

Moghadam, Hamed M.

Working Paper

The nonparametric approach to evolutionary oligopoly

Ruhr Economic Papers, No. 576

Provided in Cooperation with:

RWI – Leibniz-Institut für Wirtschaftsforschung, Essen

Suggested Citation: Moghadam, Hamed M. (2015) : The nonparametric approach to evolutionary oligopoly, Ruhr Economic Papers, No. 576, ISBN 978-3-86788-665-9, Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI), Essen, <https://doi.org/10.4419/86788665>

This Version is available at:

<https://hdl.handle.net/10419/119462>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



RUHR

ECONOMIC PAPERS

Hamed M. Moghadam

The Nonparametric Approach to Evolutionary Oligopoly



tu technische universität
dortmund

#576

Imprint

Ruhr Economic Papers

Published by

Ruhr-Universität Bochum (RUB), Department of Economics
Universitätsstr. 150, 44801 Bochum, Germany

Technische Universität Dortmund, Department of Economic and Social Sciences
Vogelpothsweg 87, 44227 Dortmund, Germany

Universität Duisburg-Essen, Department of Economics
Universitätsstr. 12, 45117 Essen, Germany

Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI)
Hohenzollernstr. 1-3, 45128 Essen, Germany

Editors

Prof. Dr. Thomas K. Bauer
RUB, Department of Economics, Empirical Economics
Phone: +49 (0) 234/3 22 83 41, e-mail: thomas.bauer@rub.de

Prof. Dr. Wolfgang Leininger
Technische Universität Dortmund, Department of Economic and Social Sciences
Economics – Microeconomics
Phone: +49 (0) 231/7 55-3297, e-mail: W.Leininger@wiso.uni-dortmund.de

Prof. Dr. Volker Clausen
University of Duisburg-Essen, Department of Economics
International Economics
Phone: +49 (0) 201/1 83-3655, e-mail: vclausen@vwl.uni-due.de

Prof. Dr. Roland Döhrn, Prof. Dr. Manuel Frondel, Prof. Dr. Jochen Kluge
RWI, Phone: +49 (0) 201/81 49-213, e-mail: presse@rwi-essen.de

Editorial Office

Sabine Weiler
RWI, Phone: +49 (0) 201/81 49-213, e-mail: sabine.weiler@rwi-essen.de

Ruhr Economic Papers #576

Responsible Editor: Wolfgang Leininger

All rights reserved. Bochum, Dortmund, Duisburg, Essen, Germany, 2015

ISSN 1864-4872 (online) – ISBN 978-3-86788-665-9

The working papers published in the Series constitute work in progress circulated to stimulate discussion and critical comments. Views expressed represent exclusively the authors' own opinions and do not necessarily reflect those of the editors.

Ruhr Economic Papers #576

Hamed M. Moghadam

The Nonparametric Approach to Evolutionary Oligopoly



Bibliografische Informationen der Deutschen Nationalbibliothek

Die Deutsche Bibliothek verzeichnet diese Publikation in der deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über:
<http://dnb.d-nb.de> abrufbar.

<http://dx.doi.org/10.4419/86788665>

ISSN 1864-4872 (online)

ISBN 978-3-86788-665-9

Hamed M. Moghadam¹

The Nonparametric Approach to Evolutionary Oligopoly

Abstract

A recent paper of Carvajal, Deb, Fenske, and Quah (Econometrica 2013)[4] applies the revealed preference approach in the context of a quantity competition oligopoly game. This paper aims to present a different solution concept for an evolutionary model in the asymmetric oligopoly setup where firms have different cost functions to produce a homogenous good. Then, using the approach introduced by Carvajal et al.(2013)[4], we derive the testable conditions of the evolutionary oligopoly model. Therefore, contrary to the typical empirical literature in IO, without making any parametric assumption regarding to the demand curve and the cost function, this nonparametric approach characterizes a set of conditions (restrictions) on observational dataset to be consistent with evolutionary oligopoly model. An empirical application to the oil market with OPEC producers and Non-OPEC producers is presented and we compare the rejection rates of both Cournot and evolutionary hypotheses.

JEL Classification: C73, D22, D43

Keywords: Evolutionary oligopoly; observable restrictions; non-parametric test; oil market

September 2015

¹ Hamed Markazi Moghadam, RGS Econ and TU Dortmund. – The author is grateful for helpful comments from Jan Heufer, Wolfgang Leininger and Lars Metzger, and seminar participants at TU Dortmund and conference participants at 49th Annual conference of Canadian Economic Association. – All correspondence to: Hamed Markazi Moghadam, TU Dortmund University, Department of Economics and Social Science, 44221 Dortmund Germany, e-mail: hamed.moghadam@tu-dortmund.de

1 Introduction

Revealed preference analysis is a practical and widely used instrument to test empirically the consistency of a theoretical model of consumer behavior with an observational dataset. This method has also been applied to analyze the firm behavior. For example, Afriat (1972) [1], Hanoch and Rothschild (1972)[9] and Varian (1984)[14] characterize a data consistency test for production analysis of profit maximization and cost minimization models. Most recently Carvajal et al.(2013 and 2014)[4],[5] and Cherchye et al. (2013)[6] derive testable conditions in the Cournot oligopoly model of firm competition. Both of papers by Carvajal et al.(2013 and 2014)[4],[5] apply a revealed preference approach in a single-product and multi-product oligopoly while Cherchye et al.(2013)[6] proposes a differential approach where the equilibrium price and quantities are functions of exogenous demand and supply shifters.

