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# Time series and spatial interaction: An alternative method to detect converging clusters

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## Abstract

In this paper an attempt is made to assess the hypothesis of regional club-convergence, using a spatial panel analysis combined with B-Splines. In this context, a ‘convergence-club’ is conceived as a group of regions that in the long-run move towards steady-state equilibrium, approximated in terms of the average per-capita income. Using data for the US states over the period 1929-2005, a pattern of club-convergence is detected. The ‘cluster’ of converging states is rather limited and a strong spatial component is detected.

**JEL Classification:** R11, R12

**Key Words:** *Club-convergence, Time-series, US States*

## 1 Introduction

Recent years have seen an increasing interest in testing for ‘club-convergence’, across the regions of individual countries (e.g. the UK, Chatterji and De-

whurst, 1996; Finland, Kangasharju, 1999). This interest is motivated by the need to identify areas that will be supported when regional policy is focused on the promotion of regional convergence. Drawing a dividing line between converging and lagging regions is not an easy task. This is mainly due to the lack of knowledge of the point at which regions are converging in the long-run. It is a primary aim of this paper to offer a methodology to detect groups of regions that follow common patterns in their convergence behaviour.

The central apparatus of our methodology derives from Nahar and Inder (2002), who implement time-series analysis to identify groups of economies moving towards a common steady-state. Although originally developed for national economies, Alexiadis and Tomkins (2004) demonstrated that it is equally suited to regional economies, although spatial dependency across observational units is not taken into account. This paper extends this methodology further by considering three major aspects. To be more precise, first, the hypothesis of spatial interaction across regional units is introduced explicitly. Second, a panel setting is implemented to control for fixed effects reflecting time and state shocks. Third, instead of simple polynomial orders, as Nahar and Inder (2002) utilize, we apply the so called Basic Splines as a functional base in order to approximate the behaviour of a region's per-capita income with respect to the average per-capita income of all the observational units over time.

The empirical application of our methodology refers to the US States over the period 1929-2005. In the case of the US States and despite a considerable literature on regional convergence (e.g. Nissan and Carter, 1998; 2001; Lall and Yilmaz, 2001; Johnson and Takeyama, 2001; Fousekis, 2007) 'club-convergence' remains a relatively unexplored area. We find for example, that a club convergence pattern is evident across the 49 US States with a significant spatial dimension.

The rest of this paper is structured as follows. Section 2 reviews alternative tests for convergence. Section 3 presents our econometrical framework and Section 4 presents our results. In order to highlight the difference in our econometrical approach we also present the results using Nahar and Inder's (2002) methodology. The last section summarises the arguments and considers possible lessons for policy-making.

## 2 Regional Convergence: Cross-section and Time-Series Tests

A useful starting point is the neoclassical model, since the assumptions of this model actually carry implications for the regional convergence debate. This model offers both a theoretical explanation and testable predictions concerning the possibility of convergence in per-capita incomes across regions. Indeed, most of the conceptual definitions of regional convergence used in empirical studies derive directly from the neoclassical model<sup>1</sup>. According to this model, absolute convergence requires that regions with low per-capita income grow faster than those with high per-capita income, indicating that poor regions catching up with rich regions.

Assuming that income per-capita ( $Y_{i,T}$ ) grows as  $Y_{i,T} = e^{g_{i,T}Y_{i,0}}$  and  $g_{i,T} = f(Y_{i,0})$ , then it is possible to express the argument of absolute convergence in terms of the following regression equation:

$$g_{i,T} = c + by_{i,0} \quad (1)$$

where  $g_{i,T} = y_{i,t} - y_{i,0}$  and  $c$  is the constant term. Absolute or  $\beta$ -convergence requires that  $f'_{g_{i,T}y_{i,0}}$  and  $b \in [-1, 0]$ .

However, this approach is too ‘aggregative’ for policy-making, given that it does not make clear if all regions are driven towards ‘steady-state’ equilibrium with equalised per-capita income. It is quite possible that the property of convergence applies only to a set of regions, or a ‘convergence-club’, while regions outside this club do not necessarily experience convergence vis-à-vis those in the club.

### 2.1 Convergence Clubs

A ‘convergence-club’ implies the existence of a set of economies that in the long-run are driven towards ‘steady-state’ equilibrium with equalised per-capita income. Encapsulated in this definition are two fundamental issues. First, there is the question of how to identify those economies which belong

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<sup>1</sup>See, for example, Kılıçaslan and Özatağan (2007), Benos and Karagiannis (2008), Gezici and Hewings (2004), among others.

to a ‘club’, and second there is the question of what is the ‘steady-state’ equilibrium towards which club-members are progressing.

Usually, tests of club-convergence implement cross-section data (e.g. Chatterji and Dewhurst, 1996; Kangasharju, 1999; Fischer and Stirböck, 2006). Such tests, however, while useful in some contexts, generally provide a rather incomplete picture of the composition of the convergence-club. This has led to tests based on the concept of stochastic convergence<sup>2</sup>. Advocates of this approach (e.g. Bernard and Jones, 1996, 1996a; Bernard and Durlauf, 1995) claim that convergence is, by definition, a dynamic concept that cannot be captured by cross-sectional studies.

The associated tests are based on whether the dispersion in per-capita income between two regions has narrowed during a time period, and all observations from that time period are used. Thus, convergence is identified, not as a property of the relationship between initial income per-capita and growth over a fixed sample period, but instead is defined by the relationship between long-run forecasts of the time-series in per-capita income. It follows, then, that this approach takes into account all the relevant information available throughout the given time period, although it might be argued that the issue of choice of time period remains. By definition, the impacts of random shocks to national and regional economies are taken into account, in predicting long-run trends. More specifically, following Bernard and Durlauf (1995), stochastic convergence between two regions  $i$  and  $j$  occurs if the long-run forecasts of per-capita income for both regions are equal. Thus,

$$\lim_{k \rightarrow \infty} E(y_{1,t+k} - y_{i,t+k} | I_t) = 0 \quad \forall i \neq 1 \quad (2)$$

where  $E$  stands for the mathematical expectation,  $y_i$  is the logarithm of income per-capita in region  $i$  ( $i = 1, \dots, n$ ) and  $I_t$  describes the information set available at time  $t$ .

Equation (2) represents the conditions required for convergence to exist between all the regions included in the test. However, a critical issue is the determination of specific econometric tests for stochastic convergence. One of the most widely used such tests, is the Augmented Dickey Fuller (ADF) test and is discussed next.

