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#### IMPRESSUM

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# Linkages between the US and European Stock Markets: A Fractional Cointegration Approach

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## Abstract

This paper analyses the long-memory properties of US and European stock indices, as well as their linkages, using fractional integration and fractional cointegration techniques. These methods are more general and have higher power than the standard ones usually employed in the literature. The empirical evidence based on them suggests the presence of unit roots in both the S&P 500 Index and the Euro Stoxx 50 Index. Also, fractional cointegration appears to hold at least for the subsample from December 1996 to March 2009 ending when the global financial crisis was still severe; subsequently, the US and European stock markets diverged and followed different recovery paths, possibly as a result of various factors such as diverging growth and monetary policy. Establishing whether the degree of cointegration has changed over time is important since past literature has shown that diversification benefits arise when markets are not cointegrated.

**Keywords:** Stock markets, linkages, fractional integration, fractional cointegration.

**JEL Classification:** C32, G15.

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## **1. Introduction**

Globalisation has led to international financial markets becoming increasingly interconnected, with equities displaying a high degree of co-movement across countries. This paper analyses linkages between US and European stock markets. Specifically, it applies fractional integration and cointegration techniques with the aim of testing co-movement between the S&P 500 Index and the Euro Stoxx 50 Index over the period from 1986 to 2013. Interestingly, we find that following the Great Recession of 2008 and early 2009, the pattern of co-movement changed, namely, after the trough in both US and European stock markets in the first quarter of 2009, the recovery paths were very different. It is well known that Europe and the US have experienced diverging growth and monetary policy in recent years (see, e.g., Pisani-Ferri and Posen, 2011). The global financial crisis that had originated in the US then led to a serious debt crisis in the Eurozone, and to the ECB eventually adopting its own version of Quantitative Easing (QE) in the form of the so-called long-term refinancing operation (LTRO) in December 2011. The initial monetary policy response had been much more expansionary in the US, the Fed immediately espousing QE; tight fiscal policy was another factor leading to much weaker growth in Europe than in the US, which also meant lower Treasury yields.

It has been shown that whether financial investors can benefit from diversification by investing in two different markets depends on their degree of cointegration (see Driessen and Laeven, 2007). This motivates our analysis, which suggests that US and European stock markets were (fractionally) cointegrated up until March 2009 (during the financial crisis), when this linkage broke down. Therefore, a European (US) investor could gain greater diversification benefits by investing in the US (European) market after that date compared to the previous period. The fractional

cointegration framework we adopt with the aim of determining when the linkages between these markets changed is more powerful and flexible than standard methods used elsewhere in the literature.

The structure of this paper is as follows. Section 2 contains a brief discussion of the literature on long memory in stock markets and cross-market linkages. Section 3 outlines the empirical methods used for the analysis. Section 4 describes the data and the main empirical results, while Section 5 offers some concluding remarks.

## **2. Literature review**

There is an extensive literature testing whether stock prices follow a random walk (as implied by the Efficient Market Hypothesis; in this case stock price changes would be unpredictable) or are instead mean-reverting. Two well-known studies by Fama and French (1988) and Poterba and Summers (1988) both found that US stock prices exhibit mean reversion. Techniques such as variance-ratio tests, regression coefficient and univariate unit root tests were used in other papers, for instance those by Fama (1995) and Choudhry (1997), also providing evidence of mean reversion. By contrast, Alvarez-Ramirez et al. (2008) concluded that both the S&P 500 and Dow Jones Industrial Average indices followed a random walk after 1972.

However, it is now well known that the unit root tests traditionally carried out (e.g., those by Dickey and Fuller (1979, 1981), Phillips and Perron (1988), and Ng and Perron (2001)) have very low power. This has led researchers to using other approaches to analyse long-run mean reversion, including ‘long memory’. The literature on long memory in stock returns has produced mixed evidence. Greene and Fielitz (1977) found evidence of persistence in daily US stock returns using R/S methods. Similar conclusions were reached by Crato (1994), Cheung and Lai (1995), Barkoulas and

Baum (1996), Barkoulas, Baum, and Travlos (2000), Sadique and Silvapulle (2001), Henry (2002), Tolvi (2003) and Gil-Alana (2006), for monthly, weekly, and daily stock market returns respectively. Several other studies, however, could not find any evidence of long memory. They include Aydogan and Booth (1988), Lo (1991), who used the modified R/S method and spectral regression methods, and Hiemstra and Jones (1997).

A number of papers have focused in particular on the Standard and Poor's (S&P) 500 Index. Granger and Ding (1995a,b) used power transformation or absolute value of the returns as a proxy for volatility, and estimated a long-memory process to examine persistence in volatility, establishing some stylized facts regarding the temporal and distributional properties of these series. However, in a following study, Granger and Ding (1996) found that the parameters of the long memory model varied considerably across subsamples. The issue of fractional integration with structural breaks in stock markets has been examined by Mikosch and Starica (2000) and Granger and Hyung (2004) among others. Stochastic volatility models using fractional integration have been estimated by Crato and de Lima (1994), Bollerslev and Mikkelsen (1996), Ding and Granger (1996), Breidt, Crato and de Lima (1997, 1998), Arteche (2004), Baillie, Han, Myers and Song (2007), etc.

