Shen, Zhiwei; Ritter, Matthias

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Forecasting volatility of wind power production

Zhiwei Shen*
Matthias Ritter*

* Humboldt-Universität zu Berlin, Germany

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Forecasting volatility of wind power production*

Zhiwei Shen** and Matthias Rittera

Abstract: The increasing share of wind energy in the portfolio of energy sources highlights its uncertainties due to changing weather conditions. To account for the uncertainty in predicting wind power production, this article examines the volatility forecasting abilities of different GARCH-type models for wind power production. Moreover, due to characteristic features of the wind power process, such as heteroscedasticity and nonlinearity, we also investigate the use of a Markov regime-switching GARCH (MRS-GARCH) model on forecasting volatility of wind power. The realized volatility, which is derived from lower-scale data, serves as a benchmark for the latent volatility. We find that the MRS-GARCH model significantly outperforms traditional GARCH models in predicting the volatility of wind power, while the exponential GARCH model is superior among traditional GARCH models.

Keywords: Wind energy, volatility forecasting, GARCH models, Markov regime-switching, realized volatility

JEL code: C22, Q42, Q47

1. Introduction

Increasing energy demand and the negative impact of fossil energy consumption on climate change impacts have led to a worldwide boom of renewable energies such as wind energy. The global cumulative installed wind energy capacity increased from 24 GW in 2001 to 370 GW in 2014 and is expected to reach 596 GW until 2018 (GWEC, 2014; 2015). However, relying on renewable energy to meet increasing energy demand is still problematic. One of the main concerns of renewable energy production is its riskiness due to changing weather conditions. This is particularly true for wind energy production, which is the most rapidly expanding energy source. Volume risk is an important economic issue since energy is a non-storable commodity. In view of the increasing share of risky renewable energies in the portfolio of

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** Corresponding author: zhiwei.shen@agrar.hu-berlin.de, Tel.: +49 (0) 30209346834, Fax: +49 (0) 30209346841.
energy sources a quantification of the production risk has gained considerable attention.\(^1\) Interest in prediction of wind energy production is manifold: In the long run (several years), investors want to predict their returns on investments in wind energy production. In the short run (several hours to days), grid operators have to make decisions about energy scheduling in order to balance supply and demand on a regional or national grid. Moreover, energy traders want to make informed decisions on how much they can offer or bid in the next trading cycle. This requires reliable forecasts of the output of wind energy farms.

Two main streams of approaches and models have been proposed to generate wind power forecasts. The first type are physical or meteorological models. They rely on Numerical Weather Prediction models to determine meteorological forecasts, which are then transformed into wind power forecast via a power curve (Monterio et al., 2009). The second type are mathematical or statistical approaches. They use the statistical models (e.g., time series models, data mining models such as neural networks or support vector machines) to identify the spatial-temporal relationship between the wind power production and explanatory variables (e.g., historical wind power data). Based on this relationship the wind power forecasts are estimated from the observed explanatory variables (Brown et al., 1984; Bilgili et al., 2007). The strengths of different models rest on the different forecast horizons. An overview about the various modelling approaches can be found in Giebel et al. (2011) and Kusiak et al. (2013).

Regardless of these approaches and models, two kinds of forecasts can be considered. Early research focused on point forecasts of wind energy production, i.e., a single value of conditional expectation of wind power production is predicted. To make optimal decisions for energy participants, however, it is not sufficient to know only the expected wind power production. The actual production most likely deviates from the forecast and their difference causes imbalance costs for market participants. Therefore, participants in the energy market need an assessment of the uncertainty involved in the prediction. To account for the uncertainty of wind power production, probabilistic forecasts of wind energy production have been proposed (Bremnes, 2004; Pinson et al., 2007; Trombe et al., 2012). Probabilistic forecasts are more flexible than point forecasts and can be quantile or interval forecasts (Bremnes, 2004; Anastasiades and McSharry, 2013) or full predictive density forecasts (Lau and McSharry, 2010).

\(^1\) Actually, the Economist Intelligence Unit (2011) reports from a survey among 280 executives and investors in the renewable energy industry that 66\% percent of the respondents were concerned about volumetric risk in wind energy production.
One crucial parameter that captures the uncertainty of wind power production in probabilistic forecasting is volatility. It is also a determinant of financial risk management instruments such as insurance or wind derivatives. To measure the volatility of wind power production, characteristic features of wind data have to be taken into account. First of all, it has to be recognized that not only wind speed but also its volatility is usually time-varying. In a medium term perspective, seasonal effects of wind activity have to be considered (Šaltytė-Benth and Benth, 2010). In addition, one can observe stochastically time-varying heteroscedasticity similar to financial markets (Lau and McSharry, 2010). Moreover, wind power production can be affected by ramp events when energy output changes by a substantial fraction of the capacity within short time. Ramp events can be caused by a passage of large scale weather systems or by thunderstorms. As a result, high wind speed shutdowns may occur, causing a rapid decrease in wind power production.

So far, a variety of volatility models have been applied either to wind speed data or to wind power data to capture time-varying heteroscedasticity. Alexandridis and Zapranis (2013) estimate an ARIMA model for daily average wind speed data and model seasonal variation of the volatility with a truncated Fourier series. The prevalent models, however, are autoregressive conditional heteroscedasticity (ARCH) and generalized autoregressive conditional heteroscedasticity (GARCH). Tastu et al. (2014) use an ARCH model to generate the variances in the probabilistic forecasts of wind power production for an offshore wind farm in Denmark. Liu et al. (2011) evaluate the effectiveness of ARMA-GARCH approaches for modeling the mean and volatility of wind speed, including different GARCH models such as EGARCH and TGARCH. Lau and McSharry (2010) identify an ARIMA-EGARCH model for aggregated wind power data in Ireland and produce forecasts of the wind power density up to 24 hours ahead. However, wind speed or wind power data exhibit random breaks and nonlinear behaviors. The classic ARMA and ARMA-GARCH models may be too restrictive to capture such nonlinear dynamic process. Recently, a Markov regime switching model has been proposed and found suitable to model the dynamic behavior of wind power (Pinson et al., 2008) and wind speed (Song et al., 2014). A nice feature of this model is that it allows for reflecting the impact of random external factors since the regime switching process is driven by a Markov chain. To account for the nonlinearity and heteroscedasticity of volatility, we propose the use of Markov regime switching GARCH (MRS-GARCH) model as an alternative for modeling the volatility of wind power production in this article.
In contrast to previous studies aiming at forecasting wind power production, this paper focuses on volatility forecasts and explores the performance of volatility forecasting within the class of GARCH models including Markov regime switching GARCH. A volatility forecast comparison can be difficult since the true, latent volatility is unobservable. As a result, the predicted value must be compared with an *ex post* proxy of volatility, e.g., realized volatility. The concept of realized volatility was initially introduced in financial market due to the availability of high frequency financial data (Andersen et al., 2003). The daily volatility of stock price is calculated by summing squared intraday returns. As a model-free estimator, realized volatility has often been used as an *ex post* proxy to evaluate the volatility forecast models in financial and energy markets (Marcucci, 2005; Brownlees et al., 2011; Byun and Cho, 2013). To our knowledge, realized volatility has not been exploited in the wind power production analysis so far.

