

Matousek, Jindrich; Cingl, Lubomir

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Institute of Economic Studies, Faculty of Social Sciences
Charles University in Prague

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Jindrich Matousek
Lubomir Cingl

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Institute of Economic Studies,
Faculty of Social Sciences,
Charles University in Prague

[UK FSV – IES]

Opletalova 26
CZ-110 00, Prague
E-mail : ies@fsv.cuni.cz
<http://ies.fsv.cuni.cz>

Institut ekonomických studií
Fakulta sociálních věd
Univerzita Karlova v Praze

Opletalova 26
110 00 Praha 1

E-mail : ies@fsv.cuni.cz
<http://ies.fsv.cuni.cz>

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Collusion in Multiobject Auctions: An Experimental Evidence

Jindrich Matousek^a
Lubomir Cingl^a

^aInstitute of Economic Studies, Faculty of Social Sciences, Charles University in
Prague, Smetanovo nábřeží 6, 111 01 Prague 1, Czech Republic

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Abstract:

We experimentally examine the attributes of two complex multi-unit auction mechanisms in the presence of an opportunity to collude among bidding participants due to a provision of a simple communication channel. The results suggest that the combinatorial bidding format does not bring higher efficiency. Allowing for communication increases efficiency in both examined auction formats. Bidders are able to split the auctioned goods in a cheap-talk collusive agreement, which results in a better allocation compared to the auction formats without the communication channel. Combinatorial bidding on packages makes the decision-making problem of bidders hard to process and cause inefficiencies, especially for designs with the large number of auctioned goods.

Keywords: Combinatorial Auction, Communication, Collusion, Experiment
JEL: C91, D43, D44, L96

1 Introduction

Multi-unit auction mechanisms are one of the most important allocation instruments for complex real-life situations. Used for the allocation of spectrum licenses, airport time slots, delivery routes, networking or furniture, they are one of the few masterpieces of modern economics (De Vries and Vohra, 2003; Guala, 2001). The main concern of every auctioneer is the efficiency of the type of the auction employed that is allocating the objects for sale to those who value them the most. This is not a simple task, especially when real auctions do not always produce the same results as predicted by theory. Moreover, theoretical literature has shown that results vary within the same types but in different settings of auctions (Holt, 2005).

One of the main issues in real auctions is a possibility of collusion among bidding participants. By using coordinated strategies, bidders are able to keep prices at low levels and decrease the revenues of the auctioneer. A variety of experimental studies therefore examined the evolution of collusion in auction mechanisms (Burtraw et al., 2009; Bachrach, 2010; Zhou and Zheng, 2010; Hu et al., 2011). Generally, collusion in auctions emerges either through repeated interaction between bidders or through bidding that occurs over multiple objects (Agranov and Yariv, 2015). In multi-unit auctions bidders may coordinate their strategies in an attempt to split the objects for sale and reach a more profitable outcome for themselves than that originating out of a competitive situation (Kwasnica and Sherstyuk, 2013).

There were special mechanisms developed particularly for spectrum auctions. Such designs requiring the ability to simultaneously allocate interrelated multiple objects are mostly a generalization of the standard English auction. There is strong evidence that ascending auctions are particularly vulnerable to the collusive behavior of bidders (Klemperer, 2004). Spectrum auctions, with their usually large number of goods and ascending simultaneous nature, are therefore typical candidates for the collusive behavior of their participants. Although many rules have been adopted since the early nineties in order to control for noncompetitive practices (not only) in simultaneous auctions, there are still opportunities for collusion through simple communication that are difficult to prevent.

Collusion in complex simultaneous auctions of multiple goods has not yet been properly examined for mechanisms with more than only a few objects

for sale (Kwasnica and Sherstyuk, 2013). On the one hand, large scale auctions may offer enough possible combinations for bidders to find profitable collusive allocation, but on the other hand it can create coordination problem that is too complicated for colluding bidders to cooperate successfully (Kwasnica and Sherstyuk, 2013). Although there is evidence that allowing for combinatorial bidding on packages of goods may break collusion in multi-unit auctions, the existing literature differs in its conclusions. The first current of literature represented by Brunner et al. (2010) discovers that combinatorial bidding increases the efficiency of simultaneous auctions. The second current of literature represented by the Bichler et al. (2014) or Goeree and Holt (2010), however, results precisely in the conclusion that combinatorial formats with a high number of goods at stake are not computationally manageable for their participants and their efficiency is therefore significantly lower compared to basic simultaneous multi-round auctions.

Therefore we fill the gap in the literature and experimentally evaluate the performance of two simultaneous auction mechanisms for selling multiple goods and combine it with the dimension of communication facilitating potential collusion among bidders in the auction. We compare the simultaneous multi-round auction format (SMR) and its combinatorial extension, the simultaneous multi-round package bidding auction format (SMRPB) within the framework of a communication channel, thereby employing a 2x2 design resulting in four treatments. We extend the design used in the previous literature (Brunner et al., 2010). Our design involves more than fifty units of goods distributed in four heterogeneous types. Although restricted by the activity limit, four bidders participating in each auction have enough possible combinations to develop a stable collusive equilibrium.¹ The possibility of communication is introduced through a simple chat window. We do not use any binding commitments or transfer promises in our design.

Our main result is that communication generally increases the efficiency of auctions in our experimental design. Regardless of the number of possible combinations, bidders tend to collusive payoff-maximizing strategies. They split the goods in stakes and reach stable noncompetitive equilibrium. The relative efficiency of the simultaneous multi-round auction format is generally higher than the one of its package bidding alternative, especially in

¹Our design provides, without restrictions, over 36 billion possible outcomes: $C(4; 6) \cdot C(4; 24) \cdot C(4; 14) \cdot C(4; 9) = 36.756.720.000$ possible combinations.

treatments allowing for communication.

2 Literature

2.1 Concepts of Multi-Unit Auctions

The SMR auction format was originally developed in the early nineties by the US Federal Telecommunications Commission (FCC) for their spectrum auctions. The description of the process of designing, testing and implementing the FCC auctions as one of the few cases of complex economic engineering is available in [Guala \(2001\)](#).

[Brunner et al. \(2010\)](#) deals with the different formats in flexible combinatorial spectrum auctions in complementarities environments. They compare a widely used simultaneous multi-round auction with three other formats: the simultaneous multi-round format with package bidding SMRPB, combinatorial clock (CC) auction and the "Resource allocation Design" the RAD auction. They used a series of laboratory experiments to evaluate these alternative multi-unit auction formats in the article. The results of their experiments suggest that all three combinatorial auction procedures are more efficient than the SMR auction format when value complementarities are present. As the interrelation of auction objects is a common feature not only of the spectrum auctions, this finding is crucially important for practice. In addition, all formats in their setting had different results in terms of efficiency as well as sellers' revenue.

[\(Bichler et al., 2013\)](#) cast doubt on mainstream literature results mentioned above and provide results favoring the combinatorial auction formats in efficiency. They compare the Combinatorial Clock auction to SMR. They analyzed the efficiency of the auction methods and the auctioneer's revenue. They also examined bidding behavior in both cases. Their experiments are based on two value models resembling single and multi-band spectrum auctions which often offer thousands of possible bundles. The efficiency of the CC auction was significantly lower than that of the SMR in the multi-band model in their case. Moreover, auctioneer revenue was lower in both value models for CC. The second recent paper dealing with high numbers of auctioned goods in the same multi-band models is [\(Bichler et al., 2014\)](#). They found that the simplicity of bid language has a substantial positive impact on the efficiency of the auction. Moreover, the simplicity of the payment

rule has a substantial positive impact on auction revenue. The CC auction scores the worst in both dimensions in their experiment, favoring the simple SMR auction format. Such results are in contradiction with [Brunner et al. \(2010\)](#) or, for example [Cramton \(2013\)](#), who prefer combinatorial bidding auction formats.

[Goeree and Holt \(2010\)](#) suggest that even though the combinatorial auction can solve the problem of license packaging by letting competition among the bidders determine the market structure, the decision-making problem in big complicated auctions that all rational bidders have to accomplish after each round can be computationally hard to do. It claims that: "*...bidders will not be able to reproduce the outcome of a round to understand why their bids did not win, unless they solve a non-deterministic polynomial-time hard problem quickly*"² ([Goeree and Holt, 2010](#), p. 3). [Goeree and Holt \(2010\)](#) then propose a new hierarchical package bidding (HPB) combinatorial auction format which should be computationally manageable. The general result of this article is that the proposed HPB format is a "*paper & pencil*" package auction format. It is trivial to implement, transparent and easily verifiable by the bidders [Goeree and Holt \(2010\)](#).

