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# Urban Agglomeration and Aggregate Economic Growth

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## Abstract

This paper presents a theoretical approach to solve the main problems faced to explain the relationship between aggregate economic growth and the urban structure. The most significant conclusion reached is the existence of a theoretical relationship between aggregate economic growth and urban concentration with an inverted-U shape. This result had been previously found in an empirical context (Henderson, 2003), but not as outcome of a theoretical model. An overlapping generations model with four different types of goods (some with both technological and local externalities) and two cities where their production could be located provides the dynamics of the movements of labor and goods across cities. The resulting system of two cities with different patterns of specialization, urban concentration and economic growth rates, makes clear how to set out the comparison of aggregate growth rates: only the aggregate growth rate between two steady states, one without migration but with trade specialization and the other after migration and specialization, makes sense.

## 1 Introduction

The endogenous growth theory is applied to any geographical entity, as if the spatial distribution of the economic units were not relevant. It considers the countries or regions as adimensional points. In other words, for the basic growth theory the economic growth of an economy is independent of its spatial structure.

On the other hand, there are reasons to think that geographic distribution is important for economic growth. An extensive literature on the urbanization process looks at both urbanization and urban concentration, asking whether and when there is under or over-urbanization or under or over urban concentration in terms of economic growth.

Urbanization researchers argue that national government policies and non-democratic institutions promote excessive concentration (the extent to which the urban population of a country is concentrated in one or two major metropolitan areas), except in former planned economies where migration restrictions were enforced. This type of research assumes that there is an optimal level of urban concentration. In fact, from the new economic geography point of view, more precisely from new urban economics, it is considered that there is a distribution of territorial structure which is most appropriate for economic growth. In fact, an inverted U-shape relationship would exist between urban concentration and economic growth.

Any deviation from the optimal level of urban concentration would imply a cost in terms of production not carried out. The evaluation of whether that deviation is important or not has been called the “so-what question” by Henderson (2003): *What are the implications of the existence of a non-optimal distribution?* (Henderson, 2003, 2005). This author has quantitatively examined the assumption and asked the basic ‘so-what’ question: *how great are the economic losses from significant deviations from any optimal degrees of urban concentration or rates of urbanization.*

Henderson shows that (1) there is a best degree of urban concentration, in terms of maximizing productivity growth (2) that best degree varies with the level of development and country size, and (3) over or under-concentration can be very costly in terms of productivity growth.

Bertinelli and Strobl (2007) also investigates how urban primacy affects economic growth using the same database as Henderson (2003). In this case, they use semi-parametric estimation techniques and find that an increasing relationship is unearthed between urban primacy and growth. This result moderates previous findings on the existence of an optimal level of urban primacy, by suggesting that if such an optimal level exists, the range of values of urban primacy for which it is reached is fairly wide. For these authors, the inverted U-shape relationship assumption should be relaxed.

These and other authors ( ) have empirically estimated the importance of the so-what question and the relationship between urban concentration and economic growth. However, the most appropriate urban concentration level for aggregate economic growth has not been showed from a theoretical point of view. Some contributions (Bertinelli and Black, 2004; Henderson and Wang, 2005) have come closer to solving the problem and shown the two forces which are relevant when determining the optimal agglomeration level. On one hand, the positive effect of economies of scale and agglomeration. On the other hand, the negative effect of the congestion caused by agglomeration creating costs that can outweigh the advantages.

Both papers outline a typical scheme of an urban system with an exogenously growing population and with human capital as the only input. The first one compares urban population growth with rural population growth, but they do not determine the urban agglomeration level. Similarly, the second paper only determines the steady state without growth or a clearly defined rate. A remarkable element of Henderson and Wang (2005) is that it explains the growth rate of the existing cities and the growth of the number of these cities. The problem is that they do not explain the decision-making process of the economic agents to reach the steady state.

Some limitations of previous theoretical research can be highlighted:

- Fixed capital stock is not considered, only human capital. The consequence of this assumption is the impossibility of considering specialization dynamics.
- The productive structure is exogenous in cities and rural areas.
- When and why previously rural areas can become urban areas is not explained.
- Migration flows are not sufficiently explained: migration choice is not based on a decision according to a utility function. The general equilibrium should contain incentives to move endogenously the population flows between cities.
- Transport costs between cities are not considered, although intra-urban costs in relation to the Central Business District are taken into account.
- The role of technology is not adequately integrated, in spite of being the key element of the growth process.
- Economic policy conclusions are not checked.

In this paper, we study the relationship between urban agglomeration and aggregate economic growth, trying to overcome some of the previous limitations. The main objective is to find out how the urban agglomeration level determines the long-term economic growth rate from a theoretical point of view. We define a procedure to find a theoretical representation of the consequences that the degree of urban concentration has on the aggregate economic growth rate, something that has not been found yet. As we have just mentioned above, this relationship has been empirically demonstrated, but a theoretical framework has not been developed, due to the complexity of the posed question. It has not been determined why the degree of concentration might affect the growth rate permanently, what is the reference of the comparison and what are

the theoretical reasons of the deviation from the best situation. Finally, it has to be established why that relationship should have an inverted U-shape as [Williamson \(1965\)](#) and [Henderson \(2003\)](#) already stated or a ‘relaxed’ version as [Bertinelli and Strobl \(2007\)](#) found.

For our purpose, we build a general equilibrium OLG model in which physical capital and technology have been more explicitly introduced to let the specialization process be endogenous. Four goods with different production technologies productions can be produced in each city. The model considers an urban system formed by two different types of cities and with a specialization hierarchy caused by technological, agglomeration or geographical reasons ([Duranton and Puga, 2001](#)). The urban concentration measure is primacy (the share of urban population represented by the largest city), which has been widely used in the literature ([Henderson, 2003](#)). An important problem to solve is how to compare the growth rate of interest because the alternatives are many. The approach makes clear that only the aggregate growth rate between two steady states: one without migration but with trade specialization and the other after migration and trade specialization.

Theoretical models of the new economic geography have traditionally explained economic agglomeration in geographical space by the interaction of some key aspects, such as increasing returns (or imperfect competition), linkages, immobile factors and transport costs. Our approach succeeds in explaining spatial concentration without invoking any of these factors. We use specialization derived from trade, urbanization, immigration processes and technological and local externalities as sources of agglomeration economies. On the other hand, we do not consider the distance in our model, such as urban economics traditionally does. That is the reason why there is not an explanation for the creation of cities. The addition of these and other topics will be the objective in further research.

The rest of the paper is structured as follows. Section 2 presents the model we develop, with its basic elements and the equilibrium of each city (closed economy equilibrium). Sections 3 to 6 develop the dynamics of the urban system formation, the specialization and migration processes. Section 7 derives the formal relationship existing between urban concentration and economic growth rate. Finally, section 8 makes some concluding remarks.

## 2 The model of closed (isolated) cities

We set up an endogenous growth model where four goods are produced with different technologies. In this section, we describe the basic elements of the model and the closed city equilibrium. The formation of the urban system and the relationship “urban concentration aggregate economic growth” will be introduced in the following sections.

### 2.1 Basic elements

#### 2.1.1 Consumers

An overlapping generation model (OLG) is considered. Every utility-maximizing individual lives for two periods (young and old).  $L$  individuals are born each period and the utility function of anyone of the generation  $t$  is given by:

$$U_t = U_t^1 + (1 + \delta)^{-1} U_{t+1}^2 \quad (1)$$

Superscripts 1 and 2 refer to the periods when the individual is young and old, respectively. The parameter  $\delta$  denotes the intertemporal discount rate.

Utility is derived from the consumption of four partially complementary goods:  $Y$ ,  $Z$ ,  $\Omega$  and  $X$ . It is assumed that the contemporaneous utility function is logarithmic in the aggregate

consumption  $(c_Y^{\nu_Y} c_Z^{\nu_Z} c_\Omega^{\nu_\Omega} c_X^{\nu_X})$ :

$$U_t^j = \nu_Y \ln c_{Y,t}^j + \nu_Z \ln c_{Z,t}^j + \nu_\Omega \ln c_{\Omega,t}^j + \nu_X \ln c_{X,t}^j \quad (2)$$

$$j = 1, 2; \nu_Y + \nu_Z + \nu_\Omega + \nu_X = 1; 0 < \nu_Y, \nu_Z, \nu_\Omega, \nu_X < 1$$

where  $c_Y, c_Z, c_\Omega$  and  $c_X$  are the consumption of goods  $Y, Z, \Omega$  and  $X$ , respectively, and  $\nu_Y, \nu_Z, \nu_\Omega, \nu_X$  are parameters indicating the weight of each good in the aggregate consumption. Young individuals work and receive a salary ( $W_t$ ) which is used in consumption spending ( $E_t^1$ ) and saving ( $s_t$ ). As good  $Y$  is taken as the numeraire, these variables are measured in units of this good. Thus, the first period budget constraint can be expressed as follows:

$$W_t = E_t^1 + s_t = c_{Y,t}^1 + P_{Z,t} c_{Z,t}^1 + P_{\Omega,t} c_{\Omega,t}^1 + P_{X,t} c_{X,t}^1 + s_t \quad (3)$$

$P_{Z,t}, P_{\Omega,t}$  and  $P_{X,t}$  are the relative prices of goods  $Z, \Omega$  and  $X$  in terms of good  $Y$ , respectively. Wages, expenditure and savings ( $W_t, E_t^1$  and  $s_t$ ) will be expressed in units of the numeraire, but not the production and consumption of goods  $Z, \Omega$  and  $X$ .

Old people do not work and consume an amount equal to the sum of the value of their savings and the returns obtained from their savings, so the budget constraint in the second period will be:

$$(1 + r_{t+1}) s_t = E_{t+1}^2 = c_{Y,t+1}^2 + P_{Z,t+1} c_{Z,t+1}^2 + P_{\Omega,t+1} c_{\Omega,t+1}^2 + P_{X,t+1} c_{X,t+1}^2 \quad (4)$$

where  $r_{t+1}$  is the real interest rate in period  $t + 1$  and  $E_{t+1}^2$  the consumption expenditure of an old individual in the same period.

Using the single-period budget constraints (3) and (4), the intertemporal budget constraint will be:

$$W_t = E_t^1 + \frac{E_{t+1}^2}{(1 + r_{t+1})} \quad (5)$$

In the first period of their lives, consumers make their consumption and savings decisions, so it is assumed that they have perfect knowledge of the values that the variables will take in the following period because no uncertainty is present in the model.

### 2.1.2 Producers

Each city can produce four goods, whose production function combine two factors, capital ( $K$ ) and labor ( $L$ ), in different ways. For simplicity, time subscripts have been avoided where possible. Production of good  $X$  is obtained according to a Cobb-Douglas production function where capital and labor have constant returns to scale and there does not exist possibility of productivity gains. This good is the ‘traditional’ one and is devoted entirely for consumption:

$$X = K_X^\alpha L_X^{1-\alpha}; 0 < \alpha < 1 \quad (6)$$

where  $K_X$  and  $L_X$  are the amounts of capital and labor used in the production of good  $X$ . For subsequent sections and interpretations we will denote  $1 - \alpha$  as  $\beta$ .

