Farzin, Y. Hossein

Working Paper
Sustainability and Hamiltonian value

Nota di Lavoro, Fondazione Eni Enrico Mattei, No. 48.2002

Provided in Cooperation with:
Fondazione Eni Enrico Mattei (FEEM)

Suggested Citation: Farzin, Y. Hossein (2002) : Sustainability and Hamiltonian value, Nota di Lavoro, Fondazione Eni Enrico Mattei, No. 48.2002

This Version is available at:
http://hdl.handle.net/10419/119654

Standard-Nutzungsbedingungen:
Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.
Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.
Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

www.econstor.eu
Sustainability and Hamiltonian Value

Y. Hossein Farzin

NOTA DI LAVORO 48.2002

JULY 2002

SUST – Sustainability Indicators and Environmental Evaluation

Y. H. Farzin, University of California-Davis

This paper can be downloaded without charge at:

The Fondazione Eni Enrico Mattei Note di Lavoro Series Index:
http://www.feem.it/web/activ/_activ.html

Social Science Research Network Electronic Paper Collection:
http://papers.ssrn.com/abstract_id=XXXXXX

The opinions expressed in this paper do not necessarily reflect the position of Fondazione Eni Enrico Mattei
Sustainability and Hamiltonian Value

Summary

The relationships among the Hamiltonian, NNP, and the level of sustainable consumption/utility have been widely misunderstood. This paper dispels the misconceptions and provides further new insight into these relationships. We show generally that for autonomous dynamic optimizing economies, a necessary and sufficient condition for sustainability is the stationarity of the current-value Hamiltonian. For autonomous cases, this stationarity condition generalizes Dixit et al.’s (1980) “zero-net-aggregate-investment” rule of sustainability, which in turn generalizes Solow-Hartwick’s sustainability rule. For non-autonomous cases, however, except when the net “pure time effect” is constant over time, the stationarity condition is unfulfilled. In non-autonomous cases, Weitzman’s (1976) “stationary equivalence” result does not hold, and the current-value Hamiltonian will underestimate (overestimate) the true welfare level when the net “pure time effect” is positive (negative). However, for the special non-autonomous case of a time-dependent utility discount rate we obtain a condition on the discount rate function that upholds the results obtained for autonomous cases. In turn, this condition extends Michel’s (1982) transversality condition for the infinite-horizon autonomous control problems to problems with time dependent discount rates.

Keywords: Sustainability, current-value Hamiltonian, net national product

JEL: D63, Q32, C61

Address for correspondence:

Y. Hossein Farzin
University of California
Department of Agricultural and Resource Economics
Davis, CA 95616
USA
Phone: +001 - 530 - 752 7610
Fax: +001 - 530 - 752 5614
E-mail: Farzin@primal.ucdavis.edu

For very helpful comments, the author is grateful to Ken Arrow, Geir Asheim, Michael Caputo, Larry Goulder, Geoff Heal, and Bob Solow. The author also thanks seminar participants at UC Berkeley, Stanford University and University of Washington for their comments. The author alone, however, is responsible for any remaining error of omission or commission.
Sustainability and Hamiltonian Value

1. Introduction

Over the past quarter of a century, the unprecedented concern about the long-run consequences of environmental and natural resource use has confronted economists with two important intertwined questions. First, how should the conventional measure of national income be modified to properly take account of depletion of natural resources and the consequent environmental quality degradation? Second, how do the concepts of economic welfare and intergenerational equity relate to the modified national income measure? In response to these concerns, a vast (and still growing) literature has emerged, providing considerable valuable insights towards both questions. Concerning the first question (green national accounting issue), the studies by Dasgupta and Heal (1979), Dasgupta (1990), Hartwick (1990), Mäler (1991), Dasgupta and Mäler (1991), Brekke (1994), Sefton and Weale (1996), and Heal and Kriström (1998) have been among the pioneering works. Regarding the second question (economic welfare and sustainability issue), original works of Solow (1974)(1986), Hartwick (1977), and Dasgupta and Mäler (1990), have been either further developed or extended in several important directions in papers by Asheim (1994), Aronsson and Löfgren (1995), Chichilnisky (1996), and Heal (1998), among others.

A starting point of most of these and related contributions has inherently been Weitzman’s (1976) seminal paper. In that paper, Weitzman showed that under the specific assumptions of his model, at any time, the optimal current-value Hamiltonian equals the economy’s net national product (NNP). Further, and perhaps more importantly, he provided the fundamental insight that, at any point in time, the optimal current-value Hamiltonian of a dynamically optimizing economy presents a (hypothetical) permanently constant consumption flow equivalent to the discounted value of the economy’s optimal consumption path. This insight is sometimes referred to as “stationary equivalence” or Weitzman’s basic result. On the other hand, Solow (1974) and Hartwick (1977) were the first to derive a condition for sustainability of a maximum constant consumption flow in the context of a closed economy using an exhaustible resource input and a reproducible capital with a constant technology to produce a consumption good. Their derived condition, known as Solow-Hartwick’s sustainability rule, required that resource rents be reinvested in reproducible capital.

Unfortunately, however, the concurrence of Weitzman’s “stationary equivalence” result with Solow-Hartwick’s sustainability rule seems to have resulted in a widespread

---

1 For an overview of the theory of green national accounting, see the special issue of Environment and Development Economics (2000).
misinterpretation of Weitzman’s result and thereby misinterpretations and confusion about the relationships among the current-value Hamiltonian, NNP, and sustainability condition. Yet, a correct understanding of these relationships is crucial to the development of a sound theoretical basis and methods for green national accounting. Building on many valuable insights from the previous literature, the present paper aims to (i) dispel the existing and potential misconceptions, (ii) generalize some of the basic results in the literature, (iii) provide further new insights into the relationships and, (iv) as a by product, extend an important transversality result in the optimal control theory due to Michel (1982). To this end, Section 2 briefly reviews the characteristics of the optimal consumption policy for the special case of a purely exhaustible resource economy. This special case greatly helps to bring out, in the clearest and simplest fashion, the prevailing misconceptions and consequent paradoxical results. Section 3 shows that, contrary to the usual misinterpretation of Weitzman’s result (see, e.g., Mäler (1991, p.5)), the current-value Hamiltonian does not represent the maximum sustainable constant utility (consumption) flow. More importantly, it shows that a necessary and sufficient condition for sustainability in this sense is that the current-value Hamiltonian must be stationary. Section 4 shows that the stationarity condition holds generally for the class of dynamically optimizing economies characterized by an infinite-horizon, time-autonomous optimal control problem, of which the economy studied by Weitzman is a special case. As such, the stationarity condition generalizes Dixit et al.’s (1980) “zero-net-aggregate-investment” rule, which in turn is a generalization of Solow-Hartwick’s “resource-rent-investment” rule of sustainability.

Section 5 considers the sustainability condition for the more general case of non-autonomous problems that arise from exogenous changes in the economy over time. We show that for such cases Weitzman’s “stationary equivalence” result no longer holds, and that the current-value Hamiltonian deviates from the true welfare level by an amount equal to the discounted value of the flow of net “pure time effect.” Furthermore, in non-autonomous cases, the stationarity of the current-value Hamiltonian no longer implies a constant utility (consumption) level unless the net pure time effect also remains constant over time. Section 6 addresses the sustainability condition for a special case of non-autonomous problems; namely, when the utility discount rate is time dependent. We obtain a new result, showing the specific condition for the discount rate function that ensures Weitzman’s “stationary equivalence” result, Dixit et al.’s rule, and hence Solow-Hartwick’s rule, all carry over from autonomous problems to such non-autonomous cases. In turn, this new condition extends Michel’s (1982) result that in infinite horizon optimal control problems the maximum of the Hamiltonian converges to zero when time goes to infinity. Section 7 concludes.
2. The Exhaustible Resource Economy Revisited

