

Dalmazzo, Alberto; Accetturo, Antonio; de Blasio, Guido

**Conference Paper**

## Skill Polarization in Local Labour Markets under Share-Altering Technical Change

52nd Congress of the European Regional Science Association: "Regions in Motion - Breaking the Path", 21-25 August 2012, Bratislava, Slovakia

**Provided in Cooperation with:**

European Regional Science Association (ERSA)

*Suggested Citation:* Dalmazzo, Alberto; Accetturo, Antonio; de Blasio, Guido (2012) : Skill Polarization in Local Labour Markets under Share-Altering Technical Change, 52nd Congress of the European Regional Science Association: "Regions in Motion - Breaking the Path", 21-25 August 2012, Bratislava, Slovakia, European Regional Science Association (ERSA), Louvain-la-Neuve

This Version is available at:

<https://hdl.handle.net/10419/120515>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# **Skill Polarization in Local Labour Markets under Share-Altering Technical Change**

**Antonio ACCETTURO**  
(Research Dept., Bank of Italy, Rome)

**Alberto DALMAZZO**  
(Dept. of Economics, University of Siena)

**Guido de BLASIO**  
(Research Dept., Bank of Italy, Rome)

**February 6, 2012**

**Corresponding author:** Alberto Dalmazzo, Dept. of Economics, University of Siena, Piazza S.Francesco 7 -53100 Siena – Italy; email: [dalmazzo@unisi.it](mailto:dalmazzo@unisi.it) ; phone: +39-0577232697; fax: +39-0577-232661.

**Abstract.** This paper considers the “share-altering” technical change hypothesis in a spatial general equilibrium model where individuals have different levels of skills. Building on a simple Cobb-Douglas production function, our model shows that the implementation of skill-biased technologies requires a sufficient proportion of highly educated individuals. Moreover, when technical progress is such to disproportionately replace middle-skill jobs, the local distribution of skill will exhibit “fat-tails”, where the proportion of both highly skilled and low-skilled workers increases. These predictions are consistent with recent existing evidence.

**Keywords:** share-altering technologies, local skill distribution, local wage premium.

**JEL Classification Numbers:** O33, R12, R23, J31.

**Acknowledgements:** We thank participants in the 57<sup>th</sup> Annual North American Meeting of the RSAI, November 2010, Denver and 51<sup>st</sup> European Congress of the RSAI, September 2011, Barcelona for comments on previous versions of this paper. The usual disclaimers apply.

## I. INTRODUCTION

A famous paper by Autor, Levy and Murnane (2003) (ALM henceforth) re-qualified the “skill-biased technical change” hypothesis by noting that, during the last three decades, new technologies have been particularly effective in replacing workers who performed “routine tasks”, that is, tasks which require a well-defined set of manual and cognitive abilities. Such new technologies have mostly raised the demand for highly-skilled workers (mainly college graduates) at the expenses of mid-skilled workers (with high-school education), while leaving basically untouched the demand for low-skills, fit to perform non-routine manual tasks such as personal services. This technological trend has generated “polarization” in employment and wages in the US labour market (Autor et al. 2008, Autor and Dorn 2009). Similar “hollowing-out” patterns in wages and employment distribution have been observed in Europe (Goos and Manning 2007, Goos et al. 2009, Michaels et al. 2010): see Van Reenen (2011) for a survey of international evidence.

How do these economy-wide findings reflect in local labour markets? Recent contributions by Moretti (2010,2011) have argued that labour market outcomes should be evaluated at the local level by exploiting spatial general equilibrium models such as Roback (1982), which account for local price levels. For instance, he observes that the educated are disproportionately located in metropolitan areas characterized by high housing costs. Some analyses of local labour markets have emphasized specific features generated by technical change. Beaudry et al. (2010) have shown that skill-intensive PC technologies have been adopted in cities where highly educated workers were abundant. Also, new developments in urban economics have argued that high levels of local human capital may attract even more educated workers, as in the “rise of the skilled city” story: see Glaeser (2008) for an overview. In particular, Berry and Glaeser (2005) have found that demand for high skilled workers has been rising in initially high skill cities. Such models, however, concentrate only on two types of workers and, for this reason, are unfit to analyze changes in skill-distribution, like rising “polarization” in the labour market. In this perspective, Eeckhout et al. (2011) observe, consistently with the findings of Lin (2011), that large cities exhibit “fat-tails” in the skill distribution. In other words, large cities disproportionately attract both the most *and* the least skilled workers. At the same time, Autor and Dorn (2011) find that skill polarization has mostly occurred in labour markets characterized both by high levels of routine activities and human capital.

This paper makes a theoretical contribution which accounts for a variety of stylized facts within a simple general equilibrium unified framework, where firms and workers are free to move across areas and markets are perfectly competitive. We argue that the ALM hypothesis on the nature of recent technological change can be given a simple representation by exploiting the idea of “share-altering” technical change. This form of technical change, modelled as a change in Cobb-Douglas share parameters, has been recently re-discovered by Seater (2005), Seater and Peretto (2008) and Zuleta (2008). The idea is rather simple: consistently with observation, most modern innovations seem to raise the share parameter of skilled workers, while keeping constant the *total* labour share. At the same time, when one considers a production function

with three types of workers (high, mid, low skills), ALM supports the view that new technologies have disproportionately substituted for “routine” tasks that were mostly occupied by mid-skill workers.

We show that our model gives similar implications about endogenous skill-biased technology adoption that Beaudry et al. (2010) derive from the “canonical”<sup>1</sup> CES approach. In particular, skill-biased technologies are adopted where high skills are relatively abundant<sup>2</sup>, adoption tends to raise the skill-premium in the short run, but migration tends to equalize the premium economy-wide. More important, by considering three types of skills, our model also predicts that places rich in human capital, by favouring the adoption of skill-biased technologies, tend to exhibit “fat-tails” in the local skill distribution, as observed by Eeckhout et al. (2011) and Autor and Dorn (2011). Intuitively, the mechanism is the following: highly skilled people locate relatively more in places which are rich in amenities that are particularly appreciated by the educated. Such places, thus, can adopt (share-altering) production technologies which shift employment from middle to low-skilled workers. Such technologies are profitable because the local skill premium is *lower* where education-specific amenities are relatively abundant. The model thus generates the fatter tails both on the right and on the left of the employment distribution.

Eeckhout et al. (2011) offer a theoretical justification of skill-polarization in big cities that is, also, entirely based on technology. They build on the presence of heterogeneity in TFP across locations and, in particular, postulate a variation<sup>3</sup> of the CES production function where complementarity between “extreme-skills” must hold. There, the local TFP level drives both city-size and skilled productivity. Under their assumptions, skilled productivity further benefits from the disproportionate presence of unskilled workers, relative to mid-skilled workers, generating fat-tails in the skill distribution of big cities. Differently from Beaudry et al. (2010) and from the model we propose here, however, Eeckhout et al. do not discuss the conditions under which their “extreme-skill complementarity” technology will be implemented at the local level. On the other hand, Autor and Dorn (2011) show that local labour markets that were mostly specialized in routine jobs have disproportionally adopted information technology leading to the reallocation of unskilled labour into service occupations. They theoretically represent this form of employment polarization by interacting the supply-side, driven by information technology replacing for routine jobs, with the demand side, where preferences place sufficiently large weight on variety in the consumption of goods and services. To this regard, our model is so simple that it does not need any specific assumption on preferences to generate fat-tails.

---

<sup>1</sup> This terminology is borrowed from Acemoglu and Autor (2010). Van Reenen (2011,p.735) emphasizes the limits of the canonical model to explain both the technology adoption mechanism and labour market polarization.

<sup>2</sup> This implication is similar to Acemoglu (1998), where investment in skill-complementary technologies depends on the proportion of skilled individuals in the workforce. In Berry and Glaeser (2005), a larger number of educated residents will generate more skilled entrepreneurs who hire skilled workers. In this perspective, the initial level of city skills crucially determines the future level of local skill demand. Our view however is closest to Beaudry’s et al. (2010) “comparative advantage” story for technology adoption.

