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# Regional Competition On Tax Administration

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# REGIONAL COMPETITION ON TAX ADMINISTRATION

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## ABSTRACT

The tax auditing parameters have scarcely been analyzed by the literature as relevant policy-making instruments; however the enforcement strategies are crucial elements of the tax burden. In this paper we show that in a federal framework the tax auditing policies could represent additional tools on which regional institutions can interact between them. We investigate the presence of this interaction by means of a spatial econometric approach. We employ a *time-space recursive model* that accounts for sluggish adjustment in the auditing policies obtaining results congruent with standard theory and corroborating the presence of horizontal competition between regions on tax auditing policies. Moreover we find that once regional governments have legal power, the opaque competition on enforcement policies disappears and supposedly it switches to a more transparent competition on statutory tax parameters.

Keywords: tax administration and auditing, fiscal competition, fiscal federalism

JEL Classification: H71, H77, H83

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## 1. Introduction

Enforcement strategies are crucial elements of the tax management process since they contribute to determine the level and distribution of effective tax rates (e.g. Johns, Slemrod, 2010; Traxler, 2011) and so the total amount of tax revenues collected. Furthermore, enforcement strategies are of particular interest in federal countries, because the auditing policies can represent a second instrument in the hand of sub-central authorities (Besfamille *et al.*, 2012) – together with the setting of statutory tax rates – on which they can interact among them. Nevertheless, the possibility of tax enforcement interdependencies has received limited attention in the literature (notable exceptions are Janeba, Peters, 1999; Cremer, Gahvari, 2000 and, Stöwhase, Traxler, 2005). In particular to the best of our knowledge, there are no empirical studies that investigate the presence of these interactions. This could be due to a lack of data on the auditing policies and/or to the difficulty to find an adequate measure to represent the level of “tax enforcement”. We aim at filling this gap in the literature by analyzing the presence of horizontal tax interdependencies among sub-central administrations in a federal context. In Spain, regional governments, the so-called “Comunidades Autonomas” (CA), have had power to administer several wealth taxes since the mid-eighties, although without legal power to modify the rule; but later on, (1997 and 2002 reforms) they obtained the legislative power to modify significant tax parameters<sup>1</sup>. In particular, we concentrate on the Inheritance and Gift Tax (IGT), the most relevant decentralized tax on wealth, which has recently become a topical issue both in Spain and in other countries<sup>2</sup>. There is evidence that the decentralization of the IGT in federal countries can induce a race to the bottom in statutory tax parameters (see e.g. Bird, 1991, Conway, Rork, 2004; Brulhart, Parchet, 2011)<sup>3</sup>. The source of this process is the mobility of tax bases<sup>4</sup>. A similar effect is documented for the Spanish case (see Durán-Cabré, Esteller-Moré 2010; López Casasnovas, Durán-Sindreu, 2008), which has provoked an academic as well as a more general debate. The Spanish press headlines on these issues are emblematic: “Cheaper gifts and inheritances”; “The regional tax competition”; “The fiscal war among regions

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<sup>1</sup> More precisely, after the 1997 reform, CAs could change the tax rate schedule keeping it close to the national ones. After the 2002 reform, CAs have a complete legislative control on the rates of the taxes ceded to them by the central government.

<sup>2</sup> Taxing wealth and wealth transfers is generally unpopular. These issues are object of debate in several OECD countries including United States and Canada. In Europe the UK case is illustrative: the IGT is popularly ostracized because it raises relatively little revenue but it is characterized by a too high flat rate (40%) and it raises issues about double taxation as well as about the absence of effects on wealth distribution (Boadway *et al.*, 2010).

<sup>3</sup> Recently there are signs of interests on these themes also from the European Commission and even if they rise from a different point of view – cross-border discrimination and double taxation – this confirms that the inheritance tax issues are becoming one of growing concern to European citizens (European Commission, 2011).

<sup>4</sup> In a decentralized framework, when the principle of residence is applied, an individual finds it profitable to move his fiscal residence to the region with the lowest IGT rate, in order to reduce the bequest tax burden.

threatens the IGT”); “Regional taxation and voting with feet”<sup>5</sup>. These articles corroborate the presence of a mobility-based competition on the regional IGT statutory tax parameters. Similarly, we hypothesize that the same type of competition among regions was occurring even before the decentralization of the legal power, in the form of opaque competition on tax enforcement since it is the effective tax rate that should condition mobility.

The objective of our paper is then to determine the form and degree of interaction among decentralized administrations when setting their policies. With this aim we develop a horizontal competition theoretical model with two tax instruments, i.e., the tax rate and the auditing rate, and we empirically test its findings. The results of the theoretical framework are in line with the literature on tax rates competition: the mobility threat tames the revenue maximizing jurisdictions that compete in a race to the bottom on both tax instruments in order to not lose the tax bases. We derive the slope of the administration’s reaction function obtaining a positive sign. We proceed testing this result through a spatial econometric approach. We estimate a *time-space recursive model* to account for possible sluggish adjustments in the auditing policies setting (see Anselin *et al.*, 2008). Our results corroborate the presence of horizontal interdependencies between the different regions and generates credible results that are coherent with the tax competition model. Moreover we obtain an additional result: after the decentralization of the legislative power on statutory tax parameters at a regional level we observe a disappearance of the competition on enforcement policies. It seems that a substitution of instruments occurs with positive implications at a normative level: somewhat paradoxically in this perspective the tax decentralization process is welcome since an opaque source of tax competition is substituted by a transparent one.

The rest of the paper is organized in this way. In the next section, we provide a summary of the relevant literature, then the theoretical framework is developed and the empirical analysis performed. Finally, we conclude.

## **2. Relevant Literature**

Our work is related to the vast literature on the taxation policy interactions among governments and in particular to the research line that deals with the horizontal tax competition (see Brennan and Buchanan, 1980; Zodrow, Mieszkosky, 1986 and Wilson, 1986). This approach analyzes a decentralized framework in which local governments compete in a race to the bottom when fixing the tax rates in order to gain or not to lose the tax bases. The mobility or simply the threat

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<sup>5</sup> The quoted articles are ABC (2008); El Periodico (2007); El País (2007) and Expansión (2011). Among other articles see El Mundo, 2007; El Pais, 2006; El Periódico, 2007a and Expansión, 2007.

of mobility of capital and people reduces the governments' discretionality to set tax rates at an optimal level with the effect of tax revenues cuts<sup>6</sup>.

This literature offered limited attention to the enforcement policies although they represent crucial elements of the tax management process. The most relevant theoretical contribution in this sense was given by Cremer and Gahvari (2000). Using a welfare maximizing framework, they examine the implications of tax evasion for fiscal competition and tax harmonization policies in an economic union. The countries have the power to set both tax rates and tax auditing probabilities. In a closed economy framework, allowing for tax evasion increases the marginal cost of public funds and reduces the level of public good provision. From our perspective the most interesting result of the paper regards the economic union of two evading countries. In this setting the states engage in mobility-based competition that produces less than optimal equilibrium values of both the tax and audit rates. Harmonization policies can theoretically circumvent this problem but, according to the authors, coordinating audit strategies may be problematic because it is difficult for the government of one country to observe and verify the enforcement efforts of the other countries. For this reason, although a harmonization policy on tax rates results effective in circumventing tax rates sub-optimality, it is not sufficient to avoid the auditing rate inefficient outcome: since member states are no more allowed to compete on tax rates, they lower their effective tax rates by cutting their auditing probabilities.

Another contribution to this literature is given by Stöwhase and Traxler (2005) that analyze the implications of different equalization systems on regional enforcement policies in a federal framework taking the statutory tax rates as exogenously fixed at the central level. The benchmark framework presents no equalization scheme and is congruent with the results by Cremer and Gahvari (2000). Their more interesting result suggests that a way to partially circumvent the enforcement inefficient outcome is to use a particular equalization scheme. Introducing a gross revenue sharing equalization scheme, under which tax revenues are shared but auditing costs are fully borne by each region, even more inefficient enforcement policy outcome is obtained. Considering instead a net revenue sharing scheme, under which both tax revenues and auditing costs are shared, the outcome results more efficient compared to both the benchmark case and the gross revenue sharing case.

