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Exchange-Rate Overshooting: An Analysis for Intermediate Macro

by

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Exchange-Rate Overshooting: An Analysis for Intermediate Macro

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Abstract

While exchange rate dynamics is an important topic in open economy macroeconomics, the standard tool commonly used to introduce exchange rate dynamics - the Dornbusch (1976) seminal paper along with phase diagram - is not well-suited for undergraduate students as most of them do not have yet a background on dynamic macroeconomic analysis.

This paper attempts to provide a graphical device – a panel IS*-LM* diagram – which can be used to teach intermediate macroeconomics students about Dornbusch’s idea of exchange rate dynamics. In addition, it also attempts to bridge the gap between undergraduate teaching and graduate teaching of exchange rate dynamics by showing the correspondence between the economy’s adjustment path in the IS*-LM* diagram and that in the phase diagram.

Key words: undergraduate teaching, graduate teaching, exchange rates, exchange rate dynamics, sticky prices, interest parity, open economy macroeconomics, fiscal policy, monetary policy.

JEL classification codes: A150, A230, F310, F410

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1. Introduction

Exchange-rate overshooting is an important phenomenon affecting households, businesses, and governments. If intermediate-level macroeconomics students are to be prepared in understanding the policy issues concerning this phenomenon, they must be provided with a tool to understand the mechanics of exchange-rate dynamics.

The pioneering and the most popular work in the literature on overshooting is the Dornbusch (1976) paper. It is an important and interesting extension of the Mundell-Fleming model\(^1\), a static small open economy \( IS-LM \) model, as it allows for price adjustment over time and rational expectations formation.\(^2\)

Dornbusch (1976, text) assumes perfect capital mobility and substitutability which together imply uncovered interest rate parity (UIP), a regressive exchange-rate expectations scheme that can be made consistent with perfect foresight\(^3\), a money demand function and equilibrium in money market, fixed income even in the short run, a demand for domestic goods function, sluggish price adjustment that depends on a measure of transitory excess demand for domestic goods over fixed income, and a small economy under flexible exchange rates which takes foreign variables as given. Given these assumptions, Dornbusch shows that when income is fixed and supply-determined even in the short-run and when the price level is a non-jumping or sluggish variable, the exchange rate will unambiguously overshoot its new long-run equilibrium value in response to an unanticipated increase in the money supply.

With the price level sticky and income fixed, an increase in the money supply creates an

\(^1\) The Mundell-Fleming model is one of the staples in intermediate macroeconomics. Also, see Boyer and Young (2010).

\(^2\) The Dornbusch model is also referred to as the Mundell-Fleming-Dornbusch model (for instance, see Obstfeld and Rogoff (1996)).

\(^3\) Perfect foresight is the deterministic/certainty analogue of rational expectations.
excess supply at the initial domestic interest rate, and, consequently, the domestic interest rate must fall to maintain money market equilibrium. At the same time, an increase in the money supply causes the expectation of depreciation of the steady-state exchange rate, since there is perfect foresight. The fall in the domestic interest rate causes incipient capital inflows, which in turn causes the exchange rate to actually depreciate. To compensate holders of domestic assets for the fall in the domestic interest rate, the instantaneous exchange rate depreciation must exceed the long-run depreciation, i.e., the exchange rate must overshoot its long-run equilibrium value. Thus, in an interdependent system where the price level is constrained not to move immediately in response to some disturbance, it is the exchange rate and other jump-variables that bear the burden of instantaneous adjustment.

However, when income is demand-determined and varies in the short run, exchange-rate overshooting is no longer an inevitable consequence of unexpected monetary expansion, as shown in Dornbusch (1976, appendix). Specifically, since an increase in the money supply temporarily stimulates income, and an increase in income in turn causes an increase in money demand, the decrease in the domestic interest rate necessary to maintain money market equilibrium is dampened. Thus, the instantaneous exchange-rate adjustment is reduced. If income expansion is sufficiently high, then the domestic interest rate rises and the instantaneous exchange-rate response is that of undershooting.

The essence of the literature (or the intuition behind exchange-rate overshooting) is best described by Cooper (1985, pp. vii-viii): “… that exchange rates can overshoot their long-term equilibrium values in response to some new disturbance. It is yet another illustration in economics of Le Chatelier’s principle, that when flexibility is constrained in some part of an interdependent system… those variables that are flexible will respond at first to an even greater extent than they would if constrained variables were also freely flexible.” In Dornbusch (1976), the variable which is constrained to adjust immediately is the price level and one of the variables
that can adjust freely is the exchange rate.

While exchange rate dynamics is an important topic in open economy macroeconomics, the standard tool commonly used to teach exchange rate dynamics (the Dornbusch (1976) model) - the phase diagram - is not well-suited for undergraduate students as most of them do not have yet a background on dynamic macroeconomic analysis.

Thus, the main objective of this paper is to attempt to provide a graphical exposition of the Dornbusch (1976, appendix) model. Specifically, it will attempt to provide a graphical device - a panel diagram, where the first three panels not only describe the three markets (foreign exchange market, money market, and goods market) in the model but also show the underpinning of the last panel (the so-called IS*-LM* diagram) - which can be used to teach intermediate macroeconomics students about exchange rate dynamics. The advantage of a panel diagram is that, given a disturbance, one would easily see the dynamic (short-run, transitional, and long-run) effects on the different markets as well as on different variables.  

This paper will also attempt to bridge the gap between undergraduate teaching and graduate teaching of exchange rate dynamics by showing the correspondence between the economy’s adjustment path in the IS*-LM* diagram and that in the phase diagram. In particular, it will show that the transitional adjustment path in the IS*-LM* model is the locus of IS*-LM* intersections as the price level changes over time and that such path corresponds to the transitional adjustment path, the saddle path, in a phase diagram.

The remainder of this paper is organized as follows. Section 2 first presents the building blocks of the Dornbusch (1976, appendix) model and shows their corresponding graphical representation – a panel IS*-LM* diagram - and then discusses the dynamic properties of the

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4 Another four-panel diagram - consisting of foreign exchange market diagram, PPP diagram, IS-LM diagram, and yd–ys diagram - is that of Harvey (2007). The PPP diagram does not seem to be necessary because whether PPP holds can be inferred from the steady-state properties while the yd–ys diagram is not a good representation of price adjustment over time.
model. Section 3 uses the panel $IS^*-LM^*$ diagram to examine the effects of a disturbances – monetary expansion and fiscal expansion. Sections 4 and 5 show that the results of the guess-and-verify method used in section 2 are the same as those when perfect foresight is imposed directly but that it is advantageous to employ a regressive expectations scheme as a guess about exchange rate expectations and verify it only later (that indeed it is an educated guess) rather than to impose perfect foresight directly because it makes possible to draw one of the schedules in the foreign exchange market diagram and, more importantly, the economy’s adjustment path right away, as their corresponding equations can be obtained readily without first resorting to some other technique for solving a system of differential equations. Finally, section 6 gives the summary and conclusions.