Consider a purely exhaustible resource economy and, following Hotelling (1931), assume that: (i) it has a fully known and fixed initial stock of the resource of size $S_0 > 0$, (ii) the resource can be extracted costlessly, (iii) no technological change, (iv) population size remains constant, and (v) citizens’ preferences are identical and presented by the representative consumer’s utility function, $u(c)$, which is a twice differentiable, increasing, and strictly concave function of the resource consumption rate (i.e., $u'(c) > 0$, $u''(c) < 0$ for all $c \geq 0$), with $\lim_{c \to 0} u'(c) = +\infty$ and $\lim_{c \to 0} u'(c) = 0$. The utilitarian social planner uses a social welfare function defined as the discounted sum of the representative consumer’s utility flow and her objective is to plan a path of resource extraction and consumption that maximizes this social welfare function given the resource stock constraint. Formally, she plans to

$$\max_{c(t)} \int_0^{\infty} e^{-\rho t} u(c(t)) dt$$

subject to

$$s.t. \quad \dot{S}(t) = -c(t) \geq 0, \quad S(t) \geq 0, \quad S_0 \ (given)$$

where $\rho > 0$ is the social time preference rate, assumed constant. Assuming the constraint $S(t) \geq 0$ holds, the current-value Hamiltonian of this problem is

$$H(c(t), S(t), \lambda(t)) = u(c(t)) - \lambda(t)c(t)$$

where $\lambda(t)$ is the utility shadow price of the resource stock. The first-order conditions for an interior optimal path are

$$\frac{\partial H}{\partial c} = u'(c(t)) - \lambda(t) = 0$$

$$-\frac{\partial H}{\partial S} = 0 = \lambda(t) - \rho \lambda(t)$$

and the transversality condition

$$\lim_{t \to \infty} e^{-\rho t} \lambda(t) S(t) = 0$$

Differentiating (3) with respect to time, using (4), and denoting the elasticity of marginal utility of consumption by $\eta(c) = -\frac{c u''(c)}{u'(c)}$, the optimal consumption path is characterized by the familiar condition
\[
\frac{c(t)}{c(t)} = -\frac{\rho}{\eta(c)}
\]

(6)

It is immediate from (6) that, in general, the optimal policy for an exhaustible resource economy does not sustain a positive constant flow of consumption and hence utility. In fact, for the class of isoelastic utility function, \( u(c) = \frac{c^{1-\eta}}{1-\eta}, \quad 0 < \eta < \infty \), along the optimal path, the consumption level declines exponentially over time at the constant rate of \( \frac{\rho}{\eta} \). That is,

\[
c(t) = c(0) e^{-\frac{\rho}{\eta} t}
\]

(7)

where from the resource stock constraint \( \int c(t) dt = S_0 \) and (7) one obtains \( c(0) = \frac{\rho}{\eta} S_0 \), so that (7) can be rewritten as

\[
c(t) = \frac{\rho}{\eta} S_0 e^{-\frac{\rho}{\eta} t}, \quad \forall t \in [0, \infty)
\]

(8)

It is important to note that for an optimal policy to exist it is necessary that \( \rho > 0 \). In particular, in the limiting cases of no utility discounting, \( \rho = 0 \), or a pure egalitarian social welfare function where \( \eta \to \infty \), a positive constant consumption path \( (c(t) = c > 0, \forall t \geq 0) \), as implied by (6) for a general utility function, \( u(c) \), cannot be sustained permanently by an exhaustible resource economy. On the other hand, the constant zero consumption path \( (c(t) = 0, \forall t \geq 0) \) implied by (8) for these limiting cases and when the utility function is isoelastic is evidently not optimal.

3. Sustainability and Current-Value Hamiltonian

In his classic paper, Weitzman (1976) investigated the welfare significance of NNP for a dynamic competitive economy that produced a single composite consumption good by utilizing services of capital, defined broadly to include a set of stocks of exhaustible natural resources and various kinds of reproductive capital stocks. A basic insight from that paper is that in a dynamically optimizing economy, along the optimal path, the current-value Hamiltonian at time \( t \),
$H^*(t)$, is related to the optimal utilitarian welfare/consumption path, $u(c^*(\tau)), \tau \in (t, \infty)$, according to the following relationship\footnote{Since $u(c)$ is a single-valued, monotonic function of $c$, sustainability can be equivalently defined in terms of a constant utility or consumption flow. In fact, Weitzman assumed a linear utility function of the form $u(c(t))=c(t)$.}

\[
\int e^{-\rho(\tau-t)} H^*(t) \, d\tau = \frac{H^*(t)}{\rho} = \int e^{-\rho(\tau-t)} u(c^*(\tau)) \, d\tau \quad (9)
\]

Unfortunately, this relationship is often misunderstood by thinking that $H^*(t)$ measures the maximum sustainable level of utility (consumption). This misunderstanding becomes evident from a seeming paradox of the exhaustible resource economy analyzed in the previous section. For that economy, using (8), (4) and (3) in (2), it is easy to calculate that

$H^*(t) = \frac{\eta \rho S_e}{1-\eta} - \eta e^{-\rho(\frac{1}{\eta}-1)t}$

for $\eta < 1$. But, as was noted in the previous section, there is no sustainable positive consumption, and hence utility, level.

The explanation for this paradox lies in a correct understanding of what $H^*(t)$ precisely measures: in utility units, $H^*(t)$ is the “stationary equivalent” of the optimal welfare path.\footnote{Note that the utility units of $H^*(t)$ can be readily converted into real consumption units by choosing a dated utility numeraire such as $u^2(0)$ or generally $u^2(c(t))$ for any $t \geq 0$.} In other words, it is the hypothetical maximum constant utility/consumption path whose time-$t$ discounted value is equivalent to that of the (generally non-constant) optimal path, $u(c^*(\tau)), \tau \in (t, \infty)$. But, “stationary equivalence” does not mean “sustainability”. That is, it does not imply, as it is often misunderstood, that our economy can actually enjoy a constant utility/consumption equal to $H^*(t)$ forever.

For the latter to be the case, $H^*(t)$ must satisfy an additional condition: it must be time invariant (or stationary). Otherwise, it does not represent an actually sustainable constant consumption level.\footnote{The stationarity condition is also necessary and sufficient for time consistency of the optimal solution path; i.e., for the optimal policy to be a sub-game perfect Nash equilibrium of the intergenerational allocation game where each generation has to decide how much to consume and how much capital stock to} The important point to note is that even for autonomous optimal control problems, which characterize most of economic problems studied in the literature, the optimal current-value Hamiltonian need not be constant over time. In fact, for the economy analyzed in Weitzman (1976), which presents an example of such problems, we can prove the following
proposition, which to our knowledge has not been shown in the previous literature, at least not explicitly

**Proposition 1:** For Weitzman’s economy, the stationarity of the optimal current-value Hamiltonian is a necessary and sufficient condition for permanently sustaining a constant utility/consumption path.

**Proof:** Differentiating the second equation in (9) w.r.t. \( t \), and using (9) again, one has

\[
H^* (t) = \rho \left[ \rho \int e^{-\rho (\tau - t)} u(c(\tau)) d\tau - u(c(t)) \right] = \rho \left[ H^* (t) - u(c(t)) \right]
\]

**Sufficient condition:** recalling that \( u'(c) > 0, \forall c \geq 0 \), it immediately follows that

\[
H^* (t) = 0, \forall t \geq 0 \implies H^* (t) = H^* (\text{cons.}) = u(c(t)), \forall t \geq 0 \implies c(t) = u^{-1}(H^*) = \text{cons., } \forall t \geq 0
\]

**Necessary condition:** letting \( c(\tau) = \bar{c} \geq 0, \forall \tau \geq 0 \), so that \( u(c(\tau)) = u(\bar{c}) \geq 0, \forall \tau \geq 0 \), and performing the integral yields \( H^* (t) = 0, \forall t \geq 0 \).

In the special case of our exhaustible resource economy, it is easy to verify that

\[
H^* (t) = -\rho \left( \frac{\rho S_0}{\eta} \right)^{-\eta} e^{-\rho \left( \frac{1}{\eta} \right) t} < 0, \forall t \geq 0
\]

That is, the stationarity condition is not satisfied, thus confirming that there is no sustainable consumption (utility) path for that economy.

4. **Sustainability Condition: Generalization**

It is quite tempting to go beyond Proposition 1 to explore if the *stationarity* of \( H^* (t) \) is a general sustainability condition for any dynamically optimizing economy characterized by an infinite-horizon optimal control problem in which the instantaneous value function may take the most general form of \( u(c(t), s(t), t) \), where \( c(t) \) is the vector of \( n \) control variables \( c_i(t), i = 1, 2, ..., n \), \( s(t) \) is the vector of \( m \) state variables, \( s_j(t), j = 1, 2, ..., m \) and the differential equations constraints take the general form of \( s_j = g_j(c(t), s(t), t), \quad j = 1, 2, ..., m \). Obviously,
a dynamic economy so characterized is general enough to present almost any interesting case that one may come across in the literature. For example, it includes cases where the utility derives not only from consumption of goods but also from capital stocks (for instance, the amenity values of environmental and natural resource stocks). It also includes cases where there is population growth or technological change over time.