We can redefine these variables in terms of labor  $\left(x = \frac{K_X}{L_X}, k_X = \frac{K_X}{L_X}\right)$  and the technology will be given by:

$$x = k_X^\alpha \quad (7)$$

To obtain good  $Y$ , capital and labor are combined according to a production function with constant returns to scale of the type used by [Kemnitz \(2001\)](#):

$$Y = K_Y^\alpha (TL_Y)^{1-\alpha}; 0 < \alpha < 1 \quad (8)$$

where  $K_Y$  and  $L_Y$  are the inputs employed in its production. For simplicity, it is assumed that all goods have the same capital intensity<sup>1</sup>. Following Kemnitz (2001), labor productivity in (8) depends on  $T$ , reflecting a positive technological externality of the capital/labor ratio in this sector. Good  $Y$  is the ‘advanced’ good. Denoting  $T = a^{-1} \frac{K_Y}{L_Y}$  ( $a > 0$ ), the following reduced form for the technology used to produce good  $Y$  can be derived:

$$Y = a^{\alpha-1} K_Y \quad (9)$$

Normalizing by the labor employed in sector  $Y$  ( $y = \frac{Y}{L_Y}$ ,  $k_Y = \frac{K_Y}{L_Y}$ ), it is obtained that:

$$y = a^{\alpha-1} k_Y \quad (10)$$

This expression means that the reduced form of (9) is equivalent to an AK model. Good  $Y$  is the capital good, but is also used for consumption. The AK technology will allow a continuous capital accumulation at stable rates in steady state and, as a consequence, long-term growth. Production function of the good  $Z$  introduces a local externality in the following way:

$$Z = K_Z^\alpha L_Z^{1-\alpha} L^\gamma; \quad 0 < \alpha < 1; \quad 0 < \gamma < 1 \quad (11)$$

where  $L$  represents the total employment in the city and  $\gamma$  the magnitude of the local externality (for further conclusions in this paper we suppose that the local externality is greater than the technological parameter  $\gamma > \alpha$ ). The function can be reduced to the following expression:

$$z = k_Z^\alpha L^\gamma \quad (12)$$

Production of the good  $\Omega$  includes both types of externalities, technological and local ones:

$$\Omega = K_\Omega^\alpha (TL_\Omega)^{1-\alpha} L^\gamma; \quad 0 < \alpha < 1; \quad 0 < \gamma < 1 \quad (13)$$

that can be reduced to:

$$\omega = a^{\alpha-1} k_\Omega L^\gamma \quad (14)$$

Finally, it will be also assumed that capital entirely depreciates every period, so the capital/labor ratio at a certain period will be equal to the total savings of each individual born in the previous period; in such a way that we can write:

$$k_{t+1} = s_t \quad (15)$$

where  $k = \frac{K}{L}$ , with  $K$  and  $L$  being the total amounts of capital and labor in the economy, such that  $K = K_X + K_Y + K_Z + K_\Omega$  and  $L = L_X + L_Y + L_Z + L_\Omega$ . As it has been said previously  $L$  is constant.

## 2.2 Equilibrium of the urban system of closed cities

We begin describing the closed equilibrium of isolated cities. We suppose that there exists two cities that do not keep any relationship between them, they do not form part of a urban system. We show the equilibrium for whichever of them.

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<sup>1</sup>It could be assumed that goods had different capital intensities, but this assumption would not make a special contribution to our objectives, only would add calculation complexity. See Sanso et al. (2005). The equality of capital intensities does not mean that finally both are the same, as it is evident from (7) and (10)

### 2.2.1 Consumers' decisions

Consumers maximize the utility function (1) subject to the constraints (5). From the first order conditions it can be derived that expenditure on each good is a fixed proportion  $\nu_Y$ ,  $\nu_Z$ ,  $\nu_\Omega$  and  $\nu_X$ , respectively, over total expenditure in the two periods:

$$P_{i,t}c_{i,t}^1 = \nu_i E_t^1; \quad i = Y, Z, \Omega, X; \quad P_{Y,t} = 1 \quad (16)$$

$$P_{i,t+1}c_{i,t+1}^2 = \nu_i E_{t+1}^2; \quad i = Y, Z, \Omega, X; \quad P_{Y,t+1} = 1 \quad (17)$$

How the salary is allocated in the first period as total consumption expenditure and savings can also be obtained:

$$E_t^1 = \frac{1 + \delta}{2 + \delta} W_t \quad (18)$$

$$s_t = \frac{1}{2 + \delta} W_t \quad (19)$$

If the discount rate increases, consumers show higher preference for current consumption. That is why the relationship between  $\delta$  and  $E_t^1$  is positive, whereas between  $\delta$  and  $s_t$  is negative.

We can also obtain the relationship between the salary and the consumption expenditure in the second period of life. It positively depends on the interest rate and wages, while negatively on the discount rate.

$$E_{t+1}^2 = \frac{1 + r_{t+1}}{2 + \delta} W_t \quad (20)$$

The expenditure gross growth rate between two consecutive periods is equal to:

$$\frac{E_{t+1}^2}{E_t^1} = \frac{1 + r_{t+1}}{1 + \delta} \quad (21)$$

### 2.2.2 Producers' decisions

Firms maximize profits in a context of perfect competition in the markets of goods and inputs. Both the marginal productivities of labor ( $W$ ) and capital ( $1 + r$ ) in each sector can be derived in terms of good  $Y$  from the optimization problem:

$$W_Y = \beta a^{-\beta} k_Y; \quad W_Z = \beta P_Z L^\gamma k_Z^\alpha; \quad W_\Omega = \beta a^{-\beta} P_\Omega L^\gamma k_\Omega; \quad W_X = \beta P_X k_X^\alpha \quad (22)$$

$$1 + r_Y = \alpha a^{-\beta}; \quad 1 + r_Z = \alpha P_Z L^\gamma k_Z^{-\beta}; \quad 1 + r_\Omega = \alpha a^{-\beta} P_\Omega L^\gamma; \quad 1 + r_X = \alpha P_X k_X^{-\beta} \quad (23)$$

As wages and interest rates in the four productive sectors are equal in equilibrium and they have the same capital intensity  $\alpha$ , the capital/labor ratios are also the same and we can obtain the relative prices:

$$k_Y = k_Z = k_\Omega = k_X = k \quad (24)$$

$$P_Z = T^\beta L^{-\gamma}; \quad P_\Omega = L^{-\gamma}; \quad P_X = T^\beta \quad (25)$$

The relative price of good  $X$  depends negatively on the technological parameter and positively on the capital/labor ratio, the relationship with the overall technology term ( $T$ ) being positive. This last property reflects the way gains in productivity in the 'advanced' sector are transmitted

to the ‘traditional’ one through relative price<sup>2</sup>. Similarly, price of good  $Z$  depends positively on the capital/labor ratio and negatively on the technological parameter, and in this case, also negatively on the level of total employment of the city. The relative price of good  $\Omega$  depends negatively on the population.

### 2.2.3 Markets clearing and steady state

As mentioned above, good  $Y$  is used both for consumption and as productive capital. Goods  $X$ ,  $Z$  and  $\Omega$  are entirely devoted to consumption. Using equations (16) and (17) we obtain their corresponding market clearing conditions:

$$\begin{aligned} l_Y y_t L_t &= \nu_Y E_t^1 L_t + \nu_Y E_t^2 L_t + k_{t+1} L_t \\ l_Z P_{Z,t} z_t L_t &= \nu_Z E_t^1 L_t + \nu_Z E_t^2 L_t \\ l_\Omega P_{\Omega,t} \omega_t L_t &= \nu_\Omega E_t^1 L_t + \nu_\Omega E_t^2 L_t \\ l_X P_{X,t} x_t L_t &= \nu_X E_t^1 L_t + (\nu_X) E_t^2 L_t \end{aligned} \quad (26)$$

where  $l_Y$ ,  $l_Z$ ,  $l_\Omega$  and  $l_X$  are the proportions of labor used in the production of goods  $Y$ ,  $Z$ ,  $\Omega$  and  $X$ , respectively. The proportion of capital used to produce each good are represented by  $q_Y$ ,  $q_Z$ ,  $q_\Omega$  and  $q_X$ . Market clearing conditions for the two productive factors imply that  $q_Y + q_Z + q_\Omega + q_X = 1$ ,  $l_Y + l_Z + l_\Omega + l_X = 1$ . From (24) and (25) it follows that:

$$\frac{q_Y}{l_Y} = \frac{q_Z}{l_Z} = \frac{q_\Omega}{l_\Omega} = \frac{q_X}{l_X} = 1 \quad (27)$$

Finally, from (15), (19), (22), (23) and (24) and (25), the equation reflecting the capital accumulation process between two consecutive periods can be obtained:

$$k_{t+1} = \frac{\beta}{2 + \delta} a^{-\beta} k_t \quad (28)$$

The steady state is characterized by a constant growth rate of capital and output in the economy. In addition, production factors will also be distributed in stable proportions between sectors. The growth rate can be derived immediately from expression (28):

$$g_t = \frac{k_{t+1}}{k_t} = \frac{\beta}{2 + \delta} a^{-\beta} \quad (29)$$

This long run growth rate depends negatively on  $a$ , implying a greater growth with a greater externality (a greater marginal productivity of capital in the ‘advanced’ sector). Also note that the growth rate depends negatively on the intertemporal discount factor. With regard to the participation of factors in the production, a larger participation of labor ( $\beta$ ) implies a larger growth rate. This is due to the fact that the capital investment depends on the wage level.

In the steady state this rate will be also the growth rate of the capital per worker in each productive sector and, therefore, according to equation (22), the one of the wage. At the same time, from equations (25) and (25) we derive, in absence of migrations, that the relative prices of  $X$  and  $Z$  grow in steady state to rate  $g^\beta$  and the price of  $\Omega$  remains the same, in relative terms to  $Y$ .

As for the consumption, since the expenditure of young and old is proportional to the wage,  $E^1$  and  $E^2$  also grow at the rate  $g$ , just like the consumption of the goods  $Y$  and  $\Omega$ . The consumption of  $X$ , however, grows at a rate  $g^\alpha$  since the expenditure in the good grows at a rate  $g$  and its relative price at a rate  $g^\beta$ , the same happens with good  $Z$ . This result is evident as from the technology of these sectors in (7) and (12), keeping in mind that in the steady state all the production of  $X$  and  $Z$  is consumed.

<sup>2</sup>This relationship can be understood as a description of the Balassa-Samuelson effect.



It can be noticed that the rate of growth the economy is able to maintain in a sustained way is, in general, different from the rhythm of growth of the expenditure along the life of an individual, this is, the expenditure during the old age in relation to the expenditure during the youth, which is given by equation (21).

As a (reduced form) AK model, the economy will always grow at rate (29), reaching the steady state immediately. No transitional dynamics will be present. Positive economic growth implies a value of  $g$  greater than unity. This will happen if the following condition is satisfied:

$$a^{-\beta} > \frac{2 + \delta}{\beta} \quad (30)$$

This condition implies that positive growth requires high productivity levels in the ‘advanced’ good and a low discount rate.