Janeba and Peters (1999) analyze the taxation of interest income in an economic union of two countries in the presence of tax evasion. In their setting the enforcement effort is proxied by the treatment of nonresidents' tax base. In fact any state can decide whether to discriminate against mobile tax base when setting the tax rate. The result is analogous to a prisoners' dilemma. The authors show that if a sequential structure of the game is considered and any country have

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<sup>6</sup> The applied literature that tests these theoretical models from an empirical point of view is vast and uses a spatial econometrics approach (among others see for example, Devereux et al., 2008; Esteller-Moré, Rizzo, 2011; Figlio et al., 1999; Overesch, Rincke, 2011).

initially to decide whether to discriminate or not and then the level of the tax rates, an equilibrium always exists: both countries discriminate offering a lower tax rate to nonresident's income with respect to resident's one. In equilibrium this strategy will allow the mobile bases to evade taxation successfully. In this sense a discrimination strategy is assimilable to a mobility-based competition on both enforcement policies and tax rates. If instead all countries had harmonized their policies deciding not to discriminate, tax competition would have led to a lower level of tax evasion. This strategy is dominated by the one in which both countries discriminate and so it is not reachable in equilibrium.

The literature on tax enforcement mobility-based competition therefore agree on the impossibility to overcome the auditing policies inefficient outcome through a harmonization policy and although some alternative strategies have been proposed, further research is needed in this field. In particular there is not any empirical study testing these models. In this perspective the case of wealth taxes seems to be particularly appropriate to investigate. Indeed the literature suggest that the cost to levy these taxes in federal systems is significantly increased by both vertical and horizontal tax competition (Bird, 1991). In Australia and Canada, for instance, the coexistence of a federal and a sub-central IGT led to the abolishment of the first one (in 1978 and 1972 respectively). This fact favored the disappearance of the local IGT too which succumbed (in 1983 in Australia and in 1986 in Canada) to the pressures of horizontal tax competition (Duff, 2005). In the U.S. the IGT has been repealed in 33 of the 48 contiguous states and its elimination is under discussion in the remaining ones. Conway and Rork (2004) show that this is a result of a mobility-based competition process exploiting historical elderly migration data. The same process occurred in the majority of Swiss cantons since early 1990s. The tax competition was the most important argument in the political debate that regarded these reforms. In particular Brulhart and Parchet (2011) find that a change in IGT tax burden has statistically significant effect on the very wealthy retirees' tax base but not on the tax base of the retirees considered as a whole. This suggests that the incentive to move regard just the upper tail of the income distribution among wealthy retirees.

The empirical evidence on wealth taxes corroborates the presence of mobility-based competition on statutory tax parameters but the possibility that these interactions may occur also at the enforcement level has not yet been investigated. In this perspective it is also useful to relate our analysis to the tax administration determinants literature. Although there is no accordance on which is the objective function of a tax administration, the dominant approach design it as a public agency that maximize tax revenues (e.g. Shaw *et al.*, 2009; Slemrod, Yitzhaki 2002, 1987). However recent empirical papers suggest that also the political as well as the budgetary variables matter in determining the tax administration's enforcement effort (see e.g. Young *et al.*, 2001; Barette *et al.*, 2002; Esteller-Moré, 2005, 2011).

The previous literature recognizes the possibility for sub-central governments to interact also on tax administration parameters and identifies the mobility of tax bases as the source of the interdependence. However further research is needed in this field to better understand the sub-central administration's behaviour. We aim at fulfilling this objective and we will empirically analyze the case of wealth taxes.

### 3. The theoretical framework: “mobility-based” competition in presence of evasion

In this theoretical section we will consider mobility-based competition as the potential source of interdependence among sub-central tax administrations: we present a simple model of tax competition in presence of tax evasion<sup>7</sup>. This will permit us to set up the basic hypotheses to be tested in the empirical analysis. The framework is modelled as a federal state constituted by two regions ( $i = 1,2$ ) of equal size in which the total population is normalized to one. At the regional level there are two institutional agents: the government that set the tax rate  $t_i \in (1, 0)$  and the tax administration that controls the auditing probability  $\beta_i \in (1, 0)$ . Following the most common approach in the literature we assume that both institutions have aligned incentives and act as Leviathans: they respectively set tax rates and auditing policies, both maximizing total tax revenues<sup>8</sup>. Since we are not interested in statutory tax parameters interactions we will not solve the government's problem and take tax rates as given. Taxpayers decide the share  $\alpha \in (1, 0)$  of wealth  $B$  to declare minimizing their tax payment. In order to let the solution to be interior tax evasion is assumed to be costly for the individual. Moreover the taxpayers are neutral risk averse in order to avoid any income effects. For sake of simplicity we will not develop the individual's problem but the results are in line with standard literature (see Allingham and Sandmo 1972; Cremer and Gahvari, 2000).

The model is developed in two *stages*:

1. Regional tax administration set tax auditing policies.
2. Individuals decide in which region set their location in the federation comparing their indirect utility function (based on their current tax burden) in the two regions. This *stage* is solved exploiting the concept of “home attachment” (see Mansoorian, Myers (1993, 1997) for the original framework and Wellisch (2000) for a recent formulation).

The solution is provided by backward induction.

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<sup>7</sup> The outline is based on Cremer, Gahvari (2000).

<sup>8</sup> This seems to be reasonable in the sense that the regional government directly set only the statutory tax rate while it just indirectly controls the auditing policy that is implemented by the administrative office. A possible extension could be for example to suppose that a bargaining process between the two institutions takes place (Fuest, 2000).

### 3.1 Stage 2: The decision about the region in which reside

To model the concept of “home” we assume that taxpayers are indexed by  $n \in (1, 0)$  and are uniformly distributed between 0 and 1<sup>9</sup>. The preferences of taxpayer  $n$  with respect to his location are given by:

$$V(n) = \begin{cases} U_1^* + a \times (1 - n) & \text{if } n \text{ lives in region 1} \\ U_2^* + a \times n & \text{if } n \text{ lives in region 2} \end{cases} \quad (1)$$

Where  $U_i^* = U_i^*(1 - \alpha^*(t_i, \beta_i))$  for  $i = 1, 2$ , represents the (pecuniary) indirect utility function<sup>10</sup> and where  $n \in (1, 0)$  measures the non-pecuniary (psychic) benefit the individual derives from living in region 2 and  $(1 - n)$  the benefit from living in region 1. Thus taxpayers indexed by  $n \in (0, \frac{1}{2})$  reside in region 1 while the ones identified by  $n \in (\frac{1}{2}, 1)$  reside in 2. The parameter  $a \in (0, +\infty)$  measures the degree of individual mobility. The interpretation of  $a$  is crucial. We assume  $a$  to represent the cost to be sustained to move from the home region<sup>11</sup>. The taxpayer’s utility to live in his own region increases with the cost of mobility: if the costs are low (high) then the relative importance that the taxpayer assigns to the psychic part of the utility function, with respect to the pecuniary one, is low (high)<sup>12</sup>.

The mobility equilibrium is characterized as:

$$\begin{aligned} U_1^* + a \times (1 - n_1) &= U_2^* + a \times n_1 \\ U_1^* + a \times (1 - n) &> U_2^* + a \times n \quad \forall n < n_1 \\ U_1^* + a \times (1 - n) &< U_2^* + a \times n \quad \forall n > n_1 \end{aligned} \quad (2)$$

Where  $n = n_1$  represents the marginal individual indifferent between living in region 1 and region 2 and, since  $\int_0^{n_1} dn = n_1$ , it also represents the population in region 1 in the migration equilibrium:

$$n_1 = n_1(t_1, \beta_1, t_2, \beta_2; a) = \frac{1}{2} + \frac{U_1^* - U_2^*}{2a} = \frac{1}{2} + \frac{B \times [\theta_2 - \theta_1 + g_2 - g_1]}{2a} \quad (3)$$

<sup>9</sup> See the Appendix 1 for a generalisation of the model that releases this assumption on the population distribution.