2.2 Collusion in Auctions

Collusion in auction mechanisms has been studied for years ([Robinson and Robinson, 1985](#); [Crawford, 1998](#); [Kwasnica and Sherstyuk, 2013](#)). Bidding rings are a theoretically well-described method of collusion ([Krishna, 2009](#)). Traditionally, various single-unit auction mechanisms where collusion has implied explicit communication among bidders have been the object of research interest. Experimental studies confirms further that collusion can and actually does occur with communication present in single-unit auctions. There is strong evidence that mainly ascending auctions are very vulnerable to collusive behavior and very likely deter entry into the auction ([Klemperer, 2004](#)). Bidders simply tend toward collusive payoff-maximizing strategies. The tacit form of collusion, during which participants silently coordinate on low-price outcomes, has been widely observed and described theoretically ([Skrzypacz and Hopenhayn, 2004](#)).

Recent auction literature concentrates on sequential and multi-round auc-

²We have observed exactly this type of situations during the execution of the experiment.

tion formats which appear to be the ones that most lead to bidder conspiracies (Kwasnica and Sherstyuk, 2013). In the multi-unit auctions, bidding agents can silently split objects and keep the competition on low levels throughout the auction. A large number of bidders are not a sufficient condition for hindering collusion as long as the bidders can share among themselves a sufficiently large number of the goods sold in the auction. Moreover, depending on the parameter setting of each individual auction, the multi-unit nature of auctions usually introduces complexities into the environment and the outcomes are therefore uncertain (Kwasnica and Sherstyuk, 2013).

Miralles (2008) analyzes a generalization of Campbells self-enforced collusion mechanism in simultaneous auctions. While Campbell (1994) based his collusion mechanism on complete comparative cheap talk and endogenous entry with only two bidders, Miralles (2008) examine cases of more than two bidders with prior symmetric design. He focuses on self-enforced and simple mechanisms without side-payments or trigger strategies. He uses just a pre-play cheap talk, which is "*clearly difficult to prosecute by competition authorities*" Miralles (2008, p. 2). Two important results arise out of the analysis: (i) a cheap talk equilibrium exists if the number of objects is large enough; and (ii) a partial cheap talk equilibrium, in which each bidder splits the objects into two sets, the favorite one and the rest, and lets the other bidders know about that split, always exists.

Agranov and Yariv (2015) experimentally study collusion through communication in one-shot first- and second-price sealed bid auctions with two bidders. The results of their research suggest that communication alone can affect auction outcomes dramatically. They documented two strategies in their simple one-by-one cheap talk auction environment. The *reveal-collude* strategy in which the players reveal their valuations and consequently collude can potentially be applicable also to our multi-unit case. Regardless of the strategy players used in their experiment, communication led to significant price drops reducing auction revenues by up to 33%.

3 Hypotheses

Based on the previous literature, we assume that in the first treatments without communication, the degree of competition should be high. We employed a setting with high complementarities among goods, which should favour the

combinatorial SMRPB format in efficiency. We expect the prices to reach competitive levels. High revenues for the auctioneer and low or even negative surpluses of bidders should occur.

The second wave of treatments introduces communication. In the extreme case of coordinated collusion we expect the prices to stay down at the upset base, similarly as in [Valley \(1995\)](#) in the case of double oral auction. If the collusion equilibrium is stable, no or very little competition among bidders should be present resulting in a shift in the distribution of rents from auctioneer revenues to bidders' surpluses as e.g. in ([Agranov and Yariv, 2015](#)). We are interested in whether allowing for combinatorial bidding will break collusion. The package-bidding format SMRPB should in that case increase competition. Prices should go up, approaching more the competitive level. Respective change in rents distribution should appear. The impact of communication on efficiency is ambiguous. Collusive behavior may, under certain circumstances, increase the efficiency, when collusive agreements result in a better allocation of goods among players than competitive processes.

Table 1: Comparison of Partial Hypotheses within Formats

	SMR			SMRPB		
	Basic		Comm	Basic		Comm
Total prices	P	$>$	P	P	$>$	P
Efficiency	E	$?$	E	E	$?$	E
Auctioneer's revenue	R	$>$	R	R	$>$	R
Bidder's surpluses	S	$<$	S	S	$<$	S

Note: Basic - without communication; Comm - with communication

Table 2: Comparison of Partial Hypotheses between Formats

	BASIC			COMM		
	SMR		SMRPB	SMR		SMRPB
Total prices	P	$<$	P	P	$<$	P
Efficiency	E	$<$	E	E	$?$	E
Auctioneer's revenue	R	$<$	R	R	$<$	R
Bidder's surpluses	S	$>$	S	S	$>$	S

Note: Basic - without communication; Comm - with communication

Table 1 summarizes comparison of partial hypotheses *within formats* and

Table 3: General Hypotheses

Null H_0		Alternative H_A	
SMR	$<$	$SMRPB$	$SMR > SMRPB$
SMR^{comm}	$<$	$SMRPB^{comm}$	$SMR^{comm} > SMRPB^{comm}$
SMR	$>$	SMR^{comm}	$SMR < SMR^{comm}$
$SMRPB$	$>$	$SMRPB^{comm}$	$SMRPB < SMRPB^{comm}$
E_{SMR}	$=$	E_{SMR}^{comm}	$E_{SMR} \neq E_{SMR}^{comm}$
E_{SMRPB}	$=$	E_{SMRPB}^{comm}	$E_{SMRPB} \neq E_{SMRPB}^{comm}$
E_{SMR}^{comm}	$=$	E_{SMRPB}^{comm}	$E_{SMR}^{comm} \neq E_{SMRPB}^{comm}$

setting *without* and *with* communication allowed in the auction. Table 2 summarizes comparison of partial hypotheses *between formats* and setting *without* and *with* communication allowed in the auction.

Table 3 summarizes main hypotheses of the experiment. Basic SMR auction should overall perform worse than its combinatorial counterpart basic SMRPB format. The same relationship should be valid also for these two formats while allowing for collusive behavior; the SMRPB should, therefore (to indistinct extent), break the collusion. Generally, both formats should naturally perform better without the collusion. However, the performance of both auction formats in terms of efficiency is in all treatments (with exception of basic SMR and SMRPB settings) uncertain.

4 Methodology

4.1 Experimental Design

In a fully computerized laboratory experiment, we employ the simultaneous multi-round auction format (SMR) and compare it with its combinatorial version, the simultaneous multi-round package bidding (SMRPB) format, to see the effect of package bidding on efficiency and revenues and evaluate the original policy format with its most natural extension. Next we incorporate the dimension of communication by implementing a simple chat window into both SMR and SMRPB auction formats and therefore allow for coordinated strategies in the experiment. The treatment matrix of the experimental design is shown in Table 4.

There are four heterogeneous types of goods in our experiment (A; B; C; D). Each type of the goods has multiple homogeneous units in stock. Each

Table 4: Treatment Matrix

	SMR	SMRPB
<i>I.set</i>	no communication	no communication
<i>II.set</i>	communication all	communication all

of the four players who were competing in the tender was assigned her own personal valuations for each type of the goods sold in the auction. These were determined randomly by the procedure specified below. At the end of the auction, players either earn profit or incur losses in the experiment.

4.2 Basic Auction Formats

The SMR auction format is a simple generalization of the ascending English auction designed for simultaneous allocation of multiple objects. The auction proceeds in a sequence of rounds in which the bidders submit their bids separately for individual items. The process continues until nobody is willing to submit a higher bid for any item. The SRMPB is a combinatorial auction format originally designed to prevent the exposure risk of bidders. The provisional winning bids in each round are calculated according to maximization of the revenues for the seller. Each bidder can have only one provisionally winning bid in each round at maximum (Brunner et al., 2010).³

In both auction formats each bidder is eligible to act in only a limited number of possible actions during each round. This number of actions is constrained by the amount of activity points she has at her disposal. This rule ensures that if the bidder wants to play seriously and win her portion of desired goods at the end, she has to maintain the activity throughout the whole auction. If not, she would lose the activity points and eligibility for the subsequent rounds. Regarding activity, we follow the procedures used for example in Cramton (2013) or Brunner et al. (2010).

In order to prevent signalling via the determination of prices, a simple system of proportional ascending bidding was introduced into the simultaneous multi-round auctions. There is a one-level raising algorithm in the program used for our experiment. Bidders can either keep their previous

³An exclusive XOR (logical exclusive-or) rule is imposed on the bids made in the auction rounds.

bid or raise their bid for some goods by 20% of the respective upset price.⁴ Bidders can withdraw their provisionally winning bids in case they would win an unwanted (e.g. incomplete) set of goods in an auction round. All bidders can withdraw their provisionally winning bids in at most 2 rounds of the auction.