<sup>3</sup> Eeckhout et al (2011) show in fact that the traditional CES production function predicts different population sizes across cities that exhibit different TFP, but it implies *the same distribution of skills in every location*.

The paper is organized as follows. Section II.1 describes the basic Roback model, where skilled individuals exhibit preferences for local amenities that are partly different from those of the mid-skilled and unskilled. Then, in Section II.2, we discuss the implications of share-altering technical change. Section III concludes.

## II. THE MODEL

We consider a standard general equilibrium model, where firms and workers are perfectly mobile across areas: see Roback (1982,1988). The economy is composed of two areas, Area 1 and Area 2, which are endowed with different characteristics, affecting residents' "quality of life". In each area, firms produce an homogeneous good by using "land" and three types of labor: unskilled, medium-skill, skilled.<sup>4</sup> The good is traded competitively across areas. All workers earn a wage and consume both the produced good and residential space, "land". For simplicity, the supply of "land" in each area is taken to be fixed and landowners are absentee. Since firms are assumed perfectly mobile between areas, profits will be equalized across the economy. Similarly, since mobility costs are taken to be zero, workers' utility will be perfectly equalized across areas. For what it concerns individuals' utility, we assume that each area possesses some local characteristics (such as sunshine or crime) that equally affect the quality of life of *all* individuals. However, we postulate that there are other local features that affect the utility of skilled individuals only. This assumption, relevant for the results that follow, is quite common in the literature (see, e.g. Glaeser 2008 and Moretti 2011) and consistent with the findings in Carlino and Saiz (2008) for the US, and Dalmazzo and de Blasio (2011) for Italy.

We start by describing the features of spatial general equilibrium in the absence of technical change.

### 1. The basic framework

The local amount of land,  $\bar{\ell}_c$ , with  $c = \{1,2\}$ , is fixed. The (endogenously-determined) local supplies of skilled, mid-skilled, unskilled labour are denoted, respectively, by  $\{n_c^s, n_c^m, n_c^u\}$ . In the aggregate, it holds that  $n_1^s + n_2^s = \bar{n}^s$ ,  $n_1^m + n_2^m = \bar{n}^m$  and  $n_1^u + n_2^u = \bar{n}^u$ , where  $\{\bar{n}^s, \bar{n}^m, \bar{n}^u\}$  define the economy-wide fixed amount of each type of labour. We first illustrate firms' optimal behavior. Then, we look at workers, so to characterize the equilibrium in the two areas.<sup>5</sup>

*Firms.* Firms in area  $c = \{1,2\}$  produce an homogeneous good by using land,  $L_c$ , and all three labor types, respectively  $\{N_c^s, N_c^m, N_c^u\}$ , with a Cobb-Douglas technology characterized by constant returns to scale:

$$Y_c = L_c^{1-\alpha-\beta-\gamma} \cdot (N_c^s)^\alpha \cdot (N_c^m)^\beta \cdot (N_c^u)^\gamma \quad (1)$$

---

<sup>4</sup> One can think of skilled workers as college graduates and mid-skill workers as individuals holding high-school diplomas.

<sup>5</sup> As in the basic spatial framework introduced by Glaeser (2008), we model both preferences and technology as Cobb-Douglas functions. However, when Glaeser proceeds to investigate local skill-premia, he drops the Cobb-Douglas specification in favour of a CES technology. We show in what follows that this step, under the share-altering hypothesis, is unnecessary.

where  $\alpha + \beta + \gamma \in (0,1)$ .<sup>6</sup> Respectively,  $\{r_c, p_c\}$  denote the local price of land (rent) and the price of the traded good. In what follows, we will assume that  $p_1 = p_2 = 1$ . The wage received by a skilled worker in area  $c$  is denoted by  $w_c^s$ , the mid-skilled wage is  $w_c^m$ , while the unskilled wage is equal to  $w_c^u$ . A competitive firm located in  $c$  will equate price to marginal cost, which is equivalent to  $\left[ \frac{(1 - \alpha - \beta - \gamma)^{1 - \alpha - \beta - \gamma} \alpha^\alpha \beta^\beta \gamma^\gamma}{r_c^{1 - \alpha - \beta - \gamma} (w_c^s)^\alpha (w_c^m)^\beta (w_c^u)^\gamma} \right] = 1$ . Since firms are perfectly mobile across areas, marginal cost must be equal to one in both areas (see Appendix A.1 for details).

*Skilled workers.* Skilled workers living in area  $c$  maximize the utility function

$$U_c^s = A_U(X_c) \cdot B_U(Z_c) \cdot L_c^{1-\mu} \cdot Y_c^\mu \quad (2)$$

subject to the budget constraint  $r_c \cdot L_c + Y_c = w_c^s$ . Skilled worker's utility (2) includes an “amenity” term  $A_U(X_c)$ , non-decreasing in the vector of local characteristics  $X_c$ , which is common to mid-skilled and unskilled utility (see below). However, skilled utility also includes an additional “amenity” term  $B_U(Z_c) \geq 1$ , non-decreasing in  $Z_c$ . The vector  $Z_c$  denotes some additional territorial characteristics that are valuable to skilled individuals, but irrelevant to the welfare of other workers' types: see, e.g., Glaeser (2008) and Moretti (2011). The optimal choice of the consumption bundle generates an indirect utility for a skilled resident in area  $c = \{1,2\}$  given by:

$$v_c^s = \eta \cdot A_U(X_c) \cdot B_U(Z_c) \cdot \frac{w_c^s}{r_c^{1-\mu}}, \quad c = \{1,2\} \quad (3)$$

where  $\eta \equiv (1 - \mu)^{1-\mu} \mu^\mu$ . In the absence of mobility costs, a skilled worker is indifferent whether to migrate or not whenever the condition  $v_1^s = v_2^s = \bar{v}^s$  holds.

*Mid-skill and unskilled workers.* Mid-skill and unskilled workers in area  $c$  receive a wage respectively equal to  $w_c^m$  and  $w_c^u$ . By maximizing utility<sup>7</sup> subject to  $r_c \cdot L_c + Y_c = w_c^m$ , indirect utility of a

<sup>6</sup> We abstract from local features, such as infrastructures, that can affect firms' local productivity through a TFP multiplicative term. Differently from Eeckhout et al. (2011) where local TFP has a central role in driving city-size and skill distribution, the inclusion of such a term is irrelevant to our main results.

<sup>7</sup> Mid-skill and unskilled utility has the same structure as skilled utility (2), except for the absence of the amenity term  $B_U(Z_c)$ .

mid-skill worker who resides in region  $c$  is given by  $v_c^m = \eta \cdot A_U(X_c) \cdot \frac{w_c^m}{r_c^{1-\mu}}$ , with  $c = \{1,2\}$ . Free mobility for mid-skill individuals implies that  $v_1^m = v_2^m = \bar{v}^m$ . Similar expressions hold for unskilled workers.

