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## Definition and measurement of natural disaster damage cost by Ramsey growth model

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**DEFINTION AND MEASUREMENT  
OF NATUAL DISTER DAMAGE COST  
BY RAMSEY GROWTH MODEL**

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## ABSTRACT

This study proposes a consistent definition of natural disaster damage costs, equivalently, of natural disaster prevention benefits in accordance with general definition of benefits, willingness to pay, more concretely, consumption or Equivalent Variation, of any policy and projects such as tax reforms, transportations, environments, infrastructures, etc. In order to formulate the damage cost this study describes the economy by the Ramsey growth model while the annual disaster physical damages of capital stock loss are reflected in the depreciation rate of capital stock. The annual damage cost is defined as the change in consumption in the steady state induced by the change in depreciation rate due to the increase in the annual disaster physical damages of capital stock loss. The change in consumption equals the change in GDP minus the change in investment, which equal to the change in depreciation of capital stock in the steady state. From the incidence form above the origin form can be derived as: The change in consumption equals the change in capital stock revenue plus the change in depreciation due to the increase in the annual disaster physical damages of capital stock loss. Thus the conventional damage calculation which is equivalent to the second term of origin formula underestimates by the first term of stock reduced revenue loss. Also sometimes the economic impacts on GDP of disaster capital stock loss are calculated but nothing is referred to the damage cost of disaster. This is because there is not yet clear definitions of damage cost and lack of recognition that capital stock is not an exogenous but an endogenous variable. Finally this study measures the impacts on the floods damage cost in terms of consumption of global warming at 2112 in Japan based on the assumption that the flood physical capital loss increases by 1.6 times. The damage cost multiplier to the direct flood physical capital loss is estimated as 1.357 which leads to that damage cost in the consumption is decreased by  $1.357 \times 1.6 = 2.16$  times.

Keywords: project evaluation, disaster damage cost, Ramsey growth model, cost benefit analysis.

JEL code: H43

## 1. INTRODUCTION

The conventional damage calculation measures only the increase in the annual natural disaster physical damages of capital stock loss (S4 Project Team(2008), IPCC(2012), Fukubayashi and Oki(2011)). However, it is not clear yet whether or not this index is consistent to the definition of natural disaster damage costs, equivalently, of natural disaster prevention benefits in accordance with general definition of benefits, willingness to pay, more concretely, consumption or equivalent variation, of any policy such as tax reforms, transportations, environments, infrastructures, etc.(Boardman, Greenberg, Vining and Weimer(2001), Hosoe, Gasawa and Hashimoto(2010)). Specifically the disaster changes the capital stock, which requires the dynamic analysis whereas usual definition and measurement of benefit are analyzed in the static framework. Therefore it needs dynamic framework because it needs to analyze the impact of depreciation rate..

In order to formulate the damage cost, this study describes the economy by the Ramsey growth model while the annual disaster physical damages of capital stock loss are reflected in the depreciation rate of capital stock (Barro and Sala-i-Martin(2004) chapter 2 and 3, Acemoglu(2009) chapter 8). The annual damage cost is defined as the change in consumption in the steady state induced by the change in depreciation rate due to the increase in the annual disaster physical damages of capital stock loss. The change in consumption equals the change in GDP minus the change in investment, i.e. the change in depreciation of capital stock. From the incidence form above, the origin form can be derived as: the change in consumption equals the change in capital stock revenue plus the change in depreciation due to the increase in the annual disaster physical damages of capital stock loss. Thus the conventional damage calculation which is equivalent to the second term of origin formula underestimates by the first term of stock reduced revenue loss. Also, sometimes the economic impacts on GDP of disaster capital stock loss are calculated but nothing is referred to the damage cost of disaster. This is because there is not yet clear definitions of damage cost and lack of recognition that capital stock is not an exogenous but an endogenous variable.

It is important to realize that disaster damages not only private capital but also output, social overhead capital and labor force, but this study pays attentions to only private capital loss for simplicity (Sawada(2007), Sawada(2010)).

This study is composed of description of Ramsey growth model in next section, definition of disaster damage costs (or disbenefits) and derivation of measurement

formula in section 3, and case study for Japan in section 4.

