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# Unit Tax versus Ad Valorem Tax: A Tax Competition Model with Cross-border Shopping\*

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## Abstract

Within the framework of spatial tax competition with cross-border shopping, we examine the choice of tax method between ad valorem tax and unit (specific) tax. The paper shows that governments endogenously choose ad valorem tax not because of a classic welfare reason, but because it is a good strategy in competing for mobile customers. Another key finding is that while governments are committed to the ad valorem tax method, the choice is not efficient; Tax-cutting competition becomes more serious when countries adopt ad valorem tax, and competition in ad valorem tax yields smaller payoffs than competition in unit tax.

*Keywords:* spatial tax competition, cross-border shopping, unit (specific) tax, ad valorem tax.

*JEL Classification Number.* H21,H77,L13,R12

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10 **1 Introduction**

11 More than 50 years have passed since economists formally compared the  
12 effects of ad valorem tax and unit (specific) commodity tax on a non-  
13 competitive market. Suits and Musgrave's (1953) path-breaking study pre-  
14 sented a formal comparison in monopoly analysis to show that ad valorem  
15 tax is welfare superior to unit tax. Prior to their analysis, Cournot (1838,  
16 1960) had already found that the two tax methods needed to be treated  
17 differently in analyzing an imperfect market, and Wicksell (1896, 1959) af-  
18 firmed the analogous argument of Suits and Musgrave: for any given tax  
19 revenue, prices in a monopoly market will be lower with ad valorem tax,  
20 indicating that ad valorem tax causes less distortion and is welfare superior  
21 to unit tax.<sup>1</sup> Researchers have reexamined and substantiated this argument  
22 through various intensive studies. Several studies have focused on the effects  
23 of unit and ad valorem taxes on the equilibrium characteristics in oligopoly  
24 and monopolistic competition, and most of them confirm that the Suits and  
25 Musgrave argument still holds, while some indicates the possibility of having  
26 a counterview.<sup>2</sup>

27 The purpose of this paper is to provide further insights into this classic  
28 argument in a two-country framework. In most of the literature examining  
29 the effects of ad valorem and unit commodity taxes, comparisons are drawn  
30 within a single country framework, in which consumers are forced to buy a  
31 domestic product no matter how high the prices and taxes are. Instead, in  
32 this paper, we explore the choice of tax method in consideration of cross-  
33 border shopping in a two-country model. We argue that each country has  
34 incentives to adopt the ad valorem tax method not because of a classic  
35 welfare reason, but because ad valorem tax is superior to unit tax from  
36 the angle of attracting cross-border consumers. In fact, we first show that  
37 governments choose the ad valorem tax method as a dominant strategy  
38 to compete for mobile consumers. Then, we show further fact that the  
39 superiority of ad valorem in attracting cross-border consumers becomes at  
40 the root of significant tax-cutting competition. In other words, tax-cutting  
41 competition is more severe when a country is committed to ad valorem  
42 tax. Therefore, competition in ad valorem tax yields lower tax revenue than  
43 competition in unit tax. This result suggests a critical implication that  
44 governments are plunged into a prisoner's dilemma when choosing their tax  
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<sup>1</sup>A recent study by Carbonnier (2011) tests this theoretical hypothesis and finds that  
54 in the alcoholic beverages market in France, the shifting of prices of per unit excise taxes  
55 was significantly larger than the shifting of ad valorem VAT.

56 <sup>2</sup>See Keen (1998) for a general review.  
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10 method, and also that the tax method used for commodity tax competition  
11 is different from the one used in capital tax competition.<sup>3</sup> Accordingly, the  
12 endogenous choice of tax method, between ad valorem tax and unit tax, in a  
13 commodity tax competition framework is one of the distinctive contributions  
14 of this paper. We aim to contribute to the consolidation of the issues on  
15 governments' choices of tax methods and tax competition.  
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17 Many studies on spatial commodity tax competition, which have been  
18 carried out since the established work of Kanbur and Keen (1993), have  
19 made their presence felt in society.<sup>4</sup> For instance, if we look at the reduction  
20 of shipping and transportation costs, as a simple example, cross-border  
21 activities have strong effects compared to earlier less open economies. Fur-  
22 thermore, political efforts to create a broad economic union also reflect the  
23 importance of cross-border shopping, and globalization has enabled firms to  
24 procure funds and buy materials from the world over. The development of  
25 global markets has significantly increased the role of cross-border shopping  
26 by consumers and cross-border material procurement by producers. Our  
27 model of spatial tax competition in a two-country framework give a de-  
28 scription of actual choice of tax method, and the insights from our analysis  
29 suggest the possible inefficiency of ad valorem tax method in the integrated  
30 market.  
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33 The paper is organized as follows. Section 2 introduces the basic model.  
34 The choice of tax rates is examined in Section 3. We derive the main propo-  
35 sitions on choosing tax methods in Section 4. Section 5 presents the discus-  
36 sion of the model, which is extended to include the government's alternative  
37 objectives and the market structure. Finally, Section 6 concludes the paper.  
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40 <sup>3</sup>The choice between ad valorem tax and unit tax in a capital tax competition model  
41 was examined in Akai *et al.* (2011) and Lockwood (2004), which show that unit tax  
42 competition is superior to ad valorem tax competition, and selecting the unit tax method  
43 is a dominant strategy for governments. The choice of tax method was also examined in  
44 a tariff war model by Jorgensen and Schröder (2005) and Lockwood and Wong (2000).

45 <sup>4</sup>See Braid (1993, 2000), Cremer and Ghavari (2000), Lucas (2004), Nielsen (2001,  
46 2002), Ohsawa (1999, 2003), Ohsawa and Koshizuka (2003), and Wang (1999). See also  
47 Leal *et al.* (2010) for an extensive summary of tax competition analyses of cross-border  
48 shopping. Non-spatial models of commodity tax competition such as de Crombrughe  
49 and Tulkens (1990), Haufler (1996), Kimberley (1999), Lockwood (1993), Lucas (2004),  
50 Mintz and Tulkens (1986), Moriconi and Sato (2009), Scharf (1999), and Trandel (1994)  
51 are similar to this paper in their tax competition structure, but contrast with this paper  
52 in that they do not have a stake in the endogenous choice of tax method.  
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## 2 Model

Our simple Hotelling economy, described in Figure 1, consists of two symmetric countries,  $i = 1, 2$ . The location space of the economy is given by  $\theta \in [-1/2, 1/2]$ , divided into two countries at  $\theta = 0$ , the length of each country is therefore  $1/2$ . In each country, there is a single private firm at both ends,  $x_1 = -1/2$ ,  $x_2 = 1/2$ , where  $x_1$  and  $x_2$  are the location points of firms 1 and 2.<sup>5</sup> The firms are fixed at their locations, and they sell their products at price  $p_i$ .