2. The Model

The model, following Dornbusch (1976, appendix), is that of a small open economy under flexible exchange rates wherein perfect capital mobility and substitutability prevail, output or income is variable, goods prices are sticky, and exchange-rate expectations can be consistent with rational expectations.

2.1 Building Blocks

The building blocks of the model are:

\begin{align}
(1.1) \quad & i = i_f + \dot{e}^E, \quad \text{uncovered interest parity (UIP)} \\
(1.2) \quad & \dot{e}^E = -\theta(e - \bar{e}), \quad \text{exchange-rate expectations} \\
(1.3) \quad & m - p = \phi y - \lambda i, \quad \text{LM} \\
(1.4) \quad & y = u + \gamma y + \delta(e - p + p_f), \quad \text{IS} \\
(1.5) \quad & \dot{p} = \beta(y - \bar{y}), \quad \text{price adjustment}
\end{align}

where $i (i_f)$ is domestic (foreign) interest rate, $e$ is the log of exchange rate measured as domestic currency per unit of foreign currency, $\dot{e}^E$ is the expected rate of rate of change in the exchange rate, $m$ is the log of exogenous money supply, $p (p_f)$ is the log of domestic (foreign) price level, $m - p$ is the log of real money supply, $y$ is the log of real income or output,
$e - p + p_f$ is the log of the real exchange rate $q$ or the relative price of domestic goods. Thus, all variables, except $i$ and $i_f$, are in natural logarithms. All parameters are positive, and $0 < \gamma < 1$. A bar on top of variable indicates long-run equilibrium value. Foreign variables $i_f$ and $p_f$ are, because the economy is assumed to be small, taken as given and therefore exogenous. $\bar{y}$ is also assumed exogenous, as it is assumed to be equal to the log of exogenous full employment level of output $y_F$.

Note that the equations describing the model are exactly the same as those in the Dornbusch (1976, appendix) model except that in this paper it is assumed, for simplicity but without loss of generality, that investment is exogenous and does not depend on the domestic interest rate (see 1.4).

2.2 Steady-State

The economy is in a long-run equilibrium, or in a steady-state, when $\dot{e}^E = 0$ and $\dot{p} = 0$, implying that the steady-state values of variables are given by

\begin{align}
(2.1) \quad & \bar{T} = i_f, \\
(2.2) \quad & \bar{p} = m - \phi \bar{y} + \lambda \bar{T}, \\
(2.3) \quad & \bar{e} = \bar{p} - p_f + (1/\delta)(-u + (1 - \gamma)\bar{y}), \\
(2.4) \quad & \bar{y} = y_F.
\end{align}

where (2.1) is obtained from (1.1), (2.2) from (1.3), and (2.3) from (1.4). and (3.4) follows from the assumption that $\bar{y} = y_F$. Thus, while income is demand-determined and varies in the short run, it is assumed to be exogenously fixed in the long-run.

2.3 A Panel IS*-LM* Diagram

The model, (1.1) to (1.5), which are relations describing the asset market (foreign exchange market and the money market) and the goods market, is illustrated graphically in Figure 1 and the details are discussed below.

*Foreign exchange market diagram.* The foreign exchange market is described by the UIP
condition (1.1) and the assumption about exchange-rate expectations (1.2). Embodied in the \( \text{UIP} \) condition are the assumptions of perfect financial capital mobility across countries and perfect financial capital substitutability from the point of view of investors, which ensure the expected rate of return on domestic-currency-denominated domestic asset \( ER \) (equal to the domestic interest rate \( i \)) and the expected rate of return on foreign-currency-denominated foreign \( ER_f \) (equal to the foreign interest rate \( i_f \)) plus the expected rate of change in the exchange rate \( \varepsilon^E \) are perfectly aligned.\(^5\) (1.2) also shows that whenever the interest differential \( i - i_f \) is negative (positive), holders of domestic (foreign) assets must be compensated by the expectation of future domestic (foreign) currency appreciation, i.e., \( \varepsilon^E < (>) 0 \).

The expected rate of change in the exchange rate \( \varepsilon^E \) is assumed to depend negatively on the deviation of the logarithm of short-run exchange rate \( e \) from its long-run equilibrium value \( \bar{e} \) (1.2). This means that if the short run exchange rate is above (equal to; below) its long run equilibrium value, the expectation is that the exchange rate will be falling (constant; rising) or, the domestic currency will be appreciating (constant; depreciating), over time. This means that (1.2) is a regressive expectations scheme.

To illustrate the foreign exchange market, notice that the LHS of (1.1) is \( ER \),

\[
(1.1) \quad ER = i,
\]

and that the RHS of (1.1) is \( ER_f \) which, when combined with (1.2), yields the downward sloping \( ER_f \) schedule,

\[
(1.1.2) \quad ER_f = i_f + \left( -\theta (e - \bar{e}) \right),
\]

or, in slope-intercept form,

\[
\]

\(^5\) The \( \text{UIP} \) condition is \( (1 + i) = (1 + i_f)E^E / E \), where \( E^E \) is expected exchange rate. It can be rewritten as \( i = i_f + ((E^E - E) / E) + i_f ((E^E - E) / E) \) and, assuming that \( i_f ((E^E - E) / E) \) is small enough, it is approximated by (1.1).
\begin{equation}
\bar{e} = \bar{e} + \frac{1}{0} i_f - \frac{1}{0} EPf.
\end{equation}

(1.2), an educated guess or a guess about \( \hat{e}^E \) which can be verified later, is an important component of the model and a convenient one because, without it, one cannot graph the \( ER_f \) schedule at the outset and no further dynamic analysis can be made.

The \( ER \) schedule is vertical at the domestic interest rate. The \( ER_f \) schedule is downward sloping because, for given \( \bar{e} \) and \( i_f \), an increase in \( e \) or a domestic currency depreciation will create an expectation of a subsequent domestic (foreign) currency appreciation (depreciation) and reduce \( ER_f \). These schedules are shown in Figure 1(a), the foreign exchange market diagram. At the intersection of the \( ER \) and \( ER_f \) schedules, the \( UIP \) condition (1.1) holds, and expectations satisfy (1.2). The \( ER \) schedule shifts to the left as \( i \) decreases while the \( ER_f \) schedule shifts to the right as \( \bar{e} \) increases and/or \( i_f \) increases. Note that the \( ER \) schedule will coincide with the \( \hat{e}^E = 0 \) line only if \( i = \bar{I} = i_f \), as in Figure 1(a).

