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Regional Unemployment in Germany: a spatial panel data analysis

Franziska Lottmann*

Abstract

This empirical paper analyzes determinants for regional differences in German unemployment rates using a spatial panel model. We provide evidence for the presence of spatial dependence in regional unemployment data. Our study covers the whole of Germany as well as East and West Germany separately. We exploit a regional data set of 24 possible explanatory variables on 412 districts for the period from 1999 until 2007. Our results suggest that the spatial dynamic panel model most appropriate for modeling regional unemployment. Furthermore, we find that German regional unemployment is of disequilibrium nature which justifies political interventions.

Keywords: Regional Unemployment, spatial dependence, spatial panel models, Germany

JEL classification: C23, R12, R23

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1 Introduction

The unemployment rate is a widely used and often discussed indicator for the economic wellbeing of a country. However, this discussion is mostly concentrated on national unemployment rates which give no information about the regional structure of unemployment. Though, data on regional unemployment rates show substantial differences between regions. According to Taylor and Bradley (1997), regional differences within a country are stronger than differences between countries. Regional differences are of particular interest in Germany as this country was divided into two separate countries with different economic systems until 1990. The division of Germany causes structural differences between both parts which partly hold until today.

This empirical paper analyzes determinants for regional differences in German unemployment rates. We identify the driving factors in the whole of Germany as well as in East and West Germany separately. Twenty years after German reunification, this study is, to our best knowledge, the first contribution investigating regional unemployment in Germany. Taking into account theoretical contributions to this literature (as e.g. Partridge and Rickman (1997)), we exploit a data set with 25 possible explanatory variables on 412 German districts (*Landkreise* and *kreisfreie Städte*) for the period from 1999 until 2007. To exploit the panel structure of the data, we apply a panel data model.

Labor market activity is not limited to a certain district. On the one hand, firms do not restrict their recruiting activities to their resident location and, on the other, job searchers might accept a job in a different area. Test results on spatial autocorrelation confirm this aspect. The spatial econometric literature shows that ignoring spatial effects yields biased and inefficient estimates. Therefore, we account for spatial dependencies in our model of regional unemployment to avoid these shortcomings. Applying a spatial panel model to the context of regional unemployment is novel to this literature. In addition to spatial dependence, labor market data exhibit temporal dynamics. Therefore, we use both a static and a dynamic model.

Regional unemployment differentials have been subject of intensive research in the literature. From a methodological point of view, the empirical literature can be divided into two strands of literature. On the one hand, models for regional unemployment are estimated using (non-spatial) panel data techniques. Examples are Partridge and Rickman (1997) who use data on state unemployment for the United States and Taylor and Bradley (1997) who provide a comparative study for regional unemployment disparities in Germany, Italy and the United Kingdom. Note that their German data covers only the Western part for the period from 1984 until 1994. They use data on the level of German *Länder* which correspond to the NUTS¹ I level. On the other hand, contributions to this literature apply spatial econometric models in a crosssectional setting. The first contribution in this direction is by Molho (1995) in which he provides evidence for the presence of significant spillovers in the adjustment to local shocks using data on 280 Local Labor Market Areas in Great Britain. Further examples for this strand of literature are Aragon et al. (2003) who analyze district-level data for the Midi-Pyrénées region of France and Cracolici et al. (2007) who explore the geographical distribution of unemployment in Italy.

¹NUTS stands for "Nomenclature of Territorial Units for Statistics" and it is a hierarchical classification of regional units for statistical purposes.

Finally, Elhorst (2003) provides a survey on theoretical models and explanatory variables for regional unemployment differences.

We contribute to the existing literature by the following two aspects: Firstly, we apply both a static and a dynamic spatial panel model. Hence, we exploit the panel dimension of the data and, in addition to that, we account for both spatial and temporal dependence in the data. Our results show that the spatial dynamic panel model fits our data in the best way. Secondly, we provide evidence that regional unemployment in Germany is of disequilibrium nature. This finding justifies political interventions on regional labor markets.

The structure of this paper is as follows: The second section briefly reviews theoretic explanations for regional unemployment differentials in the literature while the third presents the data set and explains how the spatial weights matrix is defined. The econometric model is introduced in the fourth section which covers model selection, specification testing and spatial econometric modeling. The fifth section is dedicated to the estimation results for the whole of Germany as well as for East and West Germany individually. Finally, the last section concludes.

2 Theoretic explanations for regional unemployment differentials

The empirical literature on regional unemployment is interested in both the persistence of regional unemployment differentials and the question what factors influence regional unemployment. Regarding the persistence issue, there exist various contributions for different countries providing evidence that both regions with high and low unemployment stay in its position over time (see Decressin and Fatás (1995), Jimeno and Bentolila (1998) and Martin (1997). Using time series methods, Bayer and Jüßen (2007) analyze whether changes in regional unemployment differences in West Germany are persistent over time. They provide evidence for both convergence and quick adjustment to an equilibrium distribution of regional unemployment rates.

From economic theory, we would expect that unemployed living in a region with high unemployment would move to an area with lower unemployment. A similar reasoning holds for firms which are expected to move from low-unemployment to high-unemployment regions because they can benefit from a larger pool of workers. However, due to migration and adjustment costs as well as rigidities the different regional unemployment levels do not equalize.

2.1 Why do regional unemployment rates differ?

The literature provides various explanations for the existence of regional unemployment differentials which can be summarized into two different views. The equilibrium view assumes the existence of a stable equilibrium in which regions have different unemployment rates. According to Molho (1995), this equilibrium is characterized by "uniform utility across areas for homogeneous labor group" (p. 642). In this setting, there is no incentive for further migration. Hence, households (and firms) need to be compensated for high (low) unemployment by other positive factors, so-called amenities. Such amenities can be, e.g., better climate, reasonable housing prices or better quality of life. Hence, the equilibrium rate of unemployment in region i is a function of the endowment of amenities in this region (Marston (1985)). The equilibrium view has received theoretical and empirical support from (among others) Marston (1985) drawing on ideas from Hall (1970).

Contrary to the equilibrium view, the disequilibrium view assumes that regional unemployment will equalize in the long run. However, the adjustment process might be slow. The speed of the adjustment process depends on different factors that are connected to both labor supply and labor demand. Such factors are, for example, the age structure and the educational attainment of the population. Young people are more likely to migrate as they have lower opportunity costs and as they are less risk averse (Aragon et al. (2003)). People holding a degree of higher education are also more likely to move because the labor market for high-skilled labor is larger and these people are expected to be better informed (Aragon et al. (2003)). The structure of the labor force also influences the relocation behavior of firms. Moreover, factors like population density and migration costs affect the adjustment process to the long-run equilibrium, too. Unemployment is expected to be lower in urban areas because the matching process between unemployed and vacant jobs is more efficient (Aragon et al. (2003)). The migration behavior of people is clearly affected by costs of migration. For example, housing prices and the structure of the housing market influence how easy it is for a household to change its location.

These explanations for regional unemployment differences imply different conclusions for policy makers. According to Marston (1985), government efforts to reduce regional unemployment differentials are "useless" since they cannot reduce unemployment anywhere for long when the level of regional unemployment can be considered as equilibrium situation. By contrast, the disequilibrium view delivers an "implicit justification for programs that target government funds to depressed areas" (Marston (1985), p. 58). In light of these different conclusions for policy, it is important to assess whether regional unemployment rates can be considered as equilibrium phenomenon or not.

However, both explanatory approaches for regional unemployment are not necessarily mutually exclusive. Marston (1985) states that "it may be that an equilibrium relationship exists, but that equilibrating forces are so weak that individual areas spend a long period of time away from their equilibrium" (p. 59). For the German case, there are arguments for both theoretic approaches to explain the regional labor market situation. On the one hand side, about twenty years after German reunification, there are still specific adjustment processes at work. But, on the other, we do not think that the long-run equilibrium is characterized by equalized regional unemployment rates.

In their contribution, Partridge and Rickman (1997) combine to some extent both views and extend the set of factors that might influence regional unemployment. In contrast to the equilibrium view, they do not assume that household utility in terms of income and amenities will equalize across areas in equilibrium. They add monetary and psychological costs of household relocation to the household utility function. These costs can be sufficiently high such that moving of households is limited. As regional unemployment in Germany has both equilibrium and disequilibrium aspects, we base our empirical analysis on Partridge and Rickman (1997).

2.2 Set of possible determining factors

Following Partridge and Rickman (1997), we assume that unemployment in region i in year t depends on disequilibrium variables and an equilibrium component which is a function of market equilibrium effects, demographic characteristics as well as producer and consumer amenities. For the choice of the actual variables in these categories we take into account the empirical regional unemployment literature. However, as it is often the case for empirical studies, the set of our variables is limited by data availability.

As to disequilibrium effects, we use the employment growth rate (EG_{it}) which turned out to be an important determinant for regional unemployment in the literature. This is not surprising because the change in employment directly affects unemployment.² Another variable capturing disequilibrium effects are wages or unit labor costs. But this data is not available on the desired regional level for our analysis.

Regarding market equilibrium effects, we use employment shares of different sectors ($\% IND_{it}$, $\% AGR_{it}$, ...) to account for the sectoral structure of the regions. According to Martin (1997), industrial composition effects are a "primary reason" (p. 244) for labor demand and regional unemployment to differ across regions.

To account for demographic characteristics, we use the share of young ($YOUNG_{it}$) and older (OLD_{it}) persons in the labor force. In contrast to studies on other countries, as for example the United States, German labor market data does not contain any information on ethnicity in general. However, we have data on the share of foreigners in the labor force ($FOREIGN_{it}$). Another important demographic variable is labor force participation, especially female labor force participation (FP_{it}). Due to German history, labor force participation of women differs substantially between East and West Germany. Unfortunately, the data on female labor force participation is only available for *Regierungsbezirke* which partly correspond to the NUTS II regions of Germany. To include information on human capital, we utilize data providing three levels of educational attainment which are a university degree ($H2_{it}$), a vocational qualification ($H1_{it}$) and no professional qualification ($H0_{it}$). Furthermore, to control for a region's linkages with other regions, we use the balance of incoming and outgoing commuters of district *i* ($COMM_{it}$). A positive commuting balance in region *i* indicates that labor supply in region *i* increases by incoming commuters. Moreover, a positive commuting balance gives an indication for positive demand for labor in region *i*.

On the one hand, the impact of amenities is captured by population density ($DENS_{it}$). It is a proxy for consumer and producer amenities because urban areas provide more amenities than rural areas. Unemployed persons have more employment opportunities and the matching process is expected to be more efficient in urban areas. However, urban areas are also associated with pollution and congestion. On the other hand, we consider three amenity variables which, to our best knowledge, have not been considered in the regional unemployment literature so far. We use the public debt ratio ($DEBTR_{it}$) of a district because high public debts in relation to GDP are an indication for a deficient ability of a region to finance public goods and sub-

²It would be interesting to analyze the impact of (temporally) lagged values of employment growth on regional unemployment. However, to our best knowledge, employment data on periods prior to 1999 is not available on the level of districts.

sidies. Additionally, strongly indebted communities are not attractive for firms to create new businesses. In addition to data on public debts, we utilize data on the number of business registrations (REG_{it}). This variable is a proxy for producer amenities. A higher number of new businesses will result in a higher demand for labor. Finally, we use the number of overnight stays ($STAY_{it}$) to capture a region's attractiveness for tourists. Additionally, a high number of overnight stays may be related to high business activities. A detailed description of all variables can be found in the appendix.³

3 Data and spatial weights matrix

3.1 Regional unemployment and its determining factors

The data on regional unemployment rates used in this analysis are provided online by the Federal Employment Office (*Bundesagentur für Arbeit*). As it is official data, the underlying definition of unemployment corresponds to regulations in German Social Security Code. Moreover, we utilize a huge regional data set of possible explanatory variables. All these variables are taken from the regional database of the Federal Statistical Office (*Statistisches Bundesamt*) of Germany. Since there were some values missing in this database, we requested them directly from the corresponding regional statistical institutions. A detailed description of the data and sources can be found in Table 6 in the appendix. Our data set covers the period from 1999 until 2007.⁴ The end of our sample period is determined by a change in the sectoral classification in 2008, i.e. data on employment in different sectors is not comparable before and after this change of classification. The data is available for all 412 German districts (*Landkreise* and *kreisfreie Städte*). The districts correspond to German NUTS III regions.⁵ During our sample period, there are two major reforms of district allocation. We allocate the data for the whole period in such a way that it corresponds to the situation after these reforms. Details on the district reforms can be found in the appendix.

To visualize regional differences in unemployment rates of German districts, Figure 1 presents a map of Germany which is colored according to the extent of regional unemployment in 2009.⁶ Additionally, Table 3 shows summary statistics of regional unemployment rates over time. Based on these exploratory tools, we can summarize the following major facts. First, there is substantial variation in regional unemployment rates in Germany. In 2004, the district with the lowest unemployment rate exhibited a rate of 4.4 per cent (Eichstätt district) while the highest regional unemployment rate amounted to 31.4 per cent (Uecker-Randow district). Second, the German labor market is characterized by strong differences between East and West Germany

³In contrast to other studies (as Cracolici et al. (2007) or Molho (1995)), we do not consider housing prices in our analysis for two reasons. Firstly, the majority of Germans lives in rented apartments and, secondly, until now, there is no comprehensive data base for rental prices for German districts. In 2006, 58% of the German population lived in rented apartments (see Timm (2008)).

⁴In 2005, a labor market reform ("Hartz reform") became effective which changed the definition of unemployment. Therefore, the number of unemployed increased by definition in this year.

⁵Baddeley et al. (1998) state that NUTS III regions "most closely approximate meaningful labor markets" (p. 204). However, Eckey et al. (2007b) explain that travel-to-work areas are the relevant regional level for analyses of regional production and unemployment. They discuss different allocations of districts into such areas.

⁶The map of Germany shows that some of the NUTS III regions lie within others, i.e. these districts have only one physical neighbor.

Table 1: Summary statistics of geographic distances between centroids of German districts

Min	1st Qu.	Median	Mean	3rd Qu.	Max	Std. dev.
1.18	191.7	298	310.6	417.1	845.6	155.52

which still can be considered as consequences of German division. Regional unemployment rates are higher in East Germany. However, in a ranking of German districts with respect to regional unemployment, there are some East German districts that are placed ahead of West German districts. Third, besides the East-West differences, there is a slight North-south divide.

To test for stationarity of the data, we apply panel unit root tests. The results of the Im et al. (2003) (IPS) test and the Fisher-type (ADF) test which was proposed in Maddala and Wu (1999) and in Choi (2001) clearly reject the hypothesis of a unit root in regional unemployment rates at all reasonable significance levels. In addition to that, we apply the IPS test and the Fisher-type (ADF) test to our set of explanatory variables and find that all explanatory variables are stationary as well. However, Baltagi et al. (2007a) show that there can be considerable size distortions in panel unit root tests when the true model exhibits spatial error correlation. Hence, these test results can only serve as an indication regarding stationarity of the data.

3.2 Spatial autocorrelation on German labor markets

An important component of spatial econometric models is the spatial weights matrix. It is a nonstochastic matrix that specifies exogenously the spatial relations between observations. Hence, the spatial weights matrix determines the neighborhood of district *i*. Accordingly, the term ,neighboring' always refers to the neighborhood set defined by the corresponding spatial weights matrix. On the one hand, we use a binary spatial weights matrix with entries zero and one and, on the other, matrices with general weights.

The simplest version of a spatial weights matrix is the binary contiguity matrix. When two districts are neighbors in the sense that they share a common border, the corresponding entry in the spatial weights matrix is one and zero otherwise. The elements on the main diagonal are zero by definition. This matrix induces a simple spatial structure which might not reflect actual spatial linkages in an appropriate way. Therefore, we construct spatial weights matrices with general weights. On the one hand, we utilize data on geographic distance between districts and, on the other, we use a combination of geographic distance and size, as proposed in Molho (1995), to define spatial weights.

Geographic distance has frictional effects on labor market activity. Workers prefer to find a job in their closer environment because commuting and moving entail monetary and psychological costs. Therefore, we use geographic distances between centroids of districts to define the entries of the spatial weights matrix. Summary statistics of the geographic distances are provided in Table 1.

The weights of the distance-based matrix are defined by

$$w_{ij} = \begin{cases} exp(-\tau d_{ij}), & \text{for } i \neq j \\ 0 & \text{for } i = j, \end{cases}$$
(1)

where τ is a distance decay parameter and d_{ij} is the geographic distance between districts *i* and *j*. The resulting spatial weights matrix crucially depends on the choice of τ . To determine the distance decay parameter, we use a grid search with different values for τ and decide according to the Bayesian and Akaike's information criterion which parameter value is most suitable for our data. In the literature, Niebuhr (2003) also uses a distance decay function to define the weights for her analysis of regional unemployment in Europe.