On the other hand, evolutionary oligopoly theory arises from the seminal papers of Schaffer (1988 and 1989)[12],[13]. Mainly these two papers, contrary to Friedman (1953)[7] conjecture, argue that firm survival condition does not follow absolute payoff maximizing (APM) behavior rather it tracks a relative payoff maximizing (RPM) behavior. In particular, Schaffer (1989)[13] verifies that firm survival is better demonstrated through an evolutionary model with relative payoff maximization rather than absolute payoff maximization. The appropriate evolutionary setup for firm competition is playing the field where all finite players in oligopoly game compete with each other instantaneously. So the behavior implied by RPM or spiteful behavior (Hamilton (1970)[8]) leads to more competition between firms in a quantity oligopoly game. Furthermore, Vega Redondo (1997)[15] shows that the Walrasian equilibrium turns out to be the unique stochastically stable state in the symmetric Cournot oligopoly. However, in the present paper we show that evolutionary stability leads to a *non-Walrasian* outcome in an asymmetric oligopoly.

The standard evolutionary game theory applies for identical players (with respect to their cost type) in a symmetric setup. Here, first we construct an evolutionary model in the asymmetric oligopoly where firms have different cost functions. An equilibrium concept

of *finite population evolutionary stable strategy* (FP ESS), defined by Schaffer (1988)[12], is applied. Defining a concept of evolutionary stability requires a symmetric setup with identical players. So we apply a symmetrization technique in order to transform the game with asymmetric firms into a symmetric oligopoly game and then extend Schaffer's definition (1988)[12] of a FP ESS to this setup. As a result, we identify a non-Walrasian solution concept for evolutionary stability in the asymmetric oligopoly.

Thereafter we study the consistency of evolutionary oligopoly model with an observational dataset. In particular, we attempt to answer the following questions. Whether a given set of observations is consistent with the evolutionary oligopoly model or not? Without making any parametric assumption on demand function, how can we recuperate the marginal costs from an observed behavior? In principal, inverse demand function and cost functions are not observable however we are able to observe equilibrium price and equilibrium quantities of all firms in the market. Suppose that we are given with observed dataset on the behavior of an industry consisting of K firms producing a homogenous good. Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k \in \mathcal{K}}\}_{t \in \mathcal{T}}$ where p_t^* is an observed price of homogenous market good for each $t \in \mathcal{T} = \{1, \dots, T\}$ and $q_{k,t}^*$ is observed output quantity of each firm $k \in \mathcal{K} = \{1, \dots, K\}$. For each time t , total output of industry $Q_t^* = \sum_{k=1}^K q_{k,t}^*$ is observed. Following Carvajal et al.(2013)[4], we will derive conditions on this set of observations to be consistent with the evolutionary oligopoly model. In general these testable conditions take the form of linear programming (LP) method.

Motivating Example. Consider a homogenous good market with two firms 1 and 2. Assume that we observe the produced quantities of each firm and the market price of good in the two sequential periods as follows:

$$\text{At observation } t, \quad p_t^* = 100, q_{1,t}^* = 6, \text{ and } q_{2,t}^* = 30.$$

$$\text{At observation } t', \quad p_{t'}^* = 50, q_{1,t'}^* = 9, \text{ and } q_{2,t'}^* = 30.$$

This example explains the recent falling trend of crude oil price in which OPEC (here firm 2) as the main producer in the oil market did not alter its production level in response to US

oil production increase (here firm 1). Annual average of US crude oil production due to the new extraction techniques of Shale oil raised from 5,65 million barrels per day in 2011 to 8,68 million barrels per day in 2014 despite the fact that annual average of OPEC oil production was round 32 million barrels per day without an intense variation during this period. In this example, firm 1 competes spitefully to increase its production levels and this leads to an abrupt fall of the market good price at time t' . With an easy calculation, we observe that there is a 25% decrease in US oil revenue ($p_t^* q_{k,t}^*$) (from 600 to 450) while at the same time OPEC oil revenue was decreased by 50% (from 3000 to 1500). Why did not OPEC reduce its output in response to the introduction of the US Shale oil technology? As Schaffer (1989)[13] explains if firms have market power, profit maximizers are not necessarily the best survivors because of the possibility of spiteful behavior. This spiteful behavior, where one player harms itself in order to harm another more, cannot be explained by Cournot competition. Is the evolutionary behavior the answer? And how we can test for this behavior?

This paper is organized as follows. The next section presents the evolutionary oligopoly model. Then, Section 3 discusses the data consistency test of evolutionary oligopoly model. Section 4 examines a case study in the crude oil market and concluding remarks are offered in Section 5.

2 The evolutionary oligopoly model

Consider K firms where all firms engage simultaneously to play an oligopoly stage game in each period t . Number of firms is constant for all t . The strategies for firms are their output quantities. Here we consider a static setup in which a player has the same strategy for all periods t . In evolutionary game theory, we assume that all players inherited with their strategies and they cannot change their strategies. Though, analogous to biology, there is a chance for mutation or experimenting of a new strategy. As Schaffer (1989)[13] explained the mutation can be perceived as the following situation in which the owner of one of the K

firms hires a new manager and then this new manager may with some positive probability choose a different strategy for its firm.

We define the payoff function of each firm k , that produces q_k given that all other firms in the market produce $Q_{-k} = \sum_{l \neq k}^K q_l$, as following:

$$\pi_k(q_k, Q_{-k}) = P(q_k, Q_{-k})q_k - c_k(q_k), \quad \forall k \in \mathcal{K} \quad (1)$$

$P(\cdot)$ is an inverse demand function and decreasing in its arguments and $c_k(\cdot)$ represents an increasing, twice differentiable and convex cost function.

In such a game, where players have different cost functions, defining a concept of evolutionary stability requires a symmetric setup with identical players. Leininger and Markazi (2014)[10] identify a non-Walrasian solution concept for evolutionary stability in an asymmetric oligopoly game where players are not identical with respect to their cost functions. In their model, Selten's (1980)[11] approach is applied to construct a symmetric monomorphic population game out of an asymmetric polymorphic-population game. Leininger and Markazi (2014, Proposition 4)[10] generalize the asymmetric oligopoly setup from two groups of high cost and low cost firms to K groups of firms that differ w.r.t. cost functions. In the present paper, we adopt an analogous asymmetric setup, nevertheless with only one firm in each group, i.e., K firms with K different cost functions.