In this article we use wind power production from a wind farm in Germany. Since the interest of energy market participants traditionally lie on the hourly forecast as required by the market structure (Trombe and Pinson, 2012), the hourly resolution of wind power production is chosen to generate the hourly volatility forecast through the considered models. To evaluate the forecasted volatility, we are able to access to higher frequency data of wind power production reported for an interval of 10 minutes and then derive the realized volatility as the *ex post* proxy of hourly volatility.

The contribution of this article to the existing literature is threefold. First, we develop a Markov regime switching GARCH model to describe the time-varying volatility of wind power production. This model could allow us to capture the nonlinearity of wind power production due to changing weather conditions or ramp events. Second, an assessment of the performances of the class of GARCH volatility forecasting models is provided. The results show that Markov regime switching GARCH seems to outperform other GARCH models and EGARCH perform second best. Third, this is the first time to explore the use of realized volatility for wind power production. We find that an instant ramp event within an hour causes a bias in realized volatility estimator for hourly spot volatility.

The rest of the paper is structured as follows. The next section describes the volatility forecasting methodology: we briefly give an overview of the traditional GARCH models, and then describe the Markov regime switching GARCH model in detail as well as forecast evaluation criteria. In the subsequent section, these models are applied to wind power data from
a wind farm in Germany. We present the comparison of those volatility forecasting models, in-sample and out-of-sample respectively. The last section provides conclusion and discussion on the benefit of Markov regime switching GARCH model and offers suggestions for further research.

2. Volatility Forecasting Methodology

2.1 Traditional GARCH Models

Prior to modelling the volatility of wind power production, an appropriate Autoregressive (AR) model that captures the time-varying means of wind power data is often used (Trombe et al., 2012). Considering a time series of wind power \( \{y_t\} \), we use the AR\((k)\) model given by:

\[
y_t = c + \sum_{i=1}^{k} \phi_i y_{t-i} + \varepsilon_t,
\]

where \( c \) is a constant, \( k \) the order of autoregressive terms, \( \phi_i \) the \( i \)th autoregressive coefficient and \( \varepsilon_t \) the error term. Suppose that the error term \( \varepsilon_t \) has a time-varying variance and it can be represented as:

\[
\varepsilon_t = \eta_t h_t,
\]

where \( \eta_t \) is an iid random variable with mean 0 and variance 1; \( h_t \) is the conditional standard deviation at \( t \) given the information set \( \Omega_{t-1} = \{y_{t-1}, \ldots, y_1\} \) at \( t - 1 \), i.e., \( h_t^2 = V[\varepsilon_t | \Omega_{t-1}] \).

The specification of \( h_t^2 \) determines the conditional variance evolution and the forecast of volatility at the next periods. To determine the specification of the conditional variance, a variety of GARCH models have been developed. The traditional GARCH\((p, q)\) model was proposed by Bollerslev (1986) consisting of the order \( k \) of the moving average ARCH term and the order \( m \) of the autoregressive GARCH term. It can be written as:

\[
h_t^2 = \omega + \sum_{i=1}^{k} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{m} \beta_j h_{t-j}^2,
\]

where \( \omega \) is the constant term, \( \varepsilon_{t-i}^2 \) the ARCH term, \( h_{t-j} \) the GARCH term, and \( (\alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q) \) are the coefficient parameters to be estimated. It indicates that the variance depends on previous errors and also previous conditional variances. To ensure a stationary and positive conditional variance, the parameters are confined to satisfy \( \omega > 0, \alpha_i \geq 0, \beta_t \geq 0 \) and \( \sum_{i=1}^{k} \alpha_i + \sum_{j=1}^{m} \beta_j < 1 \) (Tsay, 2010). Although any order of GARCH model is possible, in general a GARCH\((1, 1)\) is sufficient to capture the volatility clustering in the data.
It is called the standard GARCH model, denoted as SGARCH in this paper. Then, Equation (3) reduces to:

$$h_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2,$$

(4)

The important features of this GARCH model are its mean reversion due to the stationary condition ($\alpha_1 + \beta_1 < 1$) and its symmetry (i.e., the sign of the error term has no influence on the future volatility).

**Variants of GARCH models**

To account for asymmetric effects in volatility forecasting, several GARCH models have been developed. In this paper, we consider four models from the vast literature, namely exponential GARCH (EGARCH), threshold GARCH (TGARCH), Glosten-Jagannathan-Runkle GARCH (GJR GARCH) and nonlinear GARCH (NGARCH). We chose these four model for the following reasons: first, these GARCH models have been widely recommended due to their simplicities and demonstrated abilities to forecast volatility (Brownlees et al., 2011); second, Liu et al. (2011) applied the above-mentioned GARCH models for wind speed volatility and found that the volatility of wind speed has the nonlinear and asymmetric time-varying properties. Since wind speed is the main driving factor of wind power production, it is reasonable to refer to wind speed modelling. A brief overview of these GARCH models and how they deal with the asymmetry is provided below.