## 2. RAMSEY GROWTH MODEL

### 2.1 Household

The households provide labor services in exchange for wages, receive interest income on assets, purchase goods for consumption, and save by accumulating assets. Each household wishes to maximize the present value of current utility at the time zero, as given by

$$U = \int_0^{\infty} u(c(t))e^{nt} \cdot e^{-\rho t} dt, \text{ where } u(c(t)) = \frac{c(t)^{(1-\theta)} - 1}{1-\theta}, \quad (1)$$

, under the budget constraint in per capita terms:

$$\dot{a} = w + ra - c - na, \quad (2)$$

,where

$\rho$ : rate of time preference

$\theta$ : inverse of the elasticity of inter-temporal substitution

$n$ : labor growth rate

$c$ : consumption per capita

$a$ : per capita assets

$w$ : wage rate

$r$ : interest rate

The necessary condition of Hamiltonian dynamics for this mathematical problem is well known as

$$\frac{c}{c} = \frac{1}{\theta}(r - \rho) \quad (3)$$

### 2.2 Firms

Firms maximize the profit at any point

$$\pi = F(K, \hat{L}) - (r + \delta)K - wL = L[f(\hat{k}) - (r + \delta)k - we^{-xt}] \quad (4)$$

where

$$\hat{L} = L \cdot e^{xt}, \quad L = L_0 e^{nt}, \quad k = \frac{K}{L}, \quad \hat{k} = \frac{K}{\hat{L}} = \frac{K}{L \cdot e^{xt}} = ke^{-xt}, \quad k = \hat{k}e^{xt} \quad (5)$$

$x$ : rate of technological progress

$\delta$  :depreciation rate of capital stock.

$F(K, \hat{L})$  : production function with constant returns to scale

$$\hat{y} = Y / (L \cdot A(t)) = F(\hat{k}, 1) \equiv f(\hat{k})$$

The first order conditions of this problem are

$$f'(\hat{k}) = r + \delta \quad (6)$$

And

$$(f(\hat{k}) - (r + \delta)\hat{k})e^{xt} = w \quad (7)$$

Therefore zero profit of (4) holds.

### 2.3 Equilibrium

$a = k$  with (2), (3), (6), (7) determine the equilibrium value of variables  $c, k, w, r$ .

In order to express this economic system by only  $\hat{c}, \hat{k}$ , substitute  $k = \hat{k}e^{xt} + x\hat{k}e^{xt}$  and (6),(7) into (2)

$$\dot{\hat{k}}e^{xt} + x\hat{k}e^{xt} = e^{xt} \left\{ f(\hat{k}) - kf'(\hat{k}) \right\} + ke^{xt} \cdot \left\{ f'(\hat{k}) - \delta \right\} - c - nke^{xt}$$

Let  $\hat{c} = ce^{-xt}$ . Then (2) becomes

$$\dot{\hat{k}} = f(\hat{k}) - c - (n + x + \delta)\hat{k} \quad (8)$$

Also substitute  $c = \hat{c}e^{xt}$ ,  $\dot{\hat{c}} = \hat{c}e^{xt} + x\hat{c}e^{xt}$ , into (3)

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}(r - \rho - \theta x) = \frac{1}{\theta} \left\{ f'(\hat{k}) - \delta - \rho - \theta x \right\} \quad (9)$$

Thus two differential equations of (8) and (9) determine the equilibrium path.

### 2.4 The Steady State and Comparative Statics

$\dot{\hat{k}} = 0$ ,  $\dot{\hat{c}} = 0$  in (8) and (9) leads to the steady state which is expressed as

$$f(\hat{k}^*) - (n + x + \delta)\hat{k}^* = \hat{c}^* \quad (10)$$

$$f'(\hat{k}^*) - \delta = r = \rho + \theta x \quad (11)$$

The graph of (10) and (11) are shown in the Figure 1 and the steady state for depreciation rate is indicated at the point  $\delta^*$ . Now suppose that depreciation rate increases from  $\delta^*$  to  $\delta^{**}$ . Then the steady state changes from the point  $\delta^*$  to point  $\delta^{**}$  in the Figure 1 with the decrease in both capital stock and consumption per effective labor,  $\hat{k}$  and  $\hat{c}$ .

