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**Working Paper**

## Disentangling irregular cycles in economic time series

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Dominik Schober and Oliver Woll

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Zentrum für Europäische  
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Economic Research

Discussion Paper No. 15-067

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## **Disentangling Irregular Cycles in Economic Time Series\***

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## **Abstract**

Cycles play an important role when analyzing market phenomena. In many markets, both overlaying (weekly, seasonal or business cycles) and time-varying cycles (e.g. asymmetric lengths of peak and off peak or variation of business cycle length) exist simultaneously. Identification of these market cycles is crucial and no standard detection procedure exists to disentangle them. We introduce and investigate an adaptation of an endogenous structural break test for detecting at the same time simultaneously overlaying as well as time-varying cycles. This is useful for growth or business cycle analysis as well as for analysis of complex strategic behavior and short-term dynamics.

**Keywords** structural breaks, cluster analysis, filter, rolling regression, change points, model selection, cycles, economic dynamics

**JEL** C22, C24, C29, O47, L50

## 1. Introduction

From the beginning of economics, researchers have been interested in patterns of macro and microeconomic time series. Termed cycles, these may occur as seasonal, business or Kondratieff cycles in macroeconomics or Edgeworth cycles, asymmetric cost pass-through or price war cycles in microeconomics. With regard to cycles in macroeconomic “aggregate economic activity”, Burns and Mitchell (1946) developed a set of methods to summarize descriptive evidence and to date the business cycle. The methods were later adapted by the NBER for their judgments on business cycle turning points. Bry and Boschan (1971) approximated these rules by a simple non-parametric algorithm based on a sequence of lead and lag slopes as an identification procedure to determine the *reference cycle*. The major rationale is similar for many microeconomic studies using non-parametric approaches to identify dynamics. Wang (2009), for example, uses information about troughs and peaks to identify cycle lengths and, subsequently, further information, such as cycle amplitudes. In contrast to non-parametric methods, abundant contemporaneous studies use parametric methods to study cycle characteristics. Macroeconomic methods in this field are either devoted to dating structural changes, such as structural break tests and Markov Regime Switching,<sup>i</sup> or are about extracting detected or assumed characteristics of a time series, such as the application of moving averages or Fast Fourier Transforms in filtering analysis.<sup>ii</sup> At the microeconomic level, allocation and competitive processes – and also collusive practices – depend mainly on “temporary opportunities” and a “special knowledge of the opportunities”, as Hayek (1945, p.822) put it, and are, therefore, mostly short-term. In many markets, such as the sale and distribution of gasoline, food retailing and commodity markets, recurring

dynamic characteristics of prices can be observed.<sup>iii</sup> Methods in this field are similar to macroeconomic techniques but concentrate mainly on structural break tests and Markov Regime Switching.<sup>iv</sup> Filter-based methods for measuring business cycles, such as the one developed by Baxter and King (1999), require that the researcher begin by specifying the characteristics of these cyclical components. Taking seriously the authors' two questions, "How should one isolate the cyclical component of an economic time series?" and "how should one separate business-cycle elements from slowly evolving secular trends and rapidly varying seasonal or irregular components?", we try to explore the cyclical characteristics of a time series under the least restrictive set of ex ante assumptions on data generating processes. In this regard, two important facts should be mentioned about economic cycles. First, economic cycles encompass different frequency levels of economic activity. They vary in a range of 40 years for Kondratieff cycles, 1.5 to eight years for business cycles and down to several weeks or hours for asymmetric cost pass-through and Edgeworth cycles. As a matter of fact, a panoply of different cyclical dynamics may simultaneously exist. We term them overlaying cycles. Second, cycles – on each of these frequency levels such as hourly and weekly or business cycles – may be characterized by varying lengths (or frequencies) over time, depending on changes in the data generating process. We term them time-varying cycles.

In our view, the above mentioned methods lack flexibility regarding the possibility to obtain information both on the overlaying and time-varying cycles. Methods such as structural break tests, Markov Regime Switching and non-parametric methods ignore the possibility of simultaneous, overlaying cycles, whereas filtering analysis ignores the time-varying nature of

cycles. The first methods focus on determining time-varying cycles and the latter on determining overlaying cycles. Thus, these approaches are either in a sense static over time in that they (exogenously) assume constant dynamics over an investigated period or they have difficulties in disentangling different frequency levels.