<sup>10</sup> The direct utility function is defined as  $U = B \times [1 - t_i \times [\alpha + (1 - \alpha) \times \tau \times \beta_i] - g(1 - \alpha)]$ . Where  $(\tau - 1) > 0$  is the exogenous tax fine per unit of tax evaded and the function  $g(1 - \alpha)$  represents the cost of tax evasion  $(1 - \alpha)$ , such that  $g'(1 - \alpha) > 0$ ,  $g''(1 - \alpha) > 0$ ,  $g(0) = 0$ ,  $g(1) \rightarrow +\infty$ .

<sup>11</sup> Since mobility could be either real or fictitious, this could be interpreted as the cost of actual mobility or the cost of pretending the move.

<sup>12</sup> When the mobility cost is null ( $a = 0$ ) the tax bases become perfectly mobile: only the pecuniary part of the utility function matters in the taxpayer’s migration decision. Instead when the mobility costs are extremely high ( $a \rightarrow +\infty$ ) the taxpayers are perfectly immobile. This case can be interpreted as a centralized economy case in which the tax policies are set by a unique federal planner. These two limit cases are excluded to allow for imperfect mobility of individuals.



Where  $\theta_i \equiv t_i \times [\alpha + (1 - \alpha) \times \tau \times \beta_i]$  is defined as the optimal effective tax rate for the region  $i = 1, 2$ . For sake of simplicity the superscripts on the variables are omitted. The population in region 2 in the migration equilibrium is:

$$n_2 = \int_{n_1}^1 dn = 1 - n_1 \quad (4)$$

### 3.2 Stage 1: Regional administrations set tax auditing policies

The problem is symmetric: the two administrations compete “a la Cournot” simultaneously setting their tax policies. We develop the problem of the administration 1. Formally administration 1 faces the following problem given the governments’ decision on tax rates and anticipating the results of the last *stage*:

$$\text{Max}_{\beta_1} R_1(\beta_1, \beta_2; t_1, t_2, a) = n_1 \times r_1 = \left( \frac{1}{2} + \frac{B \times [\theta_2 - \theta_1 + g_2 - g_1]}{2a} \right) \times [B \times \theta_1 - d(\beta_1)] \quad (5)$$

Where  $d(\beta_i)$  represents the tax administration cost such that  $d'(\beta_1) > 0, d(\beta_1)'' > 0$  and  $r_i \equiv \frac{R_i}{n_i} = [B \times \theta_i - d(\beta_i)]$  is the unitary tax revenue.

Since the two regions are symmetric, it is possible to show that a symmetric Nash equilibrium exists and satisfies the following condition obtained from the FOC of the administrations and from the analogous FOC of the governments imposing  $t_1 = t_2 = t, \beta_1 = \beta_2 = \beta$ <sup>13</sup>:

$$\beta: \quad r'_\beta = -2n'_\beta \times r > 0 \quad (6)$$

The factor  $-2n'_\beta$  represents the expected loss in the number of taxpayers due to an increase in  $\beta$ . So the right hand side of equation (6) corresponds to the marginal mobility costs for the regional governments in term of tax revenue losses due to an increase in  $\beta$ . The left hand side represents the net marginal revenue due to an increase in  $\beta$ .

Developing the condition (6) results that  $B \times \frac{\partial \theta}{\partial \beta} - d'(\beta) = r \times \frac{B \times (\frac{\partial \theta}{\partial \beta} + \frac{\partial g}{\partial \beta})}{a}$ . It is immediate to show that in the limit case of centralization ( $a \rightarrow +\infty$ ), the marginal mobility costs are null and results that  $r'_\beta = 0$ : we are in the bliss point of the Laffer curve. Since the marginal mobility costs are positive, under decentralization ( $a \in (0, +\infty)$ ) the tax auditing implementation is more costly. In fact the net marginal tax revenue is positive ( $r'_\beta > 0$ ) and tax enforcement is less severe than under centralisation: the tax-base mobility threat tames the administration. This result replicates the one by Cremer and Ghavari (2000).

<sup>13</sup> These conditions imply  $n_1 = n_2 = n = \frac{1}{2}, r_1 = r_2 = r, \theta_1 = \theta_2 = \theta, g_1 = g_2 = g$ .

### 3.3 The slope of the reaction function and other comparative statics

Since the purpose of this paper is to empirically test the presence of regional interdependencies in the setting of tax auditing policies, we want to investigate the process that regional administrations face in order to reach the equilibrium level of the auditing probability. That is we are interested in evaluating the slope of the reaction function  $\beta_i(\beta_j)$ . A non null sign would highlight the presence of some kind of interactions among regions. In particular we expect to find a positive sign, i.e. following a game-theory definition, we guess  $\beta_i$  and  $\beta_j$  are strategic complements. If this were the case, we could conclude that in a decentralized framework, the lower equilibrium level in  $\beta$  is the consequence of a competition process due to the potential tax base mobility.

It is easy to show that:

$$\frac{\partial \beta_1}{\partial \beta_2} = - \frac{R_{1\beta_1\beta_2}(\beta_1, \beta_2; t_1, t_2, a)}{R_{1\beta_1\beta_1}(\beta_1, \beta_2; t_1, t_2, a)} = - \frac{n_{1\beta_2} \times r_{1\beta_1}}{R_{1\beta_1\beta_1}(\beta_1, \beta_2; t_1, t_2, a)} > 0 \quad (7)$$

The first term of the numerator of equation (7) represents the derivative of the population in region 1 with respect to the enforcement of region 2 and is positive: once region 2 begins to increase the auditing probability, some residents in region 2 start to move to region 1. The second factor of the numerator represents the marginal unitary tax revenue that is positive under the FOC. According to the Second Order Condition (SOC) of the administration's problem the denominator of equation (7) should be negative. The slope of the reaction function is then positive: the regional administrations set the auditing strategies in a complementary way and so they are competing on this instrument in order to attract (or to not lose) tax bases. We will test this result by means of econometric techniques. Our main research question is then: To what extent the auditing policy of each region depends on the strategies adopted by the other regions?

Moreover it is possible to show that  $\frac{\partial(\frac{\partial \beta_1}{\partial \beta_2})}{\partial a} < 0$  (please see Appendix 2 for details). This means that the competition between regions become weaker when the mobility costs are higher. Since reasonably the mobility costs are positively correlated with the distance between the regions we can then stress that two far regions will compete less than two closer regions. We will explicitly take this aspect into consideration when choosing the econometric strategy.

Another result we find and we will test is that  $\beta_1$  and  $t_2$  are strategic complements, in fact:

$$\frac{\partial \beta_1}{\partial t_2} = - \frac{R_{1\beta_1 t_2}(\beta_1, \beta_2; t_1, t_2, a)}{R_{1\beta_1\beta_1}(\beta_1, \beta_2; t_1, t_2, a)} = - \frac{n_{1t_2} \times r_{1\beta_1}}{R_{1\beta_1\beta_1}(\beta_1, \beta_2; t_1, t_2, a)} > 0 \quad (8)$$

This result is reasonable: if the government of one region increases the competition on the tax rates (i.e. it reduces  $t_j$ ), ceteris paribus, the administration of the other region will unambiguously react setting a more tolerant auditing rate (i.e. it reduces  $\beta_i$ ) in order to not loose tax bases<sup>14</sup>.

#### 4. Empirical Analysis

In this section we test the main hypothesis by means of an econometric model, we give a description of the data base and we present and comment the main results that arise from the analysis.

##### 4.1 The empirical framework

The theoretical framework presented in the previous section offers interesting insights to be empirically tested: the horizontal tax competition model suggests that revenues maximizing administrations set the auditing policies in a complementary way, interacting between them in order to not loose tax bases. This result comes from equation (7). To test this result we estimate a *time-space recursive model* that adopts a spatial econometrics specification (see Anselin *et al.* 2008).

