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Second-Best Cost-Benefit Analysis with a Microfoundation of Urban Agglomeration

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Second-Best Cost-Benefit Analysis with a **Microfoundation of Urban Agglomeration** 

Yoshitsugu Kanemoto†

Abstract

Many sources of urban agglomeration, such as the gains from variety, better

matching, and knowledge creation and diffusion, involve departures from the first-best

world. Benefit evaluation of a transportation project must then take into account changes

in excess burden along with any direct user benefits. A number of economists have

addressed this issue, and policymakers in some countries, such as in the United Kingdom,

have been attempting to include these considerations in their project assessments. By

modeling the microstructure of agglomeration economies, we derive second-best benefit

evaluation formulae for urban transportation improvements. Previous work has

investigated the same problem, but without explicitly modeling the sources of

agglomeration economies. Accordingly, our analysis examines whether earlier results

remain valid when monopolistic competition with differentiated products provides the

microfoundation of the agglomeration economies. By explicitly introducing the rural

sector and multiple cities, we also show that the agglomeration benefits depend on where

the new workers are from.

Keywords: cost-benefit analysis; agglomeration economies; monopolistic competition;

new economic geography; second-best economies

JEL Classification: D43; R12; R13

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## 1. Introduction

Many of the sources of urban agglomeration, such as gains from variety, better matching, and knowledge creation and diffusion, involve departures from the first-best world.<sup>2</sup> The benefit evaluation of a transportation project must then take into account agglomeration benefits along with any direct user benefits. A number of economists have studied this issue, and policymakers in some countries, such as in the United Kingdom, have been attempting to include these considerations in their project assessments.<sup>3</sup>

Based on past empirical work, urban agglomeration economies are substantial. For instance, a review by Rosenthal and Strange (2004, p. 2133) summarizes the empirical findings as follows: "In sum, doubling city size seems to increase productivity by an amount that ranges from roughly 3–8%." Agglomeration economies on the consumer side are also substantial, as argued by Glaeser et al. (2001), with estimates by Tabuchi and Yoshida (2000) suggesting economies in the order of 7–12 percent. Certainly, the benefit estimates could exceed 10 percent after combining production and consumption agglomeration economies.

By modeling the microstructure of agglomeration economies, this paper derives second-best benefit evaluation formulae for urban transportation improvements. Venables (2007) investigated the same problem but without explicitly modeling the sources of agglomeration economies. Accordingly, our analysis examines whether the results in this prior work remain valid when monopolistic competition with differentiated products provides the microfoundation of agglomeration economies. By explicitly introducing the rural sector and multiple cities, we also show that the agglomeration benefits depend on where the new workers are from.

Extending the Henry George Theorem to a second-best setting with distorted prices, Behrens et al. (2010) showed that the optimality condition for the number of cities (or equivalently, the optimal size of a city) must be modified to include Harberger's excess

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<sup>&</sup>lt;sup>2</sup> See Duranton and Puga (2004) for a review of the theoretical analysis of various sources of urban agglomeration, Fujita and Thisse (2002) for the New Economic Geography approach, and Kanemoto (1990) for the analysis of a nonmonocentric city model.

<sup>&</sup>lt;sup>3</sup> See, for example, Venables and Gasiorek (1999), Department of Transport (2005), (2008), Graham (2005), (2006), and Vickerman (2007).

burden, that is, the weighted sum of induced changes in consumption with weights being the price distortions. New Economic Geography (NEG)-type models of monopolistic competition contain distortions of two forms: a price distortion for each variety of the differentiated good, and a distortion associated with the number of available varieties consumed. Although the former is well known, the latter has largely escaped the attention of the existing literature. Importantly, because these two types of distortions work in opposite directions, the net effect is uncertain. In this article, we examine whether we can obtain similar results with transportation investment projects. Moreover, in yet another departure from Venables (2007), we explicitly introduce the rural sector and multiple cities. We show that the results hinge on whether the new workers are from the rural sector or other cities.