The EGARCH model was proposed by Nelson (1991) in the form of the log of variance as

$$\log(h_t^2) = \omega + \sum_{i=1}^{k} \alpha_i [|\eta_{t-i}| - E(|\eta_{t-i}|)] + \sum_{j=1}^{m} \beta_j \log(h_{t-j}^2) + \sum_{i=1}^{p} \gamma_i \eta_{t-i},$$

(5)

where $\eta_{t-i} = \varepsilon_{t-i}/h_{t-i}$, $\alpha_i$ and $\gamma_i$ capture the asymmetric effect of the sign and the magnitude of $\eta_t$ on the volatility. For example, when $\gamma_i < 0$, the negative value of $\varepsilon_{t-i}$ results in higher volatility than the positive value in EGARCH. Unlike the GARCH model in Equation (3), there is no restriction on the parameters in EGARCH.

The TGARCH developed by Zakoian (1994) models the conditional standard deviation as:

---

2 A more detailed analysis of these asymmetric models can be found in Hentschel (1995).
\[
    h_t = \omega + \sum_{i=1}^{k} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}) + \sum_{j=1}^{m} \beta_j h_{t-j},
\]

where \( \omega > 0, \alpha_i \geq 0, \beta_i \geq 0 \) and \( |\gamma_i| \leq 1 \). Similar to TGARCH, Glosten et al. (1993) introduced the GJR GARCH with the focus on the variance instead of the standard deviation. Formally stated:

\[
    h_t^2 = \omega + \sum_{i=1}^{k} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^{m} \beta_j h_{t-j}^2,
\]

Finally, the NGARCH, also known as nonlinear asymmetric GARCH, was developed by Engle and Ng (1993) in the following specification:

\[
    h_t^2 = \omega + \sum_{i=1}^{k} \alpha_i \varepsilon_{t-i} (\varepsilon_{t-i} - \gamma_i h_{t-i})^2 + \sum_{j=1}^{m} \beta_j h_{t-j}^2,
\]

where \( \omega > 0, \alpha_i \geq 0, \beta_i \geq 0 \). In the TGARCH, GJR GARCH and NGARCH, \( \gamma_i \) reflects the asymmetric effect. When \( \gamma_i > 0 \), the models indicate that negative error terms increase future volatility by a larger amount than positive ones of the same magnitude. In the application, we only use these GARCH models with the order \((1, 1)\).

### 2.2 Markov Regime Switching GARCH Model

When inspecting time series of wind power production, continuous periods with fluctuations of lower and higher magnitudes are often easily noticed and call for the use of regime-switching model. The use of Markov regime switching model proposed by Hamilton (1989) has received a growing interest in the wind power community due to its superior ability to account for structure breaks and random changes in the dynamic process (Pinson et al., 2008; Trombe et al., 2012). The basic idea of this model is allowing the parameters of the model to switch across different regimes (states or phases) according to a Markov process, which is governed by a state variable \( s_t \). In other words, it implies a mixture of processes with different characteristics according to the probability of being in each state. Early studies have mainly applied Markov regime switching models to forecast average wind power production, and the application to volatility forecasting is still rare. In this paper, we allow the regime-switching in both, average wind power production and its volatility, resulting in an extension of the Markov regime-switching model with different GARCH specifications in each regime. A difficulty of the regime switching model is the determination of the number of states since the underlying regimes themselves are unobserved and econometric tests for choosing the optimal number of regimes are still under development. As a result, almost all applications of Markov regime
switching models assume two or three different regimes (Balcilar et al., 2015; Lammerding et al., 2013; Marcucci, 2005). Similarly, we follow Trombe et al. (2012) and consider only two regimes in the Markov regime switching GARCH (MRS-GARCH) model for forecasting volatility of wind power production, i.e., \( s_t \in \{1,2\} \).

**Description of MRS-GARCH model**

The regime switching process follows a first order Markov chain with transition probability:

\[
P(s_t = j | s_{t-1} = i) = p_{ij}
\]

that indicates the probability of the regime switching from state \( i \) at time \( t - 1 \) into state \( j \) at time \( t \). By definition, the probability of switching from state \( i \) at \( t - 1 \) into state \( i \) at \( t \) is: \( p_{ii} = 1 - p_{ij} \) when only two regimes are assumed. The transition matrix \( P \), which describes all the probabilities of regimes switching from one to the other, is given by:

\[
P = \begin{bmatrix}
p_{11} & p_{21} \\
p_{12} & p_{22}
\end{bmatrix} = \begin{bmatrix}
p & 1 - q \\
1 - p & q
\end{bmatrix}.
\]

(10)

Under the framework of MRS-GARCH, we allow the regime-switching in both, the mean and the volatility of wind power production, and each regime is characterised by different parameter sets. Thus, Equation (1) for modelling conditional mean equation of wind power production changes to:

\[
y_t = c^{(i)} + \sum_{i=1}^{k} \phi_{i}^{(i)} y_{t-i} + \varepsilon_t, \tag{11}
\]

where \( i = 1, 2 \) and \( \varepsilon_t = \eta_t h_t \) as in Equation (2). However, the specification of a GARCH model for the conditional variance becomes problematic since the autoregressive structure of the conditional variance makes the specification path-dependent. The conditional variance of \( \varepsilon_t \) being in state \( (i) \) at time \( t \), given the past unobservable regime path \( (s_t, s_{t-1}, ..., s_1) \) and information set \( \Omega_{t-1} \) at time \( t - 1 \), is \( h_t^{2(i)} = V[\varepsilon_t | (s_t, s_{t-1}, ..., s_1), \Omega_{t-1}] \). In the regime switching context, a GARCH model with path dependency would be computationally intractable and infeasible because the number of regime paths grows exponentially with the number of observations. To avoid this problem of path dependence, Klaassen (2002) proposed simplifications to the GARCH model, which integrates out the past regimes to get the conditional expectation of the past conditional variance by also taking into account the current regime. As a result, the expression of GARCH(1, 1) for the conditional variance can be written as:
\[ h_t^{2(i)} = \omega^{(i)} + \alpha_1^{(i)} \varepsilon_{t-1}^2 + \beta_1^{(i)} E_{t-1}\left\{ h_{t-1}^{2(i)} | s_t \right\}. \] (12)

More details can be found in Klaassen (2002). The advantage of this approach is that it allows to calculate the log likelihood function and multi-step ahead volatility forecasts recursively as in standard GARCH models. The multi-step volatility forecasts can be computed as the weighted average of multi-step volatility forecasts in each regime, where the weights are the predicted probability of being in each regime.