*Equilibrium (without technical change).* The relative rent and wage ratios between areas are given by the following expressions (see Appendix A.1 for details):

$$\log\left(\frac{r_1}{r_2}\right) = \frac{(\alpha + \beta + \gamma) \cdot \log \frac{A_U(X_1)}{A_U(X_2)} + \alpha \cdot \log \frac{B_U(Z_1)}{B_U(Z_2)}}{1 - \mu(\alpha + \beta + \gamma)}. \quad (4)$$

$$\log\left(\frac{w_1^s}{w_2^s}\right) = \frac{-(1 - \alpha - \beta - \gamma) \cdot \log \frac{A_U(X_1)}{A_U(X_2)} - [1 - \alpha - \mu(\beta + \gamma)] \cdot \log \frac{B_U(Z_1)}{B_U(Z_2)}}{1 - \mu(\alpha + \beta + \gamma)} \quad (5)$$

$$\log\left(\frac{w_1^m}{w_2^m}\right) = \frac{-(1 - \alpha - \beta - \gamma) \cdot \log \frac{A_U(X_1)}{A_U(X_2)} + \alpha(1 - \mu) \cdot \log \frac{B_U(Z_1)}{B_U(Z_2)}}{1 - \mu(\alpha + \beta + \gamma)} \quad (6)$$

$$\log\left(\frac{w_1^u}{w_2^u}\right) = \frac{-(1 - \alpha - \beta - \gamma) \cdot \log \frac{A_U(X_1)}{A_U(X_2)} + \alpha(1 - \mu) \cdot \log \frac{B_U(Z_1)}{B_U(Z_2)}}{1 - \mu(\alpha + \beta + \gamma)} \quad (7)$$

These equilibrium expressions have standard interpretations. Equation (4) shows that local characteristics that increase welfare in Area 1 have a positive effect on rents, relative to Area 2. However, expression (5) emphasizes that relative abundance of local amenities in Area 1 reduces the relative skilled wage in this region. Skilled individuals are ready to accept a lower wage to live in places that have higher quality of life. Finally, equations (6) and (7) show that, if Area 1 is relatively richer in amenities that are mostly appreciated by the skilled,  $Z_1$ , the mid-skill and unskilled wage in Area 1 tend to be relatively higher. This occurs because such specific amenities attract skilled workers to Area 1 and, for this reason, mid-skill and unskilled workers become more productive.

*Relative population sizes across areas.* The equilibrium derived above characterizes the relative prices (rents, wages) across the economy. We now derive the relative equilibrium size of skilled, mid-skill



and unskilled populations in the two areas. Similarly to Roback (1988), the procedure to determine the equilibrium populations builds on the labour market-clearing conditions

$$n_c^s = N_c^s, \quad n_c^m = N_c^m, \quad n_c^u = N_c^u, \quad c = \{1,2\} \quad (8)$$

and market-clearing in the market for land: we leave the details to Appendix A.2.<sup>8</sup> In equilibrium, the ratio of skilled workers between areas is given by:

$$\log\left(\frac{n_1^s}{n_2^s}\right) = \log\left(\frac{\bar{\ell}_1}{\bar{\ell}_2}\right) + \frac{\log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + [1 - \mu(\beta + \gamma)] \cdot \log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right)}{1 - \mu(\alpha + \beta + \gamma)} \quad (9)$$

Expression (9) shows that skilled workers will tend to locate in Area 1 when both types of amenities  $(X_1, Z_1)$  in Area 1 are high relative to Area 2. Thus, local characteristics that enhance welfare are central factors in attracting skilled workers.

Similar calculations show that the ratios of mid-skill and unskilled workers across areas are equal, respectively, to:

$$\log\left(\frac{n_1^m}{n_2^m}\right) = \log\left(\frac{\bar{\ell}_1}{\bar{\ell}_2}\right) + \frac{\log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + \alpha\mu \cdot \log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right)}{1 - \mu(\alpha + \beta + \gamma)} \quad (10)$$

and

$$\log\left(\frac{n_1^u}{n_2^u}\right) = \log\left(\frac{\bar{\ell}_1}{\bar{\ell}_2}\right) + \frac{\log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + \alpha\mu \cdot \log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right)}{1 - \mu(\alpha + \beta + \gamma)} \quad (11)$$

Again, a higher level of general amenities (due to  $X_1$ ) in Area 1 will bias the location of mid-skill and unskilled workers toward that area. Notice that abundance of local amenities that specifically attract *skilled* individuals ( $Z_1$ ) will also tend to increase the location of mid-skill and unskilled workers in Area 1. When more skilled workers locate in Area 1, the local productivity of *mid-skill and unskilled* workers will increase, raising the demand for their labor services. This mechanism generates an interesting implication about the size of aggregate population in the area. The relative abundance of skill-specific amenities ( $Z_1$ ) in Area 1

---

<sup>8</sup> In the Appendix we also express the local population for each type of worker (say,  $n_1^s$ ) as a proportion of its economy-wide aggregate level (i.e.,  $\bar{n}^s$ ).

attracts highly skilled individuals and consequently, due to higher productivity, also mid-skill and low-skill individuals, raising aggregate population in that location. Further, the model has immediate implications about the local skill mix across areas. Consider for example the equilibrium ratio between skilled and mid-skill workers. Since it holds that  $\log\left(\frac{n_1^s}{n_2^s}\right) - \log\left(\frac{n_1^m}{n_2^m}\right) = \log\left(\frac{n_1^s}{n_1^m}\right) - \log\left(\frac{n_2^s}{n_2^m}\right)$ , equations (9) and (10) can be used to characterize the difference in the skill mix across areas, given by the following expression:

$$\log\left(\frac{n_1^s}{n_1^m}\right) - \log\left(\frac{n_2^s}{n_2^m}\right) = \log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right) \quad (12)$$

Expression (12) shows that differences in the local ratio between skilled and mid-skill workers only depend on differences in amenities that are *specific to the tastes of the skilled*, such as those included in vector  $Z_c$ .<sup>9</sup> Identical conclusions are reached when one considers the proportion between skilled and unskilled individuals. This is a relevant implication for the local “college-share”. Indeed, Area 1 will have a higher ratio of skilled vs. mid-skill and unskilled individuals only when endowed, relative to Area 2, with characteristics that are particularly appreciated by the more educated. Consequently, since the local ratio between skilled and mid-skill wages is given by  $\frac{w_c^s}{w_c^m} = \frac{\alpha}{\beta} \cdot \frac{n_c^m}{n_c^s}$ , when Area 1 is relatively richer in skills, i.e.  $\frac{n_1^s}{n_1^m} > \frac{n_2^s}{n_2^m}$ , it will also exhibit a *lower* skill premium relative to Area 2, that is,  $\frac{w_1^s}{w_1^m} < \frac{w_2^s}{w_2^m}$ . Identical considerations hold for the local ratio between skilled and unskilled wages.<sup>10</sup>

The following Result summarizes the implications reached so far:

*Result 1. In areas that are relatively rich in amenities which are particularly attractive to the educated, it holds that:*

- (i) *The aggregate population size is larger,*
- (ii) *The proportion of high-skill individuals is higher and, consequently*
- (iii) *Skilled labour is cheaper.*

---

<sup>9</sup> In fact, in this Cobb-Douglas model, local general amenities ( $X_c$ ) and productivity advantages ( $Q_c$ ) affect skilled, mid-skill and unskilled individuals in the *same* way and, thus, they are unable to affect the local skill mix. See also Glaeser (2008).

<sup>10</sup> It is immediate to show that, if Area 1 is particularly attractive to skilled workers, it holds that  $\frac{n_1^s}{n_1^u} > \frac{n_2^s}{n_2^u}$ ,

implying  $\frac{w_1^s}{w_1^u} < \frac{w_2^s}{w_2^u}$ .

Thus, this framework implies that places that disproportionately attract skilled individuals also exhibit larger populations. As we claim in what follows, the local capacity to attract skilled individuals may be crucial for the implementation of skill-biased technologies. We will show that, when adopted, such new technologies raise the local concentration of human capital even further. Thus, although local amenities remain a precondition, our conclusions are compatible with the observation of Moretti (2010) that, in the US between 1980 and 2000, changes in the geographical location of skilled workers were mostly driven by changes in their relative demand (i.e., technology), rather than supply (i.e., local amenities).