In order to construct a symmetric monomorphic population game out of an asymmetric game with non-identical players, we define a set of roles or information situations as follows

Definition 1. *A firm may find itself in a number of roles or information situations $i \in \{1, \dots, K\}$ where it must choose its action at each possible role (information situation).¹*

The set of roles here is identical with the set of cost functions $\{c_1, \dots, c_k, \dots, c_K\}$. Further we need a role assignment like so

Definition 2. *A role assignment is a map that assigns without replacement each of roles*

¹We use a dissimilar notation i for a role to be not confused with player's notation k .

$i \in \{1, \dots, K\}$ to one of the K firms.

Firm $k \in \{1, \dots, K\}$ is chosen by our role assignment with probability of $1/K$ as a firm with cost c_k . In this set-up, a firm contemplates behavior before it knows its assigned role $c_k \in \{c_1, \dots, c_K\}$, an action (local strategy) of firm k assigned at role i is to select a pure strategy of q_{ki} and hence

Definition 3. A behavior strategy for a firm k is a vector $\mathbf{q}_k = [q_{k1}, \dots, q_{ki}, \dots, q_{kK}]$ giving a local strategy q_{ki} for each role $i \in \{1, \dots, K\}$.

From this ex-ante point of view the game played in role-contingent strategies is symmetric (see Selten (1980[11], pp. 97-8.) and for a generalization Balkenborg (1994[3], pp.19-20)). Let $\pi_{ki}(q_{ki}, [\mathbf{q}]_{-k})$ be a local payoff of firm k in role i when the other firms play their behavior strategies of $[\mathbf{q}]_{-k} = [\mathbf{q}_1, \dots, \mathbf{q}_{k-1}, \mathbf{q}_{k+1}, \dots, \mathbf{q}_K]$. Therefore one can define the total (expected) payoff function of each firm as follows

Definition 4. Let \mathbf{q}_k and $[\mathbf{q}]_{-k}$ be a behavior strategy for a firm k and behavior strategies of other firms respectively. The total (expected) payoff function for each firm k in the monomorphic population game is

$$E\pi_k([q_{k1}, \dots, q_{ki}, \dots, q_{kK}], [\mathbf{q}]_{-k}) = \sum_{i=1}^K \frac{1}{K} \pi_{ki}(q_{ki}, [\mathbf{q}]_{-k}) \quad \forall k = 1, \dots, K \quad (2)$$

Consider now a finite population evolutionarily stable strategy of the game among these K firms. A strategy is evolutionary stable, if no mutant firm $l \neq k$ which chooses a different behavior strategy than $\mathbf{q}_k^* = [q_{k1}^*, \dots, q_{ki}^*, \dots, q_{kK}^*]$, say, can realize higher total profits than the firms which employ the incumbent behavior strategy \mathbf{q}^* . In other words, no mutant behavior strategy \mathbf{q}^m can invade a population of \mathbf{q}^* strategists successfully. Formally Schaffer's definition (1988)[12] then reads

Definition 5. A behavior strategy profile \mathbf{q}^* is a FPESS if

$$E\pi_k(\mathbf{q}_k^*, \mathbf{q}_l^m, [\mathbf{q}^*]_{-k-l}) > E\pi_l(\mathbf{q}_l^m, [\mathbf{q}^*]_{-l}) \quad \forall \mathbf{q}_l^m \neq \mathbf{q}_k^*, \text{ and } \forall l \neq k. \quad (3)$$

Therefore we write mutant's total payoff and incumbent's total payoff respectively as follows:

$$E\pi_l([q_{l1}^m, \dots, q_{lK}^m], [\mathbf{q}^*]_{-l}) = \sum_{i=1}^K \frac{1}{K} P(q_{li}^m, [\mathbf{q}^*]_{-l}) q_{li}^m - c_i(q_{li}^m) \quad (4)$$

$$E\pi_k([q_{k1}^*, \dots, q_{kK}^*], [q_{l1}^m, \dots, q_{lK}^m], [\mathbf{q}]_{-k-l}) = \sum_{i=1}^K \frac{1}{K} \left(\sum_{j \neq i}^K \frac{1}{K-1} (P(q_{kj}^*, q_{li}^m, [\mathbf{q}^*]_{-k-l}) q_{kj}^* - c_j(q_{kj}^*)) \right) \quad (5)$$

$E\pi_l(\cdot)$ mutant's expected payoff consists of K local payoffs where the mutant assigned to the role i with the uniform probability function of $1/K$. Accordingly, the calculation of incumbent's expected payoff $E\pi_k(\cdot)$ is slightly more complicated and consists of other terms in order to account for the mutant's role. For example, when a mutant is assigned to the role i with the probability of $1/K$ then the incumbent's role $j \neq i$ can be assigned from the $K-1$ possibility with the probability of $1/(K-1)$.

Theorem 1. *In the symmetrized game of the asymmetric oligopoly market with K firms that differ w.r.t. cost functions, there exist a non-Walrasian evolutionary equilibrium where the FPESS quantities satisfy the following equations*

$$P\left(\sum_{k=1}^K q_k^*\right) + P'\left(\sum_{k=1}^K q_k^*\right)(q_k^* - \sum_{l \neq k}^K \frac{1}{K-1} q_l^*) = c'_k(q_k^*) \quad \forall k \in \{1, \dots, K\}. \quad (6)$$

Proof. According to Schaffer (1989)[13], in a playing the field game, we can find a FPESS as the solution of following optimization problem

$$(q_{l1}^*, \dots, q_{lK}^*) = \arg \max_{q_{l1}^m, \dots, q_{lK}^m} \varphi = E\pi_l([q_{l1}^m, \dots, q_{lK}^m], [\mathbf{q}^*]_{-l}) - E\pi_k([q_{k1}^*, \dots, q_{kK}^*], [q_{l1}^m, \dots, q_{lK}^m], [\mathbf{q}]_{-k-l}) \quad (7)$$