*Consumers.* Consumers are endowed with a utility function separable in money and the utility derived from a given product, and each one is required to buy one product unit. They differ with respect to location and are uniformly distributed along a unit interval. We represent a consumer's location as  $y \in [-1/2, 1/2]$ . The utility of a consumer located at  $y$ , having money  $m$ , and buying a product sold by firm 1 is given by  $u_1 = v + m - p_1 - \tau(y - x_1)$ , where  $v$  stands for the utility of the product and  $\tau(y - x_1)$  the transportation cost ( $\tau > 0$ ).<sup>6</sup> In a similar way, the utility of a consumer buying a product sold by firm 2 can be given by  $u_2 = v + m - p_2 - \tau(x_2 - y)$ .

Utility maximization means picking the minimum of  $p_1 + \tau(y - x_1)$  and  $p_2 + \tau(x_2 - y)$ . As  $x_1 = -1/2$  and  $x_2 = 1/2$ , the utility that a consumer residing at  $\hat{y}$  derives from buying a product of either of the two firms is the same, where  $\hat{y} = 0.5(p_2 - p_1)/\tau$ . Hence, the demand function that firm  $i$  has to meet,  $D_i$ , can be expressed as

$$D_1(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2\tau} \quad \text{and} \quad D_2(p_1, p_2) = \frac{1}{2} - \frac{p_2 - p_1}{2\tau}. \quad (1)$$

*Governments.* In each country, there is a single revenue-maximizing government, which raises revenue only through commodity taxes; the government can choose either the unit tax or ad valorem tax method.<sup>7</sup> If the govern-

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<sup>5</sup>As long as the location is symmetric and exogenous, the assumption that the firms locate at both ends is not crucial. For instance, if the firms locate at the center of each country,  $x_1 = -1/4$  and  $x_2 = 1/4$ , the firms would simply compete for customers locating at somewhat short intervals  $[-1/4, 1/4]$ .

<sup>6</sup>A linear transport cost makes the analysis tractable, which is familiar in the literature of spatial tax competition. See Kanbur and Keen (1993), Ohsawa (1999), and Wang (1999), among others.

<sup>7</sup>Following Kanbur and Keen (1993), Ohsawa (1999), and Wang (1999), we begin by describing the government's objective in its simplest form, deferring discussions on generalizations until later.

ment adopts the unit tax method, taxes will be imposed on the number units sold, and if it selects the ad valorem tax method, taxes will be imposed on the amount of sales. If the government in country  $i$  employs the unit tax method, the tax revenue will be obtained by

$$R_i = T_i D_i, \quad (2)$$

where  $T_i$  denotes the unit tax rate. On the other hand, if the government imposes ad valorem tax, the tax revenue of country  $i$  will become

$$R_i = t_i p_i D_i, \quad (3)$$

where  $t_i (< 1)$  denotes the ad valorem tax rate.

With two countries, the governments can employ four possible tax method combinations: in case (i), both the countries will compete for mobile consumers in ad valorem tax; in case (ii), both the countries will compete in unit tax; in case (iii), country 1 will compete in unit tax and country 2 will compete in ad valorem tax; and in case (iv), country 1 will compete in ad valorem tax and country 2 will compete in unit tax. In Section 3, we analyze the four cases by one and clarify the possible equilibria properties.

*Firms.* Each firm tries to maximize its profits, indicated by

$$\pi_i = \begin{cases} (p_i - c)D_i - T_i D_i & \text{when country } i \text{ employs the unit tax method,} \\ (p_i - c)D_i - t_i p_i D_i & \text{when country } i \text{ employs the ad valorem tax method,} \end{cases}$$

and where  $c > 0$  is the constant unit cost of the product.

### 3 Choice of tax rates: Second-stage outcome

In this section, we consider a simple three-stage tax competition model and address the issue of governments choosing their tax method endogenously. In our three-stage game model, the governments choose either the unit tax or ad valorem tax method in the first stage. In the second stage, they choose the tax rate of the tax instrument selected in the previous stage. In the final stage, the firms choose their price.

#### 3.1 Unit tax competition by both governments

We refer to the case in which both governments employ the unit tax method as  $UU$ . In the third stage, given  $T_i$ , firm  $i$  maximizes  $\pi_i = (p_i - c)D_i - T_i D_i$ ,

where  $D_i$  is given by (1). The first-order condition is  $\partial\pi_i/\partial p_i = (\tau + c + T_i + p_j - 2p_i)/\tau = 0$ , where  $i \neq j$  and  $i, j = 1, 2$ , leading to

$$p_1 = \tau + c + \frac{2T_1 + T_2}{3} \quad \text{and} \quad p_2 = \tau + c + \frac{T_1 + 2T_2}{3}. \quad (4)$$

Substituting (4) into (2), the tax revenue of country  $i$  in the second stage can be obtained from

$$R_i = T_i \left( \frac{1}{2} + \frac{T_j - T_i}{6\tau} \right), \quad i \neq j, i, j = 1, 2. \quad (5)$$

Country  $i$ 's second-stage problem is to maximize its tax revenue with respect to  $T_i$ . The first-order condition is obtained by  $\partial R_i/\partial T_i = (3\tau + T_j - 2T_i)/\tau = 0$ , giving the equilibrium unit tax rates in the  $UU$  case as

$$T_1^{UU} = T_2^{UU} = 3\tau. \quad (6)$$

Substituting (6) into (5), we obtain the equilibrium tax revenue:

$$R_1^{UU} = R_2^{UU} = \frac{3}{2}\tau. \quad (7)$$

### 3.2 Ad Valorem tax competition by both governments

We refer to the case in which both countries adopt the ad valorem tax method as  $AA$ . In the third stage, given  $t_i$ , firm  $i$  maximizes  $\pi_i = (p_i - c)D_i - t_i p_i D_i$ . The first-order condition for the optimization problem is as follows:  $\partial\pi_i/\partial p_i = [c + (1 - t_i)(\tau + (p_j - 2p_i))]/\tau = 0$ , leading to

$$p_1 = \tau + \frac{c}{3} \left( \frac{2}{1 - t_1} + \frac{1}{1 - t_2} \right) \quad \text{and} \quad p_2 = \tau + \frac{c}{3} \left( \frac{1}{1 - t_1} + \frac{2}{1 - t_2} \right). \quad (8)$$