## ***2. Skill-biased share-altering technical change.***

In the last decades, a large body of literature has emphasized the so-called “skilled-biased technical change”<sup>11</sup>, with demand for skilled workers growing faster than the pool of educated individuals. This view has been refined by some contributions following Autor et al. (2003), which have shown that recent technological advances embodied in information and communication technology are particularly fit to execute “routine” tasks that were previously performed by middle-skill workers: see Autor and Acemoglu (2010) for an overview. This has severe consequences for the returns to certain types of skill, which we model as shifts in the factor shares associated with different types of labour. Indeed, some recent literature on economic growth has investigated technological change as directly represented by changes in share parameters. Seater (2005), Peretto and Seater (2008), and Zuleta (2008), observing the historical fall in the share of raw labour in the US, together with the stability of the share going to labour income, have explored some implications of the “share-altering” technical change hypothesis with Cobb-Douglas production functions.<sup>12</sup>

The share-altering representation of technological change is very convenient in our Cobb-Douglas spatial model. Recent technological advances are biased toward skills but, at the same time, they do not substitute for manual low-skill jobs, such as truck driving and personal services: see Autor and Dorn (2011). Thus, we can explore the impact of an increase in skilled workers’ share that is *exactly matched* by a decrease in the mid-skill share. Since one can reasonably assume that the low-skill share remains constant, the *overall* income share of labour remains unchanged. In particular, by referring to the Cobb-Douglas technology (1), we suppose that – initially - a new, share-altering technology becomes available. The new

---

<sup>11</sup> In terms of production theory, technical change is skill-biased if it increases the marginal productivity of skilled workers relative to other factors: see Acemoglu (2002, p.785). Typically, representations of skill-biased technical change rely on CES production functions, as for example in Acemoglu (2002) and Beaudry et al. (2010). For an argument about the (presumed) unsuitability of Cobb-Douglas production functions, see Acemoglu (2003, p.3).

<sup>12</sup> The share-altering hypothesis (not entirely novel, as noticed by Seater 2005), is consistent with several observations about growth history: factor-intensity is determined by factor abundance, the decrease in “raw” labour share while the total labour share does not change over economic development, underdevelopment due to inability to adopt advanced technologies in certain labour markets. Importantly, as shown in Zuleta (2008, p.838), a Cobb-Douglas function with share-altering innovations exhibits an elasticity of substitution greater than one, as commonly assumed in CES representations: see Acemoglu and Autor (2010).

technology<sup>13</sup> is such that the share of skilled labour  $\alpha$  increases by  $\Delta \geq 0$ , while the share of mid-skill labour  $\beta$  is reduced by the same amount, so that the *total* labour share,  $\alpha + \beta + \gamma$ , remains constant. Share-altering technical change is thus associated with the following production function:<sup>14</sup>

$$Y_c = L_c^{1-\alpha-\beta-\gamma} \cdot (N_c^s)^{\alpha+\Delta} \cdot (N_c^m)^{\beta-\Delta} \cdot (N_c^u)^\gamma \quad (13)$$

The following claim, an immediate application of the Envelope Theorem<sup>15</sup>, holds true:

*Lemma 1. Each individual firm will find it profitable to adopt the share-altering technology (13)*

*when it holds that  $\frac{\partial Y_c}{\partial \Delta} = Y_c \cdot \log\left(\frac{N_c^s}{N_c^m}\right) > 0$ . Under local labour market clearing, this inequality is satisfied*

*when Area c is sufficiently rich in skills, that is, when it holds that:*

$$\frac{N_c^s}{N_c^m} = \frac{n_c^s}{n_c^m} > 1. \quad (14)$$

Lemma 1 can also be given a “dual” representation in terms of factor prices. Since it initially holds that  $\frac{w_c^s}{w_c^m} = \frac{\alpha}{\beta} \cdot \frac{n_c^m}{n_c^s}$ , condition (14) implies that each individual firm will find it profitable to adopt the new

share-altering technology (13) when the skill-premium  $\frac{w_c^s}{w_c^m}$  is lower than  $\frac{\alpha}{\beta}$ . This has an immediate

explanation: when the new technology becomes available, an area rich in skills (where the ratio  $\frac{n_c^s}{n_c^m}$  is high)

is characterized by a relatively *low* skill-premium. Thus, the presence of cheap skilled labour in the local labour market may make it convenient to adopt skilled biased technologies, as in Beaudry et al. (2010).

Lemma 1 has another remarkable implication. Recall first that the initial distribution of skills between areas is accounted for by expression (12), which can be re-written as

$n_1^s / n_1^m = [B_U(Z_1) / B_U(Z_2)] \cdot \left[ \frac{\bar{n}^s - n_1^s}{\bar{n}^m - n_1^m} \right]$ : Notice that, for any given level of the skill-mix in Area

1,  $n_1^s / n_1^m$ , the skill-mix in Area 2,  $n_2^s / n_2^m$ , is *lower* when: (a) the ratio  $B_U(Z_1) / B_U(Z_2)$ , generated by the

amenity-gap between areas, is *larger* and, (b) the number of skilled people  $\bar{n}^s$  in the entire economy (given

<sup>13</sup> As in Beaudry et al. (2010) and, in general, in this kind of literature, we abstract from implementation costs of new technologies. Such costs are instead considered in the growth analysis by Seater and Peretto (2008) and Zuleta (2008).

<sup>14</sup> Notice that the production function (13) implies that the ratio between the marginal productivity of skilled labour and the marginal productivity of other factors is increasing in  $\Delta$ . Thus, this form of technical change is consistent with Acemoglu’s (2002) definition of skill-biased technical change.

<sup>15</sup> See Appendix A.3 for details of the proof.

$\bar{n}^m$ ) is *smaller*. Suppose now that  $\{Z_1, Z_2, \bar{n}^s, \bar{n}^m\}$  are such that in Area 1 the ratio between skilled and unskilled individuals is greater than 1 while, in Area 2, is less than 1.<sup>16</sup> Then, firms locating in Area 1 will find it profitable to implement the new technology (13), with  $\Delta > 0$ , while firms locating in Area 2 will stick to the “old” technology, given by (1). Thus, the spatial general equilibrium approach shows that different technologies can coexist across different areas within the *same* economy.

These implications are summarized in the following result:

*Result 2. Suppose that the regional skilled-biased amenity distribution  $\{Z_1, Z_2\}$  and the economy-wide skill-distribution  $\{\bar{n}^s, \bar{n}^m\}$  are such that condition (14) holds in Area 1 but not in Area 2. Then, the price of skills will be sufficiently low to justify adoption of the new technology ( $\Delta > 0$ ) only in Area 1. In Area 2, firms will continue to operate the “old” technology ( $\Delta = 0$ ).*

Next, we give some results which pertain to (i) the size of local skill-premia, and (ii) the skill mix of local populations *across* areas in spatial general equilibrium.

*Implications for local skill-premia, rents and wages.* When condition (14) is respected *only* in Area 1, the local skill-premium must be lower than the one in Area 2. Once adopted, the share-altering technology will have a *direct* positive impact on the local wage-premium: by taking as *given the (initial) local skill-mix*

$\frac{n_1^s}{n_1^m}$ , the ratio  $\frac{w_1^s}{w_1^m} = \frac{\alpha + \Delta}{\beta - \Delta} \cdot \frac{n_1^m}{n_1^s}$  is increasing in  $\Delta$ . Thus, areas where the skilled-biased technology is

adopted exhibit – at least, initially - an increase in the local skill-premium on the upper tail of the wage distribution. At the same time, the wage ratio between mid-skill and low-skill individuals,

$\frac{w_1^m}{w_1^u} = \frac{\beta - \Delta}{\gamma} \cdot \frac{n_1^u}{n_1^m}$  is decreasing in  $\Delta$ . Consistently with observation in Eeckhout et al. (2011) and Autor

and Dorn (2011), the variance of the wage-distribution tends to increase in places that adopt technologies unfavourable to mid-skills. However, the evidence reported in Beaudry et al. (2010) suggests that local changes in technology tend to have mostly a transitory effect on local skill premia since the workforce is mobile. They observe that during an adjustment to a new technological paradigm, the returns to skill increase most where skill is more abundant but, over the long run, the supply of skill will exert downward pressure on the skill premium. Indeed, in our model, the direct impact on the skill-premium caused by  $\Delta > 0$  tends to be entirely compensated by re-adjustments in the local skill-mix caused by migrations. The proof of this claim goes as follows (see also Glaeser 2008). Recall that the indirect utility of a skilled worker is equal to

---

<sup>16</sup> This case occurs, for example, when  $B_U(Z_1)/B_U(Z_2)=9/4$  and  $\{\bar{n}^s, \bar{n}^m\} = \{5,5\}$ , so that  $n_1^s/n_1^m=3/2$  and  $n_2^s/n_2^m=2/3$ .

