Takatsuka, Hajime; Zeng, Dao-Zhi

Conference Paper

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51st Congress of the European Regional Science Association: "New Challenges for European Regions and Urban Areas in a Globalised World", 30 August - 3 September 2011, Barcelona, Spain

Provided in Cooperation with:
European Regional Science Association (ERSA)

Suggested Citation: Takatsuka, Hajime; Zeng, Dao-Zhi (2011) : Economic Integration and Welfare: Manufacturing vs. Agricultural Markets, 51st Congress of the European Regional Science Association: "New Challenges for European Regions and Urban Areas in a Globalised World", 30 August - 3 September 2011, Barcelona, Spain, European Regional Science Association (ERSA), Louvain-la-Neuve

This Version is available at:
http://hdl.handle.net/10419/120353

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Economic Integration and Welfare: 
Manufacturing vs. Agricultural Markets

Hajime Takatsuka* Dao-Zhi Zeng †

January 19, 2011

Abstract

In the literature of new trade theory, most studies have dealt with industrial 
location by imposing an assumption of free transportation in the agricultural sector. 
We explicitly incorporate arbitrary transport costs in both the manufacturing and 
agricultural sectors into the Helpman-Krugman-Davis model of two countries and 
one production factor. The following results are obtained. First, we find a necessary 
and sufficient condition for the home market effect (HME) to be observed. Secondly, 
we find that integrating manufacturing markets has contrastive impacts on two 
countries to integrating the agricultural markets. Our results are suggestive for the 
understanding of various international trade agreements.

Key words: Transport costs, Firm location, Home market effect, Welfare.

JEL Classification: F12, Q17, R1.

*Corresponding author. Graduate School of Management, Kagawa University, Saiwai-cho 2-1, Taka-
matsu, Kagawa 760-8523, Japan. E-mail: takatsuka@gsm.kagawa-u.ac.jp
†Graduate School of Information Sciences, Tohoku University, Aoba 6-3-09, Aramaki, Aoba-ku, 
Sendai, Miyagi 980-8579, Japan. E-mail: zeng@se.is.tohoku.ac.jp
1 Introduction

One of the most remarkable economic phenomena of the modern society is the large growth of the world’s trade volume. Starting with the General Agreement on Tariffs and Trade (GATT) signed in January, 1948, the world has been reshaped by competing forces of trade integration. Trade is indeed one of the principal drivers of the growth of Newly Industrializing Economies (NIEs) and BRICs. Such a change motivates a careful investigation of the welfare effects of deeper trade integration.

Since Krugman (1980), New Trade Theory (NTT) has been developed to study the increasing returns technology and monopolistic competition in trade, and the results successfully explain why countries at the same time import and export the products of the same industry (intra-industry trade). The general framework in NTT is able to handle arbitrary manufacturing trade costs, which makes it possible to conduct research on trade integration in detail. It is noteworthy that Krugman (1980) and Helpman and Krugman (1985) find that country size is influential in international trade. In a world of two asymmetric countries, the larger one succeeds in attracting a more-than-proportionate share of manufacturing firms, and the tendency is strengthened by decreasing transport costs. This is often called the home market effect (HME). Furthermore, the HME is larger for smaller manufacturing trade costs (e.g., Head and Ries, 2001, p.866). The result of shrinking the manufacturing industry in a small country from trade integration raises a natural question: Do large/small countries gain from the trade integration from the viewpoint of welfare?

Nevertheless, most economists who examine the role of market size have not analyzed the effects of integration on national welfare (e.g., Davis, 1998; Head et al., 2002; Ottaviano and Thisse, 2004; Yu, 2005; Crozet and Trionfetti, 2008; Zeng and Kikuchi, 2009). Some studies regarding the HME have focused on the gains and losses from trade, but the comparisons are limited to completely free-trade economies with entirely autarky economies (Krugman, 1981; Venables, 1987). Although some recent empirical studies (e.g., Bernard et al., 2003; Broda and Weinstein, 2006; Beine and Coulombe, 2007; Behrens et al., 2009) suggest the importance of a welfare analysis, no such theoretical work has been published.

One reason for this research gap is the free-trade assumption imposed on the agricultural sector since Helpman and Krugman (1985). This convenient assumption makes the analysis of the manufacturing sector much easier, and, therefore, is accepted in most

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1 Venables (1987, Section VII) also examines the effects of unilateral trade policies on national welfare and shows that the national welfare is raised by an increase in its import tariff. This paper does not treat such an asymmetric reduction of trade barriers.
subsequent studies on this subject. Nevertheless, in the real world, agricultural transportation incurs positive costs, as the same as in the manufacturing sector. Furthermore, the free-trade assumption has at least two theoretical defects. First, the wages in the two countries are equalized under this assumption, failing to capture the great wage gap between developed countries and developing countries. Baldwin and Robert-Nicoud (2000) and Baldwin et al. (2003) conducted some welfare analysis under the free-trade assumption in the agricultural sector and show that a reduction of frictional barriers between asymmetric-sized nations improves the welfare of both nations. However, their results are limited in the sense that the effects of globalization on national welfare through wage incomes are ignored. Secondly, the assumption makes it impossible to examine the integration of agricultural markets. In the real world, some countries, such as Japan, carefully protect their agricultural markets. Nevertheless, most NTT papers treat globalization (or economic integration) as a reduction of barriers to trade manufacturing (differentiated) goods only (e.g., Krugman and Venables, 1990, 1995; Baldwin and Venables, 1995).

The importance of agricultural transport cost is first recognized by Davis (1998), who shows that the HME of Helpman and Krugman (1985) disappears if the agricultural good is transported with the same positive cost as the manufactured goods. Fortunately, the model of Helpman and Krugman (1985) and Davis (1998) can be used in our research. Specifically, we maintain the structure of two asymmetric-sized countries, one production factor (labor), and two sectors (manufacturing and agriculture), but we allow for arbitrary trade costs in both sectors. This makes it possible to compare the integration of manufacturing and agricultural markets. Furthermore, we are able to analyze the effects of integration on welfare at arbitrary level of trade costs. By doing so, we clarify when the progress of economic integration produces (or does not produce) a conflict of interest between the two countries.

To the best of our knowledge, the equilibrium analysis of the Helpman-Krugman-Davis model is incomplete. While Helpman and Krugman (1985) focus on the case of free transportation in the agricultural sector, Davis (1998) mainly considers the case of equal transportation costs in two sectors. The case of arbitrary trade costs in the agricultural sector remains unclear. Therefore, before the welfare analysis, we rigorously re-examine the equilibrium of industrial location and wage for arbitrary trade costs in two sectors.

We obtained the following results. First, we found a necessary and sufficient condition for observing the HME. The condition is in regard to the trade costs of manufacturing and agricultural goods, and the conclusion aids in the comprehensive understanding of some known results scattered in the literature. Secondly, when the manufacturing markets are more deeply integrated, the number of firms in the larger country (resp. the smaller
country) evolves as an inverted U-shaped curve (resp. a U-shaped curve). Meanwhile, the smaller country is definitely better off, whereas the larger one could be worse off. Thirdly, when the agricultural markets are more deeply integrated, the number of firms in the larger country (resp. the smaller country) monotonically increases (resp. decreases). Meanwhile, the smaller country is definitely worse off regarding the interior equilibrium, whereas the welfare in the larger country must be improved. In summary, the integration of manufacturing markets does not threaten the smaller country even if more firms relocate to the larger country. Rather, the integration of agricultural markets threatens the smaller country.