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<sup>2</sup>Empirical studies concentrate to a large extent on the US states (e.g. 1995; Carlino and Mills, 1996; Strauss, 2000), while some studies have also been conducted for the regions of the UK (McGuinness and Sheehan, 1998), Austria (Hofer and Wörgötter, 1997) and Italy (Proietti, 2005).

The bivariate ADF test may be used to test for convergence between two regions, and takes the following form:

$$\Delta(y_{i,t} - y_{j,t}) = \mu + \alpha(y_{i,t-1} - y_{j,t-1}) + \gamma t + \sum_{k=1}^n \delta_k \Delta(y_{i,t-k} - y_{j,t-k}) + \epsilon_t \quad (3)$$

where  $\mu$  is the constant term,  $t$  denotes the time trend and  $\epsilon_t$  is the error-term of the regression.

Of critical importance is the coefficient  $\alpha = (\rho - 1)$ , where  $\rho$  is the unit root. If  $\rho = 1$ , which implies that  $\alpha = 0$ , then income per-capita in the two regions will not converge while the absence of a unit root ( $\alpha < 0$ ) is an indication of convergence between the two regions. It might be argued that stochastic convergence in the long run implies two properties; firstly that the disparity in income per-capita between the two regions is disappearing and secondly that movement towards long run equilibrium is occurring. However, the unit root test detects ‘catching-up’ convergence only, i.e. the first of the two properties. In order to assess for convergence towards long-run equilibrium also, then it must be the case that the coefficient on the time trend is equal to zero ( $\gamma = 0$ ). Thus, long run convergence between two regions is occurring if  $\alpha < 0$  and  $\gamma = 0$  (Oxley and Greasley, 1995).

The test for stochastic convergence described thus far essentially examines the convergence possibilities between ‘pairs’ of economies only, with the presumption of convergence towards the same steady-state. Using simple ADF tests, it is therefore difficult to identify whether particular groups of economies follow a common convergence path. It comes as no surprise, therefore, that the ability of ADF tests to help policy-makers is limited. Nevertheless, a technique developed by Nahar and Inder (2002) allows such groups to be detected, using time-series data. Therefore, the remainder of this section articulates the methodology proposed by Nahar and Inder (2002).

## 2.2 Convergence Clubs and the approach of Nahar and Inder

Nahar and Inder (2002) point out that the test proposed by Bernard and Durlauf (1995) suffers from a number of weaknesses. Firstly, there is the possibility of an incorrect conclusion, as follows. The definition of convergence encapsulated in equation (2), rests on the long-run forecasts of income

per-capita differences tending to zero as the forecasting horizon approaches infinity. Thus, convergence is detected if the difference  $y_i - y_j$  is a mean zero stationary process. However, it is also possible, that certain non-stationary processes can be convergent even though the unit root hypothesis ( $\alpha = 0$ ) is accepted. The underlying process is convergent, but the ADF test suggests otherwise<sup>3</sup>. Secondly, an ADF test is able to identify convergence only between pairs of regions, as noted above. It can therefore be difficult to identify groups of regions that follow a common convergence pattern towards the same steady-state, and as a consequence the ability of ADF test to detect convergence clubs is limited.

In addition to the question of how to identify those regions which belong to a club, there is the further question of what is the steady-state equilibrium towards which club members are progressing in the long-run. In recognition of this second issue, Nahar and Inder (2002) propose a test that explicitly acknowledges a steady-state level of income per-capita and convergence is assessed with reference to this long-run equilibrium point. In doing so, it also becomes possible to identify members of a convergence-club.

In an empirical setting, convergence is said to occur when the distance of a region's per-capita income from the average approaches zero over time<sup>4</sup>. Nahar and Inder (2002), employ the following econometric test:

$$\phi_{i,t} = f(t) + u_{i,t} = \theta_0 + \theta_1 t + \theta_2 t^2 + \dots + \theta_{k-1} t^{k-1} + u_{i,t} \quad (4)$$

where  $\phi_{i,t}$  is the  $i^{\text{th}}$  economy's squared deviation from the average and  $\theta_i$ 's are parameters. According to equation (4) an estimate of the slope function, i.e.  $\frac{\partial \phi_{i,t}}{\partial t} f'(t)$ , provides an indication of stochastic convergence, which requires that the squared deviations decline through time, that is to say, the average slope of the function (4) is negative, i.e.  $\frac{1}{T} \sum_{t=1}^T \frac{\partial \phi_{i,t}}{\partial t} < 0$  and given by:

$$\frac{1}{T} \sum_{t=1}^T \frac{\partial \phi_{i,t}}{\partial t} = \theta_1 + \theta_2 r_2 + \dots + \theta_{k-1} r_{k-1} + \theta_k r_k = \theta \mathbf{r}' \quad (5)$$

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<sup>3</sup>Nahar and Inder (2002) use the following example. Suppose that the difference in income between two economies,  $y_{i,t} - y_{j,t}$ , is a non-stationary process and is represented by  $y_{i,t} - y_{j,t} = \frac{\theta}{t} + u_t$  in which  $E(u_t) = 0$  and  $u_i$  is a stationary process. As  $t \rightarrow \infty$ , then  $\frac{\theta}{t} \rightarrow 0$  and  $y_{i,t} - y_{j,t}$  is also converging since  $\lim_{k \rightarrow \infty} E(y_{1,t+k} - y_{i,t+k} | I_t) = 0$ .

<sup>4</sup>Alternatively, absolute convergence occurs when  $\lim_{n \rightarrow \infty} E_t(y_{1,t+n} - \bar{y}_{i,t+n}) = 0$ .

where  $r_2 = \frac{2}{T} \sum_{t=1}^T t, \dots, r_{k-1} = \frac{(k-1)}{T} \sum_{t=1}^T t^{k-2}, r_k = \frac{k}{T} \sum_{t=1}^T t^{k-1}$   
 $\mathbf{r}' = [0 \ 1 \ r_2 \ \dots \ r_{k-1} \ r_k]$  and  $\theta = [\theta_0 \ \theta_1 \ \dots \ \theta_{k-1} \ \theta_k]$ .