Formally, let us consider the general optimal control problem

\[
\text{Maximize } V = \int_0^\infty e^{-\rho t} u(c(t),s(t),t)dt \\
\text{s.t. } s_j = g_j(c(t),s(t),t), \quad j = 1,2,...,m, \\
\quad s_j(0) = s_{j0} \text{ (given)} \quad j = 1,2,...,m. 
\]  

Let \( c^*(t), s^*(t), \lambda^*(t) \) be the solution to this problem, where \( \lambda^*(t) \) is the vector of costate variables. Then the current-value Hamiltonian \( H(c,s,\lambda,t) = u(c,s,t) + \sum_{j=1}^m \lambda_j(t) g_j(c,s,t) \) is maximized along the optimal paths. In general, the total time derivative of the current-value Hamiltonian is (for notational convenience, superscript \( * \), denoting the optimal paths, is suppressed)

\[
\frac{dH}{dt} = \frac{\partial H}{\partial c} \cdot c + \sum_{j=1}^m \frac{\partial H}{\partial s_j} \cdot s_j + \sum_{j=1}^m \frac{\partial H}{\partial \lambda_j} \cdot \lambda_j 
\]  

Recalling that along the optimal path

\[
\frac{\partial H}{\partial c_i} \cdot c_i = 0, \forall i = 1,2,...,n 
\]  

(as either \( \frac{\partial H}{\partial c_i} = 0 \) for an interior solution or \( c_i = 0 \) for a boundary solution),

\[
-\frac{\partial H}{\partial s_j} = \lambda_j - \rho \lambda_j, \forall j = 1,2,...,m. 
\]  

5 Without loss of generality we could also have a set of, say \( r \), inequality constraints of the form \( g_k(c(t),s(t),t) \geq 0, k = 1,2,...,r \), and \( h \) equality constraints of the form \( g_j(c(t),s(t),t) = 0, l = 1,2,...,h \) on control variables, where these constraints would be assumed to satisfy the rank condition of the constraint qualifications; namely that the matrix (of order \( p,n \)) of partial derivatives of the \( p(>h) \) binding constraints with respect to control variables be of rank \( p \). For analytical convenience and to focus on the question at hand, we ignore these additional constraints and assume that the optimal control problems we are examining are all concave problems. In particular, we assume that the functions \( c(t), u \) and \( g_j \) satisfy all the continuity and differentiability conditions for the existence and uniqueness of solution to problem (10).
\[ s_j = \frac{\partial H}{\partial \lambda_j}, \forall j = 1, 2, \ldots, m \]  

(12c)

and substituting from (12a)-(12c) in (11), we have

\[
\frac{dH}{dt} = \frac{\partial H}{\partial t} + \rho \sum_{j=1}^{m} \lambda_j s_j
\]

(13)

Along an optimal path, equation (13) holds generally for both non-autonomous and autonomous cases, Weitzman’s economy being a special case of the latter. It enables us to state the following proposition, which has not appeared in the previous literature.

**Proposition 2:** For any dynamic economy characterized by an autonomous infinite-horizon control problem, the stationarity of the current-value Hamiltonian is a necessary and sufficient condition for sustainability of a constant utility path.

**Proof:** It suffices to show that Weitzman’s fundamental relationship (9) holds true for any autonomous infinite-horizon control problem, so that the proof of Proposition 1 can be invoked.

For an autonomous problem, the functions \( u \) or \( g \) take the form of \((u(t), s(t))\) and

\[ s_j = g_j(c(t), s(t)), \]

so the current-value Hamiltonian is

\[ H(c(t), s(t), \lambda(t)) = u(c(t), s(t)) + \sum_{j=1}^{m} \lambda_j(t)g_j(c(t), s(t)). \]

Since for such cases \( \frac{\partial H}{\partial t} = 0, \forall t \geq 0, \)

(11) reduces to

\[
\frac{dH}{dt} = \rho \sum_{j=1}^{m} \lambda_j s_j
\]

(13a)

Recalling that \( \dot{s}_j = g_j \), substituting (13a) in the expression for the optimal current-value Hamiltonian and rearranging terms yields the differential equation

\[
\frac{dH(t)}{dt} = \rho [H(t) - u(c(t), s(t))]
\]

(14)

which can be solved to give

\[
\int_{\tau} H(\tau) d\tau = \frac{H(\tau)}{\rho} = \int_{\tau} e^{-\rho(t-\tau)} u(c(t), s(t)) dt
\]

(15)
for any \( \tau \geq 0 \) along the optimal path. \( \square \)

**Remark 1**: It should be noted that in the general case of Proposition 2 where \( u(c(t), s(t)) \) is a vector-valued function of the flows of various consumption goods, sustainability is defined only in terms of a permanently constant utility path, and not of constant consumption paths. Accordingly, in invoking the proof of Proposition 1 only the constancy of utility flow is relevant.

**Remark 2**: Recalling that \( \sum_{j=1}^{m} \lambda_j s_j \) is the value of net aggregate investment along the optimal path at any time, it is interesting to note from (13a) that for any \( \rho > 0 \)

\[
\frac{dH}{dt} = 0, \forall t \geq 0 \iff \sum_{j=1}^{m} \lambda_j s_j = 0, \forall t \geq 0 \quad (13b)
\]

That is, our stationarity condition \( \frac{dH}{dt} = 0, \forall t \geq 0 \) for sustainability of autonomous dynamic economies generalizes the familiar “zero-net-aggregate-investment” rule which was originally derived by Dixit, Hammond, and Hoel (1980) only as a sufficient condition for sustainability (see also Solow (1986), Hartwick (1977) and Mäler (1991) among others). In turn, the latter rule generalized Solow-Hartwick’s sustainability rule of investing resources rents in a reproducible capital. It is important to note that our stationarity condition is both necessary and sufficient condition for sustaining a constant optimal utility path.

**Remark 3**: Interpreting the value of the integral \( W_t \equiv \int_0^\tau e^{-\rho(\tau-t)} u(c^*(\tau)) \, d\tau \) in (9), or its generalized version \( W_t \equiv \int_0^\tau e^{-\rho(\tau-t)} u(c(t), s(t)) \, dt \) for the class of time-autonomous economies in (15), as economy’s stock of “total wealth” (measured in utility units) at any time \( t \), we arrive at

---

6 Note that it is a necessary condition that along the optimal path \( H(t) \) is bounded above so that

\( \lim_{t \to \infty} e^{-\rho t} H(t) = 0 \), see Michel (1982).

7 Dixit, Hammond, and Hoel (1980) derived their sufficiency condition in a less general framework than that analyzed here, although they did not assume a constant discount rate. In section 6, we obtain the general sustainability condition when the discount rate is time dependent.

8 Obviously, in an economy with heterogeneous capital stocks if net aggregate investment is always positive, net national product and hence the optimal utility level can rise over time.

9 By a very different approach, Withagen and Asheim (1998) show that the converse of Solow-Hartwick’s rule (the necessary condition) holds in general for an economy with stationary preferences and technology.
another basic and familiar insight from Weitzman’s fundamental relationship (reflected by the second equality in (9), or from its generalized form here for autonomous economies (reflected by the second equality in (15). That is, along the optimal path, at any time the current-value Hamiltonian is the imputed “interest” on the economy’s stock of wealth (Solow (1986), Hartwick (1994) and others). Now, according to Proposition 2 for autonomous economies, only under the condition of stationarity of the current-value Hamiltonian \( \frac{dH}{dt} = 0, \forall t \geq 0 \), the utility level along the optimal path remains permanently constant \( u(c(t), s(t)) = \bar{u}, \forall t \geq 0 \), implying in turn that the value of wealth remains intact \( W_i = \bar{W}, \forall t \geq 0 \). In that case, the optimal current-value Hamiltonian may be interpreted as Hicksian income, in utility terms; that is, the maximum constant utility level (equal to interest on wealth, \( H = \rho \bar{W} = \bar{u} \)) that can be permanently sustained. It is important to reemphasize here that while for all autonomous economies the current-value Hamiltonian can be interpreted as interest on total wealth, it represents the sustainable constant utility (consumption) level if, and only if, it is time stationary. Unfortunately, the neglect of the latter condition in the literature has led to the common mistake of interpreting the current-value Hamiltonian as the sustainable constant utility (consumption) level. While under the specific assumptions of Weitzman’s model, the optimal current-value Hamiltonian at any time equals NNP, it does not, contrary to the prevailing mistaken belief, equal Hicksian income unless the current-value Hamiltonian is stationary.