**Estimation**

The specification of a Markov regime switching model requires the estimation of the unobservable regime sequence \( \{ s_t \} \) and the parameter set in each regime. Since the state variable \( s_t \) is unobservable, the inference of state variable can only be determined based on the data. The conditional probability of \( s_t \) being in regime \( j \), given the information set \( \Omega_t \) at time \( t \) and parameters \( \Theta \) is:

\[ \xi^j_{s_t|\Omega_t} = P(s_t = j|\Omega_t, \Theta), \] (13)

where \( \Theta \) refers to all the parameters that specify the stochastic process. An important step that allows for iteratively calculating \( \xi^j_t \) is to reformulate Equation (13) via the definition of conditional probability:

\[ \xi^j_{s_t|\Omega_t} = P(s_t = j|\Omega_t, \Theta) = \frac{f(y_t, s_t = j|\Omega_{t-1}, \Theta)}{f(y_t|\Omega_{t-1}, \Theta)}, \] (14)

where the numerator is the conditional joint density of \( y_t \) and \( s_t \) being in state \( j \) given \( \Omega_{t-1}, \Theta \), while the denominator is the conditional density of \( y_t \) given \( \Omega_{t-1}, \Theta \). The numerator can be determined by:

\[ f(y_t, s_t = j|\Omega_{t-1}, \Theta) = \xi^j_{s_t|\Omega_{t-1}} f(y_t|s_t = j, \Omega_{t-1}, \Theta), \] (15)

where \( \xi^j_{s_t|\Omega_{t-1}} = P(s_t = j|\Omega_{t-1}, \Theta) \) is the forecast of the probability of \( s_t \) being in state \( j \) given \( \Omega_{t-1} \) at \( t - 1 \) and \( \Theta \). The forecast of the probability matrix can be obtained via the transition matrix \( P \) as:

\[ \xi_{s_t|\Omega_{t-1}} = P \xi_{s_{t-1}|\Omega_{t-1}}, \] (16)

The conditional density \( f(y_t|s_t = j, \Omega_{t-1}, \Theta) \) depends on the specification of the model and the distribution of error distribution in each regime.

The denominator in Equation (14) can be calculated as the sum of conditional joint densities for all regimes (in our case the number of regimes is 2):
\[ f(y_t|\Omega_{t-1}, \Theta) = \sum_{j=1}^{2} f(y_t, s_t = j|\Omega_{t-1}, \Theta). \]  \hspace{1cm} (17)

Finally, inserting Equations (15), (16) and (17) into Equation (14) yields:
\[ \xi^j_{s_t|\Omega_t} = \mathbb{P}(s_t = j|\Omega_t, \Theta) \]
\[ = \frac{\sum_{i=1}^{2} p_{ij} \xi^i_{s_{t-1}|\Omega_{t-1}} f(y_t|s_t = j, \Omega_{t-1}, \Theta)}{\sum_{j=1}^{2} \sum_{i=1}^{2} p_{ij} \xi^i_{s_{t-1}|\Omega_{t-1}} f(y_t|s_t = j, \Omega_{t-1}, \Theta)}. \]  \hspace{1cm} (18)

The corresponding conditional log likelihood of the observations is given by:
\[ \ell(y_1, y_2, ..., y_T|\Theta) = \sum_{t=1}^{T} \log f(y_t|\Omega_{t-1}, \Theta). \]  \hspace{1cm} (19)

The maximum likelihood estimators can be obtained by maximizing Equation (19).

### 2.3 Forecast Evaluation

To evaluate the predictive accuracy of different models, statistical loss functions are typically employed. A model that has a smaller average loss is considered to be more accurate. However, many researchers have highlighted that a few extreme observations may have an excessively large impact on the outcomes of forecast evaluation and comparison tests and have suggested to use loss functions that are less sensitive to large observations (Bollerslev and Ghysels, 1994; Andersen et al., 1999; Poon and Granger, 2003). Therefore, instead of using one particular statistical loss function, we here adopt different statistical loss functions, namely Root Mean Square Error (RMSE), RMSE-LOG, Mean Absolute Error (MAE), MAE-LOG and QLIKE (Patton, 2011; Byun and Cho, 2013). With the forecasted volatility and the actual volatility at hand, RMSE, RMSE-LOG, MAE, MAE-LOG and QLIKE are defined as:

\[ RMSE = \sqrt{\frac{1}{N} \sum_{t=T+1}^{T+N} (\hat{\sigma}_t^2 - \hat{\sigma}_t^2)^2}, \]  \hspace{1cm} (20)

\[ RMSE = \sqrt{\frac{1}{N} \sum_{t=T+1}^{T+N} (\log(\hat{\sigma}_t^2) - \log(\hat{\sigma}_t^2))^2}, \]  \hspace{1cm} (21)

\[ MAE = \frac{1}{N} \sum_{t=T+1}^{T+N} |\hat{\sigma}_t^2 - \hat{\sigma}_t^2|. \]  \hspace{1cm} (22)
Where \( \hat{\sigma}^2_t \) is the ex post proxy of conditional variance representing the actual volatility at \( t, t > T \), and \( \hat{h}^2_t \) is a volatility forecast at \( t \) and \( N \) is the testing horizon. In our paper, we use realized volatility as \( \hat{\sigma}^2_t \) (see Section 3.1).

To determine if the predictive accuracies of the competing models are significantly different, we use the Diebold-Mariano (DM) test (Diebold and Mariano, 1995). Such a test is based on the loss function differential between model \( a \) and model \( b \), defined as \( d_t = [g(e_{a,t}) - g(e_{b,t})] \) where \( g(\cdot) \) means a loss function, \( e_{a,t} \) corresponding forecast errors from the competing models. In our paper we focus on two loss functions: square error loss function and absolute error loss function. The null hypothesis of DM test is no difference in the accuracy of the two competing forecasts, i.e., \( E(d_t) = 0 \). The DM test statistic takes the form of a t-statistic, i.e., \( \Delta R = \bar{d} / \sqrt{\hat{V}(\bar{d})} \sim N(0,1) \), where \( \bar{d} = \frac{1}{n} \sum_{t=1}^{n} d_t \), and \( \hat{V}(\bar{d}) \) is the asymptotic variance of \( \bar{d} \). For more details, we refer to Diebold and Mariano (1995). The null hypothesis is rejected or accepted based on the DM test statistic and the critical value.