Application of this test involves three steps. First, equation (4) is estimated using ordinary least squares (hereafter OLS) and, secondly the average slope (i.e. the  $\theta\mathbf{r}'$  vector) is estimated for each region. Finally, the null hypothesis of non-convergence, i.e.  $H_0 : \theta\mathbf{r}' \geq 0$ , is tested against the alternative  $H_a : \theta\mathbf{r}' < 0$ . This technique implies that in the long-run regions with a positive (negative) initial deviation from average, i.e. ( $y_{i,0} - \bar{y} > 0(y_{i,0} - \bar{y} < 0)$ ), move towards steady-state equilibrium, expressed in terms of a zero deviation from the average, i.e.  $(y_i - \bar{y})_t \rightarrow 0$  as  $t \rightarrow \infty$ , as shown in Figure 1.

The proposed technique by Nahar and Inder (2002) has a major advantage over the simple bivariate ADF tests discussed previously. A problem with these tests is their limited ability to distinguish groups of regions which are converging with each other because the tests are restricted to pair-wise comparisons. The advantage of a test based on deviations from a steady-state is that it identifies those regions that are converging towards a common steady-state from those which are not, thereby recognising also the concept of club-convergence. Furthermore, to analyse the composition of the convergence-club further by detecting which regions follow a faster (i.e. above the average rate) or a slower (below the average rate) path towards the steady-state equilibrium.

### 3 Estimation Method

#### 3.1 Econometrical problems of the Nahar and Inder approach

Although the approach of Nahar and Inder (2002) has considerable advantages for detecting clubs it suffers basically from three econometrical problems if it is applied for regions:

*First*, it fails to take spatial autocorrelation of the dependent variable into account. If growth data on regional level is analyzed, it is quite common to find spatial dependencies. Hence it seems plausible to assume that the convergence patterns also inhibit spatial dependence. It can be shown that ignoring spatial dependence in the dependent variable will cause biased estima-



tions in the case of OLS. In our data we find a highly significant spatial lag in the dependent variable.

*Second*, the convergence data has a panel structure and is covering shocks like the Great depression. Since not all regions will be influenced the same way from these shocks, they might act as level effects. Ignoring these level effects like we motivated for time, or generally state levels might also result in biased estimates.

*Finally*, the approximation of an unknown function via polynomials can cause numerical problems. To illustrate this problem consider the following: In the OLS estimation the  $\mathbf{X}$  matrix (the matrix containing the explanatory variables) contains the different points in time, like for example in our case the numbers zero to 75 (if we set 1929 to zero). If we want to incorporate a polynomial of order 7 then the smallest number in our last  $\mathbf{X}$  column is zero while the largest value will be approximately  $10^{13}$ . Hence our  $\mathbf{X}$ -matrix might no longer have a good conditioning and therefore finding the inverse of  $\mathbf{X}'\mathbf{X}$  can cause numerical problems. Additionally this time modelling is not translation invariant, e.g. we may get different estimates if set 1929 to zero or to one and not all polynomials of power one to  $k$  are incorporated. This numerical problem and the absence of translation invariance might also cause biased or inefficient OLS estimates.

Since biased estimations of  $\theta$  can result in biased club convergence patterns, we present in the next section an econometrical framework that is able to deal with all of these presented problems.

### 3.2 A spatial panel approach with B-Splines to Regional Convergence

In order to tackle the first two problems regarding the Nahar and Inder (2002)- approach we use the model given in 6:

$$\phi_{i,t} = \frac{\rho}{w} \sum_{j=1}^n w_{i,j} \phi_{j,t} + \gamma_t + \eta_i + f_i(t) + u_{i,t} \text{ where } u_{i,t} \sim N(0, \sigma^2) \quad (6)$$

$\phi_{i,t}$  is in (6) not only a function of the different fixed effects and the unknown function  $f_i(t)$  but also spatially autocorrelated with the neighbouring  $\phi_{j,t}$ . It is assumed that the innovations  $u_{i,t}$  are independently and identically

normal distributed with zero mean and finite variance  $\sigma^2$ . The weights  $w_{i,j}$  are assumed to be deterministic and reflect the neighbouring structure in the US. If two states  $i$  and  $j$  share a common border,  $w_{i,j}$  is set to one and zero otherwise. Additionally, we set  $w_{i,i}$  to zero. Hence no state can be its own neighbour.  $\bar{w}$  is the normalizing factor for the spatial lag and has following definition:  $\bar{w} := \max_{1 \leq i \leq n} \{\sum_{j=1}^n w_{i,j}\}$ . In (6)  $\gamma_t$  and  $\eta_i$  represent the fixed effects for time and the states respectively.  $\rho$  is the spatial autocorrelation parameter assumed to be in the set  $(-1, 1)$ . If we ignore the term  $f_i(t)$  equation 6 represents a panel data model with fixed effects and a spatial lag in the dependent variable. For more details regarding models like 6 see LeSage and Pace (2009). If we set  $\rho$  and  $\gamma_t$  equal to zero and model  $f_i(t)$  via polynomials then 6 can be seen as the Nahar and Inder (2002) where a pooled regression is used.

The model interpretation stays the same as in Nahar and Inder: We solve Eq. 6 for PHI and calculate the average derivative to time for each region. In order to find standard deviations and the distribution of these effects we use Monte Carlo Simulations.

In order to avoid the third problem of the Nahar and Inder (2002)- approach we model the unknown function  $f_i(t)$  in 6 via so called Basic Splines<sup>5</sup>. Splines are piecewise polynomials that are continuous and differentiable to a certain degree, like in our case: one. Additionally, B-Splines, like the powers of a polynomial can be seen as basis in a functional vector space (Lebesgue space). To approximate a unknown function with B-Splines a certain amount of given spline knots  $\zeta_{l,i}$  are needed. If these are given  $f_i(t)$  can be approximated by (7)

$$f_i(t) \approx \sum_{l=0}^m B_{l,i}(t)\bar{\theta}_i \text{ where } B_{l,i}(t) = \begin{cases} t^l & \text{if } l \leq 2 \\ (t - \zeta_{l,i})_+^2 & \text{if } l > 2 \end{cases} \quad (7)$$

$$\text{and } (t - \zeta_{l,i})_+^2 = \begin{cases} (t - \zeta_{l,i})^2 & \text{if } t \geq \zeta_{l,i} \\ 0 & \text{otherwise} \end{cases}$$

In our case the splineknots  $\zeta_{l,i}$  can be elements of the set  $\{1, 2, \dots, T\}$ , where  $T = 76$  is the number of time observations. Like Nahar and Inder (2002) we consider  $\sum_{l=0}^m B_{l,i}(t)\bar{\theta}_i$  a good approximation for  $f_i(t)$  if the AIC of the model is (at least locally) minimized. Hence finding suitable  $\zeta_{l,i}$  poses a combinatorial optimization problem, which can be solved by using the Genetic

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<sup>5</sup>For details regarding B-Splines see Fahrmyr et al. (2007).

algorithm developed in Koch and Krisztin (2011). If given the spline knots, B-Splines are nothing else than shifted polynomials and hence can be written in a linear fashion:  $\sum_{l=0}^m B_{l,i}(t)\bar{\theta}_l = z_i\bar{\theta}$ . Note that B-Splines are by their very construction invariant against translations in time.