**Money market diagram.** The money market is summarized by the \( LM \) equation (1.3), which assumes that money market equilibrium (\( MME \)) holds, i.e., real money supply is equal demand for real money balances \( L \), where \( L \) is assumed to be a positive function of output and a negative function of domestic interest rate. Figure 1(b), the money market diagram, shows the \( m - p \) schedule as a horizontal schedule and the \( l \) \( (= \ln L) \) function, the RHS of (1.3), as a downward sloping schedule. At the point of intersection between the \( m - p \) and \( l \) schedules, \( MME \) holds, i.e., (1.3) holds.\(^6\) The \( m - p \) schedule shifts as \( m \) or \( p \) changes while the \( l \) schedule shifts as \( y \) changes. Note that the size of the shift of the \( l \) schedule as \( y \) changes is large when parameter \( \phi \) is large.

\(^6\) Such point corresponds to a point on the upward sloping \( LM \) schedule in a \( y-i \) diagram.
**Asset market equilibrium (AME) and the LM* schedule.** The LM* (or the open economy LM) schedule is derived from the relations describing the asset market. Thus, combining (1.1), (1.2), and (1.3) yields the equation for the LM* schedule,

\[(3.1) \quad (e - \bar{e}) = -\frac{\phi}{\lambda \theta} (y - \bar{y}) - \frac{1}{\lambda \theta} (p - \bar{p}),\]

or, in slope-intercept form,

\[(3.1') \quad e = \bar{e} + \frac{1}{\theta} i + \frac{1}{\lambda \theta} (m - p) + (-\frac{\phi}{\lambda \theta}) y.,\]

which shows the \((y, e)\) combinations for which the assets market clears, i.e., both the money market and the foreign exchange market clear, and exchange-rate expectations conform with (1.2). Thus, the LM* schedule is actually the AME schedule.

The LM* schedule can be obtained graphically as follows. As \(y\) increases, \(l\) increases and the \(l\) schedule shifts up, and \(i\) increases to maintain money market equilibrium. The increase in \(i\), in turn, causes the ER schedule to shift to the right, thereby causing \(ER < ER\) at the initial \(e\), and \(e\) must fall (the domestic currency must appreciate) to increase to maintain UIP. This means that at any point on the LM* schedule, money market equilibrium and UIP hold and expectations are taken into account, and that along the LM* schedule the relationship between \(e\) and \(y\) is negative (see Figure 3(a)). Note that the size of the fall in \(e\) is also affected by parameter \(\phi\): the fall in \(e\) will be large when \(\phi\) is large, i.e., LM* is steep. The LM* schedule shifts to the right as \(\bar{e}\) increases, \(i\) increases, or \((m - p)\) increases.

**Goods market diagram.** The goods market is described by the IS* (or open economy IS) equation (1.4) and the price adjustment equation (1.5). (1.4) assumes that goods market equilibrium (GME) holds, i.e., output equals the demand for domestic output \(D\), where \(D\) is a positive function of output or real income and the real exchange rate or the relative price of
domestic goods.\(^7\)

Figure 1(c) is an open economy Keynesian cross diagram where the \(d = \ln D\) schedule, the RHS of (1.4), is drawn as upward sloping and flatter than the 45° line since its slope, \(\gamma\), is less than 1. The 45° line is the \(y = d\) line and thus is also the \(y\) line. Equilibrium output is given by the crossing of \(d\) schedule and the \(y\) line. Thus, at the point of crossing, \(GME\) (1.4) holds, and that point corresponds to a point on the \(IS^*\) schedule. The \(d\) schedule shifts up as \(u\) and/or \(e - p + p_f\) changes. Note that the size of the shift of the \(d\) schedule when \(e\) changes is large when parameter \(\delta\) is large.

The final equation of the model, (1.5), indicates that the price level adjusts only slowly over time. Specifically, price adjustment is proportional to the deviation of output from its long run equilibrium value. (1.5), whose implications are yet to be derived, cannot be linked/embedded yet in Figure 1.

**Goods market equilibrium (GME) and the IS\(^*\) schedule.** The \(IS^*\) schedule is simply the open economy \(IS\) (1.4), which can be written, in deviation form, as\(^8\)

\[
(3.2) \quad e - \bar{\varepsilon} = (p - \bar{p}) + \frac{1 - \gamma}{\delta} (y - \bar{y}),
\]

or, in slope-intercept form,

\[
(3.2') \quad e = p - p_f - \frac{1 - \gamma}{\delta} u + \frac{1 - \gamma}{\delta} y. \quad \text{[v. intercept]} \quad \text{slope}
\]

The \(IS^*\) schedule shows the \((y, e)\) combinations for which the goods market clears (see Figure 1(d)). Thus, it can also be called the \(GME\) schedule.

The \(IS^*\) schedule can be derived graphically as follows (see Figure 2(b)). As \(e\)

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\(^7\) Note that, here, the goods market is not affected directly by \(i\) as investment is assumed to be exogenous but the goods market affects the money market through \(y\). \(u\) in (1.4) captures the effects of exogenous aggregate demand such as exogenous investment and fiscal policy.

\(^8\) The \(IS^*\) schedule is the same as \(IS\) schedule in a \(y-i\) diagram which is vertical since investment is exogenous.
increases, \( e - p + p_f \) increases, net exports increase, the demand for domestic goods increases and thus the \( \ln D \) schedule shifts up, and \( y \) increases to maintain goods market equilibrium. This means that at any point on the \( IS^* \) schedule, goods market equilibrium holds and that along the \( IS^* \) schedule the relationship between \( e \) and \( y \) positive. Note that the size of the increase in \( y \) is also affected by parameter \( \delta \): the increase in \( y \) will be large when \( \delta \) is small, i.e., the \( IS^* \) schedule is steep. The \( IS^* \) schedule shifts to the right as \( p \) decreases, or \( p_f \) increases, or \( u \) increases.

**The \( IS^*-LM^* \) diagram.** Figure 1(d), the \( IS^*-LM^* \) diagram, is an open-economy \( IS-LM \) diagram drawn in a \( y-e \) diagram rather than in a \( y-i \) diagram. A shown above, it is actually a \( GME - AME \) diagram. Krugman and Obstfeld (1988) and Krugman, Obstfeld, and Melitz (2012) Krugman and Obstfeld (1988) refer to this diagram as the \( DD-AA \) diagram; however, do not use a four-panel diagram.\(^9\)

### 2.4 Equilibrium, Adjustment, and Perfect Foresight

**Long-run equilibrium and short-run equilibrium.** The economy’s short-run equilibrium occurs at the intersection of \( IS^* \) and \( LM^* \) schedules; at this point, all markets clear. If the \( IS^*-LM^* \) intersection lies on the \( y_F \) line and at the same time the \( ER - ER_f \) intersection lies on the \( \dot{e}^E = 0 \) line, then the economy is also in a long-run equilibrium where there is no tendency for both the price level and the expected exchange rate to change.