3. Empirical Volatility Forecasting Results

3.1 Wind Power Data

Volatility models have been applied to wind speed data as well as to wind power production data. Wind speed data at weather stations have the advantage of being easily accessible, and a transformation of wind speed data into energy production data is possible by means of rather simple power curves that take into account turbine types, turbine height and other technical specifications. This is found to be quite useful because mostly researchers do not have access to wind power data due to the commercial sensitivity. However, the distance between weather stations and wind turbines results in a bias on transformed production data. Alternatively, wind speed data from reanalysis data could reduce the bias, but empirical evidence showed that the \( R^2 \) of estimated power curves rarely exceeds 0.76 (Ritter et al, 2015) and thus actual production risk is underestimated. Moreover, the power curve may be different with the different time of

\[
MAE = \frac{1}{N} \sum_{t=T+1}^{T+N} |\log(\hat{\sigma}^2_t) - \log(h^2_t)|, \quad (23)
\]

\[
QLIKE = \frac{1}{N} \sum_{t=T+1}^{T+N} (\log(h^2_t) + \frac{\hat{\sigma}^2_t}{\hat{h}^2_t}), \quad (24)
\]
year and different environmental conditions (Anastasiades and McSharry, 2013). To avoid this kind of basis risk, we prefer to work with observed instead of transformed production data.

Wind power production data are collected from a wind farm located in the middle of Germany (see Figure 1). The wind farm consists of six wind turbines with same maximum capacity, namely 2.3 MW. The wind power data used in the estimation and forecasting of all models are recorded every hour from 1 October 2012 to 7 January 2014 for a total of 10,945 observations. The reason to justify the choice of the hourly resolution is that energy market participants are traditionally interested at the hourly forecast as required by the market structure (Trombe and Pinson, 2012). Following Anastasiades and McSharry (2013) and Pinson (2012), time series data are normalized and expressed as a percentage of the wind farm capacity, which range in the interval [0,1] (see Figure 2). Missing values due to technical disorder and long-time shutdown of the wind farm are neglected. For in-sample estimation, we use the data from 1 October 2012 to 31 December 2013, while the data from 1 January to 7 January 2014 are used for out-of-sample evaluation. Parameters for the considered models in the forecast are estimated on an increasing sample as new information arrives. For example, when estimating a GARCH model using data from 1 October 2012, 0:00, to 31 December 2013, 24:00, we obtain a one-step-ahead (i.e., one-hour-ahead) forecast for 1:00 on 1 January 2014. By increasing the sample by one hour and re-estimating the model for the new sample until 1 January 2014, 1:00, we obtain a forecast for 2:00 on 1 January 2014. This estimation is repeated until a forecast for 23:00 on 7 January 2014 is achieved. As a result, the estimation needs to be repeated 168 times. For multi-step-ahead forecast, the number of repetitions will reduce by the forecast horizon. We calculate the multi-step-ahead volatility forecast by summing up the hourly ahead volatility forecast over next periods since practitioners and risk managers might be more interested in the stability of the grid of wind power production over the next period than multi-step ahead one-hour volatility.

Since the true volatility is latent and unobservable, we use the realized volatility as the ex post proxy to compare with the forecast from the models. The realized volatility is based on the cumulative squared deviation over different time intervals. When we divide one hour into \( m \) periods and denote \( y_{t,i} (i = 0, ..., m) \) as an observation for the 60/\( m \)-minute wind production at time \( t \), the realized volatility within one hour can be calculated as:

\[
\sigma_{RV,t}^2 = \sum_{i=1}^{m} (y_{t,i} - y_{t,i-1})^2
\]
In this paper, we use wind power data reported for an interval of 10 minutes from 1 January to 7 January 2014 to calculate the realized volatility for hourly wind production, i.e., $m = 6$. To calculate the realized volatility at $h$ step ahead, we sum the hourly realized volatility over the $h$ steps.

![Wind farm location in Germany](image1)

Figure 1: Wind farm location in Germany

![Time series of normalized wind power production over a 10-day episode in Oct. 2012](image2)

Figure 2: Time series of normalized wind power production over a 10-day episode in Oct. 2012

### 3.2 Estimation

In this section, we present the results of estimating all the considered models in the in-sample period. The calibration of the model consists of two steps. The first step is to determine a proper autoregressive (AR) model that captures the time-varying means of wind power data (Equation
1. The order of AR term is usually chosen by using the autocorrelation function (ACF) and partial autocorrelation function (PACF). Figure 3 shows that the ACF decays very slow while the PACF becomes insignificant at lag 3, suggesting an AR(3) for modelling the mean dynamic of wind power production. The choice of the AR(3) process is also preferred according to Bayesian information criterion (BIC). After fitting an AR(3), the resulting residuals show no evidence of serial correlation (see Figure A1 in the Appendix). To detect the volatility clustering in the wind power production, the standard step is to further look at the autocorrelations of the squared residuals from the estimated model. Figure 4 plots the squared residuals obtained after fitting the AR(3) model to the wind power production time series. The volatility clustering effect can be observed in the squared residuals, meaning that large errors tend to follow large errors and small errors tend to follow small errors. The ACF and PACF in Figure 5 also illustrate the serial correlation in the squared residuals, supporting the use of GARCH models for the wind power production. In the remaining section, we will present the estimates of traditional GARCH models and MRS-GARCH model.

![Figure 3: ACF (left) and PACF (right) of wind power production](image-url)
We estimate the different GARCH models (such as SGARCH, EGARCH, TGARCH, GJR-GARCH and NGARCH) together with AR(3) model of mean process. For the sake of simplicity and to capture the volatility clustering effect sufficiently, we only consider all these GARCH models with $p = q = 1$ in our study. Figure A2 in the Appendix shows no correlation in the standardized squared residuals, implying that GARCH (1, 1) well captures the time-varying heteroscedasticity.

The parameter estimates for the traditional GARCH models are presented in Table 1. Almost all parameters for various GARCH models are significant, except for the coefficient of AR(3)
in the SGARCH and NGARCH. Moreover, the significance of $\gamma$ in these asymmetric GARCH models implies the presence of nonlinear and asymmetric effects in the volatility of wind power production data. Interestingly, the asymmetric effect is opposite to that observed in the volatility of finance data: In financial markets, a negative value of $\gamma$ is often expected for the EGARCH model, suggesting that the negative value of $\varepsilon_{t-1}$ in Equation (5) results in a higher volatility than the positive value. In contrast, we observe a positive $\gamma$ of the EGARCH model from modelling the volatility of wind power production, which means that positive changes lead to a higher volatility than negative ones. In the TGARCH, GJR GARCH, and NGARCH, $\gamma < 0$, implying that the impact of a positive change on volatility is amplified compared to negative ones, hence the results are consistent with that from the EGARCH model. This implies that increasing wind power production leads to a higher conditional volatility for the next periods than decreasing ones.