The remainder of this paper is organized as follows. Section 2 is a review of the model of Helpman and Krugman (1985) and Davis (1998). Section 3 is a detailed description of the relationship among trade costs, firm location, and the HME. Section 4 is an analysis of the welfare, and Section 5, the conclusion.

2 The Model

The economy consists of two countries (the north (N) and the south (S)), two sectors (manufacturing and agriculture), and one factor (labor). The amount of labor in country N is denoted as $L$, and its counterpart in country S is denoted with an asterisk. The worldwide endowment $L^w = L + L^*$ is fixed. Denote $\theta = L / L^w$. We assume that country N is larger so that $\theta \in (1/2, 1)$.

The manufacturing sector $M$ consists of a continuum of product varieties and is characterized by increasing returns to scale (IRS) and monopolistic competition, while the agricultural sector $A$ produces a homogeneous good under constant returns to scale (CRS) and perfect competition.

Workers are assumed to hold the same preference, which is described by a Cobb-Douglas utility for the two types of goods with a CES subutility on the varieties of good $M$:

$$U = M^\mu A^{1-\mu},$$

where

$$M \equiv \left[ \int_0^{n^w} m(i) \frac{\sigma+1}{\sigma} di \right]^{\frac{\sigma}{\sigma-1}},$$

$n^w$ is the number of varieties in the $M$ sector, and $m(i)$ is the consumption of variety $i$. 
Parameter $\sigma > 1$ is the elasticity of substitution between any two varieties of good $M$, and $\mu \in (0, 1)$ is the expenditure share on good $M$.

As in most related papers, we assume Samuelson’s iceberg trade costs. Specifically, $\tau_M$ (resp. $\tau_A$) units of the good $M$ (resp. good $A$) must be shipped for one unit to reach the other country. We assume that $\tau_M \in (1, \infty)$ and $\tau_A \geq 1$ in the paper. As in Baldwin and Robert-Nicoud (2000, p.770), we might interpret that $\tau_M$ and $\tau_A$ include (or are equal to) tariffs. However, even if such an interpretation is applied, in the following analysis, we assume that tariff revenue plays a negligible role in national welfare calculations, as is the case of all OECD nations (Baldwin and Robert-Nicoud, 2000, p.775).

Each worker owns one unit of labor. In the manufacturing production, each firm needs a marginal cost of $(\sigma - 1)/\sigma$ units of labor and a fixed cost of $f$ units of labor. Thus, there is only one production factor, which is immobile across countries in the model.

We normalize the wage in $S$ as $w^* = 1$ and denote the wage in $N$ as $w$. Meanwhile, in the agricultural production, one unit of labor produces one unit of good $A$; then, the prices of good $A$ in $N$ and $S$ are

$$p_A = w, \quad p_A^* = w^* = 1,$$

(2)

respectively. It is noteworthy that the wage is the only income of workers. Therefore, the total expenditures in the two countries are

$$E = Lw, \quad E^* = L^*,$$

(3)

respectively. On the other hand, the total costs of producing $x$ units of manufactured varieties in the two countries are $c(x) = fw + (\sigma - 1) wx/\sigma$ and $c^*(x) = f + (\sigma - 1) x/\sigma$, respectively.

Let $p$ be the price of a manufacturing variety in country $N$ made in country $N$, $p^*$ be the price of a variety in country $S$ made in country $S$, $\bar{p}$ be the price of a variety in country $N$ made in country $S$, and $\bar{p}^*$ be the price of a variety in country $S$ made in country $N$. Then, the monopolistic competition framework of Dixit and Stiglitz (1977) suggests that

$$p = w, \quad p^* = 1, \quad \bar{p} = \tau_M, \quad \bar{p}^* = w \tau_M.$$

(4)

From (1), the demand (plus iceberg costs) of each variety produced in $N$ is

$$d_M = \mu \frac{p^{-\sigma}}{P_1 - \sigma} E + \tau_M \mu \frac{(\bar{p}^*)^{-\sigma}}{(P^*)^{1-\sigma}} E^*;$$

(5)

\footnote{Because of symmetry among varieties, this price is independent of the variety name.}
where $P$ and $P^*$ are the manufacturing price indices in the two countries, respectively. They are defined by

$$P = \left[ n p^{1-\sigma} + n^* (\bar{p})^{1-\sigma} \right] ^{\frac{1}{1-\sigma}}, \quad P^* = \left[ n (\bar{p}^*)^{1-\sigma} + n^* (p^*)^{1-\sigma} \right] ^{\frac{1}{1-\sigma}} \quad (6),$$

where $n$ and $n^*$ are numbers of firms in the two countries, respectively. On the other hand, from (1), (2), and (3), the demands of good $A$ in the two countries are

$$d_A = \frac{(1 - \mu) E}{P_A}, \quad d_A^* = (1 - \mu) E^*, \quad (7)$$

respectively.

In the model, free entry and exit of firms are assumed so that firms have zero profit. The output and input of each firm in the two countries are, therefore,

$$x = x^* = f \sigma, \quad (8)$$
$$l = l^* = f \sigma, \quad (9)$$

respectively. Thus, from (3), (4), (5), (6), and (8), the market-clearing conditions for manufacturing varieties produced in N and S are

$$\mu w^{-\sigma} \left[ \frac{\theta L^w}{nw^{1-\sigma} + n^* \phi_M} + \frac{(1 - \theta) L^w \phi_M}{n^* + n \phi_M w^{1-\sigma}} \right] = f \sigma, \quad (10)$$
$$\mu \left[ \frac{(1 - \theta) L^w}{n^* + n \phi_M w^{1-\sigma}} + \frac{\theta L^w \phi_M}{nw^{1-\sigma} + n^* \phi_M} \right] = f \sigma, \quad (11)$$

where $\phi_M \equiv \tau_M^{1-\sigma}$ is the trade freeness of manufactured varieties.

### 3 Trade Costs, Firm Location, and the Home Market Effect

In this section, we examine the equilibrium firm location. Following Krugman (1980), Helpman and Krugman (1985), and Davis (1998), we apply the following definition in this paper:

**Definition 1** The home market effect (HME) is the situation of

$$\frac{n}{n + n^*} > \theta \quad (12)$$
At equilibrium.