From equation (26), the factor proportions devoted to the productive sectors can be derived:

$$\begin{aligned} q_Y = l_Y &= 1 - (1 - \nu_Y) \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right] = \left( \frac{\beta}{2 + \delta} \right) + \nu_Y \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right] \\ q_Z = l_Z &= \nu_Z \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right] \\ q_\Omega = l_\Omega &= \nu_\Omega \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right] \\ q_X = l_X &= (\nu_X) \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right] \end{aligned} \quad (31)$$

### 3 The Sequence of Urban System Dynamics

Having described how each urban economy would isolated work, the following step is to explore what happens when the two cities, namely  $A$  (poor) and  $B$  (rich), belong to the same urban system. It will be assumed that both cities have the same technology and structural characteristics in preferences ( $\nu_Y, \nu_Z, \nu_\Omega, \nu_X, \delta$ ) and productive sectors ( $\alpha, \gamma$ ). They will differ in capital/labor ratio, but an additional simplifying assumption will be that both cities have initially the same population  $L_{At} = L_{Bt} = L_t$ .

We could previously wonder what would happen if both cities were identical in all parameters and variables. According to the results of the previous section, the conclusion is that there would not be any repercussion neither of growth nor of scale. Relative prices would be the same, as wages and interest rates, and there would not be any trend to modify this initial situation. The reason is that in this model the scale has not any differential effect and the market enlargement would be the unique novelty. Therefore, our model lacks ‘Scale Effect’ if both cities have initially the same population. So there will be no consequences from an urban system formed by two or more identical cities if the the total population is evenly distributed among all cities.

We are interested in the effects that migration of workers and differences between cities have on the dynamics of population of both cities. In the end, we are interested in getting a system of cities with different sizes in order to explain theoretically why the degree of agglomeration might affect the aggregate economic growth rate as mentioned in the introduction.

The main difference is related to the capital/labor ratio, which is smaller in the poor city ( $k_A < k_B$ ). We assume that both cities have the same technological level ( $a_A = a_B = a$ ). There is not population growth in neither of both cities. Taking into account the assumption that both cities have initially the same population ( $L_{At} = L_{Bt} = L_t$ ), this means that both cities only differ in the capital stock ( $K_A < K_B$ ).

Generally, the integration of cities in an urban system can take place as a process of free factor mobility (labor and capital), trade of goods, some combination of the three elements, or all of them. For the whole paper we assume that there are perfect labor mobility across cities, free

**Table 1:** Values of the main variables by city at initial situation  $t$ 

	City $A$	City $B$	Comparison
$P_{Xt}$	$a^{-\beta}k_{At}^{\beta}$	$a^{-\beta}k_{Bt}^{\beta}$	$P_{AXt} < P_{BXt}$
$P_{Zt}$	$a^{-\beta}k_{At}^{\beta}L_{At}^{-\gamma}$	$a^{-\beta}k_{Bt}^{\beta}L_{Bt}^{-\gamma}$	$P_{AZt} < P_{BZt}$
$P_{\Omega t}$	$L_{At}^{-\gamma}$	$L_{Bt}^{-\gamma}$	$P_{A\Omega t} = P_{B\Omega t}$
$W_t$	$\beta a^{-\beta}k_{At}$	$\beta a^{-\beta}k_{Bt}$	$W_{At} < W_{Bt}$
$(1 + r_t)$	$\alpha a^{-\beta}$	$\alpha a^{-\beta}$	$(1 + r_{At}) = (1 + r_{Bt})$
$g_t$	$\frac{1-\alpha}{2+\delta}a^{-\beta}$	$\frac{\beta}{2+\delta}a^{-\beta}$	$g_{At} = g_{Bt}$

trade of goods, but not perfect capital mobility<sup>3</sup>. We consider two steps in the process of urban system dynamics. Firstly, free trade happens. Secondly, migration takes place. We have also considered the inverse sequence (first trade, then migration), in which the final result of the model would not change. The separation of the two steps makes possible the comparison of the aggregate growth rate for different level of urban concentration, related to the situation with trade specialization but without labor mobility.

The sequence of urban dynamics is characterized by the following stages, described in the subsequent sections.

- Initial situation ( $t$ ): Isolated cities.
- First stage ( $t + 1$ ): Trade and specialization.
- Second stage ( $t + 2$ ): Migration.
- Third stage ( $t + 3$ ): Specialization after migration.

The meaning of the time subscripts  $t, t + 1, t + 2, t + 3$  is not literally four successive periods of time. It is much more convenient to think about them as successive economic regimes in the urban system that makes possible a meaningful comparison of aggregate growth rates.

The initial situation (period  $t$ ) in two separated urban economies is characterized by the values of the variables shown in Table 1. The variables derived from expressions (22)-(25). Prices of  $X$  and  $Z$  are greater in city  $B$ , but price of  $\Omega$  coincides in both cities. The wage in  $B$  is greater than in city  $A$  and both the returns to capital and per capita growth rates are the same.

## 4 Trade and specialization at period $t+1$

Trade of goods will affect relative prices and people's expenditure decisions. It will be assumed that old agents have perfect foresight about variables in the first period after the formation of the urban system. This issue will not be relevant for the young generation in the economy because they are born after this process takes place.

The simultaneous production of good  $Y$  in the two cities is not feasible in the integrated equilibrium. If both cities produced good  $Y$ , input prices should equalize. Wages would be those of city  $B$ , consequently, city  $A$  would incur in higher wage costs with a lower labor productivity, thus negative profits. Therefore, city  $A$  will not produce this good. This kind of specialization in a corner equilibrium is necessary.

<sup>3</sup>As capital is the savings of the young people, it is inherent to the city of residence of the individuals when old. We assume the migrants are young and they do not return to their city of origin, in such a way that a partial mobility of capital will take place with the migration. But it is not possible to invest in the city of no residence.

On the other hand, city  $A$  has incentives to specialize in goods  $X$  and/or  $Z$  since their relative prices are lower than in city  $B$ . Producers from city  $B$  will have to decide if they are also interested in producing these goods. When city  $A$  is quite poor in comparison to city  $B$ , the former will not have enough productive capacity to satisfy city  $B$ 's demand of goods  $X$  and  $Z$ . In that case, city  $B$  will also produce these goods. On the contrary, if both cities are relatively similar in per capita income, city  $A$  will be the only producer of goods  $X$  and  $Z$  because of its competitive advantage in those goods.

Equality of relative prices of good  $\Omega$  implies that there do not exist incentives to trade good  $\Omega$  between these two cities because the relative price is the same. However, this does not necessarily mean that each city produces its own demand for that good. Again, if  $A$  is quite poor in comparison to city  $B$ , the former will not have enough productive capacity to produce good  $\Omega$ . On the contrary, if city  $A$  has a great production capacity with regard to city  $B$ , some producers from city  $A$  will also produce good  $\Omega$ .

Up to now, the only thing we know for sure is that good  $Y$  will be exclusively produced in city  $B$ , because of the differences in capital/labor ratios. With regard to the other goods, a number of combinations are possible according to the previous comments. We are going to describe those combinations and the corresponding transitional dynamics that will lead us to a more reduced number of specialization patterns in steady state. For this purpose, we start by assuming a situation in which city  $A$  is very poor with regard to city  $B$  and, from that point we study the specialization processes, the corresponding transitional dynamics and the conditions that must be hold to change to a different specialization pattern.

Two types of situations can take place:

- Convergence phase of development in city  $A$ .
- Steady differences between the cities.

## 4.1 Convergence phase of development in city A

### 4.1.1 The lower stage of development (S1): Complete and insufficient specialization of city A in the traditional good

We initially assume that the initial difference in the relative production capacity of both cities is so large ( $k_{At} \ll k_{Bt}$ ) that producers in city  $B$  manufacture all goods ( $X, Y, Z, \Omega$ ) and their counterparts in city  $A$  exclusively good  $X$ <sup>4</sup>. We name this specialization pattern as  $S1$ .

The relative prices of goods  $X$ ,  $Z$  and  $\Omega$  will be determined by city  $B$  because is the only city producing good  $Y$  ( $P_{Xt+1} = a^{-\beta} k_{Bt+1}^\beta$ ;  $P_{Zt+1} = a^{-\beta} k_{Bt+1}^\beta L^{-\gamma}$ ;  $P_{\Omega t+1} = L^{-\gamma}$ ). Taking into account these relative prices, denoting  $\bar{k} = \frac{k_A}{k_B}$  and solving the global market clearing conditions for all goods, the capital and labor proportions devoted to the production of each good are obtained and shown on the second column of Table 2.

The proportion  $q_{BX}$  is negatively related to  $\bar{k}$  because the term that multiplies  $\bar{k}^\alpha$  is smaller than zero. This means that as the relative production capacity of city  $A$  increases, the production of good  $X$  by this city will grow and, consequently, city  $B$  will be able to dedicate a lower proportion of its inputs to production of that good in which this city does not have competitive advantage, as previously mentioned. City  $B$  will not produce good  $X$  when  $\bar{k}$  is sufficiently high, that is equivalent to saying that city  $A$  will be the only one producing  $X$  whether city  $A$  is relatively rich enough to do so.

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<sup>4</sup>We could equivalently considered that producers in city  $A$  exclusively produce good  $Z$  because the quotient of relative prices leads to the same competitive advantage in both goods. However, final results in steady state do not change depending on the initial specialization chosen. For this reason, we have opted to consider only the initial specialization of city  $A$  in good  $X$ , but not in  $Z$

City  $B$  will not produce good  $X$  if the expression  $q_{BX}$  takes the extreme value 0, which requires:

$$\bar{k}_{t+1}^\alpha \geq \frac{\nu_X \left[ \frac{1+\alpha+\delta}{2+\delta} \right]}{1 - \nu_X \left[ \frac{1+\alpha+\delta}{2+\delta} \right]} = Q_1 \quad (32)$$

so  $S1$  requires that  $\bar{k}_{t+1} < Q_1^{1/\alpha}$ .

The term  $Q_1$  is the quotient between the cities  $A$ 's and  $B$ 's demand of good  $X$  when city  $A$  is the only one producing  $X$  and city  $B$  exclusively produces  $Y$ ,  $Z$  and  $\Omega$ . So, the meaning of the condition is that good  $X$  will only be produced by city  $A$  when the ratio between the average labor productivities raised to the power of  $\alpha$  ( $\bar{k}_{t+1}^\alpha$ ) is, at least, the ratio between the demands of good  $X$  when it is only produced in city  $A$ .  $Q_1$  increases with the technological parameter ( $\alpha$ ), the intertemporal discount factor ( $\delta$ ) and the weight of the traditional good in the utility function ( $\nu_X$ ). Finally, note that this expression can be greater or smaller than one. Having into account that  $Q_1 < 1$  means that, under the situation of specialization previously described, city  $B$ 's demand of good  $X$  is higher than city  $A$ 's demand of that same good, it is coherent to think that  $Q_1 < 1$  is the only possible situation. Therefore, we will assume that  $Q_1 < 1$  for the rest of the paper.

From (15), (19), (22) and relative price of good  $X$ , we obtain that city  $A$  grows at the rate :

$$g_{At+1} = \frac{\beta}{2+\delta} a^{-\beta} \bar{k}_{t+1}^{-\beta} \quad (33)$$

while growth rate at city  $B$  will be under this specialization pattern ( $S1$ ):

$$g_{Bt+1} = \frac{\beta}{2+\delta} a^{-\beta} \quad (34)$$

It can be concluded that growth will be greater in city  $A$  since  $\bar{k}_{t+1}^{-\beta}$  is larger than unity. A process of convergence will take place  $B$  ( $g_{At+1} > g_{Bt+1}$ ). This implies a rise of the relative production capacity of city  $A$ . In other words, it will be reached a point in which city  $A$  is able to manufacture the global demand of good  $X$  and, therefore, city  $B$  ceases producing it. This threshold point will be reached when  $\bar{k} = Q_1^{1/\alpha}$  and it introduces us into a new specialization situation that we denominate as  $S2$ .