The winner determination algorithm (WDA) in the case of excess demand is, in both SMR and SMRPB formats, provided by random mechanism. All four types of the goods are handled separately in SMR while in the SMRPB the items are handled in packages.⁵ At the end of each round, bidders receive information about the provisionally winning bids in the current round. The identity of the provisionally winning bidders is known. The bidders also have complete information about their own bids.

4.3 Communication

The communication channel is introduced via a simple chat window. No verbal contact between participants was allowed during the experiment. All communication was monitored and recorded. This approach was already used in for example [Phillips et al. \(2003\)](#).

A combination of [Phillips et al. \(2003\)](#); [Lopomo et al. \(2005\)](#); and [Miralles \(2008\)](#) was used to implement communication via chat into the experimental design. Enough time for communication⁶ was provided prior to each auction as in [Miralles \(2008\)](#). Since the number of objects in the auction is large, the comparative cheap-talk equilibrium should, according to [Miralles \(2008\)](#), exist. The chat window was also available during the whole auction phase as in [Phillips et al. \(2003\)](#).

In order to be consistent with [Lopomo et al. \(2005\)](#), there was only a limited amount of information available to the bidders in the pre-auction phase of communication. The bidders did not know their exact valuations

⁴Even though we are aware that setting the level at 20% was probably too high, we could not make it lower since we had to obtain at least some small number of observations per session.

⁵The winner determination algorithm applied in the program is simple. Each player involved in the problem is assigned a random number. The player with the smallest number wins and is allocated the item. There is a mechanism sorting the packages according to its highest price in the SMRPB. The loser resulting from WDA in SMRPB auction is put into a subsequent place and if she satisfies the conditions for winning her package out of remaining goods, she wins it. This process continues down to the bottom if needed.

⁶The communication window was enabled two minutes prior to the auction.

for goods and therefore were able to communicate only on the collusive mechanisms they could employ, not directly on their own private values. There was, however, no ex post budget balancing in our experiment; the effect of collusion on auction efficiency is therefore beforehand ambiguous. The only information revealed to the participants during the pre-auction communication phase was the types and numbers of auctioned goods. All three features facilitating collusion from Phillips et al. (2003) were therefore satisfied.

4.4 Valuations, Complementarities and Final Profit

There were four types of players in the experiment. Such market structure can be found in various countries and industries and was modelled for example in experiments done by Bichler et al. (2013) or by Abbink et al. (2005) who used two symmetric tetrads of bidders in their experiments. For each player-type, its valuations of goods were randomly drawn from a publicly known interval (the same interval for all players so there was an ex-ante symmetrical setting for each auctioned goods) prior to the experiment only once. Then in each session, each subject of the experiment was randomly assigned to one player type. This prevented any additional external variation between the treatments possibly caused by the random draws of valuations on place.

Valuations of goods were based on two components. The first part represents common value component (CVC), the second represents private value component (PVC) of each particular unit of goods. Common value arises from the overall market potential and is the same for all players, private value stems from the private expected profits depending on the individual potential of the bidder's business concept (Abbink et al., 2005). The bidder's total valuation for goods was therefore the sum of her CVC and PVC as in (Abbink et al., 2005).

The CVC of the signal was for each type of the goods randomly drawn from the integer interval. Players did not have the information about the exact random draw of CVC nor did they know the interval boundaries from which it was drawn. Each bidder received instead an independent private signal on the CVC and was informed about the fact that these signals were determined by uniform random draws from the integer interval $[CVC - \alpha; CVC + \alpha]$ ⁷ (Abbink et al., 2005, p. 511).

⁷The exact intervals for CVC in (Abbink et al., 2005) were [1000; 1500] for the CVC

The PVC of the signal was, for each type of the goods, randomly drawn from the integer interval $[-\beta; +\beta]$. The parameter β was proportionally lower to the CVC component. Each bidder was informed about her own PVC.

For modelling the complementarities in player-type valuations of goods we followed (Brunner et al., 2010). The interrelations among goods are modelled in a linear manner. If the player acquires multiple goods, then the value of each unit of goods raises by a factor of $[1 + \alpha(K - 1)]$, where K stands for the number of types of goods won and the α is the synergy factor. The player should, therefore, be motivated to win all four types of the goods.

Since we assume a high level of complementarities among the types of goods we set the synergy factor equal to 0.1. This setup ensures that if a bidder wins all four types of goods, his valuation for all of them rises by 30%.

The final profit was determined for each player at the end of the auction by the difference of her total valuation for all goods won in the auction and the total price paid for those goods. There was no endowment assigned to the players since the budget constraint is irrelevant in the experimental design. Bidders either earned profit or incurred losses in the experiment.

4.5 Efficiency Measurement

We measure and compare the efficiency levels of individual auction formats with different collusive properties. We use the efficiency measurement mechanism employed in the Goeree and Offerman (2002), who studied efficiency in auctions with private and common values. We generalize this approach to the multiple-object case by taking the average across all n experimental goods sold in the auction. The efficiency is therefore determined as follows:

Let PVC_{winner} denote the private value component of i -th unit of goods of the auction winner and let PVC_{max} and PVC_{min} be the private value component of i -th unit of goods of subject with maximum and minimum valuation of this type of goods in the group, respectively. Then the partial efficiency for each unit of goods i sold in the auction is measured by the Equation 1

$$e_i = \frac{PVC_{winner} - PVC_{min}}{PVC_{max} - PVC_{min}}, \forall i \in 1; \dots; n \quad (1)$$

interval and $[CVC - 200; CVC + 200]$ for the independent private signal known to the bidders. We assume quite a similar setting in our experiment.

Subsequent equation (2) represents the average taken across all partial efficiencies of the auction format and measures the efficiency E of the auction format.

$$E = \frac{\sum_1^n e_i}{n} \cdot 100\% \quad (2)$$

The absolute measure of efficiency E provided by the equation (2) may not be directly comparable with different efficiency measures in the literature. We therefore normalized this efficiency measure by the optimal allocation with the maximum possible degree of efficiency to obtain a comparable parameter. The optimal allocation was obtained by the maximization of quantities of goods subject to (I) efficiencies per unit e_i from the Equation 1 and (II) activity points at disposal for each player type.⁸ The partial efficiency is obtained by taking the sum of quantities times the efficiency per respective unit over all player types: $e^p = \sum_1^n q_i p_i$, where n is the number of player types and i number of types of goods. Total optimal efficiency is then calculated by summing up all partial efficiencies and dividing it by the total number of goods in the auction, that is by 53. The efficiency of optimal allocation resulted in 0,95692. The total absolute efficiency (e) of each observation was then normalized ($e_{optimal}$) resulting in the relative efficiency measure (e_r).

4.6 Parametrization

Experimental researchers refer to the real life situations while conducting experiments on combinatorial auction formats. [Abbink et al. \(2005\)](#) explore the design alternatives for the British 3G/UMTS auction; or more recently [Bichler et al. \(2013\)](#) use a band plan with two bands of blocks, which can be found in several European countries, in their base value model. The parameters used in our experiment are based on the real situation of the Czech Spectrum Auctions held in 2012 and 2013. The parameters were adjusted and simplified in order to be applicable in the experimental design.⁹

Since the information about the upper threshold is known only for the whole set of auctioned goods, it is necessary to determine a percentage parameter, which will always be added on top of the upset price of each particular type of the goods to obtain its common value component. The calcula-

⁸The optimal allocation is provided in the appendix to this paper.

⁹The respective table of parameters is provided in the appendix to this paper.

tion of this parameter δ in Equation 3 resulted in 5645 million CZK, which is approximately 65% of the total upset price for all goods sold in the auction.

$$\delta = \frac{B_{upp} - B_{low}}{2} \cdot \frac{1}{\sum p_{upset}} \cdot 100\%, \quad (3)$$

where B_{upp} and B_{low} are upper and lower bound respectively; and p_{upset} are upset prices of goods. The common value component of each type of the goods is therefore calculated by a 1.65-multiple of its upset price. The common value component is the same for each player. The private value component is different for each player and is determined by a random draw from the interval $[-0, 1 \cdot CVC; +0, 1 \cdot CVC]$.

The activity points per one unit of goods used in the experiment are determined by taking the respective activity per block in spectrum interval from the real parameters of the auction and rounding it up to integers. Total activity in the experiment is therefore slightly higher than in the real situation but it is more convenient for experimental purposes. Each player has her initial activity based on the $\frac{1}{4}$ of total activity in the experiment while her precise activity endowment is determined by a random draw from the interval $[-3; +3]$ ¹⁰, which is added to the $\frac{1}{4}$ of total activity in the experiment.

The following tables summarize the final experimental parameters. Table 5 shows final individual valuations for goods and the final endowment of activity points for each player. Table 6 summarizes the common value component and its private signal intervals.