We also present the estimates of the parameters for MRS-GARCH in the last two columns of Table 1. Due to the two-regime structure, 16 parameters need to be estimated in total. The estimates for the conditional mean in both regimes are significant while the estimates of $\omega$ and $\alpha_1$ for the conditional variance in regime 1 is close to 0 and thus cannot reject the null of a zero value from t test. The estimates highlight the existence of two regimes: regime 1 is characterised by a low mean and low volatility (denoted as ‘low regime’), the impact of previous shocks on the conditional volatility is nil in this state; regime 2 is characterised by a high mean, high volatility and high persistence in the conditional volatility (denoted as ‘high regime’). Table 1 shows that the transition probabilities of both regimes are significant and above 0.83, indicating that both regimes are persistent and for only less than 20% chances, the regimes will switch from one to the other. The unconditional probability of wind power production being in the low regime is 0.42, lower than that of being in the high regime, 0.58.

Figure 6 illustrate the wind power production and estimated sequence of regimes probability for the MRS-GARCH model. The plotted estimated sequence is the probability of being in regime 1 for wind power production. At most periods, these two regimes do not appear to be clearly distinguished for wind power production except the periods when the wind power production is close to 0 or maximal capacity, with the probability almost 1 or 0. Normally, we can compare the sequence probabilities with 0.5 to define the regimes for each observation more clearly. Moreover, it is not manifest to differ the volatility of each observation according to the sequence of regimes. The estimated sequence of regimes probability seems to be mainly
driven by the mean value of wind power production rather than the volatility in our case, i.e., high wind power production in high regime.

Figure 6: Wind power production and estimated sequence of probabilities of being in regime 1 (i.e., low regime)
Table 1: Estimation results of traditional GARCH models and MRS-GARCH

<table>
<thead>
<tr>
<th>Parameter</th>
<th>GARCH</th>
<th>EGARCH</th>
<th>TGARCH</th>
<th>GJR-GARCH</th>
<th>NGARCH</th>
<th>MRS-GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Regime 1</td>
</tr>
<tr>
<td>Mean equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>0.0578***</td>
<td>0.0854***</td>
<td>0.1102***</td>
<td>0.0580***</td>
<td>0.0740***</td>
<td>0.0005***</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.8130***</td>
<td>0.7893***</td>
<td>0.8172***</td>
<td>0.7949***</td>
<td>0.7977***</td>
<td>0.6825***</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.1149***</td>
<td>0.1122***</td>
<td>0.1153***</td>
<td>0.1186***</td>
<td>0.1327***</td>
<td>0.0410***</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.0086</td>
<td>0.0457***</td>
<td>0.0258***</td>
<td>0.0220***</td>
<td>0.0143</td>
<td>0.0451***</td>
</tr>
<tr>
<td>Conditional variance equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.0002***</td>
<td>-0.2137***</td>
<td>0.0025***</td>
<td>0.0002***</td>
<td>0.0001***</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.2403***</td>
<td>0.1893***</td>
<td>0.1799***</td>
<td>0.1677***</td>
<td>0.1477***</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.7587***</td>
<td>0.9587***</td>
<td>0.8491***</td>
<td>0.7842***</td>
<td>0.6810***</td>
<td>0.4401***</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2993***</td>
<td>-1.0000***</td>
<td>-0.5299***</td>
<td>-1.0737***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8352***</td>
</tr>
<tr>
<td>$q$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8357***</td>
</tr>
<tr>
<td>P($s_t = i$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.4201</td>
</tr>
<tr>
<td>Log ($L$)</td>
<td>11690.55</td>
<td>12351.78</td>
<td>12394.37</td>
<td>12043.92</td>
<td>12240.36</td>
<td>15021.21</td>
</tr>
</tbody>
</table>

Note: Asterisks *** indicate significance at the 1 per cent level.
3.3 Comparison of Models

We begin evaluating the performance of different models by comparing some statistics for both in-sample estimation and out-of-sample forecast. The in-sample goodness-of-fit statistics are presented in Table 2. The log-likelihood suggests that the MRS-GARCH model fits best our wind power production data compared to other state-independent GARCH models. The SGARCH turned out to be the least preferable although it captures well the autocorrelation of the volatility according to Figure A2. TGARCH turns out to be the second best model, followed by EGARCH. The result is also confirmed by the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), which account for the parsimony of the models. Table 2 also shows that the persistence, which indicates the influence of a shock to volatility, is rather high. In the MRS-GARCH model, the persistence difference in the two regimes is considerable for onshore wind power production observations unlike for financial data.

Having a good in-sample fit does not necessarily lead to an accurate and reliable forecast from the models. Moreover, the models with good in-sample fits likely suffer over-fitting or over-parametrization problems. To determine the best forecasting model, we conduct the out-of-sample evaluation of one- and five-step ahead volatility forecasts against the realized volatility. Figure 7 shows the realized volatility and one-step ahead forecasted volatilities from MRS-GARCH and TGARCH. From the figure, it is not able to determine the best one among the considered models. Therefore, we evaluate the forecasts by using different loss function criteria described in Section 2.3. The results are presented in Table 3. For 1-step ahead volatility forecast, the smallest values of all loss functions except QLIKE are observed for MRS-GARCH model3, suggesting that MRS-GARCH model generally outperforms the other GARCH models for volatility forecast of wind power production. EGARCH appears to be second best in the out-of-sample evaluation. Surprisingly, TGARCH turns out to be the worst one in the out-of-sample evaluation although it performed very well in the in-sample evaluation. On the other hand, SGARCH takes the third or fourth place for different selection criteria in the out-of-sample evaluation despite the worst performance in the in-sample fitting. However, the differences for all the models in terms of RMSE and MAE are rather moderate. The result is also found for the volatility forecast at 5-step ahead.