### 3.3 Computational Issues

This subsection briefly discusses some computational issues resulting from an estimation of (6). First we will discuss the Maximum Likelihood (ML) estimator for equation (6) and then the Genetic Algorithm for the approximation of  $f_i(t)$  given in (7).

Before we derive the Loglikelihood of model (6) we rewrite it by using matrix notation (see Eq. (8)). We embed the ML- estimator in the Genetic Algorithm which minimizes the AIC over possible spline knots. Hence for the ML-estimation the spline knots are given.

$$\phi|\zeta_{l,i} = \frac{\rho}{\bar{w}} (\mathbf{I}_T \otimes \mathbf{W}_n) \phi|\zeta_{l,i} + \mathbf{\Gamma}\gamma + \mathbf{H}\eta + \mathbf{Z}\bar{\theta} + \mathbf{u} \text{ where } \mathbf{u} \sim N(\mathbf{0}, \mathbf{I}_{nT}\sigma^2) \quad (8)$$

The vectors in (8) are defined by  $\phi = (\phi_{1,1}, \phi_{1,2}, \dots, \phi_{n,T})'$  and  $\mathbf{u} = (u_{1,1}, u_{1,2}, \dots, u_{n,T})'$ . The matrix  $\mathbf{\Gamma}$  reflects the fixed effects for time and is given by  $(\mathbf{I}_n \otimes \iota_T)$ , where  $\otimes$  denotes the kronecker produkt and  $\iota_T$  a  $T$  by 1 vector containing ones. The matrix for the fixed effects of the sates  $\mathbf{H}$  can for example be obtained if we delete the last<sup>6</sup> column of the matrix  $(\iota_n \otimes \mathbf{I}_T)$ . Since the spline knots are given, we can approximate the nonlinear function via the linearized form where the matrix  $\mathbf{Z}$  is given by:  $\mathbf{Z} = (z_1(1)', z_1(2)', \dots, z_n(T))'$ . For notational convenience we will define  $\mathbf{X}\beta = \mathbf{\Gamma}\gamma + \mathbf{H}\eta + \mathbf{Z}\bar{\theta}$ , where  $\mathbf{X} = [\mathbf{\Gamma}, \mathbf{H}, \mathbf{Z}]$  and  $\beta = (\gamma', \eta', \bar{\theta})$ . Since we assume that  $\rho \in (-1, 1)$  we can solve (8) for  $\phi$  and derive its Loglikelihood:

$$LL(\rho, \sigma^2, \beta) \sim -\frac{nT}{2} \log(\sigma^2) + \frac{T}{2} \log \left( \det \left( \mathbf{I}_n - \frac{\rho}{\bar{w}} \mathbf{W}_n \right) \right) - \frac{1}{2\sigma^2} (\phi - \hat{\phi}) (\phi - \hat{\phi}) \quad (9)$$

where  $\det()$  denotes the determinant of a matrix and  $\hat{\phi}$  are the fitted values of  $\phi$  given the values  $\hat{\rho}, \hat{\sigma}^2, \hat{\beta}$ . To simplify this likelihood we follow LeSage and Pace 2009 and we concentrate out the parameters  $\sigma^2$  and  $\beta$ :

$$LL(\rho) \sim \frac{T}{2} \log \left( \det \left( \mathbf{I}_n - \frac{\rho}{\bar{w}} \mathbf{W}_n \right) \right) - \frac{nT}{2} \log(S(\rho)) \quad (10)$$

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<sup>6</sup>Note that it does not matter which column is deleted. We have to delete one column, otherwise  $(\mathbf{I}_n \otimes \iota_T)$  and  $(\iota_n \otimes \mathbf{I}_T)$  would perfectly colinear.

where  $S(\rho) = e(\rho)'e(\rho)$ ,  $e(\rho) = e_0 - \rho e_d$ ,  $e_0 = \phi - \mathbf{X}\delta_0$ ,  $e_d = \frac{1}{w}(\mathbf{I}_T \otimes \mathbf{W}_n)\phi - \mathbf{X}\delta_d$ ,  $\delta_0 = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\phi$  and  $\delta_d = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\frac{1}{w}(\mathbf{I}_T \otimes \mathbf{W}_n)\phi$ . Since the weight matrix is not affected by a change of the spline knots we follow Griffith (1988) for the calculation of the logdeterminant:

$$\det\left(\mathbf{I}_n - \frac{\rho}{w}\mathbf{W}_n\right) = \prod_{i=1}^n \left(1 - \frac{\rho}{w}\omega_i\right). \quad (11)$$

In (11)  $\omega_i$  denotes the  $i$ -th eigenvalue of  $\mathbf{W}_n$ . With these simplifications one AIC calculation given the spline knots basically reduces to two ordinary least squares regressions and the simple minimization of  $LL(\rho)$ . Note that we have to calculate the eigenvalues of  $\mathbf{W}_n$  only once for the whole optimization process.

The second part of the overall optimization problem stems from choosing suitable spline knots. This is done via a combined asynchronous agents and genetic algorithm approach, outlined in Koch and Krisztin (2011). This method uses two types of asynchronous agents, which employ genetic algorithms to respectively solve the selection of splines. Based on the AIC of these results, each team of agents attempts to improve upon the solution. Each cycle the worst performing agents are deleted from the population.

## 4 Empirical Results

Regional growth may be convergent or divergent. Convergence may also be an exclusive property of a specific set of regional economies. It is the purpose of this section to provide an assessment of whether or not absolute convergence is apparent across the 49 states of the US<sup>7</sup>, and whether this applies only to a selected club.

The simplest approach to testing for convergence is to examine changes in the coefficient-of-variation for regional per-capita income over the period 1929-2005<sup>8</sup>. This test of convergence, when carried out for the 49 states, produces the outcome shown in Figure 2.

Over the examined period the long-run trend in the coefficient-of-variation suggests  $\sigma$ -convergence, although at the beginning of the period some increases are observed. The conclusion to be drawn, therefore, on the basis of

<sup>7</sup>Alaska and Hawaii are excluded, since the available data for these states begin at 1950.