Table 5: Final parameters determined by random draws

Players	Valuations for goods				Activity endowment
	A	B	C	D	
Blue	1886	53	138	48	27
Pink	1727	53	140	47	28
Red	1900	47	130	46	26
Green	1865	48	138	50	25

¹⁰Each tail of this interval represents a rounded 10% of the $\frac{1}{4}$ of total activity in the experiment.

Table 6: Common Value Component and Private Signal Intervals

Goods	CVC	CVC variance	CVC private signal interval
A	1820	200	[1620 ; 2020]
B	50	5	[45 ; 55]
C	140	10	[130 ; 150]
D	50	5	[45 ; 55]

4.7 General Procedure of the Experiment

We conducted a computerized laboratory experiment with four experimental sessions. We engaged 24 subjects per session, resulted in 96 subjects in total. The experiment was performed in the Laboratory of Experimental Economics at University of Economics in Prague.¹¹ The experiment was fully computerized using the program Z-TREE (Fischbacher, 2007).

The subject pool for the experiment was invited through the online ORSEE system of Laboratory of Experimental Economics (Greiner, 2004). Additional criteria were imposed on the selected subject pool in order to ensure they would understand the task and would be capable of taking part in the experiment; specifically we preferably invited economics majors with previous experience in auction experiments. The experiment was conducted in Czech.

Subjects were paid according to their performance in the experimental treatment. Each treatment lasted approximately two hours and the average pay for the whole treatment was expected to be on average 500 CZK¹² per subject, which was above the students' regular hourly wage rate. Prior to the experiment itself, we ran a pilot-version to verify the structure of the experiment and the functioning of the programs and to calibrate the task.

4.7.1 Instruction Procedures

The complexity of the required task was expected to be highly demanding. We were not able to train subjects specifically before the experiment and carry out the complicated procedures used for example in Abbink et al.

¹¹(LEE at VŠE); www.vse-lee.cz/eng

¹²The resulting levels of competition during the experimental auctions and random draws determining the treatments for payments resulted in a lower average payoff than 500 CZK. Payoffs from all treatments were on average 400 CZK per subject.

(2005); Brunner et al. (2010) or even Bichler et al. (2013). This was mainly due to the necessity of high over-recruitment rates in the case of such training and highly constrained funding of the research. Therefore, we used a simpler procedure instead.

The participants received an invitation five days prior to the experiment and three days prior were asked to fill in an online questionnaire based on the partial instructions available online. This online material consisted of general instructions common to all treatments of the experiment. The instructions were concluded with a 5-question quiz. Each invited participant who had filled in the questionnaire correctly was preferred in our invitation to the experiment we conduct in the lab. The whole procedure regarding the instructions in advance and the questionnaire was described in the invitation email for the experiment and was therefore publicly known. There were no difficulties with the online questionnaires since the rate of successful completion was over 95%.

4.7.2 General Procedure

For each session, a group of 24 participants in the experiment came into the lab and randomly drew the number of their seats. Each subject was seated at the respective computer station with no possibility to see anybody else's screen or to talk to each other. This rule was strictly enforced for entire experiment. The participants were provided with (I) a set of written general instructions for the experiment (the same set which they should already have seen in the online questionnaire); (II) treatment-specific supplement to the instructions¹³; (III) consent form¹⁴; (IV) pencil and blank sheet of paper for notes. Participants had 15 minutes for self-study of the instructions when they arrived to the laboratory. A computerized questionnaire with several control questions was launched for all subjects after this time expired. A practice auction round was conducted in order to be sure subjects understood the experimental interface, how to read their parameters, enter bids on the screen, and that they were acquainted with auction procedures.

To prevent misunderstandings and make the task easier, only one type of auction (one treatment) was performed in each session, that is between sub-

¹³Complete sets of instructions are available in the appendix.

¹⁴If the participant refused to give consent for the experiment, she was paid the show-up fee and sent away.

ject design was used. In each session, each participant was randomly assigned to one of four player types, which remained stable across the whole session. It ensured that no additional external variation caused by random draws was present. The player types were then randomly assigned to the groups of four players who competed in the auction among themselves. Each participant took part only in one auction format, while there were multiple auctions performed within the session and therefore within the auction format.¹⁵ All groups were randomly re-matched with the condition of stranger matching at the beginning of each auction within one session.

There was a predefined exchange rate of experimental currency units (ECU) and real money in the experiment. Participants knew this exchange rate in advance from the instructions. The payment from the experiment was not aggregated over all auctions executed in a session but rather depended on one specific session round determined by a random draw.

When the participants accomplished the experimental task and the auction was over, they were called separately to an adjacent room, where they were paid in private and then left.

4.7.3 Experimental Task

Subjects participated in the auctions within the treatments they attended. The objective of the task was to win the desired goods in the auction and gain a profit which was then converted to real money at the end of the session. At the very beginning of each auction a chat window was displayed for two minutes in the two treatments with communication. After the chat window, a screen with subject-specific experimental parameters was shown for one minute in all treatments. Then the first auction round began. The auction itself progressed in a series of simultaneous rounds where players were bidding for the collections of goods of their interest. Bidding was made by adding the goods in the bidding basket. Players could submit their baskets within the auction round time limit of two minutes. There was an auction interface with parameters for all goods; bidding basket; player's personal account; history of past rounds and in respective treatments also a chat window displayed on the auction round screen.

After all players submitted their bids, the system executed all background

¹⁵The number of manageable auctions stabilized at three per experimental session.

tasks and the summary of the auction round was displayed. Each player received complete information about her resulting situation in current the auction round and from the previous auction round (her provisionally winning goods). The history of past rounds and in respective treatments also a chat window were displayed on the summary round screen. There was a button for opening the bid withdrawal interface implemented in this stage of the round. By entering this interface, players could withdraw any of their provisionally winning goods in SMR.¹⁶ When the one minute time limit for the summary phase ran out or when all players clicked the proceed button, the next round began. The whole process was repeated until the final round of the auction in which no player submitted any higher bid.

5 Results

Table 7 shows sample statistics of the most important variables between treatments. The results of all auction treatments are different from each other. Generally, treatments allowing for communication among players score better in all experimental parameters than their non-collusive counterparts. A comparison of basic formats without the communication channel shows that both formats are statistically identical in terms of efficiencies (although SMR performs slightly better and total prices paid by the bidders (p-value 0.9518)).¹⁷ However, there arises a difference in the average final profits gained by the players of those treatments (p-value 0.0012). Generally, a lower number of goods is sold in the basic treatments since competition drives prices to higher levels and weaker players are forced to fall behind. The difference between SMR and SMRBP may be caused by the fact that facilitating the exposure risk by allowing for package bidding can further decrease the total number of goods sold in the package-bidding format. While bidding on packages of goods, the weaker players, who are outperformed by stronger ones, do not buy any goods at all rather than buying only a subset of goods they would be interested in. This decreases total quantities sold in the basic package-bidding format with respect to basic SMR and therefore affects the average of final profits in a negative manner. Lower total

¹⁶The whole package in SMRBP treatments.

¹⁷We use the two-sample t-tests with equal variances for the analysis of variable differences and check all results with a non-parametric analogy for non-normally distributed dependent variables, the Wilcoxon-Mann-Whitney test (W-M-W test).

quantities and seller revenues support the results of [Banks et al. \(2003\)](#).

Table 7: Sample Statistics Between Treatments

	SMR	SMR comm	SMRPB	SMRPB comm
Variable	Mean			
relative efficiency	0.18	0.41	0.14	0.23
final profit	174.57	597.00	-23.38	500.09
total prices	1466.28	2672.44	1445.89	1891.28
revenues	5865.11	10689.78	5783.56	7565.11
Variable	Standard Deviation			
relative efficiency	0.16	0.15	0.10	0.15
final profit	356.86	416.17	361.96	669.71
total prices	1858.58	1260.92	2168.78	1810.49
revenues	3414.89	2435.19	4080.25	3031.87

Comparing two basic formats against their representatives with communication channels favors resolutely the collusive treatments. Statistical differences arise in relative efficiencies (p-values 0.000); final profits (p-values 0.000); and total prices paid by the bidders in SMR (p-value 0.000). While allowing for communication, players do not let the prices go up. By splitting the goods in stakes, they are able to buy more goods altogether and increase their profits substantially. With more goods sold for lower prices higher revenues for the auctioneer occur. An interesting fact about efficiency can be tracked while comparing basic with collusive treatments. The relative efficiency is actually increased by the lower competition among players since the players are able to split the goods more accurately and therefore reach an allocation that is more efficient.