If good $A$ is not traded between two countries, the total output of good $A$ is $(1 - \mu)L$ in country $N$ and $(1 - \mu)L^*$ in country $S$. Therefore, from (9), we have

\[ n = \frac{\mu L}{f \sigma} = \frac{\mu \theta L^w}{f \sigma}, \quad n^* = \frac{\mu L^*}{f \sigma} = \frac{\mu (1 - \theta)L^w}{f \sigma}, \tag{13} \]

and, thus,

\[ \frac{n}{n + n^*} = \theta. \]

In other words, the HME disappears when good $A$ is nontradable. Substituting (13) into (11), we obtain

\[ F(w) \equiv (w^{1 - \sigma} - w \phi_M)\theta - (w^\sigma - \phi_M)(1 - \theta) = 0 \tag{14} \]

after simplification. Therefore, the equilibrium wage is determined by (14) when $A$ is not traded. Clearly, $F(w)$ decreases in $w$, and it holds that

\[ F(1) = (1 - \phi_M)(2\theta - 1) > 0, \]
\[ F(\tau_M^{\frac{1}{\sigma - 1}}) = -\left(\frac{1}{\phi_M} - \phi_M\right)(1 - \theta) < 0, \]

where the inequalities are from $\theta \in (1/2, 1)$ and $\phi_M < 1$. Thus, (14) has a unique solution, which is denoted by $\tilde{w}$, and lies in $(1, \tau_M^{\frac{1}{\sigma - 1}})$. On the other hand, $w = \tau_A$ when $A$ is traded. Therefore, $\tilde{w}$ is the highest value of the agricultural trade cost for $A$ to be traded. Accordingly, we sometimes use $\tilde{\tau}_A$ to denote $\tilde{w}$, indicating the fact that good $A$ is nontradable if and only if $\tau_A \geq \tilde{\tau}_A$.

We also know that $\tilde{\tau}_A$ is an increasing function of $\tau_M$ by applying the implicit function theorem to (14). Furthermore, if $\tau_M$ is large so that $\phi_M$ approaches 1, then $\tilde{\tau}_A$ approaches $[\theta/(1 - \theta)]^{1/(2\sigma - 1)}$, which is illustrated in Figure 1.

Together with the analysis for the case of tradable good $A$, we have the following results:

**Proposition 1** (i) Good $A$ is tradable if and only if $\tau_A < \tilde{\tau}_A$;

(ii) the HME is observed, and the larger country is the net exporter of good $M$ if $\tau_A < \tilde{\tau}_A$; otherwise, manufacturing firms are distributed in proportion to country size.

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3 Appendix A shows that the corner equilibrium with $n = 0$ is impossible.
\[ \frac{n}{n + n^*} - \theta = \frac{2\phi_M}{1 - \phi_M} \left( \theta - \frac{1}{2} \right) > 0, \]

where \( \phi_M \equiv \tau_M^{1-\sigma} \) is the trade freeness of manufactured varieties. Proposition 1 (ii) generalizes (15) and shows that the HME is observed in the whole shaded area of Figure 1, i.e., as long as \( \tau_A < \bar{\tau}_A \). Since \( \bar{\tau}_A < \tau_M \), the HME disappears when \( \tau_A = \tau_M \). This special result was originally provided in Davis (1998), and the above result demonstrates that the HME generally disappears for all \( \tau_A \geq \bar{\tau}_A \). Davis (1998) tried a generalization in his Section III C but did not obtain a necessary and sufficient condition. Yu (2005, p.261) shows that good \( A \) is not traded if \( \tau_A \geq \tau_M^{\frac{\sigma}{\sigma - 1}} \), which is only a sufficient condition. Crozet and Trionfetti (2008, p.313) conclude that the sufficient condition for the HME to exist is \( \tau_A < \tau_M^{\frac{\sigma}{\sigma - 1}} \) in our notations. However, their result is based on a different definition of the HME: the situation in which there exists a \( \bar{\theta} \in (1/2, 1) \) such that \( dn/d\theta > 1 \) holds
for all $\theta > \bar{\theta}$, which is neither sufficient nor necessary for (12).

Next, we consider how firms relocate when either $\tau_M$ or $\tau_A$ falls. We have the following result:

**Proposition 2** At the interior equilibrium with tradable $A$,
(i) the firm number in the larger country (resp. the smaller country) monotonically increases (resp. decreases) when $\tau_A$ falls;
(ii) the firm number in the larger country (resp. the smaller country) evolves as an inverted U-shaped curve (resp. a U-shaped curve) when $\tau_M$ falls.

**Proof**: See Appendix B. ■

To understand Proposition 2 (i), we note that the relative wage in N increases in $\tau_A$ as long as $A$ is tradable, since it holds that $w = \tau_A$. The wage differential has two effects. On the one hand, it has an impact on the production side. Firms pay the wages as production costs; thus, more firms are attracted from S to N if $w$ or $\tau_A$ falls. On the other hand, it also has an impact on the demand side. When $w$ falls, the consumption of $A$ in country N decreases. If $A$ is nontradable, then the decreased local demand of $A$ releases labor from the agricultural sector to the manufacturing sector. As a result, the manufacturing sector in country N expands in this case. However, if $A$ is tradable, then country N decreases its import of $A$ from country S, and the deducted wage income in N shrinks the market size of manufactured goods so that more firms are likely to move out from the market to save transport costs. Proposition 2 (i) shows that the effect across countries definitely dominates the effect across sectors in our setup. Therefore, the firm number in N (resp. S) monotonically increases (resp. decreases) for a falling $\tau_A$. Such a change is shown by vector (I) in Figure 1.

Helpman and Krugman (1985) conclude that a small country is de-industrialized when the manufacturing markets are more integrated. Proposition 2 (ii) shows that their result is not valid when the agricultural trade costs are positive. Specifically, there is a re-dispersion process whereby firms return to the small country for a sufficiently small $\tau_M$. This is because the dispersion force of a higher wage in the larger country dominates the agglomeration force due to the market size.\footnote{Zeng and Kikuchi (2009) analytically show this fact with a model based on Ottaviano et al. (2002).} Such a change occurs on the vector (II) in Figure 1.

In summary, the argument of Helpman and Krugman (1985) turns out to be true for a falling $\tau_A$ rather than $\tau_M$. This result is consistent with a history of industrialization in England in the 19th Century. According to Bairoch (1988, p.340), from the 1860s, the
pronounced liberalization of tariff policies for the importation of food encouraged buyers to satisfy demand by turning to the open lands overseas. This resulted in the labor release from the agricultural sector to the manufacturing sector in the country, and, thus, industrialization was accelerated.

4 Welfare

In this section, we focus on the welfare side of the model. The indirect utility of workers in N and S is expressed as

$$\omega = w^\mu P^{-\mu}, \quad \omega^* = (P^*)^{-\mu},$$

respectively.\(^5\) Furthermore, from (4) and (6), the above equations are rewritten as

$$\omega = \left[ n + n^* \phi_M w^{\sigma-1} \right] \frac{\sigma}{\sigma-1}, \quad \omega^* = \left[ \frac{n \phi_M}{w^{\sigma-1}} + n^* \right] \frac{\sigma}{\sigma-1}. \quad (16)$$

From the above equations, we know that the welfare in each country is determined by three factors: the trade freeness of good \(M (\phi_M)\), the number of firms \((n, n^*)\), and the wage in N \((w)\). Clearly, if other things are equal, the first two factors have positive effects on the welfare in both countries. On the other hand, the third one has a positive effect on local welfare and a negative one on foreign welfare. For example, a higher wage \(w\) in N implies a higher price of manufactured varieties produced there, which lowers the welfare in S. Meanwhile, in country N, the higher wage also implies a higher income, dominating the negative effect of higher prices and leading to a higher local welfare.