Remark 4: In the special case of a purely exhaustible resource economy, since by definition there is no accumulable capital stock and since no optimal policy exists for \( \rho = 0 \), it follows from (13a) that

\[
\frac{dH}{dt} = \rho \dot{\lambda}(t) S(t) = -\rho \dot{\lambda}(t) c(t) < 0
\]

i.e., the stationarity condition for sustainability is never met and hence there exists no sustainable (positive) constant utility (consumption) level. This reconfirms and generalizes the result in the previous section for the isoelastic utility function. Note that, in fact, for such an economy, along the optimal path the level of well being declines over time.

5. Sustainability Condition: Non-autonomous Cases

We now return to problem (10) and invoke equation (13) to examine the sustainability condition for the more general case of time non-autonomous economies where at least one of the
functions \( u(c(t), s(t), t) \) or \( g_j(c(t), s(t), t) \) depends explicitly on \( t \). Examples of situations giving rise to non-autonomous cases include exogenous changes over time in population size, in taste and preferences, in the state of technology, in the rate of physical stock depreciation or growth (for instance, the decay of the CO\(_2\) stock in the atmosphere or growth of forest stocks with time, or additions to reserves of mineral deposits due to exogenous new discoveries).

As in problem (10), we continue to assume a constant discount rate \( \rho > 0 \). Thus, along an optimal path, one has

\[
\frac{\partial H}{\partial t} = \frac{\partial u}{\partial t} + \sum_{j=1}^{m} \lambda_j(t) \frac{\partial g_j}{\partial t}
\]

which measures the net change in the optimal current-value Hamiltonian at time \( t \) due purely to passage of time alone. We may term this as net “pure time effect”, which may be positive (for example in the case of exogenous technological progress alone) or negative (for example when there is exogenous population growth or when the rate of stock depreciation changes with time).

Recalling that \( s_j = g_j \) and substituting for \( \sum_{j=1}^{m} \lambda_j s_j \) from the Hamiltonian expression into (13), one has along the optimal path

\[
\frac{dH}{dt} = \frac{\partial H}{\partial t} + \rho [H(t) - u(t)]
\]

Solving the differential equation (17) yields, for any \( \tau \geq 0 \)

\[
\rho \int_{\tau}^{\infty} e^{-\rho(t-\tau)} H(\tau) dt = H(\tau) = \rho \int_{\tau}^{\infty} e^{-\rho(t-\tau)} u(t) dt - \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \frac{\partial H(t)}{\partial t} dt
\]

where \( \frac{\partial H}{\partial t} \) is given by (16).

Relationship (18) is a general result, leading to further important insights.

---

10 To be sure, several interesting special cases of this general result have been studied in the literature. For instance, Weitzman (1997), Weitzman and Löfgren (1997), and Hartwick and Long (1999) have studied the conditions of a constant consumption path when technology, output prices, or interest rates change exogenously over time. Also, in an insightful paper, Aronsson and Löfgren (1995) show how the optimal Hamiltonian value as a welfare measure is modified in the presence of an exogenous technological change, stock pollution externality, or stochastic production factors. The result furnished in (18) is, however, a more general and explicit one, embracing these and many other possible specific cases where the pure time effects are present.
First, since \( \frac{\partial H}{\partial t} = \frac{\partial u}{\partial t} + \sum_{j=1}^{m} \lambda_j(t) \frac{\partial g_j}{\partial t} \) is not identically equal to zero for all \( t \geq 0 \), the second integral on the RHS of (18) does not vanish for all \( \tau \geq 0 \), so that, on comparing (18) with (15) or with (9), we have

**Proposition 3:** The “stationary equivalence” property of the current value Hamiltonian (Weitzman’s fundamental relationship) can be generalized only for time-autonomous dynamic economies but does not hold for non-autonomous cases.

It then immediately follows from (18)

**Corollary 1:** The interpretation of the optimal current-value Hamiltonian as interest (return) on economy’s wealth (see Remark 3 above), and hence as NNP, does not hold for time non-autonomous economies. For these cases, at any time \( t \), the current-value Hamiltonian will under (over) estimate the true welfare level by an amount equal to the discounted value of the net “pure time effect” \( \int_0^\infty e^{-\rho(\tau-t)} \frac{\partial H(t)}{\partial t} d\tau \) if this effect is positive (negative).

Second, by (17), one has

\[
\frac{dH}{dt} = 0, \forall t \geq 0 \quad \Rightarrow u(c(t),s(t),t) = \overline{H} + \frac{1}{\rho} \frac{\partial H}{\partial t}, \forall t \geq 0
\]  

(18a)

So that,

**Corollary 2:** In contrast to the case of time-autonomous economies, for non-autonomous cases the stationarity of the current-value Hamiltonian is not a sufficient condition for sustainability of a constant utility (consumption) level unless in the exceptional case where the net “pure time effect”, \( \frac{\partial H}{\partial t} \), also remains constant (including 0) over time.

Third, it also follows from (13) that

**Corollary 3:** For time non-autonomous economies, Dixit et al.’s “zero-net-aggregate-investment” rule, and a fortiori Solow-Hardywick’s “resource-rent-investment” rule, is not a sufficient condition for sustaining a constant utility (consumption) path.

Notice that for the non-autonomous case, the stationarity of the current-value Hamiltonian implies that Dixit et al.’s “zero-net-aggregate-investment rule” needs to be modified according to

\[
\sum_{j=1}^{m} \lambda_j s_j = -\frac{1}{\rho} \frac{\partial H}{\partial t} = -\int_0^\infty e^{-\rho(\tau-t)} \frac{\partial H(t)}{\partial t} d\tau.
\]

Accordingly, at any time, the net aggregate investment can be negative (positive) as long as the disinvestment (investment) in aggregate capital stocks is exactly made up for by a constant positive (negative) flow of “pure
time effect” of equivalent (discounted) value. Roughly speaking, this means that the economy can afford to let its national wealth run down (and hence raise its consumption level) provided it enjoys a free (windfall) flow of benefits (for example due to exogenous technological progress) of the same discounted value. Conversely, it should optimally make up for exogenous losses (for example due to transboundary environmental externalities or an exogenous deterioration in its terms of trade) by building up the aggregate capital stock.

6. Sustainability Condition: Time-dependent Discount Rate

A special non-autonomous case is when the instantaneous discount rate $\rho(t)$ varies with time, so that, denoting by $\psi(t) \equiv \int_0^t \rho(s)\,ds$ the discount rate over the interval of time $(0,t]$, the discount factor at any time $t$ is $e^{-\psi(t)}$. As is familiar, in this case the current-value Hamiltonian expression remains as before but equations (12b) and (13) are modified as

$$\frac{\partial H}{\partial s_j} = \dot{\lambda}_j - \rho(t) \lambda_j, \forall j = 1,2,...,m$$

(19)

and

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \rho(t) \sum_{j=1}^m \dot{\lambda}_j s_j$$

(20)

Concentrating on cases where, as in the general autonomous problem, none of the functions $u$ or $g_j s$ depends explicitly on $t$, so that $\frac{\partial H}{\partial t} \equiv 0$, (20) simplifies to

$$\frac{dH}{dt} = \rho(t) \sum_{j=1}^m \dot{\lambda}_j s_j$$

(20a)

which is the analog of (13a) for the case of constant discount rate. Following the same steps leading to (14), one obtains the modified version of (14) as

$$\frac{dH(t)}{dt} - \rho(t) H(t) = -\rho(t) u(c(t),s(t))$$

(21)

Solving this differential equation yields for all $\tau \geq 0$

$$H(\tau) = \int_0^\tau \rho(t) [e^{-(\psi(t) - \psi(\tau))} u(c(t),s(t))]\,dt + \lim_{t \to \tau} [e^{-(\psi(t) - \psi(\tau))} H(t)]$$

(22)

Condition (22) establishes a new result in the literature and is important in two respects. Second, from a purely technical viewpoint, with a time-dependent utility discount rate, one can
no longer necessarily use the well-known result of Michel (1982), showing that the present value Hamiltonian corresponding to a well defined optimal control problem approaches zero when time goes to infinity. Instead, the result must be defined conditional on the assumption, which in the context of the present paper is equivalent to assuming that the sum of utility discount rates approaches infinity when time goes to infinity.

Two noteworthy points emerge from (20a), (21), and (22). First, it is noted from (22) that the assumption of bounded current-value Hamiltonian along the optimal path does not ensure that the second term on the RHS of (22) vanishes. For this to be the case, the instantaneous discount rate function $\rho(s)$ must satisfy the following condition

$$\lim_{t \to \infty} \psi(t) = \lim_{s \to \infty} \int \rho(s) ds = +\infty$$

(23)

Thus, from a purely technical viewpoint, with a time-dependent utility discount rate, one can no longer necessarily use the well-known result of Michel (1982), showing that the present value Hamiltonian corresponding to a well defined optimal control problem approaches zero when time goes to infinity. Instead, the result must be defined conditional on the assumption, which in the context of the present paper is equivalent to assuming that the sum of utility discount rates approaches infinity when time goes to infinity. Accordingly, condition (23) extends Michel’s result for the case of time-dependent discount rate.