Comparing the SMR and SMRPB formats with communication generally favors the SMR format without package bidding. Statistical differences arise in relative efficiencies (p-values 0.000); and total prices paid by the bidders (p-value 0.0031). The players usually set a collusive agreement on splitting the goods in some way favorable for all. However, many try to divert from these agreements in an attempt to win more and gain higher profits. Prices gradually rise as the agreements are broken and weaker players again fall behind. Overall fewer units of goods are sold on average in the SMRPB with collusion than in the SMR with collusion. Total prices are therefore

lower and so are the revenues for the seller.¹⁸

5.1 Main Findings

In the first wave of treatments without communication, the degree of competition is higher than in the collusive treatments. Even though we employ a setting with high complementarities among goods, the combinatorial SMRPB format scores generally worse in efficiency, which is in contrary to [Brunner et al. \(2010\)](#). Prices reach competitive levels, weaker bidders fall behind the players with higher valuations. The revenues for the auctioneer are generally low due to high competition, which favors only strong bidders. Fewer goods are sold and the higher prices can not compensate the losses. Surpluses of bidders result in low or even negative levels. The SMRPB format resolves the exposure risk. The winners curse is observed in both auction formats.

The second wave of treatments introduces communication. Several cases of coordinated collusion appear during the experiment in which prices remain very low or even at the upset base. This fact confirms the results of [Valley \(1995\)](#) in the case of double oral auction or those of [Agranov and Yariv \(2015\)](#) in the case of first- and second-price sealed bid auctions. There are also cases in which some bidders try to break the agreement with others. Such cases are occasionally successful, resulting in an overall decrease in efficiency. Generally, no or very little competition among bidders is present. There is no shift in distribution of rents from revenues of the auctioneer to bidders' surpluses; the observed pattern demonstrates rising surpluses and revenues at the same time. With lower competition within the treatments with a communication channel, more goods for lower prices are sold. Higher quantities of goods sold more than compensate for lower prices.

There is evidence that allowing for combinatorial bidding in the SMRPB format breaks collusion. Some players try to divert from the collusive agree-

¹⁸Not only treatment-specific differences influence the results of the experiment. Not all players are of the same strength. The second type of player has only a limited chance to outplay others in terms of final profits due to the parameter setting. There appear to be significant differences in achieved efficiency among individual player types. The first type of player has a statistically higher rate of average cumulative efficiency (by 6 %, that is 2.5 times more) and final profit (by 49%) than others do. The situation is the opposite for the third player since her average cumulative efficiency is significantly lower (by 6 %, that is 7 times less) together with total price paid (by 24.7%); in other words the third player buys fewer goods on average.

ment in the package-bidding format in an attempt to win more goods and gain higher profits. Prices gradually rise as the agreements are broken and weaker players fall behind resulting in almost competitive situations. The package-bidding format therefore increases competition. The communication has a positive impact on efficiency in both auction formats. Players are able to make an agreement and split the goods at stake among themselves. Successful collusive agreements result in a better allocation of goods among players and therefore higher efficiencies.

Table 8 summarizes the results of partial hypotheses *within formats* and setting *without* and *with* communication allowed in the auction. Table 9 summarizes the results of partial hypotheses *between formats* and setting *without* and *with* communication allowed in the auction.

Table 8: Comparison of Partial Hypotheses within Formats - Results

	SMR		SMRPB			
	Basic	Comm	Basic	Comm		
Total prices	P	$<^{***}$	P	P	\leq	P
Efficiency	E	$<^{***}$	E	E	$<^{***}$	E
Auctioneer's revenue	R	$<^{***}$	R	R	$<$	R
Bidder's surpluses	S	$<^{***}$	S	S	$<^{***}$	S

Note: Basic - without communication; Comm - with communication

Table 9: Comparison of Partial Hypotheses between Formats - Results

	BASIC		COMM			
	SMR	SMRPB	SMR	SMRPB		
Total prices	P	$=$	P	P	$>^{***}$	P
Efficiency	E	\geq^{**}	E	E	$>^{***}$	E
Auctioneer's revenue	R	$=$	R	R	$>^{***}$	R
Bidder's surpluses	S	$>^{***}$	S	S	$=$	S

Note: Basic - without communication; Comm - with communication

Table 10 summarizes the main hypotheses of the experiment and presents their results based on the two-sample t-tests. The basic SMR auction overall performs at approximately the same level with its combinatorial counterpart basic SMRPB format. However, when allowing for communication, these two formats substantially differ. The SMR scores better in terms of efficiency and produces higher revenues to the seller with lower final prices and

approximately the same surpluses of the bidders. Even though SMRPB can prevent collusion, generally, we can say that the SMR auction format scores a better result in our experiment than the SMRPB. Both formats score better while allowing for the collusive behavior of its participants. The overall results are presented from an experimental point of view and therefore do not consider the socially inadmissible nature of collusion among players.

Table 10: General Hypotheses - Results

Null H_0		Alternative H_A		Result
SMR	$< SMRPB$	SMR	$> SMRPB$	not rej.
SMR^{comm}	$< SMRPB^{comm}$	SMR^{comm}	$> SMRPB^{comm}$	rejected
SMR	$> SMR^{comm}$	SMR	$< SMR^{comm}$	rejected
$SMRPB$	$> SMRPB^{comm}$	$SMRPB$	$< SMRPB^{comm}$	rejected
E_{SMR}	$= E_{SMR}^{comm}$	E_{SMR}	$\neq E_{SMR}^{comm}$	rejected
E_{SMRPB}	$= E_{SMRPB}^{comm}$	E_{SMRPB}	$\neq E_{SMRPB}^{comm}$	rejected
E_{SMR}^{comm}	$= E_{SMRPB}^{comm}$	E_{SMR}^{comm}	$\neq E_{SMRPB}^{comm}$	rejected

6 Conclusions

We study complex auction mechanisms under the possibility of communication between agents during the course of the auction process. We experimentally investigate two simultaneous auction formats: (i) the Simultaneous multi-round auction (SMR) format and (ii) its extension allowing for combinatorial bidding, the Simultaneous multi-round package bidding (SMRPB) auction format. Two basic treatments provide the benchmark to the experiment and two additional treatments introduce a possibility of collusion via a chat window incorporated into the auction interface; a simple self-enforced communication channel that does not require any additional procedures. Four bidders participate in an auction for multiple heterogeneous types of goods in each experimental treatment while the total number of auctioned goods exceeds fifty in each auction. We study four fundamental variables in the experiment: the relative efficiencies of the auction formats, the total price paid by the bidders, their final profits, and the auctioneer's revenue.

All auction treatments are different from each other. The competition in basic treatments drives prices to higher levels in comparison with communication treatments and consequently lower amounts of goods are sold

during the basic treatments than in treatments where communication is enabled. This situation occurs in both the basic SMR and SMRBP treatments resulting in substantially lower efficiencies, auctioneer revenues and bidder surpluses. The allocative mechanisms work much better when bidders can split the auctioned goods in a collusive agreement. Allowing for communication in the auction results in generally better results in all experimental parameters. However, there is some evidence in the experiment that combinatorial bidding on packages may break the collusion which confirms but does not strengthen the statement of [Kwasnica and Sherstyuk \(2013\)](#).

The results do not prove that the package-bidding format is significantly different from the SMR format. Allowing for combinatorial bidding does not produce higher efficiency in our parameter setting which is in contradiction to [Brunner et al. \(2010\)](#) who claim that combinatorial bidding is more efficient. Generally, there is strength in the simplicity of bidding languages in the SMR auction format. The clear and simple design of SMR makes the decision problem of players easier and manageable in comparison to its combinatorial SMRPB counterpart. The inappropriate bidding strategies in complex combinatorial mechanisms do not allow for the complete utilization of the allocative potential of the auction formats and therefore cause inefficiencies. This result corresponds to that of [Goeree and Holt \(2010\)](#), who question combinatorial formats precisely because they are not computationally manageable for their participants, and of [Bichler et al. \(2014\)](#), who suggest that with the number of goods in stock exceeding 30 the number of possible bidding combinations is immense and makes the bidder optimization problem unacceptably difficult.

The policy recommendations resulting from this research are straightforward. When suspicion of potential collusion while preparing an auction is present, the policy-makers should prefer simpler versions of auction formats, which produce higher efficiencies and revenues. This holds true especially for auctions with a high volume of goods for sale. This statement is supported not only by our research, but also by [Bichler et al. \(2013\)](#). Moreover, [Bichler et al. \(2014\)](#) further state that the efficiency of simple auction formats increases with high volumes of goods in stock. This result can not be confirmed for non-communication treatments in our research, but holds true for the auction with the presence of communication.