##### 4.1.1 Time-space recursive model

A recent literature on horizontal tax interdependences has acknowledged the possibility that policy reactions are not immediate and that inertia should be considered in the setting of statutory tax parameters (Overesch, Rincke, 2011; Esteller-Moré, Rizzo, 2011). Moreover since the tax enforcement is a policy not expected to radically change from one year to another, we consider the possibility that a sluggish adjustment in the auditing policies could play a role even stronger in our case. For this reason, we develop a *dynamic time-space recursive model* (Anselin *et al.* 2008), introducing the time-lagged endogenous variable as a regressor, in addition to the lagged spatial regressor:

$$\beta_{it} = \vartheta_i + \tau_t + \alpha\beta_{it-1} + \gamma\beta_{-it-1} + \mu TR_{it} + \delta EL_{it} + \pi Left_{it} + \rho GDPpc_{it} + \sigma Defgdp_{it} + \varphi Transfexp_{it} + \omega Prft_{it} + \xi Ded_{it} + \psi Ded_{-it-1} + \varepsilon_{it} \quad (9)$$

So the coefficient  $\alpha$  accounts for the presence of inertia and is expected to be positive (the higher the value the stronger the inertia) and less than one to be congruent to the concept of Nash equilibrium.  $\beta_{it}$  represents the total number of audits performed by region  $i$  during the year  $t$ , while the term  $\beta_{-it-1} \equiv \sum_{j=1}^N w_{ij} \beta_{jt-1}$  and  $w_{ij}$  is the spatial weight that reports the

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<sup>14</sup> Unfortunately it is not possible to unambiguously determine the sign of the slope of  $\beta_1(t_1)$ .

relative interdependence between regions  $i$  and  $j$  in such a way that  $w_{ij} \geq 0$  if  $i \neq j$  and  $w_{ij} = 0$  if  $i = j$ <sup>15</sup>. Since the amount of audits performed depends on the total number of auditable tax forms received in one year we introduce in the regression the number of tax returns received by region  $i$  during the year  $t$  ( $TR_{it}$ ), in this way we control for the size of tax administration. The specification includes fixed effects for regions  $\vartheta_i$  and years  $\tau_t$ , while  $\varepsilon_{it}$  is the error term.

So the term  $\beta_{-it-1}$  accounts for potential strategic competition on auditing policies and it is introduced with a time lag because in the practice the tax auditing policies of the other regions are not simultaneously observable by the regional administration (Fredriksson, Millimet, 2002b; Fredriksson *et al.*, 2004; Millimet, Rangaprasad, 2007). If the coefficient  $\gamma$  results significantly different from zero, the model will predict the presence of regional interactions in the setting of tax auditing policies. In particular according to the theoretical framework, eq. (7), we expect  $\gamma$  to be positive.

We assume the size of tax administration to be optimal when an increase in the number of tax returns correspond to a precisely proportional increase in the number of audits in the sense that the audited share of the tax return remains unchanged. This corresponds to  $\mu$  equal to one. The tax administration policies could be sensible to “budgetary” and “political” effects (see e.g. Esteller-Moré, 2005, 2011), as well as to other elements we control for.  $El_{it}$ , a dummy variable equal to one if there is an election in region  $i$  during the year  $t$ , is introduced to control for the electoral cycle. We expect its coefficient to be negative because regional governments could find it profitable to reduce the effective tax rate through enforcement effort cuts for a re-election purpose.  $Left_{it}$  is another dummy equal to one if the government in chair in a specific region and year is “leftish”. We use the per capita GDP ( $gdppc_{it}$ ) to control for the regional economic cycle and the regional size. The Deficit-GDP ratio ( $defgdp_{it}$ ) and the total amount of transfers received from the central government divided by the total regional expenditure ( $transfexp_{it}$ ) are introduced to account for further relevant budgetary factors. We also control for a proxy of profitability ( $prft_{it}$ ) defined as the mean revenue per audit collected by a region in a specific year. To account for possible normative modifications in the statutory tax parameters we include a dummy ( $ded_{it}$ ) equal to one if the regional government  $i$  introduces during the year  $t$  a sensible deduction in favour of the most common inheritors<sup>16</sup>. These modifications in the

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<sup>15</sup> In particular we employ a spatial matrix based on the inverse of the distance between regional capitals. This choice rises from a theoretical model’s result: when the distance between two regions – a proxy of the mobility costs – increases we observe a lower level of competition on auditing policies. Moreover although the recent literature suggests that a change in the spatial matrix should not be crucial (LeSage, Pace 2010), in our case the model could be better specified than one based on a simple natural neighbors matrix because of the presence of islands in Spain, that makes the definition of the neighbors arbitrary (see e.g. Costa-Font, Pons-Novell, 2007).

<sup>16</sup> Basically the main inheritors are the spouse, any descendant younger than 21 years or ascendants. For details on the normative aspect of the exemption regime see Durán-Cabré and Esteller 2009, 2010.

deduction regime substantially reduce the level of the effective tax rate and there is evidence that they induced a convergence process among regions congruent with a race-to-the-bottom (Durán-Cabré and Esteller-Moré 2009, 2010). We can then interpret a value equal to 1 of  $ded_{it}$  as a modification in the corresponding regional statutory tax parameters that results in a less severe effective tax rate. Finally we control for  $ded_{it-1}$ , that represents the weighted average of the neighbours' deduction policies in the previous year. Following the previous reasoning, an increase in this variable is compatible with a decrease of the lagged weighted average of the neighbours' statutory tax parameters. According to the theoretical model (equation (8)) we expect the coefficient of this variable to be negative: a value of  $ded_{it-1}$  equal to 1 (i.e. a decrease of  $t_2$  in the equation (8)) would correspond to a decreasing number of audits.

As it is well known, the lagged endogenous variable  $\beta_{it-1}$  is by definition positively correlated with the regional fixed effect  $\vartheta_i$ . This implies that estimating  $\alpha$  by means of OLS will lead to inefficient and upwards biased estimates. The Within Groups estimator eliminates this source of inconsistency by transforming the equation to eliminate  $\vartheta_i$  but leading to an estimator that is biased downwards (see Nickell 1981). As suggested by Bond (2002) “a candidate consistent estimator will lie between the OLS and Within group estimates” and the Generalized Methods of Moments (GMM) provides a convenient framework for obtaining it. In particular we employ an augmented version of the Difference GMM (Arellano and Bond, 1991) the so called System GMM estimator (Arellano and Bover, 1995) that is supposed to be more efficient (Blundell and Bond, 1998). This estimator applies a transformation to the original model taking the first differences in order to eliminate  $\vartheta_i$  and builds a stacked dataset with variables in levels and in differences. The equation in differences is then instrumented with lags 2 and up of the endogenous variables in levels while the equation in levels is instrumented using the same lags of the endogenous variable in differences.

As argued by Ziliak (1997) instrumenting the endogenous lagged variable with all the available lags is efficient but in small samples this might cause a overfitting bias in GMM estimators. To solve this problem Roodman (2009) suggests to reduce the instrument count trying to keep the number of instruments below the number of groups. We combine both the possible approaches to instruments containment: collapsing instruments and limiting the lag depth amounts (see Roodman, 2009, pp. 148-151, for more details on these techniques). Although the number of instruments results significantly reduced it is still larger than the relatively small number of groups so we cannot completely fix this problem. In any case in order to test the instruments validity we perform the Hansen and the Sargan tests of overidentifying restrictions even if the performance of these tests can be weakened by a high number of instruments.

To check the validity of the instruments we also have to check for absence of serially correlated error terms that is a required condition for the consistency of the GMM estimator. We perform tests for first (AR(1)) and second order (AR(2)) serial correlation under the null of presence of serial correlation. If there is no serial correlation in  $\varepsilon_{it}$ , the first-differenced residuals should reject the null hypothesis in AR(1) test but no in AR(2) test (Arellano and Bond, 1991).

For what concerns the estimation strategy for  $\beta_{-it-1}$  we employ System GMM using the lags of the spatial lag as instruments (reported as internal) but we also provide a robustness check adding some external instruments<sup>17</sup> in order to find to what extent the estimates are affected by the use of both set of instruments.