We therefore propose an approach to identify and disentangle the recurring patterns with regard to both overlaying as well as time-varying cyclical dynamics. Following the spirit of articles such as Zarnowitz and Ozyildirim's (2006) our approach uses a combination of methods, namely rolling endogenous structural break test regressions and an endogenous clustering of break indicators. This is applied to a mesh constructed by applying moving average filters of varying length to the time series. Inspecting and comparing the information of the entire mesh in terms of break dates and lengths between break dates (cycle lengths) allow us to identify the different dominant overlaying cycles. At the same time, these cycles are allowed to vary over time. We therefore contribute to the literature by presenting an approach to simultaneously allowing for both the disaggregation of simultaneously overlaying cycles and the determination of time-varying cycles of a time series. Thereby, the proposed adaptation of the widespread Bai and Perron test allows, to a high degree, an explorative analysis of cyclical time series characteristics. This improves the characterization of long- and short-term dynamics, such as growth or inflation analysis, that is important in the field of macroeconomic analysis as well as microeconomic analysis, such as the analysis of competitive characteristics that change over time, important for work in the fields of strategic consumer and supplier behavior, as criticized e.g. by Corts (1999).



The next section discusses the weaknesses of current approaches to cycle analysis. After presenting our extension of the Bai and Perron (1998) break test in the third section, we investigate the impact of our modifications by a direct comparison to the classical Bai and Perron, Markov Regime Switching, and Fast Fourier Transform approaches in section four. We choose an electricity price time series for an illustration in the microeconomic field. It is particularly interesting, because it has, at the same time, several regularities, such as overlaying cycles (seasonalities, weekly or hourly cycles) and time-varying cycles due to occasional deviations caused, for example, by demand shifts or unforeseen stochastic renewable electricity production. In the macroeconomic field, we choose the classical example of a national product growth time series with typical features such as seasonal and business cycles of varying lengths. Section five discusses the results and section six concludes. The Online Appendix contains more simulation studies to give better understanding of the characteristics of our approach.

## **2. Approaches to Cycle Analysis**

We require that our approach meet one central objective, extract time series cyclical information endogenously thereby indicating both dominant overlaying and time-varying cycles. It is, therefore, of major importance to us which assumptions a researcher will have to accept using contemporaneous methods in the field of cycle analysis and how these methods perform with regard to the simultaneous analysis of simultaneously overlaying and time-varying cycles.

Typical time series analytical methods such as the classical Bai and Perron structural break test (BP), Markov Regime Switching (MRS) or non-parametric methods focus on analyzing time-varying cycles. They mostly ignore the simultaneous, overlaying cycles of a time series and identify the cycle with the largest amplitude (e.g. seasonal, daily depending on the time series). Also, more advanced structural break tests, such as those of Kejriwal and Perron (2008) and Bataa et al. (2013) as well as MRS methods such as Chauvet and Hamilton (2005), Chauvet and Pizer (2008) and Altuğ et al. (2013) use parametric tests based on the variance of a time series.

The much simpler non-parametric approach chosen by Harding and Pagan (2002) follows a similar intuition in terms of exploiting the variance of a time series. They applied the Bry and Boschan algorithm to several time series showing that this method performs well compared to MRS methods. This triggered a discussion in Hamilton (2003), Harding and Pagan (2003), and Harding and Pagan (2006), making clear that much of the information obtained by applying MRS methods is equivalent to the information from a Bry and Boschan algorithm application.<sup>v</sup> Non-parametric approaches, therefore, suffer the same weakness, which is that they ignore simultaneous, overlaying cycles.

Other typical methods such as the Fast Fourier Transformation (FFT) and filtering techniques focus on simultaneous, overlaying cycles. In contrast to structural break tests and MRS they ignore time-varying characteristics. For example, they have significant problems identifying asymmetric phases, such as peak and off peak phases, which are typically characterized by different interval lengths. However, in many areas, moving averages, Fourier Transforms,

Hodrick-Prescott- and Baxter-King-filters are state of the art.<sup>vi</sup> Recent developments in this field have tried to address this problem but have not found their way to application yet.<sup>vii</sup>

Similarly, studies based on dynamic factor model analysis assume certain stochastic processes remaining constant over time. For example, Kose et al. (2012) investigate the convergence of business cycles, separating them into different components (global, national, etc.) and then search for different cyclicalities. Also, similar to the spirit of this article, some authors have used combinations of different methods to better capture the characteristics of time series. Following this line, Zarnowitz and Ozyildirim (2006) filter a time series, thereby deriving a phase average trend (75-months-trend), and then apply the Bry and Boschan algorithm. Compared to typical filtering techniques, they found that their approach captured short-term details of the time series better.

All of these methods are problematic due to several reasons concerning their respective necessary assumptions. In addition to their focus on either simultaneously overlaying or time-varying cycles (see Table 1), they require substantial arbitrary ex ante judgment on assumed stochastic processes – except maybe for the case of the non-parametric methods.