Substituting (8) into (3), we obtain the following tax revenue:

$$R_i = \frac{t_i}{2\tau} \left[ \tau + \frac{c}{3} \left( \frac{2}{1 - t_i} + \frac{1}{1 - t_j} \right) \right] \left[ \tau - \frac{c}{3} \left( \frac{1}{1 - t_i} - \frac{1}{1 - t_j} \right) \right]. \quad (9)$$

Country  $i$ 's second-stage problem is to maximize its tax revenue given by (9) with respect to  $t_i$ . The first-order condition is given by

$$\begin{aligned} \frac{\partial R_i}{\partial t_i} &= \frac{1}{\tau} \left[ \tau + \frac{c}{3} \left( \frac{2}{1-t_i} + \frac{1}{1-t_j} \right) \right] \left[ \tau - \frac{c}{3} \left( \frac{1}{1-t_i} - \frac{1}{1-t_j} \right) \right] \\ &\quad + \frac{ct_i}{3\tau(1-t_i)^2} \left[ \tau - \frac{c}{3} \left( \frac{4}{1-t_i} - \frac{1}{1-t_j} \right) \right] = 0. \end{aligned}$$

In the symmetric equilibrium, the tax rate in the  $AA$  case,  $t_1 = t_2 = t^{AA}$ , satisfies

$$3(1-t^{AA})^3 + k(1-t^{AA})(3-2t^{AA}) - k^2 t^{AA} = 0, \quad (10)$$

where  $k \equiv c/\tau > 0$ . Solving (10) with respect to  $k$ , we obtain

$$k(t^{AA}) = \frac{1-t^{AA}}{2t^{AA}} \left( 3 - 2t^{AA} + \sqrt{9 - 8(t^{AA})^2} \right). \quad (11)$$

$k(t^{AA})$  is a monotone decreasing function, and  $\lim_{t^{AA} \rightarrow +0} k(t^{AA}) = \infty$  and  $\lim_{t^{AA} \rightarrow 1} k(t^{AA}) = 0$ ; thus, the equilibrium ad valorem tax rate,  $t^{AA}$ , is an inverse function of  $k$ . Conversely,  $t^{AA}(k)$  is a monotone decreasing function, and  $\lim_{k \rightarrow +0} t^{AA}(k) = 1$  and  $\lim_{k \rightarrow \infty} t^{AA}(k) = 0$ . Figure 2 depicts the graph of  $t^{AA}(k)$  for reference. Substituting (11) into (9), we obtain the tax revenue under the given equilibrium tax rate in the  $AA$  case:

$$R_1^{AA} = R_2^{AA} = \frac{3 + \sqrt{9 - 8(t^{AA}(k))^2}}{4} \tau. \quad (12)$$

### 3.3 Preliminary results

At this stage, we obtain some remarkable results.

**Lemma 1.** *The price of products in unit tax competition is higher than that in ad valorem tax competition,  $p^{UU} > p^{AA}$ .*

**Proof.** From (4) and (6),  $p^{UU} = \tau(4+k)$ . From (8),  $p^{AA} = \tau(1 + \frac{k}{1-t^{AA}(k)})$ .

Using (11),  $p^{UU} - p^{AA} = 0.5\tau(2t^{AA}(k) + 3 - \sqrt{9 - 8(t^{AA}(k))^2})$ . Since  $3 - \sqrt{9 - 8(t^{AA}(k))^2} > 0 \forall t^{AA}(k) \in [0, 1]$ ,  $p^{UU} > p^{AA}$ . (Q.E.D.)

**Lemma 2.** *The tax rate in both unit tax competition and ad valorem tax competition has a positive relationship with transportation costs.*



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10 **Proof.** From (6),  $\partial T^{UU}/\partial\tau > 0$ . From (11), we can find that  $t^{AA}(k)$  is  
11 a monotone decreasing function. As  $k \equiv c/\tau$ ,  $t^{AA}$  increases with  $\tau$ .  
12 (Q.E.D.)  
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14 The first lemma confirms that the classic argument presented by Suits  
15 and Musgrave (1953, p.603) in a single country model still preserves in the  
16 two-country tax competition model: consumers have to pay a higher price  
17 when their countries use the unit tax method. The second lemma also  
18 presents a common view: as the mobility of consumers increase, represented  
19 by a decrease in  $\tau$ , governments are likely to engage in tax-cutting compe-  
20 tition.  
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22 Next, we discuss equivalent translation between ad valorem tax and unit  
23 tax.<sup>8</sup> Assume that country  $i$  adopts an ad valorem tax method  $t_i$ . We define  
24 the effective unit tax rate for country  $i$  as  
25

$$26 \quad T_i = t_i p_i, \quad (13)$$

27 by replacing ad valorem tax with unit tax. Using the equivalence tax rate,  
28 we obtain the following proposition.  
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32 **Proposition 1.** *Compared with ad valorem tax, tax competition is less*  
33 *severe when countries adopt the unit tax method:  $T^{AA} < T^{UU}$ .*  
34

35 **Proof.** Substituting  $p^{AA} = \tau(1 + \frac{k}{1-t^{AA}(k)})$  and (11) into (13), under  
36 symmetric equilibrium, we obtain  
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$$39 \quad T^{AA} = \left( \frac{3 + \sqrt{9 - 8t^{AA}(k)}}{2} \right) \tau. \quad (14)$$

40 Comparing (6) with (14), for  $t^{AA}(k) \in [0, 1)$ , we obtain  $T^{AA} - T^{UU} =$   
41  $(\sqrt{9 - 8(t^{AA}(k))^2} - 3)/2 < 0$ . (Q.E.D.)  
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46 Furthermore, we obtain the following result, which is directly linked to  
47 Proposition 1.  
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50 **Proposition 2.** *The tax revenue in unit tax competition is larger than the*  
51 *revenue in ad valorem tax competition:  $R_i^{AA} < R_i^{UU}$ .*  
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53 <sup>8</sup>See, for instance, Lockwood (2004), Salanié (2002, p.21), and Suits and Musgrave  
54 (1953, p.601).  
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**Proof.** Comparing (7) with (12), we obtain

$$R_i^{AA} = \frac{3 + \sqrt{9 - 8(t^{AA}(k))^2}}{4} \tau < \frac{3}{2} \tau = R_i^{UU}. \quad (\text{Q.E.D.})$$