First, we derive the following result for the welfare comparison:

**Proposition 3** The welfare in the larger country is always higher than that in the smaller country.

**Proof.** The previous section shows that \(w = \tau_A\) if good \(A\) is tradable and \(w = \tilde{\tau}_A\) when \(A\) is nontradable. In either case, we have \(w < \tau_M^{\frac{\sigma-1}{\sigma}} < \tau_M\), which implies \(w^{1-\sigma} > \phi_M > \phi_M w^{1-\sigma} \). Therefore, it holds that

$$\frac{n}{n^*} > 1 > \frac{w^{1-\sigma} - \phi_M}{w^{1-\sigma} - \phi_M (w^{1-\sigma})^2} = \frac{w^{1-\sigma} - \phi_M}{w^{1-\sigma} (1 - \phi_M w^{1-\sigma})}, \quad (17)$$

\(^5\)For simplicity, a constant multiplier, \(\mu^{\mu}(1-\mu)^{1-\mu}\), is omitted in each equation.
where the first inequality is from Proposition 1 (ii). The inequalities of (17) imply

\[ nw^{1-\sigma} (1 - \phi_M w^{1-\sigma}) > n^* (w^{1-\sigma} - \phi_M) , \]

which derives \( \omega > \omega^* \) according to (16).

Subsequently, we examine how the welfare in each country changes when either trade cost \( \tau_M \) or \( \tau_A \) decreases.

### 4.1 Falling \( \tau_M \)

This subsection focuses on the decreasing \( \tau_M \), as illustrated by vector (II) in Figure 1. We consider the case of tradable \( A \) (the shaded area in Figure 1) in Section 4.1.1 and the case of nontradable \( A \) in Section 4.1.2.

#### 4.1.1 The case of tradable good \( A \)

In this case, the large country imports good \( A \) from the small country (see Appendix A (ii)), and, thus, we have \( w = p_A = \tau_A \). Therefore, (16) could be rewritten as

\[
\omega = \left( n + n^* \frac{\phi_M}{\phi_A} \right) \frac{n^{\mu}}{\sigma - 1}, \quad \omega^* = \left[ n\phi_M \phi_A + n^* \right] \frac{n^{\mu}}{\sigma - 1},
\]

where \( \phi_A \equiv \tau_A^{1-\sigma} \) is the trade freeness of good \( A \). From the fact that \( w = \tau_A \), (10), and (11), we have

\[
n = \frac{\tau_A \mu L^w (1 - \theta) \tau_A^{\sigma - \sigma} \phi_M^2}{f \sigma} \left( \phi_A - \tau_A \phi_M \right) (\phi_A - \phi_M), \quad n^* = \frac{\tau_A \phi_M L^w \theta \tau_A^{\sigma} \phi_M^2 - [1 + (\tau_A - 1) \theta] \phi_M + \theta \phi_A}{f \sigma} \left( \phi_A - \tau_A \phi_M \right) (\phi_A - \phi_M). \]

It is noteworthy that the above equations are true only if the RHSs of (19) and (20) are nonnegative. Otherwise (see footnote 3),

\[
n = \frac{\mu (\theta \tau_A + 1 - \theta) L^w}{f \sigma \tau_A}, \quad n^* = 0. \]

By (18), (19), and (20), we have

\[
\frac{\partial \omega}{\partial \phi_M} = \frac{\mu \theta L^w \tau_A \left( \phi_M^2 + 1 - 2 \tau_A \phi_M \right)}{f \sigma (\tau_A^{\sigma} - \phi_M)^2 \phi_A},
\]

where \( \phi_A \) is the trade freeness of good \( A \). From the fact that \( w = \tau_A \), (10), and (11), we have

\[
n = \frac{\tau_A \mu L^w (1 - \theta) \tau_A^{\sigma - \sigma} \phi_M^2}{f \sigma} \left( \phi_A - \tau_A \phi_M \right) (\phi_A - \phi_M), \quad n^* = \frac{\tau_A \phi_M L^w \theta \tau_A^{\sigma} \phi_M^2 - [1 + (\tau_A - 1) \theta] \phi_M + \theta \phi_A}{f \sigma} \left( \phi_A - \tau_A \phi_M \right) (\phi_A - \phi_M). \]

It is noteworthy that the above equations are true only if the RHSs of (19) and (20) are nonnegative. Otherwise (see footnote 3),

\[
n = \frac{\mu (\theta \tau_A + 1 - \theta) L^w}{f \sigma \tau_A}, \quad n^* = 0. \]

By (18), (19), and (20), we have

\[
\frac{\partial \omega^*}{\tau_A} = \frac{\mu \theta L^w \tau_A \left( \phi_M^2 + 1 - 2 \tau_A \phi_M \right)}{f \sigma (\tau_A^{\sigma} - \phi_M)^2 \phi_A},
\]
\[
\frac{\partial (\omega^*)}{\partial \phi_M} \frac{\omega_{\mu}^{\sigma-1}}{(\omega^*)^{\frac{\sigma-1}{\mu}}} = \frac{\mu(1 - \theta)L^w \left[ \tau_A^\sigma (\phi_M^2 + 1) - 2\phi_M \right]}{f \sigma (\tau_A^\sigma \phi_M - 1)^2} > 0,
\]

(23)
at the interior equilibrium. Inequality (23) implies that \(\omega^*\) increases in \(\phi_M\). Furthermore, it holds that

\[
\frac{\partial}{\partial \phi_M} \left[ \frac{\omega_{\mu}^{\sigma-1}}{(\omega^*)^{\frac{\sigma-1}{\mu}}} \right] = -\frac{\theta \tau_A (\tau_A^{2\sigma} - 1)}{(1 - \theta)(\tau_A - \phi_M)^2 \phi_A} < 0
\]

(24)
from (19) and (20). Thus, we know that \(\omega/\omega^*\) decreases in \(\phi_M\) at the interior equilibrium.

On the other hand, for a corner equilibrium with \(n^* = 0\), we have

\[
\frac{\partial \omega}{\partial \phi_M} = 0, \\
\frac{\partial (\omega^*)}{\partial \phi_M} = \frac{\mu (\theta \tau_A + 1 - \theta) L^w}{f \sigma \tau_A} \phi_A > 0,
\]

from (18) and (21). These imply that \(\omega\) is independent of \(\phi_M\), while \(\omega^*\) must increase in \(\phi_M\).