First order conditions with respect to q_{li}^m respectively are as follows:

$$\frac{1}{K} (P(\cdot) + P'(\cdot) q_{li}^m - c'_i(q_{li}^m)) - \frac{1}{K} \sum_{j \neq i}^K \frac{1}{K-1} P'(\cdot) q_{kj}^* = 0 \quad \forall i \in \{1, \dots, K\}$$

$P'(\cdot)$ is the derivative of inverse demand function. For the reason that the solution must be symmetric in players and satisfies definition (5), we impose $q_{ki}^m = q_{ki}^* = q_i^*$. Then after rearranging, the following set of equations are obtained

$$P\left(\sum_{i=1}^K q_i^*\right) + P'\left(\sum_{i=1}^K q_i^*\right)\left(q_i^* - \sum_{j \neq i} \frac{1}{K-1} q_j^*\right) = c'_i(q_i^*) \quad \forall i \in \{1, \dots, K\}$$

The set of roles can be identified with the set of players in our asymmetric setup and the proof is complete. \square

In fact the strategy that survives in economic natural selection under playing the field conditions is the relative, not absolute, payoff maximizing strategy. A firm needs to beat the average of expected payoffs over all firms rather than to maximize its absolute expected payoff to be evolutionarily successful.

This theorem represents that, in a homogenous good market with asymmetric cost modeling, the equilibrium price is determined such that a low cost firm obtains a positive markup over its marginal cost while a high cost firms sells in a price lower than its marginal cost. However as it has shown in proposition 2 of Leininger and Markazi (2014)[10], all types of firms can coexists and make a positive profit if the cost functions are sufficiently convex.

It is important to note that the evolutionary equilibrium in the asymmetric setup is different from both Walraisan equilibrium and Cournot equilibrium. Though, in a monomorphic population of firms when they have the same cost functions, FPESS equilibrium reproduces the Walrasian result as the term $(q_k^* - \sum_{l \neq k} \frac{1}{K-1} q_l^*)$ turns out to be zero.

3 Testing data consistency of evolutionary oligopoly model (characterization)

Deriving testable implication from an evolutionary model of oligopoly is important because it better explains conditions for firm survival in the market. Particularly, we are able

to test empirically whether the model described in the previous section effectively holds or not and it also permits us to compare whether it is distinguishable from the competitive Walrasian behavior and the best reply Cournot Nash model of firm behavior. In principal, the inverse demand function and cost functions are not observable however we are able to observe the equilibrium quantities of all firms and the market equilibrium price. Suppose that we are given with the observed dataset on the behavior of an industry or a market with K firms producing a homogenous single good. Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k \in \mathcal{K}}\}_{t \in \mathcal{T}}$ where $p_t^* > 0$ is an observed price of homogenous market good at time $t \in \mathcal{T} = \{1, \dots, T\}$ and $q_{k,t}^* > 0$ is observed output quantity of each firm $k \in \mathcal{K} = \{1, \dots, K\}$ in every period of t . Total output of industry $Q_t^* = \sum_{k=1}^K q_{k,t}^*$ is also observed for each time t . Market demand of this single good is determined by a continuous and differentiable inverse demand function P_t at each time t and we assume that it is decreasing in its argument. In addition, each firm k has also a continuous and increasing function of c_k .

Before addressing the data consistency test for the evolutionary oligopoly model, we first give the definitions for the rationalization of Cournot Nash model and perfect competition (Walrasian) model of firm behavior with an observational dataset. Carvajal et al.(2013)[4] formally define and characterize the consistency of a dataset with the Cournot model under convex cost functions² as follows:

²As it has been argued in the paper Cournot rationalizability on its own does not impose operational constraints on the observation set across time and it requires assuming a convexity property for cost functions.

Definition 6 (Cournot rationalizability with convex cost functions). *Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k \in \mathcal{X}}\}_{t \in \mathcal{T}}$. This dataset is Cournot rationalizable if there exist convex cost functions $c_k(\cdot) \forall k \in \mathcal{K}$ and decreasing inverse demand functions $P_t(\cdot)$ for each $t \in \mathcal{T}$ that satisfy the following two conditions*

1. $P_t(Q_t^*) = p_t^*$
2. $P_t(Q_t^*) + P_t'(Q_t^*)q_{k,t}^* = c_k'(q_{k,t}^*)$

The first condition connects unobserved inverse demand function evaluated at total output of industry Q_t^* to observed prices in each time t and the second condition states that $q_{k,t}^*$, given the output of other firms (best responses of other firms $q_{l,t}^*, l \neq k$), must solve the first order condition of firm k 's profit maximization problem at each time t .

Note that the approach explained here, without making any parametric assumption about demand curve and cost functions, checks the consistency of Cournot model with a set of observations. Similarly we can define the rationalization of our evolutionary oligopoly model with an observational dataset as follows:

Definition 7 (Evolutionary rationalizability). *Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k \in \mathcal{X}}\}_{t \in \mathcal{T}}$. This dataset is evolutionary rationalizable if there exist convex cost functions $c_k(\cdot) \forall k \in \mathcal{K}$ and decreasing inverse demand functions $P_t(\cdot)$ for each $t \in \mathcal{T}$ that satisfy the following two conditions*

1. $P_t(Q_t^*) = p_t^*$
2. $P_t(Q_t^*) + P_t'(Q_t^*)\widehat{q}_{k,t}^* = c_k'(q_{k,t}^*)$ where $\widehat{q}_{k,t}^* = (q_{k,t}^* - \sum_{l \neq k} \frac{1}{K-1} q_{l,t}^*)$

The difference between Definition 6 and Definition 7 comes from the second condition which it is inferred directly from equation 6. Here the relative quantity terms of $\widehat{q}_{k,t}^*$ are substituted in the markup term as opposed to the absolute quantity terms of $q_{k,t}^*$. In a relative contest implied by evolutionary successfulness, relative position of firm k at time t in market i.e. $\widehat{q}_{k,t}^*$ is determined by its market share $q_{k,t}^*$ subtracted from the market share

average of the rest of firms in the market $\sum_{l \neq k}^K \frac{1}{K-1} q_{l,t}^*$. That is why, in an oligopolistic competition, a low cost firm chooses larger market share in order to benefit a higher markup over its marginal cost and it pushes a high cost firm to choose a smaller market share with a marginal cost higher than market price.