Another strand of the literature focuses not only on individual time series, but also on the co-movement between international stock markets. It dates back to Panto et al. (1976), who used correlations to test for stock market interdependence. Subsequent studies relied on the cointegration framework developed by Engle and Granger (1987) and Johansen (1991, 1996) to examine long-run linkages. For instance, Taylor and Tonks (1989) showed that markets in the US, Germany, Netherlands and Japan exhibited cointegration over the period October 1979 - June 1986. Jeon and Von-Furstenberg (1990) used the VAR approach and found an increase in cross-border

cointegration since 1987. For post-crash periods and times of heightened volatility, Lee and Kim (1994) showed that the US and Japanese markets had tighter linkages. Copeland and Copeland (1998) and Jeong (1999) found a leadership role for the US relative to smaller markets. Wong et al. (2005) used fractional cointegration and reported linkages between India and the US, the UK and Japan. Syllignakis and Kouretas (2010) studied instead the integration of European and US stock markets, finding strong long-run linkages between US and German stock prices. Bastos and Caiado (2010) found evidence of cointegration for a wider sample of forty-six developed and emerging countries. The present study contributes to this literature by using fractional cointegration techniques to test for long-run linkages between the US and European financial markets and highlighting a change in their relationship.

Cointegration has also been used to determine if there are diversification benefits from investing in different stock markets: if cointegration does not hold, markets are not linked in the long run and therefore it is possible to gain from diversification. For this reason testing for cointegration and any changes over time in its degree is important. Richards (1995), for example, showed the absence of cointegration between various national stock markets and therefore the existence of diversification benefits for investors. By contrast, Gerrits and Yuce (1999) found that the US stock market is cointegrated with the German, UK, and Dutch ones, and Syriopoulos (2004) identified linkages between the US stock market and various Central European stock ones; in both cases the implication is that diversification cannot produce benefits.

### **3. Empirical methodology**

The empirical analysis is based on the concepts of fractional integration and cointegration. For our purposes, we define an  $I(0)$  process as a covariance stationary



process with a spectral density function that is positive and finite at the zero frequency. Therefore, a time series  $\{x_t, t = 1, 2, \dots\}$  is said to be I(d) if it can be represented as:

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, \dots \quad (1)$$

with  $x_t = 0$  for  $t \leq 0$ , where  $L$  is the lag-operator ( $Lx_t = x_{t-1}$ ) and  $u_t$  is I(0). By allowing  $d$  to be fractional, we introduce a much higher degree of flexibility in the dynamic specification of the series in comparison to the classical approaches based on integer differentiation, i.e.,  $d = 0$  and  $d = 1$ .

Processes with  $d > 0$  in (1) are characterized by a spectral density function which is unbounded at the origin. They were initially analysed in the 1960s, when Granger (1966) and Adelman (1965) pointed out that most aggregate economic time series have a typical shape where the spectral density increases sharply as the frequency approaches zero. However, differencing the data frequently leads to over-differencing at the zero frequency. Fifteen years later, Robinson (1978) and Granger (1980) showed that aggregation could be a source of fractional integration. Since then, fractional processes have been widely employed to describe the dynamics of many economic and financial time series (see, e.g. Diebold and Rudebusch, 1989; 1991a; Sowell, 1992; Baillie, 1996; Gil-Alana and Robinson, 1997; etc.).

Given the parameterisation in (1), different models can be obtained depending on the value of  $d$ . Thus, if  $d = 0$ ,  $x_t = u_t$ ,  $x_t$  is said to be “*short memory*”, and the observations may be weakly autocorrelated, i.e. with the autocorrelation coefficients decaying at an exponential rate; if  $d > 0$ ,  $x_t$  is said to be “*long memory*”, so named because of the strong association between observations far apart in time. If  $d$  belongs to the interval  $(0, 0.5)$   $x_t$  is still covariance stationary, while  $d \geq 0.5$  implies nonstationarity. Finally, if  $d < 1$ , the series is mean reverting, implying that the effect of

the shocks disappears in the long run, in contrast to what happens if  $d \geq 1$ , when the effects of shocks persist forever.

There exist many methods for estimating and testing the fractional differencing parameter  $d$ . Some of them are parametric while others are semiparametric and can be specified in the time or in the frequency domain. In this paper, we use a parametric Whittle function in the frequency domain (Fox and Taqqu, 1986; Dahlhaus, 1989) along with a Lagrange Multiplier (LM) test developed by Robinson (1994a) that has the advantage that it remains valid even in the presence of nonstationarity.<sup>1</sup> Some semi-parametric methods (Robinson, 1995a,b) will also be used for the analysis.