Second, the economic interpretation of condition (22) reveals that, if the utility discount rate is time dependent, then in general the current-value Hamiltonian along the optimal trajectory no longer represents the discounted value of the imputed interest income (in utility terms), but that plus the limit of the Hamiltonian value as time approaches infinity. Consequently, for the time-dependent discount rate Proposition 3 is modified as

**Corollary 4:** When the discount rate varies with time, the “stationary equivalence” property of the current value Hamiltonian (i.e., Weitzman’s fundamental result generalized as

$$\int \rho(t) e^{-\psi(t)} H(t) dt = \int \rho(t) e^{-\psi(t)} u(c(t), s(t)) dt$$

holds if and only if the discount rate function satisfies the condition $\lim_{t \to \infty} \int \rho(s) ds = +\infty$.

This is an important result for it modifies the claims in the literature (see, for example, Svensson (1986) and Asheim (1994, P. 261)) that Weitzman’s fundamental result does not hold without the assumption of a constant utility discount rate. It shows that the result holds provided
the discount rate function satisfies the condition \( \lim_{t \to \infty} \int_s^t \rho(s) \, ds = +\infty \), which is obviously the case as long as the discount rate does not decline too fast with time. One example of such a discount rate function which has recently received considerable attention in the economic literature (see, e.g., Liabson (1996)(1997) among others) is the hyperbolic discount function. Presented generally in the form of \( \rho(t) = \frac{k_1}{1 + k_2 t} \), \((0 < k_1 < 1, k_2 > 0)\), it is readily checked that \( \lim_{t \to \infty} \int_s^t \rho(s) \, ds = \frac{k_1}{k_2} \lim_{t \to \infty} \ln(1 + k_2 t) = \infty \). Note that this condition is not satisfied, for example, by the exponentially declining function \( \rho(t) = k_1 e^{-k_2 t} \), for which \( \lim_{t \to \infty} \psi(t) = \frac{k_1}{k_2} \).

However, it should be noted that even if the utility discount rate function satisfies condition (23), the implied optimal sustainable consumption path will be time inconsistent (see, Strotz (1955-1956)), unless the social planner can somehow precommit to it.

Further, it is noted that the integral \( \int_t^\infty \rho(t)e^{-(\psi(t)-\psi(s))} u(c(t),s(t)) \, dt \) can no longer be interpreted as the interest on stock of wealth in the same precise sense as in the case of constant discount rate \( \rho(t) = \rho \) (see Remark 3 above), for it now presents the discounted value of the stream of interests on the optimal utility path. Thus, by (22) and (23), we can state

**Corollary 5:** When the discount rate varies with time, the optimal current-value Hamiltonian (or \( NNP \)) does not in general represent the interest on the economy’s wealth. It presents the discounted value of the flow of interest on the optimal utility path only if the discount rate function satisfies the condition \( \lim_{t \to \infty} \int_s^t \rho(s) \, ds = +\infty \).

This corollary has an important implication for green national accounting: while it cautions us against equating the interest on wealth as green \( NNP \) when the discount rate (or the consumption rate of interest) varies with time (as noted by Svensson (1986, 155), Hung (1993, p.381), and Asheim (1994, p. 261)), it also shows the condition under which such a practice would be valid.

Second, it easily follows from (20a) and (21) that

\[
\frac{dH}{dt} = 0, \quad \forall t \geq 0 \Rightarrow \sum_{j=1}^m \lambda_j \dot{s}_j = 0, \quad \forall t \geq 0, \quad \Rightarrow H(t) = \bar{H}(\text{cons}) = u(t), \forall t \geq 0
\]

That is, as in the case of constant discount rate, the stationarity of the current-value Hamiltonian , and hence the “zero-net-aggregate-investment” rule is still sufficient for sustainability of a
constant positive utility level (equal to the constant Hamiltonian value). However, contrary to the case of constant discount rate, the reverse is no longer generally true. This latter is seen by noting from (22) that for a constant utility flow, \( u(t) = \bar{u} > 0 \), one has for all \( \tau \geq 0 \) (recalling that \( \psi(t) = \rho(t) \))

\[
H(\tau) = \bar{u} + \lim_{t \to \infty} e^{-(\psi(t)-\psi(\tau))} (H(t)-\bar{u})
\]  

(24)

So that unless \( \bar{u} = \lim_{t \to \infty} H(t) \) or condition (23) is met, \( H(\tau) \neq \bar{u} \) for all \( \tau \geq 0 \), i.e., a constant utility level does not generally imply a constant current-value Hamiltonian (equal to the constant utility level). We can therefore state the following

**Proposition 4:** Even when the discount rate varies with time, the stationarity of the current-value Hamiltonian, and hence Dixit et al.’s “zero-net-aggregate-investment” rule, (a fortiori Solow-Hardwick’s “resource-rent-investment” rule) is still a sufficient condition for sustainability of a constant utility (consumption) path (equal to the optimal current-value Hamiltonian), but the converse is no longer true unless either \( \lim_{t \to \infty} \int \rho(s) \, ds = +\infty \) or \( \lim_{t \to \infty} H(t) = \bar{u} \).

According to the first part of Proposition 4, it is incorrect to think that Dixit et al.’s rule, or Solow-Hardwick’s rule, of sustainability is valid only if the utility discount rate is constant. The second part of the Proposition shows the specific condition under which the reverse of these rules also holds despite a variable discount rate. On both accounts, Proposition 4 weakens Svensson’s (1986, p.154, p.155) claim of the contrary. As we have seen, in general, for any autonomous problem, the stationarity of the current-value Hamiltonian is a sufficient condition for sustainability regardless of whether the discount rate is constant or time-dependent. But, while for a constant discount rate, the stationarity is also a necessary condition, for a time-dependent discount rate, it is so provided as time goes to infinity, either the discount factor approaches zero or the optimal Hamiltonian approaches the constant utility level. Obviously, these results also extend to Dixit et al’s and Solow-Hartwick’s rules.

7. Conclusions

This paper has scrutinized the fundamental relationships among the concepts of current-value Hamiltonian, sustainability, and NNP. Building on a body of insightful pioneering works, it has clarified some of the misconceptions surrounding these relationships in the green accounting literature, generalized and extended some of the previous basic results obtained in that literature for special cases, and provided new insights into the relationships.
Specifically, we have argued that contrary to common interpretation, the current–value Hamiltonian does not represent the maximum sustainable level of consumption (utility). We have shown generally that for any dynamic optimizing economy presented by an autonomous optimal control problem, a necessary and sufficient condition for sustainability in that sense is that the current-value Hamiltonian should be stationary over time. Accordingly, even when the optimal current-value Hamiltonian equals NNP, it is only under the stationarity condition that it can be interpreted as Hicksian income. For the more general case of time non-autonomous economies, characterized by exogenous changes in the economy over time, we have shown that the “stationary equivalence” property of the current-value Hamiltonian does not carry over, with two important implications. First, the optimal current-value Hamiltonian can no longer be interpreted as interest on the economy’s wealth, and hence as NNP. In fact, equating NNP with the current-value Hamiltonian will lead to an underestimation (overestimation) of the true level of well being if the net “pure time effect” is positive (negative). Second, the stationarity of the current-value Hamiltonian, and hence the “zero-net-aggregate-investment” rule, will no longer be a sufficient condition for permanently sustaining a constant utility (consumption) level. While these results pose conceptual and measurement difficulties for green national accounting, few economists may view continued exogenous changes, such as technological progress, population growth, preference shifts, or environmental externalities, as realistic possibilities. Interestingly, for one special, but important, non-autonomous case—namely, a time dependent discount rate— we have shown that the results obtained under the general autonomous case do prevail provided the discount rate function satisfies a certain mild condition, which is satisfied, for example, by a hyperbolic discount rate function.
References


NOTE DI LAVORO DELLA FONDAZIONE ENI ENRICO MATTEI
Fondazione Eni Enrico Mattei Working Papers Series
Our working papers are available on the Internet at the following addresses:

Server WWW: WWW.FEEM.IT
Anonymous FTP: FTP.FEEM.IT
http://papers.ssrn.com/abstract_id=XXXXXX