To compare with Cournot model and evolutionary model, we further consider the perfect competition model and provide also a definition of Walrasian rationalizability. Price taking behavior Walrasian (Perfect competition) model sets the marginal cost of each firm k at time t equal to market price at time t .

Definition 8 (Walrasian rationalizability). *Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k \in \mathcal{K}}\}_{t \in \mathcal{T}}$. This dataset is Walrasian rationalizable if there exist convex cost functions $c_k(\cdot) \forall k \in \mathcal{K}$ and decreasing inverse demand functions $P_t(\cdot)$ for each $t \in \mathcal{T}$ that satisfy the following condition $P_t(Q_t^*) = p_t^* = c'_k(q_{k,t}^*)$.*

Analogous to Carvajal et al.(2013)[4], we define $c'_k(q_{k,t}^*)$ as a set of subgradients of $c_k(\cdot)$ at $q_{k,t}^*$ and $P'_t(Q_t^*)$ as a set of gradients of inverse demand function $P_t(\cdot)$ at Q_t^* and assume that the set of observations $\{p_t^*, (q_{k,t}^*)_{k \in \mathcal{K}}\}_{t \in \mathcal{T}}$ is consistent with evolutionary oligopoly model. Let's say there exist a set of numbers $x_{k,t} \geq 0$ and $y_t \leq 0$ respectively belong to the subsequent sets of $c'_k(q_{k,t}^*)$ and $P'_t(Q_t^*)$ that satisfy the first order condition 2) in definition 7 of firm k at each time t . Then after substituting condition 1) into the condition 2), we obtain the following property

$$y_t = \frac{x_{1,t} - p_t^*}{\widehat{q_{1,t}^*}} = \frac{x_{2,t} - p_t^*}{\widehat{q_{2,t}^*}} = \dots = \frac{x_{K,t} - p_t^*}{\widehat{q_{K,t}^*}} \leq 0 \quad (8)$$

We denote equation 8 as the Joint demand slope property³ if it is satisfied for each $t \in \mathcal{T}$. Moreover we have another type of restrictions imposed by convexity of cost functions, that is,

$$\text{if } q_{k,t'}^* < q_{k,t}^* \text{ then } x_{k,t'} \leq x_{k,t} \quad \forall k \in \mathcal{K}$$

³This property is known by Carvajal et al. (2013)[4] as common ratio property.

This set of restrictions, the so-called *co-monotone property*, imposes across time for each firm k and it can also be expressed as

$$(q_{k,t'}^* - q_{k,t}^*)(x_{k,t'} - x_{k,t}) \geq 0 \quad (9)$$

This says that the set of $\{x_{k,t}\}_{(\forall t \in \mathcal{T} \& \forall k \in \mathcal{K})}$ obeys increasing marginal costs. So we say that a non-increasing (decreasing) inverse demand function $P_t(\cdot)$ and convex cost functions $c_k(\cdot)$ *evolutionarily* rationalize the dataset if the set of $\{x_{k,t}\}_{(\forall t \in \mathcal{T} \& \forall k \in \mathcal{K})}$ satisfies the above two properties. Hence the following theorem summarizes the above discussion.

Theorem 2. *The set of observations $\{p_t^*, (q_{k,t}^*)_{k \in \mathcal{K}}\}_{t \in \mathcal{T}}$ is consistent with evolutionary model under convex cost functions if and only if there exist two number sets of $\{y_t \leq 0\}_{(\forall t \in \mathcal{T})}$ and $\{x_{k,t} \geq 0\}_{(\forall t \in \mathcal{T} \& \forall k \in \mathcal{K})}$ that satisfy the following properties.*

1. $y_t = \frac{x_{k,t} - p_t^*}{q_{k,t}^*} \leq 0$ where $\widehat{q_{k,t}^*} = (q_{k,t}^* - \sum_{l \neq k}^K \frac{1}{K-1} q_{l,t}^*) \quad \forall t \in \mathcal{T} \text{ and } \forall k \in \mathcal{K}$
2. $(q_{k,t'}^* - q_{k,t}^*)(x_{k,t'} - x_{k,t}) \geq 0 \quad \forall t, t' \in \mathcal{T} \text{ and } \forall k \in \mathcal{K}$

Proof. Assume that the set of observations is consistent with cost functions $\{c_k\}_{\forall k \in \mathcal{K}}$ and demand functions $\{P_t\}_{\forall t \in \mathcal{T}}$ then we have already proved that there exist $x_{k,t} \in c'_k(q_{k,t}^*)$ & $y_t \in P'_t(Q_t^*)$ that satisfy the properties of 1 and 2.

To show the reverse, first of all, it is required to show that if we have positive scalars $\{x_{k,t}\}_{(\forall t \in \mathcal{T})}$ that are increasing with $q_{k,t}^*$ for some firm k ; then there exist a convex cost function c_k with $x_{k,t} \in c'_k(q_{k,t}^*)$. The proof of this statement follows exactly from Carvajal et al.(2013)[4]. (See Carvajal et al.(2013, Lemma 2)[4]).