Finally, from (22), we have

\[
\frac{\partial \omega}{\partial \phi_M} \bigg|_{\phi_M=0} = \frac{\mu \theta L^w}{f \sigma \tau_A^\sigma} > 0,
\]

which implies that \(\omega\) increases in \(\phi_M\) for a small \(\phi_M\) at the interior equilibrium. However, for a large \(\phi_M\), \(\omega\) might increase or decrease in \(\phi_M\). For a further examination, let

\[
T(\tau_A) \equiv \left( \frac{1 - \theta}{\theta} \tau_A^{-\sigma-1} - 1 \right) \left[ \left( 2\tau_A - \frac{1 - \theta}{\theta} \right) \tau_A^{-\sigma-1} - 1 \right] + 2(\tau_A^\sigma - 1),
\]

which increases in \(\tau_A\) because

\[
T'(\tau_A) = \frac{2(1 - \theta)}{\theta^2} \tau_A^{-2\sigma-3} \left[ \theta (2\sigma - 1) \tau_A - (1 - \theta) (\sigma - 1) \right] > 0.
\]

Meanwhile, it holds that

\[
T(1) = -\left( 2 - \frac{1}{\theta} \right)^2 < 0, \\
T\left( \frac{\theta}{1 - \theta} \frac{\sigma-1}{\sigma} \right) = 2\left( \frac{\theta}{1 - \theta} \right) \frac{\sigma-1}{\sigma} - 1 > 0.
\]

Therefore, \(T(\tau_A) = 0\) has a unique solution \(\tau_A^S \in (1, [\theta/(1 - \theta)]^{1/(\sigma - 1)})\), which is illustrated in Figure 1. We summarize the results as follows:
Proposition 4 If good $A$ is costly tradable (i.e., $\tau_A \in (1, \tilde{\tau}_A)$),

(i) $\omega^*$ increases in $\phi_M$, and the ratio $\omega/\omega^*$ decreases in $\phi_M$;

(ii) at the interior equilibrium, $\omega$ increases in $\phi_M$ when $\tau_A > \tau^*_A$ and has an inverted U-shaped relationship with $\phi_M$ when $\tau_A < \tau^*_A$;

(iii) for a corner equilibrium, $\omega$ is independent of $\phi_M$.

Proof: Both (i) and (iii) are already shown in the context, so we here prove (ii). The sign of (22) depends on the term of $(\phi^2_M + 1 - 2\tau^*_A\phi_M)$, which decreases in $\phi_M \in (0, 1)$. Thus, we need to evaluate (22) at $\phi_M = \tilde{\phi}_M$, which is the maximum value of $\phi_M$ making good $A$ tradable, given by (36) in Appendix B. Result (ii) then follows from sign $(\tilde{\phi}^2_M + 1 - 2\tau^*_A\tilde{\phi}_M) = \text{sign } T(\tau_A)$. ■

While Proposition 2 (ii) shows that the firm number in the smaller country evolves as a U-shaped curve when $\tau_M$ decreases, Proposition 4 shows that the welfare in the country monotonically increases. This is basically because falling $\tau_M$ lowers the prices of imported goods and contributes to the welfare improvement. This effect dominates the negative effect of decreasing firms in the smaller country. Interestingly, this is not true for the larger country. Intuitively, when $\tau_A < \tau^*_A$, most firms (or all firms) could agglomerate in the larger country because the dispersion force based on the wage differential is small. Nevertheless, the firm share returns to the population share when $\tau_M$ becomes so small that good $A$ becomes nontraded. The negative effect of a firm relocation is relatively large for a small $\tau_A$, which dominates the positive effect of falling $\tau_M$, so the welfare in N becomes lower.

The model of Helpman and Krugman (1985) is the case of $\tau_A = 1$. In a similar manner, we conclude that the following results hold for their case: (i) at the interior equilibrium, both $\omega$ and $\omega^*$ increase in $\phi_M$, and their ratio $\omega/\omega^*$ is independent of $\phi_M$;\(^6\) (ii) for a corner equilibrium, $\omega$ is independent of $\phi_M$, $\omega^*$ increases in $\phi_M$, and their ratio $\omega/\omega^*$ decreases in $\phi_M$. Therefore, an integration of manufacturing markets basically improves the welfare of both countries, which is similar to the result of Baldwin and Robert-Nicoud (2000, Section 3) based on a two-factor model. This is because, in both papers, the assumption of costless agricultural transportation, which equalizes wages in two countries and does not capture the U-shaped evolution of firm location, is imposed.

\(^6\)For $\tau_A = 1$, (24) becomes 0.
4.1.2 The case of nontradable good $A$

In this case, manufacturing firms are distributed in proportion to country size. Specifically, Appendix A shows that the firm numbers in N and S are

$$n = \frac{\mu \theta L^w}{\sigma f}, \quad n^* = \frac{\mu (1 - \theta)L^w}{\sigma f},$$

respectively, and the equilibrium wage in N is $\tilde{w}$, the unique solution of (14). Recall that $\tilde{w} = \tilde{\tau} \in (1, \tau^{\frac{\sigma - 1}{\sigma}}_M)$ holds and $\tilde{w}$ decreases in $\phi_M$.

In this case, from (16), we have

$$\omega^\frac{\sigma - 1}{\mu} = \frac{\mu L^w}{\sigma f} \left[ \theta + (1 - \theta)\tilde{w}^{\sigma - 1}\phi_M \right], \quad (25)$$

$$\left(\omega^*\right)^{\frac{\sigma - 1}{\mu}} = \frac{\mu L^w}{\sigma f} \left[ \theta \tilde{w}^{1 - \sigma} \phi_M + (1 - \theta) \right]. \quad (26)$$

According to (26), $\omega^*$ increases in $\phi_M$ since $\tilde{w}$ decreases in $\phi_M$.

On the other hand,

$$\frac{\partial \tilde{w}^{\sigma - 1}\phi_M}{\partial \phi_M} = (\sigma - 1) \tilde{w}^{\sigma - 2}\phi_M \frac{d \tilde{w}}{d \phi_M} + \tilde{w}^{\sigma - 1}$$

$$= \frac{\tilde{w}^{2\sigma}(1 - \theta)\sigma + \tilde{w}\theta(\sigma - 1) + \phi_M \tilde{w}^{\sigma} [(\sigma - 1)(1 - \theta) - \tilde{w}\theta(\sigma - 2)]}{-\tilde{w}^{2}F'(w)}, \quad (27)$$

where the second equality is obtained by the implicit function theorem. Since $F'(w) < 0$, the denominator is positive. Then, if $(\sigma - 1)(1 - \theta) - \tilde{w}\theta(\sigma - 2) \geq 0$, (27) is evidently positive. Otherwise, we have

$$\tilde{w}\theta(\sigma - 1) + \phi_M \tilde{w}^{\sigma} [(\sigma - 1)(1 - \theta) - \tilde{w}\theta(\sigma - 2)]$$

$$>\tilde{w}\theta(\sigma - 1) + [(\sigma - 1)(1 - \theta) - \tilde{w}\theta(\sigma - 2)]$$

$$=\tilde{w}\theta + (\sigma - 1)(1 - \theta)$$

$$>0,$$

where the first inequality is from $\tilde{w} < \tau^{\frac{\sigma - 1}{\sigma}}_M$. Therefore, (27) is always positive, which implies that $\omega$ increases in $\phi_M$.