Finally we enrich the model in order to better understand to what extent the reforms that occurred in Spain starting from the mid nineties have affected the horizontal interdependency on tax auditing. More precisely to test whether  $\beta_{-it-1}$  is affected by the decentralization process that gave more legislative tax power to the regional governments, we interact  $\beta_{-it-1}$  with  $post97_{it}$  a dummy equal to one for years posteriors to the first IGT reform year (1997). In this way we identify the effect of the devolution process as a whole. In order to disentangle the specific role of any reform, we interact  $\beta_{-it-1}$  with a dummy that identify the second reform (2002) and, in the same model, with another dummy associated with the period between the two reforms. We finally introduce a last model where  $\beta_{-it-1}$  is interacted with  $ded_{it}$ . The aim in this case is to emphasize the effects of the actual implementation of the second IGT reform on the enforcement competition process.

If the coefficients of these alternative interacted terms result to be negative this would mean that after the reforms that gradually decentralized the legislative power on the statutory tax parameters the regions reduced the competition on their auditing policies. In other words: a reduction in the opaque competition on enforcement strategies would be the result of the possibility to compete on the statutory tax parameters.

#### **4.1.2 Data and Sources**

Our panel is constituted by the information about the 15 Spanish common regime autonomous communities<sup>18</sup> for the period 1987-2009<sup>19</sup>. In Table 1 we present the summary statistics. The

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<sup>17</sup> We used some (lagged) exogenous explanatory variables as instruments employing the same weighting scheme for the instruments as we do for  $\beta_{-it-1}$  (see e.g. Kelejian, Robinson, 1993; Kelejian, Prucha, 1998).

<sup>18</sup> Both the communities of Navarre and Basque Country belongs to the Foral System, a special regime of independence that allows them to have own laws and institutions. For this reason have not been included in the survey.

<sup>19</sup> The information about auditing in the Madrid community starts from 1996 because this is the year in which it received the administrative power. We do not have information about the administration policies in 1993 because in 1995 the budget was not approved.

information about the regional tax enforcement policies is annually released inside the report “Informe sobre la cesión de tributos a las Comunidades Autónomas” published together with the Spanish National Budget “Proyecto de Presupuestos Generales del Estado”. The report registers the number of audits performed year by year by each region. We will use this information to define our endogenous variable  $\beta_{it}$  together with the number of Tax Return ( $TR_{it}$ ) and the proxy of auditing profitability ( $prft_{it}$ ). The other variables are obtained from different statistical sources. The per capita GDP ( $gdppc_{it}$ ) is extracted from the Spanish National Institute of Statistics, INE. The deficit data used to construct the variable  $defgdp_{it}$  is calculated as the difference between current availability and current expenditure extracted from the Ministry of the Economy and Finance database. The  $transfexp_{it}$  is constructed as the ratio between the total amount of transfers received from the central government (extracted from the INE database) and the total regional expenditure (extracted from the Ministry of the Economy and Finance database). The information on election years  $el_{it}$  is obtained from the Interior Ministry website ([http://www.mir.es/DGPI/Elecciones/Procesos\\_Electorales\\_Celebrados/proceso\\_por\\_tipoyfecha.html](http://www.mir.es/DGPI/Elecciones/Procesos_Electorales_Celebrados/proceso_por_tipoyfecha.html)) while the information about the political colour of the regional governments necessary to construct the dummy  $left_{it}$  is obtained from the Zarate’s Political Collections website (<http://www.terra.es/personal2/monolith>).

The information to construct the dummy  $ded_{it}$  that accounts for the introduction of IGT tax deductions is extracted from Durán-Cabré, Esteller-Moré (2009).

[TABLE 1]

#### 4.2 Basic results

In Table 2 we report the results of the *time-space recursive model* expressed in equation (9). Column (1) is a baseline estimation to test auditing interactions without accounting for inertia (i.e. assuming  $\alpha = 0$  in equation 9)<sup>20</sup>. The autoregressive coefficient is significant and positive: this is congruent with the theoretical model although the coefficient is not credible because it is much higher than one, which is not compatible with the Nash equilibrium. Moreover even if there are not specific papers analyzing these issues with which to compare these estimates, the closer literature on spatial interactions in statutory tax parameters setting suggests that the slope of the reaction function should be pretty lower<sup>21</sup>. For this reason we think that not taking into

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<sup>20</sup> This is a *pure-space recursive model* estimated through a two-stage-least-square procedure. We used just the external instruments. We report an endogeneity test of  $\beta_{-it-1}$  (under the null hypothesis that the regressor can actually be treated as exogenous), we perform the Hansen (1982) overidentifying restrictions test and finally we report the first-stage F-statistic to check the performance and the potential weakness of the instruments.

<sup>21</sup> They should be around 0.2 – 0.35 according to Revelli, (2001, 2006).

account the inertia leads to a misspecification of the model: otherwise, its role is picked up by the spatial lag.

In fact the inertia plays an important role in the setting of regional auditing policies: the coefficients are strongly significant and positive<sup>22</sup>. The coefficient of the spatial lag is positive and significant, which confirms that horizontal interactions between regional administrations take place when setting the auditing policies and it is congruent with the hypothesis of tax competition adopted in the theoretical model. In columns (2) to (5) we present the basic model without interactions and the spatial lag coefficient is within the range (0.38 – 0.66) depending on which controls are introduced and which instruments are employed. Even if there is not a benchmark in the literature to which compare these results, these values are much more credible than the ones obtained without accounting for sluggish adjustments in the endogenous variable. In particular in column (5) we exploit both internal and external instruments and we can see that this does not change much the results on the estimates of inertia and horizontal interactions.

Looking at the controls we can highlight three robust results. First of all, the variable “election year” results to be significant and negatively associated with the setting of the auditing policies: in presence of election the regional administrations reduces the enforcement on IGT. This suggests that there is a political link between the regional governments and regional tax administrations. The politicians in office have the incentive to reduce the IGT effective tax rate in order to gain votes and be re-elected and operate through the tax administration, which reducing the enforcement policies comply the politician’s objective. This supports the hypothesis of identical incentives of the two institutions adopted in the theoretical model. The second result regards the variable  $ded_{it}$  that accounts for the introduction of IGT tax deductions by the regional governments. This variable results to be significant and positively associated with the enforcement strategies of the regional administrations. This suggests that once the governments are directly entitled to modify the statutory tax parameters (that is after the 2002 reform) and decide to introduce generous deductions, the administrations increase the number of audits: statutory tax parameters and auditing policies are strategic substitutes. Lastly, the coefficient of the term  $ded_{it-1}$  is negative and significant corroborating the result of the theoretical framework (equation 8): neighbour’s statutory tax parameters and auditing policies are strategic complements.

[TABLE 2]

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<sup>22</sup> The coefficients of the lagged endogenous variable are pretty high but they are lower than the ones obtained through the OLS estimator (that are around 0,96) and higher than the ones obtained through the Within group estimator (that are around 0,83) so they are within the range indicated by Bond (2002) to contain a consistent estimator. Moreover note that AR(1) and AR(2) tests detect first order but not second order serial correlation for first-differenced residuals.