<sup>8</sup>The source for our data is the US Bureau of Economic Analysis.

the  $\sigma$ -convergence test alone is that the states of the US have moved closer together as a group in that the dispersion of income per-capita at the end of the period is narrower than at the beginning.

However, the coefficient-of-variation is only a simple descriptive tool and is not based on a model of regional convergence. There is no underlying explanation of a catching up process implicit in the measure. The concept of  $\beta$ -convergence is derived from an underlying model, however and is examined next.

The potential or otherwise for such  $\beta$ -convergence is indicated in Figure 3, which shows a scatterplot of the average annual growth rate against the initial level of per-capita income. Prior to the formal convergence test, casual inspection of the data in Figure 3 provides a clear indication of an inverse relationship between the average annual growth rate and initial level of income per-capita. The presence of  $\beta$ -convergence, however, cannot be confirmed by visual inspection alone. Therefore, the cross-section test, based on estimation of equation (1) for the 49 US States, is applied to the whole period 1929-2005.

The results, presented in Table 1, show the convergence coefficient ( $b$ ) to be negative and significant at the 95% level. The presence of absolute convergence in the form of a negative relationship between the rate of growth and initial per-capita income is suggested by this evidence, and US states have, on average, shown a strong tendency to converge over the period 1929 to 2005, at a rate of 1.4% per annum.

Nevertheless, it is possible (and necessary given the concerns of this paper) to reconstruct a more precise account of the nature of regional convergence in the US, especially in regard to the relevance of a notion of convergence, as encapsulated in equation (4).

Equation (4) is estimated by OLS for various values of  $k$ . The specific choice of the polynomial order is then made using the Akaike and the Schwartz-Bayesian information criteria. They are used extensively as selection criteria, and are generally considered as reliable, especially in the analysis of time-series data.

Estimation of equation (4) leads to the average slope estimates for each state shown in Table 2. The interpretation of these results is as follows. A state is deemed to be converging to the common steady-state, which is the average level of per-capita income, when the slope estimate is both statistically significant and negative in sign.

Thus, significant convergence towards average per-capita income is iden-

tified for 38 states<sup>9</sup>. These areas, thus, constitute a convergence-club moving towards average per-capita income at an average rate of about 0.25% per annum, with the convergence rates for individual members varying from 0.0027% per annum in the case of West Virginia to 1.4% in the case of District-of-Columbia. About 45% of the club-members exhibit an annual rate of convergence above the average rate. For six states, the average slope estimates are statistically significant but positive in sign to indicate divergence, although the rate of divergence varies somewhat from case to case with South Carolina exhibiting the highest rate of divergence from the average (0.84%).

A closer examination of the evolution of per-capita income in the club suggests that states with an initial deviation above (below) the average per-capita income move towards zero. This is in accordance with the predictions of the neoclassical model and is shown on Figure 4. The location of the convergence-club is shown on Figure 5, together with the diverging regions. A tendency for clustering of the states with similar convergence pattern is apparent. This suggests that regional location is a significant factor in the growth process.

The hypothesis of club convergence and spatial interaction is examined more formally, using the framework introduced in section 3. The relevant results are reported in Table 3. Using the extended framework, the convergence club includes fewer states, as shown in Figure 6. A further distinction of the club can be identified in Figure 6. To be more precise, the convergence club includes two distinct geographical clusters. The first is located in the West Coast while the second in the north-east areas. Given that the spatial results are based on a more robust methodology, it might be argued that the results using the methodology by Nahar and Inder (2002) might be biased, since spatial interaction is omitted.

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<sup>9</sup>It should be mentioned that the size of the states might cause some problems, given the differential performance among areas within the states. For example, estimation of equation (4) using data for metropolitan areas or cities might reveal a different picture. Such a case is quite obvious in European NUTS-2 and NUTS-3 regions. However, in the case of the US states the size of states seems to be appropriate for examining convergence tendencies, since their size allows a relatively detailed decomposition of the pattern of regional convergence.

## 5 Conclusion

This paper offers an empirical assessment of the predictions of the neoclassical model regarding regional convergence. Using two standard measures of regional convergence, namely  $\sigma$  and  $\beta$ -convergence, it is established that the states of the US exhibit strong tendencies towards convergence. This hypothesis is tested further using an alternative method using time series data set. Application of this method has shown that most of the US states exhibit a similar tendency of convergence towards a steady-state level of per-capita income, approximated in terms of the average per-capita income, providing considerable support to the predictions of the neoclassical model across the US states. Nevertheless, the process of convergence revealed in this paper is more complicated. Introducing spatial interaction in a time series context leads to different conclusions, regarding the composition of the convergence club. As the results indicate, the cluster of converging states is limited and includes states located in certain geographical areas. This suggests an intra-club variation. The contribution of this paper's empirical findings, however, is not just limited to adding to the list of tests for regional club-convergence, but most importantly to provide a methodology that will allow policy-makers to detect different groups of regions, introducing a spatial dimension in a time series context, overcoming thus a shortcoming of the standard cross-section and time-series tests. From a policy perspective, this evidence is useful at two levels. Firstly, given a general focus at national and regional level upon support for lagging regions and the promotion of convergence, the identification of a convergence-club clearly assists in drawing a dividing line between regions which might be deemed eligible for assistance and those which are not. Regional assistance should, to a substantial extent, be diverted towards those regions that do not belong to the convergence-club. Secondly, the greater part of effort and assistance should be directed to improve the underlying conditions of lagging regions and thereby generate an economic environment that more closely resembles the combination of characteristics found in the convergence-club.

While the empirical results are serious in their own right, they must be placed in perspective. There is a little pretence that the forgoing analysis provides an exhaustive account of all factors that affect the process of regional club convergence. For example, additional complications arise from the multidimensional nature of the institutional structure of each region; a factor that, indubitably, has important spatial implications. Considerably

more research, therefore, is required before the issue of regional convergence in agricultural productivity can be discussed with confidence. What then is the purpose of this paper? Perhaps the main purpose of this paper should be to provoke interest in further work on the underlying mechanisms of regional club convergence.