**Short-run equilibrium and adjustment: the SP schedule as the locus of \( IS^*-LM^* \) intersections as \( p \) changes.** At any point in time, the economy is in short-run equilibrium, i.e., on a point of intersection between the \( IS^* \) (or \( GME \)) schedule and \( LM^* \) (or \( AME \)) schedule.

---

\(^9\) On ‘the inconsistencies which one finds in identifying models in the open macroeconomy’, Boyer and Young (2010) remark that Obstfeld and Rogoff (1996) use the expression ‘the Mundell-Fleming-Dornbusch model’ at least seven times in their graduate textbook but the same expression ‘is not to be found in the undergraduate textbook by Krugman and Obstfeld’ (1988 and later editions), which employs the mechanism identical to that in the so-called ‘the Mundell-Fleming-Dornbusch model’ as the central mechanism in their key framework, which they call the \( DD-AA \) model.
Using the IS* and LM* equations in deviation form, (3.2) and (3.1), yields

\[(4) \quad (e - \bar{e}) = -\left(\frac{a_{12}}{a_{11} + \theta}\right) (p - \bar{p}),\]

or, in slope-intercept form,

\[(4') \quad e = \left(\bar{e} + \frac{a_{12}}{a_{11} + \theta} \bar{p}\right) - \frac{a_{12}}{a_{11} + \theta} p,\]

where \(a_{11} = \frac{\phi \delta / (1 - \gamma)}{\lambda} > 0\) and \(a_{12} = \frac{1}{\lambda} - a_{11} = \frac{1 - \phi \delta / (1 - \gamma)}{\lambda} \leq 0\) as \(\phi \delta / (1 - \gamma) \leq 1.\)

(4) is the key equation of the model and illustrated as the schedule labeled SP. It determines, for given \(\bar{e}\) and \(\bar{p}\), the short-run equilibrium value of \(e\) as a function of \(p\). It is the key equation because, as shown below, it is the equation for the adjustment path and thus it is the one which determines the size of exchange-rate jump at \(t = 0\) and also the one which describes the relation between \(e\) and \(p\) during adjustment.

(4), using (3.2) for \((p - \bar{p})\), can be rewritten equivalently as

\[(5) \quad (e - \bar{e}) = \left(\frac{a_{12}}{1/\lambda + \theta}\right) \frac{1 - \gamma}{\delta} (y - \bar{y}),\]

or, in slope-intercept form,

\[(5') \quad e = \bar{e} - \left(\frac{a_{12}}{1/\lambda + \theta}\right) \frac{1 - \gamma}{\delta} \bar{y} + \left(\frac{a_{12}}{1/\lambda + \theta}\right) \frac{1 - \gamma}{\delta} y,\]

which determines, for given \(\bar{e}\) and \(\bar{y}\), the short-run equilibrium value of \(e\) as a function of \(y\).

Thus, (4) or (5) is the equation for the locus of IS*-LM* intersections. Graphically, the locus of points of IS*-LM* intersection in \(y - e\) diagram labeled SP (see Figure 3(a)) corresponds to a locus in \(p - e\) diagram which is also labeled SP (see Figure 3(b)).

Figures 3(a) and 3(b) show the case where \(a_{12} > 0\). Note that when \(a_{12}\) is positive (zero; negative) or, equivalently, when \(\phi \delta / (1 - \gamma)\) is less than (equal to; greater than) unity, the SP
schedule in $p-e$ diagram is downward sloping (horizontal; upward sloping) whereas the $SP$ schedule in $y-e$ diagram is upward sloping (horizontal; downward sloping). Note also that both SP schedules in $p-e$ diagram and in $y-e$ diagram shifts up when either $m$ or $i_f$ increases (and therefore both $\bar{p}$ and $\bar{e}$ increase) or when either $p_f$ or $u$ decreases (and therefore $\bar{p}$ increases).

The $SP$ schedule in the $p-e$ ($y-e$) diagram shifts to the right (left) whenever there is an increase in $\bar{e}$ and/or $\bar{p}$, thereby changing the economy’s long-run equilibrium position.

During adjustment from short-run to the new long-run equilibrium, or when the price level is changing over time, both the $IS^*$ and $LM^*$ schedules are shifting over time but the economy is always on an $IS^*-LM^*$ intersection and therefore always on the $SP$ schedule. Thus, the SP schedule ((4) or (5)) is the graphical representation of the economy’s convergent adjustment path.

**Dynamics of $p$ and $e$ along the adjustment path and convergence.** The equation describing the dynamics of $p$ along the adjustment path can be obtained using the assumed price adjustment (1.5), (3.2) for $(y-ar{y})$, and (4) for $(e-\bar{e})$,

$$\dot{p} = \beta \left( \frac{\delta}{1-\gamma} (e-\bar{e}) - \frac{\delta}{1-\gamma} (p-\bar{p}) \right) \quad \text{(y-\bar{y}), using (2.2')}$$

$$= a_{21} (e-\bar{e}) + a_{22} (p-\bar{p})$$

$$= a_{21} \left( \frac{a_{12}}{a_{11} + \theta} \right) (p-\bar{p}) + a_{22} (p-\bar{p}) \quad \text{(e-\bar{e}), using (4)}$$

which yields

$$(6.1) \quad \dot{p} = -v (p-\bar{p}),$$

where $v = \left( \frac{a_{12}a_{21} - (a_{11} + \theta)a_{22}}{a_{11} + \theta} \right)$ and $a_{21} = -a_{22} = \beta \delta / (1-\gamma) > 0.$
(6.1) shows that $p$ will be increasing (constant; decreasing) over time whenever $p$ is below (equal to; above) $\bar{p}$. (6.1) can be solved to yield the time path of $p$ ,

$$p(t) = \bar{p} + (p(0) - \bar{p}) \exp^{-vt},$$

which shows that $p$ will converge towards $\bar{p}$ at the rate $v$.