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Appendices

A Optimal Allocation

Table 11: Optimal Efficiency Allocation

Goods	Player type				
	I	II	III	IV	
A	Quantity	1	0	2	1
	PVC	66	-93	80	45
	Efficiency per unit	0.9190	0	1	0.7976
B	Quantity	10	14	0	0
	PVC	3	3	-3	-2
	Efficiency per unit	1	1	0	0.16666
C	Quantity	0	14	0	0
	PVC	-2	0	-10	-2
	Efficiency per unit	0.8	1	0	0.8
D	Quantity	0	0	0	9
	PVC	-2	-3	-4	0
	Efficiency per unit	0.5	0.25	0	1
Total activity		20	28	20	19
Activity disposed		27	28	26	15
Partial efficiency		10.9190	28	2	9.7976
Optimal efficiency		0.95692			

B Additional Parameters of the Experiment

Table 12: Additional Parameters of the Experiment

Category of Goods	A	B	C	D	Total
Goods in stock	6	24	14	9	53
Upset price per unit	1400	40	180	40	-
Total upset price per category	8400	960	2520	360	12240
CVC	1820	50	140	50	-
PVC interval	±182	±5	±14	±5	-
Activity per unit	10	1	1	1	-
Total activity for category	60	24	14	9	107

Table 12 shows actual parameters used in the experiment. Blocks A3 and B1 from real auctions were adjusted compared to the original settings

in order to be homogeneous with other blocks in respective categories. One specific real block A3 of 2x10 MHz was split up into two blocks of 2x5 MHz which is in accordance with other category A blocks. One specific real block B1 of 2x15 MHz was split up into fifteen blocks of 2x1 MHz which is in accordance with the other category B block. The upset prices, activity points per block etc. were also homogenized according to this principle.

The setting with the original price vector had to be changed after the pilot experiment since the whole task would have been too long and therefore unfeasible, that is the upset prices were increased together with the percentage level of the price-raising algorithm. The experimental upset prices per unit were multiplied by the coefficient 1.3 and the price-raising algorithm was increased to 20% due to these experimental reasons.

Table 13: Random Draws for PVC and Activity Points

Players	PVC random draws				Activity random draw
	A	B	C	D	
Blue	66	3	-2	-2	-1
Pink	-93	3	0	-3	0
Red	80	-3	-10	-4	-2
Green	45	-2	-2	0	-3

Table 14: Random Draws for Private Signals on CVC

Players	PVC random draws			
	A	B	C	D
Blue	1718	51	141	48
Pink	1859	50	145	46
Red	1770	50	148	54
Green	1947	49	141	50

C Additional Literature to the Topic

Apart from the initial experimental evaluation of the SMR design and performance, one of the first experimental tests of the SMR auction was conducted by [Banks et al. \(2003\)](#). At the request of the Congress of the United States, they compared the SMR auction to the design which allowed for combinatorial bids. In its first part, the paper provides an unusually extensive review

of previous experiments and conceptual issues relevant to spectrum auctions. The main body is concentrated on the experimental study which evaluates the SMR auction type and examines its alternatives that might promote the allocation of efficient combinations of complementary licenses. The Combinatorial Multi-round Auction (CMA), an alternative developed for the FCC as a combinatorial mechanism to allocate spectrum intervals, led to more efficient allocations but lower revenues in comparison to SMR. The SMR auction should have had particular features that decrease its efficiency and create a trade-off between efficiency and the length of the auction. The CMA leads to more efficient allocations but lower revenues in comparison to SMR, because many bidders experienced losses in their SMR design due to the exposure risk (Banks et al., 2003).

Peter Cramton (2013) also analyzes the standard simultaneous multi-round ascending auction used to assign the licenses for spectrum intervals in telecommunications. He examines the strengths and weaknesses of the standard approach using examples from the US spectrum auctions. Moreover he presents the alternative of the combinatorial clock auction (CC) as a more precise and fitting format for the spectrum auctions. His paper suggests that the CC auction is a large step ahead over the simultaneous ascending auction. The CC should eliminate the exposure risk and most of the gaming behavior and it should encourage competition. The author recommends the CC format for settings in which the local regulator does not know in advance how the spectrum should be organized. This format should also be highly transparent which is useful for the allocation of public resources (Cramton, 2013).

The winning of some valuable package of interest or a particular set of licenses with specific value to the bidder can get complicated in the combinatorial auctions. Bidders with high value complementarities may have to bid more for some licenses than they are actually worth individually for them. When only a part of a desired package is won, the bidder can incur big losses. This is considered exposure risk in the auction literature. It may lead to conservative bidding in the auction and therefore lower revenues and inefficient allocation of the auctioned goods. An exposure risk may lead to conservative bidding in the auction and therefore lower revenues and inefficient allocation (Brunner et al., 2010). Goeree and Lindsay (2012) deal extensively with the value complementarities and exposure risk. They eval-

uate the impact of exposure risk on an imaginary real estate market and its performance where complementarities arise when the selling of an old house must precede the purchase of a new one. The authors however suggest important implications for many other markets and situations among which the allocation of telecommunication frequencies is not absent. Their paper reports a series of laboratory experiments comparing a variety of auction formats and treatments. It provides a setting of continuous double auction (CDA) with a high and low degree of exposure risk. Even though the CDA is shown to be very effective in a wide range of settings, it produces poor results in the treatment with a high degree of exposure risk (around 20%). The article introduces a simple package market and shows that it effectively handles the exposure risk. This package market is only a simple extension of the continuous double auction and, as noted above, could be applied in various situations (Goeree and Lindsay, 2012).

(Phillips et al., 2003) documented the impact of practices that may facilitate low final prices in repeated English auctions with multiple units. They have created laboratory markets of English auctions with a symmetric structure of bidders. By employing two sizes of the market (two and six bidder structures) they control for competitive and rivalry environments. Three practices are identified as potentially facilitating collusion among the bidders: (i) knowledge about the number of units for sale; (ii) familiarity through repeated interaction; and (iii) communication. The repeated interaction, according to their results, should allow buyers to learn the bidding strategies of their opponents even without communication. Moreover, if the agents can talk or exchange the information, agreements become easier and bid prices lower.

Lopomo et al. (2005) dispute in their paper the idea that collusion creates inefficiencies in sealed-bid auctions, but not in ascending bid auctions. They show that if there is no communication before the auction and the ex-post budget balance is satisfied by the collusive mechanism, collusion actually does affect auction efficiency. They state in particular: *"Any collusive mechanism that increases cartel members expected payoffs relative to non-cooperative play results in inefficiency either in the allocation among cartel members or in the allocation between cartel and non-cartel bidders, or both"* (Lopomo et al., 2005, p. 4).

Valley (1995) studied a double oral auction both without communica-

tion and with communication via cheap talk, either written or face-to-face. They found that either written or face-to-face communication often allowed subjects to coordinate on single price or split-the-difference outcomes that revealed enough of their private information to take them outside the incentive-efficiency boundary for unrestricted mechanisms (Crawford, 1998, p. 296).

D Instructions

The original instructions were written in Czech. This is an English translation. The instructions were divided into three parts: (I) the introduction in subsection D.1 ; (II) general instructions common to all treatments in subsection D.2; and (III) a treatment specific supplement for respective treatments in subsection D.3.

D.1 Introduction

Welcome to the Laboratory of Experimental Economics. My name is Jindřich Matoušek and my colleague's name is Lubomír Cingl. Thank you for participating in today's experiment.

Please, put all your belongings away so we can have your full attention.

In the course of the experiment, please do not talk to other participants and do not drink water. Please shut down your mobile phones. Violation of these rules will cause immediate exclusion from the experiment without any payment.

You cannot lose any money in this experiment. You will be given 100 CZK for coming on time. This 150 CZK and any money that you earn during the experiment will be paid to you, privately in cash, at the end of the experiment. The average expected payment in today's experiment is 400 CZK and the average length of the experiment is 2 hours. The length of the experiment depends on the speed of participants, therefore please be patient.

All amounts in this experiment will be given in Experimental Currency Units (ECU). The exchange rate to Czech Crowns is one CZK for three ECUs.

You can make notes on the enclosed sheet of paper. With the control questions placed at the beginning, we only want to make sure you understand the experiment; you will not be excluded nor discriminated against in any manner because of them.

Please note that you commit yourself to participation in the whole experiment and if you leave before the end, you receive no payment at all. For your participation on the experiment, we need you to sign the consent form. Please take the consent form provided on a separate sheet of paper, read it and when you sign it, raise your hand and the experimenters will collect them. If you are not willing to participate and not sign the consent form, please leave the experiment now and your participation fee of 100 CZK will be paid to you.

If you have any question now or during the experiment, please raise your hand and we will answer it in private.

D.2 General Instructions

THE EXPERIMENT

The experiment will involve a series of auctions. Each auction will consist of multiple rounds.

In each auction, you will be competing with others for a set of multiple goods, which will contain various types and quantities. There are several important rules in this auction that encompasses (i) the way you can bid for the goods; (ii) provisional winners in each round of the auction; (iii) your eligibility; and (iv) the possibility of bid withdrawal. These rules are not trivial but crucial for your participation in the auction and also for your payoff from the experiment. Therefore, please, devote to them the utmost attention.