SUST 1.2001  Inge MAYERES and Stef PROOST: Should Diesel Cars in Europe be Discouraged?
SUST 2.2001  Paola DORIA and Davide PETTENELLA: The Decision Making Process in Defining and Protecting Critical Natural Capital
CLIM 3.2001  Alberto PENCH: Green Tax Reforms in a Computable General Equilibrium Model for Italy
CLIM 4.2001  Maurizio BUSSOLO and Dino PINELLI: Green Taxes; Environment, Employment and Growth
CLIM 5.2001  Marco STAMPINI: Tax Reforms and Environmental Policies for Italy
ETA 6.2001  Walid OUESLATI: Environmental Fiscal Policy in an Endogenous Growth Model with Human Capital
CLIM 7.2001  Umberto CIORBA, Alessandro LANZA and Francesco PAULI: Kyoto Commitment and Emission Trading: a European Union Perspective
MGMT 8.2001  Brian SLACK (xlv): Globalisation in Maritime Transportation: Competition, uncertainty and implications for port development strategy
VOL 9.2001  Giulia PESARO: Environmental Voluntary Agreements: A New Model of Co-operation Between Public and Economic Actors
ETA 11.2001  Sergio CURRARINI and Marco MARINI: A Sequential Approach to the Characteristic Function and the Core in Games with Externalities
ETA 12.2001  Gaetano BLOISE, Sergio CURRARINI and Nicholas KIKIDIS: Inflation and Welfare in an OLG Economy with a Privately Provided Public Good
KNOW 13.2001  Paolo SURICO: Globalisation and Trade: A "New Economic Geography" Perspective
ETA 14.2001  Valentina BOSETTI and Vincenzo MESSINA: Quasi Option Value and Irreversible Choices
CLIM 15.2001  Guy ENGELEN (xlii): Desertification and Land Degradation in Mediterranean Areas: from Science to Integrated Policy Making
SUST 16.2001  Julie Catherine SORS: Measuring Progress Towards Sustainable Development in Venice: A Comparative Assessment of Methods and Approaches
SUST 17.2001  Julie Catherine SORS: Public Participation in Local Agenda 21: A Review of Traditional and Innovative Tools
CLIM 18.2001  Johan ALBRECHT and Niko GOBRIN: Schumpeter and the Rise of Modern Environmentalism
VOL 19.2001  Rinaldo BRAU, Carlo CARRARO and Giulio GOLFETTO (xliii): Participation Incentives and the Design of Voluntary Agreements
ETA 20.2001  Paola ROTA: Dynamic Labour Demand with Lumpy and Kinked Adjustment Costs
ETA 21.2001  Paola ROTA: Empirical Representation of Firms' Employment Decisions by an (S,s) Rule
ETA 22.2001  Paola ROTA: What Do We Gain by Being Discrete? An Introduction to the Econometrics of Discrete Decision Processes
PRIV 23.2001  Stefano BOSI, Guillaume GIRMANS and Michel GUILLARD: Optimal Privatisation Design and Financial Markets
KNOW 24.2001  Giorgio BRUNELLO, Claudio LUPI, Patrizia ORDINE, and Maria Luisa PARISI: Beyond National Institutions: Labour Taxes and Regional Unemployment in Italy
ETA 25.2001  Klaus CONRAD: Locational Competition under Environmental Regulation when Input Prices and Productivity Differ
CLIM 27.2001  Frédéric BROCHIER and Emiliano RAMIERI: Climate Change Impacts on the Mediterranean Coastal Zones
ETA 28.2001  Nunzio CAPPUCCIO and Michele MORETTO: Comments on the Investment-Uncertainty Relationship in a Real Option Model
KNOW 29.2001  Giorgio BRUNELLO: Absolute Risk Aversion and the Returns to Education
CLIM 30.2001  ZhongXiang ZHANG: Meeting the Kyoto Targets: The Importance of Developing Country Participation
ETA 31.2001  Jonathan D. KAPLAN, Richard E. HOWITT and Y. Hossein FARZIN: An Information-Theoretical Analysis of Budget-Constrained Nonpoint Source Pollution Control
Coalition Theory Network
ETA 33.2001  Shlomo WEBER and Hans WIESMETH: From Autarky to Free Trade: The Impact on Environment
ETA 34.2001  Margarita GENIUS and Elisabetta STRAZZERA: Model Selection and Tests for Non Nested Contingent Valuation Models: An Assessment of Methods
Carlo GIUPPONI: The Substitution of Hazardous Molecules in Production Processes: The Atrazine Case Study in Italian Agriculture

Raffaele PACI and Francesco PIGLIARU: Technological Diffusion, Spatial Spillovers and Regional Convergence in Europe

Bernardo BORTOLOTTI: Privatisation, Large Shareholders, and Sequential Auctions of Shares


Giacomo CALZOLARI and Carlo SCARPA: Regulation at Home, Competition Abroad: A Theoretical Framework

Giorgio BRUNELLO: On the Complementarity between Education and Training in Europe

Alain DESDOIGTS and Fabien MOIZEAU: Technological Diffusion, Spatial Spillovers and Regional Convergence in Europe

Wietze LISE and Richard S.J. TOL and Bob van der ZWAAN (xlvi): Negotiating Climate Change as a Social Situation

Mohammad R. KHAWLIE (xlvii): The Impacts of Climate Change on Water Resources of Lebanon-Eastern Mediterranean

Mutase EL-FADEL and E. BOU-ZEID (xlvii): Climate Change and Water Resources in the Middle East: Vulnerability, Socio-Economic Impacts and Adaptation

Eva IGLESIAS, Alberto GARRIDO and Almudena GOMEZ (xlvii): An Economic Drought Management Index to Evaluate Water Institutions’ Performance Under Uncertainty and Climate Change

Wietze LISE and Richard S.J. TOL (xlviii): Impact of Climate on Tourist Demand

Francesco BOSELLO, Barbara BUCHNER, Carlo CARRARO and Davide RAGGI: Can Equity Enhance Efficiency? Lessons from the Kyoto Protocol

Roberto ROSON (xlviii): Carbon Leakage in a Small Open Economy with Capital Mobility

Edwin WOERDMAN (xlviii): Developing a European Carbon Trading Market: Will Permit Allocation Distort Competition and Lead to State Aid?

Richard N. COOPER (xlviii): The Kyoto Protocol: A Flawed Concept

Kari KANGAS (xlviii): Trade Liberalisation, Changing Forest Management and Roundwood Trade in Europe

Xueqin ZHU and Ekko VAN IERLAND (xlviii): Effects of the Enlargement of EU on Trade and the Environment

M. Ozgur KAYALICA and Sajal LAHIRI (xlviii): Strategic Environmental Policies in the Presence of Foreign Direct Investment

Savas ALPAY (xlviii): Can Environmental Regulations be Compatible with Higher International Competitiveness? Some New Theoretical Insights

Roldan MURADIAN, Martin O’CONNOR, Joan MARTINEZ-ALER (xlviii): Embodied Pollution in Trade: Estimating the “Environmental Load Displacement” of Industrialised Countries

Matthew R. AUER and Rafael REUVENY (xlviii): Foreign Aid and Direct Investment: Key Players in the Environmental Restoration of Central and Eastern Europe

Onno J. KUIK and Frans H. OOSTERHUIS (xlviii): Lessons from the Southern Enlargement of the EU for the Environmental Dimensions of Eastern Enlargement, in particular for Poland

Carlo CARRARO, Alessandra POME and Domenico SINISCALCO (xlxi): Science vs. Profit in Research: Lessons from the Human Genome Project

Efrem CASTELNUOVO, Michele MORETTO and Sergio VERGALLI: Global Warming, Uncertainty and Endogenous Technical Change: Implications for Kyoto

Gian Luigi ALBANO, Fabrizio GERMANO and Stefano LOVO: On Some Collusive and Signaling Equilibria in Ascending Auctions for Multiple Objects

Elbert DIJKGRAAF and Herman R.J. VOLLEBERGH: A Note on Testing for Environmental Kuznets Curves with Panel Data

Paolo BUONANNO, Carlo CARRARO and Marzio GALEOTTI: Endogenous Induced Technical Change and the Costs of Kyoto

Guido CAZZÁVILLAN and Ignazio MUSU (1): Transitional Dynamics and Uniqueness of the Balanced-Growth Path in a Simple Model of Endogenous Growth with an Environmental Asset

Giovanni BÀIOCCHI and Salvatore DI FALCO (1): Investigating the Shape of the EKC: A Nonparametric Approach

Marzio GALEOTTI, Alessandro LANZA and Francesco PAULI (1): Desperately Seeking (Environmental) Kuznets: A New Look at the Evidence

Alexey VIKHYTAEV (xlviii): The Use of Trade Measures for Environmental Purposes – Globally and in the EU Context

Gary D. LIBECAP and Zeynep K. HANSEN (li): U.S. Land Policy, Property Rights, and the Dust Bowl of the 1930s

Barbara BUCHNER

On the Complementarity between Education and Training in Europe

Alain DESDOIGTS and Fabien MOIZEAU: Technological Diffusion, Spatial Spillovers and Regional Convergence in Europe

Wietze LISE and Richard S.J. TOL: Impact of Climate on Tourist Demand

Francesco BOSELLO, Barbara BUCHNER, Carlo CARRARO and Davide RAGGI: Can Equity Enhance Efficiency? Lessons from the Kyoto Protocol

Roberto ROSON (xlviii): Carbon Leakage in a Small Open Economy with Capital Mobility

Edwin WOERDMAN (xlviii): Developing a European Carbon Trading Market: Will Permit Allocation Distort Competition and Lead to State Aid?