The remainder of the paper is as follows. In Section 2, we present a model of urban agglomeration economies based on monopolistic competition in differentiated intermediate products. Section 3 derives second-best benefit measures of transportation investment assuming the number of cities is fixed. In Section 4, we extend the analysis to the situation where transportation investment changes the number of cities. Section 5 concludes.

#### 2. The model

Our model adds three elements to Venables (2007): (i) an explicit rural sector, (ii) multiple cities, and (iii) the microstructure of agglomeration in the urban sector. We ignore income tax distortion because Venables' analysis is applicable to our model without modification. The economy contains n cities and a rural area, where all cities are monocentric, that is, all workers commute to the central business district (CBD). All cities also have the same topographical and technological conditions. We assume that the economy is initially in a symmetric equilibrium where all cities have identical allocations, and examine the effect of a small improvement in urban transportation in one of the cities.

We model the microstructure of agglomeration in such a way as to produce the same aggregate production function as Venables (2007). Here, the production of an urban final good requires differentiated intermediate products only and we assume the final good is homogeneous. Final good producers compete with producers in other cities as well as other producers located in the same city. Furthermore, for simplicity, we assume

the rural area produces the same final product albeit with a different technology. While the final good is the only good that consumers directly consume, it is also used in the production of housing and transportation services. The final good producers are competitive but the intermediate good producers are monopolistically competitive. We assume free-entry (zero-profit) conditions for both types of producers. For transportation and housing, we assume a simple monocentric city model again following Venables (2007). As noted above, these sectors use final products as their sole inputs. Also for simplicity, we assume absentee landlords own land in both urban and rural areas. Usefully, it is not difficult to modify this particular assumption.

## Production in cities

The production of the urban final good requires only differentiated intermediate inputs and the production function is additively separable with respect to the inputs:

(1) 
$$y_0 = \left(\int_0^m F(y_i)di\right)^{1+\rho},$$

where  $y_i$ ,  $i \in M$  is the input of differentiated intermediate good i, parameter  $\rho$  is positive  $(\rho > 0)$  reflecting economies of variety in the intermediate inputs, and  $F(y_i)$  exhibits increasing returns-to-scale initially though scale economy declines as production increases, eventually attaining decreasing returns-to-scale. More specifically, the scale elasticity of production,  $\theta(y) = yF'(y)/F(y)$  is assumed to satisfy:

(2) 
$$\theta'(y) < 0, \ \theta(0) > \frac{1}{1+\rho}, \text{ and } \theta(y) > \frac{1}{1+\rho} \text{ for some } y.$$

The number (mass) of intermediate goods m is endogenous. The final good,  $y_0$ , is homogeneous and its cost of transportation is zero. We normalize the price of the final good as one (1). There are many final good producers in each city and they take both the final good and intermediate good prices as given. Let k denote the number of final good producers.

The profit maximization problem for a final good producer is:

(3) Max 
$$\pi = \left\{ \left( \int_0^m F(y_i) di \right)^{1+\rho} - \int_0^m p_i y_i di \right\}.$$

The first-order conditions for  $y_i$  and m are respectively:

(4) 
$$p_i = F'(y_i)(1+\rho) \left( \int_0^m F(y_i) di \right)^{\rho},$$

$$(5) F(y_m)(1+\rho)\left(\int_0^m F(y_i)di\right)^\rho \ge p_m y_m.$$

We can interpret the left-hand side of the inequality in (5) as the willingness to pay for a new variety by a final good producer and the right-hand side as its purchase cost. Because all final good producers benefit from a new variety, we sum these over producers to obtain the demand price of diversity:

$$(6) P_m \equiv kF(y_m)(1+\rho)\left(\int_0^m F(y_i)di\right)^\rho,$$

and its supply price as:

$$(7) P_m - \tau_m \equiv k p_m y_m,$$

where  $\tau_m$  is the 'price' distortion of variety. Note that the demand price may exceed the supply price because supply side conditions determine the number of intermediate goods in addition to the demand side conditions. The first-order condition (4) for  $y_i$  yields the demand function for intermediate good i:

(8) 
$$y_i = d_i(p_i; \int_0^m F(y_i) di)$$
, with  $\frac{\partial d_i}{\partial p_i} = \frac{1}{F'(y_i)(1+\rho) \left(\int_0^m F(y_i) di\right)^{\rho}}$ .