$\bar{v}^s = \eta \cdot A_U(X_c) \cdot B_U(Z_c) \cdot (w_c^s / r_c^{1-\mu})$ , where  $\bar{v}^s$  denotes the “reservation utility” of skilled individuals across the economy (with free-mobility). Similarly, the indirect utility of an mid-skilled worker is given by  $\bar{v}^m = \eta \cdot A_U(X_c) \cdot (w_c^m / r_c^{1-\mu})$ , where  $\bar{v}^m$  denotes the “reservation utility” of the mid-skill individuals in the economy. Thus, in equilibrium, the wage-premium in area  $c$  is given by:

$$\frac{w_c^s}{w_c^m} = \left( \frac{1}{B_U(Z_c)} \right) \cdot \frac{\bar{v}^s}{\bar{v}^m} \quad (15)$$

Expression (15) shows that the wage-gap between high-skilled and mid-skilled workers only depends on amenities that affect skilled utility, and it does *not* depend on technological factors such as  $\Delta$ . Similar considerations hold for the wage-ratio of mid-skill and unskilled workers. Thus, as will be confirmed in what follows, there must occur re-adjustments in the skill-mix across areas which exactly compensate for the direct effect generated by the adoption of the new technology. In other words, consistently with the evidence reported in Beaudry et al. (2010), local skill premia tend to revert to their pre-implementation level, but technological change has *permanent* effects on the distribution of skills within each area. In order to show this, we first derive some spatial general equilibrium implications of share-altering technical change on relative prices across areas. As shown in Appendix A.4, the following result holds:

*Result 3 (Relative rents and wages). Share-altering technical change localized in Area 1 will raise rents and wage levels relative to Area 2.*

We can now analyse what happens to equilibrium skill distributions in the two areas when share-altering technological change occurs. All derivations are reported in Appendix A.5.

*Implications for local skill distributions.* The impact on the ratio between high-skill populations in Area 1 and 2 is given by:

$$\left. \frac{d \log(n_1^s / n_2^s)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \frac{1}{\alpha} + \frac{\mu \cdot \log \Sigma_0}{1 - \mu(\alpha + \beta + \gamma)} > 0 \quad (16)$$

where  $\Sigma_0 \equiv \frac{n_1^s}{n_1^m} > 1$  is the pre-implementation skill-ratio between high and mid-skill individuals in Area 1.

Thus, a localised skill-biased technological change will generate a relative increase in the skilled population of that area. The opposite result generally holds for the mid-skilled, which we mainly identify with high-

school educated individuals. Share-altering technical change has the following effect on the territorial distribution of such local populations:

$$\left. \frac{d \log(n_1^m / n_2^m)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \frac{-1}{\beta} + \frac{\mu \cdot \log \Sigma_0}{1 - \mu(\alpha + \beta + \gamma)} \quad (17)$$

The sign of expression (17) is ambiguous in principle. However, plausible values of the parameters imply that technological change will generally *reduce* the relative size of the mid-skilled population in Area 1.<sup>17</sup> Low-skilled population will instead *increase* in the area that undergoes technical change:

$$\left. \frac{d \log(n_1^l / n_2^l)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \frac{\mu \cdot \log \Sigma_0}{1 - \mu(\alpha + \beta + \gamma)} > 0 \quad (18)$$

By inspection of (16), (17), and (18), thus share-altering technical change localized in Area 1 implies that the inequality  $\left. \frac{d \log(n_1^m / n_2^m)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} < \left. \frac{d \log(n_1^l / n_2^l)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} < \left. \frac{d \log(n_1^s / n_2^s)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1}$  holds, generating thicker tails in the distribution of skills in Area 1, relative to Area 2. This conclusion is summarized in the following:

*Result 4 (Local skills distributions). If the skill-biased share-altering technology is adopted only in Area 1, the skill distribution in that area will exhibit “fatter tails”: The local population of the low-skilled and –to a greater extent– the population of the high-skilled increase, relative to the population endowed with mid-skills.*

Thus, our simple model has implications consistent with the finding in Eeckhout et al (2011) that large cities have fatter tails in skill-distribution. Similar conclusions are reached by Lin (2010, p.565). Also, Autor and Dorn (2011) show that polarization in local skill distribution emerges in areas characterized both by past specialization in routine activities *and greater relative supply of college graduates*.

In our model, the mechanism is the following. Places that are particularly attractive to high-skilled individuals are larger and richer in human capital (as shown by Result 1). As a consequence, such locations will be more prone to endogenous adoption of skill-biased technologies that, as documented by Autor et al. (2003) and Goos and Manning (2007), come at a disadvantage for jobs typically occupied by mid-skill

---

<sup>17</sup> With  $\mu=2/3$  and  $\alpha+\beta+\gamma=2/3$ , expression (17) is negative when  $\log \Sigma_0 < \frac{5}{6} \frac{1}{\beta}$ ; for example, when  $\beta=2/9$ , (17) is negative when  $\log \Sigma_0 < \frac{15}{4}$ . Thus, if  $(\log \Sigma_0) > 0$  is not implausibly large, this derivative will have a negative sign. The parameterization of factor-shares is consistent with Mankiw et al. (1992).

individuals. Productivity of both high-skill and low-skill individuals rises relative to mid-skill workers, generating hollowing-out in the local skill distribution.

Result 4 can be restated in terms of an improvement of the *skill mix* in Area 1. Since it holds<sup>18</sup> that

$$\left. \frac{d[\log(n_1^s / n_1^m) - \log(n_2^s / n_2^m)]}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \frac{1}{\alpha} + \frac{1}{\beta} > 0, \text{ skilled-biased share-altering technical change localized}$$

in Area 1 will lead to a *further* concentration of high-skills in that region. Hence, when Area 1 is fit to adopt skilled-biased share-altering technologies (i.e., when condition (14) is satisfied), later on it will be ready to implement *additional technological advances of the same kind*. By contrast, if the innovation could not be profitably adopted in Area 2 (and the economy-wide aggregate supply of educated individuals,  $\bar{n}^s$ , does not adequately rise), this region is going to remain stuck with the old technology also in the future. This implies that, within the same country, output will grow in Area 1, while Area 2 stagnates (see also Seater 2005).

---

<sup>18</sup> Combining (16) and (17) gives  $d[\log(n_1^s / n_2^s) - \log(n_1^m / n_2^m)] / d\Delta = d[\log(n_1^s / n_1^m) - \log(n_2^s / n_2^m)] / d\Delta$ .



### III. CONCLUSIONS

Under the hypothesis of skill-biased share-altering technical change, places that are particularly attractive to the skilled may permit the adoption of technologies which, by reducing the relative productivity of mid-skill workers, generate fat tails in the local skill distribution. From a technical viewpoint, we can represent skill-bias and reduction in demand for mid-skill individuals even with the simplest Cobb-Douglas production function. Thus, in order to model some main stylized facts that have been occurring in local labor markets during the last three decades, there is no need to use more cumbersome CES technologies, as in the spatial analysis by Glaeser (2008) or in the labor-market analysis by Beaudry et al. (2010), or even CES with varying degrees of complementarity as in Eeckhout et al. (2011), or CES with specific consumer preferences as in Autor and Dorn (2011). Further, while Beaudry et al. (2010) only consider two types of labour while explicitly analyzing the conditions for implementation of new technologies, Eeckhout et al. (2011) consider three types of labour but do not say under which conditions their “extreme-skill complementarity” technology is going to be implemented. Both instances, local implementability and skill-distribution, are jointly considered here.