Each auction will have an indefinite number of rounds, which depends on the decisions of its participants.

Let us explain the individual rules of the auction more closely.

INDIVIDUAL AUCTION ROUNDS

Each auction will consist of a series of a preliminary inexactly determined number of auction rounds. Each round has a time limit in which you have to submit your bid. After each round, the system will evaluate all submitted bids and show the round summary. This process repeats until the auction ends.

GROUPS AND BIDDERS

At the beginning of each auction, you will be randomly assigned to a group of four bidders (you and 3 others). Within these groups, you will be competing in all rounds of this auction. After this auction ends, you will be randomly assigned to a new group of four bidders.

GOODS FOR SALE

In each group of four players, four types of goods labeled A, B, C, and D, will be auctioned off. Each type of the goods is offered in multiple homogeneous units. You can submit bids for as many units of each type of the goods and for as many types as you want to. You will submit bids by adding the units in your bidding basket.

PRICES AND VALUES OF GOODS

Each type of goods A; B; C; a D offered for sale has a different upset price. The price of each type can increase gradually throughout the auction, in case an offer was made for this type in the previous auction round. Each increase will be implemented at a volume of 20% of the upset price of a respective type of the goods. If an offer was not made, the price of the goods remains the same. The price of goods within each type will be always the same for all units.

Each player will have different valuations for all types of goods. Your total personal value for each type of the goods will be known only to you.

The total valuation of each type of the goods consists of two components: the common value and the private value components of the goods. The total valuation is then the sum of these two components.

Each unit of the goods has its own common value, which is identical for all units of the goods. No bidder has the precise information about this common value. Each bidder receives only her private estimate of the common value determined by a random draw. The estimate of the common value is drawn separately for each player, but always out of the same interval.

Each player is further informed about her own private valuation of the goods, which she receives upon each unit bought in the auction. The private value is typically different for each player and is determined by a random draw from the interval in the range of $\pm 10\%$ of the common value component (which you do not know, but which is the same for all players); that is:

$$\mathbf{PVC} \in [-0.1 \cdot \mathbf{CVC}; +0.1 \cdot \mathbf{CVC}].$$

Even though your private component can be negative, your total value of each type of the goods is always positive.

The following table summarizes your knowledge of each type of the goods in the auction:

PVC	$\mathbf{PVC} \in [-0.1 \cdot CVC; +0.1 \cdot CVC].$
CVC	Estimate
Total information	PVC + Estimate of CVC

EXAMPLE:

Private value component PVC	20
Estimate of CVC	300
Total signal	PVC + Estimate of CVC = 320

COMPLEMENTARITIES OF GOODS

All types of goods offered in the auction are complements. It means that a set of multiple goods containing more types (A; B; C; or D) has higher value than each type separately; thus the winning of more than one type of goods at once gives you the advantage of higher profit.

If you win one type of goods (in an arbitrary quantity), your profit is equal to the value of this goods. However, if you win more than one type of goods at once, your profit will rise according to following formula:

$$valuation = [1 + 0.1 \cdot (X - 1)] \cdot \text{sum of valuations of goods won,}$$

where X stands for the number of goods types acquired. Thus:

- 1 type value is equal to the valuation of goods;
- 2 types value raises by 10% of the valuation of goods;
- 3 types value raises by 20% of the valuation of goods;
- 4 types value raises by 30% of the valuation of goods.

EXAMPLE:

Total value of the goods of type A is 300, of type B is 100.

If a player wins goods A, her profit is 300.

If a player wins goods B, her profit is 100.

If a player wins goods A and B, her profit is $1.1 \cdot (300 + 100) = 1,1 \cdot 400 = 440$.

PROVISIONAL WINNERS

The system automatically process all submitted bids when the auction round is finished. In the auction round summary you will be informed if and for how many units of goods are you currently the provisional winner. A situation that more than one player submits the same bid can occur in the course of the auction round. If the sum of such bids in your group exceeds the number of goods sold in the auction, the system determines the winner of given units randomly, since the price is the same for all players. You therefore do not have to win a complete set of goods on which you have submitted your bid.

After the time limit runs out or when each player submits her bid a new auction round occurs.

You will win precisely such goods in the last auction round for which you are currently the provisional winner. Only the final offers out of the last auction round are used for the calculation of auction profits and therefore your real payoff out of the experiment.

THE RULE OF ELIGIBILITY

Each participant in the auction has a certain number of activity points at her disposal, which represents her eligibility to submit bids in the auction. The activity points determine the maximum number of goods on which a player is able to submit bids.

Each unit of the goods costs a certain number of activity points. Your total bid cannot exceed your current level of activity points.

The number of your activity points can decrease during the auction, since it depends on your behavior in previous auction rounds. In each round, you will gain the same number of activity points as you have used in the previous one. If you submit a bid in a given round with a total activity cost lower than your current level of activity at your disposal, your eligibility for subsequent rounds will diminish your number of activity points will fall.

There is no way of acquiring the activity points back throughout the auction, nor to acquire more of them.

EXAMPLE:

You have 10 points of activity in a given auction round at your disposal. One unit of goods A costs 3 activity points, one unit of goods B costs 1 activity point.

If you submit a bid for 3 units of goods A and for 1 unit of goods B in a given round, you will pay 10 activity points in total ($3 \cdot 3 + 1 \cdot 1$), by which you will use up your activity for this round. You will have 10 points of activity at your disposal in a subsequent round of the auction.

If you submit a bid for 2 units of goods A and for 2 units of goods B in a given round, you will pay 8 activity points in total ($2 \cdot 3 + 2 \cdot 1$). You will have 8 points of activity at your disposal in a subsequent round of the auction.

WITHDRAWING WINNING BIDS

A situation could arise during the course of an auction, in which you win in some auction round only a subset of goods on which you have placed your bid. You can therefore win only a subset of goods for a price which exceeds the actual value of the goods.

If such a situation occurs, you have the possibility to withdraw your provisionally winning bid. Bid withdrawal is always available during the auction round summary. You can withdraw your bid for as many goods (both types and units), for which you are currently the provisional winner.

The possibility of bid withdrawal is limited in its volume. In particular, each bidder can use the right withdraw in at most two auction rounds, without any reference to the number of withdrawn goods in each particular round. However, the number of activity points for subsequent rounds will be appropriately decreased during each bid withdrawal by the sum of activity points for all respective withdrawn bids.

FINAL AUCTION ROUND

A final auction round arises when no participant submits an additional bid on any goods. Technically this situation means that all four participants in a group submit a bid for "empty bidding basket." The auction ends with this situation.

If you submit an empty bidding basket in some auction round during the course of the auction, your activity will fall to zero. You will not be able to

participate in the auction any further. Submit, therefore, an empty bidding basket only in the situation when you wish to terminate your participation in the auction.

HISTORY

There is a history box present during the whole auction in the bottom left corner of the auction interface. It displays, for each player, the number of individual types of goods in this box for which this player was a provisional winner in a given auction round. The history, due to space constraints, is displayed with abbreviations (1-A; 1-B; 1-C; 1-D; 2-A; 2-B; etc.). The abbreviation "1-A" means "player 1 goods of type A; the abbreviation "2-B" means "player 2 goods of type B" etc.

Figure 1: The Auction Interface

Period		1		Remaining time (sec): 0	
AUCTION INTERFACE					
GOOD A	GOOD B	GOOD C	GOOD D	BIDDING BASKET	YOUR ACCOUNT
<p>price per unit</p> <p>goods in stock</p> <p>activity per unit</p> <p>CVC Signal A</p> <p>PVC A</p> <p>your valuation</p> <p>upset price</p> <p>bid increment</p> <p>YOUR BID QUANTITY</p> <p>-1 +1</p>	<p>price per unit</p> <p>goods in stock</p> <p>activity per unit</p> <p>CVC Signal B</p> <p>PVC B</p> <p>your valuation</p> <p>upset price</p> <p>bid increment</p> <p>YOUR BID QUANTITY</p> <p>-1 +1</p>	<p>price per unit</p> <p>goods in stock</p> <p>activity per unit</p> <p>CVC Signal C</p> <p>PVC C</p> <p>your valuation</p> <p>upset price</p> <p>bid increment</p> <p>YOUR BID QUANTITY</p> <p>-1 +1</p>	<p>price per unit</p> <p>goods in stock</p> <p>activity per unit</p> <p>CVC Signal D</p> <p>PVC D</p> <p>your valuation</p> <p>upset price</p> <p>bid increment</p> <p>YOUR BID QUANTITY</p> <p>-1 +1</p>	<p>GOOD A</p> <p>GOOD B</p> <p>GOOD C</p> <p>GOOD D</p> <p>total valuation</p> <p>total price</p> <p>profit if won</p> <p>activity at disposal</p> <p>activity used</p> <p>activity loss</p> <p>ERASE</p> <p>SUBMIT</p>	<p>current activity</p> <p>LAST ROUND RESULTS</p> <p>provisionally winning</p> <p>GOOD A</p> <p>GOOD B</p> <p>GOOD C</p> <p>GOOD D</p> <p>total valuation</p> <p>total price paid</p> <p>total profit made</p> <p>total penalty in the auction</p>
HISTORY				CHAT	