Richard N. COOPER (xlviii): The Kyoto Protocol: A Flawed Concept

Kari KANGAS (xlviii): Trade Liberalisation, Changing Forest Management and Roundwood Trade in Europe

Xueqin ZHU and Ekko VAN IERLAND (xlviii): Effects of the Enlargement of EU on Trade and the Environment

M. Ozgur KAYALICA and Sajal LAHIRI (xlviii): Strategic Environmental Policies in the Presence of Foreign Direct Investment

Savas ALPAY (xlviii): Can Environmental Regulations be Compatible with Higher International Competitiveness? Some New Theoretical Insights

Roldan MURADIAN, Martin O’CONNOR, Joan MARTINEZ-ALER (xlviii): Embodied Pollution in Trade: Estimating the “Environmental Load Displacement” of Industrialised Countries

Matthew R. AUER and Rafael REUVENY (xlviii): Foreign Aid and Direct Investment: Key Players in the Environmental Restoration of Central and Eastern Europe

Onno J. KUIK and Frans H. OOSTERHUIS (xlviii): Lessons from the Southern Enlargement of the EU for the Environmental Dimensions of Eastern Enlargement, in particular for Poland

Carlo CARRARO, Alessandra POME and Domenico SINISCALCO (xlxi): Science vs. Profit in Research: Lessons from the Human Genome Project

Efrem CASTELNUOVO, Michele MORETTO and Sergio VERGALLI: Global Warming, Uncertainty and Endogenous Technical Change: Implications for Kyoto

Gian Luigi ALBANO, Fabrizio GERMANO and Stefano LOVO: On Some Collusive and Signaling Equilibria in Ascending Auctions for Multiple Objects

Elbert DIJKGRAAF and Herman R.J. VOLLEBERGH: A Note on Testing for Environmental Kuznets Curves with Panel Data

Paolo BUONANNO, Carlo CARRARO and Marzio GALEOTTI: Endogenous Induced Technical Change and the Costs of Kyoto

Guido CAZZÁVILLAN and Ignazio MUSU (1): Transitional Dynamics and Uniqueness of the Balanced-Growth Path in a Simple Model of Endogenous Growth with an Environmental Asset

Giovanni BÀIOCCHI and Salvatore DI FALCO (1): Investigating the Shape of the EKC: A Nonparametric Approach

Marzio GALEOTTI, Alessandro LANZA and Francesco PAULI (1): Desperately Seeking (Environmental) Kuznets: A New Look at the Evidence

Alexey VIKHYTAEV (xlviii): The Use of Trade Measures for Environmental Purposes – Globally and in the EU Context

Gary D. LIBECAP and Zeynep K. HANSEN (li): U.S. Land Policy, Property Rights, and the Dust Bowl of the 1930s
Lee J. ALSTON, Gary D. LIBECAP and Bernardo MUHELLER (li): Land Reform Policies, The Sources of Violent Conflict and Implications for Deforestation in the Brazilian Amazon

Claudia KEMFTERT: Economy-Energy-Climate Interaction – The Model WIAGEM -

Paulo A.L.D. NUNES and Jovanes E. RYANTO: Policy Instruments for Creating Markets for Biodiversity: Certification and Ecolabeling

Paulo A.L.D. NUNES and Erik SCHOKKAERT (lii): Warm Glow and Embedding in Contingent Valuation

Paulo A.L.D. NUNES, Jeroen C.J.M. van den BERGH and Peter NIJKAMP (lii): Ecological-Economic Analysis and Valuation of Biodiversity

Johan EYCKMANS and Henry TULKENS (li): Simulating Coalitionally Stable Burden Sharing Agreements for the Climate Change Problem


Bernardo BORTOLOTTI, Marcella FANTINI and Domenico SINISCALCO: Privatisation around the World: New Evidence from Panel Data

Toke S. AIDT and Jayasri DUTTA (li): Transitional Politics. Emerging Incentive-based Instruments in Environmental Regulation

Alberto PETRUCI: Consumption Taxation and Endogenous Growth in a Model with New Generations

Pierre LASSERRE and Antoine SOUBEYRAN: Elena BONTEMPI, Alessandra DEL BOCA, Alessandra FRAN: Economic Evaluation of Smallholder Subsistence Livestock Production: Lessons from an Ethiopian Goat Development Program

Riccardo SCARPA, Adam DRUCKER, Simon ANDERSON, Nancy FERRAES-MICHELE MORETTO and Gianpaolo ROSSINI: 

Claudia KEMFTERT: A Ricardian Model of the Tragedy of the Commons

SUST 72.2001: On the Consequences of the U.S. Withdrawal from the Kyoto/Bonn Protocol

Clemens B.A. WOLNY: The Need to Conserve Farm Animal Genetic Resources Through Community-Based Management in Africa: Should Policy Makers be Concerned?


Gianni CICIA, Elisabetta D’ERCOLE and Davide MARINO: Valuing Farm Animal Genetic Resources by Means of Contingent Valuation and a Bio-Economic Model: The Case of the Pentro Horse

Clem TISCELL: Socioeconomic Causes of Loss of Animal Genetic Diversity: Analysis and Assessment

M.A. JABBAR and M.L. DIEDHOU: Does Breed Matter to Cattle Farmers and Buyers? Evidence from West Africa

K. TANO, M.D. FAMINOW, M. KAMUANGA and B. SWALLOW: Using Conjoint Analysis to Estimate Farmers’ Preferences for Cattle Traits in West Africa

Efrem CASTELNUOVO and Paolo SURICO: What Does Monetary Policy Reveal about Central Bank’s Preferences?

Duncan KNOWLER and Edward BARBIER: The Economics of a “Mixed Blessing” Effect: A Case Study of the Black Sea

Andreas LÖSCHEL: Technological Change in Economic Models of Environmental Policy: A Survey

Carlo CARRARO and Carmen MARCHIORI: Stable Coalitions

Marzo GALEOTTI, Alessandro LANZA and Matteo MANERA: Rockets and Feathers Revisited: An International Comparison on European Gasoline Markets

Effrosyni DIAMANTOUDI and Efthichios S. SARTZETAKIS: Stable International Environmental Agreements: An Analytical Approach

Alain DESDOIGTS: Neoclassical Convergence Versus Technological Catch-up: A Contribution for Reaching a Consensus

Giuseppe DI VITA: Renewable Resources and Waste Recycling

Giorgio BRUNELLO: Is Training More Frequent when Wage Compression is Higher? Evidence from 11 European Countries

Mordecai KURZ, Hehui JIN and Maurizio MOTOLESE: Endogenous Fluctuations and the Role of Monetary Policy

Reyer GERLAGH and Marjan W. HOFKES: Escaping Lock-in: The Scope for a Transition towards Sustainable Growth?

