θ then

$$\int_{\theta^*}^{\hat{\theta}} (q - pR\theta) \frac{\partial D_\theta}{\partial P} d\theta < - \int_{\hat{\theta}}^1 (q - pR\theta) \frac{\partial D_\theta}{\partial P} d\theta,$$

and therefore

$$\int_{\theta^*}^1 (q - pR\theta) \frac{\partial D_\theta}{\partial P} d\theta < 0,$$

as desired. □

Proof of Corollary 4. The result directly follows from the fact that $\partial D_\theta / \partial P > 0$. □

Proof of Corollary 5. Implicit differentiation of the fixed-point condition for P yields

$$\frac{dP}{dr} = \frac{f'(q) \partial q / \partial r}{1 - f'(q) \partial q / \partial P}.$$

Stability of the fixed point requires that $f'(q) \partial q / \partial P < 1$ so the denominator is positive. From Corollary 2 we know that $\partial q / \partial r > 0$ so with $f'(q) > 0$ by assumption, the numerator is positive as well. □