Some authors argue that fractional integration and non-linear models are closely related. Therefore, we also apply a procedure recently developed by Cuestas and Gil-Alana (2015) for analysing the degree of integration of a series in the presence of non-linear deterministic terms. The estimated model is

$$y_t = \sum_{i=0}^m \theta_i P_i(t) + x_t, \quad (1 - L)^d x_t = u_t, \quad (2)$$

where  $P_{i,T}(t)$  are the Chebyshev time polynomials, defined by:

$$P_{0,T}(t) = 1, \\ P_{i,T}(t) = \sqrt{2} \cos(i\pi(t-0.5)/T), \quad t = 1, 2, \dots, T; \quad i = 1, 2, \dots \quad (3)$$

Here,  $m$  indicates the order of the Chebyshev polynomial: if  $m = 0$  the model contains an intercept, if  $m = 1$  it also includes a linear trend, and if  $m > 1$  it becomes non-linear, and the higher  $m$ , the less linear the approximated deterministic component becomes.<sup>2</sup>

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<sup>1</sup> In addition, the tests of Robinson (1994a) are the most efficient ones in the Pitman sense against local departures from the null; in other words, against local departures from the null the limit distribution is also normal with a minimum variance.

<sup>2</sup> See Hamming (1973) and Smyth (1998) for a detailed description of these polynomials.

For the multivariate case, we apply fractional cointegration methods. This is a generalisation of the standard concept initially introduced by Engle and Granger (1987) and later extended by Johansen (1991, 1996) and others. First we test for homogeneity in the orders of integration of the two series by using an adaptation of the Robinson and Yajima (2002) statistic  $\hat{T}_{xy}$  to log-periodogram estimation. This is a test of the homogeneity in the orders of integration in a bivariate system (i.e.,  $H_0: d_x = d_y$ ), where  $d_x$  and  $d_y$  are the orders of integration of the two individual series. It is calculated as:

$$\hat{T}_{xy} = \frac{m^{1/2}(\hat{d}_x - \hat{d}_y)}{\left(\frac{1}{2}(1 - \hat{G}_x - \hat{G}_y + \hat{G}_{xy})\right)^{1/2} + h(n)} \quad (4)$$

where  $h(n) > 0$  and  $\hat{G}_{xy}$  is the  $(xy)^{\text{th}}$  element of:

$$\hat{G} = \frac{1}{m} \sum_{j=1}^m \mathbf{R} \left[ \hat{\Lambda}(\lambda_j)^{-1} \mathbf{I}(\lambda_j) \hat{\Lambda}(\lambda_j)^{-1*} \right], \quad \hat{\Lambda}(\lambda_j) = d \left\{ e^{i\pi \hat{d}_x / 2} \lambda^{-\hat{d}_x}, e^{i\pi \hat{d}_y / 2} \lambda^{-\hat{d}_y} \right\},$$

with a standard normal limit distribution (see Gil-Alana and Hualde (2009) for evidence on the finite sample performance of this procedure). Then, since the two parent series appear to be  $I(1)$ , we run a standard OLS regression of one variable against the other, and examine the order of integration of the estimated errors. A Hausman test of the null hypothesis of no cointegration against the alternative of fractional cointegration (Marinucci and Robinson, 2001) is also carried out. This method compares the estimate  $\hat{d}_x$  of  $d_x$  with the more efficient bivariate one of Robinson (1995), which uses the information that  $d_x = d_y = d^*$ . Marinucci and Robinson (2001) show that:

$$H_i = \frac{1}{m} 8m(\hat{d}^* - \hat{d}_i)^2 \rightarrow_d \chi_1^2 \quad \text{as} \quad \frac{1}{m} + \frac{m}{T} \rightarrow 0, \quad (5)$$

with  $i = x, y$ , and where  $m < [T/2]$  is again a bandwidth parameter, analogous to that introduced earlier;  $\hat{d}_i$  are univariate estimates of the parent series, and  $\hat{d}_*$  is a restricted estimate obtained in the bivariate context under the assumption that  $d_x = d_y$ . In particular,

$$\hat{d}_* = -\frac{\sum_{j=1}^s \mathbf{1}_2' \hat{\Omega}^{-1} Y_j v_j}{2 \mathbf{1}_2' \hat{\Omega}^{-1} \mathbf{1}_2 \sum_{j=1}^s v_j^2}, \quad (6)$$

where  $\mathbf{1}_2$  indicates a  $(2 \times 1)$  vector of 1s, and with  $Y_j = [\log I_{xx}(\lambda_j), \log I_{yy}(\lambda_j)]^T$ , and  $v_j = \frac{1}{s} \sum_{j=1}^s \log I_j$ . The limiting distribution above is presented heuristically, but Marinucci and Robinson (2001) argue that it seems sufficiently convincing for the test to warrant serious consideration.

#### 4. Data and empirical results

The series used for the analysis are the S&P 500 Index and the Euro Stoxx 50 Index (downloaded from Yahoo! Finance), representing two of the most liquid markets in the world. In addition, they are closely followed by market participants and are the most informative about dynamics in the US and European markets respectively. The frequency is monthly and the sample period goes from December 31, 1986, to December 31, 2013.