The free-entry condition for the final good producers is:

(9) 
$$\pi = \left(\int_0^m F(y_i)di\right)^{1+\rho} - \int_0^m p_i y_i di = 0.$$

Next, let us turn our attention to the producers of the intermediate good. Production of an intermediate good requires only labor as an input. The labor input required for producing  $Y_i$  of variety i is:

$$(10) N_i = cY_i + f,$$

where the fixed cost is f and the constant marginal cost is c in labor units.

Intermediate good producers have monopoly power because their products are differentiated, but an individual producer is sufficiently small not to influence the number of intermediate goods, m, and the number of final good producers, k. Then the demand function that producer i is faced with can be written as:

(11) 
$$Y_i = D_i(p_i) = kd_i(p_i)$$
,

where from (8) the price elasticity of demand is:

(12) 
$$\zeta = -\frac{\partial D_i}{\partial p_i} \frac{p_i}{Y_i} = -\frac{F'(y)}{yF''(y)} = \frac{1}{R_R(y)},$$

and  $R_R(y) \equiv -yF''/F'$  is equivalent to the measure of relative risk aversion in expected utility theory. The profit of an intermediate good producer is then:

(13) 
$$\pi_i = p_i Y_i - w(cY_i + f),$$

where w is the wage rate. Profit maximization yields:

$$(14) \qquad \frac{p_i - wc}{p_i} = \frac{1}{\zeta(Y_i)}.$$

From free entry, the profit of a firm is zero:

(15) 
$$(p_i - wc)Y_i - wf = 0$$
.

Now we concentrate on a symmetric equilibrium and omit subscript i for an intermediate good. The following lemma obtains the equilibrium conditions for the urban production sector.

**Lemma 1 (Urban Production Sector)** The quantity of an intermediate input, *y*, used by a final good producer satisfies:

(16) 
$$\theta(y) = \frac{yF'(y)}{F(y)} = \frac{1}{1+\rho}$$
.

The production of the differentiated good, *Y*, is determined by:

(17) 
$$(cY+f)R_R(Y)=f$$
,

where  $R_R(y) \equiv -yF''/F'$ . The number of final good producers and the price of the intermediate good satisfy:

$$(18)$$
  $k = Y/y$ ,

and

(19) 
$$\frac{p}{w} = \frac{c}{1 - R_R(Y)} = \frac{cY + f}{Y}$$
,

respectively. Furthermore, the number of urban workers required for production is related to variety by:

$$(20)$$
  $N = m(cY + f)$ .

The price of an intermediate good, the wage rate, and the total production of the final good are respectively:

(21) 
$$p = p(N) = \frac{cY + f}{Y} \omega N^{\rho},$$

$$(22) w = w(N) = \omega N^{\rho},$$

(23) 
$$Y_0 = Y_0(N) = \omega N^{1+\rho}$$
,

where  $\omega$  is a parameter defined by:

(24) 
$$\omega = \frac{Y}{y} \left( \frac{F(y)}{cY + f} \right)^{1+\rho}$$
.

## **Proof:**

The production of the final good by a producer is:

(25) 
$$y_0 = (mF(y))^{1+\rho}$$
.

Substituting this into the first-order condition for a final good producer (4) and the free-entry condition (9) yields:

(26) 
$$p = F'(y)(1+\rho)(mF(y))^{\rho}$$
,

(27) 
$$(mF(y))^{1+\rho} = mpy$$
.