Propositions 1 and 2 show that unit tax competition is superior to ad valorem tax competition in terms of raising tax revenue. This is simply because ad valorem tax competition induces more severe competition for customers; prices and tax rates are relatively lower in ad valorem tax competition.<sup>9</sup> Intensified tax competition simply produces significant battle to attract cross-border consumers among regions, resulting in inefficiently lower levels of tax rates and smaller amounts of tax revenue. Hence, selecting the unit tax method is desirable for both countries in terms of raising tax revenue. Proposition 2, however, leaves the possibility open for the stage 1 game to choose tax methods that have a prisoner's dilemma structure, where the ad valorem tax is a dominant strategy even though it leaves both countries worse off in equilibrium. Thus, in the next subsection, we characterize the *mixed* equilibria in which one country sets ad valorem tax, and the other sets unit tax.

### 3.4 Ad valorem tax versus unit tax

We derive the equilibrium outcome when countries employ different tax methods. We denote the case in which country 1 adopts the ad valorem (unit) tax method and country 2 selects the unit (ad valorem) tax method as  $AU(UA)$ . However, as the two cases,  $AU$  and  $UA$ , are symmetric, we examine only the  $AU$  case.

The objective function of firms 1 and 2 in the third stage are, respectively, given by  $\pi_1 = (p_1 - c)D_1 - t_1 p_1 D_1$  and  $\pi_2 = (p_2 - c)D_2 - T_2 D_2$ . The first-order conditions of the optimization problem give

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1} &= \frac{c}{2\tau} + \frac{1 - t_1}{2} + \frac{(1 - t_1)(p_2 - 2p_1)}{2\tau} = 0, \\ \frac{\partial \pi_2}{\partial p_2} &= \frac{1}{2} + \frac{c + T_2}{2\tau} - \frac{2p_2 - p_1}{2\tau} = 0, \end{aligned}$$

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<sup>9</sup>The reason ad valorem tax induces more severe competition for customers is that a tax-cut incentive is stronger in ad valorem tax than in unit tax, which is explained in Section 4.

leading to

$$p_1 = \tau + \frac{1}{3} \left[ \frac{2c}{1-t_1} + (c + T_2) \right] \quad \text{and} \quad p_2 = \tau + \frac{1}{3} \left[ \frac{c}{1-t_1} + 2(c + T_2) \right]. \quad (15)$$

Using (2), (3), and (15), tax revenues are obtained as follows.

$$R_1 = \frac{t_1}{2\tau} \left\{ \tau + \frac{1}{3} \left[ \frac{2c}{1-t_1} + (c + T_2) \right] \right\} \left\{ \tau - \frac{1}{3} \left[ \frac{c}{1-t_1} - (c + T_2) \right] \right\}, \quad (16)$$

$$R_2 = \frac{T_2}{2\tau} \left\{ \tau + \frac{1}{3} \left[ \frac{c}{1-t_1} - (c + T_2) \right] \right\}. \quad (17)$$

Given the tax rate of the other country, each country maximizes its tax revenue. The first-order conditions are given by

$$\begin{aligned} \frac{\partial R_1}{\partial t_1} &= \frac{1}{\tau} \left\{ \tau + \frac{1}{3} \left[ \frac{2c}{1-t_1} + (c + T_2) \right] \right\} \left\{ \tau - \frac{1}{3} \left[ \frac{c}{1-t_1} - (c + T_2) \right] \right\} \\ &\quad + \frac{ct_1}{3\tau(1-t_1)^2} \left\{ \tau - \frac{1}{3} \left[ \frac{4c}{1-t_1} - (c + T_2) \right] \right\} = 0, \end{aligned} \quad (18)$$

$$\frac{\partial R_2}{\partial T_2} = 1 + \frac{1}{3\tau} \left[ \frac{c}{1-t_1} - (c + 2T_2) \right] = 0. \quad (19)$$

From (19), we obtain

$$T_2 = \frac{3}{2}\tau + \frac{ct_1}{2(1-t_1)}. \quad (20)$$

Substituting (20) into (18), we obtain country 1's ad valorem tax rate in the  $AU$  case,  $t^{AU}$  which satisfies

$$\begin{aligned} 81(1-t^{AU})^3 &+ 18k(1-t^{AU})(3-3t^{AU}+(t^{AU})^2) \\ &- k^2t^{AU}(18-5t^{AU}+(t^{AU})^2) = 0, \end{aligned} \quad (21)$$

where  $k \equiv c/\tau > 0$ . Solving (21) with respect to  $k(\equiv c/\tau)$ , we obtain

$$k(t^{AU}) = \frac{9(1-t^{AU}) \left[ 3-3t^{AU}+(t^{AU})^2 + \sqrt{9-8(t^{AU})^2} \right]}{t^{AU}(18-5t^{AU}+(t^{AU})^2)}. \quad (22)$$

$k(t^{AU})$  is a monotone decreasing function, and  $\lim_{t^{AU} \rightarrow +0} k(t^{AU}) = \infty$  and  $\lim_{t^{AU} \rightarrow 1} k(t^{AU}) = 0$ ; thus, country 1's tax rate in the equilibrium,  $t^{AU}$ , is an inverse function of  $k$ . Viewed from the opposite side,  $t^{AU}(k)$  is a monotone decreasing function, and  $\lim_{k \rightarrow +0} t^{AU}(k) = 1$  and  $\lim_{k \rightarrow \infty} t^{AU}(k) = 0$ . Figure 3 depicts the graph of  $t^{AU}(k)$  for reference. From (11) and (22), we find  $t^{AA}(k) < t^{AU}(k)$  for any  $k > 0$ .<sup>10</sup>

Substituting (20) and (22) into (16) and (17), we obtain the tax revenue under the given equilibrium tax rate in the  $AU$  case:

$$\begin{aligned}
 R_1^{AU} &= \frac{9 \left[ 18 - 3t^{AU} + 4(t^{AU})^2 + (6 - t^{AU})\sqrt{9 - 8(t^{AU})^2} \right]}{8(18 - 5t^{AU} + (t^{AU})^2)^2} \\
 &\quad \times \left( 15 - 2t^{AU} - \sqrt{9 - 8(t^{AU})^2} \right) \tau \tag{23}
 \end{aligned}$$

$$R_2^{AU} = \frac{3 \left( 27 - 14t^{AU} + 4(t^{AU})^2 + 3\sqrt{9 - 8(t^{AU})^2} \right)^2}{8(18 - 5t^{AU} + (t^{AU})^2)^2} \tau, \tag{24}$$

where  $t^{AU}$  in (23) and (24) is a function of  $k$ . While we do not explicitly present the  $UA$  case,  $R_1^{AU} = R_2^{UA}$  and  $R_1^{UA} = R_2^{AU}$  hold.