#### 4.1.2 Second stage of development ( $S2$ ): Complete and sufficient specialization of city $A$ in the traditional good

This new situation is characterized by a productive specialization in which producers of city  $A$  manufacture exclusively good  $X$  and those from city  $B$  do the same with goods ( $Y$ ,  $Z$ ,  $\Omega$ ). From market clearing condition of good  $X$  and taking into account that  $l_{AXt+1} = 1$ , we obtain the relative price of good  $X$  in  $S2$ <sup>5</sup>:

$$P_{Xt+1} = a^{-\beta} \frac{k_{Bt+1}}{k_{At+1}^\alpha} Q_1 \quad (35)$$

and solving the global market clearing conditions for the rest of the goods, the proportions of capital and labor in city  $B$  in their corresponding productions are shown in the third column of Table 2.

<sup>5</sup>Relative prices of goods  $Z$  and  $\Omega$  are the same as those in  $S1$ .

Analogously to  $S1$  per capita growth rates can be calculated for  $S2$ . The growth rate of city  $B$  remains the same as (34), while the rate in city  $A$  holds the expression:

$$g_{At+1} = \frac{\beta}{2 + \delta} a^{-\beta} \frac{Q_1}{\bar{k}_{t+1}} \quad (36)$$

in such a way that it can be easily checked that  $g_{At+1} > g_{Bt+1}$  since  $\bar{k} < Q_1$ . Thus the process of convergence continues, improving city  $A$ 's relative production capacity.

Therefore, producers from city  $A$  gain enough capacity to start manufacturing another good, apart from good  $X$ . That good will be  $Z$  since it is the other good in which city  $A$  has competitive advantage with regard to city  $B$ . This leads us to specialization pattern  $S3$ , characterized by city  $A$  producing goods  $(X, Z)$  and city  $B$   $(Y, Z, \Omega)$ .

### 4.1.3 Third stage of development (S3): City A overcomes the complete specialization

The condition that must be fulfilled in order to city  $A$  starts producing good  $Z$  ( $l_{AZt+1} > 0$ ) can be obtained from the global market clearing condition of good  $Z$ . This conditions requires that:

$$\bar{k}_{t+1}^\alpha > \frac{\nu_X \left[ \frac{1+\alpha+\delta}{2+\delta} \right]}{1 - \nu_X \left[ \frac{1+\alpha+\delta}{2+\delta} \right]} = Q_1 \quad (37)$$

Considering both expressions (32) and (37) it can be observed that specialization pattern  $S2$  only takes place at the point where  $\bar{k}_{t+1} = Q_1^{1/\alpha}$ . Beyond that point, situation  $S3$  will be reached.

Under specialization  $S3$ , the relative price of goods  $Z$  and  $\Omega$  are the same as in the two previous situations. The price of good  $X$  is obtained from the equilibrium of input markets ( $P_{Xt+1} = a^{-\beta} k_{Bt+1}^\beta$ ). The proportions of capital and labor used in the production of every good by each city under  $S3$  are obtained from the global market clearing conditions and displayed on the fourth column of Table 2.

Growth rates, calculated in an analogous way to previous situations, are the same as in (33) and (34). Thus, the convergence process continues and the production capacity of city  $A$  keeps its rise enabling that city to satisfy the global demand of good  $Z$ . City  $B$  will not produce good  $Z$  any more if the expression  $l_{BZ}$  takes the extreme value 0, which requires:

$$\bar{k}_{t+1}^\alpha \geq \frac{(1 - \nu_Y - \nu_\Omega) \left[ \frac{1+\alpha+\delta}{2+\delta} \right]}{1 - (1 - \nu_Y - \nu_\Omega) \left[ \frac{1+\alpha+\delta}{2+\delta} \right]} = Q_2 \quad (38)$$

The term  $Q_2$  is the quotient between the cities  $A$ 's and  $B$ 's demand of goods  $X$  and  $Z$  when city  $A$  produces  $X$  and  $Z$  and city  $B$  produces  $Y$  and  $\Omega$ . So, the meaning of the condition is that goods  $X$  and  $Z$  will be exclusively produced by city  $A$  when the ratio between the average labor productivities raised to the power of *alpha* ( $\bar{k}_{t+1}^\alpha$ ) is, at least, the ratio between the demands of goods  $X$  and  $Z$  when they are only produced in city  $A$ .  $Q_2$  increases with the technological parameter ( $\alpha$ ), the intertemporal discount factor ( $\delta$ ) and the weight of  $X + Z$  goods in the utility function ( $\nu_X + \nu_Z$ ). Finally, note that this expression can be greater or smaller than unity. Taking into account that  $Q_2 < 1$  means that, under the situation of specialization previously described, city  $B$ 's demand of goods  $X$  and  $Z$  is higher than city  $A$ 's demand of those same goods, it is coherent to think that  $Q_2 < 1$  is the only possible situation. Therefore, we consider for the rest of the paper that  $Q_2 < 1$ .  $Q_1$  will be lower than  $Q_2$  and both magnitudes will be lower than unity.

Table 2: Stages in the convergence phase of development of city A

	S1	S2	S3	S4	S4'
	$\bar{k} < Q_1^{1/\alpha}$	$\bar{k} = Q_1^{1/\alpha}$	$Q_1^{1/\alpha} < \bar{k} < Q_2^{1/\alpha}$	$Q_2^{1/\alpha} \leq \bar{k} < Q_2$	$\bar{k} = Q_2$
$P_X$	$a^{-\beta} k_B^\beta$	$\frac{a^{-\beta} k_{B,t+1}}{k_{A,t+1}^\alpha} Q_1$	$a^{-\beta} k_B^\beta$	$\frac{a^{-\beta} k_B^\beta}{k_A^\alpha} Q_2$	$a^{-\beta} k_A^\beta$
$P_Z$		$a^{-\beta} k_B^\beta L^{-\gamma}$		$\frac{a^{-\beta} k_B^\beta}{k_A^\alpha} Q_2 L^{-\gamma}$	$a^{-\beta} k_A^\beta L^{-\gamma}$
$P_\Omega$	$L^{-\gamma}$				
$l_{AX}$	1	1	$\nu_X \Psi \left( 1 + \frac{1}{k^\alpha} \right)$	$\nu_X \Psi \left( 1 + \frac{1}{Q_2} \right)$	
$l_{AY}$	—	—	—	—	—
$l_{AZ}$	—	—	$1 - \nu_X \left[ \frac{1+\alpha+\delta}{2+\delta} \right] \left( 1 + \frac{1}{k_{t+1}^\alpha} \right)$	$\nu_Z \Psi \left( 1 + \frac{1}{Q_2} \right)$	
$l_{A\Omega}$	—	—	—	—	—
$l_{BX}$	$\nu_X \Psi (1 + \bar{k}^\alpha) - \bar{k}^\alpha$	—	—	—	—
$l_{BY}$	$[1 - (1 - \nu_Y) \Psi] (1 + \bar{k}^\alpha)$	$[1 - (1 - \nu_Y) \Psi] (1 + Q_1)$	$[1 - (1 - \nu_Y) \Psi] (1 + \bar{k}_{t+1}^\alpha)$	$[1 - (1 - \nu_Y) \Psi] (1 + Q_2)$	
$l_{BZ}$	$\nu_Z \Psi (1 + \bar{k}^\alpha)$	$\nu_Z \Psi (1 + Q_1)$	$(1 - \nu_Y - \nu_\Omega) \left[ \frac{1+\alpha+\delta}{2+\delta} \right] (1 + \bar{k}_{t+1}^\alpha) - \bar{k}_{t+1}^\alpha$	—	
$l_{B\Omega}$	$\nu_\Omega \Psi (1 + \bar{k}^\alpha)$	$\nu_\Omega \Psi (1 + Q_1)$	$\nu_\Omega \Psi (1 + \bar{k}_{t+1}^\alpha)$	$\nu_\Omega \Psi (1 + Q_2)$	
$g_A$	$\frac{\beta}{2+\delta} a^{-\beta} \bar{k}^{-\beta}$	$\frac{\beta}{2+\delta} a^{-\beta} \frac{Q_1}{k}$	$\frac{\beta}{2+\delta} a^{-\beta} \bar{k}^{-\beta}$	$\frac{\beta}{2+\delta} a^{-\beta} \frac{Q_2}{k}$	$\frac{\beta}{2+\delta} a^{-\beta}$
$g_B$	$\frac{\beta}{2+\delta} a^{-\beta}$				
	$g_A > g_B$				$g_A = g_B$
	$\Psi = \frac{1+\alpha+\delta}{2+\delta}$				

#### 4.1.4 The end of convergence process (S4 and S4')

A new specialization pattern, namely  $S4$ , is reached when  $\bar{k} = Q_2^{1/\alpha}$ <sup>6</sup>. This stage is characterized by city  $A$  producing goods  $(X, Z)$  and city  $B$  producing  $(Y, \Omega)$

Under these circumstances, relative prices of goods  $X$  and  $Z$  can be derived from the global market clearing conditions of these goods besides the condition that  $l_{AXt+1} + l_{AZt+1} = 1$ . The proportions of capital and labor devoted to the production of each good can be obtained in a similar way. The expression of prices and capital and labor proportions are shown in the fifth column of Table 2.

Per capita growth rates in each city are calculated as in previous stages of the process. City  $B$  continues growing at the same rate (34), but city  $A$  now grows a rate:

$$g_{At+1} = \frac{\beta}{2 + \delta} a^{-\beta} \frac{Q_2}{\bar{k}_{t+1}} \quad (39)$$

in such a way that  $g_{At+1} > g_{Bt+1}$  causing  $\bar{k}_{t+1}$  increases until reaches  $Q_2$ . Both rates equalize in that moment and the steady state is reached, but some disequilibriums remain in the labor markets because of wage differentials. Therefore, specialization  $S4$  holds in the interval  $Q_2^{1/\alpha} \leq \bar{k} \leq Q_2$ .

Summarizing, whenever the initial quotient of relative capital/labor ratio is low enough ( $\bar{k} < Q_2$ ), trade between cities in an urban system leads to a convergence process between both cities until a situation of steady state is reached ( $g_A = g_B$ ) with a specialization pattern characterized by city  $A$  producing goods  $(X, Z)$ , city  $B$   $(Y, \Omega)$  and  $\bar{k} = Q_2$ . For later reference in this paper we refer to this final situation as  $S4'$ . Table 2 shows all the specialization patterns commented until now and the main magnitudes, such as relative prices, proportions of capital and labor devoted to each industry and growth rates. In order to simplify expressions we have named  $\left[ \frac{1+\alpha+\delta}{2+\delta} \right]$  as  $\Psi$ .

## 4.2 Steady differences between the cities

What happens if  $\bar{k}$  is larger than  $Q_2$  in the initial starting point? In this case, city  $A$  has enough production capacity to manufacture goods  $X, Z$  and  $\Omega$ . None of the cities has a special competitive advantage in producing this good, but city  $A$  will not produce good  $Y$  because of the corner solution mentioned above. Under this situation city  $B$  continues producing goods  $Y$  and  $\Omega$ . This specialization pattern is named as  $S5$ . The proportions of capital and labor, the relative prices of goods and the growth rates can be derived in an analogous way to previous states. Table 3 shows these magnitudes under specialization pattern  $S5$ .