From these two equations, we obtain (16).

The profit maximization and the zero-profit conditions, (14) and (15), for an intermediate good producer are now:

$$(28) \qquad \frac{p-wc}{p} = R_R(Y),$$

(29) 
$$(p-wc)Y - wf = 0$$
.

These two conditions immediately yield (17).

Now that once Y and y are determined, the number of final good producers is given by k = Y/y. Equation (20) follows immediately from (10).

Substituting (20) into the first-order condition for the final good producer (26) and noting (16), we can rewrite the price of an intermediate good as (21). From (19) we can also see that the wage rate satisfies (22). From k = Y/y, and (25) we obtain the total production of the final good (23).

This lemma shows that the total production of an intermediate good, Y, the amount of an intermediate good used by a final good producer, y, the number of final good producers, k, and the relative price between an intermediate good and labor, p/w, are all determined by the equilibrium conditions within the urban production sector and do not depend on city size or conditions in other parts of the economy. We assume that conditions (16) and (17) uniquely determine y and Y. The lemma also shows that differentiated intermediate goods yield agglomeration economies: that is, the total production in a city exhibits increasing returns-to-scale. The price of an intermediate good and the wage rate are also increasing in city size.

Commuting costs, housing, and the rural sector

An urban worker consumes housing with quality  $\overline{h}$ , which is assumed exogenously fixed. We ignore  $\overline{h}$  as it is fixed, and for simplicity assume housing only requires land as an input.

Each worker has one unit of labor and supplies it to the producers of the differentiated consumer goods. The total endowment of labor in the economy is  $\overline{N}$ . There are n cities with labor force N. The labor force in the rural area is denoted by  $N_A$ . As noted before, we assume that the rural product is the same as the urban final product. The production function of the rural sector is  $Y_A = G(N_A)$ , where the marginal product is  $W_A(N_A) \equiv G'(N_A)$ .

As in Venables (2007), we assume that urban workers do not receive a share of the land rent revenue. The budget constraint for an urban worker is:

(30) 
$$w = x_0 + t(z) + r(z)$$
 for all  $z \in [0, \hat{z}]$ ,

where t(z) is the commuting cost for a worker living at distance z from the CBD, r(z) is housing rent, and  $\hat{z}$  is the edge of a city. We assume that the rent is zero at the periphery of the city. Note that commuting requires only the final good as an input. In equilibrium, the housing rent differentials completely offset the commuting cost differentials such that:

(31) 
$$w = x_0 + t(\hat{z}) \text{ for all } z \in [0, \hat{z}].$$

In order for the housing market to be in equilibrium, all urban residents must find a

place to live in the city:

(32) 
$$N = \int_0^{\hat{z}} n(z) dz = \int_0^{\hat{z}} (1+\theta) z^{\theta} dz = \hat{z}^{(1+\theta)},$$

where as in Venables (2007) we have assumed  $n(z) = (1 + \theta)z^{\theta}$ . From this equation,  $\hat{z}$  satisfies:

(33) 
$$\hat{z} = N^{1/(1+\theta)}$$
.

Transport costs are assumed to satisfy  $t(z) = tz^{\lambda}$ . The transport costs of a worker at the periphery of the city are:

$$(34)$$
  $t(\hat{z}) = tN^{\gamma-1}$ ,

where we define:

(35) 
$$\gamma = \frac{1+\theta+\lambda}{1+\theta} > 1.$$

The aggregate transportation costs in a city (measured in terms of the final product) are:

(36) 
$$TC = \int_0^{\hat{z}} kz^{\theta} tz^{\lambda} dz = \frac{tN^{\gamma}}{\gamma} \equiv TC(N, t).$$

The aggregate housing (land) rent is:

$$(37) LR = \int_0^{\hat{z}} (1+\theta) z^{\theta} (t\hat{z}^{\lambda} - tz^{\lambda}) dz = \left(1 - \frac{1}{\gamma}\right) tN^{\gamma}.$$

Free mobility of workers requires that the utility levels equalize across cities and between the rural area and the cities:

(38) 
$$w(N) - tN^{\gamma - 1} = w_A(\overline{N} - nN)$$
.