Secondly, suppose that there are $\{x_{k,t}\}_{(\forall t \in \mathcal{T} \& \forall k \in \mathcal{K})}$ such that the joint demand slope property and co-monotone property hold and moreover there are convex cost functions c_k with $x_{k,t} \in c'_k(q_{k,t}^*)$. Then, we show that $\{(q_{k,t}^*)_{k \in \mathcal{K}}\}_{t \in \mathcal{T}}$ form an evolutionary equilibrium if there exist a non-increasing demand function $P_t(\cdot)$ such that $P_t(Q_t^*) = p_t^*$ and with firms having cost functions c_k .

Following Carvajal et al.(2013, Lemma 1)[4], let's define P_t by $P_t(Q) = \alpha_t + \beta_t Q$, where $\beta_t = \frac{x_{k,t} - p_t^*}{\widehat{q_{k,t}^*}}$ and we can choose α_t such that $P_t(Q_t^*) = p_t^*$. A mutant firm l in our symmetrized version of evolutionary game selects a different behavior strategy $\mathbf{q}_{l,t}^m = [q_{l1,t}^m, \dots, q_{lK,t}^m]$ from other incumbent firms $[\mathbf{q}^*]_{-l,t} = [\mathbf{q}_{1,t}^*, \dots, \mathbf{q}_{l-1,t}^*, \mathbf{q}_{l+1,t}^*, \dots, \mathbf{q}_{K,t}^*]$ at time t . Here a mutant firm l chooses a local strategy $q_{li,t}^m \geq 0$ to maximize the following relative total payoff (equation 7) at each role i and time t

$$\varphi_{l,t} = \sum_{i=1}^K \frac{1}{K} P_t(q_{li,t}^m, [\mathbf{q}^*]_{-l,t}) q_{li,t}^m - c_i(q_{li,t}^m) - \sum_{i=1}^K \frac{1}{K} \left(\sum_{j \neq i}^K \frac{1}{K-1} (P_t(q_{kj,t}^*, q_{li,t}^m, [\mathbf{q}^*]_{-k-l,t}) q_{kj,t}^* - c_j(q_{kj,t}^*)) \right)$$

Since $\varphi_{l,t}$ is concave, $q_{li,t}^m$ is optimal if and only if it satisfies the following FOC evaluated at $q_{li,t}^m = q_{ki,t}^* = q_{i,t}^*$

$$P(Q_t^*) + q_{i,t}^* P'(Q_t^*) - \sum_{j \neq i}^K \frac{1}{K-1} q_{j,t}^* P'(Q_t^*) - c'_i(q_i^*) \quad \forall i \in \{1, \dots, K\}$$

As the set of roles is equivalent with the set of players and we also have $q_{k,t}^* - \sum_{j \neq i}^K \frac{1}{K-1} q_{l,t}^* = \widehat{q_{k,t}^*}$, $x_{k,t} \in c'_k(q_{k,t}^*)$ and $P'(Q_t^*) = \beta_t = \frac{x_{k,t} - p_t^*}{\widehat{q_{k,t}^*}}$

$$p_t^* + \widehat{q_{k,t}^*} \left(\frac{x_{k,t} - p_t^*}{\widehat{q_{k,t}^*}} \right) - x_{k,t} = 0$$

Therefore we have proved that $q_{k,t}^*$ constitute an evolutionary equilibrium for firm k at observation t and this also completes the proof that the set of observations $\{p_t^*, (q_{k,t}^*)_{k \in \mathcal{X}}\}_{t \in \mathcal{T}}$ for all t constitutes the evolutionary equilibrium. \square

Note that the relative quantity terms of $\widehat{q_{k,t}^*}$ may be negative or positive and knowing that the slope of demand curve is negative at each time t , we must have $x_{k,t} < p_t^*$ if $\widehat{q_{k,t}^*} > 0$ and $x_{k,t} > p_t^*$ if $\widehat{q_{k,t}^*} < 0$. So this condition can be summarized in the following form $(x_{k,t} - p_t^*) \widehat{q_{k,t}^*} < 0$.

In general, these properties impose linear restrictions with unknowns $x_{k,t}$ that can be

checked by linear programming (LP) or quadratic programming (QP) methods. Therefore the test takes the form of mathematical optimization problem in which the consistency of a dataset with the evolutionary model would be verified if the linear constraints could produce a convex feasible region of possible values for those unknowns. This feasible region is a convex polytope that formed as the intersection of finitely many half spaces defined by the following sets of linear restrictions of i-iii

- i. $\frac{-x_{k,t}}{\widehat{q_{k,t}^*}} + \frac{x_{l,t}}{\widehat{q_{l,t}^*}} = \frac{-p_t^*}{q_{k,t}^*} + \frac{p_t^*}{q_{k,t}^*} \quad \forall k, l \in \mathcal{K}, k \neq l \text{ and } \forall t \in \mathcal{T}$
- ii. $\widehat{q_{k,t}^*} x_{k,t} < p_t^* \widehat{q_{k,t}^*} \quad \forall k \in \mathcal{K} \text{ and } \forall t \in \mathcal{T}$
- iii. $(q_{k,t'}^* - q_{k,t}^*)(x_{k,t'} - x_{k,t}) \geq 0 \quad \forall k \in \mathcal{K}, \forall t, t' \in \mathcal{T} \text{ and } t \neq t'$

where $\widehat{q_{k,t}^*} = \frac{K}{K-1} q_{k,t}^* - \frac{\sum_{i=1}^K q_{k,t}^*}{K-1}$ Note that Cournot rationalizability imposes a different set of restrictions of iv-v on the dataset nevertheless the co-monotone condition iii is the same.