### 4.3 Further results

In Table 3 we perform various interactions. In the first regression we interact  $\beta_{-it-1}$  with a dummy that capture the effect of the first relevant IGT reform introduced in 1997 obtaining a negative and significant coefficient for the interacted term. This reform seems to have a negative effect on the enforcement strategies of the administrations. Moreover the absolute values of the two coefficients (the interacted and the un-interacted) are pretty similar and in fact looking at the linear combination between them we found that the total effect is strongly not significantly different from zero<sup>23</sup>. This result can be interpreted as a corroboration of the hypothesis of horizontal tax competition as source of the auditing interdependence. In particular this analysis seems to suggest that during the period in which the legal power on IGT was decentralized, the competition on the enforcement policies disappeared and supposedly switched to the setting of the statutory tax parameters. This result implicitly provides a way to elude the unfeasibility of harmonization policies on enforcement strategies raised by Cremer and Ghavari (2000). As argued in section 2, their argument is that a coordination strategy between different sub-central administrations is unreachable because of the difficulty to establish whether a specific region's enforcement effort is adequate or not<sup>24</sup>. This leads to an unavoidable enforcement inefficient outcome. Our result seems to suggest that the devolution process can play a crucial role. Decentralizing the legislative power on statutory tax parameters has the positive and welcome effect to switch from an opaque tax competition on tax enforcement to a clearer and more transparent competition on statutory tax parameters making a harmonization policy more feasible. To better specify this aspect, in column (2) we interact – in the same regression –  $\beta_{-it-1}$  with the dummies that identify respectively, the second wave of reform and the period between the two reforms. In this way it is possible to disentangle which is the different effect of any reform. We obtain analogous results but in this case although both the interacted terms are negative, the one that identifies the second reform results lower in absolute value and not significant<sup>25</sup>. This means that the first wave of reform had a stronger negative impact on the auditing competition. We interpret this result as an evidence that to observe a substantial disappearance of the competition on enforcement strategies, it is sufficient for the regions to have the possibility to weakly set (and compete on) statutory tax parameters<sup>26</sup>. This last result is also corroborated interacting  $\beta_{-it-1}$  with the dummy that identify the actual implementation of a substantial reduction in the statutory tax parameters ( $ded_{it}$ ): the interacted

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<sup>23</sup> The coefficient is -0.042 and the t-statistic -0.25.

<sup>24</sup> In fact, for instance, a low auditing rate could be identified as inefficient just because is low when it is actually low as a result of an improvement process that made the enforcement effort much more precise and efficient.

<sup>25</sup> The linear combination of the two effect with the un-interacted one is still not significant: the coefficient is -0.262 and the t-statistic is -0.67.

<sup>26</sup> This could also simply depend on the fact that the observations involved in the evaluation of the second and more recent wave of reform are less.

term also results negative and even if it is not significant, the linear combination between the interacted and the un-interacted term is not significantly different from zero.

[TABLE 3]

## 5. Conclusions

In this paper we analyze the presence of another level of tax interdependence that may occur in federal contexts: horizontal competition on enforcement policies among regional administrations. Through a theoretical framework we derive a regional auditing reaction function that results to be positively sloped: regional administrations compete on auditing policies. This result is tested by means of spatial econometric techniques. The *time-space recursive model* produces outcomes that corroborate the theory. In particular allowing for sluggish adjustment we obtain a high degree of inertia in the auditing policy setting and credible coefficients for the spatial lag (around 0.38 – 0.66), which are congruent with the Nash equilibrium condition. This is our main contribution and recall us to the results of Cremer and Gahvari (2000).

They suggest that in presence of horizontal competition since auditing strategies are not publicly or not easily observable, it could be pretty difficult to set a binding agreement between sub-central governments in order to harmonize them. This is a reasonable point in the sense that even if the policies are publicly observable, because for instance they are recorded in a publicly available report – as in the Spanish case – it could be difficult to establish whether a specific region's enforcement effort is sufficient or not. Regarding this point we implicitly find a way to circumvent this problem. Our empirical evidence suggests in fact that if the decentralization process is gradually implemented and the administration power is decentralized before the normative power, the competition on the enforcement policy disappears exactly when it is possible to compete on more powerful instruments: the statutory tax parameters. In this perspective the decentralization process is welcome: a higher decentralized framework has the advantage to switch from an opaque competition to a transparent one.

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**TABLES:**

**Table 1: Summary statistics**

Variable	Observations	Mean	Median	SD	Max	Min
Audits	307	370.5961	195	486.5621	2550	0
Tax Returns	308	21187	13442	18234.62	88528	1641
Leftish government	322	0.4627329	0	0.4993853	1	0
Election year	322	0.2546584	0	0.4363471	1	0
Deduction	322	0.1335404	0	0.3406872	1	0
Deficit-GDP ratio	308	-0.0028976	-0.0017705	0.0070989	0.0298811	-0.026144
Transfers-GDP ratio	294	0.3977149	0.3853665	0.1348314	1.373906	0.1117062
Per Capita GDP	322	11.52553	11.35349	5.497171	23.01702	2.174576
Auditing Profitability	280	8.936545	4.650814	12.75857	108.2774	0

**Table 2: Tax Auditing interdependence: Time-space recursive model  
(System-GMM/One-step; Fixed effects & Time Effects in all specifications)**

VARIABLES	(1)	(2)	(3)	(4)	(5)
	Audits	Audits	Audits	Audits	Audits
L.audits	-	0.912***	0.888***	0.896***	0.932***
		(5.969)	(6.265)	(6.733)	(8.989)
L.Waudits	2.114*	0.659**	0.521**	0.381*	0.468**
	(1.718)	(2.205)	(2.214)	(1.685)	(2.329)
Leftish government	-233.735**	-53.907	-34.527	-3.522	-4.096
	(-2.193)	(-0.982)	(-0.804)	(-0.170)	(-0.208)
Election year	-101.380	-34.505**	-37.684**	-33.477**	-33.315**
	(-1.102)	(-2.254)	(-2.210)	(-2.158)	(-2.114)
Deficit/GDP	5740.946	-270.497	27.531	137.939	-393.251
	(1.061)	(-0.141)	(0.014)	(0.067)	(-0.180)
Transfers/expenditure	-130.466	226.583	233.558	225.265	252.761
	(-0.530)	(1.136)	(1.168)	(1.219)	(1.410)
Tax Return	-0.001	0.003	0.003	0.003	0.002
	(-0.108)	(1.421)	(1.486)	(1.041)	(1.027)
Deduction			76.970**	59.932***	49.657***
			(2.307)	(2.724)	(2.774)
L.WDeduction			-168.552*	-271.726**	-332.964***
			(-1.726)	(-2.226)	(-2.748)
Per capita GDP				4.151	9.031
				(0.324)	(1.007)
L.audit_profitability				-0.418	-0.490
				(-0.316)	(-0.372)
Tax Returnxd_foral				0.004	0.004
				(1.147)	(1.136)
_cons		-211.897*	-175.954	-195.919	-279.633
		(-1.671)	(-1.187)	(-0.678)	(-1.170)
Observations	237	237	237	237	237
Internal Instruments	NO	YES	YES	YES	YES
External Instruments	YES	NO	NO	NO	YES
# Instruments	5	32	34	37	40
Gmm lag limits	-	(2, 5)	(2, 5)	(2, 5)	(2, 5)
AR(1) ( <i>p</i> -value)	-	0.039	0.040	0.037	0.026
AR(2) ( <i>p</i> -value)	-	0.987	0.951	0.981	0.964
Sargan-test	-	8.673	8.126	9.008	13.533
Sargan-test ( <i>p</i> -value)	-	0.371	0.421	0.342	0.260
Hansen-test	2.105	0.000	0.000	0.000	0.000
Hansen-test ( <i>p</i> -value)	0.716	1.000	1.000	1.000	1.000
Endogeneity-test	10.864	-	-	-	-
Endogeneity-test ( <i>p</i> -value)	0.001	-	-	-	-
First stage F-statistic	54.46	-	-	-	-
First stage F-statistic ( <i>p</i> -value)	0.0000	-	-	-	-

Note: *t* statistics in parentheses; \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . Model (1) is a *pure-space recursive model* estimated through a 2SLS procedure.