This determines the city size, N. The stability of migrational equilibrium requires that:

(39) 
$$w'(N) - t(\gamma - 1)N^{\gamma - 2} + nw_A'(\overline{N} - nN) \le 0.$$

## 3. Benefits of transportation investment: a fixed number of cities

First, assuming that the number of cities is fixed, we obtain the benefits of an improvement in transportation. Starting from a symmetric equilibrium with equal-sized cities, we examine the effects of a transportation improvement that occurs in only one of the cities, that is, City 1. Utility levels equalize between the cities and the rural area. As the consumption of an urban worker is  $x_0 = w - t(\hat{z}) = w - tN^{\gamma-1}$  and the consumption of a

rural worker is  $w_A(\overline{N} - nN)$ , the equilibrium conditions are:

(40) 
$$w^1 - t^1(N^1)^{\gamma - 1} = w_A(\overline{N} - N^1 - (n-1)N),$$

$$(41) w - tN^{\gamma - 1} = w_{\Lambda}(\overline{N} - N^{1} - (n-1)N),$$

where superscript 1 denotes City 1 and variables without a superscript are other cities. Given the lemma holds in each city, the wage rates satisfy  $w^1 = w(N^1)$  and w = w(N). The above equations then determine the populations of City 1 and other cities as functions of transport costs:

$$(42)$$
  $N^1 = N^1(t^1)$ 

$$(43) N = N(t^1).$$

Given there is only a single consumption good in our model, we can define the social surplus as the total amount of the good available for consumption:

(44) 
$$S = Y_0(N^1(t^1)) - TC(N^1(t^1), t^1) + (n-1)[Y_0(N(t^1)) - TC(N(t^1), t)] + G(\overline{N} - N^1(t^1) - (n-1)N(t^1))$$

The following theorem obtains the impact of a transportation improvement on the social surplus.

**Theorem 1 (Second-Best Cost–Benefit Analysis)** When the number of cities is fixed, a marginal reduction in transportation costs changes the social surplus by:

$$(45) -\frac{dS}{dt^{1}} = MB_{T} - \tau_{w} \left( \frac{dN^{1}}{dt^{1}} + (n-1) \frac{dN}{dt^{1}} \right) \ge MB_{T},$$

where  $MB_T$  is the direct benefit of a transportation investment in City 1 defined as the reduction in transportation costs in City 1:

$$(46) MB_T = \frac{\partial TC(N^1, t^1)}{\partial t^1} = \frac{(N^1)^{\gamma}}{\gamma},$$

and  $\tau_w$  is the distortion in the wage rate:

(47) 
$$\tau_w = Y_0'(N) - \frac{Y_0(N)}{N} = \rho \frac{Y_0(N)}{N} > 0.$$

## **Proof:**

Differentiating the social surplus (44) with respect to  $t^1$  yields:

$$-\frac{dS}{dt^{1}} = MB_{T} - \left(\frac{dY_{0}}{dN^{1}} - \frac{\partial TC}{\partial N^{1}} - G'\right)\frac{dN^{1}}{dt^{1}} - (n-1)\left(\frac{dY_{0}}{dN} - \frac{\partial TC}{\partial N} - G'\right)\frac{dN}{dt^{1}}$$

$$= MB_{T} - \tau_{w}\left(\frac{dN^{1}}{dt^{1}} + (n-1)\frac{dN}{dt^{1}}\right)$$

From:

$$(49) \qquad \frac{dN^{1}}{dt^{1}} = \frac{(N^{1})^{\gamma-1} \left(w' - t(\gamma - 1)N^{\gamma-2} + (n-1)w_{A}'\right)}{\left(w' - t^{1}(\gamma - 1)N^{\gamma-2}\right)\left(w' - t(\gamma - 1)N^{\gamma-2} + nw_{A}'\right)},$$