Furthermore, from (25) and (26), we have

$$\frac{\partial}{\partial \phi_M} \left[ \frac{\omega^\frac{\sigma - 1}{\mu}}{(\omega^*)^{\frac{\sigma - 1}{\mu}}} \right] = \frac{\mu L^w}{\sigma f} \left( 1 - \theta \right) \left( \omega^* \right)^{\frac{\sigma - 1}{\mu}} \frac{\partial \tilde{w}^{\sigma - 1}\phi_M}{\partial \phi_M} - \theta \omega^* \frac{\partial \tilde{w}^{1 - \sigma} \phi_M}{\partial \phi_M} \frac{(\omega^*)^2(\sigma - 1)}{\mu} < 0$$
since $\theta > 1/2$, $\omega > \omega^*$, and

$$\frac{\partial \tilde{w}^{1-\sigma} \phi_M}{\partial \phi_M} > \frac{\partial \tilde{w}^{\sigma-1} \phi_M}{\partial \phi_M},$$

where the last inequality is from the fact that $\tilde{w}$ decreases in $\phi_M$. Thus, we know that $\omega/\omega^*$ decreases in $\phi_M$.

The above results are summarized as follows:

**Proposition 5** If good $A$ is nontradable (i.e., $\tau_A \geq \tilde{\tau}_A$), both $\omega$ and $\omega^*$ increase in $\phi_M$, and their ratio $\omega/\omega^*$ decreases in $\phi_M$.

It is noteworthy that $w$ decreases, while the number of firms in each country does not change when $A$ is nontradable. Thus, the welfare in $S$ clearly increases, since the price of imported varieties in country $S$ (i.e., $\bar{p}^* = w \tau_M$) decreases. With respect to country $N$, the decreasing $w$ reduces the income as well as the price index of the manufactured goods. Proposition 5 concludes that the positive effect dominates the negative one and the welfare in $N$ also increases.

Our model is general enough to include the model of Krugman (1980, Section II) as a special case of $\mu = 1$ (without sector $A$). Thus, Proposition 5 also holds for his setup.

### 4.2 Falling $\tau_A$

Next, we examine the effects of $\tau_A \in [1, \tilde{\tau}_A)$ on the welfare. If $\tau_A \geq \tilde{\tau}_A$, good $A$ is not traded, and, thus, decreasing $\tau_A$ does not change the equilibrium.

From (18), (19) and (20), we have

$$\frac{\partial \omega}{\partial \tau_A} = \frac{\mu \sigma^2 L^w \phi_M (1 - \phi_M)}{f} < 0,$$

$$\frac{\partial (\omega^*)}{\partial \tau_A} = \frac{\mu (1 - \theta) L^w \phi_M (1 - \phi_M)}{f} > 0,$$

at the interior equilibrium. In other words, $\omega$ increases in $\phi_A (= \tau_A^{1-\sigma})$, while $\omega^*$ decreases in $\phi_A$. On the other hand, for a corner equilibrium with $n^* = 0$, we have

$$\frac{\partial \omega}{\partial \tau_A} = \frac{\mu (1 - \theta) L^w}{f \sigma \tau_A^2} < 0,$$

$$\frac{\partial (\omega^*)}{\partial \tau_A} = \frac{-\mu L^w \phi_M [\sigma (1 - \theta) + (\sigma - 1) \theta \tau_A]}{f \sigma \tau_A^{\sigma+1}} < 0,$$

If $\tau_A \geq \tilde{\tau}_A$, good $A$ is not traded, and, thus, decreasing $\tau_A$ does not change the equilibrium.
from (18) and (21). They imply that both $\omega$ and $\omega^*$ increase in $\phi_A$. Furthermore, it holds that

$$\frac{\partial}{\partial \tau_A} \left[ \frac{\omega^{\sigma_A^{-1}}}{(\omega^*)^{\sigma_A^{-1}}} \right] = \frac{\partial}{\partial \tau_A} \left[ \frac{\tau_A^{\sigma_A^{-1}}}{\phi_M} \right] > 0$$

from (16) and the fact that $w = \tau_A$. Thus, we know that $\omega/\omega^*$ decreases in $\phi_A$.

The above results are summarized as follows:

**Proposition 6** If good $A$ is tradable (i.e., $\tau_A \in [1, \tilde{\tau}_A]$),

(i) $\omega$ increases in $\phi_A$;

(ii) at the interior equilibrium, $\omega^*$ decreases in $\phi_A$, and $\omega/\omega^*$ increases in $\phi_A$;

(iii) for a corner equilibrium, $\omega^*$ increases in $\phi_A$, and $\omega/\omega^*$ decreases in $\phi_A$.

When $\tau_A$ falls, $w$ decreases, and $n$ (resp. $n^*$) increases (resp. decreases), while $\phi_M$ does not change. With respect to the welfare in country N, Proposition 6 shows that the positive effect of increasing $n$ dominates the negative effect of decreasing $w$. On the other hand, with respect to the welfare in S, the negative effect of decreasing $n^*$ dominates the positive effect of decreasing the price of imported varieties in the country. At first glance, the above result (ii) might be counterintuitive because increasing $\phi_A$ improves S’s export of $A$. However, our model captures the sectoral labor movement. A large agricultural sector in S results in a small manufacturing sector and, finally, decreases the welfare in S.

4.3 Discussion

For the interior equilibrium case, our results in Propositions 1-6 are summarized in Table 1, where a falling $\tau_M$ is represented by “$\phi_M \uparrow$” and a falling $\tau_A$ is represented by “$\phi_A \uparrow$”.

<table>
<thead>
<tr>
<th>$\phi_M \uparrow$</th>
<th>$\phi_A \uparrow$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>tradable $A$</strong></td>
<td><strong>nontradable $A$</strong></td>
</tr>
<tr>
<td>small $\phi_M$</td>
<td>large $\phi_M$</td>
</tr>
<tr>
<td>$n$</td>
<td>$n^*$</td>
</tr>
<tr>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes: +: increase; -: decrease; 0: no change
Immediately, we find that integrating the agricultural markets is very contrastive to integrating the manufacturing markets from the viewpoint of welfare. In fact, the welfare in the smaller country is improved, and the welfare differential (in terms of welfare ratio) becomes smaller when the manufacturing markets are more integrated. However, the welfare in the smaller country is lowered, and the welfare differential becomes larger when the agricultural markets are more integrated. In other words, while the integration of manufacturing markets does not threaten the smaller country, the integration of agricultural markets does. This implies that increasing the trade freeness of differentiated varieties is beneficial to small countries even if it drives firms there to relocate to a larger country. Meanwhile, although increasing $\phi_A$ also contributes to decreasing the price of imported varieties via decreasing the wage in the larger country, such a positive effect is not sufficiently large to dominate the negative effect of decreasing firm share.

The above result, revealing the importance of balancing the trade barriers of two sectors, is interesting to trade policy makers. In the United Kingdom of Great Britain and Ireland, the Corn Laws protected corn prices against competition from less expensive foreign imports between 1815 and 1846. It was thought to be dangerous for Britain to rely on imported corn as lower prices would reduce labor wages and manufacturers would lose out due to the fall in purchasing power of landlords and farmers. The abolition of the Corn Laws marked a significant step towards free trade. While the immediate impact of repealing the Corn Laws was not great (Grigg, 1989, p.21), it paved the way for the agricultural depression after 1870 on the one hand and increased industrial activities leading to the industrial agglomeration in the UK on the other.