## 4 Choice of tax method: First-stage outcome

Table 1 shows the payoff matrix in the first stage. To obtain the subgame perfect Nash equilibrium, we compare the tax revenues in each case and obtain the following results.

**Lemma 3.**  $R_1^{AU} > R_1^{UU}$  and  $R_1^{AA} > R_1^{UA}$ . Symmetrically,  $R_2^{UA} > R_2^{UU}$  and  $R_2^{AA} > R_2^{AU}$ .

Proof. See Appendix B.

To explain the result  $R_1^{AU} > R_1^{UU}$ , suppose country 2 selects the unit tax method. If country 1 changes its method from the unit tax to the ad valorem tax method and sets its ad valorem tax rate at a level that would attain the same price level of  $p^{UU}$ , the equivalent tax rate would become higher with the same demand, thus, country 1's tax revenue would increase.

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<sup>10</sup>See Appendix A.

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Country 1/Country 2	Unit tax	Ad valorem tax
Unit tax	$R_1^{UU}, R_2^{UU}$	$R_1^{UA}, R_2^{UA}$
Ad valorem tax	$R_1^{AU}, R_2^{AU}$	$R_1^{AA}, R_2^{AA}$

Table 1. Payoff Matrix

Note. First (second) coordinate in each pair is payoff to country 1 (2).

This feature—the superiority of the ad valorem tax method in raising tax revenue with the same demand—is well known, and the insight behind this result is identical with that presented by Suits and Musgrave (1953, p.599-600). Thus, country 1, which maximizes its tax revenue, has an incentive to select ad valorem tax. In addition, in our model, which allows for cross-border shopping, country 1 attracts cross-border consumers and increase its tax revenue further by reducing its ad valorem tax rate from the level that attains  $p^{UU}$ . This additional tax-cut incentive produces significant fiscal externalities, and ad valorem tax competition results in an inferior outcome, compared with unit tax competition.

The result  $R_1^{AA} > R_1^{UA}$  can also be explained simply. Suppose country 2 selects ad valorem tax. This choice sends the signal to country 1 that country 2 has an additional tax-cutting incentive, as explained above. Country 1 can avoid this severe tax-cutting competition by selecting the unit tax method in the first stage. However, even if country 1 avoids the tax-cutting competition and yields higher prices by selecting unit tax, the demand will fall, and consumers will flow out from country 1 to country 2, resulting in lower tax revenue in country 1. Thus, country 1 has no incentive to select unit tax when country 2 uses ad valorem tax. The same argument applies to country 2, and therefore both the countries select ad valorem tax in the first stage.

Based on Lemma 3 and Proposition 2, we obtain the following result.

**Proposition 3.** *Choosing ad valorem tax is a dominant strategy. However, an equilibrium in which both countries adopt the method is not efficient.*

Proof. Lemma 3 shows that ad valorem tax is the dominant strategy for both governments. Furthermore, Proposition 2 leads directly to the

latter result. (Q.E.D.)

In sum, ad valorem tax method succeeds in lowering monopoly price in the domestic market, as suggested by Suits and Musgrave (1953). This is a positive side of using ad valorem tax. However, there exists a dark side of ad valorem tax method when the cross-border shopping is allowed: any decrease in price leads directly to inter-regional competition for mobile consumers, bringing down the fiscal externality. Since the governments' objective is the revenue maximization, the dark side prevails only in our model, and choosing the ad valorem tax method is always inefficient.

## 5 Discussion

### 5.1 Benevolent government

So far, we assumed that each country maximizes its revenue. This is a reasonable assumption under most circumstances and can be justified by assuming a Leviathan-type government. Alternatively, revenue maximization objectives of governments can be justified when tax-competing governments face severe revenue shortfalls, so that their tax revenue becomes sufficiently more important than private good consumption. In this section, we check the robustness of our results, assuming an alternative government objective. To show our main argument as simple as possible, we present a simple numerical example by assuming that each country maximizes its domestic welfare,  $W_i$ , given by

$$W_1 = \begin{cases} \int_{-1/2}^{\hat{y}} [v + m - p_1 - \tau(y + 0.5) + \beta \ln R_1] dy \\ \quad + \int_{\hat{y}}^0 [v + m - p_2 - \tau(0.5 - y) + \beta \ln R_1] dy + \pi_1 & \text{if } \hat{y} < 0 \\ \int_{-1/2}^0 [v + m - p_1 - \tau(y + 0.5) + \beta \ln R_1] dy + \pi_1 & \text{if } \hat{y} \geq 0 \end{cases} ,$$

$$W_2 = \begin{cases} \int_0^{1/2} [v + m - p_2 - \tau(0.5 - y) + \beta \ln R_2] dy + \pi_2 & \text{if } \hat{y} < 0 \\ \int_0^{\hat{y}} [v + m - p_1 - \tau(y + 0.5) + \beta \ln R_2] dy \\ \quad + \int_{\hat{y}}^{1/2} [v + m - p_2 - \tau(0.5 - y) + \beta \ln R_2] dy + \pi_2 & \text{if } \hat{y} \geq 0 \end{cases} .$$

Note that a country's tax revenue is meant for public good provision in order to benefit its residents.  $\beta > 0$  indicates the weight for public good consumption. Assuming that  $\tau = c = 1$  and  $v = m = 3$ , we compare two polar cases; (i) individuals receiving significant benefits from public good

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Country 1/Counrty 2	Unit tax	Ad valorem tax
Unit tax	1.96, 1.96	0.73, 3.03
	1.47*, 1.47*	1.41*, 1.56*
Ad valorem tax	3.03, 0.73	1.91, 1.91
	1.56*, 1.41*	1.50*, 1.50*

Table 2. Payoff Matrix under Benevolent Government

Note. The first (upper) pair shows the social surplus to country 1 and 2 when  $\beta = 12$ ; the second (lower) pair, marked with an asterisk, shows the social surplus when  $\beta = 1$ .

provision financed by commodity taxation (e.g.,  $\beta = 12$ ), and (ii) individuals receiving moderate benefits (e.g.,  $\beta = 1$ ). Using simple calculations, we obtain the domestic welfare in each case, given in Table 2. The first (upper) pair shows the welfare obtained in countries 1 and 2 when  $\beta = 12$ ; the second (lower) pair, marked with an asterisk, shows the countries' domestic welfare when  $\beta = 1$ . The table shows that, for both examples, selecting ad valorem tax is the dominant strategy in the first stage.