However, the distance decay function neglects the district-level labor market size. The spatial influence differs when the extent of employment opportunities differs although distances between districts are the same. We expect that the spatial impact of a district with high employment on a low-employment district is stronger than vice versa. Therefore, we utilize the weighting scheme proposed by Molho (1995) which combines size with the distance decay effect. According to Molho (1995), the spatial weights are defined by

$$w_{ij} = \begin{cases} \frac{E_j \exp(-\eta d_{ij})}{\sum_{k \neq i} E_k \exp(-\eta d_{ik})}, & \text{for } i \neq j \\ 0 & \text{for } i = j, \end{cases}$$
(2)

where *E* denotes the employment level and η is the distance decay parameter. As Molho (1995) points out, this weighting scheme implies that the spillover effect of the labor market situation in region *j* on the setting in region *i* increases with size (measured in terms of employment) of region *j* and decreases with the distance between both districts. Again, the impact of distance on the strength of the spatial relation crucially depends on the distance decay parameter η . To decide which value for η is appropriate for our unemployment regression, we perform again a grid search for η and decide on the appropriate value according to information criteria.⁷

Labor market activity and hence labor market data is expected to be correlated over space. To justify this aspect, we perform the Moran I test for spatial autocorrelation using regional unemployment rates. As this test is not specified for a particular spatial process, we can apply it directly to our data. The null hypothesis of this test is the absence of spatial autocorrelation while the alternative is not exactly specified. The test statistic can be expressed by (see Moran (1950))

$$I = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}(u_{i} - \bar{u})(u_{j} - \bar{u})}{\sum_{i=1}^{n} (u_{i} - \bar{u})^{2}}$$
(3)

where u_i and u_j are the regional levels of unemployment in district *i* and *j*. \bar{u} is defined by $\bar{u} = \frac{1}{n} \sum_n u_i$ and w_{ij} is the element of the spatial weights matrix indicating the spatial impact of region *j* on region *i*. For the computation of the Moran *I* statistic we use the binary contiguity matrix.⁸

As the Moran *I* statistic is designed to detect spatial autocorrelation in cross-sectional data, we compute it for every year of our sample separately. The results of the Moran *I* test are presented in Table 2. They show that regional unemployment rates are positively spatially autocorrelated during the period from 1999 until 2009. Furthermore, they show a decreasing

⁷We also experimented with other measures of similarity between districts as, e.g., population density to control for the type of a region (urban vs. rural) or political orientation to capture social tendencies. However, the resulting spatial weights matrices do not have desired properties.

⁸We also used the other spatial weights matrix to compute the Moran I statistic and got qualitatively the same results.

	Moran I	Ζ	<i>p</i> -value
1999	0.874	26.48	0
2000	0.875	29.02	0
2001	0.890	29.51	0
2002	0.882	29.25	0
2003	0.863	28.61	0
2004	0.846	28.05	0
2005	0.799	26.5	0
2006	0.810	26.86	0
2007	0.793	26.29	0
2008	0.772	25.61	0
2009	0.744	24.66	0

Table 2: Results of the Moran *I* test for spatial autocorrelation (1999-2009)

Notes: *Z* denotes the standard deviate of the Moran *I* statistic, i.e. $Z = \frac{I - E[I]}{sd(I)}$. The null hypothesis is the absence of spatial autocorrelation whereas the alternative is positive spatial autocorrelation. The Moran *I* values are computed assuming normality.

	Min	1st Qu.	Median	Mean	3rd Qu.	Max	Std.dev.	national
1999	4	7.8	10	11.41	14.3	24.8	4.815	11.7
2000	3	6.7	8.8	10.46	13.3	25.6	5.158	10.7
2001	3	6.3	8.4	10.19	12.7	26.7	5.356	10.3
2002	3.9	6.9	9	10.69	12.9	27.6	5.279	10.8
2003	4.6	7.7	9.8	11.57	13.9	29.7	5.424	11.6
2004	4.4	7.7	9.8	11.66	14	31.4	5.467	11.7
2005	4.7	8.7	11.4	12.84	16.1	29.7	5.323	13
2006	3.7	7.7	10.5	11.81	15	27.6	5.084	12
2007	2.4	6.1	8.5	9.868	12.6	24.2	4.733	10.1
2008	1.9	4.8	7.2	8.435	11	21.5	4.306	8.7
2009	2.5	5.7	7.9	8.843	11.4	20.1	3.908	9.1
			6	E 1 1E	1.000			

Table 3: Summary statistics of regional unemployment rates (2000-2009)

Source: Federal Employment Office

trend in the values of the Moran I statistic, i.e. the extent of spatial autocorrelation in regional unemployment rates decreases during 1999 and 2009.

4 Econometric Model

4.1 Model selection

In order to find an appropriate model for our data, we divide our set of explanatory variables into three groups. The first group contains variables which are essential for our model. We include in this group the employment share in manufacturing and in the construction industry (%*IND_{it}* and %*CON_{it}*), the age-related demographic variables as well as one of the human capital variables and employment growth (EG_{it}) to account for disequilibrium effects.⁹ The second group contains variables that are expected to be important for the explanation of regional unemployment rates. We assign to this group three of our four amenity variables (DENS_{it},

⁹Note that we have not assigned female labor force participation to this group as its regional variation is small because of limited data availability.



Figure 1: Regional unemployment in Germany in 2009

 $DEBTR_{it}$ and REG_{it}). Furthermore, the employment shares of agriculture, electricity, gas and water supply, financial business, transport, storage and communication, real estate and public administration are considered for this group. Additionally, female labor force participation as well as the remaining educational variables are part of this group. The last group consists of variables that are expected to have a weaker influence on regional unemployment. These variables are the share of foreign employed persons (*FOREIGN*_{it}), the number of overnight stays (*STAY*_{it}) and the employment shares of mining and quarrying, wholesale and retail trade, hotels and restaurants as well as education, health and social work. The exact division of variables can be found in Table 4.

To select our model, we regress regional unemployment rates on different combinations of variables where the variables of the first group are always contained. These regressions are based on the standard two-way fixed effects panel model as our data constitutes an exhaustive sample of all German districts. This model controls neither for spatial autocorrelation nor for temporal dynamics in the data. Therefore, we refer to this model as basic model. We compute Akaike's (AIC) and the Bayesian information criterion (BIC) to assess the goodness-of-fit of the regressions. Our model selection procedure selects a model containing thirteen variables. The summary statistics of these variables are in Table 12 in the appendix. To check for possible multicollinearity in our model, we analyze both the correlation matrix of the regressors and variance inflation factors where both give no indication for multicollinearity. Hence, our final best model is

$$\begin{aligned} U_{it} &= \beta_1 E G_{it} + \beta_2 \% IND_{it} + \beta_3 \% ENERW_{it} + \beta_4 \% CON_{it} + \beta_5 \% HOT_{it} \\ &+ \beta_6 \% FIN_{it} + \beta_7 \% PUB_{it} + \beta_8 YOUNG_{it} + \beta_9 OLD_{it} + \beta_{10} H0_{it} \\ &+ \beta_{10} H1_{it} + \beta_{12} REG_{it} + \beta_{13} DEBTR_{it} + \mu_i + \alpha_t + \epsilon_{it}, \\ &i = 1, \dots, n, \quad t = 1, \dots, T, \quad (4) \end{aligned}$$

where the variables are defined as before. μ_i denote the district-specific effects and α_t represent time effects. The time effects capture national effects as, e.g., business cycle effects that affect all regions in the same way and are strongly correlated with the national unemployment rate (correlation: 0.95). The indices of the variables denote district *i* and year *t*.

Our final model contains all variables of group one. The model selection procedure selects the share of employed persons holding a vocational qualification as additional demographic variable. Hence, we account for two of three educational variables. As all educational variables together cover all employed persons, the results for *H*0 and *H*1 are referred to as deviations from the base category *H*2. Only the public debt ratio and the number of business registrations out of our amenity variables are contained in our model. Hence, our model selection results reveal a first indication that regional unemployment is a disequilibrium phenomenon. Furthermore, the age-related demographic variables and the educational variables are contained in our final model. Regarding the market equilibrium effects, employment shares in power generation, hotels and restaurants, financial business and public administration are selected to be part of our model in addition to the sectoral variables of group one. The significance of the

employment share in hotels and restaurants can be explained by the fact that a significant part of the work in this industry is done by persons holding no specific training qualification for this field. Hence, it might be easier for unemployed persons to get a job in this industry.

Our basic model neglects any spatial dependence in the data. Results of Pesaran's *CD* test (Pesaran (2004))¹⁰ confirm this aspect. If spatial dependence in the data is ignored, standard OLS regression will provide biased parameter estimates in case of spatial lag dependence and in case of spatially lagged exogenous variables. In case of spatial error dependence, OLS estimation produces unbiased but inefficient estimates. Neglecting the spatial lag term is similar to an omitted variable bias (Franzese and Hays (2007)). As the spatial lag term is correlated with the error term, OLS estimation of the associated coefficient will be inconsistent (Franzese and Hays (2007), Anselin (1988)). To obtain unbiased and efficient estimates, we apply a spatial econometric model to our data.

¹⁰Details on this test can be found in the appendix.

nodel selection	group 3	 - share of foreign employed persons (<i>FUKEIGNit</i>) - number of overnight stays (<i>STAY_{it}</i>) share of persons working - in mining and quarrying (<i>MINE_{it}</i>), - in hotels and restaurants (<i>HOT_{it}</i>), - in wholesale and retail trade (<i>TRADE_{it}</i>), - in education, health and social work (<i>EDUHEALTH_{it}</i>)
Table 4: Division of explanatory variables for m	group 2	 - temale labor force participation (<i>FY</i>_{it}) share of employed persons - with vocational training (<i>H</i>1_{it}) - and with a university degree (<i>H</i>2_{it}) - population density (<i>DENS</i>_{it}) - public debt ratio (<i>DEBTR</i>_{it}) - business registrations (<i>REG</i>_{it}) share of persons working - in agriculture, hunting and forestry (<i>AGR</i>_{it}), - in electricity, gas and water supply (<i>ENERW</i>_{it}), - in financial business (<i>FIN</i>_{it}), - in financial business (<i>FIN</i>_{it}), - in real estate, renting and business activities (<i>REAL</i>_{it}), - in public administration and defence; compulsory social security (<i>PUB</i>_{it})
	group 1	 employment growth (<i>EG_{it}</i>) share of persons working in manufacturing (<i>IND_{it}</i>) and in construction industry (<i>CON_{it}</i>) share of young (<i>YOUNG_{it}</i>) and old persons (<i>OLD_{it}</i>) employed persons without any vocational training (<i>HO_{it}</i>)

Table 4: Division of explanatory variables for model selection

4.2 Spatial econometric modeling

To control for spatial autocorrelation in the data, we specify a spatial econometric model for our analysis of regional unemployment. As we have panel data, we need a spatial panel model. The spatial econometric literature provides different models for data with spatial autocorrelation: the model with spatially lagged exogenous variables, the spatial error model, the spatial lag model and combinations of them. Firstly, we consider the model with spatially lagged exogenous regressors as it is the simplest spatial panel model. Secondly, we perform a specification test to choose from the other models.

In stacked notation, the model with spatially lagged exogenous variables can be written as

$$U_t = X_t \beta_1 + W X_t \beta_2 + \mu + \alpha_t + \epsilon_t \mathbb{1}_n; \quad t = 1, \dots, T,$$
(5)

where $U_t = (U_{1,t}, U_{2,t}, ..., U_{n,t})'$ is a $(n \times 1)$ vector containing regional unemployment rates. X_t contains all regressors from selected our model (equation 4) and is of dimension $(n \times k)$, W is the $(n \times n)$ spatial weights matrix, μ is the $(n \times 1)$ vector of individual effects and $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, ..., \epsilon_{nt})'$ where ϵ_{it} are i.i.d. across *i* and *t* with zero mean and constant variance σ^2 . $\mathbb{1}_n$ represents a *n*-dimensional vector of ones. The coefficient β_2 measures the external effect of the explanatory variables on regional unemployment in neighboring districts. For the estimation of this model, there is no specific estimation methodology needed as the additional regressors are exogenous and the error term fulfills Gauss-Markov assumptions. We performed this regression with our data but the coefficients of all spatially lagged regressors are not significant and the results are, according to information criteria, slightly worse than those of the basic model.¹¹ However, the results of the *CD* test by Pesaran (2004) give a slight indication for cross-sectional independence in the residuals with a *p*-value of 0.052.¹²

4.2.1 Testing for the spatial model specification

As the model with spatially lagged exogenous variables is not appropriate for our data, we need to specify a more complex spatial panel model. We perform the specification test by Debarsy and Ertur (2010) to differentiate between the different spatial processes. To our best knowledge, the test by Debarsy and Ertur (2010) is the only specification test at the moment that allows to discriminate between the spatial lag model, the spatial error model and the combined model. Baltagi et al. (2003) extend the langrange multiplier (LM) test by Breusch and Pagan (1980) to the spatial error component model to test simultaneously for the existence of spatial error correlation as well as for random region effects. Additionally, they derive conditional tests for spatial error correlation and random region effects. Baltagi et al. (2007b) generalize the underlying model to a spatial panel model that controls for serial correlation over time for each spatial unit. We use this test to motivate our spatial dynamic model. Finally, Baltagi and Liu (2008) derive a test for autoregressive spatial lag dependence instead of spatial error terms.

The starting point of the test by Debarsy and Ertur (2010) is the general model with both a spatial lag term and spatially autocorrelated errors including fixed effects. It is called spatial

¹¹The results can be obtained from the author upon request.

¹²Further information on the test by Pesaran (2004) are in the appendix.

	H_0^a	H_0^b	H_0^c	H_0^d	H_0^e
LM	1353.8	1285.7	967.19	7.86	3771.1
<i>p</i> -value	0	0	0	0.0051	0

Table 5: Test results of the specification test by Debarsy and Ertur (2010) using the binary contiguity matrix

autoregressive model with spatially autocorrelated disturbances of order (1, 1) (SARAR (1, 1) model) and can be described by

$$U_t = \lambda W U_t + X_t \beta + \mu + V_t, \quad V_t = \rho W V_t + \Xi_t, \quad t = 1, \dots, T,$$
(6)

where X_t is the $(n \times k)$ matrix containing all explanatory variables and β is the $(k \times 1)$ coefficient vector. $\Xi_t = (\xi_{1,t}, \ldots, \xi_{n,t})$ is the $(n \times 1)$ vector of innovations where $\xi_{i,t}$ are i.i.d. across *i* and *t* and $\xi_{i,t} \sim (0, \sigma^2)$.¹³ Finally, λ is the spatial autoregressive coefficient and ρ is the spatial autocorrelation coefficient.

Debarsy and Ertur (2010) consider five different hypotheses in their paper:

- H_0^a : $\rho = \lambda = 0$. This joint hypothesis tests whether there is spatial dependence in the data at all. If it cannot be rejected, there is no need for a spatial econometric model.
- H_0^b : $\lambda = 0$. Under the alternative, the specification is the spatial lag model. However, it is possible that spatial errors may exist.
- H_0^c : $\rho = 0$. Under the alternative, the model contains spatially autocorrelated errors. However, it is possible that a spatial lag term may exist.
- *H*^d₀ : *ρ* = 0, with λ possibly different from 0. Under the alternative, the general specification (equation 6) has to be estimated.
- *H*^e₀ : λ = 0, with ρ possibly different from 0. Under the alternative, the general specification (equation 6) has to be estimated.

The test statistics for the hypotheses H_0^a until H_0^e are in the appendix. Table 5 shows the results of the Debarsy/Ertur (2010) test using the binary contiguity matrix.¹⁴ According to the results, we can reject all five hypotheses even on the 1% significance level. Hence, the SARAR(1,1) model is the most appropriate for our data.

4.2.2 Static model specification

In accordance with the results of the test by Debarsy and Ertur (2010), we include a spatial lag term and spatially autocorrelated errors in our model. Additionally, we incorporate time effects in our static spatial panel model in order to have a two-way specification as in our basic model. The static model specification is

¹³Debarsy and Ertur (2010) specify the model in their original contribution using different spatial weights matrices for the spatial lag and spatial error part. But they note that the test also works when these are equal.

¹⁴We also performed this test using the other spatial weights matrices and obtained qualitatively the same results.

$$U_{t} = \lambda W U_{t} + \beta_{1} E G_{t} + \beta_{2} \% I N D_{t} + \beta_{3} \% E N E R W_{t} + \beta_{4} \% C O N_{t} + \beta_{5} \% H O T_{t} + \beta_{6} \% F I N_{t} + \beta_{7} \% P U B_{t} + \beta_{8} Y O U N G_{t} + \beta_{9} O L D_{t} + \beta_{10} H 0_{t} + \beta_{11} H 1_{t} + \beta_{12} R E G_{t} + \beta_{13} D E B T R_{t} + \mu + \alpha_{t} 1_{n} + V_{t}, V_{t} = \rho W V_{t} + \Xi_{t} \quad t = 1, \dots, T \quad (7)$$

where the variables are defined as before. The elements of the $(n \times 1)$ disturbance vector $\Xi_t = (\xi_{1,t}, \ldots, \xi_{n,t})'$ are assumed to be i.i.d. across *i* and *t* with zero mean and constant variance σ^2 .