**[Insert Figure 1 about here]**

Figure 1 displays the two series. They exhibit very similar behaviour from the beginning of the sample until 2009, with two peaks occurring in 2000 and 2007, followed by a sharp decline in 2001 and 2008. After equity prices reached their trough

during the global financial crisis in March 2009, the S&P 500 Index recovered strongly (from the end of March 2009 till the end of December 2013 it increased by 132%). During this period the performance of the Euro Stoxx 50 lagged behind (it only increased by 50%).

As a preliminary step we estimate the order of integration of the series using standard (unit root) methods, specifically ADF (Dickey and Fuller, 1979); PP (Phillips and Perron, 1988), ERS (Elliot et al., 1996) and NP (Ng and Perron, 2001) tests; these provide strong evidence of unit roots. However, such tests have very low power under certain types of alternatives, including structural breaks, non-linearities and fractional integration. In particular, it has been shown that if a series is integrated of order  $d$  and  $d$  is different from 0 or 1, standard methods might not be appropriate (see Diebold and Rudebusch (1991), Hassler and Wolters (1994), Lee and Schmidt (1996) and others).

We start then by estimating the fractional differencing parameter in the following model,

$$y_t = \beta_0 + \beta_1 t + x_t, \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (7)$$

where  $y_t$  is the observed series,  $\beta_0$  and  $\beta_1$  are the coefficients corresponding to an intercept and a linear time trend, and  $x_t$  is assumed to be  $I(d)$ , where  $d$  can take any real value. Therefore the error term,  $u_t$ , is  $I(0)$ , and is assumed in turn to be a white noise, a non-seasonal and seasonal (monthly) AR(1) process and to follow the exponential spectral model of Bloomfield (1973), which is a non-parametric approach that produces autocorrelations decaying exponentially as in the AR case.

Table 1 shows the estimates of the fractional differencing parameter  $d$  for the log-transformed data, along with their corresponding 95% confidence intervals, in the three cases of no regressors ( $\beta_0 = \beta_1 = 0$  a priori in (7)), an intercept ( $\beta_0$  unknown and  $\beta_1 = 0$  a priori) and an intercept with a linear trend ( $\beta_0$  and  $\beta_1$  unknown).

**[Insert Table 1 about here]**

If  $u_t$  is assumed to be a white noise, the estimates of  $d$  are about 1 or slightly above 1, and the unit root null hypothesis cannot be rejected in case of the US stock market; however, for the European stock markets, this hypothesis is rejected in favour of  $d > 1$  in the model with an intercept and/or a linear time trend.. The results are very similar with seasonal AR disturbances. By contrast, if  $u_t$  is assumed to be autocorrelated (either following a non-seasonal AR(1) process or the more general model of Bloomfield), the unit root null hypothesis is almost never rejected. When using the Bloomfield's (1973) specification for the disturbances, the estimated value of  $d$  is 0.98 for the log S&P 500 Index, and slightly higher, 1.01, for the log-Euro Stoxx 50 Index. In both cases, an intercept seems to be sufficient to describe the deterministic components.<sup>3</sup>

**[Insert Table 2 about here]**

Table 2 displays the estimates of  $d$  obtained using a “local” Whittle semiparametric approach (Robinson, 1995) for a selected range of bandwidth parameters  $m = (T)^{0.5 \pm 3}$ ; the unit root hypothesis cannot be rejected in any case for either series.<sup>4</sup> These results are consistent with those of other papers also providing evidence of unit roots in stock indices in most developed economies (Huber, 1997; Liu et al., 1997; Ozdemir, 2008; Narayan, 2005, 2006; Narayan and Smyth, 2004, 2005; Qian et al., 2008; etc.).

Various studies in the literature have documented non-linear dynamics in stock prices. For instance, Hsieh (1991) explored ‘Chaos Dynamics’ in stock prices not

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<sup>3</sup> Very similar results were obtained with the raw series. The results are available from the authors upon request.

<sup>4</sup> These tests are robust against conditional heteroskedasticity, and the estimates were obtained using first-differenced data, then adding 1 to obtain the proper estimates of  $d$ . Alternative semiparametric methods also based on the Whittle function (Velasco and Robinson, 2000; Abadir et al., 2007) produced essentially the same results.

following a normal distribution; Abhyankar et al. (1995) provided evidence of non-linearity in the London Financial Times Stock Exchange (FTSE) index that cannot be fully explained by a GARCH model; Kosfeld and Robé (2001) showed various types of non-linearities in German bank stocks. Therefore we also carried out some non-linearity tests following the procedure developed by Cuestas and Gil-Alana (2015) briefly described above for the estimation of  $d$  in the context of fractional integration with non-linear deterministic terms.