The time path of $e$, using (4) and (6.2), is given by

$$(7.1) \quad e(t) - \bar{e} = -\frac{a_{12}}{a_{11} + \theta}(p(0) - \bar{p}) \exp^{-vt} = (e(0) - \bar{e}) \exp^{-vt},$$

indicating that $e$ will converge towards $\bar{e}$ also at the rate $v$. Along the SP schedule, the dynamics of $e$ is given by

$$(7.2) \quad \dot{e} = -v(e - \bar{e}),$$

implying that the domestic currency will be appreciating (constant; depreciating) over time for as long as $e$ is above (below) its long-run level.10

**Consistency with perfect foresight.** Expectations would be consistent with perfect foresight if the expected rate of change in the exchange rate is equal to the actual rate of change in the exchange rate, $E(\dot{e}) = \dot{e}$, i.e., if $\theta$ in (1.2) is equal to $v$ in (7.2). Equating $\theta$ and $v$

$$\theta = \left( \frac{a_{12}a_{21} - (a_{11} + \theta)a_{22}}{a_{11} + \theta} \right)$$

$$\theta^2 + a_{11}\theta = a_{12}a_{21} - a_{11}a_{22} - \theta a_{22}$$

$$0 = \theta^2 + (a_{11} + a_{22})\theta + (a_{11}a_{22} - a_{12}a_{21})$$

yields an equation that is quadratic in $\theta$,

$$(8.1) \quad \theta^2 + (a_{11} + a_{22})\theta + (a_{11}a_{22} - a_{12}a_{12}) = 0,$$

with solution

---

10 The equations describing the behavior of other variables, $i$ and $y$, during adjustment can be similarly obtained (see Appendix, available upon request).
\[
\theta_1, \theta_2 = \frac{-(a_{11} + a_{22}) \pm \left[(-a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})\right]^{1/2}}{2},
\]

\( (4) \) is the equation for the unique \( SP \) schedule or adjustment path provided that

\[
\nu = 0 = \theta_1 = \frac{-(a_{11} + a_{22}) + \left[(-a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})\right]^{1/2}}{2} > 0.
\]

\( \theta \) is the economy’s speed of adjustment, and convergence will be faster the lower is \( \lambda \), the higher is \( \phi \), and the higher is \( \delta \).

**Adjustment process.** The economy’s process of adjustment can be described using Figure 3(a) or 3(b). At any point in time, all markets clear, i.e., there is short-run equilibrium (see (4)). This means that the economy is always on the \( SP \) schedule.

The adjustment process is also illustrated in Figure 4, which is the same as Figure 3(b) with the \( \dot{p} = 0 \) locus in it. The \( \dot{p} = 0 \) locus shows the \((p, e)\) combinations for which the goods market is in equilibrium and the price level does not change.\(^{11}\) At any point to the left (right) of and above (below) the \( \dot{p} = 0 \) schedule, \( y \) is above (below) \( \bar{y} \) and thus the price level is rising (falling). The \( \dot{p} = 0 \) locus is positively sloped and parallel to the 45° line because an increase in \( e \) creates an excess demand and \( p \) will have to increase equi-proportionately to restore equilibrium since an increase in \( p \) affects aggregate demand only through the real exchange rate and not through interest rate.

Long-run equilibrium is a point on the vertical \( y_F \) line or on the \( \dot{p} = 0 \) locus. Specifically, long-run equilibrium occurs at the point where the \( IS^* \) and \( LM^* \) schedules and the vertical \( y_F \) line intersect (i.e., when \( y = \bar{y} = y_F \) and thus \( \dot{p} = 0 \)) or, alternatively, at the point where the \( SP \) intersects the \( \dot{p} = 0 \) locus. This occurs at point C.

\(^{11}\) Since the semi-reduced form price adjustment equation is \( \dot{p} = a_{21}(e - \bar{e}) + a_{22}(p - \bar{p}) \), the \( \dot{p} = 0 \) locus is given by \( (e - \bar{e}) = (p - \bar{p}) \) or, in slope-intercept form, \( e = \bar{e} - \bar{p} + (1) \ p \). In the case, the \( \dot{p} = 0 \) may either coincide with or be parallel to the 45° line.
Consider point B, a point on the \( SP \) schedule but above the \( p = 0 \) locus. At this point, \( p \) will be rising over time and thus both the \( IS^* \) and \( LM^* \) schedules are shifting, with the points of intersection tracing the line with series of arrows from point B to point C. The path from point B to point C is the locus of points of intersection between the \( IS^* \) schedule and the \( LM^* \) schedule as \( p \) changes over time. Thus, the economy’s adjustment path is the \( SP \) schedule in Figures 3(a), (3(b), and 4. As shown below, the \( SP \) schedule is the same as the stable arm of the saddlepoint in a model where perfect foresight is imposed directly. A detailed discussion of the adjustment process is covered in the next section.

3. Effects of Disturbances

3.1 Monetary Expansion

Assume that the economy is initially in a long-run equilibrium. All initial positions are labeled A while all initial values are labeled with the subscript 1 (see Figures 5a and 5b for the overshooting case and Figures 6a and 6b for the undershooting case). Now consider a monetary expansion in the form of an increase in \( m \) at \( t = 0 \), a disturbance which affects long-run equilibrium price level.

**Steady-state effects.** The steady-state effects of a monetary expansion, shown as the movement from A to C in Figures 5a and 6a, using (3.1)-(3.4), are

\[
\begin{align*}
(8.1) \quad & \frac{di}{dm} = \frac{df}{dm} = 0, \\
(8.2) \quad & \frac{dp}{dm} = 1, \\
(8.3) \quad & \frac{\delta e}{dm} = \frac{\delta p}{dm} = 1, \\
(8.4) \quad & \frac{\delta y}{dm} = \frac{\delta y_f}{dm} = 0,
\end{align*}
\]

and, by implication, \( d(\bar{e} - \delta p + p_f) / dm = 0 \). In the long run, money is neutral (see 8.4), and affects \( e \) and \( p \) equipropotionately (see (8.2) and (8.3)). This means that an \( x\% \) increase in the money supply will lead to an \( x\% \) increase in the price level and an \( x\% \) domestic currency depreciation, but will leave the relative price or real exchange and therefore output unchanged.
implying that the relative purchasing power parity (PPP) holds. Thus, the model is monetarist/classical in the long-run.