**Table 1** Focus of methods analyzing time-varying and simultaneously overlaying cycles

	Bai and Perron	Markov Regime Switching	Fast Fourier Transformation	Adapted Bai and Perron
Vertical, overlaying dimension			x	x
Horizontal, time-varying dimension	x	x		x

With regard to MRS, Altug et al. (2013), for example, mention judgment issues concerning the number of states or the variability of transition probabilities. The detection of overlaying cycles is also not possible. Temporal variance dependent MRS could partly resolve problems

of static state definition. However, this would also be an *ex ante*, exogenous assumption because variance dependence would be estimated on the basis of the entire time series. In a similar way, Harding and Pagan (2003) criticize the implicit assumption of a reference cycle being necessary for MRS. The other methods involve arbitrary judgment with respect to filter specification, pre-selection of frequency levels and cycle lengths<sup>viii</sup> or the static application of test criteria, such as is the case for BP structural break tests, MRS and also non-parametric dating methods. By this token, these methods define test characteristics for the entire time series. Break tests, Regime Switching or factor analysis, in this sense, remain static instead of dynamically adapting to a time series' evolution and a possibly changing data generating process. We see this as a serious restriction of time series data analysis because the *ex ante* assumptions regarding the data generating process made by these methods pre-determine their results. When coming to real life applications, these processes typically change over time due to economic reasons. This is the case, for example, for structural demand changes, the introduction of substitutes or new technologies and the habit formation of consumers. It may, therefore, be equally important for both macroeconomic and microeconomic analysis to have methods at hand imposing less severe restrictions and assumptions on the data generating process.

### **3. Modeling Framework**

The modeling framework is laid out in three steps. First, filtering by means of moving averages is described. This serves to isolate time series characteristics at different average cycle lengths or frequency levels. Furthermore, the intuition of the classical Bai and Perron

(1998) test statistic is sketched. Second, the identification of time-varying characteristics by means of rolling endogenous break test regressions for each of the filtered versions of a time series is demonstrated. How the distribution of all cumulated break dates can be used to identify definite, robust and representative break dates is discussed. For this purpose, endogenous cluster analysis is applied to the results of the rolling break tests for the corresponding averaged time-series. The section lengths between these definite break dates then provide the corresponding economic cycles. Repeating this for different averaging windows, we obtain the distribution of the frequency of occurrence of corresponding cycles. This is described in a last, third step.

### **3.1. Adaption of Bai and Perron's Endogenous Break Test for Detecting Overlaying Cycles**

Bai and Perron (1998) developed a structural break test, which estimates coefficients in a structural break model.<sup>ix</sup> For each partition  $(T_1, \dots, T_m)$  of structural break dates, an associated least-squares estimate is obtained by minimizing the sum of squared residuals ( $SSR = \sum u_t^2$ ) with  $m$  as the break index and  $t$  as the usual time index. Here, the intuition is to identify additional breaks whenever the reduction of  $SSR$  is significant. Bai and Perron developed the following test statistic for the  $n^{\text{th}}$  additional break to measure the significance of an additional structural break:  $[SSR(n-1) - SSR(n)]/MSE(n)$  with  $MSE$  as the mean squared error, respectively the mean of the squared residuals  $(\frac{1}{T} \sum u_t^2)$ . Hence, the structural break dates identify partitions with similar patterns or, in other words, partitions with the most significant variance reduction separating the longest continuous horizontal intervals.<sup>x</sup> The length of the time

interval between two successive break dates can be regarded as a certain cycle of a time series, which encompasses a relatively homogenous pattern. For the purposes of this article, an economic cycle is defined as a relatively homogenous time interval with respect to some measure (mean, variance and so forth), which is separated by structural changes. Moreover, cycles shall be defined in reference to certain average cycle lengths or frequency levels as borrowed from Fourier Transform terminology. Each of these frequency levels shall comprise sufficiently homogenous cycles. We call these frequency levels a time series' overlaying characteristics. At each of these levels, however, the cycles may alter their duration. These are the economic cycles or the time-varying characteristics of a time series.

Real data time series are usually not smooth and are often characterized by overlaying and varying short-term, high frequency cycles. Due to this fact, the ratio between the *SSR* reduction with an additional break and the *MSE* can vary within a wide range. This is caused by several overlaying cycles. In the case of a sufficiently large *SSR* reduction in relation to the *MSE*, the sum of squared residuals reduces significantly and the test statistic will indicate an additional break date. In the case of a small *SSR* reduction in relation to the *MSE*, the test will stop with less structural break dates.<sup>xi</sup> Therefore, on the one hand, the Bai and Perron test may identify a relatively large number of structural breaks and only very small cycles. Phases with high volatility or strongly increasing or decreasing data values are typical phases for which this effect will occur. On the other hand, there is the possibility that the Bai and Perron test will identify relatively few structural breaks. This will occur in phases with low volatility or decreasing/increasing trends relative to the time series' length, or, in other words, the entire time series' *SSR*. To cope with these problems and to extend the Bai and Perron test for

identifying structural breaks in overlaying cycles, we will adapt this test using rolling averages before applying the structural break test to each of the averaged series. The adapted structural change model is then