However, both cases offer different welfare implication. When individuals receive significant benefits from public good provision financed by commodity taxation, that is, when  $\beta = 12$ , the countries encounter a prisoner's dilemma game, as shown in the previous section. In contrast, when individuals receive moderate benefits, that is, when  $\beta = 1$ , the ad valorem tax choice is efficient.

Compared with unit tax, ad valorem tax produces less dead weight loss in the domestic market under the same amount of tax revenue. This is a positive aspect of using ad valorem tax. However, there is a negative factor associated with ad valorem tax competition in the globalized market, as stated in the previous section: when consumers cross borders, the resulting severe tax competition leads to inefficiently lower levels of tax rates. If  $\beta$  is sufficiently large, the objective of governments corresponds approximately to tax revenue maximization. Thus, the negative aspect of ad valorem tax dominates the positive aspect, and the welfare in ad valorem tax becomes

less than that in unit tax. In contrast, if  $\beta$  is sufficiently small, distortions in the domestic market will become more important than those caused by tax competition. In this case, ad valorem tax competition produces less dead weight loss and yields higher welfare.

## 5.2 Competitive product market

In Section 4, we assumed that a single producer in each country. By examining a perfectly competitive market, which is antithetical to monopoly, we can find out, roughly, how incentives to adopt an ad valorem tax method would change as the market structure changes. In this regard, Braid (1993, p.89-90) mentioned that unit tax is equivalent to ad valorem tax in a perfectly competitive market. In our framework, this can be easily confirmed as follows.

First, consider the *UU* case, in which both the countries adopt a unit tax method. As the present price in country  $i$  can be given by  $p_i^* = T_i + c$ , each country's tax revenue will be as follows:

$$R_i = T_i D_i(p_i^*, p_j^*) = T_i \left( \frac{1}{2} + \frac{T_j - T_i}{2\tau} \right), i \neq j, \text{ and } i, j, = 1, 2. \quad (25)$$

Maximization of (25) gives the first-order condition as  $\partial R_i / \partial T_i = (\tau + T_j - 2T_i) / \tau = 0$ , giving the tax rate in the equilibrium:  $T_1^* = T_2^* = T^{UU} = \tau$ . Substituting  $T_i = \tau$  into (25), we get the tax revenue in *UU* equilibrium as  $R_i^{UU} = \tau/2$ .

In *AA* case in which both countries adopt the ad valorem tax method, the price obtained is  $p_i^* = t_i p_i^* + c$ , from this we obtain  $p_i^* = c / (1 - t_i) = c + \hat{t}_i$ , where  $\hat{t}_i \equiv ct_i / (1 - t_i)$ . The tax revenue of each country can be calculated in analogy with (25). The first-order condition for the tax revenue maximization problem is satisfied when  $\hat{t}_1 = \hat{t}_2 = \tau$ , therefore  $t_i = \tau / (c + \tau)$ . This yields the tax revenue in the *AA* equilibrium as  $R_i^{AA} = \tau/2$ .

The equilibrium outcome when one country competes in unit tax and the other country selects ad valorem tax can be derived in the same manner. We find that the equilibrium tax revenue in the *AU* case is the same as that in the *AA* and *UU* cases.<sup>11</sup> This shows that as the magnitude of the market's incompleteness is reduced, the ad valorem tax method loses its superiority.

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<sup>11</sup>Available from the author upon request.



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10 **6 Conclusion**

11 In this paper, we examined the tax method selected in a model of cross-  
12 border shopping model. The results show that revenue-maximizing govern-  
13 ments could use the ad valorem tax method, but this choice is not efficient:  
14 choosing the unit tax method yields higher tax revenue for governments.  
15 Our result can provide an alternative explanation as to why almost every  
16 country facing cross-border shopping employs the ad valorem tax method  
17 in tax competition.  
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20 We extend our analysis to an alternative model of benevolent government  
21 objective and market structure to check the robustness of our results. Our  
22 extension on the government objective has proved that most of the results  
23 still hold in many instances. Specifically, governments adopt the ad valorem  
24 tax method to compete for mobile demand. The welfare implication should,  
25 however, be slightly modified. When individuals receive significant benefits  
26 from public good provision financed by commodity taxation, the main results  
27 will still hold; countries encounter the prisoner's dilemma game. By contrast,  
28 when individuals receive moderate benefits from public good provision, the  
29 choice of ad valorem tax is efficient.  
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32 An examination of the effects of market structures suggests that coun-  
33 tries are indifferent about choosing between ad valorem tax and unit tax in  
34 perfectly competitive environment, although the ad valorem tax is usually  
35 adopted.  
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38 **Appendices**

39 **Appendix A**

40 *Proof of  $t^{AA}(k) < t^{AU}(k)$ .* Denote the left-hand side of (11) and (22) as  $k_{AA}$   
41 and  $k_{AU}$ , respectively. Then, for  $t^{AA} = t^{AU} = t$ , we have  
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$$k_{AU}(t) - k_{AA}(t) = -\frac{(1-t)[f(t) - g(t)]}{2(18 - 5t + t^2)},$$
  
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50 where  $f(t) \equiv 3 - 5t - 2t^2$  and  $g(t) \equiv (5 - t)\sqrt{9 - 8t^2}$ . When  $0 < t \leq 1/2$ ,  
51  $f(t) \geq 0$  and  $g(t) > 0$ ; thus,  
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$$\begin{aligned} \text{sgn}[k_{AU}(t) - k_{AA}(t)] &= \text{sgn} \left[ - \left\{ (f(x))^2 - (g(x))^2 \right\} \right] \\ &= \text{sgn}[12(1 - t^2)(18 - 5t + t^2)] > 0. \end{aligned}$$
  
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When  $1/2 < t < 1$ ,  $f(t) < 0$  and  $g(t) > 0$ ; thus,  $k_{AU}(t) - k_{AA}(t) > 0$ . Accordingly, we obtain that  $k_{AA}(t) < k_{AU}(t)$  for any  $t \in (0, 1)$ . Since  $k_{AA}(t)$  and  $k_{AU}(t)$  are monotone decreasing, we have  $t^{AA}(k) < t^{AU}(k)$ .