**Impact effects.** Assume the case where \( a_{12} > 0 \), i.e., the case where \( SP \) schedule in the \( p - e \) diagram is downward sloping. The impact effects, or the effects which occurs at \( t = 0 \), are shown as the jump from A to B in Figure 5a and 5b, the overshooting case. Economic agents, because they are endowed with perfect foresight, see the monetary expansion and know that it will lead to a long-run domestic currency depreciation. They respond accordingly by altering their expectations, which causes the \( ER_f \) schedule to shift up by an amount equal to \( d\bar{e} (=\bar{e}_2 - \bar{e}_1) \). With \( i \) unchanged yet, \( UIP \) no longer holds but this may not be possible because \( i \) may change. To see why, notice that an increase in \( m \) results in an increase in \( (m - p) \) since \( p \) does not change at \( t = 0 \), i.e., the \( (m - p) \) schedule shifts up by an amount equal to \( dm \). At the same time, the increase in \( e \) will, since \( p \) is sticky and \( p_f \) is exogenously fixed, lead to a real depreciation, an increase in net exports, and increase in the demand for domestic output and thus the \( d \) schedule shifts up; to maintain GME, \( y \) must increase. This increase in \( y \), in turn, will increase \( l \) and shift up the \( l \) schedule. If, at the initial \( i \), the increase in \( (m - p) \) is greater than the resulting increase in \( l \),

\[
1 - \phi / (1 - \gamma) \frac{d(m - p)}{dm} = \frac{dl}{dy} \frac{dy}{de} \frac{de}{d\bar{e}} \frac{d\bar{e}}{dm} = \frac{d(m - p)}{dm} - \frac{dl}{dm} > 0, \\
\text{at the initial } i = 1
\]

or the v. shift of the \( (m - p) \) schedule > the v. shift of the \( l \) schedule, then will be an excess supply of money at the initial \( i \); to maintain money market equilibrium, \( i \) must fall.

If \( i \) decreases, the \( ER \) schedule shifts to the left and, with \( i_f \) remaining the same, \( UIP \) will be maintained if \( \dot{e}^E < 0 \), i.e., if there is an expectation of a subsequent domestic currency appreciation (constant exchange rate; domestic currency depreciation) so as to compensate holders of domestic assets. Such an expectation will be created only if \( de > d\bar{e} \), i.e., \( e \)
overshoots $\bar{e}$ initially.

In Figure 1(d), the $LM^*$ schedule shifts up because the $(m - p)$ schedule shifts up and also because the $ER$ schedule shifts up, but the $IS^*$ schedule remains the same. Since the asset market is in continuous equilibrium, the economy jumps from A on $LM^*_1$ to point B on $LM^*_2$.

At point B, the $e$ has risen but is greater than the new $\bar{e}$, $i$ is lower, $y$ is higher, and $p$ which has not changed yet is lower than the new $\bar{p}$. Using (4), when $t = 0$,

$$
(9) \quad (e(0) - \bar{e}) = -\left( \frac{a_{12}}{a_{11} + \theta} \right)(p(0) - \bar{p}),
$$

The resulting exchange rate jump is

$$
(9.1) \quad \frac{de(0)}{dm} = \frac{d\bar{e}}{dm} + \left( \frac{a_{12}}{a_{11} + \theta} \right) \frac{d\bar{p}}{dm} = \frac{\theta + 1}{\lambda} \left( \frac{\phi \delta}{1 - \gamma} \right) + 0 > 0,
$$

and thus the extent of overshooting/undershooting is

$$
(9.2) \quad \frac{de(0)}{dm} - \frac{d\bar{e}}{dm} = \frac{\left( \frac{a_{12}}{a_{11} + \theta} \right) \frac{d\bar{p}}{dm}}{\lambda} = \frac{1 - \phi \delta / (1 - \gamma)}{\lambda} + \theta
$$

where the numerator of $a_{12}$, $1 - \phi \delta / (1 - \gamma)$, has already been interpreted above.

Clearly, the impact effects shows that the model exhibits Keynesian properties in the short-run in the sense that money affects output and in this model, like in the Mundell-Fleming model, monetary policy is effective in influencing output in the short-run under conditions of flexible exchange rates and perfect capital mobility.

**Transitional dynamics.** After the jump, the economy moves continuously along the adjustment path, from point B to point C in Figures 5a and 5b. This happens because at point B, $y > \bar{y} = y_F$, causing $p$ to rise over time which, in turn, causes other variables to change over
time. Thus, during transition, dynamics is driven by $p$.

Specifically, rising $p$ over time leads to falling $(m - p)$ and downward shifting $(m - p)$ schedule over time. Rising $p$ also causes falling $q(\equiv e - p + p_f)$ and downward shifting $d$ schedule, falling $y$ and thus falling $l$ and downward shifting $l$ schedule. In the overshooting case, this means that $i$ will be rising and correspondingly the $ER$ schedule is shifting to the right and, since there is no change in $i_f$, $\dot{e}^E < 0$. Since with perfect foresight expectations are self-fulfilling, subsequent adjustment is indeed characterized by actual appreciation, $\dot{e} < 0$.

Adjustments continue until the new steady-state is reached, at point C.

**Other cases.** The previous analysis assumes that $a_{i2} > 0$. It can be shown that if $a_{i2} < (=) 0$, then the $SP$ schedule is $p - e$ upward sloping (horizontal). Following a monetary expansion, $e$ will jump but will undershoot (neither overshoot nor undershoot) $\bar{e}$ at $t = 0$; during transition, $\dot{e}^E = \dot{e} < (=) 0$, i.e., adjustment is characterized by an expectation of and an actual domestic currency depreciation (constant exchange rate). This is so because when $a_{i2} < (=) 0$, i.e., when $1 - \phi \delta / (1 - \gamma) < (=) 0$, the increase in $(m - p)$ is less than (equal to) the resulting increase in $l$ at the initial $i$, or the v. shift of the $(m - p)$ schedule $< (=)$ the v. shift of the $l$ schedule, and an excess demand for money (neither an excess supply of or demand for money) is created at the initial $i$; to maintain money market equilibrium, $i$ must rise (remain the same). An increase in (An unchanged) $i$, with $i_f$ remaining the same, requires $\dot{e}^E > (=) 0$ for the $UIP$ condition to be maintained. But such an expectation of a subsequent domestic currency depreciation (constant exchange rate) can only be created if $e$ undershoots (neither overshoots nor undershoots) the new $\bar{e}$. In this case, the transition is characterized by actual depreciation (constant exchange rate), $\dot{e} > (=) 0$, and a rising price level. The undershooting overshooting case is shown in Figures 6a and 6b.
Limiting case of $\phi = 0$. If $\phi = 0$, then given a monetary expansion, there will be no increase in money demand even when there is a resulting income expansion and the result will be exchange rate overshooting as in the Dornbusch (1976, text) overshooting model.