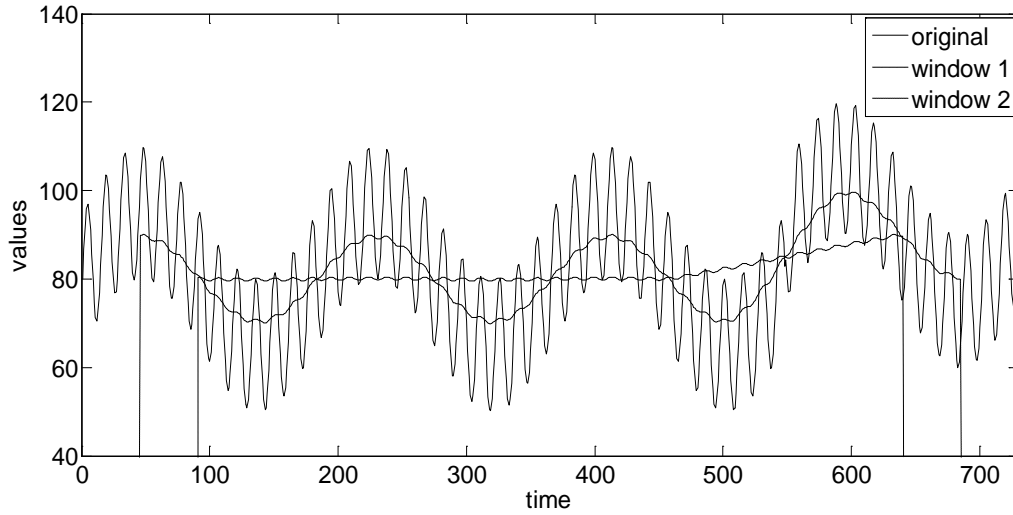
$$y_t^w = x_t' \beta + z_t^{w'} \delta_j^w + u_t^w, \quad (1)$$

$$t = T_{j-1} + 1, \dots, T_j, \text{ for } j = 1, \dots, m + 1, T_0 = \frac{w}{2} \text{ and } T_{m+1} = T - \frac{w}{2}$$

$x_t^{\text{xii}}$  and  $z_t^w$  = covariates;  $\beta$  and  $\delta_j^w$  = coefficients;  $m$  = number of breaks and  $y_t^w$  is calculated as

$$y_t^w = \frac{1}{w} \sum_{\tau=t-\frac{w}{2}}^{\tau=t+\frac{w}{2}} y_\tau$$

Here,  $w$  denotes the length of the moving window for the calculation of the rolling averages. The size of  $w$  has a number of obvious effects. An increasing  $w$  reduces the number of observations for the structural break test and reduces the volatility of the time series. Diagram 1 illustrates the application of different lengths of  $w$  on a simulated time series consisting of different overlaying sine cycles (15 days, 180 days and a vertical shift of 10 for the last 180-day sine cycle). Here, the size of window 1 is greater than the size of window 2.



**Diagram 1** Original values of a simulated time series; rolling averages of the same time series with averaging windows of  $w=90$  and  $w=180$  days

Regarding the test statistic, it is obvious that possible reductions of the *SSR* are much smaller for large window lengths  $w$ . Thus, the question of which sizes of  $w$  to employ in the analysis arises. A priori, each and every possible size of  $w$  could be used for the determination of structural break dates. Subsequently, significant changes in the number of structural breaks between the different versions of the filtered time series could be identified. Depending on the length of the time series regarded, this could lead to many calculations and high computational effort. In this case, grid approaches are useful to overcome problems of computational time. Usually, equidistant grids are most suitable when no information on a time series' economic cycles exists. In cases of searching for specific cycles, the grid can be adapted accordingly.

It has to be mentioned that the different smoothing windows do not change the overlay of the different dynamics. Averaging is centered, leading to a symmetric extension of the averaging window around a certain date  $t$ . The different averaging windows help to visualize the



movements at the respective, envisaged cycle length (or frequency level). In section 3.3, it will be shown how these identified cycles from respective filtered versions of the time series, i.e. at different envisaged average cycle lengths or on different frequency levels, and their cumulated number of occurrences will be used as a measure for the importance of the respective cycle lengths. As this is done simultaneously with the identification of time-varying cycles, the next section describes how the rolling structural break tests applied to the different filtered versions of the time series will help identify respective cycles on each frequency or average cycle length level.