Another possible specialization can be that in which city  $A$  has enough production capacity to satisfy the global demand of good  $\Omega$ , in such a way that city  $B$  will not produce it any more. This situation requires the following condition:

$$\bar{k}_{t+1} \geq \frac{(1 - \nu_Y) \left[ \frac{1+\alpha+\delta}{2+\delta} \right]}{1 - (1 - \nu_Y) \left[ \frac{1+\alpha+\delta}{2+\delta} \right]} = Q_3 \quad (40)$$

Therefore, we define the pattern  $S5$  in the interval  $Q_2 < \bar{k}_{t+1} < Q_3$ . If  $\bar{k}_{t+1}$  equals  $Q_3$  a different specialization is reached, namely  $S6$ , where city  $A$  produces goods  $(X, Z, \Omega)$  and  $B$  only good  $Y$ . However, we must take into account that this specialization pattern is quite unlikely because it implies that  $Q_3$  must be lower than one, which requires  $(1 - \nu_Y) \left[ \frac{1+\alpha+\delta}{2+\delta} \right] < 0.5$ , and  $\bar{k}$  must be large enough to reach  $Q_3$ .

<sup>6</sup>Situation  $S3$  holds if  $Q_1^{1/\alpha} < \bar{k}_{t+1} < Q_2^{1/\alpha}$

The term  $Q_3$  is the quotient between the cities  $A$ 's and  $B$ 's demand of goods  $X$ ,  $Z$  and  $\Omega$  when city  $A$  produces  $X$ ,  $Z$  and  $\Omega$  and city  $B$  produces  $Y$ . So, the meaning of the condition is that goods  $X$ ,  $Z$  and  $\Omega$  will only be produced by city  $A$  when the ratio between the average labor productivity ( $\bar{k}_{t+1}$ ) is higher than the ratio between the demands of goods  $X$ ,  $Z$  and  $\Omega$  when city  $A$  is the only one producing them.  $Q_3$  increases with the technological parameter ( $\alpha$ ), the intertemporal discount factor ( $\delta$ ) and the weight of  $X + Z + \Omega$  goods in the utility function ( $\nu_X + \nu_Z + \nu_\Omega$ ).

Given that  $q_{B\Omega}$  decreases with  $\bar{k}$ , city  $B$  will not produce good  $\Omega$  when  $\bar{k}$  is sufficiently high. This is equivalent to saying that city  $A$  will be the only one producing  $X$ ,  $Z$  and  $\Omega$  if city  $A$  is rich enough. Finally, note that this expression can be greater or smaller than unity. Having into account that  $\bar{k}$  is lower than unity, then  $Q_3$  must be also lower than 1 under  $S6$ . This would mean that city  $B$ 's demand of goods  $X$ ,  $Z$  and  $\Omega$  will be higher than city  $A$ 's demand of those same goods. This would be a very strange/rare situation, since a larger demand of city  $B$  would mean also a larger producing capacity of this city, what is contradictory with the fact of city  $A$  producing three goods by its own. Therefore, is coherent to think that  $Q_3$  can be higher than unity.  $Q_2$  will be lower than  $Q_3$ .

Relative prices, proportions of capital and labor and growth rates under  $S6$  are shown in Table 3. As in previous situation growth rates are the same in both cities, so no transitional dynamics take places under these circumstances.

Finally, there is the very unlikely possibility of  $\bar{k}$  greater than  $Q_3$ . This would imply a complete specialization patter, namely  $S7$ , with city  $A$  producing all four goods and city  $B$  only producing good  $Y$ . Even though this situation is really improbable, we show the theoretical magnitudes for relative prices, proportions of capital and labor and growth rates on the fifth column of Table 3. Again, steady state is directly reached with no possibility of transitional dynamics.

To conclude, the urban system can be placed in different situations at the end of this period of specialization, according to the value of  $\bar{k}$  as regards  $Q_1$ ,  $Q_2$  and  $Q_3$ . The four possible final situations correspond to a steady state characterized by the same growth rate ( $S4'$  to  $S7$ ) for the two cities as shown in Table 3. AS the capital labor ratios are different in the two cities, the gap between them will be permanent in absence of other type of incentives. In what follows we consider the consequences of migration flows once the specialization process has taken place. For the rest of the paper we consider uniquely specialization  $S5$ , given that the results and conclusions are the same for the other three specialization patterns in steady state.

## 5 Migration at period $t+2$

Migration takes place in the period  $t+2$  and it is assumed that variables with superscripts  $(t+2)^0$  or  $(t+2)$  represent the values before and after migration, respectively. We suppose, without losing of generality, the urban system locates in the situation  $S5$  at the beginning of period  $t+2$ . We could have chosen whatever of the four situations with same growth rates, because final results remain the same. We assume that only young people migrate with no reversal migrations when old.

The marginal productivity of capital will be the same in both cities,  $\alpha a^{-\beta}$ , but wages will be lower in city  $A$  than in city  $B$ . Perfect mobility of labor implies that workers will move to the city offering a higher income (measured as the sum of wages and capital yields):  $I_{A(t+2)^0} = W_{A(t+2)^0} + (1 + r_{A(t+2)^0})k_{A(t+2)^0} < W_{B(t+2)^0} + (1 + r_{B(t+2)^0})k_{B(t+2)^0} = I_{B(t+2)^0}$ . This labor



**Table 3:** Stages of development of city  $A$  in the phase of steady differences

	$S4'$	$S5$	$S6$	$S7$
	$\bar{k} = Q_2$	$Q_2 < \bar{k} < Q_3$	$\bar{k} = Q_3$	$\bar{k} > Q_3$
$P_X$	$a^{-\beta} k_A^\beta$			
$P_Z$	$a^{-\beta} k_A^\beta L^{-\gamma}$			
$P_\Omega$	$L^{-\gamma}$			
$l_{AX}$	$\nu_X \Psi \left(1 + \frac{1}{Q_2}\right)$	$\nu_X \Psi \left(1 + \frac{1}{\bar{k}}\right)$	$\nu_X \Psi \left(1 + \frac{1}{Q_3}\right)$	$\nu_X \Psi \left(1 + \frac{1}{\bar{k}}\right)$
$l_{AY}$	—	—	—	$1 - l_{AX} - l_{AZ} - l_{A\Omega}$
$l_{AZ}$	$\nu_Z \Psi \left(1 + \frac{1}{Q_2}\right)$	$\nu_Z \Psi \left(1 + \frac{1}{\bar{k}}\right)$	$\nu_Z \Psi \left(1 + \frac{1}{Q_3}\right)$	$\nu_Z \Psi \left(1 + \frac{1}{\bar{k}}\right)$
$l_{A\Omega}$	—	$1 - (\nu_X + \nu_Z) \Psi \left(1 + \frac{1}{\bar{k}}\right)$	$\nu_\Omega \Psi \left(1 + \frac{1}{Q_3}\right)$	$\nu_\Omega \Psi \left(1 + \frac{1}{\bar{k}}\right)$
$l_{BX}$	—	—	—	—
$l_{BY}$	$[1 - (1 - \nu_Y) \Psi] (1 + Q_2)$	$[1 - (1 - \nu_Y) \Psi] (1 + \bar{k})$	1	1
$l_{BZ}$	—	—	—	—
$l_{B\Omega}$	$\nu_\Omega \Psi (1 + Q_2)$	$1 - [1 - (1 - \nu_Y) \Psi] (1 + \bar{k})$	—	—
$g_A$	$\frac{\beta}{2+\delta} a^{-\beta}$			
$g_B$	$\frac{\beta}{2+\delta} a^{-\beta}$			
	$g_A = g_B$			

migration will change  $k_A$  and  $k_B$  in the following way:

$$\begin{aligned}
L_{A(t+2)^1} &= (1 - m)L_t \\
L_{B(t+2)^1} &= (1 + m)L_t \\
k_{A(t+2)^0} &= \frac{K_A}{L_t} = \frac{K_A}{L_{A(t+2)^1}} \frac{L_{A(t+2)^1}}{L_{At}} = k_{A(t+2)^1} (1 - m) \Rightarrow k_{A(t+2)^1} = \frac{k_{A(t+2)^0}}{(1 - m)} \\
k_{B(t+2)^0} &= \frac{K_B}{L_{Bt}} = \frac{K_B}{L_{B(t+2)^1}} \frac{L_{B(t+2)^1}}{L_{Bt}} = k_{B(t+2)^1} (1 + m) \Rightarrow k_{B(t+2)^1} = \frac{k_{B(t+2)^0}}{(1 + m)}
\end{aligned}$$

Assuming perfect mobility of labor, both capital/labor ratios will be equalized  $k_{A(t+2)^1} = k_{B(t+2)^1} \Rightarrow \bar{k}_{(t+2)^1} = 1$ . This allows us to get the value of the proportion of people who move from city  $A$  towards city  $B$ :

$$\frac{k_{At}}{1 - m} = \frac{k_{Bt}}{1 + m} \Rightarrow m = \frac{k_{B(t+2)^0} - k_{A(t+2)^0}}{k_{A(t+2)^0} + k_{B(t+2)^0}} \quad (41)$$

and dividing (41) by  $k_{Bt}$  we get the expression:

$$m = \frac{1 - \bar{k}}{1 + \bar{k}} \quad (42)$$

These changes will affect the relative prices of goods  $Z$ ,  $\Omega$  and  $X$ . This new situation in prices will lead to a new process of specialization through trade in the following period ( $t + 3$ ).

## 6 Specialization after migration at period $t+3$

The relative price of good  $\Omega$  will differ between cities after migration due to different total population in each city.  $P_{\omega t+3}$  will be given by city  $B$  because its producers can offer it at a lower price due to the greater urbanization externality in that city.

$$P_{\omega t+3} = L_{Bt+3}^{-\gamma} \quad (43)$$

From (43) and the required equilibrium in factor markets materialized in the equality of interest rates ( $1 + r_{AX} = 1 + r_{AZ} = 1 + r_{A\Omega}$ ), the relative prices for  $X$  and  $Z$  are obtained:

$$P_{Zt+3} = a^{-\beta} k_{At+3}^{\beta} L_{Bt+3}^{-\gamma} \quad (44)$$

$$P_{Xt+3} = a^{-\beta} k_{At+3}^{\beta} \left( \frac{1-m}{1+m} \right)^{\gamma} = a^{-\beta} k_{At+3}^{\beta} \bar{k}^{\gamma} \quad (45)$$

It must be noted that we name  $\bar{k}$  to the quotient of capital/labor ratios before migration ( $\bar{k} = \bar{k}_{t+1} = \bar{k}_{(t+2)^0}$ ).