Lee and Yu (2010b) show that for the (static) model with fixed individual and time effects the direct quasi-maximum likelihood estimation method yields inconsistent estimates for the common parameters unless n is large. In addition to that, they show that even in the case when both n and T are large, the distribution of the estimates of common parameters is not properly centered.

Moreover, Lee and Yu (2010b) show that the use of the typical within transformation to eliminate fixed effects causes the errors in the within-transformed model to be linearly dependent. Therefore, they apply an orthogonal transformation to eliminate the individual effects which produces an error variance with desirable properties. The standard within transformation uses the deviation from time mean operator, i.e. $J_T = I_T - \frac{1}{T} \mathbb{1}_T \mathbb{1}_T'$. Lee and Yu (2010b) define the orthonormal eigenvector matrix of J_T , i.e. $[F_{T,T-1}, \frac{1}{\sqrt{T}} \mathbb{1}_T]$. $F_{T,T-1}$ is the $(T \times (T - 1))$ submatrix corresponding to the eigenvalues of one. They suggest to transform the original data by $F_{T,T-1}$, i.e.

$$[Y_{n1}^*, \dots, Y_{n,T-1}^*] = [Y_{n1}, \dots, Y_{nT}]F_{T,T-1}.$$
(8)

Note that the dimension of the transformed model is n(T-1). To remove the time effects from the model, they propose a similar transformation which is based on the orthogonal transformation using $J_n = I_n = \frac{1}{n} \mathbb{1}_n \mathbb{1}'_n$. Correspondingly, the model has dimension (n-1)(T-1) after both transformations. Lee and Yu apply this transformation approach in various contributions (Lee and Yu (2010a), Lee and Yu (2010b), Lee and Yu (2010c)). We apply this approach for both our static and our dynamic model. Finally, the transformed model can be estimated by quasi-maximum likelihood.¹⁵

4.2.3 Dynamic model specification

Labor market data is not only correlated over space but also over time. To motivate the dynamic approach, we use the test by Baltagi et al. (2007b) because it allows for serial correlation in the error terms (in addition to spatial autocorrelation). Details on hypotheses and test statistics are in the appendix. The test results clearly show the following three aspects of our data. Firstly, there is serial dependence in our data. Hence, a dynamic model specification is reasonable in our context. Secondly, the test results give an indication for the presence of spatially autocorrelated errors. This is in line with the results of the Moran *I* test that also show significant spatial autocorrelation in both regional unemployment rates and explanatory variables.

¹⁵For more details on the estimation methodology, see Lee and Yu (2010b).

Thirdly, the test results support our assumption of a fixed effects model because we cannot reject the hypothesis that the standard deviation of the fixed effects is equal to zero.

The literature on spatial dynamic panel models provides various model specifications. Elhorst (2012) provides a survey of the literature on specification and estimation of spatial dynamic panel data models. For our analysis of regional unemployment, we include a spatial lag term, a temporally lagged term as well as a combined spatially and temporally lagged term in our dynamic model. The resulting model can be described by

$$U_{t} = \lambda W U_{t} + \gamma U_{t-1} + \delta W U_{t-1} + \beta_{1} E G_{t} + \beta_{2} \% I N D_{t} + \beta_{3} \% E N E R W_{t} + \beta_{4} \% C O N_{t} + \beta_{5} \% H O T_{t} + \beta_{6} \% F I N_{t} + \beta_{7} \% P U B_{t} + \beta_{8} Y O U N G_{t} + \beta_{9} O L D_{t} + \beta_{10} H 0_{t} + \beta_{11} H 1_{t} + \beta_{12} R E G_{t} + \beta_{13} D E B T R_{t} + \mu + \alpha_{t} \mathbb{1}_{n} + \Xi_{t}, \quad t = 1, \dots, T \quad (9)$$

where γ captures the pure time-dynamic effects and δ captures the combined spatial-temporal effect. The assumptions about the error term Ξ_t are as before.

Yu et al. (2008) propose a bias corrected quasi-maximum likelihood estimator for the spatial dynamic panel data model including a spatial lag, a temporal lag and a combined spatial-temporal term. However, they only allow for individual-specific fixed effects but not for fixed time effects. Lee and Yu (2010a) provide an estimator for this model extended to include time period fixed effects. Lee and Yu (2010a) show that direct quasi-maximum likelihood estimation of all parameters in the model with time effects yields an additional bias of order $O(n^{-1})$. They apply their transformation approach and show that it can avoid the additional bias with the same asymptotic efficiency as the direct quasi-maximum likelihood estimates when *n* is not relatively smaller than *T*. Furthermore, Lee and Yu (2010a) show that the direct estimates have a degenerate limit distribution while the transformed estimates are properly centered and asymptotically normal. Therefore, we apply the estimation methodology of Lee and Yu (2010a) to our dynamic model.

5 Estimation results

Firstly, we estimate the basic model, i.e. the model without any terms controlling for spatial dependence. The basic model is specified according to a two-way fixed effects panel data model and it is estimated using the standard within-estimator (see Baltagi (2008)). Secondly, we estimate the static spatial panel specification and, thirdly, the spatial dynamic model, both using the binary contiguity matrix, the distance decay matrix as well as the Molho (1995) weights matrix. Hence, we perform seven regressions for the whole of Germany. The regression results for the basic and the static model are in Table 6 and the results for the dynamic model are in Table 7. In addition to that, we perform the same regressions for the Eastern and Western part of Germany individually. Elhorst (2012) discusses stationarity issues and proposes conditions for stationarity for spatial dynamic panel data models. These conditions as well as the conditions stated in Lee and Yu (2010c) are satisfied in the regression results for the whole of Germany. However, the regression results for East and West Germany in case of the distance decay matrix do not meet the stationarity conditions. Therefore, we only present the results using the other spatial weights matrices for the separate analyses.

5.1 Results for the whole of Germany

Economic interpretations

As expected, regional unemployment rates are influenced negatively by employment growth. Furthermore, the shares of employed persons working in manufacturing and in the construction industry also have a negative impact on regional unemployment. Hence, districts that are specialized in these industries exhibit lower unemployment than districts with a different sectoral structure. Our estimation results reveal no indication for a discrimination of older workers as the associated coefficient is also negative. Though, this coefficient should not be overinter-preted because it can simply be related to effects of demographic change, i.e. an aging labor force. By contrast, the impact of younger employees on regional unemployment is positive. But this implies not necessarily youth unemployment because the majority of persons aged 15 until 25 is still in the educational system. The share of persons without any professional qualification influences regional unemployment positively which is not surprising. Interestingly, this also holds for the share of persons with vocational training.

Our model contains only a few of the amenity variables. Additionally, the signs of the amenity variables are against expectation from theory. According to the equilibrium view, consumers are expected to stay in regions with high unemployment when there is a high level of amenities. Hence, high unemployment should be related negatively to public debts because heavily indebted districts are not able to finance public goods to improve life quality. If high public debts result from high investments in the past, consumers expect less expenditures in the future. However, our results show a significant positive coefficient for the public debt ratio. A similar reasoning holds for producer amenities. Firms are expected to move to districts with high unemployment, i.e. the level of producer amenities should be higher when regional unemployment is lower. But the coefficient of business registrations is positive in our empirical results. Even if the public debt ratio is interpreted as proxy for producer amenities, its coefficient has not the desired sign. Thus, our results reveal no indication for German regional unemployment to be of equilibrium nature. Nonetheless, some of the market equilibrium variables, i.e. employment shares, are significant in our model.

Spatial econometric interpretations

Ignoring spatial dependence in the data, results in biased and inefficient estimates. The estimated coefficients of the basic model are mostly upward-biased in absolute value in comparison with the results of the static model. In an earlier contribution (Lottmann (2012)) we get a similar result for the estimation of matching functions. The existence of this bias is theoretically shown in Franzese and Hays (2007). They argue that neglecting a spatial lag process results in an omitted-variable bias. In addition to that, the information criteria show that the spatial models are more appropriate for our data than the basic one.

The dynamic model fits our data better than the static model according to information criteria. Thus, in order to model regional unemployment, a dynamic modeling approach needs to be applied. To our best knowledge, most of the contributions to the regional unemployment literature apply only a static model. However, most of the explanatory variables are not significant in the dynamic model. Hence, the temporal lag is able to explain a lot of the variability in regional unemployment rates. Only employment growth, the employment shares of manufacturing, construction industry and power generation as well as the age-related demographic variables have a significant impact on regional unemployment. Interestingly, the sign of the coefficient for the share of people working in construction industry differs between the static and the dynamic model.

The spatial autoregressive (λ) and the spatial autocorrelation coefficient (ρ) measuring the spatial influence in our static spatial panel model are both significant while the influence of both coefficients is in most cases positive. Hence, district-level unemployment is influenced by unemployment in neighboring districts. The spatial autocorrelation coefficient indicates regional effects that affect a region consisting of more than one district. Examples in the context of regional unemployment are exogenous shocks as the closure of a production site. The spatial autoregressive coefficient of the dynamic model is also significant and positive. The same holds for the pure time-dynamic effect. This result underlines the fact that our data exhibit not only spatial but also temporal autocorrelation. Contrary to this, the combined spatial-time effect is negative and significant.

Furthermore, the results are fairly sensitive to the choice of the spatial weights matrix. In the spatial econometric literature, Bell and Bockstael (2000) also find that estimation results are more sensitive to the specification of the spatial weights matrix than to the estimation technique. According to information criteria, the binary spatial weights matrix captures the spatial structure of the data in the best way for the static model while the distance decay function is most appropriate in case of the dynamic model.

5.2 Differences between East and West Germany

Due to the specific history of Germany, it is worthwhile to analyze the differences between the Western and Eastern part of the country. To analyze this issue, we use a two-regime regression, i.e. we estimate the model for both parts separately. This procedure rests on the assumption that coefficients of the explanatory variables differ between East and West Germany. From an economic perspective, we find no reason why a particular coefficient, e.g. the coefficient of the employment share in manufacturing, should be similar in East and West Germany. We also tested for the coefficients to be different between East and West Germany. The test results show that most of the coefficients differ significantly between East and West Germany.

The Eastern part of Germany consists of 87 districts and the Western part of 325 districts. Note that the spatial model specifications of the separate regressions take only the spatial relations *within* the areas into account, but no spatial interactions *between* them. The results for East Germany are in Tables 8 and 9 while the results for Western Germany can be found in Tables 10 and 11.

Employment growth and the employment share in manufacturing are negatively related to regional unemployment in both parts of Germany. Beyond that, the estimation results differ between East and West Germany. Firstly, the employment share of the construction industry has only a significant (negative) impact on regional unemployment rates in Eastern Germany.

	basic	basic static		
		binary	Molho (1995)	distance
			$(\eta = 0.01)$	$(\tau = 0.02)$
EG _{it}	-0.066^{***}	-0.033^{***}	-0.05^{***}	-0.04^{***}
	(-7.12)	(-6.2)	(-6.15)	(-5.41)
%IND _{it}	-0.11^{***}	-0.071^{***}	-0.09^{***}	-0.08^{***}
	(-7.95)	(-7.35)	(-7.05)	(-7.11)
%ENERW _{it}	0.17^{***}	0.098^{**}	0.12^{*}	0.08
	(2.6)	(1.98)	(1.93)	(1.47)
%CON _{it}	-0.29^{***}	-0.133^{***}	-0.17^{***}	-0.12^{***}
	(-11.85)	(-10.73)	(-7.46)	(-5.58)
%HOT _{it}	0.16***	0.072^{*}	0.09^{*}	-0.01
	(2.96)	(1.95)	(1.96)	(-0.17)
%FIN _{it}	0.17***	0.046	0.14^{***}	0.102**
	(3.06)	(1.13)	(2.75)	(2.21)
$%PUB_{it}$	0.12***	0.053***	0.073***	0.056**
	(4.36)	(2.74)	(3.05)	(2.49)
<i>YOUNG_{it}</i>	0.35***	0.021	0.057	-0,008
	(9.75)	(0.96)	(1.64)	(-0.24)
OLD_{it}	-0.16^{***}	-0.13^{***}	-0.22^{***}	-0.2^{***}
	(-5.86)	(-7.28)	(-8.73)	(-8.03)
$H0_{it}$	0.103***	0.098***	0.089***	0.088^{***}
	(3.8)	(7.78)	(4.15)	(4.52)
$H1_{it}$	0.081^{***}	0.081^{***}	0.084^{***}	0.079***
	(4.14)	(7.56)	(4.74)	(4.72)
REG_{it}	0.17***	0.08***	0.11***	0.14^{***}
	(4.44)	(3.35)	(3.96)	(4.44)
$DEBTR_{it}$	0.054^{**}	0.015	0.026	0.02
	(2.2)	(0.87)	(1.16)	(0.97)
λ	—	0.83***	0.78^{***}	0.79***
		(71.59)	(14.56)	(16.41)
ho	—	-0.46^{***}	0.71***	0.67***
		(-13.68)	(9.06)	(8.77)
σ^2	0.61	0.34	0.5	0.44
log-like	-4123.08	-3274.95	-3525.43	-3361.05
AIC	2.23	1.78	1.82	1.82
BIC	2.25	1.80	1.85	1.85
obs.	3708	3708	3708	3708

Table 6: Estimation results for the basic and the static model

Notes: *t*-statistics are in parentheses. t-statistics for the static model are computed according to Anselin (1988). λ is the spatial autoregressive coefficient and ρ is the spatial autocorrelation coefficient. * * * , * * and * indicate coefficients that are significant at 1%, 5% and 10%, respectively.

	Molho (1995)	distance	
	billary	(n - 0.01)	$(\tau - 0.02)$
	_0.055***	-0.069^{***}	-0.063^{***}
$LO_{l\bar{l}}$	(-7.93)	(-9.23)	-0.005
%IND.	_0 0 2 1**	-0.015	(-0.98) -0.011
$/011ND_{it}$	-0.021	-0.013	(1.05)
% ENERW.	0 209***	(-1.01) 0 2/1***	0.23***
	(4.25)	(4.56)	(4.63)
%CON:	0.058***	0.047**	0.019
/oconv _{it}	(27)	(2.03)	(0.87)
%HOT.	-0.08*	-0.056	-0.02
/011011	(-1.85)	(-1.22)	(-0.36)
%FIN.	0.055	0.065	0.07
701 11 v _{lt}	(1.16)	(1.29)	(1.39)
%PHB.	0.0107	0.0078	0.0095
	(0.53)	(0.36)	(0.47)
YOUNG	0.046*	0.078***	-0.0001
	(1.74)	(2.7)	(-0.0039)
OLD;+	-0.08^{***}	-0.104^{***}	-0.104^{***}
11	(-3.8)	(-4.61)	(-4.9)
$H0_{it}$	0.006	-0.016	0.0091
**	(0.28)	(-0.71)	(0.43)
$H1_{it}$	-0.0149	-0.029^{*}	-0.014
	(-0.97)	(-1.73)	(-0.91)
REG_{it}	0.0073	0.013	0.013
	(0.28)	(0.48)	(0.51)
$DEBTR_{it}$	0.0088	0.0081	0.0043
	(0.48)	(0.41)	(0.23)
λ	0.5***	0.79***	0.88^{***}
	(26.55)	(32.87)	(42.41)
γ	0.78^{***}	0.8^{***}	0.78^{***}
	(49.04)	(55.29)	(52.11)
δ	-0.42^{***}	-0.71^{***}	-0.68^{***}
	(-15.98)	(-14.31)	(-17.55)
σ^2	0.27	0.31	0.27
log-like	-2270.7	-1444.4	-1251.6
AIC	1.39	0.89	0.77
BIC	1.42	0.92	0.80
obs.	3296	3296	3296

Table 7: Estimation results of the dynamic model

Notes: *t*-statistics are in parentheses. *t*-statistics of the dynamic spatial panel model are computed using the asymptotic distribution derived in Lee and Yu (2010c). λ is the spatial autoregressive coefficient, γ captures the pure time effect and δ captures the combined spatial-time effect. * * * , ** and * indicate coefficients that are significant at 1%, 5% and 10%, respectively. The reduced number of observations results from Lee and Yu's transformation approach.

During the 1990s, economic growth in Eastern Germany was strongly driven by an expansion of the construction industry. Though, it contracted towards the end of the decade (Davies and Hallet (2001)). In 2000, the share of the construction industry in gross value added (using current prices) amounted to 8.1 per cent in Eastern Germany (including Berlin) whereas it amounted to only 4.7 per cent in Western Germany.¹⁶ Secondly, regional unemployment in Eastern Germany is only influenced by some of the factors which we account for in our model. Only the educational variables, the number of business registrations and the employment share in hotels and restaurants have a significant (positive) impact on Eastern German regional unemployment rates. Contrary to this, the age variables as well as the employment shares in financial business and in public administration are also significant in our model for Western Germany. Thirdly, the influence of the employment share in hotels and restaurants is positive in East Germany while being negative in West Germany.