Even now, although many countries reduce the trade barriers in both sectors and harmonize the numerous regulations that govern international trade, Japan continues to protect its agricultural markets. Indeed, the existing bilateral economic partnership agreements into which Japan has entered have all exempted agricultural products from any elimination of tariffs. A recent trade organization, called the Trans-Pacific Partnership (TPP), requires member countries to open both markets; thus, Japan is very reluctant to participate. Given the large trade surplus in the manufacturing sector, Japan corresponds to the large country in our model, as did the UK in the 19th Century. Besides the theory of comparative advantage (first nature), our NTT results based on the second nature (imperfect competition and the technology of increasing returns to scale) also suggest that integrating the agricultural markets should improve the national welfare.

Finally, our analysis reveals that $\omega$ and $\omega^*$ may change in different directions, as shown by two $[-]$ in Table 1. This suggests possible conflicts of interest within the free trade policy. With respect to the Free Trade Agreements (FTA), the conflict of interest oc-
curs when manufacturing firms go from a developed country to a developing country in order to save labor costs, resulting in the emergence of NIEs. To prevent the relocation of firms, developed economies sometimes oppose the freer trade of manufacturing goods. For example, the United States and Korea ratified a U.S.-Korea FTA in November 2010. However, its negotiation was long delayed due to the controversial issues regarding automobiles. By a U.S. request, the agreement finally allowed for the United States to retain a 2.5 percent tariff on vehicle imports until the fifth year.

5 Concluding Remarks

This study is an examination of the effects of globalization (i.e., falling trade costs) on industrial location and national welfare. We use the Helpman-Krugman-Davis model with two countries, one factor, and two industries, both of which incur trade costs. The following results were obtained.

First, we found a necessary and sufficient condition for the HME to be observed. The condition is in regard to the trade costs of manufacturing and agricultural goods, and the result is helpful for a comprehensive understanding of some known results throughout the literature. Second, when the manufacturing markets are more integrated, the firm number in the larger country (resp. the smaller country) evolves as an inverted U-shaped curve (resp. a U-shaped curve). Meanwhile, the welfare in the smaller country must be better off, while the welfare in the larger country could be worse off. Third, when the agricultural markets are more integrated, the firm number in the larger country (resp. the smaller country) monotonically increases (resp. decreases). Meanwhile, the welfare in the smaller country must be worse off at the interior equilibrium while the welfare in the larger country must be improved.

In summary, the integration of manufacturing markets does not threaten the smaller country even if more firms relocate to the larger country. Rather, the integration of agricultural markets threatens the smaller country.

References


Behrens, K., Mion, G., Murata, Y. and Südekum, J. (2009) Trade, wages, and productivity, Discussion Paper No.7369, CEPR.


**Appendix A. Proof of Proposition 1**

The case of nontraded $A$ is already considered in the text, so we analyze the case of traded $A$ here. First, we show that it is impossible that all firms agglomerate in country $S$. Otherwise, country $N$ imports good $M$, which implies that country $N$ exports good $A$ and $w = 1/\tau_A$ by the trade balance. Noting $n = 0$ and $w = 1/\tau_A$, the market-clearing condition for good $M$ (11) gives

\[ n^* = \frac{\mu L_w}{\frac{f}{\sigma} \left[ (1 - \theta) + \frac{\theta}{\tau_A} \right]}. \]
The number of agricultural workers in S is

\[(1 - \theta)L^w - n^*f\sigma = (1 - \theta)L^w - \mu L^w \left[ (1 - \theta) + \frac{\theta}{\tau_A} \right].\]

from (9), and, thus, the import of good A in S is

\[(1 - \mu)(1 - \theta)L^w - \left\{ (1 - \theta)L^w - \mu L^w \left[ (1 - \theta) + \frac{\theta}{\tau_A} \right] \right\} = \mu L^w \left[ 2(1 - \theta) + \frac{\theta}{\tau_A} \right].\]

On the other hand, the export of good A from N is

\[\frac{\theta L^w - (1 - \mu)\theta L^w}{\tau_A} = \frac{\mu \theta L^w}{\tau_A},\]

which is always smaller than (28). Therefore, it is impossible that all firms agglomerate in country S.

Second, we show that it is impossible that all firms agglomerate in country N for \(\tau_A \geq \tilde{\tau}_A\). Otherwise, country N imports good A, which implies \(w = \tau_A\). From (10), (11), \(n^* = 0\) and \(w = \tau_A\), the market-clearing condition for each manufacturing variety produced in N and the condition for no firms in S are written as

\[\frac{L^w}{n} \left( \theta + \frac{1 - \theta}{\tau_A} \right) = f\sigma,\]

\[\frac{L^w}{n} \left[ \frac{(1 - \theta)}{\phi M \phi_A} + \frac{\tau_A \theta \phi_M}{\phi_A} \right] < f\sigma,\]

respectively. From (29), we obtain

\[n = \frac{\mu L^w}{f\sigma} \left[ \theta + \frac{1}{\tau_A} (1 - \theta) \right].\]

Substituting this into (30), we have

\[\theta \left( \tau_A \phi_M - \tau_A^{1 - \sigma} \right) + (1 - \theta) \left( \frac{1}{\phi_M} - \tau_A^{-\sigma} \right) < 0.\]

The LHS is increasing in \(\tau_A\). On the other hand, when \(\tau_A = \tilde{\tau}_A\), the LHS of (31) is

\[\theta (\tilde{\tau}_A \phi_M - \tilde{\tau}_A^{1 - \sigma}) + (1 - \theta) \left( \frac{1}{\phi_M} - \tilde{\tau}_A^{-\sigma} \right)\]

\[= - (1 - \theta)(\tilde{\tau}_A - \phi_M) + (1 - \theta)\left( \frac{1}{\phi_M} - \tilde{\tau}_A^{-\sigma} \right)\]
\[ (1 - \theta) \left( \frac{\sigma_{\tau-1}^M - \sigma_A}{\sigma_{\tau-1}^A} \right) = (1 - \theta) \left( \frac{\sigma_{\tau-1}^M - \sigma_A}{\sigma_{\tau-1}^A} \right) > 0, \]

where the first equality is from \( F(\sigma_A) = 0 \) and the last inequality is from \( \sigma_A \in (1, \tau_M^{-1}) \).

Therefore, the LHS of (31) is positive for \( \tau_A \geq \sigma_A \), which implies that the agglomeration never occurs for \( \tau_A \geq \sigma_A \).