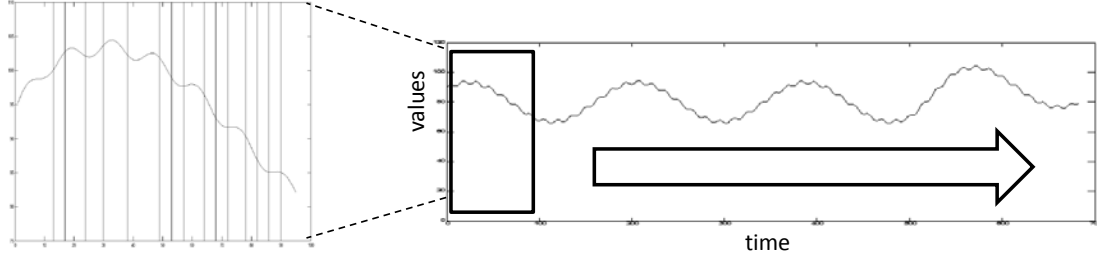
### **3.2. Adaption of Bai and Perron's Endogenous Break Test for Detecting Time-Varying Cycles**

The above adaptation addresses the problem of detecting overlaying cycles but still ignores time-varying characteristics. To address this problem, we again use a rolling estimation method according to the work of Officer (1973) and Fama and MacBeth (1973). Analogous to the methods proposed by these authors, we take a subsample of the time series and apply the structural break test for this subsample. This is done for every subsample that can be constructed from the time series. The number of subsamples – and thereby structural break tests – depends on the ratio of the lengths of the original time series and the subsample. The resulting break dates for each regression will be memorized. The structural change model evolves to

$$y_t^r(\tau) = x_t'(\tau)\beta + z_t^{r'}(\tau)\delta_j^r(\tau) + u_t^r(\tau), \forall \tau \in \left\{\frac{r}{2}, \dots, T - \frac{r}{2}\right\} \quad (2)$$

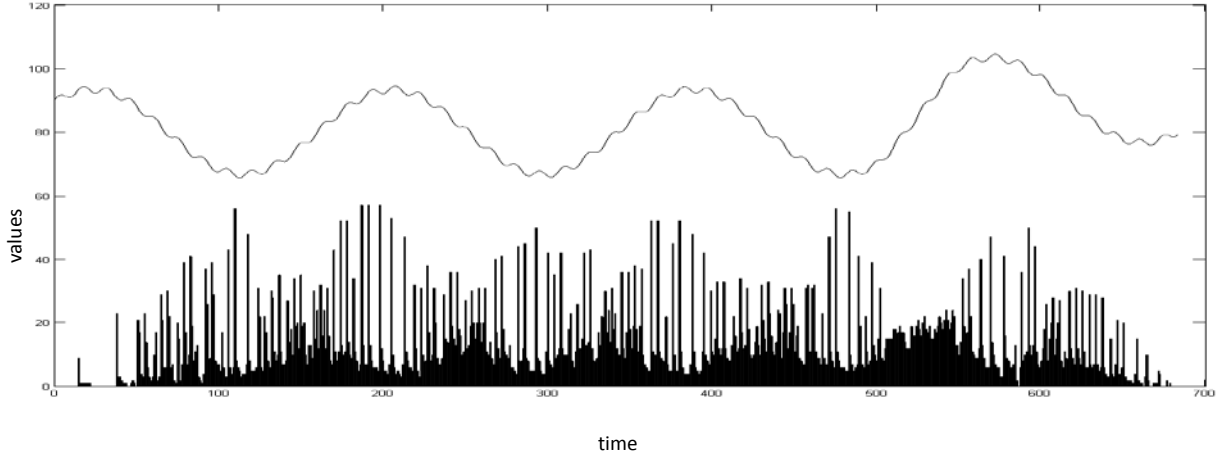
$$t = T_{j-1} + 1, \dots, T_j, \text{ for } j = 1, \dots, m + 1, T_0 = \tau - \frac{r}{2} \text{ and } T_{m+1} = \tau + \frac{r}{2}$$

$y_t^r$  = observed variable;  $x_t$  and  $z_t^r$  = covariates;  $\beta$  and  $\delta_j^r$  = coefficients;  $m$  = number of breaks.



**Diagram 2** Rolling endogenous structural break test window and averaged time series (at the level of  $w=90$  days rolling average)

Diagram 2 illustrates the principle of the rolling regression in our case, where the small black box indicates the subsample. The break test regressions are repeated along the time dimension starting on each of the successive dates of the entire time series from  $1+w/2$  up to  $T-w/2$ . This is then repeated for each of the filtered versions of the time series. The choice of the size of this subsample is crucial and can be optimized according to Foster and Nelson (1996). Similar to the case of using rolling averages, one could calculate the rolling tests for all possible subsample sizes – for each of the filtered versions of the time series. Again, a grid approach is useful to avoid computational problems. As a result, we obtain the structural break dates of each subsample as a result of the rolling regressions applied to the filtered time series. Diagram 3 illustrates the distribution of the occurrence of all *cumulated* break dates.