(50) 
$$\frac{dN}{dt^{1}} = -\frac{(N^{1})^{\gamma-1}w_{A}'}{(w'-t^{1}(\gamma-1)N^{\gamma-2})(w'-t(\gamma-1)N^{\gamma-2}+nw_{A}')},$$

we obtain:

$$(51) \qquad \frac{dN^{1}}{dt^{1}} + (n-1)\frac{dN}{dt^{1}} = \frac{(N^{1})^{\gamma-1}}{w' - t(\gamma - 1)N^{\gamma-2} + nw_{A}'} \le 0,$$

where the inequality results from the stability condition for migrational equilibrium (39). This immediately yields (45).

This theorem shows that a transportation improvement increases the total urban population and there will be positive additional benefits. As shown, a transportation improvement in City 1 tends to increase its population. Moreover, this creates agglomeration benefits in addition to the direct user benefits because the social value of an additional worker exceeds the wage rate. However, this process also reduces the size of other cities and these adverse effects in other cities (partially) offset the benefits in City 1. If the population of the rural area (or equivalently, the total population of the urban areas) is fixed, then these effects cancel each other out and there will be no extra benefits:

$$(52) \qquad -\frac{dS}{dt^1} = MB_T.$$

Note that the Harberger formula is valid in our model: that is, the benefit of a given transportation improvement is the sum of the reduction in transportation costs (i.e., the direct benefit) and the changes in the excess burden. In our model of differentiated intermediate goods, however, the production of an intermediate good is uniquely determined by profit maximization and free-entry conditions in the urban sector and is unaffected by any transportation improvement. The only distortions that then matter relate to variety and the wage rate. Furthermore, these two distortions are in proportion to

each other and so in effect, we only have a single distortion. This distortion makes the urban population (or equivalently, the variety of intermediate products) too small and so a transportation improvement yields additional benefits if it increases urban population.

# 4. Transportation investment with an induced change in the number of cities

Next, we examine the case where a transportation investment induces a change in the number of cities, that is, the creation of new cities or the disappearance of existing cities. We first examine the condition for optimal city size.

The condition for optimal city size: the second-best Henry George Theorem If all cities are identical, the net social surplus is:

(53) 
$$S = nY_0(N) - nTC(N,t) + G(\overline{N} - nN),$$

where:

$$(54) TC(N,t) = \frac{tN^{\gamma}}{\gamma}.$$

The following theorem derives the condition for the optimal number of cities, assuming the number of cities is a continuous variable.

**Theorem 2 (Second-Best Henry George Theorem)** The change in surplus caused by a change in the number of cities is:

$$(55) \qquad \frac{dS}{dn} == LR + n\tau_{w}N_{n}.$$

If the number of cities is optimal, we have:

$$(56) \qquad \frac{dS}{dn} == LR + n\tau_{w}N_{n} = 0.$$

## **Proof:**

Differentiating the social surplus with respect to the number of cities yields:

$$\frac{dS}{dn} = Y_0 - TC - NG' + n(Y_0' - TC_N - w_A)N_n.$$

Combining  $Y_0 = wN$  with migrational equilibrium condition (38) yields  $Y_0 = wN = tN^{\gamma} + w_A N$ . Hence, the change in surplus is:

(57) 
$$\frac{dS}{dn} = tN^{\gamma} - \frac{1}{\gamma}tN^{\gamma} + n(Y_0' - tN^{\gamma - 1} - (w - tN^{\gamma - 1}))N_n$$

$$= LR + n\tau_w N_n$$

which must be zero if the number of cities is optimal.