## Appendix B

*Proof of  $R_1^{AU} > R_1^{UU}$ .* Since  $0 < t < 1$  is satisfied for any  $k > 0$ , it is sufficient to show  $R_1^{AU}(t) > 3\tau/2 (= R_1^{UU})$  for  $0 < t < 1$ . Using (23), we have

$$\begin{aligned} R_1^{AU}(t) - \frac{3\tau}{2} &= \frac{9 \left[ 18 - 3t + 4t^2 + (6-t)\sqrt{9-8t^2} \right]}{8(18-5t+t^2)^2} \\ &\quad \times \left( 15 - 2t - \sqrt{9-8t^2} \right) \tau - \frac{3\tau}{2} \\ &= -\frac{3\tau[h(t) - l(t)]}{4(18-5t+t^2)^2}, \end{aligned}$$

where  $h(t) \equiv 324 - 252t - 49t^2 + 4t^3 + 2t^4$  and  $l(t) \equiv 3(36 - 12t - t^2)\sqrt{9 - 8t^2}$ . Since  $h(t) > 0$  and  $l(t) > 0$  for  $0 < t < 1$ ,

$$\begin{aligned} \text{sgn}[R_1^{AU}(t) - \frac{3\tau}{2}] &= \text{sgn}[-\{h^2(t) - l^2(t)\}] \\ &= \text{sgn}[4t(18-5t+t^2)(72-52t-14t^2-t^3)] > 0. \end{aligned}$$

Thus,  $R_1^{AU}(t^{AU}(k)) > 3\tau/2 = R_1^{UU}$  is satisfied for any  $k > 0$ . For reference, Figure A.1 represents the tax revenue of country 1 in *AU* case and shows  $R_1^{AU}(t^{AU}) > 3\tau/2 = R_1^{UU}$ .

*Proof of  $R_1^{AA} > R_1^{UA}$ .* First, we show  $R_1^{AA}(t) > R_1^{UA}(t)$  for  $0 < t < 1$ .

$$\begin{aligned} R_1^{AA}(t) - R_1^{UA}(t) &= R_1^{AA}(t) - R_2^{AU}(t) \\ &= \left( \frac{3 + \sqrt{9-8t^2}}{4} - \frac{3 \left( 27 - 14t + 4t^2 + 3\sqrt{9-8t^2} \right)^2}{8(18-5t+t^2)^2} \right) \tau \\ &= -\frac{\tau[m(t) - n(t)]}{4(18-5t+t^2)^2}, \end{aligned}$$

where  $m(t) \equiv 3(81 - 198t + 109t^2 - 46t^3 + 7t^4)$  and  $n(t) \equiv (81 - 54t + 25t^2 - 10t^3 + t^4)\sqrt{9 - 8t^2}$ .  $m(t)$  is a monotone decreasing function of  $t$  for  $0 < t < 1$

and  $m(0) > m(1/2) > 0 > m(2/3) > m(1)$ .  $n(t)$  is a monotone decreasing function of  $t$  and  $n(t) > 0$  for  $0 < t < 1$ . When we denote  $\alpha$  as the root of equation  $m(t) = 0$  for  $1/2 < t < 2/3$ ,  $1/2 < \alpha < 2/3$ , and if  $t \geq \alpha$ ,  $m(t) < 0$ ; thus,  $R_1^{AA}(t) - R_1^{UA}(t) > 0$ . Moreover, if  $t < \alpha$ ,  $m(t) > 0$ ; thus,

$$\begin{aligned} \operatorname{sgn}[R_1^{AA}(t) - R_1^{UA}(t)] &= \operatorname{sgn}[-\{m^2(t) - n^2(t)\}] \\ &= \operatorname{sgn}[4t(18 - 5t + t^2)^2s(t)], \end{aligned}$$

where  $s(t) \equiv 162 - 297t + 180t^2 - 86t^3 + 20t^4 - 2t^5$ . It is easily find  $s(t)$  is a monotone decreasing function of  $t$  and is greater than zero for  $0 < t < 2/3$ ; thus,  $R_1^{AA}(t) - R_1^{UA}(t) > 0$  for  $0 < t < \alpha < 2/3$ . Accordingly,  $R_1^{AA}(t) > R_1^{UA}(t)$  for  $0 < t < 1$ . Figure A.2 is depicted, for reference, to represent the tax revenue of country 1 under the given equilibrium tax rate in case *AA* and *UA*. Moreover, regarding equilibrium tax rate, we get  $t^{AA}(k) < t^{AU}(k) = t^{UA}(k)$  for any  $k$ . Therefore, we have  $R_1^{AA}(t^{AA}(k)) > R_1^{AA}(t^{UA}(k)) > R_1^{UA}(t^{UA}(k))$  for any  $k$ .