Our framework has also relevant spatial general equilibrium implications. In particular, the model gives some specific suggestions about the possibility of path-dependency in regional development. Only areas that are sufficiently rich in human capital will be ready to adopt skilled-biased share-altering technological advancements. This is consistent with the “comparative advantage” story of Beaudry et al. (2010), who find that skill abundance is associated with more intense skilled-biased technology adoption. Moreover, our model suggests that, after the implementation of new technologies, human capital tends to move away from areas that were exhibiting a relatively poor skill mix, to areas that were *already* relatively rich in educated workers. As a result, there will be further polarization in the composition of the local labour force across the economy. This prediction is consistent with the empirical results reported in Berry and Glaeser (2005) and Glaeser and Gottlieb (2008): areas with high levels of human capital (a pre-condition for the adoption of skilled-biased technologies, here) will attract a disproportionate number of skilled workers. At the same time, the migratory movements induced by the adoption of the new technology will generate a “fat-tail” also in the *lower* end of the skill distribution, consistently with evidence in Eeckhout et al. (2011) and Autor and Dorn (2011).

Finally, the model raises some policy questions. The existence of relevant non-linear effects of local skills, documented in Glaeser and Gottlieb (2008), may suggest that there can be returns from pushing skilled workers into areas that are *already* relatively skilled. However, this would mean to subsidize places that are rich in human capital. Although such a policy seems inequitable and improper, our approach may suggest that in an economy characterized by a low average level of education, it may be desirable to concentrate human capital in few specific places<sup>19</sup>, so to allow for the adoption of techniques that would otherwise be unprofitable. In our perspective, this kind of intervention can be implemented by subsidizing local amenities that prove to be particularly attractive to educated individuals.

---

<sup>19</sup> Examples of such policies in the US are given by Glaeser and Gottlieb (2008, p.224-25).

## APPENDIX

### *A.1 Derivation of relative local rents and wages in equilibrium.*

Equalization of marginal cost to one in both areas implies that the following condition holds:

$$\left(\frac{r_1}{r_2}\right) = \left(\frac{w_2^s}{w_1^s}\right)^{\frac{\alpha}{1-\alpha-\beta-\gamma}} \cdot \left(\frac{w_2^m}{w_1^m}\right)^{\frac{\beta}{1-\alpha-\beta-\gamma}} \cdot \left(\frac{w_2^u}{w_1^u}\right)^{\frac{\gamma}{1-\alpha-\beta-\gamma}} \quad (\text{A1.1})$$

Equalization of skilled utility across areas implies that:

$$\left(\frac{w_1^s}{w_2^s}\right) = \frac{A_U(X_2)}{A_U(X_1)} \cdot \frac{B_U(Z_2)}{B_U(Z_1)} \cdot \left(\frac{r_1}{r_2}\right)^{1-\mu} \quad (\text{A1.2})$$

Equalization of mid-skill utility and unskilled utility across areas delivers, respectively, the following two expressions:

$$\left(\frac{w_1^m}{w_2^m}\right) = \frac{A_U(X_2)}{A_U(X_1)} \cdot \left(\frac{r_1}{r_2}\right)^{1-\mu} \quad (\text{A1.3})$$

and

$$\left(\frac{w_1^u}{w_2^u}\right) = \frac{A_U(X_2)}{A_U(X_1)} \cdot \left(\frac{r_1}{r_2}\right)^{1-\mu} \quad (\text{A1.4})$$

By substituting (A1.2), (A1.3) and (A1.4) into (A1.1), one obtains expression (4) in the text. Then, exploiting (4) to substitute into (A1.2), (A1.3) and (A1.4) one obtains, respectively, (5), (6) and (7) in the text.

### *A.2. Derivation of relative population sizes in equilibrium.*

Profit maximization for firms located in area  $c$  implies that the demands for highly skilled labour  $N_c^s$ , mid-skilled labour  $N_c^m$ , unskilled labour  $N_c^u$ , and land  $L_c$  are given, respectively, by:

$$N_c^s = \frac{\alpha \cdot Y_c}{w_c^s}, \quad N_c^m = \frac{\beta \cdot Y_c}{w_c^m}, \quad N_c^u = \frac{\gamma \cdot Y_c}{w_c^u}, \quad L_c = \frac{(1-\alpha-\beta-\gamma) \cdot Y_c}{r_c} \quad (\text{A2.1})$$

In equilibrium, skilled labour demand  $N_c^s$  must be equal to its local supply  $n_c^s$ . Also, mid-skill labor demand  $N_c^m$  and unskilled labour demand  $N_c^u$  must be equal, respectively, to local mid-skill labor supply

$n_c^m$  and unskilled supply,  $n_c^u$ . Finally, the local supply of land,  $\bar{\ell}_c$ , must be equal to the total demand for land, which is given by the sum of land demanded by firms (as from A2.1), plus the land demanded by the skilled workers, equal to  $n_c^s \cdot (1 - \mu) \cdot \frac{w_c^s}{r_c}$ , plus the land demanded by the mid-skill and unskilled workers, respectively,  $n_c^m \cdot (1 - \mu) \cdot \frac{w_c^m}{r_c}$  and  $n_c^u \cdot (1 - \mu) \cdot \frac{w_c^u}{r_c}$ . Thus, the following four equations constitute a system in  $\{Y_c, n_c^s, n_c^m, n_c^u\}$ , given the price vector  $\{r_c, w_c^s, w_c^m, w_c^u\}$ :

$$n_c^s = \frac{\alpha \cdot Y_c}{w_c^s} \quad (\text{A2.2})$$

$$n_c^m = \frac{\beta \cdot Y_c}{w_c^m} \quad (\text{A2.3})$$

$$n_c^u = \frac{\gamma \cdot Y_c}{w_c^u} \quad (\text{A2.4})$$

$$\bar{\ell}_c = \frac{1}{r_c} \left\{ (1 - \alpha - \beta - \gamma) \cdot Y_c + (1 - \mu) \cdot n_c^s \cdot w_c^s + (1 - \mu) \cdot n_c^m \cdot w_c^m + (1 - \mu) \cdot n_c^u \cdot w_c^u \right\} \quad (\text{A2.5})$$

By using (A2.2), (A2.3) and (A2.4) to substitute  $\{n_c^s \cdot w_c^s, n_c^m \cdot w_c^m, n_c^u \cdot w_c^u\}$  away in (A2.5), one obtains:

$$Y_c = \frac{\bar{\ell}_c \cdot r_c}{1 - \mu(\alpha + \beta + \gamma)} \quad (\text{A2.6})$$

which can be substituted back into (A2.2), (A2.3) and (A2.4) to obtain:

$$n_c^s = \frac{\alpha}{w_c^s} \left[ \frac{\bar{\ell}_c \cdot r_c}{1 - \mu(\alpha + \beta + \gamma)} \right], \quad c = 1, 2 \quad (\text{A2.7})$$

$$n_c^m = \frac{\beta}{w_c^m} \left[ \frac{\bar{\ell}_c \cdot r_c}{1 - \mu(\alpha + \beta + \gamma)} \right], \quad c = 1, 2 \quad (\text{A2.8})$$

$$n_c^u = \frac{\gamma}{w_c^u} \left[ \frac{\bar{\ell}_c \cdot r_c}{1 - \mu(\alpha + \beta + \gamma)} \right], \quad c = 1, 2 \quad (\text{A2.9})$$

Thus, the relative population sizes of skilled individuals across areas will be given by:

$$\frac{n_1^s}{n_2^s} = \frac{\bar{\ell}_1}{\bar{\ell}_2} \cdot \frac{r_1}{r_2} \cdot \frac{w_2^s}{w_1^s} \quad (\text{A2.10})$$

Taking logs of (A2.10) and using (4) and (5), one obtains equation (9) in the text. Using a similar procedure, one can exploit (A2.8) and (A2.9) to obtain, respectively, (10) and (11) in the text.