Figure 1. A panel IS*-LM* diagram
Figure 2a. Graphical derivation of the $LM^*$ schedule

\[ e = \bar{e} + \frac{1}{\theta} i_f + \frac{1}{\lambda \theta} (m - p) + \left( -\frac{\phi}{\lambda \theta} \right) y \]

Figure 2b. Graphical derivation of the $IS^*$ schedule

\[ e = p - p_f - \frac{1 - \gamma}{\delta} u + \frac{1 - \gamma}{\delta} y \]
Figure 3. $SP$ as the locus of $IS^*-LM^*$ intersections (overshooting case)

Figure 4. The $SP$ schedule and the $\dot{p} = 0$ locus (overshooting case)
Figure 5a. Effects of a permanent monetary expansion
(overshooting case)

Figure 5b. Effects of a permanent monetary expansion:
\( IS^*-LM^* \) diagram vs. phase diagram
(overshooting case, \( a_{12} > 0 \))
Figure 6a. Effects of a permanent monetary expansion
(undershooting case, \(a_{12} < 0\))

Figure 6b. Effects of a permanent monetary expansion:
\(IS^*\)-\(LM^*\) diagram vs. phase diagram
(overshooting case, \(a_{12} > 0\))
3.2 Fiscal Expansion

The four-panel $IS^*-LM^*$ diagram can also be used to analyze the effects of other disturbances. In the case of real disturbances (disturbances in aggregate demand such as changes in $p_f$ or $u$ in the $IS^*$ equation) and, therefore, fiscal disturbances (captured by $u$), it can be shown that there are no transitional dynamics since equilibrium price level is not affected by these disturbances; the system simply jumps from the initial steady state to the new steady state. Thus, the Dornbusch framework and therefore the model presented here preserve the Mundell-Fleming result that fiscal policy is ineffective under conditions of perfect capital mobility and flexible exchange rates.\(^{12}\)

4. Assuming A Perfect-Foresight Consistent Regressive Expectations Scheme and the Method of Undetermined Coefficient (Guess and Verify Method)

Dornbusch, by assuming a perfect-foresight-consistent regressive exchange-rate expectations scheme ($\hat{e}^E$ in (1.1) and specified in (1.2)), and a price adjustment over time (1.5), essentially turned the static Mundell-Fleming model into a dynamic model. He uses a regressive expectations scheme (1.2) as a guess, and later verifies that it can be consistent with rational expectations provided that the exchange-rate expectations coefficient satisfies a unique relationship among all the parameters of the model. As there is no uncertainty in the model, rational expectations are equivalent to perfect foresight, and perfect foresight implies that the expected rate of change in the exchange rate is the same as the actual rate of change in the exchange rate, i.e., $\hat{e}^E = \hat{e}$.

This paper has applied the same procedure and, as shown above, the advantage of having a guess such as (1.2) and verifying it only later (that indeed it is an educated guess) is that it is possible to draw the $ER_f$ schedule in the foreign exchange market diagram and, more

\(^{12}\) The four-panel $IS^*-LM^*$ diagram can also be used to analyze the effects of a change in the foreign interest rate.
importantly, the saddle path (Dornbusch’s QQ schedule) or the economy’s adjustment path right away, as their corresponding equations can be obtained readily without first resorting to some technique for solving a system of differential equations.

5. **Imposing Perfect Foresight Directly**

   If, instead of assuming a perfect-foresight consistent regressive expectations scheme, perfect foresight – that the expected and the actual rate of change in the exchange rate are the same - is imposed directly into the model, i.e., (1.2) is replaced by

   \[(1.2') \quad \dot{e}^F = \dot{e},\]

   then \(\dot{e}\) equation is derived from the entire model for consistency with perfect foresight and the model now has two dynamic equations, the \(\dot{e}\) equation and the \(\dot{p}\) equation, which constitutes a system of differential equations. \(^{13}\)

   It can be shown that the reduced-form short run static equations of the model, (1.1) to (1.5), are

   \[(10.1) \quad (i(t) - \bar{i}) = a_{11}(e(t) - \bar{e}) + a_{12}(p(t) - \bar{p}),\]

   \[(10.2) \quad (y(t) - \bar{y}) = \frac{\delta}{1-\gamma}(e(t) - \bar{e}) - \frac{\delta}{1-\gamma}(p(t) - \bar{p}),\]

   and that the dynamic equations are

   \[(11.1) \quad \dot{e} = a_{11}(e(t) - \bar{e}) + a_{12}(p(t) - \bar{p}),\]

   \[(11.2) \quad \dot{p} = a_{21}(e(t) - \bar{e}) + a_{22}(p(t) - \bar{p}),\]

   where \(a_{ij}\)’s are as defined before. \((10.1)\), obtained using the IS (1.4) and the LM (1.3), shows the \(i\) that is consistent with the simultaneous clearing of money and goods markets, while \((10.2)\) is obtained using the IS (1.4) Note that the assets markets are in continuous equilibrium.

   Specifically, in the short run, \(i\) is the equilibrating factor in the money market and \(e\) in the

\(^{13}\) See Bhandari (1982).
foreign exchange market the exchange rate. Note that that the goods market is always in
‘equilibrium’ due to the assumption that short run income or output is demand-determined.

(11.1), because of perfect foresight assumption, is derived from the entire model (or
from UIP (1.2) and (10.1)), i.e., derived from the condition of short run equilibrium in all
markets. Thus, the IS, the LM, the UIP condition, and the perfect foresight assumption yields the
\( \dot e \) equation. The equation for \( \dot p \) is obtained using an assumed feature of the model (1.5) and
(10.2).

The differential equations in \( e \) and in \( p \) (11.1) and (11.2) along with the reduced form
short-run static equations for \( i \) and \( y \) ((10.1) and (10.2)), fully describe the economy’s motion
over time, contingent upon some set of initial conditions. However, (11.1) and (11.2) do not
explicitly show the system’s dynamic properties. It is therefore necessary to derive the
quantitative and/or the qualitative solution to these dynamic equations.

**Qualitative solution: phase diagram.** (11.1) and (11.2) constitute a system of two first-
order linear differential equations with constant coefficients, whose solution can be found using a
phase diagram.

Using the \( \dot e \) equation (11.1), and setting \( \dot e \) equal to zero gives

\[
\dot e(t) = \ddot e + \frac{a_{12}}{a_{11}} \bar p - \frac{a_{12}}{a_{11}} p(t),
\]

which the equation for the \( \dot e = 0 \) locus. It is downward sloping (horizontal; upward sloping) as
\( a_{12} \geq 0 \). Using (A4.1) again,

\[
\frac{d\dot e}{de} = a_{11} > 0,
\]

implying that all points above (below) the \( \dot e = 0 \) locus, where \( \dot e \) is positive (negative), \( e \) is
increasing (decreasing) as denoted by the vertical arrows pointing upward (downward). The
\( \dot e = 0 \) locus shifts as \( \bar e \) and/or \( \bar p \) changes.