In line with our results for the whole country, we find that both spatial and temporal dynamics have to be accounted for when modeling regional unemployment. According to information criteria, the binary contiguity matrix captures the spatial relations in the best way for East Germany. Contrary to this, the binary contiguity matrix captures the spatial structure best for the static model while the Molho (1995) matrix is the best weights matrix in case of the dynamic model for Western Germany. The spatial coefficients are mostly significant for both parts of Germany. The signs of the coefficients are in line with the results for the whole country. Both the spatial autoregressive and the spatial autocorrelation coefficient are positive. Likewise, the pure time-dynamic is significant and positive while the combined space-time effect is negative and mostly significant.

6 Conclusion

In this paper, we analyze the determinants for regional unemployment in Germany. Regional unemployment rates in Germany are characterized by substantial regional differences. We show that there are significant spatial spillovers in regional unemployment data. To avoid biased and inefficient estimates, we apply a spatial panel model to our data. Our analysis is based on theoretical considerations proposed in Partridge and Rickman (1997). Our data set covers 24 possible explanatory variables for the period from 1999 until 2007.

We derive our spatial panel model in two steps. Firstly, we divide our set of explanatory variables into three groups according to theoretical importance. Then, we regress regional unemployment rates on combinations of explanatory variables in which the essential variables are always contained. Secondly, we apply the specification test by Debarsy and Ertur (2010) in order to decide which spatial process is appropriate for our data. We also apply a dynamic model to capture the dynamics in labor market data. For the estimation, we use the methodology proposed in Lee and Yu (2010b) for the static model and in Lee and Yu (2010a) for the dynamic model. We incorporate the spatial structure of the data by constructing three different spatial weights matrices. On the one hand, we use the binary contiguity matrix and, on the other, we apply a distance-based spatial weights matrix as well as the matrix proposed in Molho (1995).

¹⁶The values are taken from Volkswirtschaftliche Gesamtrechnungen der Länder, http://www.vgrdl.de/ Arbeitskreis_VGR/.

	basic	s	tatic
		binary	Molho (1995)
			$(\eta = 0.01)$
EG _{it}	-0.084^{***}	-0.057^{***}	-0.078***
	(-4.25)	(-3.6)	(-4.3)
$\% IND_{it}$	-0.075^{**}	-0.061^{**}	-0.048
	(-2.09)	(-1.96)	(-1.35)
%ENERW _{it}	0.14	-0.12	0.0059
	(0.7)	(-0.68)	(0.03)
%CON _{it}	-0.24^{***}	-0.23^{***} -0.2	
	(-4.11)	(-6.39)	(-4.63)
$\% HOT_{it}$	0.27***	0.13**	0.15**
	(3.64)	(1.97)	(2.01)
%FIN _{it}	-0.098	-0.35^{*}	-0.23
	(-0.45)	(-1.88)	(-1.09)
%PUB _{it}	0.066	0.0016	0.029
	(1.58)	(-0.04)	(0.71)
<i>YOUNG_{it}</i>	0.036	0.12*	0.045
	(0.4)	(1.72)	(0.53)
OLD_{it}	-0.046	-0.08^{*}	-0.069
	(-0.83)	(-1.72)	(-1.31)
$H0_{it}$	0.19**	0.14^{**}	0.18^{**}
	(2.35)	(2.22)	(2.36)
$H1_{it}$	0.092^{*}	0.069^{*}	0.12***
	(1.86)	(1.78)	(2.64)
REG_{it}	0.086	0.083^{*}	0.13***
	(1.56)	(1.82)	(2.89)
$DEBTR_{it}$	0.11**	0.072	0.06
	(2.01)	(1.49)	(1.09)
λ		0.67^{***}	0.76^{***}
		(13.84)	(12.44)
ρ	—	-0.03	0.42***
-		(-0.31)	(2.73)
σ^2	0.79	0.59	0.75
log-like	-965.54	-824.36	-868.21
AIC	2.5	2.15	2.26
BIC	2.59	2.24	2.35
obs.	783	783	783

Table 8: Estimation results for the basic and static model for East Germany

Notes: t-statistics are in parentheses. t-statistics for the static model are computed according to Anselin (1988). λ is the spatial autoregressive coefficient and ρ is the spatial autocorrelation coefficient. ** *, ** and * indicate coefficients that are significant at 1%, 5% and 10%, respectively.

	binary	Molho (1995)
	5	$(\eta = 0.01)$
EG _{it}	-0.089***	-0.1***
	(-5.71)	(-6.08)
%IND _{it}	0.0074	0.035
	(0.25)	(1.11)
%ENERW _{it}	0.21	0.23
	(1.23)	(1.3)
%CON _{it}	-0.0094	-0.0061
	(-0.19)	(-0.12)
%HOT _{it}	-0.021	0.0035
	(-0.32)	(0.05)
%FIN _{it}	-0.26	-0.27
	(-1.59)	(-1.55)
%PUB _{it}	-0.0024	-0.0059
	(-0.08)	(-0.18)
<i>YOUNG_{it}</i>	-0.018	-0.015
	(-0.25)	(-0.2)
OLD_{it}	-0.15^{***}	-0.16^{***}
	(-3.42)	(-3.54)
$H0_{it}$	0.056	0.048
	(0.89)	(0.73)
$H1_{it}$	-0.014	-0.015
	(-0.37)	(-0.38)
REG_{it}	-0.029	-0.026
	(-0.74)	(-0.62)
DEBTR _{it}	0.039	0.045
	(0.89)	(0.96)
λ	0.38***	0.33***
	(8.83)	(3.74)
γ	0.74^{***}	0.72***
	(21.24)	(23.44)
δ	-0.31^{***}	-0.15
2	(-5.29)	(-1)
σ^2	0.37	0.41
log-like	-594.99	-766.51
AIC	1.76	2.25
BIC	1.87	2.36
obs.	696	696

Table 9: Estimation results of the dynamic model for East Germany

Notes: *t*-statistics are in parentheses. *t*-statistics of the dynamic spatial panel model are computed using the asymptotic distribution derived in Lee and Yu (2010c). λ is the spatial autoregressive coefficient, γ captures the pure time effect and δ captures the combined spatial-time effect. ** *, ** and * indicate coefficients that are significant at 1%, 5% and 10%, respectively. The reduced number of observations results from Lee and Yu's transformation approach.

	basic	static		
		binary	Molho (1995)	
		2	$(\eta = 0.01)$	
EG_{it}	-0.043^{***}	-0.016^{**}	-0.036***	
	(-4.28)	(-2.26)	(-4.05)	
%IND _{it}	-0.12^{***}	-0.051^{***}	-0.1^{***}	
	(-8.9)	(-5.37)	(-8.26)	
%ENERW _{it}	0.12^{*}	0.047	0.12**	
	(1.91)	(1.11)	(2.11)	
%CON _{it}	-0.061^{*}	0.0066	-0.052^{*}	
	(-1.89)	(0.29)	(-1.82)	
%HOT _{it}	-0.28^{***}	-0.14^{**}	-0.27^{***}	
	(-3.22)	(-2.28)	(-3.42)	
%FIN _{it}	0.16***	0.07**	0.13***	
	(3.29)	(2.14)	(2.86)	
%PUB _{it}	0.14***	0.08***	0.12***	
	(4.22)	(3.46)	(3.83)	
<i>YOUNG_{it}</i>	-0.04	-0.12^{***}	-0.11^{**}	
	(-0.73)	(-2.72)	(-2.38)	
OLD_{it}	-0.31***	-0.25***	-0.36***	
	(-10.12)	(-8.66)	(-11.79)	
$H0_{it}$	0.034	0.065***	0.037*	
	(1.19)	(3.31)	(1.72)	
$H1_{it}$	0.038*	0.078***	0.081***	
	(1.88)	(4.51)	(4.54)	
REG_{it}	0.25***	0.08**	0.2***	
	(4.65)	(1.98)	(4.37)	
$DEBTR_{it}$	0.01	-0.0019	0.0084	
	(0.41)	(-0.1)	(0.36)	
λ		-0.62***	0.76***	
		(-20.09)	(16.52)	
ρ	_	0.96***	0.6***	
		(189.74)	(6.72)	
σ^2	0.45	0.26	0.39	
log-like	-2820.29	-2404.88	-2457.77	
ĂIC	1.94	1.66	1.69	
BIC	1.97	1.69	1.72	
obs.	2925	2925	2925	

Table 10: Estimation results of the basic and static model for West Germany

Notes: t-statistics are in parentheses. t-statistics for the static model are computed according to Anselin (1988). λ is the spatial autoregressive coefficient and ρ is the spatial autocorrelation coefficient. ** *, ** and * indicate coefficients that are significant at 1%, 5% and 10%, respectively.

	binary	Molho (1995)
	Diffuly	(n = 0.01)
FG	-0.06***	0077***
LO_{lt}	(-7 55)	(-9 34)
%IND.	-0.012	-0.0082
$/011 V D_{ll}$	(-1.12)	(-0.73)
% ENERW.	0 22***	0.25***
/oliverv _{it}	(4.51)	(4.9)
%CON.	0.07**	0.062**
/oconv _{it}	(25)	(2 11)
%HOT.	-0.064	-0.063
/01101 _{1t}	(-0.92)	-0.005
%FIN.	0.09*	(-0.07)
701 11 v _{1t}	(1.92)	(2.06)
%PHB.	0.021	0.0062
$701 \ \text{GD}_{ll}$	(0.69)	(0.2)
YOUNG	(0.07)	(0.2)
rearro _{lt}	(1.62)	(1.51)
OLD_{24}	-0.01	-0.03
0 22 11	(-0.4)	(-1.12)
$H0_{ii}$	0.014	-0.0002
110/1	(0.6)	(-0.01)
$H1_{it}$	-0.034^{**}	-0.054^{***}
11	(-2)	(-3.05)
REG_{it}	0.05	0.056
- 11	(1.25)	(1.35)
$DEBTR_{it}$	-0.0043	-0.012
**	(-0.21)	(-0.58)
λ	0.47***	0.83***
	(21.1)	(33.95)
γ	0.805***	0.84***
	(44.66)	(50.95)
δ	-0.37***	-0.84^{***}
	(-11.5)	(-16.2)
σ^2	0.23	0.25
log-like	-2028.3	-841.5
ĂIC	1.57	0.66
BIC	1.61	0.7
obs.	2600	2600

Table 11: Estimation results of the dynamic model for West Germany

Notes: *t*-statistics are in parentheses. *t*-statistics of the dynamic spatial panel model are computed using the asymptotic distribution derived in Lee and Yu (2010c). λ is the spatial autoregressive coefficient, γ captures the pure time effect and δ captures the combined spatial-time effect. * * *, ** and * indicate coefficients that are significant at 1%, 5% and 10%, respectively. The reduced number of observations results from Lee and Yu's transformation approach.

We apply these models for the whole of Germany as well as for the Eastern and Western part separately. Our results clearly show that the spatial panel model fits our data better than the basic model. Moreover, the dynamic modeling is more appropriate for our analysis of regional unemployment than the static one. Hence, the spatial dynamic panel model is the best model for the analysis of regional unemployment.

Our study leads to several conclusions for policy. Firstly, policy measures to reduce unemployment should account for regional differences in unemployment in Germany. In addition to framework conditions that are set on the national level, policy makers have to take regional differences in economic structure into account. Secondly, according to our results, regional unemployment in Germany is of disequilibrium nature. Hence, our results provide justification for policy makers to intervene on regional labor markets. However, thirdly, we find significant spatial spillovers, i.e. political decisions do not only affect the district on which they are targeted but also the neighboring districts. This aspect motivates political cooperations between different districts. The definition of labor market regions as, e.g., proposed by Eckey et al. (2007a) can offer guidance for this process.

Appendix

District reforms

There are two district reforms during the period from 1999 until 2009. In the federal state of Saxony, the reform became effective as from August 1st, 2008. The reform resulted in a reduced number of districts from 29 to 13. The other reform affects the federal state of Saxony-Anhalt where it became effective as from July 1st, 2007. The reform reduced the number of districts from 21 to 11. In order to have a data set which corresponds to the same regional division over the entire period, we aggregated the data according to the reforms. Hence, we use the regional structure after the reforms for the entire period. Regional unemployment rates of the 'new' district are weighted averages of the corresponding regional unemployment rates using the associated labor force as weights.

	Min	1st Qu.	Median	Mean	3rd Qu.	Max	Std. dev.
EG	-4.084	-0.859	-0.002	-0.234	0.537	2.724	1.178
%IND	3.455	19.580	26.350	27.650	35.990	67.240	11.254
%ENERW	0	0.478	0.739	0.904	1.074	7.974	0.732
%CON	2.130	5.681	7.559	7.938	10.1	17.4	3.035
%HOT	0.849	2.002	2.407	2.97	3.148	22.08	1.989
%FIN	0.445	2.096	2.612	2.941	3.232	16.95	1.73
%PUB	2.334	4.882	6.012	6.742	8.136	19.32	2.679
YOUNG	0.137	0.165	0.175	0.175	0.184	0.234	0.015
OLD	0.215	0.267	0.279	0.279	0.292	0.337	0.019
H0	7.351	14.44	17.91	17.1	20.32	30.57	4.63
H1	50.1	61.3	64.38	64.35	67.25	77.93	4.983
REG	0.335	0.923	1.404	1.992	2.258	39.27	2622
DEBTR	0.025	3.002	4.363	4.71	6.127	13.49	2.24

Summary statistics of explanatory variables

Table 12: Summary statistics of the explanatory variables of our model using averages over the period from 1999 until 2007

Pesaran's diagnostic test for cross-sectional dependence in panels (Pesaran (2004))

Pesaran (2004) develops the *LM* test by Breusch and Pagan (1980) further and proposes a diagnostic test that does not require the specification of a spatial weights matrix. Its null hypothesis is cross-sectional independence of the regressors. His test is applicable to a variety of panel data models, including stationary dynamic and unit root heterogeneous panels with short T and large N. Furthermore, his test has better small sample properties than the test of Breusch and Pagan (1980) especially for N large and T small. The test is based on the pair-wise correlation coefficients of the OLS residuals from the individual regressions in the panel. The test statistic is given by

$$CD = \sqrt{\frac{2T}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij} \right)$$
(10)

where $\hat{\rho}_{ij}$ is the sample estimate of the pair-wise correlation of the residuals. It is defined by

$$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\sum_{t=1}^{T} e_{it} e_{jt}}{\left(\sum_{t=1}^{T} e_{it}^2\right)^{1/2} \left(\sum_{t=1}^{T} e_{jt}^2\right)^{1/2}}.$$
(11)

where e_{it} are the residuals in the model

$$y_{it} = \mu_i + \beta'_i x_{it} + \epsilon_{it}; \ i = 1, \dots, N; \ t = 1, \dots, T$$
 (12)

where y_{it} is the observation of region *i* at time period *t* of the dependent variable and x_{it} is a $(k \times 1)$ vector of observed time-varying regressors. The test statistic is asymptotically standard normally distributed (under the null hypothesis). Furthermore, Pesaran (2004) shows that the test is robust to single or multiple breaks in the slope coefficients and/or error variances as long as the unconditional means of variables in the panel remain constant over time. When the cross section units can be ordered a priori (as for spatial observations), the test might not be sufficiently powerful. Pesaran (2004) also proposes a generalization of his test for that case.

Test statistics of test by Debarsy and Ertur (2010)

Consider the SARAR(1,1) model¹⁷

$$Y_t = \lambda W Y_t + X_t \beta + \mu + U_t; \quad U_t = \rho M U_t + V_t; \quad t = 1, ..., T,$$
 (13)

where $Y_t = (y_{1,t}, y_{2,t}, ..., y_{n,t})$ is the $(n \times 1)$ vector of the dependent variable for all individuals in period t, X_t is the $(n \times k)$ matrix of exogenous regressors and β is the associated vector of unknown regression coefficients. $V_t = (v_{1,t}, ..., v_{n,t})$ is the innovation term where $v_{i,t}$ is i.i.d. across i and t with zero mean and constant variance σ^2 . μ is the $(n \times 1)$ vector of individual effects. W and M are $(n \times n)$ spatial weights matrices, λ and ρ are the unknown spatial parameters. Applying the transformation approach by Lee and Yu yields the transformed model

$$Y_t^* = \lambda W Y_t^* + X_t^* \beta + U_t^*; \quad U_t^* = \rho M U_t^* + V_t^*; \quad t = 1, \dots, T-1$$
(14)

Denoting $\theta' = [\beta', \rho, \lambda, \sigma^2]$ and $\eta' = [\beta', \rho, \lambda]$, the log-likelihood function is given by

$$l(\theta) = -\frac{n(T-1)}{2}ln(2\pi) - \frac{n(T-1)}{2}ln(\sigma^2) + (T-1)ln|S(\lambda)| + (T-1)ln|R(\rho)|$$
(15)

$$-\frac{1}{2\sigma^2}\sum_{t=1}^{T-1} V_t^{*'}(\eta) V_t^{*}(\eta)$$
(16)

where $S(\lambda) = I_n - \lambda W$, $R(\rho) = I_n - \rho M$ and $V_t^* = R(\rho)[S(\lambda)Y_t^* - X_t^*\beta]$. Debarsy and Ertur (2010) consider five different hypotheses (H_0^a until H_0^e) for their specification test.