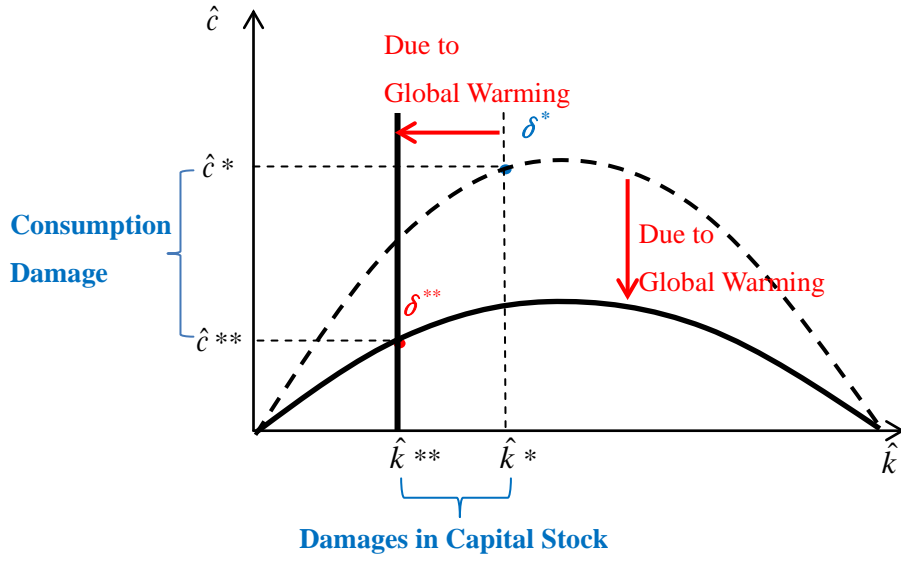


Figure 1 Steady State

Thus comparative statics can be carried as follows.

From (11),

$$f''(\hat{k}^*) d\hat{k}^* = d\delta \quad \frac{d\hat{k}^*}{d\delta} = \frac{1}{f''(\hat{k}^*)} < 0 \quad (12)$$

(by Inada condition)

From (10),

$$\begin{aligned} \{f'(\hat{k}^*) - (n + x + \delta)\} d\hat{k}^* - \hat{k}^* d\delta &= d\hat{c}^* \\ \Leftrightarrow d\hat{c}^* &= \{\rho + \theta x - (n + x)\} d\hat{k}^* - \hat{k}^* d\delta \end{aligned} \quad (13)$$

$$\Leftrightarrow \frac{d\hat{c}^*}{d\delta} = \frac{\{\rho + (\theta - 1)x - n\}}{f''(\hat{k}^*)} - \hat{k}^* \quad (14)$$

Finally the level variables are

$$L = L_0 e^{nt}, K = \hat{k} L_0 e^{(n+x)t}, C = \hat{c} L_0 e^{(n+x)t}, Y = F(K, L) = f(\hat{k}) L_0 e^{(n+x)t}, \quad (15)$$

Therefore (10) and (11) can be expressed as

$$Y^* - (n + x + \delta)K^* = C^* \quad (10a)$$

$$F_K(K^*, L_0 e^{-(n+x)t}) - \delta = r = \rho + \theta x \quad (11a)$$

And from (13) the change in consumption level at t caused by the depreciation is expressed as

$$dC^* = \{\rho + \theta x - (n + x)\} dK^* - K^* d\delta \quad (16)$$

### 3. DAMAGE COST and MEASUREMENT

The annual disaster physical damages of capital stock loss caused by floods, high tide and landslide in Japan fluctuates from US\$2 to 20 million, and average for recent 30 years is US\$5 million. Thus the disaster depreciation rate at present  $\delta_D^*$  is the physical

damage  $D_{t=0}^*$  US\$5 million, divided by the present capital stock  $K_{t=0}^*$ , US\$1800 billion, which is.

$$\delta_D^* = \frac{D_{t=0}^*}{K_{t=0}^*} = \frac{0.5}{1800} = 0.0278\%$$

It is assumed that this disaster depreciation rate continues constant forever if the global warming would not occur. Now it is expected that by the end of 21 century with 3 degree increase due to the global warming the disaster depreciation rate become 2.6 times of present level, i.e. US\$13 million.

$$\delta_D^{**} = 2.6\delta_D^* = 0.0722\%$$



It is assumed that this disaster depreciation rate continues constant forever if the global warming in fact occurs. Thus the change in depreciation rate due to the global warming is expressed as

$$\delta^{**} - \delta^* = \delta_D^{**} - \delta_D^* \quad (17)$$

, where  $*$  and  $**$  indicate the case for no global warming and global warming, respectively.

The damage cost (disbenefit) per effective labor of increase (decrease) in depreciation rate is defined as the decrease (increase) in consumption per effective labor:

$$\hat{c}^{**} - c^* = (y^{**} - y^*) - \{(n+x+\delta^{**})\hat{k}^{**} - (n+x+\delta^*)k^*\} \quad (18)$$

Or from (13)

$$\begin{aligned} \hat{c}^{**} - c^* &= \int_{c^*}^{\hat{c}^{**}} dc = \{\rho + \theta x - (n+x)\} \int_{k^*}^{\hat{k}^{**}} d\hat{k} - \int_{\delta^*}^{\delta^{**}} k d\delta \\ &\approx \{\rho + \theta x - (n+x)\} (\hat{k}^{**} - k^*) - \frac{1}{2} (k^{**} + k^*) (\delta^{**} - \delta^*) \end{aligned} \quad (19)$$