3 The different rank using QLIKE is expected since its loss function depends on the multiplicative forecast error instead of additive forecast error as in RMSE and MAE. Moreover, this is common in the volatility forecast literature, e.g. Marcucci (2005); Patton (2011); Byun and Cho (2013).
### Table 2. In-sample goodness-of-fit statistics

<table>
<thead>
<tr>
<th>Model</th>
<th># Param.</th>
<th>Persistence</th>
<th>AIC</th>
<th>BIC</th>
<th>Log(L)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>SGARCH</td>
<td>7</td>
<td>0.999</td>
<td>-2.167</td>
<td>-2.163</td>
<td>11690.55</td>
<td>6</td>
</tr>
<tr>
<td>EGARCH</td>
<td>8</td>
<td>0.959</td>
<td>-2.290</td>
<td>-2.284</td>
<td>12351.78</td>
<td>3</td>
</tr>
<tr>
<td>TGARCH</td>
<td>8</td>
<td>0.993</td>
<td>-2.297</td>
<td>-2.292</td>
<td>12394.37</td>
<td>2</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>8</td>
<td>0.999</td>
<td>-2.233</td>
<td>-2.227</td>
<td>12043.92</td>
<td>5</td>
</tr>
<tr>
<td>NGARCH</td>
<td>8</td>
<td>0.999</td>
<td>-2.269</td>
<td>-2.264</td>
<td>12240.36</td>
<td>4</td>
</tr>
<tr>
<td>MRS-GARCH</td>
<td>16</td>
<td>0.440/0.999</td>
<td>-2.783</td>
<td>-2.773</td>
<td>15021.21</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: The persistence of MRS-GARCH is reported for Regimes 1 and 2, respectively. AIC is calculated as $(-2\log(L) + 2k)/T$, where $k$ is the number of parameters and $T$ is the number of observations. BIC is calculated as $(-2\log(L) + k\log(T))/T$.

---

**Figure 7:** Realized volatility and 1-step forecasted volatility from MRS-GARCH and TGARCH
Table 3. Out-of-sample evaluation for 1-step ahead volatility forecast

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
<th>RMSE-LOG</th>
<th>MAE</th>
<th>MAE-LOG</th>
<th>QLIKE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Rank)</td>
<td>(Rank)</td>
<td>(Rank)</td>
<td>(Rank)</td>
<td>(Rank)</td>
</tr>
<tr>
<td>SGARCH</td>
<td>0.0335</td>
<td>1.9332</td>
<td>0.0244</td>
<td>0.9195</td>
<td>-2.4430</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(3)</td>
</tr>
<tr>
<td>EGARCH</td>
<td>0.0328</td>
<td>1.9845</td>
<td>0.0243</td>
<td>0.9117</td>
<td>-2.4592</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>(3)</td>
<td>(2)</td>
<td>(2)</td>
<td>(2)</td>
</tr>
<tr>
<td>TGARCH</td>
<td>0.0416</td>
<td>2.3421</td>
<td>0.0308</td>
<td>1.0139</td>
<td>-2.4227</td>
</tr>
<tr>
<td></td>
<td>(6)</td>
<td>(6)</td>
<td>(6)</td>
<td>(6)</td>
<td>(6)</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>0.0390</td>
<td>2.1738</td>
<td>0.0283</td>
<td>0.9687</td>
<td>-2.4321</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(5)</td>
<td>(5)</td>
<td>(5)</td>
<td>(4)</td>
</tr>
<tr>
<td>NGARCH</td>
<td>0.0344</td>
<td>2.0626</td>
<td>0.0254</td>
<td>0.9134</td>
<td>-2.4813</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>(4)</td>
<td>(4)</td>
<td>(3)</td>
<td>(1)</td>
</tr>
<tr>
<td>MRS-GARCH</td>
<td>0.0295</td>
<td>1.7597</td>
<td>0.0207</td>
<td>0.8406</td>
<td>-2.4244</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(1)</td>
<td>(1)</td>
<td>(1)</td>
<td>(5)</td>
</tr>
</tbody>
</table>

Table 4. Out-of-sample evaluation for 5-step ahead volatility forecast

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
<th>RMSE-LOG</th>
<th>MAE</th>
<th>MAE-LOG</th>
<th>QLIKE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Rank)</td>
<td>(Rank)</td>
<td>(Rank)</td>
<td>(Rank)</td>
<td>(Rank)</td>
</tr>
<tr>
<td>SGARCH</td>
<td>0.1260</td>
<td>0.5487</td>
<td>0.0882</td>
<td>0.5694</td>
<td>-2.3679</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>(4)</td>
<td>(4)</td>
<td>(4)</td>
<td>(4)</td>
</tr>
<tr>
<td>EGARCH</td>
<td>0.1001</td>
<td>0.4279</td>
<td>0.0701</td>
<td>0.4770</td>
<td>-2.3737</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>(2)</td>
<td>(2)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>TGARCH</td>
<td>0.1732</td>
<td>0.6709</td>
<td>0.1219</td>
<td>0.6300</td>
<td>-2.3677</td>
</tr>
<tr>
<td></td>
<td>(6)</td>
<td>(6)</td>
<td>(6)</td>
<td>(6)</td>
<td>(5)</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>0.1600</td>
<td>0.6468</td>
<td>0.1062</td>
<td>0.5951</td>
<td>-2.3507</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(5)</td>
<td>(5)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>NGARCH</td>
<td>0.1281</td>
<td>0.4805</td>
<td>0.0881</td>
<td>0.5133</td>
<td>-2.4104</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>(3)</td>
<td>(3)</td>
<td>(3)</td>
<td>(2)</td>
</tr>
<tr>
<td>MRS-GARCH</td>
<td>0.0982</td>
<td>0.4066</td>
<td>0.0676</td>
<td>0.4696</td>
<td>-2.4287</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(1)</td>
<td>(1)</td>
<td>(1)</td>
<td>(1)</td>
</tr>
</tbody>
</table>

Note: The volatility at 5-step ahead is calculated as $\sigma^2_{t+5|t} = \sum_{j=1}^{5} \hat{\sigma}^2_{t+j|t}$, where $\hat{\sigma}^2_{t+j|t}$ is the forecast volatility at $t+j$ given information at $t$. 