Using the \( \dot p \) equation (11.2), and setting \( \dot p \) equal to zero gives
(13.1) \[ e(t) = \bar{e} + \frac{a_{22}}{a_{21}} \bar{p} - \frac{a_{22}}{a_{21}} p(t) \]
\[ = \bar{e} - \bar{p} + p(t) \]

which is the equation for the \( \dot{p} = 0 \) locus. It is upward sloping with slope equaling 1, since \( a_{21} = -a_{22} \). Using (11.2),

(13.2) \[ \frac{d\dot{p}}{dp} = a_{22} < 0, \]

implying that at all points to the left (right) of the \( \dot{p} = 0 \) locus, where \( \dot{p} = 0 > (\leq;\leq) 0, \ p \) is increasing (decreasing) as denoted by the horizontal arrows pointing rightward (leftward). The \( \dot{p} = 0 \) locus shifts/rotates as either \( p_f \) or \( i_f \) changes, but not when \( m \) changes since the \( \dot{p} = 0 \) locus is simply \( e = \bar{e} - \bar{p} + p \) and \( d\bar{e} = d\bar{p} = dm \) (see (2.1), (2.2), and (2.3)).

Combining the sets of horizontal and vertical arrows gives an idea about the trajectories and, in particular, the totally unstable path and the saddle path (see Figure 7).

Figure 7. Phase diagram for the model with perfect foresight imposed directly (overshooting case)

The phase diagram for the model in which perfect foresight is imposed directly (Figure 7) is the standard diagram showing the \( \dot{p} = 0 \) and \( \dot{e} = 0 \) loci and the stable arm of the saddlepoint. The phase diagram in a model which assumes a regressive expectations scheme does not have the \( \dot{e} = 0 \) locus as it is not necessary in deriving the adjustment path.
Quantitative solution. Consider first the nature of the model’s stationary equilibrium. Differentiating the \( \dot{p} \) equation (11.2) with respect to time, and using the \( \dot{\varepsilon} \) equation (11.1) to substitute out for \( \dot{\varepsilon} \) and the \( \dot{p} \) equation (11.2) for \( (e(t) - \bar{e}) \), yields

\[
\dot{p} - (a_{11} + a_{22}) \dot{p} + (a_{11}a_{22} - a_{21}a_{12})(p(t) - \bar{p}) = 0,
\]

which is a second-order differential equation in \( p \).

The characteristic equation associated with (14) is

\[
r^2 - (a_{11} + a_{22})r + (a_{11}a_{22} - a_{21}a_{12}) = 0,
\]

and the characteristic roots of (14) are given by

\[
r_1, r_2 = \frac{(a_{11} + a_{22}) \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{21}a_{12})}}{2}
\]

where \( tr(A) = r_1 + r_2 = a_{11} + a_{22} \), \( det(A) = r_1r_2 = a_{11}a_{22} - a_{21}a_{12} = \frac{\beta\delta / (1 - \gamma)}{\lambda} < 0 \), and \( A \) is the matrix of coefficients associated with (11.1) and (11.2). Since \( det(A) = r_1r_2 < 0 \), then whatever is the sign of \( tr(A) = r_1 + r_2 \), the two roots are real and opposite in signs. This ensures that the perfect-foresight stationary equilibrium of the model is a saddle point and that there exists a unique path converging to that point.

Now let \( r_1 < 0 \) and \( r_2 > 0 \). Provided that \( \theta = \nu \), then

\[
r_1 = \frac{(a_{11} + a_{22}) - \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{21}a_{12})}}{2} = -\theta = -\nu = -\theta_i < 0.
\]

It can be shown that the condition that the coefficient associated with the positive root must be zero, which is satisfied when

\[
(e(0) - \bar{e}) = \frac{a_{12}}{(a_{11} - r_i)} (p(0) - \bar{p}),
\]

\(14\) Techniques for solving a system of first-order linear differential equations can be found in Gandolfo (1980) and Dixit (1980).
along with the condition that $p$ cannot jump at $t = 0$ converts the general solution to (11.1) and (11.2) into particular or bounded solution, which should be exactly the same as (7.1) and (6.2), respectively, with $v$ is replaced by $-r_i$. Thus, provided that $\theta = v = \theta_1$, then indeed $r_i = -\theta$ and, consequently, all the results when a regressive expectations scheme is assumed – equation for the saddle path (4), time paths of $e$ and $p$ ((7.1) and (6.2)), dynamics of $e$ and $p$ along the saddle path ((7.2) and (6.1)), and the exchange rate jump as well as the extent of overshooting/undershooting resulting from a monetary expansion ((9.1) and (9.2)) – are identical to the results when perfect foresight is imposed directly (see Bhandari (1982) and Appendix (available upon request)).

6. Summary

The pioneering and the most popular work in the literature on overshooting is the Dornbusch (1976) paper. It is an important and interesting extension of the Mundell-Fleming model, a static small open economy $IS-LM$ model, as it allows for price adjustment over time and rational expectations formation.

While exchange rate dynamics is an important topic in open economy macroeconomics, the standard tool commonly used to teach exchange rate dynamics - the phase diagram - is not well-suited for undergraduate students as most of them do not have yet a background on dynamic macroeconomic analysis.

The main objective of this paper has been to attempt to provide a graphical exposition of the Dornbusch (1976, appendix) model. Specifically, it has attempted to provide a graphical device - a panel diagram, where the first three panels not only describe the three markets (foreign exchange market, money market, and goods market) in the model but also show the underpinning of the last panel (the so-called $IS^*-LM^*$ diagram) - which can be used to teach intermediate macroeconomics students about exchange rate dynamics. The advantage of a panel diagram is that, given a disturbance, one would easily see the dynamic (short-run, transitional,
and long-run) effects on the different markets as well as on different variables.

This paper has also attempted to bridge the gap between undergraduate teaching and graduate teaching of exchange rate dynamics by showing the correspondence between the economy’s adjustment path in the $IS^*-LM^*$ diagram and that in the phase diagram. In particular, it has shown that the transitional adjustment path in the $IS^*-LM^*$ model is the locus of $IS^*-LM^*$ intersections as the price level changes over time and that such path corresponds to the transitional adjustment path, the saddle path, in a phase diagram. Furthermore, this paper has shown that the results of the guess-and-verify method are the same as those when perfect foresight is imposed directly but that it is advantageous to employ a regressive expectations scheme as a guess about exchange rate expectations and verify it only later (that indeed it is an educated guess) rather than to impose perfect foresight directly because the former technique makes possible to draw not only one of the schedules in the foreign exchange market diagram but also, more importantly, the economy’s adjustment path right away, as their corresponding equations can be obtained readily without first resorting to some other technique for solving a system of differential equations.
References