Joint test statistic for $H_0^a: \rho = \lambda = 0$

Under the null hypothesis, the specification to be estimated is the standard fixed effects panel model. Debarsy and Ertur (2010) show that the transformed model can also be estimated by OLS. The joint *LM* statistic is then given by

$$LM_a = \tilde{Q}^{-1} [T_{22} \tilde{R}_y^2 - 2T_{12} \tilde{R}_v \tilde{R}_y + (\tilde{D} + T_{11}) \tilde{R}_v^2]$$
(17)

where

$$\begin{split} \tilde{R}_{v} &= \frac{\sum_{t=1}^{T-1} \tilde{V}_{t}^{*'} M \tilde{V}_{t}^{*}}{\tilde{\sigma}^{2}}; \\ \tilde{R}_{y} &= \frac{\sum_{t=1}^{T-1} \tilde{V}_{t}^{*'} W Y_{t}^{*}}{\tilde{\sigma}^{2}}; \\ \tilde{D} &= \tilde{\sigma}^{-2} \sum_{t=1}^{T-1} (W X_{t}^{*} \tilde{\beta})' M_{X^{*}} (W X_{t}^{*} \tilde{\beta}); \\ \tilde{Q} &= (\tilde{D} + T_{11}) T_{22} - T_{12}^{2} \end{split}$$

Also, $T_{11} = (T-1)tr[(W+W')W]$, $T_{22} = (T-1)tr[(M+M')M]$, $T_{12} = (T-1)tr((M'+M)W)$

¹⁷All explanations are taken from the original paper by Debarsy and Ertur (2010).

and $M_{X^*} = I_n - X_t^* (X_t^* X_t^*)^{-1} X_t^*$. Finally, $\tilde{V}_t^* = Y_t^* - X_t^* \tilde{\beta}$ is the residual of the constrained model and $\tilde{\sigma}^2$ is the associated OLS residual variance. LM_a is expected to be asymptotically distributed as χ_2^2 under the joint null hypothesis H_0^a .

Marginal test statistic for $H_0^b: \lambda = 0$ (assuming $\rho = 0$)

Under this hypothesis, the constrained model is

$$Y_t^* = X_t^* \beta + V_t^*, \quad t = 1, \dots, T - 1,$$
(18)

where V_t^* is distributed according to a multivariate normal distribution with zero mean and the variance-covariance matrix $\sigma^2 I_{n(T-1)}$. The unconstrained model is

$$Y_t^* = \lambda W Y_t^* + X_t^* \beta + V_t^*; \quad t = 1, \dots, T - 1,$$
(19)

and should be estimated using maximum likelihood. The LM statistic for this hypothesis is

$$LM_b = \frac{\sum_{t=1}^{T-1} (\tilde{V}_t^{*'} W Y_t^{*} / \tilde{\sigma}^2)^2}{\tilde{D} + T_{11}}$$
(20)

where the variables are defined as before. \tilde{V}_t^* are the OLS residuals of equation (18). This *LM* statistic is asymptotically distributed as χ_1^2 under the null hypothesis.

Marginal test statistic for $H_0^c: \rho = 0$ (assuming $\lambda = 0$)

For this test, the restricted model is again equation (18). The specification under the alternative is

$$Y_t^* = X_t^* \beta + U_t^*; \quad U_t^* = \rho M U_t^* + V_t^*; \quad t = 1, \dots, T - 1$$
(21)

The LM statistic for this hypothesis is

$$LM_{c} = \frac{\sum_{t=1}^{T-1} (\tilde{V}_{t}^{*'} M \tilde{V}_{t}^{*} / \tilde{\sigma}^{2})^{2}}{T_{22}}$$
(22)

Again, \tilde{V}_t^* are the residuals of equation (18) and $\tilde{\sigma}^2$ is the estimate of the corresponding residual variance. Under the null hypothesis, LM_c is asymptotically distributed as χ_1^2 .

Conditional test statistic for $H^d_0: \rho=0$ given $\lambda\neq 0$

The appropriate specification under the null is model (19). When the null is rejected, the correct specification is the general model (14). The disturbances of the restricted model are given by

$$V_t^* = S(\lambda)Y_t^* - X_t^*\beta; \quad t = 1, \dots, T - 1,$$
(23)

These disturbances can be estimated by ML in model (19). The *LM* statistic for the conditional test for spatially autocorrelated errors in the presence of an endogenous spatial lag is given by

$$LM_{d} = \frac{(\sum_{t=1}^{T-1} \tilde{V}_{t}^{*'} M \tilde{V}_{t}^{*} / \tilde{\sigma}^{2})^{2}}{T_{22} - (\tilde{T}_{\lambda\rho})^{2} * var(\tilde{\rho})}$$
(24)

where $var(\tilde{\rho})$ is the variance of the autoregressive coefficient estimated under the constrained model and $\tilde{T}_{\lambda\rho} = (T-1)tr[M'WS(\tilde{\rho})^{-1} + MWS(\tilde{\rho})^{-1}]$. LM_d is asymptotically distributed as χ_1^2 under the null hypothesis.

Conditional test statistic for $H^{\boldsymbol{\varrho}}_0: \boldsymbol{\lambda} = 0$ given $\boldsymbol{\rho} \neq 0$

For this test, the unconstrained model is the general specification (equation 14) whereas the constrained model is model (21). Its error term is given by

$$V_t^* = R(\rho)[Y_t^* - X_t^*\beta]; \quad t = 1, \dots, T - 1$$
(25)

where $R(\rho) = I_n - \rho M$. The conditional *LM* statistic for the conditional test for an endogenous spatial lag in the presence of spatially autocorrelated errors is

$$LM_{e} = \frac{\sum_{t=1}^{T-1} (\tilde{V}_{t}^{*'} R(\tilde{\rho}) W Y_{t}^{*} / \tilde{\sigma}^{2})^{2}}{\tilde{I}_{11} - \tilde{I}_{12} \tilde{I}_{22} \tilde{I}_{21}}$$
(26)

with \tilde{I}_{22} being the variance-covariance matrix of the non-constrained parameters, namely $\tilde{\rho}$, $\tilde{\beta}$ and $\tilde{\sigma}^2$. The other terms are defined as follows

$$\tilde{I}_{11} = (T-1)tr(W)^{2} + \frac{1}{\tilde{\sigma}^{2}} \sum_{t=1}^{T-1} [(R(\tilde{\rho})WX_{t}^{*}\tilde{\beta})'(R(\tilde{\rho})WX_{t}^{*}\tilde{\beta})] + (T-1)tr[(R(\tilde{\rho})WR(\tilde{\rho})^{-1})'(R(\tilde{\rho})WR(\tilde{\rho})^{-})]$$
(27)

$$\tilde{I}_{12} = \begin{bmatrix} \frac{1}{\tilde{\sigma}^2} \sum_{t=1}^{T-1} X_t^{*'} R(\tilde{\rho})' R(\tilde{\rho}) W X_t^* \tilde{\beta} \\ (T-1) tr[(MR(\tilde{\rho})^{-1})' R(\tilde{\rho}) W R(\tilde{\rho})^{-1} + MW R(\tilde{\rho})^{-1}] \\ 0 \end{bmatrix}$$
(28)

All parameters involved in this test come from the contrained model. The LM_e statistic is asymptotically distributed as χ_1^2 .

Test for serial correlation, spatial autocorrelation and random effects by Baltagi et al. (2007b)

Consider the panel data model¹⁸

$$y_{ti} = x'_{ti}\beta + u_{ti}; \quad i = 1, \dots, n; \quad t = 1, \dots, T,$$
 (29)

where y_{ti} is the observation of the *i*th region for the *t*th time period, x_{ti} denotes the $(k \times 1)$ vector of observations on the nonstochastic regressors and u_{ti} is the regression disturbance. In vector form, the disturbance vector of equation (29) is assumed to have random region effects, spatially autocorrelated residual disturbances and a first-order autoregressive remainder disturbance term and it is written by

$$u_t = \mu + \epsilon_t \tag{30}$$

with

$$\epsilon_t = \lambda W \epsilon_t + v_t \tag{31}$$

and

$$v_t = \rho v_{t-1} + e_t \tag{32}$$

where $u'_t = (u_{t1}, ..., u_{tN})$ and ϵ_t , v_t and e_t are similarly defined. $\mu' = (\mu_1, \mu_2, ..., \mu_N)$ denote the vector of random region effects which are assumed to be $IIN(0, \sigma_{\mu}^2)$. λ is the scalar spatial autoregressive coefficient while ρ is the time-wise serial correlation coefficient. W is the $(n \times n)$ spatial weights matrix.

Baltagi et al. (2007b) consider joint, marginal and conditional hypotheses for spatial error dependence, random region effects as well as for serial error dependence. Table 13 presents an overview of the hypotheses, test statistics and asymptotic distributions. To save space, we define all terms only once in the table.

 $^{^{18}\}mathrm{All}$ explanations are taken from the original paper by Baltagi et al. (2007b).

	One-dimensional conditional tests continued
Hypothesis	$ \rho = 0 (allowing \ \lambda \neq 0 \ and \ \sigma_{\mu}^2 > 0) $
	$\mathrm{LM}_{ ho/\lambda\mu}=\widehat{D}^2(ho)J_{33}^{-1}$
	where $\hat{D}(\rho) = \frac{\partial L}{\partial \rho} _{H_0}$
	$= -\frac{(T-1)}{T} \big(\hat{\sigma}_{e}^{2} \operatorname{tr}(Z_{0}(B'B)^{-1}) - n \big) + \frac{1}{2} \hat{\sigma}_{e}^{2} \hat{u}' \Big(\frac{1}{\hat{\sigma}_{e}^{2}} (E_{T} G E_{T}) \otimes (B'B) + \frac{1}{\hat{\sigma}_{e}^{2}} (\bar{I}_{T} G E_{T}) \otimes Z_{0} + \frac{1}{\hat{\sigma}_{e}^{2}} (E_{T} G \bar{I}_{T}) \otimes Z_{0} + (\bar{I}_{T} G \bar{I}_{T}) \otimes Z_{0} + (\bar{I}_{T} G \bar{I}_{T}) \otimes Z_{0} (B'B)^{-1} Z_{0} \Big) \hat{u}$
	with $Z_0 = (T\sigma_{\theta}^2 I_n + \sigma_{e}^2 (B'B)^{-1})^{-1}$, $B = I_n - \lambda W$, $\bar{I}_T = J_T/T$ and $E_T = I_T - \bar{I}_T$
	J_{33}^{-1} is the (3,3) element of the inverse of the information matrix $\widehat{f}_ heta$ evaluated under H_0 .
LM statistic	$\hat{f}_{\theta} = \begin{pmatrix} \frac{1}{2} \left(\frac{n(T-1)}{\tilde{\sigma}_{e}^{2}} + d_{1} \right) & \frac{T}{2} d_{2} & \frac{(T-1)}{T} \left(\tilde{\sigma}_{e}^{2} d_{1} - \frac{n}{\tilde{\sigma}_{e}^{2}} \right) & \frac{1}{2} \left[\frac{(T-1)}{\tilde{\sigma}_{e}^{2}} d_{3} + \tilde{\sigma}_{e}^{2} d_{4} \right] \\ \frac{T}{2} d_{2} & \frac{T}{2} tr \left[Z_{0} \right]^{2} & (T-1) \tilde{\sigma}_{e}^{2} d_{2} & \frac{T \tilde{\sigma}_{e}^{2}}{2} d_{5} \\ \frac{T-1}{2} \left(\tilde{\sigma}_{e}^{2} d_{1} - \frac{n}{\tilde{\sigma}_{e}^{2}} \right) & (T-1) \tilde{\sigma}_{e}^{2} d_{2} & \tilde{f}_{\rho\rho} & \frac{T-1}{T} \left(\tilde{\sigma}_{e}^{4} d_{4} - d_{3} \right) \\ \frac{1}{2} \left(\frac{T-1}{\tilde{\sigma}_{e}^{2}} d_{3} + \tilde{\sigma}_{e}^{2} d_{4} \right) & \frac{T \tilde{\sigma}_{e}^{2}}{2} d_{5} & \frac{T-1}{T} \left(\tilde{\sigma}_{e}^{4} d_{4} - d_{3} \right) & \frac{1}{2} \left[(T-1) d_{6} + \tilde{\sigma}_{e}^{4} d_{7} \right] \end{pmatrix}$
	with $d_1 = \operatorname{tr} (Z(B'B)^{-1})^2$ $d_2 = \operatorname{tr} (Z(B'B)^{-1}Z)$
	$d_{3} = \operatorname{tr} \left((W'B + B'W)(B'B)^{-1} \right)$ $d_{4} = \operatorname{tr} \left(Z(B'B)^{-1}(W'B + B'W)(B'B)^{-1}Z(B'B)^{-1} \right)$
	$\begin{aligned} d_5 &= \operatorname{tr}\left(Z(B'B)^{-1}(W'B + B'W)(B'B)^{-1}Z\right) \\ d_6 &= \operatorname{tr}\left((W'B + B'W)(B'B)^{-1}\right)^2 \\ d_{+}\left(Z(B'B)^{-1}(M'B + B'W)(B'B)^{-1}\right)^2 \end{aligned}$
	and $\widehat{f}_{\rho\rho} = \frac{n}{T^2} (T^3 - 3T^2 + 2T + 2) + \frac{2(T-1)^2 \widehat{\sigma}_1^4}{T^2} d_1$
asymptotic distribution under H_0	X12

Two-dimensional conditional tests	hesis $\lambda = \rho = 0 \text{ (allowing } \sigma_{\mu}^2 > 0 \text{)}$	$\text{LM}_{\lambda\rho/\mu} = \widehat{D}[\theta]_{\mu}^{2} n_{\gamma}^{2} r_{\gamma}^{2} (r-1)] + \frac{\widehat{D}[\rho]_{\mu}^{2} n_{\gamma}^{2} r_{\gamma}^{2} (r-1)]}{[(r-1) + \frac{\sigma_{\gamma}^{2}}{2}]} + \frac{\widehat{D}[\rho]_{\mu}^{2} n_{\gamma}^{2} r_{\gamma}^{2} (r-1)]}{[(r-1) + \frac{\sigma_{\gamma}^{2}}{2}]} + \frac{\widehat{D}[\rho]_{\mu}^{2} n_{\gamma}^{2} n_{\gamma}^{2} (r-1)]}{[(r-1) + \frac{\sigma_{\gamma}^{2}}{2}]} + \frac{\widehat{D}[\rho]_{\mu}^{2} n_{\gamma}^{2} (r-1)]}{[(r-1) + \frac{\sigma_{\gamma}^{2}}{2}]} + \frac{\widehat{D}[\rho]_{\mu}^{2} n_{\gamma}^{2} (r-1)]}{[(r-1) + \frac{\sigma_{\gamma}^{2}}{2}]} + \frac{\widehat{D}[\rho]_{\mu}^{2} n_{\gamma}^{2} n$	where $\hat{f}_{\rho\rho} = n \left[2a^2 (T-1)^2 + 2a(2T-3) + T-1 \right]$ with $a = \frac{\hat{\sigma}_c^2 - \hat{\sigma}_1^2}{T\hat{\sigma}_1^2}$	totic disonant under χ^2_2
	Hypothesis	LM statistic		asymptotic di tribution und