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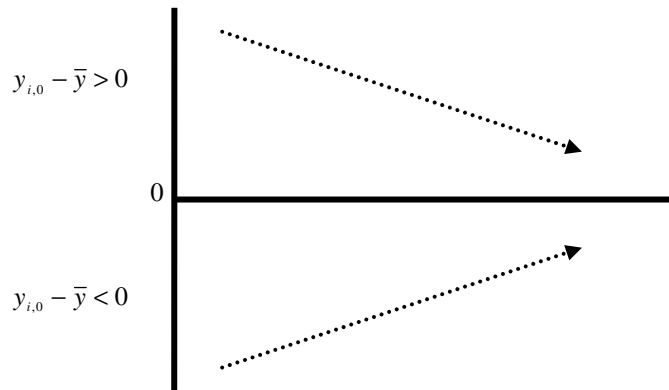
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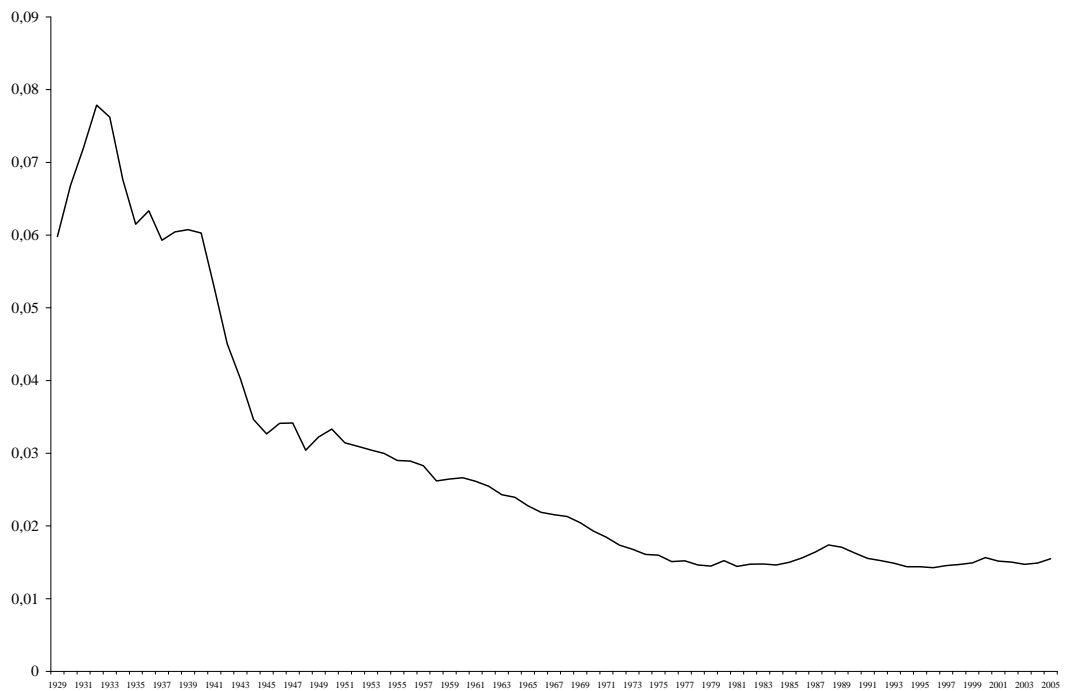
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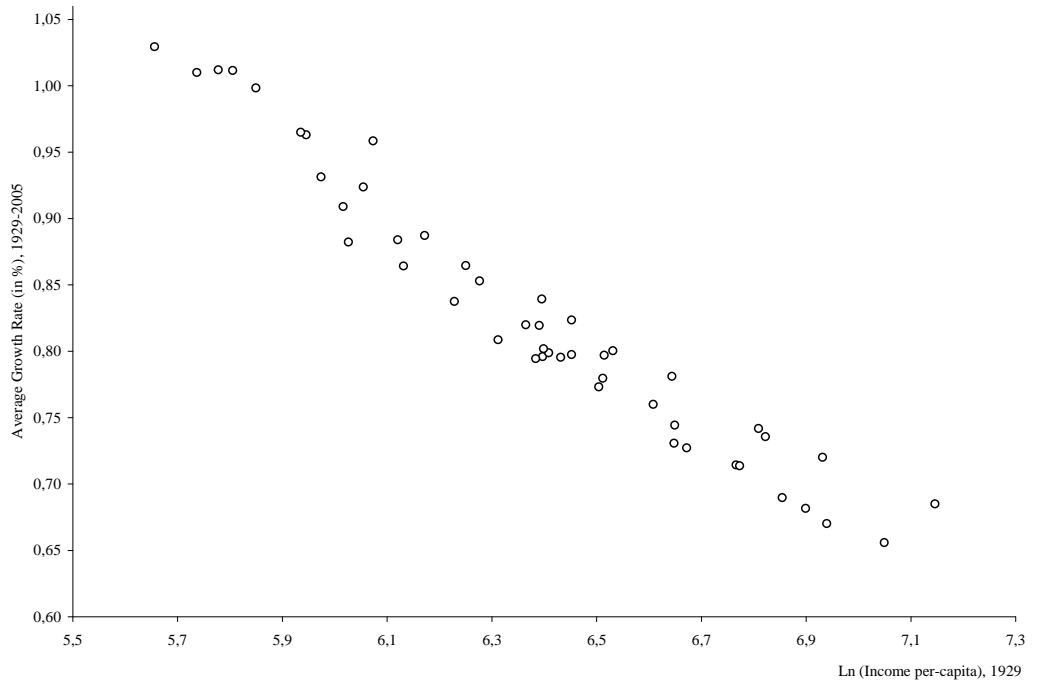
**Figure 1: Convergence towards Steady-state Equilibrium**