YOUR PROFIT AND EARNINGS FROM THE EXPERIMENT

At the end of the auction your earnings for this auction are determined. Your profit will be equal to the total value of the goods you won at the end of the auction (that is in the final round of the auction), minus the total cost you paid for them. Thus:

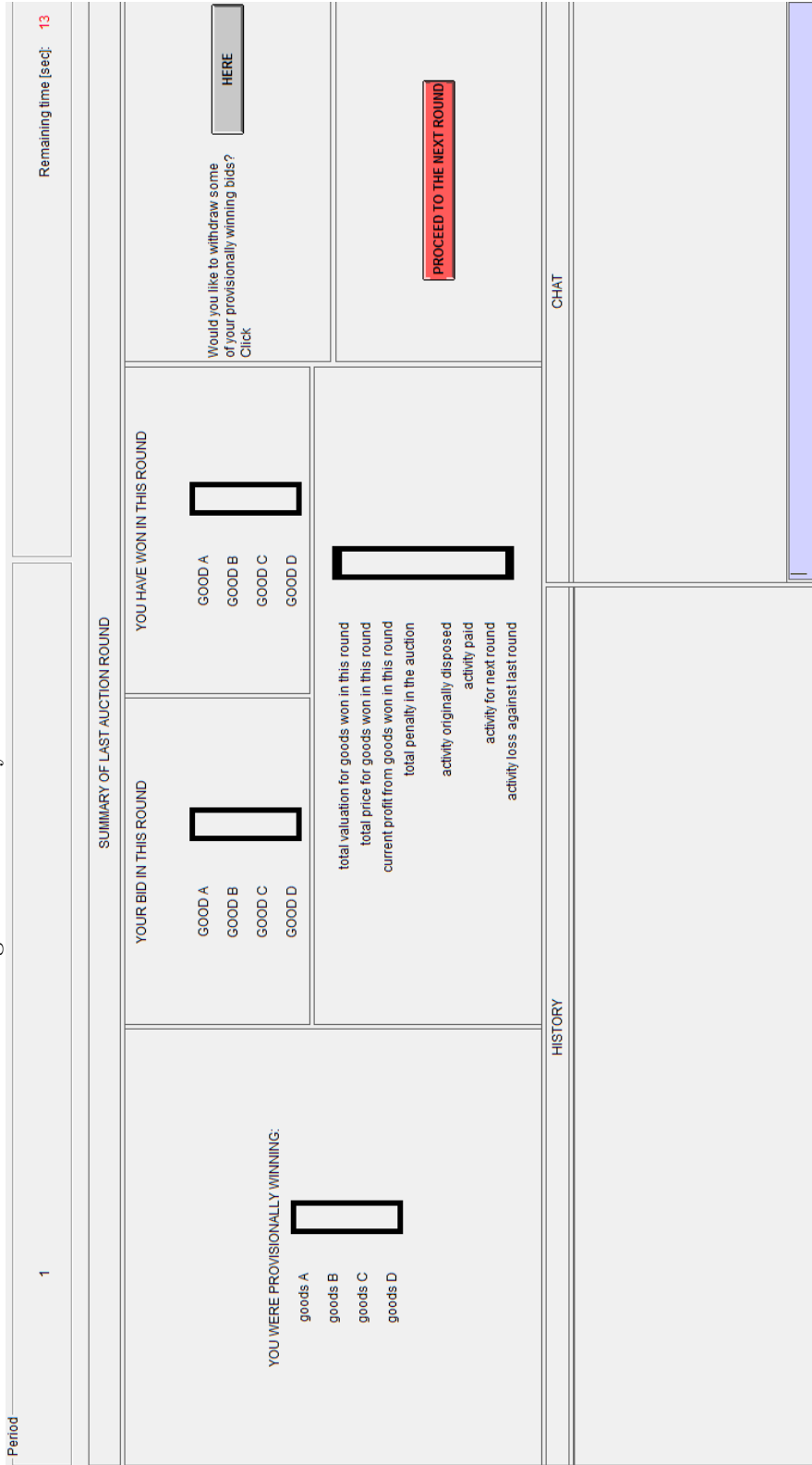
$$\textit{Profit} = \textit{total value of the goods won} - \textit{price paid for all goods won}$$

Your final earnings will depend on one of the auctions held in this experiment. Which auction will be determined randomly at the end of the experiment.

SUMMARY

1. The experiment will consist of a series of auctions. The first auction is a trial and will not influence your payoff from the experiment. Each auction will consist of a series of a preliminary inexactly determined number of auction rounds. A final auction round arises when no participant submits an additional bid on any goods.
2. You will submit your bids by adding the units of goods in your bidding basket.
3. The price of a particular type of the goods can rise during the auction if there is positive demand for this type of goods. Your payoff from the experiment will depend on your ability to win the desired goods but also on the luck and abilities of others.
4. Provisional winning bids are announced after each auction round. However, these do not affect the final profit from the auction until they became final winning bids in the last round of the auction.
5. The rule of eligibility says, in principal, that you cannot wait to submit your bid until the end of the final rounds of the auction. If you want to win your desired portion of goods in the auction, you have to submit bids already from the beginning.

Figure 2: Summary of the Auction Round



6. During the course of the auction, but not at its end, you will have the possibility to renounce your provisionally winning bid. This possibility will however be limited.
7. Individual valuations of goods are determined randomly for each player. It consists of a common and private value components of the goods, where the private component is known individually to all bidders. The common value component is, on the other hand, not known and the players have only a private signal about its value.
8. Your profit out of each auction will be determined only based on the situation from the final auction round and will be equal to the difference of the total value of goods you have won and the total price of your final bidding basket. Only one of the auctions held today will be chosen for your payoff at the end of the experiment.

D.3 Treatment-Specific Supplements

The treatment-specific supplements to the instructions were presented to the participants in the following sequence. There were three basic parts of the supplement: (i) notice; (ii) communication window; and (iii) the set of goods as a package. The following table summarizes which parts were presented in which treatment. There was a simple one-sentence introduction "treatment specific supplement introduction" present at the beginning in all treatments.

	Notice	Communication window	Set of goods as a package
SMR Basic	✓	✗	✗
SMR Collusion	✓	✓	✗
SMRPB Basic	✓	✗	✓
SMRPB Collusion	✓	✓	✓

TREATMENT-SPECIFIC SUPPLEMENT INTRODUCTION

Hereby presented additional rules were not stated in the online questionnaire.

NOTICE

1. Price of the goods gradually rises throughout the auction (in case the offers are made for this type of goods). If you are not able to find an optimal situation with positive profit in any round and you will incur a loss, it is highly improbable that you would find such a situation in subsequent rounds.
2. If you incur a loss out of the auction used for the calculation of your payoff from the experiment, it will appear in that payoff. Potential loss will be adequately subtracted from your payment for timely arrival. We therefore strongly recommend not submitting bids that can incur losses.
3. If you submit a bid in any round, your bidding basket will reset and assigns the goods freshly again according to your new offer. It is not possible to add some goods in your existing bidding basket. You always have to submit an offer for a complete set of desired goods.
4. Your task in the experiment is to gain a positive profit at the end of each auction, not to maintain your full level of activity points.

COMMUNICATION WINDOW

There will be a communication window present in the bottom right corner of the auction interface. You can send any messages to other participants in your group through this window. Such messages will be visible only to the players in your own group. The communication window will also be displayed for two minutes before each auction.

SET OF GOODS AS A PACKAGE

You will be bidding for a set of goods of your preference in each round of today's auctions. The system will handle this set as one compact package. Your bid will be either accepted as a package or refused as a package; you will therefore win the complete set you were bidding for or nothing.

At the end of each auction round, the system processes all bid packages submitted in the current auction round and displays information about the provisionally winning bids of this round. The processing runs based on the package with highest price. Even the players who did not submit an offer with the highest price, but whose offer was, after the processing stage

and determination of other provisionally winning bids, still available from the perspective of the quantity, can become the provisional winners of their packages.

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Univerzita Karlova v Praze, Fakulta sociálních věd

Institut ekonomických studií [UK FSV – IES] Praha 1, Opletalova 26

E-mail : ies@fsv.cuni.cz

<http://ies.fsv.cuni.cz>