The skilled population level in Area 1,  $n_1^s$ , can also be expressed as a fraction of the economy-wide skill supply,  $\bar{n}^s$ . Since it holds that  $n_2^s = \bar{n}^s - n_1^s$ , then  $\frac{n_1^s}{n_2^s} = \frac{n_1^s}{\bar{n}^s - n_1^s} \equiv K$ , where  $K$  is equal to the antilog of the right-hand side of (9) in the text. Thus, it immediately follows that  $n_1^s = \left( \frac{K}{1+K} \right) \cdot \bar{n}^s$ . A similar argument can be used to characterize the local levels of the mid-skill and low-skill populations in each area.

### A.3. Proof of Lemma 1.

Share-altering technical change, summarized by  $\Delta > 0$ , will be adopted by local firms when it has a positive impact on profit, given by  $\pi_c = Y_c - r_c \cdot L_c - w_c^s \cdot N_c^s - w_c^m \cdot N_c^m - w_c^u \cdot N_c^u$ . By Envelope Theorem, it holds that:

$$\frac{d\pi_c}{d\Delta} = \frac{\partial Y_c}{\partial \Delta} + \left[ \frac{\partial Y_c}{\partial L_c} - r_c \right] \frac{dL_c}{d\Delta} + \left[ \frac{\partial Y_c}{\partial N_c^s} - w_c^s \right] \frac{dN_c^s}{d\Delta} + \left[ \frac{\partial Y_c}{\partial N_c^m} - w_c^m \right] \frac{dN_c^m}{d\Delta} + \left[ \frac{\partial Y_c}{\partial N_c^u} - w_c^u \right] \frac{dN_c^u}{d\Delta} = \frac{\partial Y_c}{\partial \Delta};$$

Thus, if condition (14) holds true, profit is increasing in  $\Delta$ , making the share-altering technology convenient to adopt.

### A.4. Derivation of relative rents and wages under technical change.

We now explore the effects of share-altering technological change localized *only* in Area 1 on relative local prices across the two regions. To this purpose, we will evaluate the results for an *initially* given skill-ratio,  $N_1^s / N_1^m$ , set equal to the constant  $\Sigma_0 > 1$ . Moreover, derivatives in comparative statics results

will be calculated by setting  $\Delta \approx 0$ , that is, evaluating the impact of change by starting with the same production function in both areas.

Since condition (14) is satisfied in Area 1, but not in Area 2, firms in the former region (costlessly) adopt the share-altering innovation, while firms in the latter one continue to use the old technology. Under perfect competition in tradable good production, price (the *numeraire*) equals marginal cost, implying that firms locating in Area 1 will respect the following condition:

$$\left[ \frac{(1 - \alpha - \beta - \gamma)^{1 - \alpha - \beta - \gamma} (\alpha + \Delta)^{\alpha + \Delta} (\beta - \Delta)^{\beta - \Delta} \gamma^\gamma}{r_1^{1 - \alpha - \beta - \gamma} \cdot (w_1^s)^{\alpha + \Delta} \cdot (w_1^m)^{\beta - \Delta} \cdot (w_1^u)^\gamma} \right] = 1, \quad (\text{A4.1})$$

For competitive firms locating in Area 2, the condition:

$$\left[ \frac{(1 - \alpha - \beta - \gamma)^{1 - \alpha - \beta - \gamma} \alpha^\alpha \beta^\beta \gamma^\gamma}{r_2^{1 - \alpha - \beta - \gamma} \cdot (w_2^s)^\alpha \cdot (w_2^m)^\beta \cdot (w_2^u)^\gamma} \right] = 1 \quad (\text{A4.2})$$

will continue to hold. Free mobility implies that, in equilibrium, firms must make zero profit no matter where they choose to locate. Thus, by combining (A4.1) and (A4.2), one obtains:

$$\frac{r_1}{r_2} = \left[ \left( \frac{(\alpha + \Delta)^{\alpha + \Delta} (\beta - \Delta)^{\beta - \Delta}}{\alpha^\alpha \beta^\beta} \right) \cdot \left( \frac{w_2^s}{w_1^s} \right)^\alpha \cdot \left( \frac{w_2^m}{w_1^m} \right)^\beta \cdot \left( \frac{w_2^u}{w_1^u} \right)^\gamma \cdot \left( \frac{w_1^m}{w_1^s} \right)^\Delta \right]^{\frac{1}{1 - \alpha - \beta - \gamma}} \quad (\text{A4.3})$$

By substituting equations (A1.2), (A1.3) and (A1.4) into (A4.3), one obtains (the log of) the equilibrium rent-ratio between Area 1 and Area 2:

$$\log\left(\frac{r_1}{r_2}\right) = \frac{1}{1 - \mu(\alpha + \beta + \gamma)} \left\{ \begin{aligned} & \left[ (\alpha + \Delta) \cdot \log(\alpha + \Delta) + (\beta - \Delta) \cdot \log(\beta - \Delta) - \log(\alpha^\alpha \beta^\beta) \right] + \Delta \cdot \log\left(\frac{w_1^m}{w_1^s}\right) + \\ & + (\alpha + \beta + \gamma) \cdot \log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + \alpha \cdot \log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right) \end{aligned} \right\} \quad (\text{A4.4})$$

where profit-maximization implies that  $\left(\frac{w_1^m}{w_1^s}\right) = \frac{\beta - \Delta}{\alpha + \Delta} \left(\frac{N_1^s}{N_1^m}\right)$ . Differentiating (A4.4) with respect to  $\Delta$ ,

and evaluating the result for  $\Delta \approx 0$  and  $\frac{N_1^s}{N_1^m} = \Sigma_0 > 1$ , one obtains:

$$\left. \frac{d \log(r_1 / r_2)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \left( \frac{1}{1 - \mu(\alpha + \beta + \gamma)} \right) \cdot \log \Sigma_0 \quad (\text{A4.5})$$

Since  $\Sigma_0 > 1$ , the sign of expression (A4.5) is positive. Thus, localized skill-biased technical change will increase rents in Area 1 relative to Area 2.

Consider now the impact of the change in the skilled share on the skilled wage-ratio across areas. By exploiting (A1.2), differentiating with respect to  $\Delta$ , and evaluating the result for  $\Delta \approx 0$  and  $\frac{N_1^s}{N_1^u} = \Sigma_0 > 1$ , one obtains that:

$$\left. \frac{d \log(w_1^s / w_2^s)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = (1 - \mu) \cdot \left. \frac{d \log(r_1 / r_2)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \left( \frac{1 - \mu}{1 - \mu(\alpha + \beta + \gamma)} \right) \cdot \log \Sigma_0 > 0 \quad (\text{A4.6})$$

Similarly, one can use (A1.3) and (A1.4) to assess the impact of share-altering change on relative mid-skill and unskilled wages. It turns out that the effect is the same as for skilled wages:

$$\left. \frac{d \log(w_1^m / w_2^m)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \left. \frac{d \log(w_1^u / w_2^u)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \left( \frac{1 - \mu}{1 - \mu(\alpha + \beta + \gamma)} \right) \cdot \log \Sigma_0 > 0 \quad (\text{A4.7})$$

Expressions (A4.6) and (A4.7) confirm that, with labor mobility, local skill premia are unaffected by technical change, as emphasized by (15).