- iv. $\frac{-x_{k,t}}{\widehat{q_{k,t}^*}} + \frac{x_{l,t}}{\widehat{q_{l,t}^*}} = \frac{-p_t^*}{q_{k,t}^*} + \frac{p_t^*}{q_{k,t}^*} \quad \forall k, l \in \mathcal{K}, k \neq l \text{ and } \forall t \in \mathcal{T}$
- v. $q_{k,t}^* x_{k,t} < p_t^* q_{k,t}^* \quad \forall k \in \mathcal{K} \text{ and } \forall t \in \mathcal{T}$
- iii. $(q_{k,t'}^* - q_{k,t}^*)(x_{k,t'} - x_{k,t}) \geq 0 \quad \forall k \in \mathcal{K}, \forall t, t' \in \mathcal{T} \text{ and } t \neq t'$

Example 1 illustrates a dataset that is not consistent with the Cournot model but can be rationalized by the evolutionary model. In this example, firm 1 competes spitefully to increase its production levels and this leads to an abrupt fall of the market good price at time 2. This spiteful behavior, where one player harms itself in order to harm another more, cannot be explained by Cournot competition. Furthermore this example explains the recent dropping trend of crude oil price in which OPEC as the main oil markets producer did not alter its production level in response to growing of USA production caused by new extraction techniques of shale oil. This observed behavior cannot be rationalized by the Cournot competition model but rather by the relative payoff maximizers of the evolutionary model.

Example 1. Consider a homogenous good market with two firms 1 and 2 and assume that we observe the produced quantities of each firm and the market price of good in the two sequential periods as follows:

$$\text{At observation } t, \quad p_t^* = 100, q_{1,t}^* = 6, \text{ and } q_{2,t}^* = 30.$$

$$\text{At observation } t', \quad p_{t'}^* = 50, q_{1,t'}^* = 9, \text{ and } q_{2,t'}^* = 30.$$

To see whether this dataset is Cournot rationalizable with convex cost function, it is required to find a set of numbers assigned to marginal costs i.e. $x_{1,t}, x_{1,t'}, x_{2,t}, x_{2,t'} \geq 0$ that satisfy the restrictions (iii-iv). So we have

$$\begin{aligned} 5x_{1,t} - x_{2,t} &= 400, & 10x_{1,t'} - 3x_{2,t'} &= 350 \\ x_{1,t}, x_{2,t} &\leq 100, & x_{1,t'}, x_{2,t'} &\leq 50 \\ x_{1,t} &\leq x_{1,t'} \end{aligned}$$

Note that co-monotone property does not impose a restriction on firm 2. So it is straightforward to check that the solution space defined by these restrictions has not a feasible region. (Since $5x_{1,t} - x_{2,t} = 400$ does not intersect with the region $0 \leq x_{1,t} \leq 50, 0 \leq x_{2,t} \leq 100$.) As a result, this dataset cannot be rationalized by Cournot model.

However the evolutionary rationalizability (linear restrictions of i-iii) leads to

$$\begin{aligned} x_{1,t} + x_{2,t} &= 200, & x_{1,t'} + x_{2,t'} &= 100 \\ x_{1,t} \geq 100, x_{2,t} &\leq 100, & x_{1,t'} \geq 50, x_{2,t'} &\leq 50 \\ x_{1,t} &\leq x_{1,t'} \end{aligned}$$

Solving for a feasible region, the set of candidate solutions narrows down as follows

$$\begin{aligned} x_{1,t} &= 100, x_{2,t} = 100, y_t = 0 \\ x_{1,t'} &= 100, x_{2,t'} = 0, y_{t'} = -50/21 \end{aligned}$$

Along to the identification of marginal costs, the slopes of demand curve in each time i.e. $y_t, y_{t'} \leq 0$ have been identified by formula $y_t = \frac{x_{k,t} - p_t^*}{q_{k,t}^*}$.

4 A case study in the oil market

In this section we apply the consistency test explained in the section 3 to a dataset of the crude oil market. In the beginning we use exactly the same dataset of Carvajal et al.(2013)[4] to compare the effectiveness of our evolutionary model with the Cournot model. This dataset contains monthly series of crude oil production in thousands of barrels per day by 7 non-OPEC countries of Canada, China, Egypt, Mexico, Norway, USA and UK and 12 OPEC countries of Algeria, Angola, Ecuador, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, UAE, and Venezuela. The data sources include oil production series from Monthly Energy Review (MER) and West Texas Intermediate (WTI) crude oil price series in dollars per barrel taken from St. Louise Federal Reserve from January 1973 until April 2009. Furthermore, the whole dataset is split up into the several subsets so that each set is made of W sequential months (as time windows) and I countries. Then we modify their MATLAB code so as to capture the features of our relative evolutionary test. Table 1 displays a comparison between rejection rates of both tests. As we see the rejection rates are not very different.

OPEC countries produce approximately at least 40 percent of the world's oil since its formation in 1960 and this share is even more for the internationally traded oil. The OPEC member countries decide about the production levels during their regular meetings held twice every year and therefore they do not choose their production level monthly. So then we test both hypotheses with a less frequent dataset of the oil market in which we use an annual dataset from 1973 until 2015. Average annual crude oil productions are from the same source of MER where Russia is added to the non-OPEC dataset as well.⁴ Furthermore we employ another source for series of oil price. In addition to annual nominal WTI crude oil prices, we use also price series of annual averages of selected OPEC crude oils (OPEC basket) published by association of German petroleum industry (MWV). The rejection rates

⁴Russia (formerly Soviet Union) is a major oil producer that can be used as a one unit since most of oil production in Soviet Union (around more than 95 percent) was produced in the present-day territory of Russia.

are reported in Table 2. Here again we do not observe a strictly different rejection rates between two models.

Lastly we test both hypothesis among three major players in the oil market that is, OPEC total production as a single unit, Russia and USA. These three top oil producers are accounted for over than 85% of the world's oil production in 1973 and over than 66% in 2014. Table 3 illustrates the rejection rates for the subsets of this data (with the number of countries $I = 2, 3$ and windows W from 2 years up to 6 years). Comparing the evolutionary model to the Cournot model, we see that the rejection rates jump down for the evolutionary model. For example, in case of $I=2$ countries and $T=2$ years window, the drop in rejection rates is more than 50 percent (from 0.398 to 0.187). Put side by side both Cournot and evolutionary models, we conclude that evolutionary oligopoly model explains better the dataset of main producers (OPEC, Russia and USA) in the oil market.