If the number of cities is too large (small), then the city size is too small (large), in which case  $dS/dn = LR + n\tau_m \frac{m}{N} N_n$  is negative (positive). Note that in Venables,  $w_A$  is constant and hence  $N_n = 0$ . Hence, dS/dn cannot be zero, implying that either the rural workers or the urban workers disappear when the number of cities is optimal.

## Transportation investment

We have seen that when the number of cities is fixed, the distortion associated with agglomeration externalities yields additional benefits. If the number of cities changes, equations (42) and (43) must be modified to:

$$(58) N^1 = N^1(t^1, n(t^1)),$$

(59) 
$$N = N(t^1, n(t^1)).$$

Theorem 1 is now as follows.

**Theorem 3 (Variable Number of Cities)** When the number of cities is variable, a marginal reduction in transportation costs changes the social surplus by:

$$(60) -\frac{dS}{dt^{1}} = MB_{T} - (LR + n\tau_{w}N_{n})\frac{dn}{dt^{1}} - \tau_{w}\left(\frac{dN^{1}}{dt^{1}} + (n-1)\frac{dN}{dt^{1}}\right).$$

### **Proof:**

Substituting (58) and (59) into the social surplus (44) and differentiating it with respect to  $t^1$  immediately yields the theorem.

If the number of cities is optimal, then  $LR + n\tau_w \frac{\partial N}{\partial n} = 0$ , and we obtain the same results as with a fixed number of cities. If the city size is too large (i.e., the number of cities is too small), then  $LR + n\tau_w \frac{\partial N}{\partial n}$  is positive. A decrease in transport costs in City 1

will then be more likely to reduce the number of cities because City 1 becomes more attractive than these other cities. In such a case, the induced effect has a tendency to reduce the social surplus if the city size is too large.

# 5. Concluding remarks

This article obtained cost–benefit measures for the situation where monopolistic competition with differentiated products provides the microfoundation of agglomeration economies. The major results are as follows. First, while the Harberger formula for excess burden is also valid in our model, the production of the intermediate good is unaffected by a transportation improvement if the number of cities is fixed. Furthermore, the variety and wage rate distortions are in proportion, and so in effect, we have only a single distortion. This distortion makes the urban population too small and so a given transportation improvement yields additional benefits if it increases urban workers. Our measure of excess burden is equivalent to the agglomeration externality measure in Venables (2007) with a reduced-form aggregate production function. Note that we obtain equivalence because the production of a differentiated good is unaffected by transportation investment. In a richer model where this does not hold, the equivalence will be broken.

Second, an improvement in urban transportation in one city increases the population in that city, but reduces the populations in other cities. If the population of the rural area (or equivalently, the total population of the urban areas) is fixed, then the changes in the excess burden cancel each other out and only the direct benefit remains. Further, if migration between the rural area and cities is possible, then a transportation improvement increases the total urban population and there will be positive additional benefits. If the number of cities changes, we have an additional change in the excess burden, but the result depends on whether the city size is too large. In the former, the induced effect on the number of cities has a tendency to reduce the social surplus.

There are two practical implications of our findings. First, at least in a model of differentiated intermediate products, one can use a reduced-form aggregate production function, as in Venables (2007), to estimate the benefits of transportation improvements. Second, whether or not substantial agglomeration benefits exist depends on where the new workers are from. If they are from another city with similar agglomeration

economies, there will be little additional benefit. Conversely, if they are from rural areas with no agglomeration economies, or from small cities with only small agglomeration economies, the additional benefits may be substantial.<sup>4</sup>

Graham (2005, 2006, 2007a, 2007b) and Department of Transport (2005, 2008) employ a framework unlike Venables (2007) in modeling urban agglomeration. These particular studies use the concept of 'effective density' to measure relative proximity to urban activities, as defined for each location using a gravity model-type equation; for example, the weighted sum of the number of workers with weights determined as a decreasing function of distance. However, even in a model of this type, we need to consider the adverse effects on areas that lose workers. We defer to future work the analysis of a second-best benefit measure based on the microfoundation for effective density.

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