Next, we consider an interior equilibrium. We first assume that the larger country \( N \) is the importer of good \( A \), so that \( p_A = \tau_A \) and \( E = wL = \tau_A L \) hold. From (10) and (11), we have

\[ \mu \left[ \frac{\theta L^w \phi_A}{n \phi_A + n^* \phi_M} + \frac{1}{\tau_A} \frac{(1 - \theta)L^w \phi_M \phi_A}{n^* + n \phi_M \phi_A} \right] = f \sigma, \]  
\[ \mu \left[ \frac{(1 - \theta)L^w}{n^* + n \phi_M \phi_A} + \frac{\tau_A \theta L^w \phi_M}{n \phi_A + n^* \phi_M} \right] = f \sigma. \]  

The above equations immediately derive

\[ \frac{\theta L^w}{n \phi_A + n^* \phi_M} = \frac{f \sigma (\tau_A - \phi_A \phi_M)}{\mu \tau_A \phi_A (1 - \phi_M^2)}, \quad \frac{(1 - \theta)L^w}{n^* + n \phi_M \phi_A} = \frac{f \sigma (\phi_A - \tau_A \phi_M)}{\mu \phi_A (1 - \phi_M^2)}. \]

Since the above two terms are positive, we obtain a necessary condition for the interior equilibrium:

\[ \phi_A - \tau_A \phi_M > 0. \]  

From (32) and (33), the numbers of firms in two countries are expressed as (19) and (20). Then, using these equations, we have

\[ n - \theta(n + n^*) = \frac{\mu L^w \phi_M}{f \sigma} \left[ (\tau_A - 1) \theta + 1 \right] F(\tau_A). \]  

where \( F(\cdot) \) is defined in (14). The denominator of (35) is positive from (34). Since \( F(\cdot) \) is a decreasing function and \( F(\sigma_A) = 0 \), we have \( n - \theta(n + n^*) > 0 \) for all \( \tau_A \in [1, \sigma_A] \), which implies that the HME exists for all \( \tau_A \in [1, \sigma_A] \). Noting that the labor input of firms in both countries are the same given by (9), the inequality, \( n - \theta(n + n^*) > 0 \), implies that the the share of good \( A \) produced in \( N \) is less than \( \theta \). Meanwhile, the share of good \( A \) consumed in \( N \) is \( \theta \) from (2), (3) and (7). Therefore, we confirm that country \( N \) is an importer of good \( A \).
On the other hand, when $\tau_A > \tilde{\tau}_A$, we have $n - \theta(n + n^*) < 0$. By the same logic used above, country N must be an exporter of good A, which contradicts the assumption that N is the importer of good A. Thus, a necessary condition for N to be the importer of good A is that $\tau_A < \tilde{\tau}_A$.

Here, we show that it is impossible for the smaller country S to be the importer of good A. Otherwise, the equilibrium wage in N is $w = 1/\tau_A < 1$. Similar to the previous arguments, we have

$$n - \theta(n + n^*) = \frac{\mu L^w \phi_M}{f \sigma} \frac{[(\tau_A^{-1} - 1)\theta + 1]F(\tau_A^{-1})}{(\phi_A^{-1} - \phi_M \tau_A^{-1})(\tau_A^{-\sigma} - \phi_M)} > 0,$$

where the inequality is from the following facts: (a) $\tau_A^{-\sigma} > \phi_M$, which can be derived similar to (34); (b) $F(\cdot)$ is decreasing; (c) $F(1) > 0$ and $\tau_A > 1$. Therefore, country N must be an importer of good A, which contradicts the assumption that N is the exporter of good A.

Summing up the above discussion, good A is not traded and the HME disappears if $\tau_A \geq \tilde{\tau}_A$. Otherwise, country N imports (resp. exports) good A (resp. good M) and the HME appears. ■

Appendix B. Proof of Proposition 2

(i) Noting $\phi_A \equiv \tau_A^{1-\sigma}$, we have

$$\frac{\partial n}{\partial \tau_A} = -\frac{\mu L^w \phi_M \tau_A^{-2}}{f \sigma} (1 - \theta)(\tau_A^{-\sigma} - \phi_M)^2(\sigma - 1 + \tau_A^{-\sigma} \phi_M) + \sigma \theta \tau_A (1 - \tau_A^{-\sigma} \phi_M)^2 < 0,$$

$$\frac{\partial n^*}{\partial \tau_A} = \frac{\mu L^w \phi_M \sigma (1 - \theta) \tau_A^{-1} (\tau_A^{-\sigma} - \phi_M)^2 + \theta (1 - \tau_A^{-\sigma} \phi_M)^2 [(\sigma - 1) \tau_A^{-\sigma} + \phi_M]}{(1 + \tau_A^{2\sigma}) \phi_M - \tau_A^{-\sigma} (1 + \phi_M^2)^2} > 0$$

from (19) and (20). In other words, the firm number in the larger (resp. smaller) country monotonically increases (resp. decreases) when $\tau_A$ decreases.

(ii) Even if good A is traded initially, it becomes nontradable when $\phi_M$ increases and reaches a threshold, as can be seen in Figure 1. The threshold is obtained by solving $F(\tau_A) = 0$ for $\phi_M$, where $F(\cdot)$ is defined by (14) in Appendix A. Specifically, the threshold is expressed as

$$\tilde{\phi}_M = \frac{\tau_A^{-\sigma} [\theta \tau_A - (1 - \theta) \tau_A^{2\sigma}]}{\theta \tau_A - (1 - \theta)}.$$ (36)
If $\tau_A < \bar{\tau}_A$, it must hold that

$$\theta \tau_A > (1 - \theta) \tau^2_A,$$

(37)

and, then, $\bar{\phi}_M \in (0, 1)$. From (19) and (20), we have

$$\frac{\partial n}{\partial \phi_M} \bigg|_{\phi_M=0} = \frac{\mu L^w \tau^{-\sigma -1}_A [\theta \tau_A - (1 - \theta) \tau^2_A]}{f \sigma} > 0,$$

$$\frac{\partial n^*}{\partial \phi_M} \bigg|_{\phi_M=0} = -\frac{\mu L^w \tau^{-\sigma}_A [\theta \tau_A - (1 - \theta) \tau^2_A]}{f \sigma} < 0,$$

$$\frac{\partial n}{\partial \phi_M} \bigg|_{\phi_M=\bar{\phi}_M} = -\frac{\mu L^w \tau^{-\sigma -2}_A (\theta \tau_A + \theta - 1)^2 [\theta \tau_A - (1 - \theta) \tau^2_A]}{f \sigma \theta (1 - \theta)(\tau^2_A - 1)^2} < 0,$$

$$\frac{\partial n^*}{\partial \phi_M} \bigg|_{\phi_M=\bar{\phi}_M} = \frac{\mu L^w \tau^{-\sigma -1}_A (\theta \tau_A + \theta - 1)^2 [\theta \tau_A - (1 - \theta) \tau^2_A]}{f \sigma \theta (1 - \theta)(\tau^2_A - 1)^2} > 0,$$

where all inequalities are from (37).

Furthermore, we know that both

$$\frac{\partial n}{\partial \phi_M} (\phi_M) = 0$$

and

$$\frac{\partial n^*}{\partial \phi_M} (\phi_M) = 0$$

have at most two roots. Thus, we conclude that $\frac{\partial n}{\partial \phi_M} (\phi_M)$ (resp. $\frac{\partial n^*}{\partial \phi_M} (\phi_M)$) is concave (resp. convex) for $\phi_M \in [0, 1]$. ■