**Table 3: Tax Auditing interdependence: Time-space recursive model – Interactions  
(System-GMM/One-step; Fixed effects & Time Effects in all specifications)**

VARIABLES	(1) Audits	(2) Audits	(3) Audits
L.audits	0.879*** (10.933)	0.869*** (13.432)	0.904*** (6.721)
L.Waudits	0.641** (2.540)	0.688*** (2.667)	0.398* (1.818)
L.Waudits×post97	-0.683*** (-3.458)		
L.Waudits×D97-01		-0.716*** (-3.337)	
L.Waudits×post01		-0.235 (-0.849)	
L.Waudits×deduction			-0.530 (-1.052)
Leftish government	-4.669 (-0.264)	-4.895 (-0.230)	-3.485 (-0.163)
Election year	-27.053* (-1.655)	-31.638* (-1.815)	-37.983** (-1.996)
Deficit/GDP	-319.061 (-0.152)	-763.501 (-0.354)	26.451 (0.012)
Transfers/expenditure	218.924 (1.156)	208.353 (1.078)	232.217 (1.271)
Tax Return	0.002 (1.232)	0.003 (1.589)	0.002 (0.964)
Deduction	44.531*** (2.960)	72.272*** (3.483)	91.448** (2.185)
L.WDeduction	-198.526 (-1.487)	-278.812* (-1.834)	-341.017*** (-3.060)
Per capita GDP	-4.910 (-0.758)	-1.843 (-0.220)	5.686 (0.446)
L.audit_profitability	-0.113 (-0.095)	-0.158 (-0.134)	-0.425 (-0.307)
Tax Return×d_foral	0.003 (1.227)	0.004* (1.688)	0.004 (1.117)
_cons	36.460 (0.170)	-105.899 (-0.396)	-197.627 (-0.689)
Observations	237	237	237
Internal Instruments	YES	YES	YES
External Instruments	NO	NO	NO
# Instruments	42	45	40
Gmm lag limits	(2, 5)	(2, 5)	(2, 5)
AR(1) ( <i>p</i> -value)	0.017	0.017	0.036
AR(2) ( <i>p</i> -value)	0.985	0.946	0.922
Sargan-test	14.496	14.704	9.126
Sargan-test ( <i>p</i> -value)	0.270	0.399	0.520
Hansen-test	0.000	0.000	0.000
Hansen-test ( <i>p</i> -value)	1.000	1.000	1.000

Note: *t* statistics in parentheses; \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

### Appendix 1: Generalized results with not uniform distribution of taxpayers

Now we assume that the distribution of taxpayers along the home attachment is not uniform, i.e. we assume that  $n \in (0,1) \sim f(n)$  where  $f(n)$  represents a generic density function. Graphically we have that:

[figure about here]

The value  $n_1(t_1, \beta_1, t_2, \beta_2; a) = \frac{1}{2} + \frac{U_1^* - U_2^*}{2a}$  represents the marginal individual indifferent between living in region 1 and region 2. Below  $n_1$  we have all the taxpayers that settle in region 1, while above  $n_1$  there are all the taxpayers that live in region 2. The respective shares of each group are  $F(n_1) = \int_0^{n_1} f(n)dn$  and  $1 - F(n_1) = \int_{n_1}^1 f(n)dn$ .

At *stage 2* the problem of the administration of region 1 becomes:

$$\text{Max}_{\beta_1} R_1 = F(n_1) \times r_1 = F(n_1) \times [B \times \theta_1 - d(\beta_1)]$$

The FOC of this problem is:

$$n_1'_{\beta_1} \times f(n_1) \times r_1 + r_1'_{\beta_1} \times F(n_1) \equiv P(\beta_1, \beta_2; t_1, t_2, a) = 0$$

The SOC is:

$$P_{\beta_1}(\beta_1, \beta_2; t_1, t_2, a) < 0$$

The slope of the reaction function becomes:

$$\frac{\partial \beta_1}{\partial \beta_2} = - \frac{P_{\beta_2}(\beta_1, \beta_2; t_1, t_2, a)}{P_{\beta_1}(\beta_1, \beta_2; t_1, t_2, a)}$$

That is positive as long as  $f'(n_1) \leq 0$ .

### Appendix 2: Comparative statics on $a$

It is possible to express  $\frac{\partial \beta_1}{\partial \beta_2}$  as a function of  $a$  in order to perform a comparative statics:

$$\frac{\partial \beta_1}{\partial \beta_2} = - \frac{N}{A + a \times \frac{\partial^2 r_1}{\partial \beta_1^2}} = -N \times \left( A + a \times \frac{\partial^2 r_1}{\partial \beta_1^2} \right)^{-1} \quad (A1)$$

Where:

$$A = -2B \times \left[ \frac{\partial \theta_1}{\partial \beta_1} + \frac{\partial g_1}{\partial \beta_1} \right] \times \left[ B \times \frac{\partial \theta_1}{\partial \beta_1} - d'(\beta_1) \right] + B \times [\theta_2 - \theta_1 + g_2 - g_1] \\ \times \left[ B \times \frac{\partial^2 \theta_1}{\partial \beta_1^2} - d''(\beta_1) \right] - [B \times \theta_1 - d(\beta_1)] \times B \times \left[ \frac{\partial^2 \theta_2}{\partial \beta_1^2} + \frac{\partial^2 g_2}{\partial \beta_1^2} \right] \quad (A2)$$

and

$$N = B \times \left[ \frac{\partial \theta_2}{\partial \beta_2} + \frac{\partial g_2}{\partial \beta_2} \right] \times \left[ B \times \frac{\partial \theta_1}{\partial \beta_1} - d'(\beta_1) \right] \quad (A3)$$

Results that under conditions (16a) and (17a),  $N > 0$  and:

$$\frac{\partial \left( \frac{\partial \beta_1}{\partial \beta_2} \right)}{\partial a} = \frac{N}{\left( A + a \times \frac{\partial^2 r_1}{\partial \beta_1^2} \right)^2} \times \frac{\partial^2 r_1}{\partial \beta_1^2} < 0 \quad (A4)$$

### Appendix 3: Interdependencies between different instruments

The derivative  $\frac{\partial \beta_1}{\partial \beta_2}$  with respect to  $t_2$  could be written as:

$$\frac{\partial \left( \frac{\partial \beta_1}{\partial \beta_2} \right)}{\partial t_2} = \frac{-R_{1\beta_1\beta_2 t_2} \times R_{1\beta_1\beta_1} + R_{1\beta_1\beta_1 t_2} \times R_{1\beta_1\beta_2}}{\left( R_{1\beta_1\beta_1} \right)^2}$$

That is positive as long as:

$$M \equiv R_{1\beta_1\beta_1 t_2} \times R_{1\beta_1\beta_2} - R_{1\beta_1\beta_2 t_2} \times R_{1\beta_1\beta_1} > 0 \Leftrightarrow R_{1\beta_1\beta_1 t_2} \times R_{1\beta_1\beta_2} > R_{1\beta_1\beta_2 t_2} \times R_{1\beta_1\beta_1}$$

Where:

$$R_{1\beta_1\beta_1 t_2} = \frac{\partial n_1}{\partial t_2} \times \frac{\partial^2 r_1}{\partial \beta_1^2} < 0,$$

$$R_{1\beta_1\beta_1} = 2 \frac{\partial n_1}{\partial \beta_1} \times \frac{\partial r_1}{\partial \beta_1} + n_1 \times \frac{\partial^2 r_1}{\partial \beta_1^2} + r_1 \times \frac{\partial^2 n_1}{\partial \beta_1^2} < 0 \text{ under the SOC;}$$

$$R_{1\beta_1\beta_2 t_2} = \frac{\partial^2 n_1}{\partial \beta_2 t_2} \times \frac{\partial r_1}{\partial \beta_1} > 0 \text{ under FOC;}$$

$$R_{1\beta_1\beta_2} = \frac{\partial n_1}{\partial t_2} \times \frac{\partial r_1}{\partial \beta_1} > 0 \text{ under FOC.}$$

So in general it is not possible to establish an unambiguous relationship between  $\frac{\partial \beta_1}{\partial \beta_2}$  and  $t_2$ .

The same results hold for the relationship between  $\frac{\partial \beta_1}{\partial \beta_2}$  and  $t_1$ .