Table 1: Rejection rates of Cournot and evolutionary models(Monthly datasets)

OPEC monthly datasets								
W	Cournot model				Evolutionary model			
	I=2	I=3	I=6	I=12	I=2	I=3	I=6	I=12
3 months	0.284	0.539	0.890	0.995	0.318	0.587	0.912	0.995
6 months	0.646	0.887	0.995	1.000	0.685	0.919	0.998	1.000
12 months	0.901	0.988	1.000	1.000	0.930	0.995	1.000	1.000

Non-OPEC monthly datasets								
W	Cournot model				Evolutionary model			
	I=2	I=3	I=6	I=7	I=2	I=3	I=6	I=7
3 months	0.439	0.751	0.988	0.995	0.433	0.748	0.974	0.981
6 months	0.831	0.978	1.000	1.000	0.829	0.982	0.998	0.998
12 months	0.961	0.998	1.000	1.000	0.971	0.999	1.000	1.000

WTI oil price series are applied.

Table 2: Rejection rates of Cournot and evolutionary models(Annual datasets)

OPEC Annual datasets with OPEC basket price series								
W	Cournot model				Evolutionary model			
	I=2	I=3	I=6	I=12	I=2	I=3	I=6	I=12
2 years	0.246	0.451	0.761	0.902	0.262	0.467	0.773	0.902
3 years	0.508	0.762	0.941	0.975	0.500	0.767	0.949	0.975
4 years	0.705	0.895	0.987	1.000	0.701	0.914	0.992	1.000

Non-OPEC Annual datasets with WTI oil price series								
W	Cournot model				Evolutionary model			
	I=2	I=3	I=6	I=8	I=2	I=3	I=6	I=8
2 years	0.290	0.527	0.858	0.927	0.294	0.525	0.830	0.902
3 years	0.510	0.782	0.979	1.000	0.495	0.748	0.949	1.000
4 years	0.646	0.878	0.994	1.000	0.653	0.860	0.986	1.000

Table 3: Rejection rates of Cournot and evolutionary models with main oil producers

OPEC, Russia and USA, Annual datasets				
W	Cournot model		Evolutionary model	
	I=2	I=3	I=2	I=3
2 years	0.398	0.683	0.183	0.439
3 years	0.608	0.850	0.333	0.750
4 years	0.744	0.923	0.487	0.872
5 years	0.833	0.974	0.596	0.921
6 years	0.919	1.000	0.694	0.946

OPEC basket price series are applied.

5 Conclusion

The contributions of present study are twofold. Firstly we show that a static evolutionary model offers a different solution than a competitive Walrasian equilibrium. In fact, here we take issue with the result by Vega-Redondo (1997)[15] that the imitation of successful strategies leads to the competitive equilibrium outcome in the symmetric quantity game of a homogenous good market. Apesteguia et al. (2010)[2] also show that Vega-Redondo's result is not robust to the slightest asymmetry in fixed costs. Then secondly we design for practical purposes a revealed preference test to check the consistency of developed model with a generic set of observations based on the work of Carvajal et al.(Econometrica 2013)[4]. Therefore, contrary to the typical empirical literature in IO without making any parametric assumption regarding to the demand curve and the cost function, this approach characterizes a set of conditions (restrictions) on observational dataset to be consistent with evolutionary oligopoly model. Finally, this nonparametric revealed preference test has been applied to a dataset for oil market and we conclude that the behavior of top oil producers in the market is more consistent with an evolutionary game than a Cournot game.

References

- Afriat, S.: Efficiency Estimates of Production Functions. *International Economic Review* 13, 568-598 (1972).
- Apestequia, J., Huck, S., Oechssler, J., Weidenholzer, S.: Imitation and the evolution of Walrasian behavior: Theoretically fragile but behaviorally robust. *Journal of Economic Theory* 145, 1603-1617 (2010).
- Balkenborg, D.: Strictness and evolutionary stability. Mimeo, University of Bonn (1994)
- Carvajal, A., Deb, R., Fenske, J., Quah, J.K.-H.: Revealed preference tests of the Cournot model. *Econometrica* 81(6), 2351-2379 (2013)
- Carvajal, A., Deb, R., Fenske, J., Quah, J.K.-H. : A nonparametric analysis of multi-product oligopolies. *Econ Theory* 57, 253-277 (2014)
- Cherchye, L., Demuynck, T., De Rock, B.: The empirical content of Cournot competition. *Journal of Economic Theory* 148, 1552-1581 (2013)
- Friedman, M.: *Essays in positive economics*. University of Chicago Press, Chicago. (1953)
- Hamilton, W.D.: Selfish and spiteful behaviour in an evolutionary model. *Nature* 228, 1218-1220 (1970)
- Hanoch, G., Rothschild, M.: Testing the Assumptions of Production Theory: A Nonparametric Approach, *Journal of Political Economy* 80, 256-275 (1972)
- Leininger, W., Moghadam, H.M.: Evolutionary stability in asymmetric oligopoly: a non-Walrasian result. *Ruhr economic papers #497* (2014)
- Selten, R.: A note on evolutionarily stable strategies in asymmetric animal conflict. *J. Theor. Biol.* 84, 93-101 (1980)

Schaffer, M.: Evolutionarily stable strategies for a Finite Population and a variable contest size. *J Theor Biol* 132, 469-478 (1988)

Schaffer, M.: Are profit-maximisers the best survivors? *J Econ Behav Org* 12, 29-45 (1989)

Varian, H.: The nonparametric approach to production analysis. *Econometrica* 52, 579-597 (1984)

Vega-Redondo, F.: The evolution of Walrasian behavior. *Econometrica* 65, 375-384 (1997)