**[Insert Table 3 about here]**

Table 3 displays the  $d$ -coefficient estimates and their 95% confidence bands for different degrees of linear ( $m = 1$ ) and non-linear ( $m = 2, 3$ ) behaviour in the logged-transformed series. It can be seen that the unit root model cannot be rejected in any case; the estimated coefficients for the linear and non-linear trends (not reported) were found to be statistically insignificant in all cases, which implies a rejection of the hypothesis of non-linear trends in the two series.<sup>5</sup>

Next, we investigate the issue of time variation in the fractional differencing parameter  $d$  by carrying out recursive analysis, starting with the first 120 observations (the first 10 years of the sample), and then adding one at a time. In particular, we focus on the log-transformed series and the specification with Bloomfield disturbances, with an intercept but not a linear trend, which is the model chosen on the basis of various diagnostic tests on the residuals.<sup>6</sup>

**[Insert Figure 2 about here]**

The two series appear to behave in a very similar way, although the estimates of  $d$  are slightly higher for the Euro Stoxx 50 Index. Those for the S&P 500 are all below

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<sup>5</sup> Very similar results were obtained with the unlogged data, and allowing for autocorrelated errors.

<sup>6</sup> In addition to t-tests for the deterministic terms, LR tests and various likelihood information criteria were used for model selection.

1, but the unit root null cannot be rejected. The estimated value of  $d$  increases when extending the sample recursively up to the 141<sup>st</sup> observation (the month following the 1998 Russian financial crisis); then it remains stable before jumping after the 191<sup>st</sup> observation (the start of the recovery in stock markets after the early 2000s recession), and is stable again till reaching 265 observations (right before the start of the recovery in global financial markets), when a new shift occurs.<sup>7</sup>

A similar behaviour of  $d$  is found in the case of the Euro Stoxx 50 Index, namely an upward trend for the first 191 observations (despite a downward shift after 143 observations), and then a jump after 266 observations. The unit root null hypothesis, i.e., the I(1) case, cannot be rejected for any subsample, which confirms the results from the full sample analysis; since both series appear to be I(1) throughout the sample it is legitimate to test for cointegration.

A necessary condition for cointegration in a bivariate context is that the two parent series must have the same degree of integration. In our case, the confidence intervals reported in Table 1 and 2 clearly suggest that the unit root (I(1)) hypothesis cannot be rejected for either series. However, we also perform the test of Robinson and Yajima (2002) for the homogeneity in the orders of integration of the two series. As expected, the results strongly support the hypothesis that the two orders of integration are the same, with a unit root being present in both cases.

Next, we examine the cointegrating relationship by estimating the following regression,

$$y_{1t} = \beta_0 + \beta_1 y_{2t} + x_t, \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, \dots \quad (8)$$

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<sup>7</sup> The sample containing the first 141 observations ends in August 1998, the one with 191 ends in October 2002, and finally, the sample containing 265 observations ends in December 2008.



where  $y_{1t}$  is the logged S&P 500 Index and  $y_{2t}$  the logged Euro Stoxx 50 Index. We consider the two cases of uncorrelated (white noise) and correlated (Bloomfield) errors. The fact that the two individual series are  $I(1)$  validates the use of standard OLS methods under the standard setting of cointegration (Phillips and Durlauf, 1986). In a fractional setting, things are more complicated and the properties depend on the specific orders of integration of the parent series and that of the cointegrating regression (Gil-Alana and Hualde, 2009).<sup>8</sup>

**[Insert Table 4 about here]**

Table 4 displays the estimated value of  $d$  in the cointegrating regression along with the other parameters in the cointegrating relationship. The estimated value of  $d$  in the residuals from the above regressions is 0.97 with white noise errors and 0.98 with autocorrelated disturbances, and the unit root null cannot be rejected in either case. This constitutes strong evidence against the hypothesis of cointegration, since the cointegrating residuals display a similar order of integration to the original series.

**[Insert Figure 3 about here]**

Next, we carry out recursive cointegration analysis, again starting with a sample of 121 observations. The results for  $d$  are displayed in Figure 3. It can be seen that the estimated value of  $d$  is below 1 (implying fractional cointegration and mean-reverting errors) in all the subsamples before reaching 268 observations, when the confidence intervals start including the unit root case, thus rejecting the hypothesis of cointegration. This point in the sample corresponds to March 2009, namely the trough of the financial crisis and the moment when global markets began to exit it. Our analysis indicates that at that stage the pattern of co-movement that had existed for the previous 22 years

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<sup>8</sup> Alternative methods for the estimation of  $\beta_0$  and  $\beta_1$  in (8) were also employed including a Narrow Band Least Squared (NBLS) estimator as proposed in Robinson (1994b) and a Fully Modified NBLS as in Nielsen and Frederiksen (2011).

between the US and European stock markets began to break down, and different recovery paths were followed. As mentioned before, different policy responses, namely the very prompt adoption of QE by the Fed in contrast to fiscal tightening and very limited monetary easing in Europe in the presence of a serious debt crisis, have led to different growth experiences on the two sides of the Atlantic, the European economies lagging behind and their stock markets underperforming.