By multiplying the effective labor force at t  $L_0 e^{-(n+x)t}$  to the both side, the annual total damage cost (benefit) at t of increase (decrease) in depreciation rate is defined as the decrease (increase) in consumption:

$$C^{**} - C^* = (Y^{**} - Y^*) - \{(n+x+\delta^{**})K^{**} - (n+x+\delta^*)K^*\} \quad (18a)$$

Or from (18)

$$\begin{aligned} C^{**} - C^* &= \int_{C^*}^{C^{**}} dC = \{\rho + \theta x - (n+x)\} \int_{K^*}^{K^{**}} dK - \int_{\delta^*}^{\delta^{**}} K d\delta \\ &\approx \{\rho + \theta x - (n+x)\} (K^{**} - K^*) - \frac{1}{2} (K^{**} + K^*) (\delta^{**} - \delta^*) \end{aligned} \quad (19a)$$

(18a) says simply that change in consumption per effective labor equals the change in per capita GDP minus the change in investment per effective labor due to the change in depreciation rate. This way of measurement is called incidence form. On the other hand (19a) says that the change in consumption equal direct damage (2<sup>nd</sup> term) plus revenue loss due to the decrease in capital stock (1<sup>st</sup> term). Thus conventional method which measures the damage cost by the 2<sup>nd</sup> term underestimates by the first term, i.e. the change in returns to the capital stock.

#### 4. ESTIMATION of FLOOD DAMAGE COST BY GLOBAL WARMING in JAPAN

##### 4.1 Japan economy at the present

This study assumes that the present Japanese economy is in the steady state with the variables and parameters such that:

$$K_{t=0} = \text{US\$1800billion}, Y_{t=0} = \text{US\$500billion}, L_{t=0} = 60\text{million} \quad (20)$$

$$\rho = 0.01, n = 0, x = 0.01, \theta = 2, \delta = 0.05 \quad (21)$$

And production function of Cobb-Douglas type as

$$Y_t = B(A_0 e^{xt} \cdot L_0 e^{nt})^{1-b} K_t^b, \quad (22)$$

By assuming  $b=0.3$ ,  $A_0=1$ ,  $t=0$  and substituting (20), (21) into (22), we obtain that  $B=47608.26751$ .

Furthermore production function, output and capital stock per effective labor are

$$\hat{y}^* = f(\hat{k}) = Bk^{*b} \quad \hat{y}^* = \text{US\$0.0833million}, \hat{k}^* = \text{US\$0.3million} \quad (23)$$

##### 4.2 The annual disaster physical damages of capital stock loss caused by floods, high tide and landslide in Japan

The disaster depreciation rate at present  $\delta_D^*$  is the physical damage  $D_{t=0}^*$  US\$5 million, divided by the present capital stock  $K_{t=0}^*$ , US\$1800billion, which is.

$$\delta_D^* = \frac{D_{t=0}^*}{K_{t=0}^*} = \frac{0.5}{1800} = 0.0278\%$$

It is assumed that this disaster depreciation rate continues constant forever if the global warming would not occur. It is expected that by the end of 21 century with 3 degree increase due to the global warming the disaster depreciation rate become 2.6 times of present level, i.e. US\$13 million.

$$\delta_D^{**} = 2.6\delta_D^* = 0.0722\%$$

It is assumed that this disaster depreciation rate continues constant forever if the global warming in fact occurs. Thus the change in depreciation rate due to the global warming is expressed as

$$\delta_D^{**} - \delta_D^* = \delta_D^{**} - \delta_D^*, \text{ or } \delta^{**} = \delta_D^{**} - \delta_D^* + \delta^* = 0.0444\% + 5\% = 5.0444\%$$

, where \* and \*\* indicate the case for no global warming and global warming,

respectively, and D is the physical damage level assuming that capital stock level continues as same as the present level.