tribution under	accumutatio die	$J(\theta_1)$ is the block-diagonal information matrix with respect to the parameters $(\sigma_e^2, \sigma_{\mu'}^2, \rho)$.	\hat{u} are the restricted maximum likelihood residuals under H_0	The information matrix is given by $\widehat{T}_{c} = \begin{bmatrix} \frac{ng}{2\sigma_{c}^{2}} & \frac{ng^{2}}{2} \\ \frac{2\sigma_{c}^{2}}{2} & \frac{\sigma_{c}^{2}(1+\rho)}{2} \left[(2-T)\rho^{2} + \rho + (T-1) \right] & 0 \end{bmatrix}$	M statistic $\left(\begin{array}{cc} \frac{nT}{2\sigma_{e}^{2}} & \frac{ng}{2\sigma_{e}^{2}} \\ \frac{nT}{2\sigma_{e}^{2}} & \frac{ng}{2\sigma_{e}^{2}} \end{array} \right)$	$D(\lambda) = rac{g_L}{\partial \lambda} _{H_0} = rac{1}{2\sigma_c^2} \hat{u}' (V_{ ho}^{-1} \otimes (W'+W)) \hat{u}$		$\left \; \widehat{D}(\sigma_{\mu}^2) \; = \; rac{\partial L}{\partial \sigma_{\mu}^2} ight _{H_0} = - rac{n_S}{2} + rac{1}{2\sigma_{\mu}^2} \hat{u}'(V_{ ho}^{-1})_T V_{ ho}^{-1} \otimes I_n) \hat{u}$	where	$\lim_{\tau \to 0} \frac{1}{2} \int \frac{1}$	$1 M = -\widehat{B} (\widehat{r}^2) = \widehat{D}^2 (\widehat{r}^2) = n^2 = \int T (2n^2 - 2\pi + \pi - 4) = n^2 \int -\widehat{D}^2 (A)$	ypothesis $\lambda = \sigma_{\mu}^2 = 0$ (allowing $\rho \neq 0$)	Two-dimensional conditional tests continued
\hat{u} are the restricted maximum likelihood residuals under H_0 $J(\theta_1)$ is the block-diagonal information matrix with respect to the parameters $(\sigma_c^2, \sigma_{\mu}^2, \rho)$.	\hat{u} are the restricted maximum likelihood residuals under H_0 $J(\theta_1)$ is the block-diagonal information matrix with respect to the parameters $(\sigma_e^2, \sigma_{\mu'}^2, \rho)$.	\hat{u} are the restricted maximum likelihood residuals under H_0		$\left(\begin{array}{ccc} \frac{n\rho}{\sigma_{e}^{2}(1-\rho^{2})} & \frac{n\rho}{\sigma_{e}^{2}(1-\rho^{2})} & \frac{n\rho}{\sigma_{e}^{2}(1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1) \right] & \frac{n}{(1-\rho^{2})^{2}} \left(3\rho^{2} - \rho^{2}T + T - 1 \right) & 0 \\ 0 & 0 & Tb \end{array}\right)$	The information matrix is given by $\widehat{j}_{\theta} = \begin{cases} \frac{ng}{2\sigma_{\tau}^2} & \frac{ng^2}{2} & \overline{\sigma_{\tau}^2(1+\rho)} \left[(2-T)\rho^2 + \rho + (T-1) \right] & 0 \\ \frac{n\rho}{\sigma_{\tau}^2(1-\rho^2)} & \overline{\sigma_{\tau}^2(1+\rho)} \left[(2-T)\rho^2 + \rho + (T-1) \right] & \frac{n}{(1-\rho^2)^2} (3\rho^2 - \rho^2T + T - 1) & 0 \\ 0 & 0 & Tb \end{pmatrix}$	$ \text{The information matrix is given by } \widehat{j}_{\theta} = \begin{pmatrix} \frac{nT}{2\sigma_{\theta}^{2}} & \frac{nR}{2\sigma_{\theta}^{2}} & \frac{nR}{2\sigma_{\theta}^{2}} \\ \frac{nR}{2\sigma_{\theta}^{2}} & \frac{nR}{2\sigma_{\theta}^{2}} & \frac{nR}{2\sigma_{\theta}^{2}} \\ \frac{nR}{2\sigma_{\theta}^{2}(1-\rho^{2})} & \frac{nR}{\sigma_{\theta}^{2}(1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1) \right] & \frac{n}{(1-\rho^{2})^{2}} (3\rho^{2} - \rho^{2}T + T - 1) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} $	$ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_0} = \frac{1}{2\sigma_c^2} \hat{u}'(V_{\rho}^{-1} \otimes (W' + W))\hat{u} $ $ \text{M statistic} \qquad \qquad$	$ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_0} = \frac{1}{2\sigma_r^2} \hat{u}'(V_{\rho}^{-1} \otimes (W' + W))\hat{u} $ M statistic The information matrix is given by $\hat{f}_{\theta} = \begin{cases} \frac{\pi T}{2\sigma_r^4} & \frac{\pi g}{2\sigma_r^2} \\ \frac{\pi g}{2\sigma_r^2} & \frac{\pi g}{2\sigma_r^2} \\ \frac{\pi g}{\sigma_r^2(1+\rho)} & \frac{\pi g}{\sigma_r^2(1+\rho)} \end{bmatrix} \begin{pmatrix} 2\pi T & \frac{\pi g}{\sigma_r^2(1+\rho)} \\ \frac{\pi g}{\sigma_r^2(1+\rho)} \end{bmatrix} \begin{pmatrix} \frac{\pi T}{\sigma_r^2(1+\rho)} & \frac{\pi g}{\sigma_r^2(1+\rho)} \\ \frac{\pi g}{\sigma_r^2(1-\rho^2)} & \frac{\pi g}{\sigma_r^2(1+\rho)} \end{bmatrix} \begin{pmatrix} 2\pi T & \frac{\pi g}{\sigma_r^2(1+\rho)} \\ \frac{\pi g}{\sigma_r^2(1-\rho^2)} & \frac{\pi g}{\sigma_r^2(1+\rho)} \end{bmatrix} \begin{pmatrix} \pi T & \frac{\pi g}{\sigma_r^2(1+\rho)} \\ \frac{\pi g}{\sigma_r^2(1-\rho^2)} & \frac{\pi g}{\sigma_r^2(1+\rho)} \end{bmatrix} \begin{pmatrix} \pi T & \frac{\pi g}{\sigma_r^2(1+\rho)} \\ \frac{\pi g}{\sigma_r^2(1-\rho^2)} & \frac{\pi g}{\sigma_r^2(1+\rho)} \end{bmatrix} \begin{pmatrix} \pi T & \frac{\pi g}{\sigma_r^2(1+\rho)} \\ \frac{\pi g}{\sigma_r^2(1-\rho^2)} & \frac{\pi g}{\sigma_r^2(1+\rho)} \end{bmatrix} \begin{pmatrix} \pi T & \frac{\pi g}{\sigma_r^2(1+\rho)} \\ \frac{\pi g}{\sigma_r^2(1-\rho^2)} & \frac{\pi g}{\sigma_r^2(1+\rho)} \end{bmatrix} \end{pmatrix} $	$\begin{split} \widehat{D}(\sigma_{\mu}^{2}) &= \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{n_{S}}{2} + \frac{1}{2\sigma_{\mu}^{2}}\hat{u}'(V_{\rho}^{-1}J_{T}V_{\rho}^{-1}\otimes I_{n})\hat{u} \\ \widehat{D}(\lambda) &= \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1}\otimes (W'+W))\hat{u} \\ \mathbf{M} \text{ statistic} \\ \mathbf{M} \text{ statistic} \\ The information matrix is given by \widehat{J}_{\theta} = \begin{cases} \frac{n_{T}}{2\sigma_{\rho}^{2}} & \frac{n_{T}}{2\sigma_{\rho}^{2}} \\ \frac{n_{T}}{2\sigma_{\rho}^{2}} & \frac{n_{T}}{2\sigma_{\rho}^{2}} \\ \frac{n_{T}}{2\sigma_{\rho}^{2}} & \frac{n_{T}}{2\sigma_{\rho}^{2}} \\ \frac{n_{T}}{2\sigma_{\rho}^{2}} & \frac{n_{T}}{2\sigma_{\rho}^{2}} \\ \frac{n_{T}}{\sigma_{\rho}^{2}(1-\rho^{2})} & \frac{n_{T}}{\sigma_{\rho}^{2}(1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1) \right] & \frac{n_{T}}{(1-\rho^{2})^{2}} (\rho^{2} - \rho^{2}T + T-1) & 0 \\ 0 & 0 & 0 \end{cases}$	$\begin{array}{l} \text{where} \\ \widehat{D}(\sigma_{\mu}^{2}) &= \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} &= -\frac{ng}{2} + \frac{1}{2}\sigma'(V_{\rho}^{-1}f_{T}V_{\rho}^{-1} \otimes I_{n})\hat{a} \\ \\ \widehat{D}(\lambda) &= \frac{\partial L}{\partial \lambda} _{H_{0}} &= \frac{1}{2\sigma_{e}^{2}}\hat{a}'(V_{\rho}^{-1} \otimes (W'+W))\hat{a} \\ \\ \widehat{D}(\lambda) &= \frac{\partial L}{\partial \lambda} _{H_{0}} &= \frac{1}{2\sigma_{e}^{2}}\hat{a}'(V_{\rho}^{-1} \otimes (W'+W))\hat{a} \\ \\ \text{The information matrix is given by } \widehat{I}_{\theta} &= \begin{pmatrix} \frac{nT}{2\sigma_{e}^{2}} & \frac{ng}{2\sigma_{e}^{2}} \\ \frac{ng}{2\sigma_{e}^{2}(1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1) \right] \\ \frac{ng}{(1-\rho^{2})} & \frac{n}{\sigma_{e}^{2}(1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1) \right] \\ \end{pmatrix} \\ \end{array} $	$ \begin{aligned} \text{where} & \hat{D}(\sigma_{\mu}^{2}) = \frac{\partial U_{\mu}}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{ng}{2} + \frac{1}{2}n'(V_{\rho}^{-1}I_{T}V_{\rho}^{-1} \otimes I_{n})n \\ \hat{D}(\sigma_{\mu}^{2}) = \frac{\partial U_{\mu}}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{ng}{2} + \frac{1}{2}n'(V_{\rho}^{-1}I_{T}V_{\rho}^{-1} \otimes I_{n})n \\ \hat{D}(\lambda) = \frac{\partial U_{\mu}}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\tau}^{2}}n'(V_{\rho}^{-1} \otimes (W' + W))n \\ \hat{D}(\lambda) = \frac{\partial U_{\mu}}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\tau}^{2}}n'(V_{\rho}^{-1} \otimes (W' + W))n \\ \text{The information matrix is given by } \hat{f}_{\theta} = \begin{cases} \frac{n}{2\sigma_{\tau}^{2}} & \frac{n}{2\sigma_{\tau}^{2}} \\ \frac{n}{\sigma_{\tau}^{2}(1-\rho)} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} [(2-T)\rho^{2} + \rho + (T-1)] \\ \frac{n}{(1-\rho^{2})^{2}} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} [(2-T)\rho^{2} + \rho + (T-1)] \end{cases} \xrightarrow{0} \\ \text{The information matrix is given by } \hat{f}_{\theta} = \begin{cases} \frac{n}{\sigma_{\tau}^{2}} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} [(2-T)\rho^{2} + \rho + (T-1)] \\ \frac{n}{(1-\rho^{2})^{2}} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} [(2-T)\rho^{2} + \rho + (T-1)] \end{cases} \xrightarrow{0} \\ \text{The information matrix is given by } \hat{f}_{\theta} = \begin{cases} \frac{n}{\sigma_{\tau}^{2}} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} [(2-T)\rho^{2} + \rho + (T-1)] \\ \frac{n}{(1-\rho^{2})^{2}} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} (0 \\ 0 \end{pmatrix} \end{pmatrix} \\ \text{The information matrix is given by } \hat{f}_{\theta} = \begin{cases} \frac{n}{\sigma_{\tau}^{2}} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} [(2-T)\rho^{2} + \rho + (T-1)] \\ \frac{n}{(1-\rho^{2})^{2}} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} (0 \\ 0 \end{pmatrix} \\ \text{The information matrix is given by } \hat{f}_{\theta} = \begin{cases} \frac{n}{\sigma_{\tau}^{2}} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} [(2-T)\rho^{2} + \rho + (T-1)] \\ \frac{n}{(1-\rho^{2})^{2}} & \frac{n}{\sigma^{2}(1+\rho)} (0 \\ 0 \\ 0 \end{pmatrix} \\ \text{The information matrix is given by } \hat{f}_{\theta} = \begin{cases} \frac{n}{\sigma_{\tau}^{2}} & \frac{n}{\sigma_{\tau}^{2}(1+\rho)} [(2-T)\rho^{2} + \rho + (T-1)] \\ \frac{n}{(1-\rho^{2})^{2}} & \frac{n}{\sigma^{2}(1+\rho)} (0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $	$ \text{LM}_{\lambda\mu/\rho} = \widehat{D}_{\rho}^{\prime} \widehat{f}_{\rho}^{-1} \widehat{D}_{\rho} = \frac{\widehat{D}_{\gamma}^{\prime} (\tau_{\rho}^{\prime})}{\operatorname{det}(I(\theta_{1}))} \frac{n^{2}}{\sigma_{\tau}^{\prime} (1-\rho^{\prime})} \left\{ \frac{T}{2} (3\rho^{2} - \rho^{2}T + T - 1) - \rho^{2} \right\} + \frac{\widehat{D}_{\gamma}^{\prime} (\lambda)}{Tp} $ where $ \widehat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \overline{\sigma}_{\mu}^{2}} _{H_{0}} = -\frac{n^{2}}{2} + \frac{1}{2\sigma_{\tau}^{\prime}} n' (V_{\rho}^{-1} J_{T} V_{\rho}^{-1} \otimes I_{n}) \hat{n} $ $ \widehat{D}(\Lambda) = \frac{\partial L}{\partial \Lambda} _{H_{0}} = \frac{1}{2\sigma_{\tau}^{\prime}} n' (V_{\rho}^{-1} \otimes (W' + W)) \hat{n} $ $ \widehat{D}(\Lambda) = \frac{\partial L}{\partial \Lambda} _{H_{0}} = \frac{1}{2\sigma_{\tau}^{\prime}} n' (V_{\rho}^{-1} \otimes (W' + W)) \hat{n} $ $ \text{M statistic} $ $ \text{M statistic mentry is given by } \widehat{f}_{\theta} = \left\{ \begin{array}{cc} \frac{nT}{2\sigma_{\tau}^{\prime}} & \frac{nS}{\sigma_{\tau}^{\prime} (1+\rho)} \\ \frac{nS}{\sigma_{\tau}^{\prime} (1-\rho)} & \frac{nS}{\sigma_{\tau}^{\prime} (1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1) \right] \\ \frac{nS}{\sigma_{\tau}^{\prime} (1+\rho)} & \frac{nS}{(1-\rho)^{\prime}} & \frac{nS}{\sigma_{\tau}^{\prime} (1-\rho)} \\ 0 & 0 \end{array} \right\} $	$\begin{array}{c c} \text{ypothesis} & \lambda = \sigma_{\mu}^{2} = 0 \text{ (allowing } \rho \neq 0) \\ & \qquad \qquad$
\hat{n} are the restricted maximum likelihood residuals under H_0 $J(\theta_1)$ is the block-diagonal information matrix with respect to the parameters $(\sigma_e^2, \sigma_\mu^2, \rho)$.	\hat{u} are the restricted maximum likelihood residuals under H_0 $J(\theta_1)$ is the block-diagonal information matrix with respect to the parameters $(\sigma_{e}^2, \sigma_{\mu'}^2, \rho)$.	\hat{u} are the restricted maximum likelihood residuals under H_0		$\frac{1}{\left[\frac{\sigma_{p}^{2}(1-\rho^{2})}{\sigma_{p}^{2}(1-\rho^{2})} - \frac{\eta\rho}{\sigma_{p}^{2}(1-\rho^{2})} - \frac{\eta\rho}{\sigma_{p}^{2}(1+\rho)} \right]}{\left[\frac{\sigma_{p}^{2}(1-\rho^{2})}{\sigma_{p}^{2}(1-\rho^{2})} - \frac{\sigma_{p}^{2}(1+\rho)}{\sigma_{p}^{2}(1-\rho^{2})} - \frac{\eta}{\sigma_{p}^{2}(1-\rho^{2})} - \frac{\eta}{\sigma_{p}^{2}(1-$	The information matrix is given by $\widehat{j}_{\theta} = \begin{vmatrix} \frac{ng}{2\sigma_{\theta}^{2}} & \frac{ng^{2}}{2} & \overline{\sigma_{\theta}^{2}(1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1) \right] & 0 \\ \frac{n\rho}{\sigma_{\theta}^{2}(1-\rho^{2})} & \frac{ng}{\sigma_{\theta}^{2}(1-\rho^{2})} & \frac{ng}{\sigma_{\theta}^{2}(1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1) \right] & \frac{n}{(1-\rho^{2})^{2}} (3\rho^{2} - \rho^{2}T + T - 1) & 0 \end{vmatrix}$	$ \text{The information matrix is given by } \widehat{j}_{\theta} = \left\{ \begin{array}{ccc} \frac{nT}{2\sigma_{\theta}^{2}} & \frac{ng}{2\sigma_{\theta}^{2}} & \frac{ng}{2\sigma_{\theta}^{2}} \\ \frac{ng}{2\sigma_{\theta}^{2}} & \frac{ng}{2\sigma_{\theta}^{2}} & \frac{ng}{2\sigma_{\theta}^{2}} \\ \frac{ng}{2\sigma_{\theta}^{2}(1-\rho^{2})} & \frac{ng}{\sigma_{\theta}^{2}(1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1) \right] & \frac{n}{(1-\rho^{2})^{2}} (3\rho^{2} - \rho^{2}T + T - 1) & 0 \end{array} \right\} $	$\begin{split} \widehat{D}(\lambda) &= \frac{\partial L}{\partial \lambda} _{H_0} = \frac{1}{2\sigma_c^2} \widehat{u}' (V_{\rho}^{-1} \otimes (W' + W)) \widehat{u} \\ \text{M statistic} \\ \text{The information matrix is given by } \widehat{j}_{\theta} &= \left\{ \begin{array}{cc} \frac{nT}{2\sigma_c^4} & \frac{ng}{2\sigma_c^2} & \frac{ng}{2\sigma_c^2} \\ \frac{ng}{2\sigma_c^2} & \frac{ng}{2\sigma_c^2} & \frac{ng}{2\sigma_c^2} \\ \frac{ng}{2\sigma_c^2(1-\rho^2)} & \frac{ng}{2\sigma_c^2(1+\rho)} \left[(2-T)\rho^2 + \rho + (T-1) \right] & \frac{ng}{(1-\rho^2)^2} (3\rho^2 - \rho^2 T + T - 1) & 0 \\ \frac{ng}{(1-\rho^2)^2} (3\rho^2 - \rho^2 T + T - 1) & 0 \\ \end{array} \right\}$	$\begin{split} \widehat{D}(\lambda) &= \frac{\partial L}{\partial \lambda} _{H_0} = \frac{1}{2\sigma_r^2} \hat{u}' (V_\rho^{-1} \otimes (W' + W)) \hat{u} \\ \text{M statistic} \\ \text{The information matrix is given by } \widehat{j}_{\theta} &= \begin{cases} \frac{nT}{2\sigma_r^2} & \frac{nR}{2\sigma_r^2} \\ \frac{nT}{2\sigma_r^2} & \frac{nR}{2\sigma_r^2} \\ \frac{nT}{2\sigma_r^2} & \frac{nR}{2\sigma_r^2} \\ \frac{nT}{2\sigma_r^2} & \frac{nR}{2\sigma_r^2} \\ \frac{nT}{2\sigma_r^2(1-\rho^2)} & \frac{n}{\sigma_r^2(1+\rho)} \left[(2-T)\rho^2 + \rho + (T-1) \right] \\ \frac{nT}{(1-\rho^2)^2} (3\rho^2 - \rho^2 T + T - 1) & 0 \end{cases}$	$\begin{split} \widehat{D}(\sigma_{\mu}^{2}) &= \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{n_{g}}{2} + \frac{1}{2\sigma_{\mu}^{2}}\widehat{u}'(V_{\rho}^{-1}S_{I_{n}})\widehat{u} \\ \widehat{D}(\lambda) &= \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\widehat{u}'(V_{\rho}^{-1}\otimes(W'+W))\widehat{u} \\ \widehat{D}(\lambda) &= \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\widehat{u}'(V_{\rho}^{-1}\otimes(W'+W))\widehat{u} \\ \end{split}$ $\begin{aligned} M \text{ statistic} \\ \text{The information matrix is given by } \widehat{f}_{\theta} &= \left(\begin{array}{c} \frac{nT}{2\sigma_{\rho}^{2}} & \frac{ng}{2\sigma_{\rho}^{2}}\\ \frac{ng}{2\sigma_{\rho}^{2}} & \frac{ng}{2\sigma_{\rho}^{2}(1+\rho)} \left[(2-T)\rho^{2}+\rho+(T-1)\right] & 0\\ \frac{ng}{(1-\rho^{2})^{2}} & \frac{ng}{(1-\rho^{2})^{2}}(\beta\rho^{2}-\rho^{2}T+T-1) & 0 \end{array}\right) \end{split}$	$ \text{ where } \underbrace{ \hat{\mathcal{D}}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{ng}{2} + \frac{1}{2\sigma_{\ell}^{2}} \hat{u}'(V_{\rho}^{-1} f_{T} V_{\rho}^{-1} \otimes I_{n}) \hat{u} \\ \hat{\mathcal{D}}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\ell}^{2}} \hat{u}'(V_{\rho}^{-1} \otimes (W' + W)) \hat{u} \\ \hat{\mathcal{D}}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\ell}^{2}} \hat{u}'(V_{\rho}^{-1} \otimes (W' + W)) \hat{u} \\ \text{ The information matrix is given by } \hat{f}_{\theta} = \left\{ \begin{array}{c} \frac{nT}{2\sigma_{\ell}^{2}} & \frac{ng}{2\sigma_{\ell}^{2}} \\ \frac{ng}{2\sigma_{\ell}^{2}} & \frac{ng}{2\sigma_{\ell}^{2}} \\ \frac{ng}{2\sigma_{\ell}^{2}(1-\rho^{2})} & \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} \end{array} \right\} \xrightarrow{n} \\ \text{ The information matrix is given by } \hat{f}_{\theta} = \left\{ \begin{array}{c} \frac{nT}{2\sigma_{\ell}^{2}} & \frac{ng}{2\sigma_{\ell}^{2}} \\ \frac{ng}{2\sigma_{\ell}^{2}(1-\rho^{2})} & \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} \end{array} \right\} \xrightarrow{n} \\ \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} & \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} \\ \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} & \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} \end{array} \right\} \xrightarrow{n} \\ \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} = \left\{ \begin{array}{c} \frac{n}{\sigma_{\ell}^{2}} & \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} \\ \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} & \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} \end{array} \right\} \xrightarrow{n} \\ \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} & \frac{n}{\sigma_{\ell}^{2}(1-\rho^{2})} \end{array} \right\}$	$\begin{array}{l} \text{where} \\ \widehat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{ng}{2} + \frac{1}{2\sigma_{e}^{2}}u'(V_{\rho}^{-1}J_{T}V_{\rho}^{-1} \otimes I_{n})\hat{a} \\ \widehat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \lambda} _{H_{0}} = -\frac{ng}{2} + \frac{1}{2\sigma_{e}^{2}}u'(V_{\rho}^{-1}J_{T}V_{\rho}^{-1} \otimes I_{n})\hat{a} \\ \widehat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{e}^{2}}u'(V_{\rho}^{-1} \otimes (W' + W))\hat{a} \\ \widehat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{e}^{2}}u'(V_{\rho}^{-1} \otimes (W' + W))\hat{a} \\ \text{M statistic} \\ \text{The information matrix is given by } \widehat{f}_{\theta} = \left\{ \begin{array}{c} \frac{nT}{2\sigma_{e}^{2}} & \frac{ng}{2\sigma_{e}^{2}} \\ \frac{nT}{\sigma_{e}^{2}(1+\rho)} \left[(2-T)\rho^{2} + \rho + (T-1)\right] & \frac{n}{(1-\rho^{2})^{2}} (3\rho^{2} - \rho^{2}T + T-1) & 0 \end{array} \right\} \end{array}$	$ \text{I.M}_{\lambda\mu/\rho} = \widehat{D}_{\theta} \widehat{f}_{\theta}^{-1} \widehat{D}_{\theta} = \frac{\widehat{D}^2(\sigma_1^2)}{\det(I(\theta_1))} \frac{n^2}{\sigma_1^2(1-\rho^2)} \left\{ \frac{T}{2} (3\rho^2 - \rho^2 T + T - 1) - \rho^2 \right\} + \frac{\widehat{D}^2(\lambda)}{T D} $ where $ \widehat{D}(\sigma_1^2) = \frac{\partial L}{\partial \sigma_1^2} \Big _{H_0} = -\frac{1}{2} + \frac{1}{2\sigma_1^2} \vartheta'(V_{\rho}^{-1} I_T V_{\rho}^{-1} \otimes I_n) \vartheta$ $ \widehat{D}(\lambda) = \frac{\partial L}{\partial \lambda} \Big _{H_0} = \frac{1}{2\sigma_1^2} \vartheta'(V_{\rho}^{-1} \otimes (W' + W)) \vartheta $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} \Big _{H_0} = \frac{1}{2\sigma_1^2} \vartheta'(V_{\rho}^{-1} \otimes (W' + W)) \vartheta$ $ \text{A statistic} $ $ \text{M statistic} $ The information matrix is given by $\widehat{f}_{\theta} = \left\{ \begin{array}{c} \frac{n^2}{2\sigma_1^2} \\ \frac{n^2}{2\sigma_1^2} \\ \frac{n^2}{2\sigma_1^2(1-\rho^2)} \\ \frac{n^2}{\sigma_2^2(1-\rho^2)} \\ \frac{n^2}{\sigma_2^2$	$\begin{array}{l lllllllllllllllllllllllllllllllllll$
$\begin{array}{c c} & 0 & 0 & Tb \\ \hline \hat{u} \text{ are the restricted maximum likelihood residuals under } H_0 \\ \hline J(\theta_1) \text{ is the block-diagonal information matrix with respect to the parameters } (\sigma_{e}^2, \sigma_{\mu}^2, \rho). \end{array}$	n η are the restricted maximum likelihood residuals under H_0 $J(\theta_1)$ is the block-diagonal information matrix with respect to the parameters $(\sigma_e^2, \sigma_\mu^2, \rho)$.	\hat{n} are the restricted maximum likelihood residuals under H_0			The information matrix is given by $\widehat{I}_0 = \begin{bmatrix} \frac{ng}{2\sigma_1^2} & \frac{ng^2}{2} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots$	$ \text{M statistic} \qquad \qquad$	$ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_0} = \frac{1}{2\sigma_{\tilde{c}}^2} \hat{u}'(V_{\rho}^{-1} \otimes (W' + W))\hat{u} $ $ \mathbf{M} \text{ statistic} $ $ \mathbf{M} \text{ statistic} $ $ \mathbf{T} \text{ he information matrix is given by } \hat{h}_0 = \begin{bmatrix} \frac{\pi T}{2\sigma_{\tilde{c}}^2} & \frac{\pi R}{2\sigma_{\tilde{c}}^2} & \frac{\pi R}{2\sigma_{\tilde{c}}^2} & 0 \\ \frac{\pi R}{2\sigma_{\tilde{c}}^2} & \frac{\pi R}{2\sigma_{\tilde{c}}^2} & \frac{\pi R}{2\sigma_{\tilde{c}}^2} & 0 \end{bmatrix} $	$ \widehat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_0} = \frac{1}{2\sigma_{\tilde{e}}^2} \widehat{u}'(V_{\rho}^{-1} \otimes (W' + W))\widehat{u} $ M statistic $ \widehat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_0} = \frac{1}{2\sigma_{\tilde{e}}^2} \widehat{u}'(V_{\rho}^{-1} \otimes (W' + W))\widehat{u} $ The information matrix is given by $\widehat{I}_0 = \begin{bmatrix} \frac{nT}{2\sigma_{\tilde{e}}^2} & \frac{nR}{2\sigma_{\tilde{e}}^2} \\ \frac{nR}{2\sigma_{\tilde{e}}^2} & \frac{nR}{2\sigma_{\tilde{e}}^2} \end{bmatrix} \xrightarrow{\frac{n}{2}\sigma_{\tilde{e}}^2} \underbrace{\frac{n}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} = \underbrace{\frac{n}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{n}{2}\sigma_{\tilde{e}}^2} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{n}{2}\sigma_{\tilde{e}}^2} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2\sigma_{\tilde{e}}^2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2\sigma_{\tilde{e}}^2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2\sigma_{\tilde{e}}^2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2\sigma_{\tilde{e}}^2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2\sigma_{\tilde{e}}^2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2\sigma_{\tilde{e}}^2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2\sigma_{\tilde{e}}^2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}} \xrightarrow{\frac{nR}{2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}} \underbrace{\frac{nR}{2\sigma_{\tilde{e}}^2}}_{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}} \underbrace{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}} \underbrace{\frac{nR}{2}} \xrightarrow{\frac{nR}{2}} $	$ \hat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{n_{S}}{2} + \frac{1}{2\sigma_{\mu}^{2}}\hat{u}'(V_{\rho}^{-1}J_{T}V_{\rho}^{-1} \otimes I_{n})\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ M \text{ statistic} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} $ $ \hat{D}(\lambda) = \frac{1}{2\sigma_{\rho}^{2}}\hat{u}'($	$\begin{array}{c} \text{where} \\ \widehat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{ng}{2} + \frac{1}{2\sigma_{e}^{2}}\hat{u}'(V_{\rho}^{-1}I_{T}V_{\rho}^{-1} \otimes I_{n})\hat{u} \\ \widehat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{e}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} \\ \widehat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{e}^{2}}\hat{u}'(V_{\rho}^{-1} \otimes (W'+W))\hat{u} \\ \text{M statistic} \\ \begin{array}{c} \frac{nT}{2\sigma_{e}^{2}} & \frac{ng}{2\sigma_{e}^{2}} \\ \frac{ng}{2\sigma_{e}^{2}} & \frac{ng}{2\sigma_{e}^{2}} \\ \frac{ng}{2\sigma_{e}^{2}} & \frac{ng}{2\sigma_{e}^{2}} \\ \end{array} \end{array}$	$\mathbf{M} \text{ tatistic} \qquad \qquad$	$\operatorname{LM}_{\lambda\mu/\rho} = \widehat{D}_{j}\widehat{f}_{\theta}^{-1}\widehat{D}_{\theta} = \frac{\widehat{D}^{2}(\sigma_{p}^{2})}{\operatorname{det}(I(\theta_{1})]}\frac{n^{2}}{\sigma^{2}(1-\rho^{2})}\left\{\frac{T}{2}(3\rho^{2}-\rho^{2}T+T-1)-\rho^{2}\right\} + \frac{\widehat{D}^{2}(\Lambda)}{Tb}$ where $\widehat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \widetilde{\sigma}_{\mu}^{2}} _{H_{0}} = -\frac{ng}{2} + \frac{1}{2\sigma_{p}^{2}}\widehat{a}^{\prime}(V_{\rho}^{-1})_{T}V_{\rho}^{-1} \otimes I_{n})\widehat{a}$ $\widehat{D}(\Lambda) = \frac{\partial L}{\partial \Lambda} _{H_{0}} = \frac{1}{2\sigma_{p}^{2}}\widehat{a}^{\prime}(V_{\rho}^{-1} \otimes (W'+W))\widehat{a}$ $M \text{ statistic}$ $M \text{ statistic}$ $\operatorname{The information matrix is given by } \widehat{I}_{0} = \left[\begin{array}{c} \frac{nT}{2\sigma_{p}^{2}} \\ \frac{2\sigma_{p}^{2}}{2\sigma_{p}^{2}} \\ \frac{ng}{2} \\ ng$	$\begin{array}{c c} \text{ypothesis} & \lambda = \sigma_{\mu}^{2} = 0 \text{ (allowing } \rho \neq 0) \\ \\ \mathbb{L}M_{\lambda\mu/\rho} = \widehat{D}_{\theta}^{\prime} \widehat{\beta}_{\theta}^{-1} \widehat{D}_{\theta} = \frac{\widehat{D}^{2}(\sigma_{\mu}^{2})}{\det[(\theta_{1})]} \frac{n^{2}}{\sigma_{\tau}^{\prime}(1-\rho^{2})} \left\{ \frac{\gamma}{2}(3\rho^{2}-\rho^{2}T+T-1)-\rho^{2} \right\} + \frac{\widehat{D}^{2}(\lambda)}{Tb} \\ \\ \text{where} \\ \\ \text{where} \\ \widehat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \sigma_{\mu}^{2}} \Big _{H_{0}} = -\frac{ng}{2} + \frac{1}{2\sigma_{\tau}^{2}} \hat{u}'(V_{\rho}^{-1}J_{T}V_{\rho}^{-1} \otimes I_{n}) \hat{u} \\ \\ \widehat{D}(\lambda) = \frac{\partial L}{\partial \Lambda} \Big _{H_{0}} = \frac{1}{2\sigma_{\tau}^{2}} \hat{u}'(V_{\rho}^{-1} \otimes (W'+W)) \hat{u} \\ \\ \widehat{D}(\lambda) = \frac{\partial L}{\partial \Lambda} \Big _{H_{0}} = \frac{1}{2\sigma_{\tau}^{2}} \hat{u}'(V_{\rho}^{-1} \otimes (W'+W)) \hat{u} \\ \\ \text{M statistic} \\ \text{The information matrix is eiven by } \widehat{I}_{0} = \left\{ \begin{array}{c} \frac{nT}{2\sigma_{\tau}^{2}} & \frac{ng}{2} \\ \frac{ng}{2\sigma_{\tau}^{2}} & \frac{ng}{2} \\ \frac{ng}{2\sigma_{\tau}^{2}} & \frac{ng}{2\sigma_{\tau}^{2}(1+\rho)} \Big[(2-T)\rho^{2} + \rho + (T-1)] & 0 \end{array} \right\} \end{array}$
pothesis $\lambda = \sigma_{\mu}^{2} = 0$ (allowing $\rho \neq 0$)representation continued $\lambda = \sigma_{\mu}^{2} = 0$ (allowing $\rho \neq 0$) $M_{N_{\mu}/\rho} = \hat{D}_{\theta}\hat{D}_{\theta}^{-1}\hat{D}_{\theta} = \frac{D^{2}(\sigma_{\mu}^{2})}{aet(16))} \frac{\pi^{2}}{\sigma_{\ell}^{2}(1-\rho^{2})} \left\{ \frac{\pi}{2} (3\rho^{2} - \rho^{2}T + T - 1) - \rho^{2} \right\} + \frac{D^{2}(\Lambda)}{T\rho^{2}}$ where $\hat{D}(\sigma_{\mu}^{2}) = \frac{2\mu}{\delta\sigma_{\mu}^{2}} _{H_{\theta}} = -\frac{\pi}{2\sigma_{\mu}^{2}} \frac{1}{\sigma_{\ell}^{2}(1-\rho^{2})} (V_{\rho}^{-1} \otimes (W^{-1}M)) d$ where $\hat{D}(\Lambda) = \frac{2\mu}{\delta\sigma_{\ell}^{2}} _{H_{\theta}} = -\frac{\pi}{2\sigma_{\ell}^{2}} \frac{1}{\sigma_{\ell}^{2}(1-\rho^{2})} (V_{\rho}^{-1} \otimes (W^{-1}M)) d$ M statistic $\hat{D}(\Lambda) = \frac{2\mu}{\delta\sigma_{\ell}^{2}} _{H_{\theta}} = \frac{2\pi}{\sigma_{\ell}^{2}} d(V_{\rho}^{-1} \otimes (W^{-1}M)) d$ M statistic $\hat{D}(\Lambda) = \frac{2\mu}{\delta\sigma_{\ell}^{2}(1-\rho^{2})} \frac{\pi}{\sigma_{\ell}^{2}(1-\rho^{2})} \frac{\pi}{\sigma_{\ell}^{2}(1+\rho)} \left[(2 - T)\rho^{2} + \rho + (T - 1) \right] $ $\hat{D}(\Lambda) = \frac{2\mu}{\delta\sigma_{\ell}^{2}} \frac{\pi}{\rho_{\rho}^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} + \rho + (T - 1) \right]$ $\hat{D}(\Lambda) = \frac{2\mu}{\delta\sigma_{\ell}^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} + \rho + (T - 1) \right]$ $\hat{D}(\Lambda) = \frac{2\mu}{\delta\sigma_{\ell}^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} + \rho + (T - 1) \right]$ $\hat{D}(\Lambda) = \frac{2\mu}{\sigma_{\ell}^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} + \rho + (T - 1) \right]$ $\hat{D}(\Lambda) = \frac{2\mu}{\sigma_{\ell}^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} + \rho + (T - 1) \right]$ $\hat{D}(\Lambda) = \frac{2\mu}{\sigma^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} \frac{\pi}{\rho^{2}} + \rho + (T - 1) \right]$ $\hat{D}(\Lambda) = \mu^{2}$ $\hat{D}(\Lambda) = \frac{\pi}{\rho^{2}} $	pothesis $\lambda = \sigma_{\mu}^{2} = 0$ (allowing $\rho \neq 0$)regime of the set of	ypothesis $\lambda = \sigma_{\mu}^{2} = 0$ (allowing $\rho \neq 0$)Proprint respective to the set of the set	ypothesisA $= \sigma_{\mu}^{2} = 0$ (allowing $\rho \neq 0$)pophenesisLMA_{\mu/\rho} = $\hat{D}_{\rho}^{f} \hat{f}_{\rho}^{-1} \hat{D}_{\theta} = \frac{\hat{D}^{2}(\sigma_{p}^{2})}{\det(P_{0}^{-1}) \tau^{2}(1-\rho^{2})} \left\{ \tilde{T}(3\rho^{2} - \rho^{2}T + T - 1) - \rho^{2} \right\} + \frac{\hat{D}^{2}(\Lambda)}{T\bar{D}}$ where $\hat{D}(\sigma_{p}^{2}) = \frac{\partial T}{\partial \tilde{\sigma}_{p}^{2}} _{H_{0}} = -\frac{\pi}{2} + \frac{1}{2\sigma_{p}^{2}} \hat{a}((V_{\rho}^{-1})_{T}V_{\rho}^{-1} \otimes I_{n})\hat{a}$ where $\hat{D}(\sigma_{p}^{2}) = \frac{\partial T}{\partial \tilde{\sigma}_{p}^{2}} _{H_{0}} = -\frac{\pi}{2} + \frac{1}{2\sigma_{p}^{2}} \hat{a}((V_{\rho}^{-1})_{T}V_{\rho}^{-1} \otimes I_{n})\hat{a}$ $\hat{D}(\sigma_{p}^{2}) = \frac{\partial T}{\partial \tilde{\lambda}} _{H_{0}} = \frac{1}{2\sigma_{p}^{2}} \hat{a}((V_{\rho}^{-1})_{P}V_{\rho})\hat{a}$ $\hat{D}(\Lambda) = \frac{\partial T}{\partial \tilde{\lambda}} _{H_{0}} = \frac{1}{2\sigma_{p}^{2}} \hat{a}((V_{\rho}^{-1})_{P}V_{\rho})\hat{a}$ \hat{M} statistic $\frac{\pi}{\tilde{\Omega}_{p}^{2}} = \frac{\pi}{\sigma_{p}^{2}} \hat{a}((V_{\rho}^{-1})_{P}V_{\rho})\hat{a} + \frac{\pi}{\sigma_{p}^{2}} \hat{a}((V_{\rho}^{-1}))\hat{a}$ $\hat{D}(\Lambda) = \frac{\partial T}{\partial \tilde{\lambda}} _{H_{0}} = \frac{1}{2\sigma_{p}^{2}} \hat{a}((V_{\rho}^{-1}) \otimes (W^{-1}W))\hat{a}$ $\hat{D}(\Lambda) = \frac{\partial T}{\partial \tilde{\lambda}} _{H_{0}} = \frac{1}{2\sigma_{p}^{2}} \hat{a}((V_{\rho}^{-1}))\hat{a}$ $\hat{D}(\Lambda) = \frac{\partial T}{\partial \tilde{\lambda}} _{H_{\rho}} \otimes (W^{-1}W))\hat{a}$ $\hat{D}(\Lambda) = \frac{\partial T}{\partial \tilde{\lambda}} _{H_{\rho}} \otimes (W^{-1}W)\hat{a}$ $\hat{D}(\Lambda) = $	ypothesisA = σ_{μ}^{2} = 0 (allowing $\rho \neq 0$)ypothesis $\lambda = \sigma_{\mu}^{2} = 0$ (allowing $\rho \neq 0$)LM $LM_{\lambda\mu/\rho} = \hat{D}_{\rho}\hat{I}_{\rho}^{-1}\hat{D}_{\theta} = \frac{\hat{D}^{2}(\sigma_{\mu}^{2})}{\det[I(\theta_{1})]} \frac{n^{2}}{\sigma_{\ell}^{2}(1-\rho^{2})} \left\{ \frac{T}{2}(3\rho^{2} - \rho^{2}T + T - 1) - \rho^{2} \right\} + \frac{\hat{D}^{2}(\lambda)}{Tb}$ where $\hat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{ng}{2} + \frac{1}{2\sigma_{\ell}^{2}}\hat{n}'(V_{\rho}^{-1}I_{T}V_{\rho}^{-1} \otimes I_{n})\hat{n}$ $\hat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{ng}{2\sigma_{\ell}^{2}} + \frac{1}{2\sigma_{\ell}^{2}}\hat{n}'(V_{\rho}^{-1}I_{T}V_{\rho}^{-1} \otimes I_{n})\hat{n}$ $\hat{D}(\Lambda) = \frac{\partial L}{\partial \Lambda} _{H_{0}} = \frac{1}{2\sigma_{\ell}^{2}}\hat{n}'(V_{\rho}^{-1} \otimes (W' + W))\hat{n}$ M statistic	ypothesis $\lambda = \sigma_{\mu}^{2} = 0$ (allowing $\rho \neq 0$) $\lambda = \sigma_{\mu}^{2} = 0$ (allowing $\rho \neq 0$) $LM_{\lambda\mu/\rho} = \widehat{D}_{\rho}^{2} \widehat{f}_{\rho}^{-1} \widehat{D}_{\rho} = \frac{\widehat{D}^{2} (\sigma_{\mu}^{2})}{\det[1(0_{1})]} \frac{n^{2}}{\sigma_{\rho}^{2}(1-\rho^{2})} \left\{ \frac{T}{2} (3\rho^{2} - \rho^{2}T + T - 1) - \rho^{2} \right\} + \frac{\widehat{D}^{2}(\lambda)}{Tb}$ where $\widehat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{n^{2}}{2} + \frac{1}{2\sigma_{\mu}^{2}} \hat{a}' (V_{\rho}^{-1})_{T} V_{\rho}^{-1} \otimes I_{n}) \hat{a}$ $\widehat{D}(\lambda) = \frac{\partial L}{\partial \lambda} _{H_{0}} = \frac{1}{2\sigma_{\mu}^{2}} \hat{a}' (V_{\rho}^{-1} \otimes (W' + W)) \hat{a}$	ypothesis $\frac{1}{\rho \circ the s} = \frac{1}{\rho \circ t} \frac{1}{\rho \circ t} $	ypothesisA = $\sigma_{\mu}^2 = 0$ (allowing $\rho \neq 0$)Notesis $\lambda = \sigma_{\mu}^2 = 0$ (allowing $\rho \neq 0$) $M_{\lambda\mu/\rho} = \widehat{D}' \widehat{J}_{\theta}^{-1} \widehat{D}_{\theta} = \frac{\widehat{D}^2 (\sigma_{\mu}^2)}{\det[f(\theta_1)]} \frac{n^2}{\sigma_{\theta}^2 (1-\rho^2)} \left\{ \frac{T}{2} (3\rho^2 - \rho^2 T + T - 1) - \rho^2 \right\} + \frac{\widehat{D}^2 (\lambda)}{T^b}$ where $\widehat{D} (\sigma_{\mu}^2) = \frac{\partial L}{\partial \sigma_{\mu}^2} _{H_0} = -\frac{ng}{2} + \frac{1}{2\sigma_{\theta}^2} \hat{u}' (V_{\rho}^{-1} J_T V_{\rho}^{-1} \otimes I_n) \hat{u}$	ypothesis LM _{Aµ/ρ} = $\hat{D}_{\theta}^{\prime} \hat{j}_{\theta}^{-1} \hat{D}_{\theta} = \frac{\hat{D}^{2}(\sigma_{\mu}^{2})}{\det[f(\theta_{1})]} \frac{n^{2}}{\sigma_{\theta}^{2}(1-\rho^{2})} \left\{ \frac{T}{2} (3\rho^{2} - \rho^{2}T + T - 1) - \rho^{2} \right\} + \frac{\hat{D}^{2}(\lambda)}{T^{b}}$ where	ypothesis $\lambda = \sigma_{\mu}^2 = 0$ (allowing $\rho \neq 0$)LM $\lambda_{\mu/\rho} = \hat{D}' \hat{\beta}^{-1} \hat{D}_{\theta} = \frac{\hat{D}^2 (\sigma_{\mu}^2)}{\det[f(\theta_{\mu})]} \frac{n^2}{\sigma_{\ell}^4 (1-\rho^2)} \left\{ \frac{T}{2} (3\rho^2 - \rho^2 T + T - 1) - \rho^2 \right\} + \frac{\hat{D}^2 (\lambda)}{Tb}$	ypothesis 1 M. $\lambda = \hat{\rho}_{\mu}^{2} = 0$ (allowing $\rho \neq 0$) 1 M. $\lambda = \hat{\rho}_{\mu}^{2} = 0$ (γ_{μ}^{2}) $\mu^{2} = \hat{\rho}_{\mu}^{2} + 0$)	ypothesis $\lambda = \sigma_{\mu}^2 = 0$ (allowing $\rho \neq 0$)		Two dimonstrand conditional tech continued