**[Insert Table 5 about here]**

Finally, we perform the Hausman test for no cointegration of Marinucci and Robinson (2001). The results are displayed in Table 5. The null hypothesis of no cointegration cannot be rejected for the full sample, the estimated order of integration for the cointegrating error being about 1.01, which is very close to the values obtained for the individual series. By contrast, the null is rejected in favour of fractional cointegration for the subsample ending in December 2008, although the estimated value of  $d$  in the cointegrating error is close to 1, which implies highly persistent deviations from the long-run equilibrium relationship.

## **5. Conclusions**

This paper analyses the long-memory properties of US and European stock indices, as well as their linkages, using fractional integration and fractional cointegration techniques. The empirical evidence, based on both standard unit roots and  $I(d)$  methods, suggests the presence of unit roots in both the S&P 500 Index and the Euro Stoxx 50 Index. This result is robust to using a variety of parametric and semi-parametric methods. Given the fact that the two series exhibit the same order of integration, we also examine the possibility of a long-run equilibrium relationship linking them. The results indicate that cointegration does not hold over the full sample; however, there is

evidence of fractional cointegration over the subsample from December 1996 to March 2009, indicating that the effects of shocks affecting the long-run relationship vanish at a very slow rate.

It appears that the recovery paths followed by US and European stock markets after reaching their lowest price level (as a result of the Great Recession) have been very different. The Eurozone debt crisis combined with fiscal tightening and no significant monetary easing led to much weaker growth in Europe than in the US, where the Fed immediately embarked on an extensive QE programme. This has also affected European financial markets, with downward pressures on both bond yields and stock prices.

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**Table 1: Estimates of d for each series using the logged transformed data**

i) White noise disturbances			
	No regressors	An intercept	A linear time trend
U.S. stock market	1.01 (0.94, 1.10)	1.06 (0.99, 1.16)	1.06 (0.99, 1.16)
Euro stock market	0.99 (0.92, 1.07)	1.09 (1.02, 1.19)	1.09 (1.02, 1.19)
ii) AR(1) disturbances			
U.S. stock market	1.39 (1.27, 1.55)	0.98 (0.85, 1.12)	0.98 (0.87, 1.11)
Euro stock market	1.37 (1.25, 1.52)	1.01 (0.89, 1.15)	1.01 (0.89, 1.15)
iii) Bloomfield disturbances			
U.S. stock market	0.99 (0.87, 1.14)	<b>0.98 (0.87, 1.11)</b>	0.97 (0.88, 1.11)
Euro stock market	0.98 (0.86, 1.12)	<b>1.01 (0.90, 1.14)</b>	1.01 (0.90, 1.14)
iv) monthly AR(1) disturbances			
U.S. stock market	1.01 (0.93, 1.10)	1.06 (0.98, 1.16)	1.06 (0.98, 1.16)
Euro stock market	0.99 (0.92, 1.07)	1.09 (1.02, 1.19)	1.09 (1.02, 1.19)

We report the estimates of d in the model given by equation (2). The values in parenthesis refer to the 95% band for the non-rejection values of d. In bold, the most significant model for each series according to the deterministic terms and the type of I(0) disturbances.

**Table 2: Estimates of d based on the “local” Whittle semiparametric approach**

Bandwidth nb.	Log SP&500	Log Euro Stock	Lower 95%	Upper 95% I(1)
15	1.08	1.09	0.78	1.21
16	1.13	1.12	0.79	1.20
17	1.12	1.12	0.80	1.19
18	1.03	1.06	0.80	1.19
19	1.04	1.07	0.81	1.18
20	1.05	1.10	0.81	1.18
21	1.06	1.14	0.82	1.17

We report the estimates of d using the semiparametric method of Robinson (1995). The first column refers to the bandwidth number. The second and third columns report the estimated values of d. The fourth and the fifth columns refer to the 95% lower and upper confidence bands for the I(1) hypothesis.

**Table 3: Estimates of d based on a model with non-linear deterministic trends**

	m = 1 (linear)	m = 2 (non-linear)	m = 3 (non-linear)
Log of U.S. stock	1.07 (0.99, 1.16)	1.06 (0.98, 1.16)	1.05 (0.97, 1.14)
Log of Euro stock	1.09 (1.00, 1.19)	1.08 (0.99, 1.17)	1.08 (0.99, 1.16)

We report the estimated values of d for the model given by equation (2). The values in parenthesis refer to the 95% band for the non-rejection values of d

**Table 4: Estimates of d in the cointegrating regression**

	d	Intercept	Slope
White noise errors	0.97 (0.92, 1.05)	1.373 (6.864)	0.601 (20.656)
Bloomfield errors	0.98 (0.86, 1.11)	1.131 (5.676)	0.641 (22.108)