We wonder whether the specialization pattern reached in  $S5$  keeps after the changes in the relative prices: does city  $A$  continue producing good  $\Omega$ ? The proportions of capital and labor used in the production of goods  $X$  and  $Z$  in city  $A$  are obtained from the global market clearing conditions for those goods:

$$l_{AXt+3} = \nu_X \left[ \frac{1+\alpha+\delta}{2+\delta} \right] \left[ 1 + \frac{1}{\bar{k}^{1+\gamma}} \right] \quad (46)$$

$$l_{AZt+3} = \nu_Z \left[ \frac{1+\alpha+\delta}{2+\delta} \right] \left[ 1 + \frac{1}{\bar{k}^{1+\gamma}} \right] \quad (47)$$

in such a way that  $l_{A\Omega t+3} = 1 - l_{AXt+3} - l_{AZt+3}$  and the condition to this expression being zero is:

$$\bar{k}^{1+\gamma} \leq \frac{(1 - \nu_Y - \nu_{\Omega}) \left[ \frac{1+\alpha+\delta}{2+\delta} \right]}{1 - (1 - \nu_Y - \nu_{\Omega}) \left[ \frac{1+\alpha+\delta}{2+\delta} \right]} = Q_2 \quad (48)$$

and this expression is equivalent to  $\bar{k} \leq Q_2^{\frac{1}{1+\gamma}}$ .

As our starting point is  $S5$ ,  $\bar{k} > Q_2$ , and taking into account that  $Q_2^{\frac{1}{1+\gamma}} > Q_2$ , we can not directly conclude if city  $A$  will continue producing good  $\Omega$  or not. If city  $A$  continues producing good  $\Omega$ , per capita growth rates in each city are given by the following expressions:

$$\begin{aligned} g_{At+3} &= \frac{\beta}{2+\delta} a^{-\beta} \bar{k}^{\gamma} \\ g_{Bt+3} &= \frac{\beta}{2+\delta} a^{-\beta} \end{aligned} \quad (49)$$

where  $g_{At+3} < g_{Bt+3}$ , resulting in a divergence process. The reason rests on the fact that city  $A$  now is less productive than city  $B$  in the production of good  $\Omega$ . This is due to the lower population (employment force) in city  $A$  and this city can not take advantage of the urbanization economies. The consequence will be a new divergence in the quotient of capital/labor ratios which would lead to a wage differential causing a new migration process. Larger differences in population between cities will produce larger urbanization economies in city  $B$  that will again lead to a higher growth rate in city  $B$  whether city  $A$  continues producing good  $\Omega$ . As  $m$  grows, the quotient of capital/labor ratios before migration decreases. Therefore, a situation in which condition (48) will be fulfilled and city  $A$  will cease from producing good  $\Omega$ . Consequently, the

specialization pattern so far would be characterized by the city  $A$  producing  $X$  and  $Z$  and the city  $B$  producing  $Y$  and  $\Omega$ . In the following step, we wonder whether city  $B$  will produce good  $Z$  under the new situation of relative prices after migration. In an analogous way to previous stage, we calculate the relative prices for each good from the capital market equilibrium.

$$P_{\omega t+3} = L_{Bt+3}^{-\gamma} \quad (50)$$

$$P_{Zt+3} = a^{-\beta} k_{Bt+3}^{\beta} L_{Bt+3}^{-\gamma} \quad (51)$$

$$P_{Xt+3} = a^{-\beta} k_{Bt+3}^{\beta} \left( \frac{1-m}{1+m} \right)^{\gamma} = a^{-\beta} k_{Bt+3}^{\beta} \bar{k}^{\gamma} \quad (52)$$

The proportions of capital and labor devoted to goods  $Y$  and  $\Omega$  are derived from the global market clearing conditions of these goods.

$$l_{BYt+3} = \left[ 1 - (1 - \nu_Y) \left( \frac{1 + \alpha + \delta}{2 + \delta} \right) \right] (1 + \bar{k}^{1+\gamma}) \quad (53)$$

$$l_{B\Omega t+3} = \nu_{\Omega} \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right] (1 + \bar{k}^{1+\gamma}) \quad (54)$$

in such a way that  $l_{BZt+3} = 1 - l_{BYt+3} - l_{B\Omega t+3}$  and the condition to be this expression larger than zero and, there fore,  $B$  produces  $Z$  would be:

$$\bar{k}^{1+\gamma} < \frac{(1 - \nu_Y - \nu_{\Omega}) \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right]}{1 - (1 - \nu_Y - \nu_{\Omega}) \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right]} = Q_2 \quad (55)$$

This condition is practically the same as the one in expression (48) but with strict inequality. Thus, city  $A$  will not produce good  $\Omega$  when migration takes place, whereas city  $B$  will produce good  $Z$ . The next question is whether city  $A$  will continue producing good  $Z$ , although city  $B$  has a competitive advantage in its production due to the existence of urbanization economies.

The proportion of capital and labor devoted to the production of good  $Z$  by city  $A$  is given by the same expression obtained in equation (46). The proportion devoted to good  $Z$  in that city will be given by  $l_{AZt+3} = 1 - l_{AXt+3}$  and the condition to be zero, that is, city  $A$  does not produce  $Z$  is given by:

$$\bar{k}^{1+\gamma} \leq \frac{\nu_X \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right]}{1 - \nu_X \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right]} = Q_1 \quad (56)$$

is equivalent to  $\bar{k} \leq Q_1^{\frac{1}{1+\gamma}}$ . From  $S5$  we know that  $\bar{k} > Q_1$  and, considering that  $Q_1^{\frac{1}{1+\gamma}} > Q_1$ , we cannot directly conclude whether city  $A$  will produce  $Z$  or not.

If city  $A$  continues producing this good, growth rates in each city are given by expressions in (49): and, as it holds before, a divergence process would take place between both cities, causing a lower productivity of city  $A$  in the production of good  $Z$  because of the urbanization economies. This situation will lead to a new divergence in wages and, consequently, to a new migration process, losing even more work force city  $A$  and increasing the urbanization economies in city  $B$  even more. As it previously happened, as  $m$  grows, the quotient of capital/labor ratios before migration decreases. Therefore, a situation in which condition in equation (56) will be fulfilled and city  $A$  will cease from producing good  $Z$ . Consequently, the specialization so far would be characterized by city  $A$  producing  $X$  and city  $B$  producing  $Y$ ,  $Z$  and  $\Omega$ .

The migration stopping condition implies that capital/labor ratios equalize in both cities. For this reason, a corner equilibrium with regard to good  $Y$  is not compulsory any more. That is why we must wonder under which circumstances city  $A$  will produce this good. The proportion of capital and labor devoted in each city to the production of good  $Y$  can be devoted from global

**Table 4:** Final situation after migration and specialization

	City A	City B
$P_X$	$a^{-\beta} k_A^\beta$	
$P_Z$	$a^{-\beta} k_B^\beta L_B^{-\gamma}$	
$P_\Omega$	$L_B^{-\gamma}$	
$l_X$	$\nu_X \Psi \left(1 + \frac{1}{k}\right)$	—
$l_Y$	$1 - \nu_X \Psi \left(1 + \frac{1}{k}\right)$	$1 - (\nu_Z + \nu_\Omega) \Psi (1 + \bar{k})$
$l_Z$	—	$\nu_Z \Psi (1 + \bar{k})$
$l_\Omega$	—	$\nu_\Omega \Psi (1 + \bar{k})$
$g$	$\frac{\beta}{2+\delta} a^{-\beta}$	

market clearing condition of that good, given by:

$$l_{AXt+3} = \nu_X \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right] \left[ 1 + \frac{1}{\bar{k}} \right] \quad (57)$$

in such a way that  $l_{AYt+3} = 1 - l_{AXt+3}$  and the condition to be positive and, therefore,  $A$  produces  $Y$  is:

$$\bar{k} > \frac{\nu_X \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right]}{1 - \nu_X \left[ \frac{1 + \alpha + \delta}{2 + \delta} \right]} = Q_1 \quad (58)$$

From stage  $S5$  we know that  $\bar{k} > Q_2$  and, taking into account that  $Q_2 > Q_1$ , we can conclude directly that city  $A$  will produce  $Y$ . The per capita growth rates in both cities will be the same in this case and the steady state will be reached. Finally, we wonder whether city  $B$  will produce good  $X$ . The condition to answer no to this question coincides with (58). That is, if  $\bar{k} > Q_1 \Rightarrow l_{BXt+3} = 0$ .

Corollary: From the previous argumentation it is concluded that, given the characteristics of this economy (2 cities, 4 goods, same preferences, etc.) if there is trade and migration between cities, a steady state situation will be reached and characterized by both cities with the same level of per capita income and a specialization pattern in which city  $A$ , initially poorer city, produces  $X$  e  $Y$  and city  $B$ , richer city in origin, produces  $Y$ ,  $Z$  and  $\Omega$ . Table 4 summarizes the magnitudes of the situation reached after the process.

As we have commented previously, the same conclusion is reached from whatever of the other three possibilities just before migration ( $S4'$ ,  $S6$ ,  $S7$ ) following a similar reasoning.

Finally, we wonder what would be the development of the model in case we assumed that migration takes place firstly and then trade. Final result would be the same as previously, the difference would be that steady state would be reached in one period less. Initial situation (period  $t$ ) is identical to the previous one, the difference in incomes per capita leads to migration (period  $t+1$ ) equivalent to the one taking place in the previous case at  $t+2$ . After that, trading produce a specialization process at period  $t+2$  as the one that before took place at  $t+3$ . Therefore, assuming that migration or trade take place firstly is indifferent. The conclusion is the same, but first trade allows for a one period longer transitional dynamics to steady state and more model richness.

## 7 Urban concentration and aggregate economic growth

In the previous sections we have identified the steady state situations after a process of trade specialization and after specialization and migration. Only the second situation has to do with the urban concentration, because it affect to the population of the cities. Then, if we are interested in the different aggregate growth rates that are associated to each urban concentration level we need a situation from which the calculation must be done. The obvious election is the steady situation after the trade specialization. In other words,  $S4'$ ,  $S5$ ,  $S6$  or  $S7$ , depending on the initial value of  $\bar{k}$ .

With this comparison we obtain an aggregate growth rate depending on the initial value of  $\bar{k}$ . As migration depends precisely on this variable and the relative population of the two cities depends on the migration flows, the relationship between the aggregate growth rate and the level of urban concentration is immediate. The properties of this relationship will be examined from the point of view of the “so-what question”.

As until now only the variables related to every city have been considered we need an aggregation of the magnitudes in order to define the aggregate growth rate. In the first subsection the price index of the aggregate production is identified and the GDP is defined. The properties of the aggregate growth rate and its relation to the “so-what question” is examined in the second subsection.

### 7.1 Price index and Gross Domestic Product

The price index minimizes the cost of a unity of aggregate good.

$$\left. \begin{array}{l} \text{Min}_{Y,Z,\Omega,X} P = P_Y Y + P_Z Z + P_\Omega \Omega + P_X X \\ \text{s.t. } 1 = Y^{\nu_Y} Z^{\nu_Z} \Omega^{\nu_\Omega} X^{\nu_X} \end{array} \right\} \Rightarrow P = \frac{P_Y^{\nu_Y} P_Z^{\nu_Z} P_\Omega^{\nu_\Omega} P_X^{\nu_X}}{\nu_Y^{\nu_Y} \nu_Z^{\nu_Z} \nu_\Omega^{\nu_\Omega} \nu_X^{\nu_X}} \quad (59)$$

we can name  $M = \frac{1}{\nu_Y^{\nu_Y} \nu_Z^{\nu_Z} \nu_\Omega^{\nu_\Omega} \nu_X^{\nu_X}}$  and in this way prices are  $P = M P_Y^{\nu_Y} P_Z^{\nu_Z} P_\Omega^{\nu_\Omega} P_X^{\nu_X}$   
Gross Domestic Product is calculated as the product of quantities by prices:

$$GDP = P \cdot q = \underbrace{M \cdot P_Y^{\nu_Y} P_Z^{\nu_Z} P_\Omega^{\nu_\Omega} P_X^{\nu_X}}_P \underbrace{Y^{\nu_Y} Z^{\nu_Z} \Omega^{\nu_\Omega} X^{\nu_X}}_q \quad (60)$$

where we take as reference the prices of the final period ( $t + 3$ ) in order to calculate the aggregate country GDP in constant terms. We are interested in obtaining a growth rate between period  $t + 3$  and the initial moment  $t + 1$  in real terms, as it has been commented previously.