	Two-dimensional conditional	sts continued
othesis	$\sigma_{\mu}^2 = \rho = 0$ (allowing	≠ 0)
	$\mathrm{LM}_{\mu ho/\lambda}=\widehat{D}_{ heta}^{\prime}\widehat{f}_{ heta}^{-1}$.	
ei iii iiii iiii	where $\hat{D}_{\theta}^{i} = (0, \hat{D}(\sigma_{\mu}^{2}), \hat{D}(\rho), 0)$ with $\hat{D}(\sigma_{\mu}^{2}) = \frac{\partial L}{\partial \sigma_{\mu}^{2}} _{H_{0}} = -\frac{T}{2\sigma_{\mu}^{2}} \operatorname{tr}(B'B) + \frac{1}{2\sigma_{\mu}^{2}} \hat{a}'(J_{T} \otimes (B'B)^{2}) \hat{a}$ $\hat{D}(\alpha) = \frac{\partial L}{\partial -1} = -\frac{1}{2} \hat{a}' [G \otimes (B'B)] \hat{a}$	
	The information matrix is given by $\hat{f}_{\theta} = \begin{cases} \frac{nT}{2\sigma_{\theta}^{2}} & \frac{nT}{2\sigma_{\theta}^{2}} & \frac{T}{2\sigma_{\theta}^{2}} \operatorname{tr}(B'B) \\ \frac{T}{2\sigma_{\theta}^{2}} \operatorname{tr}(B'B) & \frac{T}{2\sigma_{\theta}^{2}} \operatorname{tr}(B'B)^{2} \end{bmatrix}$	$egin{array}{c} 0 & rac{T}{2 a_{e}^{4}} d_{3} & \ & rac{-1}{2 c_{e}^{2}} \mathrm{tr}(B'B) & rac{T}{2 c_{e}^{2}} \mathrm{tr}[W'B+B'W] & \ & n(T-1) & 0 & \ & 0 & rac{T}{2 c_{e}^{4}} d_{6} & \ & \end{array} ight)$
	\hat{u} are the restricted maximum likelihood residuals under H_0	
nptotic dis- ution under	X ²	