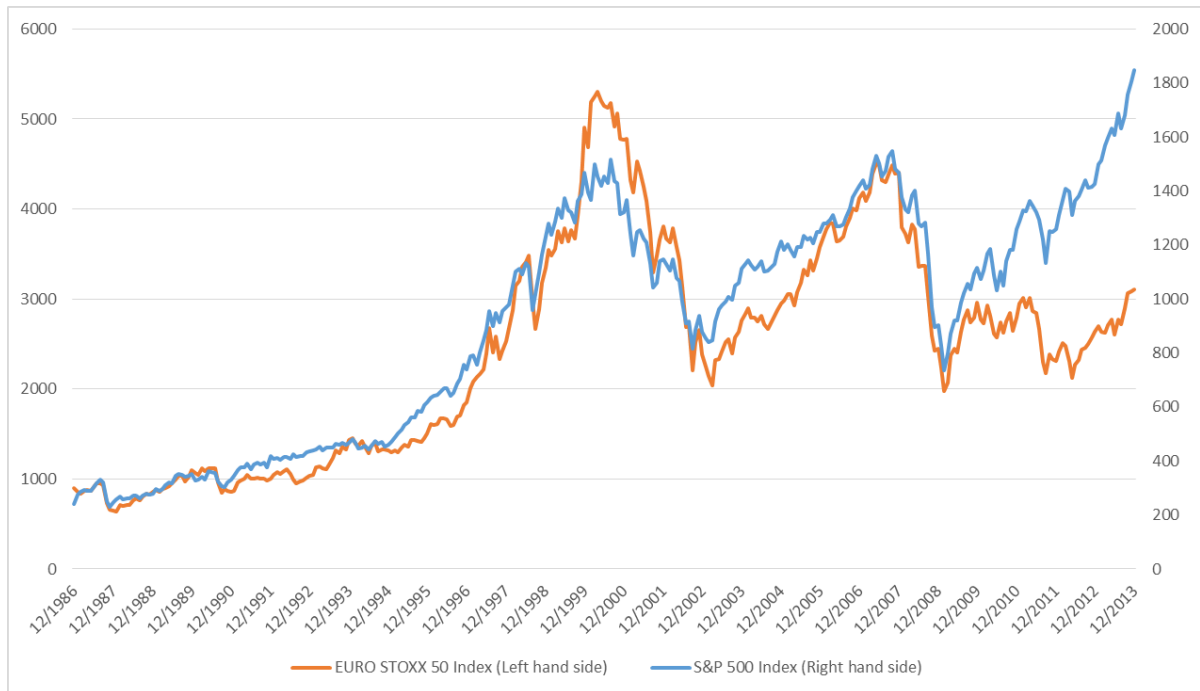
The values in parenthesis in the second column refers to the 95% band for the non-rejection values of d. In the third and fourth columns t-values are reported.

**Table 5: Testing the null of no cointegration against fractional cointegration**

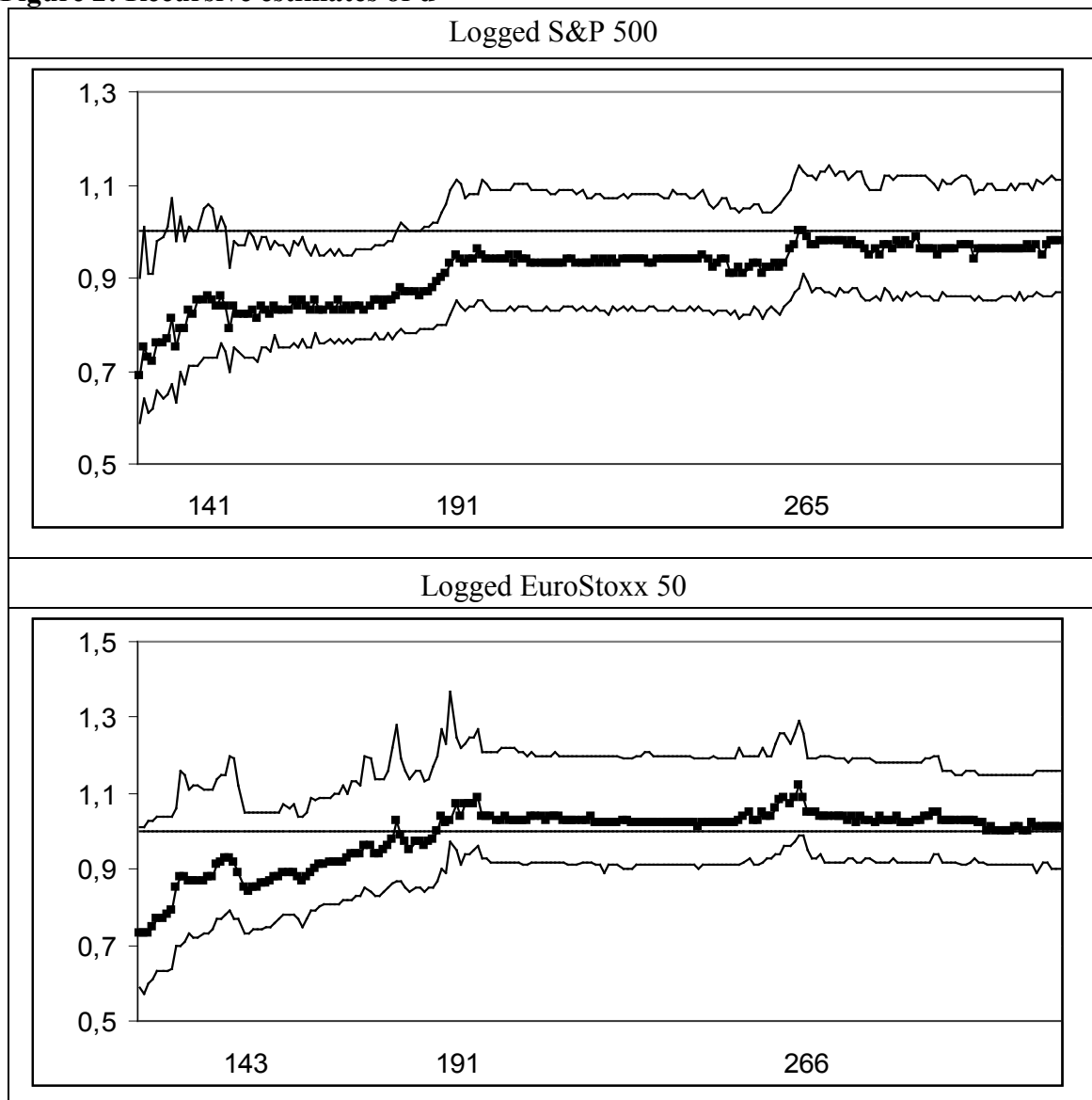
Log SP&500 / Log Euro Stock	$H_x$	$H_y$	$\hat{d}_*$
Whole sample (1986 – 2013)	0.0057	0.360	1.011
Sub-sample (1986 – 2008)	3.343	4.701	0.938