Michele MORETTO and Paolo ROSATO: The Use of Common Property Resources: A Dynamic Model

Philippe QUIRION: Macroeconomic Effects of an Energy Saving Policy in the Public Sector

Roberto ROSON: Dynamic and Distributional Effects of Environmental Revenue Recycling Schemes: Simulations with a General Equilibrium Model of the Italian Economy

Francesco RICCI (liv): Environmental Policy Growth when Inputs are Differentiated in Pollution Intensity

Alberto PETRUCCI: Devaluation (Levels versus Rates) and Balance of Payments in a Cash-in-Advance Economy

László A. KÓCZY (liv): The Core in the Presence of Externalities

Steven J. BRAMS, Michael A. JONES and D. Marc KILGOUR (liv): Single-Peakedness and Disconnected Coalitions

Guillaume HAERINGER (liv): On the Stability of Cooperation Structures

Fausto CAVALLARO and Luigi CIRAOLO: Economic and Environmental Sustainability: A Dynamic Approach

Barbara BUCHNER, Carlo CARRARO, Igor CERSOSIMO and Carmen MARCHIORI: Back to Kyoto? US Participation and the Linkage between R&D and Climate Cooperation

Andreas LÖSCHEL and ZhongXIANG ZHANG: The Economic and Environmental Implications of the US Repudiation of the Kyoto Protocol and the Subsequent Deals in Bonn and Marrakech

Hannes EGLI: Are Cross-Country Studies of the Environmental Kuznets Curve Misleading? New Evidence from Time Series Data for Germany

Adam B. JAFFE, Richard G. NEWELL and Robert N. STAVINS: Environmental Policy and Technological Change

Joseph C. COOPER and Giovanni SIGNORELLO: Farmer Premiums for the Voluntary Adoption of Conservation Plans

The ANSEA Network: Towards An Analytical Strategic Environmental Assessment

Paolo SURICO: Geographic Concentration and Increasing Returns: a Survey of Evidence

Robert N. STAVINS: Lessons from the American Experiment with Market-Based Environmental Policies

Carlo GIUPPONI and Paolo ROSATO: Technological Change in Economic Models of Environmental Policy: A Survey

G. BRUNELLO, M.L. PARISI and Daniela SONEDDA: Labor Taxes, Wage Setting and the Relative Wage Effect

<table>
<thead>
<tr>
<th>Volume</th>
<th>Year</th>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLIM</td>
<td>36.2002</td>
<td>T. Tietenberg</td>
<td>The Tradable Permits Approach to Protecting the Commons: What Have We Learned?</td>
</tr>
<tr>
<td>CLIM</td>
<td>38.2002</td>
<td>C. Fischer</td>
<td>Multinational Taxation and International Emissions Trading</td>
</tr>
<tr>
<td>NRM</td>
<td>40.2002</td>
<td>S. M. Cavanagh, W. M. Hanemann and R. N. Stavins</td>
<td>Muffled Price Signals: Household Water Demand under Increasing-Block Prices</td>
</tr>
<tr>
<td>NRM</td>
<td>41.2002</td>
<td>A. J. Plantinga, R. N. Lubowski and R. N. Stavins</td>
<td>The Effects of Potential Land Development on Agricultural Land Prices</td>
</tr>
<tr>
<td>CLIM</td>
<td>42.2002</td>
<td>C. Ohl</td>
<td>Inducing Environmental Co-operation by the Design of Emission Permits</td>
</tr>
<tr>
<td>CLIM</td>
<td>43.2002</td>
<td>J. Eyckmans, D. Van Regemorter and V. Van Steenberghe</td>
<td>Is Kyoto Fatally Flawed? An Analysis with MacGEM</td>
</tr>
<tr>
<td>CLIM</td>
<td>44.2002</td>
<td>A. Antoci and S. Borghesi</td>
<td>Working Too Much in a Polluted World: A North-South Evolutionary Model</td>
</tr>
<tr>
<td>ETA</td>
<td>45.2002</td>
<td>Per G. Fredriksson, John A. List and Daniel L. Millimet</td>
<td>Chasing the Smokestack: Strategic Policymaking with Multiple Instruments</td>
</tr>
<tr>
<td>CLIM</td>
<td>46.2002</td>
<td>Zhihao Yu</td>
<td>A Theory of Strategic Vertical FDI and the Missing Pollution-Haven Effect</td>
</tr>
<tr>
<td>SUST</td>
<td>47.2002</td>
<td>Y. Hossein Farzin</td>
<td>Can an Exhaustible Resource Economy Be Sustainable?</td>
</tr>
<tr>
<td>SUST</td>
<td>48.2002</td>
<td>Y. Hossein Farzin</td>
<td>Sustainability and Hamiltonian Value</td>
</tr>
</tbody>
</table>
This paper was presented at the International Workshop on "Climate Change and Mediterranean Coastal Systems: Regional Scenarios and Vulnerability Assessment" organised by the Fondazione Eni Enrico Mattei in co-operation with the Istituto Veneto di Scienze, Lettere ed Arti, Venice, December 9-10, 1999.

This paper was presented at the International Workshop on “Voluntary Approaches, Competition and Competitiveness” organised by the Fondazione Eni Enrico Mattei within the research activities of the CAVA Network, Milan, May 25-26, 2000.

This paper was presented at the International Workshop on “Green National Accounting in Europe: Comparison of Methods and Experiences” organised by the Fondazione Eni Enrico Mattei within the Concerted Action of Environmental Valuation in Europe (EVE), Milan, March 4-7, 2000

This paper was presented at the International Workshop on “New Ports and Urban and Regional Development. The Dynamics of Sustainability” organised by the Fondazione Eni Enrico Mattei, Venice, May 5-6, 2000.

This paper was presented at the International Workshop on “Voluntary Approaches, Competition and Competitiveness” organised by the Fondazione Eni Enrico Mattei within the research activities of the CAVA Network, Milan, May 25-26, 2000.

This paper was presented at the International Workshop on “Voluntary Approaches, Competition and Competitiveness” organised by the Fondazione Eni Enrico Mattei within the research activities of the CAVA Network, Milan, May 25-26, 2000.

This paper was presented at the Sixth Meeting of the Coalition Theory Network organised by the Fondazione Eni Enrico Mattei and the CORE, Université Catholique de Louvain, Louvain-la-Neuve, Belgium, January 26-27, 2001

This paper was presented at the RICAMARE Workshop “Socioeconomic Assessments of Climate Change in the Mediterranean: Impact, Adaptation and Mitigation Co-benefits”, organised by the Fondazione Eni Enrico Mattei, Milan, February 9-10, 2001

This paper was presented at the International Workshop “Trade and the Environment in the Perspective of the EU Enlargement”, organised by the Fondazione Eni Enrico Mattei, Milan, May 17-18, 2001

This paper was presented at the International Conference “Knowledge as an Economic Good”, organised by Fondazione Eni Enrico Mattei and The Beijer International Institute of Environmental Economics, Palermo, April 20-21, 2001

(i) This paper was presented at the Workshop “Growth, Environmental Policies and Sustainability” organised by the Fondazione Eni Enrico Mattei, Venice, June 1, 2001

(ii) This paper was presented at the Fourth Toulouse Conference on Environment and Resource Economics on “Property Rights, Institutions and Management of Environmental and Natural Resources”, organised by Fondazione Eni Enrico Mattei, IDEI and INRA and sponsored by MATE, Toulouse, May 3-4, 2001

(iii) This paper was presented at the International Conference on “Economic Valuation of Environmental Goods”, organised by Fondazione Eni Enrico Mattei in cooperation with CORILA, Venice, May 11, 2001

(iii) This paper was circulated at the International Conference on “Climate Policy – Do We Need a New Approach?”, jointly organised by Fondazione Eni Enrico Mattei, Stanford University and Venice International University, Isola di San Servolo, Venice, September 6-8, 2001

(iv) This paper was presented at the Seventh Meeting of the Coalition Theory Network organised by the Fondazione Eni Enrico Mattei and the CORE, Université Catholique de Louvain, Venice, Italy, January 11-12, 2002

(iv) This paper was presented at the First Workshop of the Concerted Action on Tradable Emission Permits (CATEP) organised by the Fondazione Eni Enrico Mattei, Venice, Italy, December 3-4, 2001

(vi) This paper was presented at the ESF EURESCO Conference on Environmental Policy in a Global Economy “The International Dimension of Environmental Policy”, organised with the collaboration of the Fondazione Eni Enrico Mattei, Acquafrredda di Maratea, October 6-11, 2001.
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Editor</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLIM</td>
<td>Climate Change Modelling and Policy</td>
<td>Marzio Galeotti</td>
</tr>
<tr>
<td>VOL</td>
<td>Voluntary and International Agreements</td>
<td>Carlo Carraro</td>
</tr>
<tr>
<td>SUST</td>
<td>Sustainability Indicators and Environmental Evaluation</td>
<td>Carlo Carraro</td>
</tr>
<tr>
<td>NRM</td>
<td>Natural Resources Management</td>
<td>Carlo Giupponi</td>
</tr>
<tr>
<td>KNOW</td>
<td>Knowledge, Technology, Human Capital</td>
<td>Dino Pinelli</td>
</tr>
<tr>
<td>MGMT</td>
<td>Corporate Sustainable Management</td>
<td>Andrea Marsanich</td>
</tr>
<tr>
<td>PRIV</td>
<td>Privatisation, Regulation, Antitrust</td>
<td>Bernardo Bortolotti</td>
</tr>
</tbody>
</table>