### ***A.3. Populations and share-altering technical change.***

We first analyze the impact of skill-biased share-altering technical change on the relative size of the skilled population. It is immediate to show that, in Area 1, skilled population is now given by:

$$n_1^s = \frac{\alpha + \Delta}{w_1^s} \left[ \frac{\bar{\ell}_1 \cdot r_1}{1 - \mu(\alpha + \beta + \gamma)} \right], \quad (\text{A5.1})$$

while for Area 2 equation (A2.7) still holds. Hence, with share-altering technical change, the skilled-population ratio is given by:

$$\frac{n_1^s}{n_2^s} = \frac{\alpha + \Delta}{\alpha} \cdot \frac{\bar{\ell}_1}{\ell_2} \cdot \frac{r_1}{r_2} \cdot \frac{w_2^s}{w_1^s}. \quad (\text{A5.2})$$

Taking the logs of (A5.2), differentiating with respect to  $\Delta$ , and calculating the resulting expression for  $\Delta \approx 0$ , one obtains:

$$\left. \frac{d \log(n_1^s / n_2^s)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \left( \frac{1}{\alpha} \right) + \left. \frac{d \log(r_1 / r_2)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} - \left. \frac{d \log(w_1^s / w_2^s)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \frac{1}{\alpha} + \frac{\mu \cdot \log \Sigma_0}{1 - \mu(\alpha + \beta + \gamma)} > 0 \quad (\text{A5.3})$$

which gives expression (16) in the text.

Mid-skilled population in Area 1 is given by

$$n_1^m = \frac{\beta - \Delta}{w_1^m} \left[ \frac{\bar{\ell}_1 \cdot r_1}{1 - \mu(\alpha + \beta + \gamma)} \right], \quad (\text{A5.4})$$

while for Area 2 equation (A2.8) still holds. Hence, the mid-skill population ratio is equal to:

$$\frac{n_1^m}{n_2^m} = \frac{\beta - \Delta}{\beta} \cdot \frac{\bar{\ell}_1}{\ell_2} \cdot \frac{r_1}{r_2} \cdot \frac{w_2^m}{w_1^m}. \quad (\text{A5.5})$$

Differentiating the log of (A5.5) with respect to  $\Delta$  and calculating the result for  $\Delta \approx 0$ , one obtains:

$$\left. \frac{d \log(n_1^m / n_2^m)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \left( \frac{-1}{\beta} \right) + \left. \frac{d \log(r_1 / r_2)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} - \left. \frac{d \log(w_1^m / w_2^m)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \frac{-1}{\beta} + \frac{\mu \cdot \log \Sigma_0}{1 - \beta - \alpha\mu} \quad (\text{A5.6})$$

which gives expression (17) in the text.

Since low-skilled workers' share is unaffected we can use (A2.4) for both areas, so to obtain:

$$\frac{n_1^u}{n_2^u} = \frac{\bar{\ell}_1}{\ell_2} \cdot \frac{r_1}{r_2} \cdot \frac{w_2^u}{w_1^u}. \quad (\text{A5.7})$$

Differentiating the log of (A5.7) with respect to  $\Delta$  and calculating the result for  $\Delta \approx 0$ , one obtains:

$$\left. \frac{d \log(n_1^u / n_2^u)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \left. \frac{d \log(r_1 / r_2)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} - \left. \frac{d \log(w_1^u / w_2^u)}{d\Delta} \right|_{\Delta \approx 0, \Sigma_0 > 1} = \frac{\mu \cdot \log \Sigma_0}{1 - \beta - \alpha\mu} > 0 \quad (\text{A5.6})$$

which gives expression (18).

## REFERENCES

- Acemoglu D. (1998), "Why do new technologies complement skills? Directed technical change and wage inequality", *Quarterly Journal of Economics*, 113(4), 1055-1089.
- Acemoglu D. (2002), "Directed technical change", *Review of Economic Studies*, 69, 781-809.
- Acemoglu D. (2003), "Labour- and capital-augmenting technical change", *Journal of the European Economic Association*, 1(1), 1-37.
- Acemoglu D. and D. Autor (2010), "Skills, tasks and technologies: Implications for employment and earnings", forthcoming in *Handbook of Labor Economics (Vol. 4)*, D. Ashenfelter and D. Card (eds.), Amsterdam: Elsevier.
- Autor D. , F. Levy, R. Murnane (2003), "The skill content of recent technological change: An empirical exploration", *Quarterly Journal of Economics*, 118, 1279-1333.
- Autor D., L. Katz, M. Kearney (2008), "Trends in U.S. wage inequality: Revising the revisionists", *Review of Economics and Statistics*, 90(2), 300-323.
- Autor D. and D. Dorn (2009), "This job is 'getting old': Measuring changes in job opportunities using occupational age structure", *American Economic Review, Papers and Proceedings*, 99(2), 45-51.
- Autor D. and D. Dorn (2011), "The growth of low skill service jobs and the polarization of the U.S. labor market", mimeo, MIT (revised version NBER WP 15150).
- Beaudry P., M. Doms, E. Lewis (2010), "Should the PC be considered a technological revolution? Evidence from US Metropolitan Areas", *Journal of Political Economy*, 118(5), 988-1036.
- Berry C. and E. Glaeser (2005), "The divergence of human-capital levels across cities", *Papers in Regional Science*, 84, 407-444.
- Carlino G. and A. Saiz (2008), "Beautiful City", IZA Discussion Paper 3778, Bonn.
- Dalmazzo A. and G. de Blasio (2011), "Skill-biased agglomeration effects and amenities: Theory with an application to Italian cities", *Papers in Regional Science*, 90(3), 503-527.
- Eeckhout J., R. Pinheiro, K. Schmeideiny (2011), "Spatial sorting: Why New York, Los Angeles and Detroit attract the greatest minds as well as the unskilled?", mimeo, UPF and UPenn.
- Glaeser E. (2008), *Cities, agglomeration and spatial equilibrium*, Oxford University Press, Oxford.
- Glaeser E. and J. Gottlieb (2008), "The economics of place-making policies", *Brooking Papers in Economic Activity*, Spring, 155-239.
- Goos M. and A. Manning (2007), "Lousy and lovely jobs: The rising polarization of work in Britain", *Review of Economics and Statistics*, 89(1), 118-133.
- Goos M., A. Manning, A. Salomons (2009), "Job polarization in Europe", *American Economic Review, Papers and Proceedings*, 99(2), 58-63.
- Lin J. (2011) "Technological adaptation, cities and new work", *Review of Economics and Statistics*, 93(2), 554-574.
- Mankiw G., D. Romer and D. Weil (1992), "A contribution to the empirics of economic growth", *Quarterly Journal of Economics*, 107, 407-437.



- Michaels G., A. Natraj and J. Van Reenen (2010), “Has ICT polarized skill demand? Evidence from eleven countries over 25 years”, CEP Discussion Paper No.987, London School of Economics.
- Moretti E. (2010), “Local labor markets”, forthcoming in *Handbook of Labor Economics*. <http://emlab.berkeley.edu/~moretti/handbook.pdf>
- Moretti E. (2011), “Real wage inequality”, mimeo, <http://emlab.berkeley.edu/~moretti/inequality.pdf>.
- Roback J. (1982), “Wages, rents and the quality of life”, *Journal of Political Economy*, 90(6), 1257-78.
- Roback J. (1988), “Wages, rents, and amenities: Differences among workers and regions”, *Economic Inquiry*, 26(1), 23-41.
- Seater J. (2005), “Share-altering technical progress”, in *Economic Growth and Productivity*, L.A. Finley (editor), Nova Science Publisher, NY.
- Seater J. and P. Peretto (2008), “Factor-eliminating technical change”, mimeo, Duke University, NC.
- Van Reenen J. (2011), “Wage inequality, technology and trade: 21<sup>st</sup> century evidence”, *Labour Economics*, 18, 730-41.
- Zuleta H. (2008), “Factor saving innovations and factor income shares”, *Review of Economic Dynamics*, 11(4), 836-51.