Country GDP at the period  $t + 1$ , for whatever of the four situations ( $S4'$ ,  $S5$ ,  $S6$ ,  $S7$ ) in which per capita growth rates equalize before migration is given by:

$$GDP_{t+1}^{t+3} = \chi \left( \frac{\beta}{2 + \delta} a^{-\beta} \right)^{\alpha \nu_{XZ} + \nu_{Y\Omega}} (\bar{k} + 1) \bar{k}^{-\beta \nu_{XZ}} k_{Bt}^{\alpha \nu_{XZ} + \nu_{Y\Omega}} \quad (61)$$

where  $\chi = P_{t+3} a^{-\beta \nu_{Y\Omega}} l_{Yt}^{\nu_Y} l_{Zt}^{\nu_Z} l_{\Omega t}^{\nu_\Omega} l_{Xt}^{\nu_X} L_t^{1 + \gamma \nu_{Z\Omega}}$ ,  $\nu_{XZ} = \nu_X + \nu_Z = 1 - \nu_Y - \nu_\Omega$ ,  $\nu_{Y\Omega} = \nu_Y + \nu_\Omega$  and  $\nu_{Z\Omega} = \nu_Z + \nu_\Omega$ .

For situations  $S1$  a  $S4$ , a convergence process will take place between cities prior to any migration movement, thus, the starting point for the analysis will be the one reached in  $S4'$ .

Country GDP at period  $t + 3$ , once steady state has been reached:

$$GDP_{t+3}^{t+3} = \chi \left( \frac{\beta}{2 + \delta} a^{-\beta} \right)^{3[\alpha \nu_{XZ} + \nu_{Y\Omega}]} (\bar{k} + 1)^{\alpha \nu_{XZ} + \nu_{Y\Omega} - \gamma \nu_{Z\Omega}} k_{Bt}^{\alpha \nu_{XZ} + \nu_{Y\Omega}} 2^{\beta \nu_{XZ} + \gamma \nu_{Z\Omega}} \quad (62)$$

## 7.2 The “so-what question” and the optimal urban concentration rate

We can derive the national long-term economic growth rates from (62) and (61) as the quotient  $\frac{GDP_{t+3}^{t+3}}{GDP_{t+1}^{t+3}}$ .

$$g(\bar{k}) = \left( \frac{\beta}{2+\delta} a^{-\beta} \right)^{2[\alpha\nu_{XZ} + \nu_{Y\Omega}]} (\bar{k} + 1)^{-\beta\nu_{XZ} - \gamma\nu_{Z\Omega}} \bar{k}^{\beta\nu_{XZ}} 2^{\beta\nu_{XZ} + \gamma\nu_{Z\Omega}} \text{ if } \bar{k} \geq \frac{\nu_{XZ}\Psi}{1 - \nu_{XZ}\Psi} \quad (63)$$

If  $\bar{k} < \frac{\nu_{XZ}\Psi}{1 - \nu_{XZ}\Psi}$  a convergence process would take place previously and the starting point for the calculation of the growth rate would be  $S4'$ . Then the aggregate growth rate for all those points is the value of the rate for  $\bar{k} = \frac{\nu_{XZ}\Psi}{1 - \nu_{XZ}\Psi}$ .

We can put the expression (63) in terms of migration rate  $m$ , taking into account (42).

$$g(m) = \left( \frac{\beta}{2+\delta} a^{-\beta} \right)^{2[\alpha\nu_{XZ} + \nu_{Y\Omega}]} (1+m)^{\gamma\nu_{Z\Omega}} (1-m)^{\beta\nu_{XZ}} \text{ if } m \leq 1 - 2\nu_{XZ}\Psi \quad (64)$$

that can be expressed in terms of the urban concentration rate defined as the primacy rate  $l_{t+3} = \frac{L_{Bt+3}}{L_{At+3} + L_{Bt+3}} = \frac{(1+m)}{2}$  and  $1 - l_{t+3} = \frac{L_{At+3}}{L_{At+3} + L_{Bt+3}} = \frac{(1-m)}{2}$ :

$$g(l) = \left( \frac{\beta}{2+\delta} a^{-\beta} \right)^{2[\alpha\nu_{XZ} + \nu_{Y\Omega}]} l_{t+3}^{\gamma\nu_{Z\Omega}} (1 - l_{t+3})^{\beta\nu_{XZ}} 2^{\beta\nu_{XZ} + \gamma\nu_{Z\Omega}} \text{ if } l \leq 1 - \nu_{XZ}\Psi \quad (65)$$

It is observed that the aggregate growth rate in the long run is related to the technological level of each city, the urban concentration rate, the initial relative wealth level of cities (measured as per capita income) and the parameters of consumer preferences according to the specialization level of cities. (Put this into relation with [Henderson \(2003\)](#) and others).

The aggregate country growth rate depends positively on the parameters  $(\nu_Y, \nu_Z, \nu_\Omega, \gamma)$ . This means that the larger the consumer preferences for more technologically advanced goods, the larger the national GDP growth. In a similar way, the larger the urbanization externalities, the larger the growth rate. On the other hand, the growth rate shows a negative relationship with the parameters  $(\alpha, \delta, a)$ . Explain the relationship with alfa and with delta. A negative relationship between the growth rate and  $a$  means that an improvement in technology (lower  $a$ ) leads to a higher growth rate.

From (65), the existence of a theoretical relationship between urban concentration ( $l_{t+3}$ ) and long-term economic growth rate ( $g_{LT}$ ) is demonstrated. The interval defined by situations from  $S1$  to  $S4$  is characterized by a constant growth rate, independent from urban concentration rate. Subsequently, we deduce the shape of the relationship defined in the second interval of the function (65). The first derivative of long-term growth rate with respect to urban concentration rate (primacy) is given by:

$$\frac{dg_{LT}}{dl_{t+3}} = \left( \frac{\beta}{2+\delta} a^{-\beta} \right)^{2[\alpha\nu_{XZ} + \nu_{Y\Omega}]} 2^{\beta\nu_{XZ} + \gamma\nu_{Z\Omega}} \left[ \frac{\gamma\nu_{Z\Omega} (1 - l_{t+3})^{\beta\nu_{XZ}}}{l_{t+3}^{1 - \gamma\nu_{Z\Omega}}} - \frac{\beta\nu_{XZ} l_{t+3}^{\gamma\nu_{Z\Omega}}}{(1 - l_{t+3})^{1 - \beta\nu_{XZ}}} \right] \quad (66)$$

There is not a concluding positive or negative sign from these expression. The first derivative will be equal to zero in an local optimum, whose expression is given by:

$$l^* = \frac{\gamma\nu_{Z\Omega}}{\gamma\nu_{Z\Omega} + \beta\nu_{XZ}} \quad (67)$$

This urban concentration rate will range from 0 to 1 and it will be a maximum (optimal concentration rate) provided that the second derivative of the aggregate growth rate with regard to urban concentration rate  $\left( \frac{d^2 g_{LT}}{dl_{t+3}^2} \right)$  is negative, in such a way that the curve representing that

**Table 5:** Parameter values

Parameters	Simulation 1	Simulation 2
Consumer preferences for good $Y$ ( $\nu_Y$ )	0.25	0.30
Consumer preferences for good $Z$ ( $\nu_Z$ )	0.25	0.20
Consumer preferences for good $\Omega$ ( $\nu_\Omega$ )	0.25	0.20
Consumer preferences for good $X$ ( $\nu_X$ )	0.25	0.30
Capital intensity ( $\alpha$ )	0.40	0.40
Labor intensity ( $\beta$ )	0.60	0.60
Intertemporal discount factor ( $\delta$ )	0.03	0.03
Technological parameter ( $a$ )	0.10	0.10
Urbanization externality ( $\gamma$ )	0.80	0.80
Optimal concentration ( $l^*$ )	0.571	0.516

relationship is concave or inverted U-shape. However, the sign of the second derivative is not concluding. The condition for  $\frac{d^2 g_{LT}}{dl_{t+3}^2}$  being negative is that the primacy rate ranges in the following interval:

$$\frac{c(c+b-1) - ((c+b-1)cb)^{0.5}}{(c+b)(c+b-1)} < l_{t+3} < \frac{c(c+b-1) + ((c+b-1)cb)^{0.5}}{(c+b)(c+b-1)} \quad (68)$$

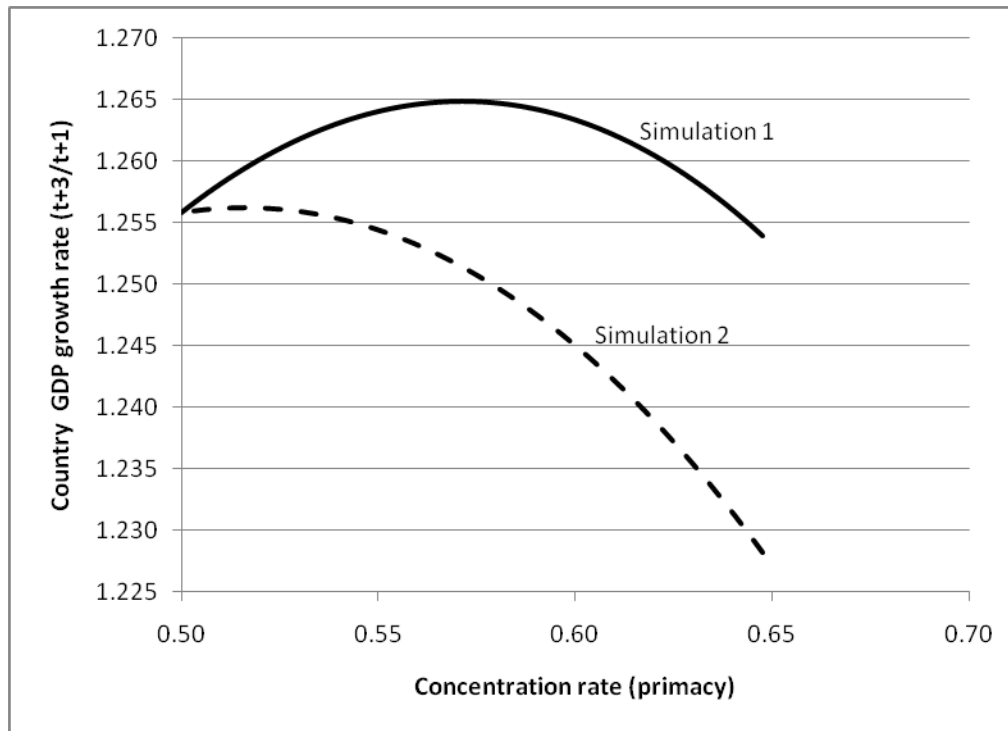
where  $c = \gamma\nu_{Z\Omega}$  and  $b = \beta\nu_{XZ}$ . Moreover, the condition  $c+b-1 > 0$  must be imposed in order not to have imaginary solutions, or in an equivalent way,  $\gamma\nu_{Z\Omega} + \beta\nu_{XZ} > 1$ . Given this property, condition (68) becomes  $\frac{c}{c+b} - \frac{1}{c+b} \left(\frac{cb}{c+b-1}\right)^{0.5} < l_{t+3} < \frac{c}{c+b} + \frac{1}{c+b} \left(\frac{cb}{c+b-1}\right)^{0.5}$  what means that  $l^*$  will pertain always to the internal.