**Diagram 3** Cumulated break date indicators of rolling endogenous structural break tests (*black bars*) and  $w=180$  days rolling average time series (*blue line*)

In a second step, definite break dates of the time series will be extracted from this cumulative distribution of break dates. Accumulation points have to be determined to derive definite break dates. Endogenous cluster analysis will serve for this purpose.<sup>xiii</sup> In most cluster techniques, the number of clusters is predetermined. The optimal number of clusters can be determined e.g. by using error measures (cf. Tibshirani et al. 2001). Here, we choose the number of clusters according to the silhouette method (cf. Rouseeuw 1987). The silhouette's value is a measure of similarity within one cluster and is calculated as

$$S_i = \frac{(b_i - a_i)}{\max(a_i, b_i)}$$

with  $a_i$  being the average distance from point  $i$  to other points in the same cluster, and  $i$  and  $b_i$  being the minimum average distance from point  $i$  to points in the different clusters. A high silhouette value is evidence of a good matching of point to cluster. Per definition, the silhouette value is between -1 and 1. Here, 1 indicates a perfect match. To derive the optimal number of clusters, we repeat the calculation of the silhouette values for different numbers of

clusters and calculate the average silhouette value for the partition. The partition with the highest silhouette value indicates the optimal number of clusters.

These clusters determine the definite break dates for each of the filtered versions of the time series or, in other words, for each of the different frequency or average cycle length levels.

### 3.3. Simultaneous Detection of Overlaying and Time-Varying Cycles

To address both the varying behavior over time and the overlaying cycles, rolling averages and rolling structural break regressions from structural change models (1) and (2) will be combined accordingly.

$$y_t^{w,r}(\tau) = x_t'(\tau)\beta + z_t^{w,r'}(\tau)\delta_j^{w,r}(\tau) + u_t^{w,r}(\tau), \forall \tau \in \left\{\frac{r}{2}, \dots, T - \frac{r}{2}\right\} \quad (3)$$

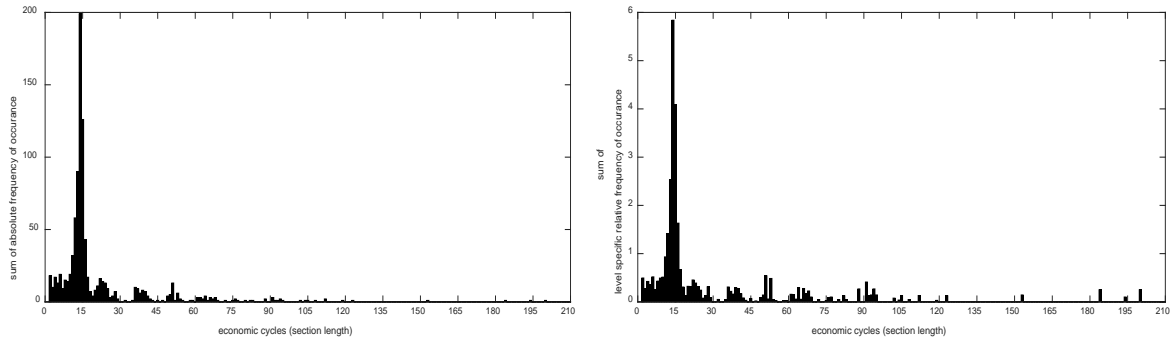
$$t = T_{j-1} + 1, \dots, T_j, \text{ for } j = 1, \dots, m + 1, T_0 = \tau - \frac{r}{2} \text{ and } T_{m+1} = \tau + \frac{r}{2}$$

$$y_t^{w,r} = y_t^w(T_0, \dots, T_{m+1}); x_t \text{ and } z_t^{w,r} = \text{covariates}; \beta \text{ and } \delta_j^{w,r} = \text{coefficients};$$

$m$  = number of breaks and  $y_t^w$  is calculated as the rolling average mentioned above.

As a result of this model, we obtain the definite break dates by applying the cluster analysis on the results of the rolling break tests for the corresponding averaged time-series. The section lengths between these definite break dates then provide the corresponding economic cycles. Repeating this for different averaging windows, we obtain the distribution of the frequency of occurrence of corresponding cycles.