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Data description

Variable	Description	Data Source
	dependent variable	
unemployment rate (U_{it})	Regional unemployment rate at district level. Unemployment is defined according to regulations in Social Security Code, i.e. a person is officially registered as unem- ployed when certain requirements are ful- filled as this status is connected to the right to receive public benefits.	Federal Employment Office (Bun- desagentur für Arbeit), data can be downloaded from regional data base of Federal Statistical Office (www.regionalstatistik.de).
	explanatory variables	
market equilibrium effect	S	
Industry mix (%AGR _{it} , %IND _{it} ,)	Number of employed persons which have a job that is subject to social insurance con- tribution according to the sector in which they are working in relation to total num- ber of employed persons with a job that is subject to social contributions. The classi- fication of sectors in the version of 2003 is used which bases upon the European clas- sification (NACE Rev. 1.1)). As there is a change in the sector classification in 2008, we only use data until 2007.	Regional Database of Fed- eral Statistical Office (www. regionalstatistik.de). Some values for the federal country of Saxony-Anhalt are missing in this database. We requested them di- rectly from the regional statistical office of Saxony-Anhalt.
demographic variables		
Age structure of the population $(OLD_{it}, YOUNG_{it})$	50 until 65, respectively. The labor force consists of employed and unemployed per-	regional Database of Fed- eral Statistical Office (www. regionalstatistik.de). Some values for the federal country of Saxony-Anhalt and Saxony are missing in this database. We requested them directly from the regional statistical office of Saxony-Anhalt and of Saxony.
Foreigners (FOREIGN _{it})	Extent of foreign labor force in relation to the whole labor force in district <i>i</i> .	Regional Database of Fed- eral Statistical Office (www. regionalstatistik.de). Some values for the federal country of Saxony-Anhalt are missing in this database. We requested them di- rectly from the regional statistical office of Saxony-Anhalt.

Variable	Description	Data Source
Female labor force	Ratio of female labor force (aged 15-65) to	Unfortunately, data on female la-
participation (FP_{it})	the female resident population in the same	bor force participation is only avail-
	age.	able on the level of <i>Regierungsbezirke</i>
		(partly corresponding to German
		is the microconcus (Mikrozawawa)
		is the microcensus (<i>Mikrozensus</i>)
		tive statistics of the population and
		the labor market in Germany How-
		ever, data is only available until
		2002. We obtained the missing val-
		ues by extrapolation.
Educational attain-	Share of employed persons which have a	Regional Database of Fed-
ment of population	job that is subject to social insurance con-	eral Statistical Office (www.
$(H0_{it}, H1_{it}, H2_{it})$	tribution according to the level of educa-	regionalstatistik.de). Some
	tion. The statistic provides three levels of	values for the federal country of
	educational attainment: without any pro-	Saxony-Anhalt are missing in this
	fessional training, with a certificate of a vo-	database. We requested them di-
	cational school or certificate of a univer-	effice of Sevenu Anhalt
Commuting	Balance of incoming and outgoing com	Pagional Database of Fod
$(COMM_{\rm e})$	muters of district <i>i</i> is a if the value is posi-	eral Statistical Office (www
$(COMM_{lt})$	tive there are more people that commute in	regionalstatistik de) Some
	the district than people that commute out	values for the federal country
	of the district.	of Saxony-Anhalt and Saxony
		are missing in this database. We
		requested them directly from
		the regional statistical office of
		Saxony-Anhalt and of Saxony.
amenities		1
Population density	Population density in persons per km ² , val-	Regional Database of Fed-
$(DENS_{it})$	ues are calculated from the average popu-	eral Statistical Office (www.
	lation in every district divided by the area of every district.	regionalstatistik.de).
Public debts of dis-	Sum of public debts of all communities be-	Regional Database of Fed-
tricts ($DEBTR_{it}$)	longing to district <i>i</i> in relation to the re-	eral Statistical Office (www.
	gional domestic product of district <i>i</i> .	regionalstatistik.de).
Number of business	Number of all newly registered businesses	Regional Database of Fed-
registrations (REG_{it})	in district <i>i</i> .	eral Statistical Office (www.
		regionalstatistik.de).
Number of overnight	Number of persons that stay over night in	Regional Database of Fed-
stays in hotels	hotels, hostels, etc.	eral Statistical Office (www.
$(SIAY_{it})$		regionaistatistik.de).

Table 14: Descriptions and data sources of the dependent and all explanatory variables

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