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Previous literature has shown that the performance of various models might differ with the forecast horizon (Marcucci, 2005). Hence, we plot the RMSE with regard to different forecast horizons (Figure 8). It shows that the RMSE of the MRS-GARCH model is consistently lowest within 24 hours ahead, followed by that of the EGARCH model. This difference from previous literature may be due to the higher frequency of wind power production data. To evaluate the predictive accuracies of the competing models, we use the Diebold-Mariano (DM) test reported in Table 5 where the benchmark is the best model for 1-step ahead horizon (MRS-GARCH). It is observable that the MRS-GARCH model significantly outperforms every other model in terms of square error loss function and absolute error loss function. Moreover, the values of DM test statistics are negative, implying that the MRS-GARCH model has a lower loss than other models. Therefore, the MRS-GARCH model is superior in predicting the volatility of wind power production. Moreover, we are also interested in the predictive accuracy of the EGARCH model, so we also use the EGARCH model as the benchmark in the DM test. The results in Table 6 show the EGARCH model performs significantly better than all other GARCH models except the MRS-GARCH model. The same finding is also observed for 5-step ahead volatility forecasts.
Table 5. Diebold-Mariano Test (1-step ahead)

<table>
<thead>
<tr>
<th>Model</th>
<th>Square error loss</th>
<th>Absolute error loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRS-GARCH</td>
<td></td>
<td>Benchmark</td>
</tr>
<tr>
<td>SGARCH</td>
<td>-2.49***</td>
<td>-3.64***</td>
</tr>
<tr>
<td>EGARCH</td>
<td>-2.02***</td>
<td>-3.44***</td>
</tr>
<tr>
<td>TGARCH</td>
<td>-4.24***</td>
<td>-5.54***</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>-3.86***</td>
<td>-4.82***</td>
</tr>
<tr>
<td>NGARCH</td>
<td>-2.86***</td>
<td>-3.79***</td>
</tr>
</tbody>
</table>

Note: The negative sign implies that the benchmark’s loss is lower than that implied by other models. Asterisks *** denote significance at the 1 per cent level.

Table 6. Diebold-Mariano Test (1-step ahead)

<table>
<thead>
<tr>
<th>Model</th>
<th>Square error loss</th>
<th>Absolute error loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGARCH</td>
<td></td>
<td>Benchmark</td>
</tr>
<tr>
<td>MRS-GARCH</td>
<td>2.01***</td>
<td>3.44***</td>
</tr>
<tr>
<td>SGARCH</td>
<td>-0.48*</td>
<td>-0.06*</td>
</tr>
<tr>
<td>TGARCH</td>
<td>-4.60***</td>
<td>-5.81***</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>-4.13***</td>
<td>-4.67***</td>
</tr>
<tr>
<td>NGARCH</td>
<td>-1.12***</td>
<td>-1.58***</td>
</tr>
</tbody>
</table>

Note: The negative sign implies that the benchmark’s loss is lower than that implied by other models. Asterisks *** and * denote significance at the 1 and 10 per cent levels, respectively.

Although the Markov regime switching GARCH model outperforms all the other models, it is observed in Figure 7 that its forecast volatility deviates largely from the realized volatility. It seems that none of the considered models can predict the sudden jumps in hourly volatility of wind power production. This may be due to the fact that wind power production recorded at 10 minutes fluctuates more than the production data at every hour. To understand those two spikes of realized volatility between 3rd and 4th of January, we plot the wind power production recorded at 10 min (Figure 9). On 3rd of January, wind power production experiences two instantaneous rapid decreases, resulting in the big increases of realized volatility. Thus, jumps at 10 minutes cause a bias in the realized volatility estimator for hourly spot volatility.
4. Conclusions
In this article, we have studied the specification of different GARCH models to forecast the volatility of wind power production. Besides the comprehensive comparison of traditional GARCH models in the context of wind power production, an important contribution of this paper is that we propose and examine the use of Markov regime-switching GARCH (MRS-GARCH) model to better account for nonlinear and heteroscedastic effects of wind power production. The comparison in the in-sample fitting according to model selection criteria (Log-likelihood, AIC, BIC) suggests that the MRS-GARCH model captures the dynamic process of wind power production better than other models. However, good in-sample fitting does not necessarily imply a good predictive ability in out-of-sample comparison. To evaluate the forecasting performances of different models, the ‘true volatility’ would be needed. Since the true volatility is latent and unobservable, we resort to the concept of realized volatility introduced in financial market as an ex post proxy of the volatility of wind power production. Empirical results show that the MRS-GARCH model significantly outperforms other GARCH models in forecasting volatility according to a set of statistical loss functions and tests. The difference at 1-step ahead volatility forecast, however, is moderate. Taking into account the computational effort and information gain, the EGARCH model might also be a fair choice to forecast the volatility of wind power production. Moreover, although the MRS-GARCH model has already outperformed other considered models, it cannot predict the abrupt changes in the realized volatility due to the big instantaneous jumps in high frequency wind power production.
Further research on modelling volatility forecast with jumps to improve the predictive ability may be of considerable interests.

This study is of particular relevance for energy traders to make decision on the balance of supply and demand of wind energy and for investors to develop financial risk management tools against changing weather conditions. Our results support the use of Markov regime-switching GARCH models for forecasting volatility of wind power production since the model can better capture the dynamic nature of wind power production. The suggested model shows superior performance on the short-term volatility forecast of wind power production and gives more accurate forecast information on the uncertainty about future wind power production. Accordingly, it will be helpful for risk managers and market traders to determine Value-at-Risk of their energy portfolios and to price the related derivatives or insurances on wind power production.

There are several extension of this study that may improve the volatility forecast. First, one may apply the models with heavy-tailed distributions instead of the normal distribution in our case. The assumption of a heavy-tailed distribution might improve the results by considering the fact that the wind power production data are bounded and mass data points are generated around the boundary. Second, the Markov regime-switching model was only incorporated with a standard GARCH model in our case. It would be interesting to consider the combination with other asymmetric GARCH models as well. Last but not least, given substantial geographical difference of wind conditions, the validation of models using wind farm data from different locations is of great importance and relevance to its potential implementation for energy traders and investors.

5. References


6. Appendix

Figure A1: ACF (left) and PACF (right) of residuals from a AR(3) model for wind power production

Figure A2: Square residuals (left, top), conditional variance estimated from SGARCH(1,1) (left, bottom), and the ACF of standardized squared residual (right)
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