**Figure 2: Coefficient of Variation: 49 US States: 1929-2005**



**Figure 3: Absolute Convergence, 49 US States: 1929-2005**



**Figure 4: Movements towards Average per-capita Income**

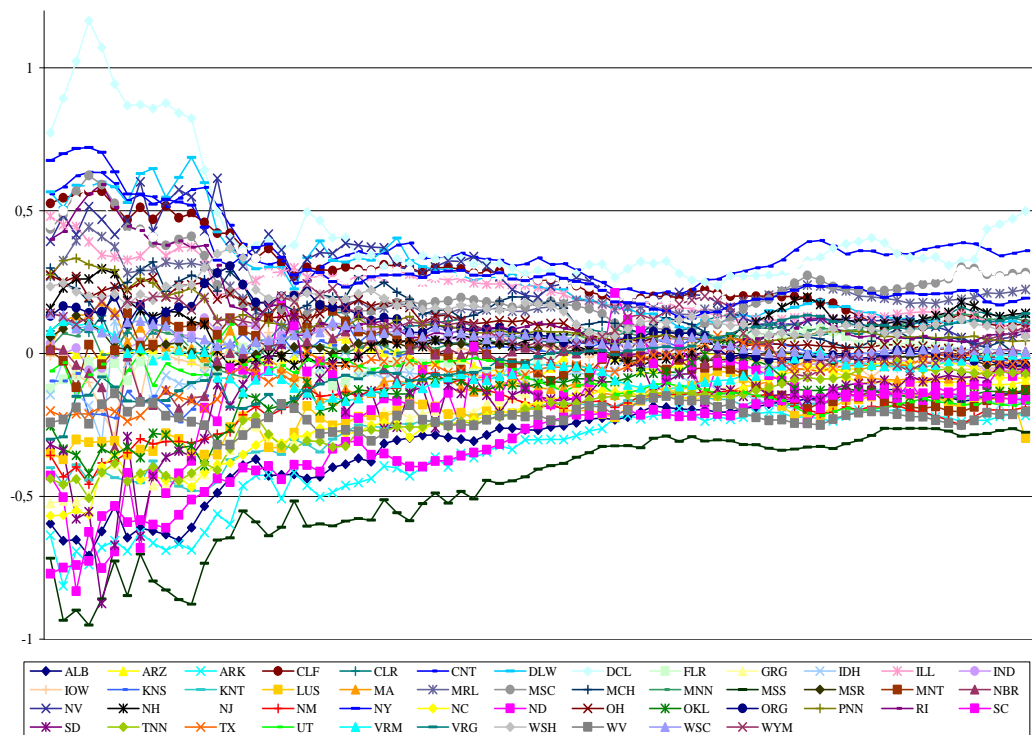


Figure 5: The 'Convergence-club'

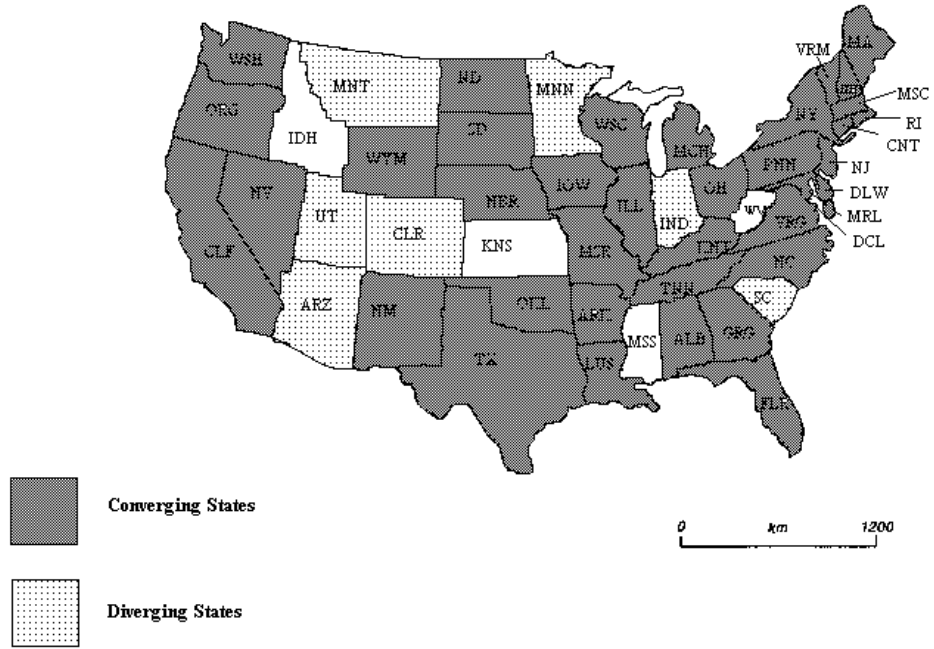
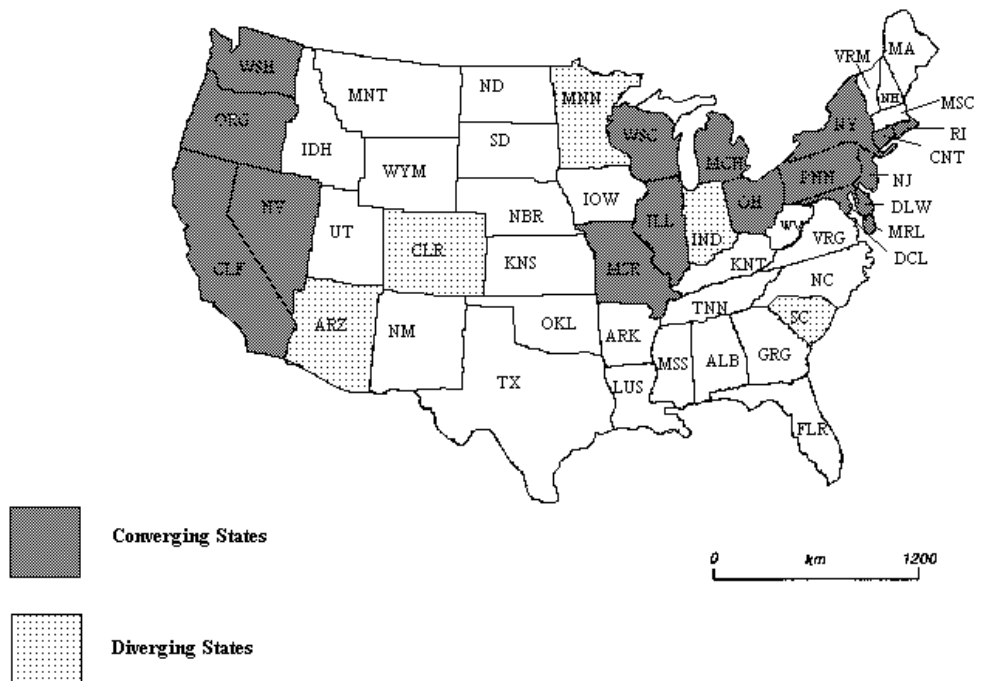


Figure 6: The 'Convergence-club', extended spatial specification



**Table 1: Absolute  $\beta$ -convergence and the speed of convergence: US States, 1929-2005**

OLS, Estimated equation:  $g_i = c + by_{i,0}$

$c$	8.3217*	(34.702)
$b$	-0.6714*	(-18.356)
Implied $\beta$	1.4456*	(18.022)

Notes: Figures in brackets are t-ratios. An asterisk (\*) indicates statistical significance at 95% level of confidence. The rate of convergence is defined as  $\beta = -\frac{\ln(b+1)}{T}$ .