The number of maximum possible break dates and the corresponding cycles differ at each (subsequently changing) size of the averaging window and according to the subsample's size. Thus, the absolute frequency of a certain cycle has to be evaluated in relation to the total number of cycles detected at a certain frequency level. The maximum absolute frequency at a certain level is given by the number of possible definite break dates plus one. Thereby, we can calculate the relative, level-specific frequency of occurrence of a certain economic cycle. Diagram 4 illustrates the comparison of the absolute frequency to the relative frequency of occurrence. Longer economic cycles have an increased frequency when relative frequencies are used for the analysis (rhs diagram).



**Diagram 4** Distribution of frequencies of identified economic cycles (section lengths between definite break dates); a) unweighted (lhs), b) weighted relative to the level-specific maximum frequency of occurrence of breaks (rhs)

The dominant cycles of the time series can then be derived from this distribution. The 15-day cycles are dominant, whereas the longer cycle is only moderately detected. This can be attributed to the fact that long (180-day) cycles only occur four times in the original time series.<sup>xiv</sup> It is clear that this structural change model can be estimated for every size of the moving window for the rolling averages and also for every size of the subsample for the rolling break tests. Since economic cycles are regularities and, thus, should not change

extremely fast, in our view, it is sufficient to run the model for an equidistant grid of moving window and corresponding subsample sizes.

An illustration of the operation of the method for three different simulated time series is given in the Online Appendix.

## **4. Application**

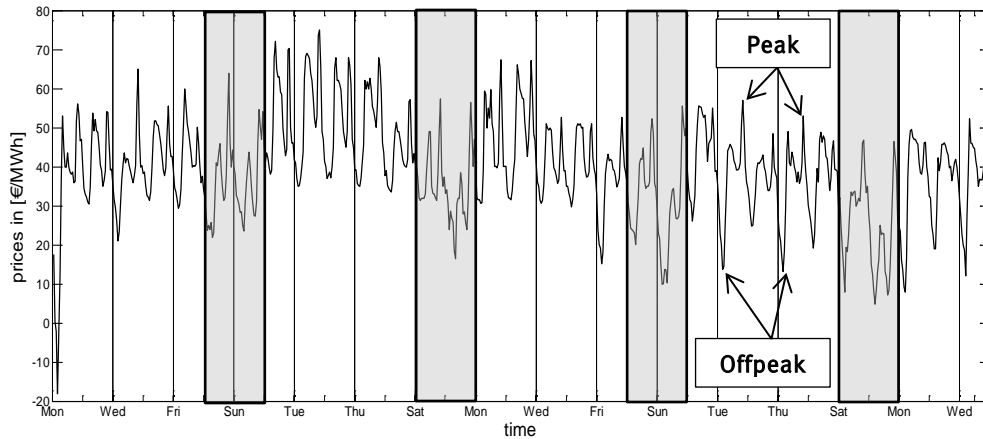
To demonstrate the impact of our approach to identify and disentangle the recurring patterns with regard to both overlaying as well as time-varying cycles, we chose a time series in a microeconomic and a macroeconomic area for the application. The results will be compared to other approaches such as the classical Bai and Perron (BP), Markov Regime Switching (MRS) and Fast Fourier Transform (FFT) methods.

### **4.1. Data**

#### *Microeconomic Time Series: Hourly Electricity Prices*

For an illustration of the properties of the developed approach for dating breaks and detecting economic cycles and for a subsequent comparison to the other methods, we take data for electricity spot prices in Germany traded at the European Energy Exchange (EEX). Spot prices for electricity have some periodic characteristics. First, prices can be classified as working and weekend days and have different demand characteristics. Second, within days, a further distinction in peak and off-peak hours is possible, with peak hours from 8 a.m. to 8 p.m. on working days and off-peak hours in the complementary time interval according to the EEX definition. Larger cycles such as seasonalities do exist but will not be addressed in our

application for reasons of clarity. We will concentrate on hourly and daily cycles and, therefore, choose the prices during March 2010 for our application.