The partial derivative of  $l^*$  with regard to each of the parameters in the equation (67) is positive. Therefore, increases in the value of whatever of the parameters ( $\nu_Y, \nu_Z, \nu_\Omega, \gamma, \alpha$ ) will lead to increases in the urban concentration rate which makes maximum the aggregate country GDP growth rate. From (67) it is observed that the optimal primacy rate does not depend on the technological level ( $a$ ) neither the discount rate ( $\delta$ ).

### 7.3 Numerical simulations

Results obtained above reflects the theoretical background for the existence of the so-what question. In this subsection, we present two numerical simulations for the urban system we have just described. Their objective is to show the different possibilities of the relationship “aggregate growth rate-urban concentration” after the dynamic processes of trade and migration across cities. Table 5 displays the values of the parameters chosen to characterize the economy in both simulations. The difference between the two simulated cases has to do with the four parameters of the preferences on the different goods. Preferences in simulation 1 weight less goods  $X$  and  $Y$  than in simulation 2 and more the other two goods.

The relationships for both simulations between the aggregate growth rate and the urban concentration, measured by the primacy, are calculated from equation (65) and shown in Figure 1. It contains in the two cases all the possible combinations between the two variables. As it can be seen, the highest aggregate growth rate in simulation 1 corresponds to a primacy of 0.571 and the inverted U-shape is very clear. In simulation 2 this form is not so evident but a primacy with the best aggregate growth rate exists for the value 0.516. These relationships show an inverted

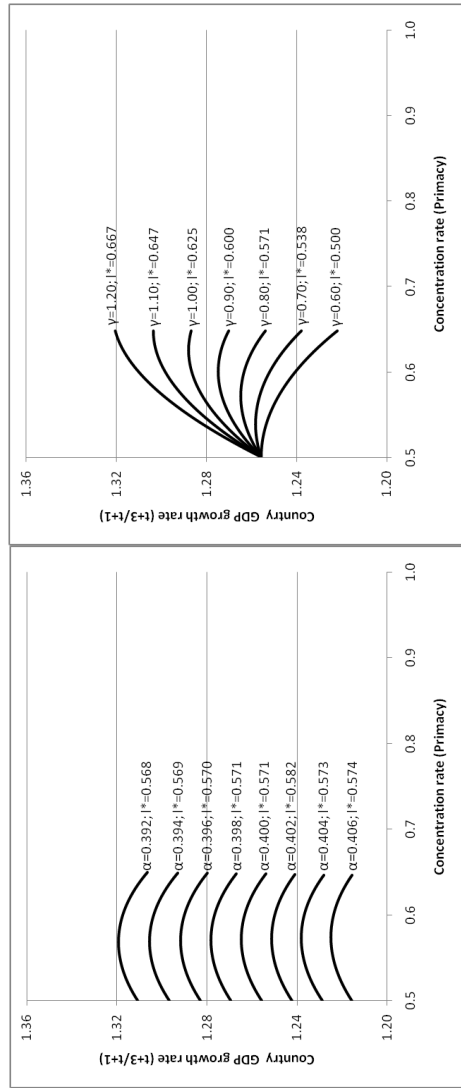


**Figure 1:** The relationship between growth and urban concentration

U-shape as it could be expected from the empirical literature (Henderson, 2003; Williamson, 1965).

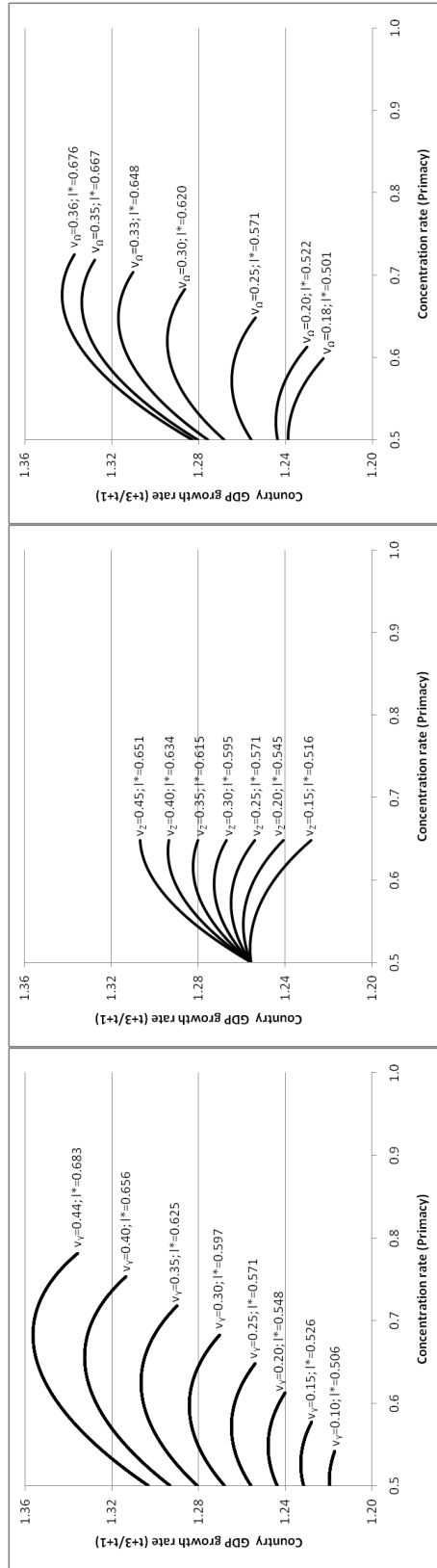
Figures 2 and 3 display a sensitivity analysis of the relationship between the aggregate growth rate and the primacy when the different parameters change. Each line displayed in the Figures corresponds to a value of the parameter which is indicated at the bottom. This value is adequately printed besides the line, as well as the value of the primacy with the greater aggregate growth rate. Figure 2a shows how the parameter  $\alpha$  is negatively related to the growth rate for each primacy value and positively to the primacy with the greater aggregate growth rate. In Figure 2b we can see that the parameter  $\gamma$  is not only positively related to the primacy with the best growth rate but also to the growth rate for each primacy value. This is also the case for the parameters  $\nu_Y$ ,  $\nu_Z$  and  $\nu_\Omega$ , as it can be seen in the Figures 2c, 2d and 2e. In all these cases the parameters affect the value of the primacy with the highest growth rate. This is not the case for the technological parameter  $a$  and the discount rate  $\delta$ . Both parameters are negatively related to the growth rate for each primacy value without affecting the primacy with the highest growth rate, as can be seen in Figures 3a and 3b.





(a) Changes in the contribution of capital to production ( $\alpha$ )

(b) Changes in urbanization externality ( $\gamma$ )

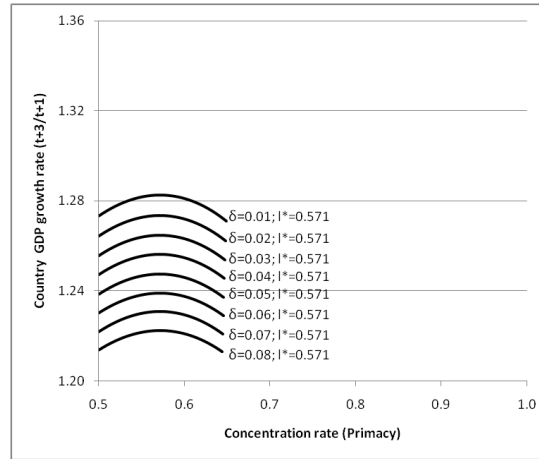
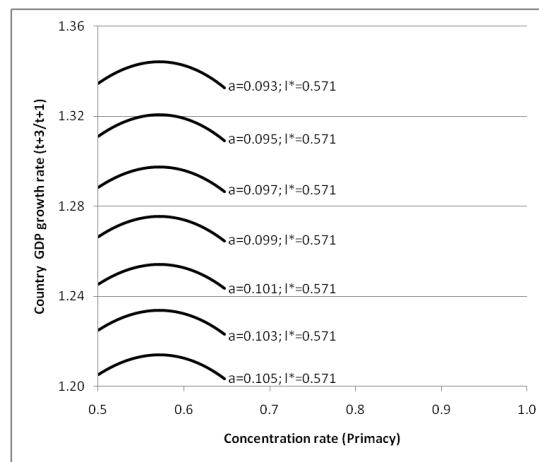


(c) Changes in ( $\nu\gamma$ )

(d) Changes in ( $\nu z$ )

(e) Changes in ( $\nu\Omega$ )

Figure 2: Sensibility analysis to changes in model parameters

(a) Changes in discount rate ( $\delta$ )(b) Changes in technology ( $a$ )**Figure 3:** Sensibility analysis to changes in model parameters not affecting optimal primacy rate

## 8 Conclusions

A theoretical approach to solve the main problems faced to explain the relationship between aggregate economic growth and the urban structure has been presented in the previous Sections. This approach consists of an overlapping generations model with four different types of goods (some with both technological and local externalities) and two cities where their production could be located. The equilibrium of the model provides the dynamics of the movements of labor and goods across cities and the corresponding urban structure. The resulting system of two cities with different patterns of specialization, urban concentration and economic growth rates, has made clear how to set out the comparison of aggregate growth rates: only the aggregate growth rate between two steady states, one without migration but with trade specialization and the other after migration and specialization, makes sense. As a result, the main conclusion obtained has been the existence of a theoretical relationship between aggregate economic growth and urban concentration with an inverted-U shape.

## References

- Bertinelli, L. and Black, D. (2004), "Urbanization and Growth," *Journal of Urban Economics*, 56, 80–96.
- Bertinelli, L. and Strobl, E. (2007), "Urbanisation, Urban Concentration and Economic Development," *Urban Studies*, 44, 2499–2510.
- Duranton, G. and Puga, D. (2001), "Nursery Cities: Urban Diversity, Process Innovation, and the Life Cycle of Products," *American Economic Review*, 91, 1454–1477.
- Henderson, J. (2005), "Urbanization, Economic Geography, and Growth," in *Handbook of Economic Growth, Volume 1*, eds. Aghion, P. and Durlauf, S. e., Amsterdam: Elsevier-North Holland.
- Henderson, J. and Wang, H. (2005), "Aspects of the rural-urban transformation of countries," *Journal of Economic Geography*, 5, 23–42.
- Henderson, V. (2003), "The Urbanization Process and Economic Growth: The So-What Question," *Journal of Economic Growth*, 8, 47–71.
- Kemnitz, A. (2001), "Endogenous growth and the gains from immigration," *Economics Letters*, 72, 215–218.
- Sanso, M., Pueyo, F., and Ponz, M. (2005), "Efectos de la integración económica sobre el bienestar, el crecimiento y los flujos de bienes y de factores," Mimeo.
- Williamson, J. (1965), "Regional Inequality and the Process of National Development: A Description of the Patterns," *Economic Development and Cultural Change*, 13, 3–84.