**Table 2: Convergence towards the Average per-capita Income, US States**

State	Polynomial Order	Slope	t-ratio	
Alabama (ALB)	5	-0.0050118	-10.11	**
Arizona (ARZ)	4	0.0001950	8.707	**
Arkansas (ARK)	2	-0.0065412	-28.546	**
California (CLF)	5	-0.0040241	-14.433	**
Colorado (CLR)	2	0.0000686	2.149	**
Connecticut (CNT)	3	-0.0035808	-8.785	**
Delaware (DLW)	2	-0.0047440	-15.795	**
District of Columbia (DCL)	3	-0.0145569	-12.007	**
Florida (FLR)	5	-0.0002230	-5.482	**
Georgia (GRG)	3	-0.0039248	-22.34	**
Idaho (IDH)	3	-0.0000929	-0.787	
Illinois (ILL)	5	-0.0029664	-14.042	**
Indiana (IND)	4	0.0000517	1.695	
Iowa (IOW)	2	-0.0001636	-2.622	**
Kansas (KNS)	5	-0.0000846	-0.671	
Kentucky (KNT)	5	-0.0017234	-6.854	**
Louisiana (LUS)	2	-0.0013329	-14.600	**
Maine (MA)	4	-0.0001544	-2.244	**
Maryland (MRL)	3	-0.0017413	-8.758	**
Massachusetts (MSC)	4	-0.0036279	-10.132	**
Michigan (MCH)	2	-0.0010014	-12.062	**
Minnesota (MNN)	5	0.0002703	8.738	**
Mississippi (MSS)	2	-0.0093797	-23.459	**
Missouri (MSR)	5	-0.0001496	-5.425	**
Montana (MNT)	5	0.0004839	4.445	**
Nebraska (NBR)	2	-0.0002756	-3.775	**
Nevada (NV)	5	-0.0019381	-3.206	**
New Hampshire (NH)	3	-0.0006480	-5.585	**
New Jersey (NJ)	3	-0.0028700	-10.896	**
New Mexico (NM)	5	-0.0014699	-7.540	**
New York (NY)	4	-0.0072033	-22.982	**
North Carolina (NC)	4	-0.0043490	-19.069	**
North Dakota (ND)	3	-0.0067489	-8.414	**
Ohio (OH)	2	-0.0008952	-21.881	**
Oklahoma (OKL)	5	-0.0009309	-3.565	**
Oregon (ORG)	4	-0.0001874	-1.622	
Pennsylvania (PNN)	4	-0.0015209	-19.777	**
Rhode Island (RI)	5	-0.0031111	-6.953	**
South Carolina (SC)	5	0.0084863	-19.181	**
South Dakota (SD)	3	-0.0051511	-5.787	**
Tennessee (TNN)	2	-0.0028139	-26.77	**
Texas (TX)	3	-0.0007966	-13.807	**
Utah (UT)	4	0.0002896	5.201	**
Vermont (VRM)	2	-0.0001045	-2.949	**
Virginia (VRG)	5	-0.0009432	-6.248	**
Washington (WSH)	4	-0.0003804	-2.251	**
West Virginia (WV)	5	-0.0000279	-0.129	
Wisconsin (WSC)	5	-0.0002679	-7.522	**
Wyoming (WYM)	4	-0.0003295	-2.754	**

Note: \*\* indicates that the average slope is statistically significant at 95% level. The final choice on the polynomial order is based on the Akaike and the Schwartz-Bayesian information criterion.

**Table 3: Convergence Clubs and spatial interaction, US States, panel data**

State	Average Slope	Standard Deviation	p-value
Alabama (ALB)	0.511	0.041	0.000
Arizona (ARZ)	0.063	0.025	0.005
Arkansas (ARK)	0.502	0.041	0.000
California (CLF)	-0.247	0.029	0.000
Colorado (CLR)	0.175	0.030	0.000
Connecticut (CNT)	-0.060	0.027	0.012
Delaware (DLW)	-0.339	0.032	0.000
District of Columbia (DCL)	-0.019	0.041	0.316
Florida (FLR)	0.244	0.027	0.000
Georgia (GRG)	0.597	0.028	0.000
Idaho (IDH)	0.186	0.047	0.000
Illinois (ILL)	-0.247	0.031	0.000
Indiana (IND)	0.022	0.034	0.263
Iowa (IOW)	0.086	0.042	0.020
Kansas (KNS)	0.224	0.044	0.000
Kentucky (KNT)	0.379	0.033	0.000
Louisiana (LUS)	0.266	0.025	0.000
Maine (MA)	0.014	0.028	0.315
Maryland (MRL)	0.071	0.030	0.009
Massachusetts (MSC)	0.049	0.036	0.086
Michigan (MCH)	-0.201	0.036	0.000
Minnesota (MNN)	0.265	0.043	0.000
Mississippi (MSS)	0.448	0.042	0.000
Missouri (MSR)	-0.007	0.026	0.388
Montana (MNT)	-0.031	0.041	0.227
Nebraska (NBR)	0.091	0.027	0.002
Nevada (NV)	-0.219	0.031	0.000
New Hampshire (NH)	0.180	0.035	0.000
New Jersey (NJ)	-0.063	0.029	0.018
New Mexico (NM)	0.278	0.042	0.000
New York (NY)	-0.397	0.030	0.000
North Carolina (NC)	0.554	0.045	0.000
North Dakota (ND)	0.357	0.041	0.000
Ohio (OH)	-0.155	0.032	0.000
Oklahoma (OKL)	0.211	0.042	0.000
Oregon (ORG)	-0.032	0.030	0.150
Pennsylvania (PNN)	-0.127	0.027	0.000
Rhode Island (RI)	-0.181	0.031	0.000
South Carolina (SC)	0.769	0.025	0.000
South Dakota (SD)	0.338	0.045	0.000
Tennessee (TNN)	0.498	0.041	0.000
Texas (TX)	0.346	0.030	0.000
Utah (UT)	-0.005	0.041	0.458
Vermont (VRM)	0.058	0.028	0.020
Virginia (VRG)	0.645	0.036	0.000



Washington (WSH)	-0.012	0.029	0.346
West Virginia (WV)	0.120	0.041	0.002
Wisconsin (WSC)	-0.026	0.034	0.227
Wyoming (WYM)	0.155	0.021	0.000

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