#### Appendix 4: Derivatives computation

From equations (2) and (3) and by means of the implicit function theorem it is possible to show that:

$$\frac{\partial \alpha^*_i}{\partial t_i} = \frac{\partial \alpha^*(t_i, \beta_i)}{\partial t_i} = \frac{-(1 - \beta_i \times \tau)}{g''} < 0$$

$$\frac{\partial \alpha^*_i}{\partial \beta_i} = \frac{\partial \alpha^*(t_i, \beta_i)}{\partial \beta_i} = \frac{t_i \tau}{g''} > 0$$

From these results and the equations (6) and (8) we have that:

$$\frac{\partial \theta_i^*}{\partial t_i} = \alpha^*_i + (1 - \alpha^*_i) \times \beta_i \times \tau + \frac{\partial \alpha^*_i}{\partial t_i} \times (1 - \beta_i \times \tau) \times t_i < 0$$

$$\frac{\partial \theta_i^*}{\partial \beta_i} = (1 - \alpha^*_i) \times t_i \times \tau + \frac{\partial \alpha^*_i}{\partial \beta_i} (1 - \beta_i \times \tau) \times t_i > 0$$

$$\frac{\partial g_i}{\partial t_i} = -\frac{\partial \alpha^*_i}{\partial t_i} (1 - \beta_i \times \tau) \times t_i > 0$$

$$\frac{\partial g_i}{\partial \beta_i} = -\frac{\partial \alpha^*_i}{\partial \beta_i} (1 - \beta_i \times \tau) \times t_i < 0$$

$$\left[ \frac{\partial \theta_i^*}{\partial t_i} + \frac{\partial g_i}{\partial t_i} \right] = \alpha^*_i + (1 - \alpha^*_i) \times \beta_i \times \tau > 0$$

$$\left[ \frac{\partial \theta_i^*}{\partial \beta_i} + \frac{\partial g_i}{\partial \beta_i} \right] = (1 - \alpha^*_i) \times t_i \times \tau > 0$$

$$\frac{\partial n_1}{\partial \beta_1} = -\frac{B \times \left[ \frac{\partial \theta_1}{\partial \beta_1} + \frac{\partial g_1}{\partial \beta_1} \right]}{2a} < 0$$

$$\frac{\partial n_1}{\partial \beta_2} = \frac{B \times \left[ \frac{\partial \theta_2}{\partial \beta_2} + \frac{\partial g_2}{\partial \beta_2} \right]}{2a} > 0$$

$$\frac{\partial n_1}{\partial t_1} = -\frac{B \times \left[ \frac{\partial \theta_1}{\partial t_1} + \frac{\partial g_1}{\partial t_1} \right]}{2a} < 0$$

$$\frac{\partial n_1}{\partial t_2} = \frac{B \times \left[ \frac{\partial \theta_2}{\partial t_2} + \frac{\partial g_2}{\partial t_2} \right]}{2a} > 0$$

$$\frac{\partial r_1}{\partial \beta_1} = \left[ B \times \frac{\partial \theta_1}{\partial \beta_1} - d'(\beta_1) \right] < 0$$

$$\frac{\partial r_1}{\partial t_1} = \left[ B \times \frac{\partial \theta_1}{\partial t_1} \right] < 0$$

$$\frac{\partial^2 \alpha^*(t_i, \beta_i)}{\partial \beta_i^2} = \frac{\frac{\partial \alpha^*}{\partial \beta_i} \times \tau \times t_i}{(g'')^2} \times g''' \leq 0 \quad \text{iff} \quad g''' \leq 0$$

$$\frac{\partial^2 \alpha^*(t_i, \beta_i)}{\partial t_i^2} = \frac{\frac{\partial \alpha^*}{\partial t_i} \times (1 - \beta_i \times \tau)}{(g'')^2} \times g''' \leq 0 \quad \text{iff} \quad g''' \geq 0$$

We assume  $g''' = 0$ .

$$\frac{\partial^2 \theta_i^*}{\partial t_i^2} = \frac{\partial \alpha^*}{\partial t_i} \times 2 \times (1 - \beta_i \times \tau) + \frac{\partial^2 \alpha^*}{\partial t_i^2} (1 - \beta_i \times \tau) \times t_i < 0 \quad \text{iff} \quad g''' = 0$$

$$\frac{\partial^2 \theta_i^*}{\partial \beta_i^2} = -2 \times \frac{\partial \alpha^*}{\partial \beta_i} \times \tau \times t_i + \frac{\partial^2 \alpha^*}{\partial \beta_i^2} (1 - \beta_i \times \tau) \times t_i < 0 \quad \text{iff} \quad g''' = 0$$

$$\frac{\partial^2 r_1}{\partial \beta_1^2} = \left[ B \times \frac{\partial^2 \theta_1}{\partial \beta_1^2} - d''(\beta_1) \right] < 0 \quad \text{iff} \quad g''' = 0$$

$$\frac{\partial^2 r_1}{\partial t_1^2} = \left[ B \times \frac{\partial^2 \theta_1}{\partial t_1^2} \right] < 0 \quad \text{iff} \quad g''' = 0$$

$$\frac{\partial^2 g_i}{\partial t_i^2} = -\frac{\partial^2 \alpha^*}{\partial t_i^2} (1 - \beta_i \times \tau) \times t_i - \frac{\partial \alpha^*}{\partial t_i} (1 - \beta_i \times \tau) > 0$$

$$\frac{\partial^2 g_i}{\partial \beta_i^2} = -\frac{\partial^2 \alpha^*}{\partial \beta_i^2} (1 - \beta_i \times \tau) \times t_i + \frac{\partial \alpha^*}{\partial \beta_i} \times \tau \times t_i > 0$$

$$\left[ \frac{\partial^2 \theta_i^*}{\partial t_i^2} + \frac{\partial^2 g_i}{\partial t_i^2} \right] = \frac{\partial \alpha^*}{\partial t_i} \times (1 - \beta_i \times \tau) < 0$$

$$\left[ \frac{\partial^2 \theta_i^*}{\partial \beta_i^2} + \frac{\partial^2 g_i}{\partial \beta_i^2} \right] = -\frac{\partial \alpha^*}{\partial \beta_i} \times \tau \times t_i < 0$$

$$\frac{\partial^2 n_1}{\partial \beta_1^2} = -\frac{B \times \left[ \frac{\partial^2 \theta_2}{\partial \beta_1^2} + \frac{\partial^2 g_2}{\partial \beta_1^2} \right]}{2a} > 0$$

$$\frac{\partial^2 n_1}{\partial t_1^2} = -\frac{B \times \left[ \frac{\partial^2 \theta_2}{\partial t_1^2} + \frac{\partial^2 g_2}{\partial t_1^2} \right]}{2a} > 0$$

$$\frac{\partial^2 n_1}{\partial \beta_1 \partial t_1} = -\frac{B}{2a} \times \left[ \frac{\partial^2 \theta_i^*}{\partial \beta_i \partial t_i} + \frac{\partial^2 g_i}{\partial \beta_i \partial t_i} \right] = -\frac{B}{2a} \times \left( (1 - \alpha^*) \times \tau - \frac{\partial \alpha^*}{\partial t_i} \times \tau \times t_i \right) < 0$$

$$\frac{\partial^2 r_1}{\partial \beta_1 \partial t_1} = \left[ B \times \frac{\partial^2 \theta_1}{\partial \beta_1 \partial t_1} \right] > 0$$

$$\frac{\partial^2 n_1}{\partial t_1 \partial \beta_1} = -\frac{B}{2a} \times \left[ \frac{\partial^2 \theta_i^*}{\partial t_i \partial \beta_i} + \frac{\partial^2 g_i}{\partial t_i \partial \beta_i} \right] = -\frac{B}{2a} \times \left( \frac{\partial \alpha^*}{\partial \beta_i} \times (1 - \beta_i \times \tau) + (1 - \alpha^*) \times \tau \right) < 0$$

$$\frac{\partial^2 r_1}{\partial t_1 \partial \beta_1} = \left[ B \times \frac{\partial^2 \theta_1}{\partial t_1 \partial \beta_1} \right] > 0$$

$$\frac{\partial r_1}{\partial \beta_2} = \frac{\partial^2 r_1}{\partial \beta_1 \partial \beta_2} = \frac{\partial^2 n_1}{\partial \beta_1 \partial \beta_2} = \frac{\partial^2 n_1}{\partial \beta_2 \partial \beta_1} = \frac{\partial r_1}{\partial t_2} = \frac{\partial^2 r_1}{\partial t_1 \partial t_2} = \frac{\partial^2 n_1}{\partial t_1 \partial t_2} = \frac{\partial^2 n_1}{\partial t_2 \partial t_1} = \frac{\partial^2 n_1}{\partial t_1 \partial \beta_2} = 0$$