In order to calculate the change in capital per effective labor, from (13) and (23)

$$f' = bB\hat{k}^{b-1} = bfk^{*-1} = \rho + \theta x + \delta$$

$$f'' = (b-1)bB\hat{k}^{b-2} = (b-1)(bfk^{*-1}) / k^* = (b-1)(\rho + \theta x + \delta) / k^*$$

Therefore

$$d\hat{k}^* / d\delta = 1 / f'' = k^* / \{(b-1)(\rho + \theta x + \delta)\}$$

And the percentage of capital decrease for depreciation rate increase is 0.793%

$$\frac{d\hat{k}^*}{\hat{k}^*} = \frac{d\delta}{(b-1)(\rho + \theta x + \delta)} = -\frac{0.0444}{0.7(0.01 + 2 \times 0.01 + 0.05)} = \frac{0.0444}{0.056} = 0.793\%$$

The damage cost per effective labor is

$$\begin{aligned} d\hat{c}^* &= \{\rho + \theta x - (n+x)\} d\hat{k}^* - k^* d\delta = -\left(\frac{\rho + \theta x - (n+x)}{(1-b)(\rho + \theta x + \delta)} + 1\right) k^* d\delta \\ &= -\left(\frac{0.01 + 2 \times 0.01 - 0.01}{0.7(0.01 + 2 \times 0.01 + 0.05)} + 1\right) \hat{k}^* d\delta = -\left(\frac{0.02}{0.056} + 1\right) k^* d\delta = -(0.357 + 1) k^* d\delta \\ &= -1.357 \times \$300,000 \times 0.000444 = \$180.752 \end{aligned} \tag{24}$$

Therefore the total annual damage cost at today (t=0) and at 2112 (t=100) are

$$\begin{aligned} dC_t &= d\hat{c}L_0 e^{(n+x)t} = (\$180.752 \times 0.06 \text{ billion}) e^{(0+0.01)t} = (\$1.085 \text{ billion}) e^{(0.01)t} \\ dC_{t=0} &= \$1.085 \text{ billion} \\ dC_{t=100} &= (\$1.084 \text{ billion}) e = (\$1.0845 \text{ billion}) \times 2.718 = \$2.95 \text{ billion} \end{aligned}$$

It is possible to check the value of  $dC_{t=0}$  above by multiplying  $L_0$  to both sides of (24)

with noting that  $K_{t=0} d\delta = (D^{**} - D^*)$

$$\begin{aligned} dC_{t=0} &= -\left(\frac{\rho + \theta x - (n+x)}{(1-b)(\rho + \theta x + \delta)} + 1\right) K_{t=0} d\delta = -(0.357 + 1)(D^{**} - D^*) \\ &= -1.357 \times 1.6 \times \$0.5 \text{ billion} = \$1.085 \text{ billion} \end{aligned}$$

The calculation above says that

- (1) The annual flood damage cost measured in the consumption of today and 100 years ahead due to the global warming in Japan are \$1.1billion and \$3.0 billion, respectively.
- (2) The damage can be composed of the direct physical capital stock loss and the repercussion effect of decrease in net interest revenue due to the decrease in capital stock in the steady state. The size of latter item is 0.357 relative to the size of direct loss so that the multiplier is 1.357.
- (3) Conventional method underestimates by about 26%.

## 5. CONCLUTING REMARKS

This study proposes a consistent definition of natural disaster damage costs, equivalently, of natural disaster prevention benefits in accordance with general definition of benefits, willingness to pay, more concretely, consumption or Equivalent Variation, of any policy and projects such as tax reforms, transportations, environments, infrastructures, etc.

In order to formulate the damage cost this study describes the economy by the Ramsey growth model while the annual disaster physical damages of capital stock loss are reflected in the depreciation rate of capital stock. The annual damage cost is defined as the change in consumption in the steady state induced by the change in depreciation rate due to the increase in the annual disaster physical damages of capital stock loss. The change in consumption equals the change in GDP minus the change in investment, which equal to the change in depreciation of capital stock in the steady state. From the incidence form above the origin form can be derived as: the change in consumption equals the change in capital stock revenue plus the change in depreciation due to the increase in the annual disaster physical damages of capital stock loss. Thus the conventional damage calculation which is equivalent to the second term of origin formula underestimates by the first term of stock reduced revenue loss.

Also sometimes the economic impacts on GDP of disaster capital stock loss are calculated but nothing is referred to the damage cost of disaster. This is because there is not yet clear definitions of damage cost and lack of recognition that capital stock is not an exogenous but an endogenous variable.

Finally this study measures the impacts on the floods damage cost in terms of consumption of global warming at 2112 in Japan based on the assumption that the flood physical capital loss increases by 1.6 times. The damage cost multiplier to the direct

flood physical capital loss is estimated as 1.357 which leads to that damage cost in the consumption is decreased by  $1.357 \times 1.6 = 2.16$  times.

There are several works remaining for future. First it is important to realize that disaster damages not only private capital but also output, social overhead capital and labor force, but this study pays attentions to only private capital loss for simplicity. Second this study assumes the floods occur constantly every year at the average level. But in reality it fluctuates just as business cycle. Third the extension to the multi-regional or -sector model is also important for practical applications.

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