$H_x$  and  $H_y$  refer respectively to the hypothesis in (7) for each one of the two series using the Hausman test of Marinucci and Robinson (2001). The values in the fourth column is the estimated value of  $d^*$ .  $\chi_1^2(5\%) = 3.84$ .

**Figure 1: Time series plots: US and European stock market indices**

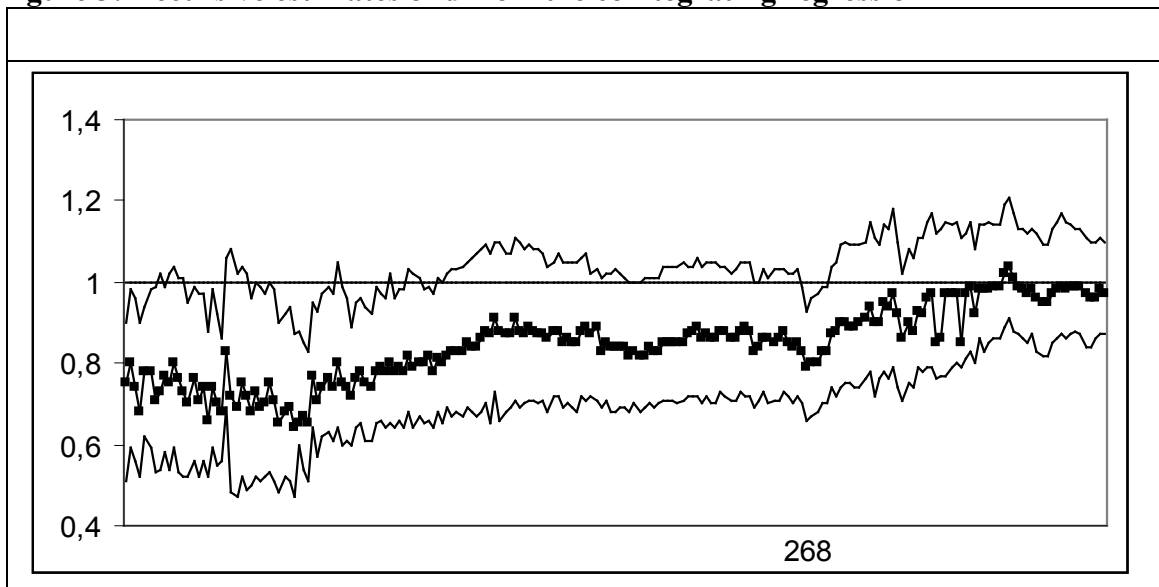


**Figure 2: Recursive estimates of d**



The thick line refers to the estimated values of  $d$ . The thin lines are the 95% confidence intervals. The horizontal axe refers to the number of observations used in each estimation. The vertical axe refers to the estimated values of  $d$ .

**Figure 3: Recursive estimates of  $d$  from the cointegrating regression**



The thick line refers to the estimated values of  $d$ . The thin lines are the 95% confidence intervals. The horizontal axe refers to the number of observations used in each estimation. The vertical axe refers to the estimated values of  $d$ .