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Figure 1

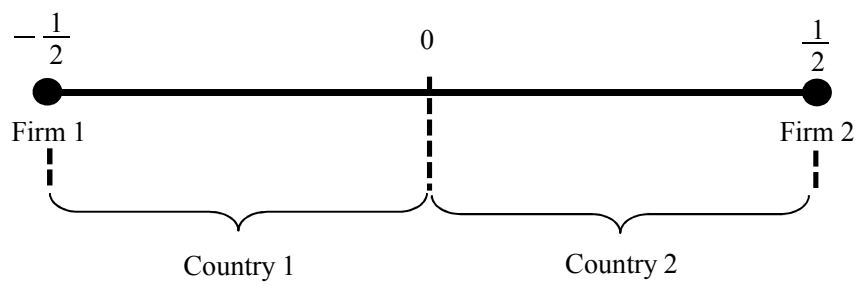


Figure 1. The location space

Figure 2

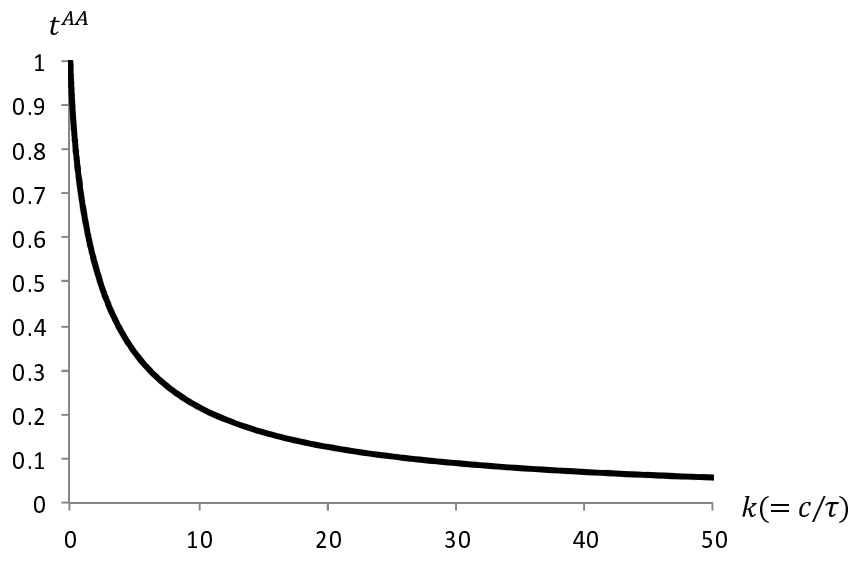


Figure 2. The graph of  $t^{AA}(k)$



Figure 3

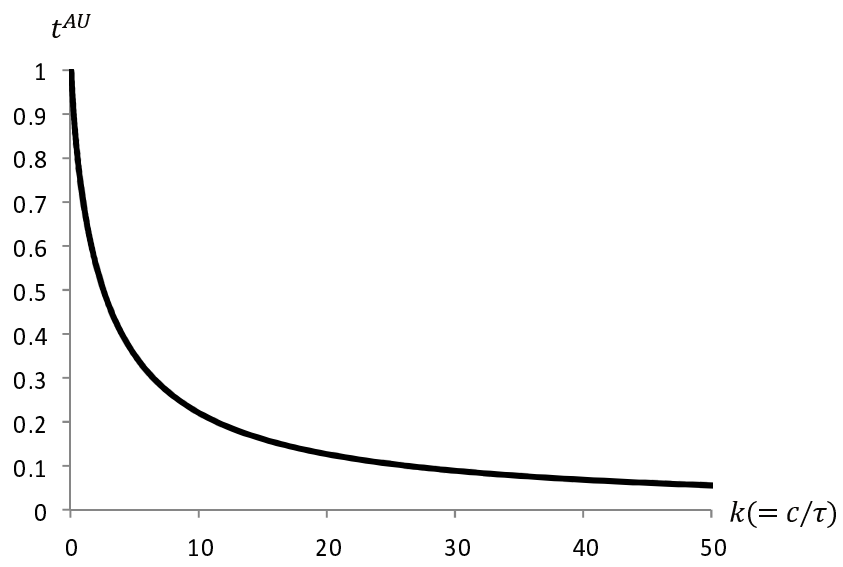


Figure 3. The graph of  $t^{AU}(k)$

Figure A.1

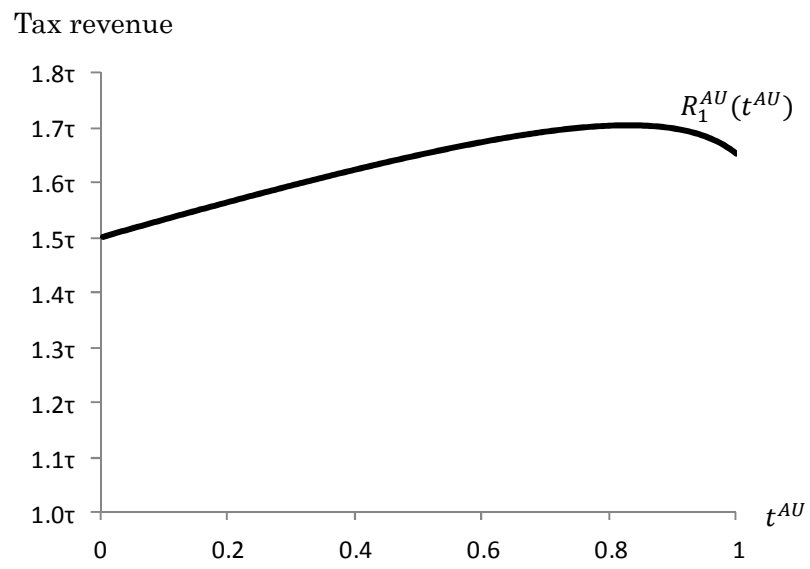


Figure A.1. The graph of  $R_1^{AU}(t^{AU})$

Figure A.2

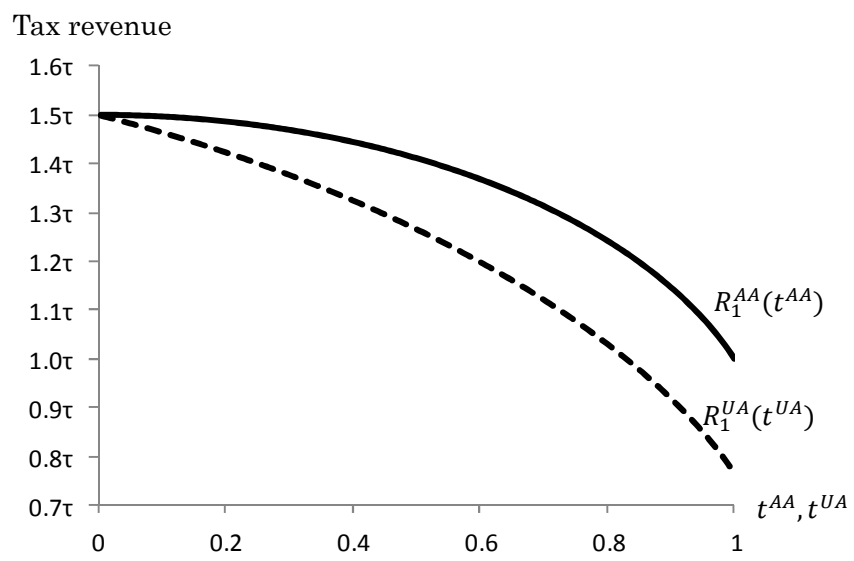


Figure A.2. The graphs of  $R_1^{AA}(t^{AA})$  and  $R_1^{UA}(t^{UA})$