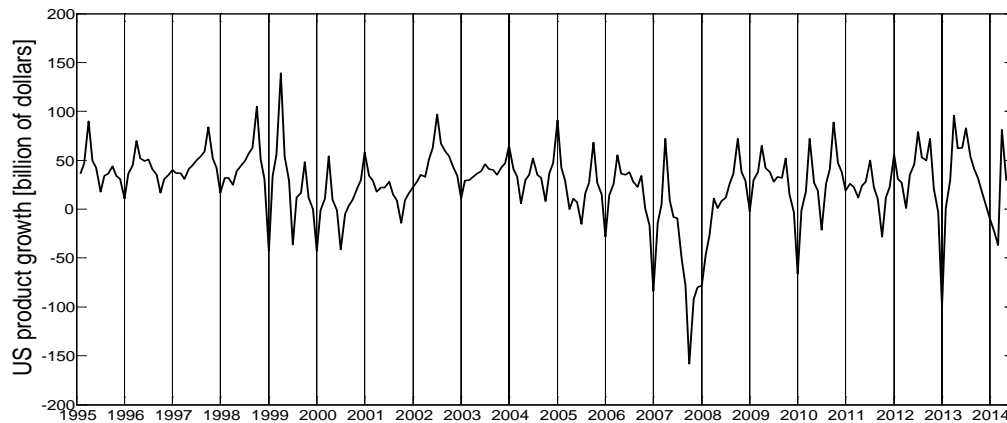


**Diagram 5** EEX day ahead electricity prices, March 2010; source: EEX

Diagram 5 shows the EEX price data and illustrates the periodic weekend cycle as well as peak and off-peak prices. The latter vary with demand (in contrast to the 8 a.m. to 8 p.m. definition of the Energy Exchange) and, therefore, show a noon and an afternoon peak. Moreover, the time series has properties favorable for our analysis. For example, the working day-weekend cycle is recognizable, but deviations from the regular cycles do exist, such as the Friday of the second week, which is more similar to the weekend than to the other working days. The different methods shall identify exactly such time-varying irregularities of regular economic cycles, in addition to the separation of overlaying cycles such as weekly, working day or hourly cycles.

*Macroeconomic Time Series: National Product Growth*

For application in the macroeconomic area, we take data for US product growth from January 1996 to July 2015 on a monthly basis. Product growth reflects the value of different products and services. Thus, product growth contains different cycles, such as seasonalities or longer business cycles.



**Diagram 6** US product growth, January 1996 to July 2015; source: Reuters

Diagram 5 shows the product growth data and illustrates the existence of seasonal and yearly cycles as well as longer cycles of five or six years. Shorter seasonalities are irregular, but, frequently, very short-term cycles of about six months are observable. This is especially true for the turbulent years after 2000 and after the dip of the economic crisis in 2008. However, cycles are not stable during the remainder of the time series either. Short-term cycles typically range from six months to one year.

## 4.2. Comparison: Fast Fourier Transform and Markov Regime Switching

### *Fast Fourier Transform*



The FFT converts a finite list of equally spaced samples of a function into a list of coefficients of a finite combination of complex sinusoids. It is ordered by frequencies (of the sample values). Thereby, it converts the sampled function from the original domain (often a time or position along a line) to the frequency domain. The Discrete Fourier Transform is part of the Fourier analysis and used for empirical purposes.  $H(f) = \sum_{t=1}^N h(t)\omega_N^{(t-1)(f-1)}$  is the Fourier Transformation of  $h(t) = \frac{1}{N} \sum_{f=1}^N H(f)\omega_N^{-(t-1)(f-1)}$  where  $\omega_N = e^{\frac{-2\pi i}{N}}$  is an  $N^{\text{th}}$  root of unity,  $h(t)$  is a function of time  $t$  and  $H(f)$  is a function of frequency  $f$ . Inverting the support (frequencies) provides signals (amplitudes, prices) in the period space.

Frequencies directly refer to different cycle lengths of the time series in the time domain, and amplitudes give information regarding their significance.

### *Markov Regime Switching*

MRS regressions assume several states indicating a different data generating process in each state. In the simplest form, level changes are investigated, but more complicated formulations considering e.g. autoregressive characteristics are possible. For the purpose of this article, the formulation considering level switches with two states is sufficient.

$$p_t = \begin{cases} \alpha_0 + \beta p_{t-1} + \varepsilon_t, & s_t = 0 \\ \alpha_0 + \alpha_1 + \beta p_{t-1} + \varepsilon_t, & s_t = 1 \end{cases} \quad (4)$$

with transition probabilities

$$\mathbf{P} = \begin{bmatrix} P(s_t = 0 | s_{t-1} = 0) & P(s_t = 1 | s_{t-1} = 0) \\ P(s_t = 0 | s_{t-1} = 1) & P(s_t = 1 | s_{t-1} = 1) \end{bmatrix} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}. \quad (5)$$

States can, thus, take the values  $s_t = \{0,1\}$ , and  $p_t$  denotes the price in period  $t$ . Matrix  $\mathbf{P}$  contains the transition probabilities between the respective states. The price equation is characterized by a regime switch, which leads to a level shift of  $\alpha_1$ . The absolute term is  $(\alpha_0 + \alpha_1)$  for state 1 instead of  $\alpha_0$  for state 0. The parameters of the model,  $\mathbf{P}$  and  $\boldsymbol{\theta} = (\alpha_0, \alpha_1, \beta)$ , can be estimated via maximum likelihood estimation.

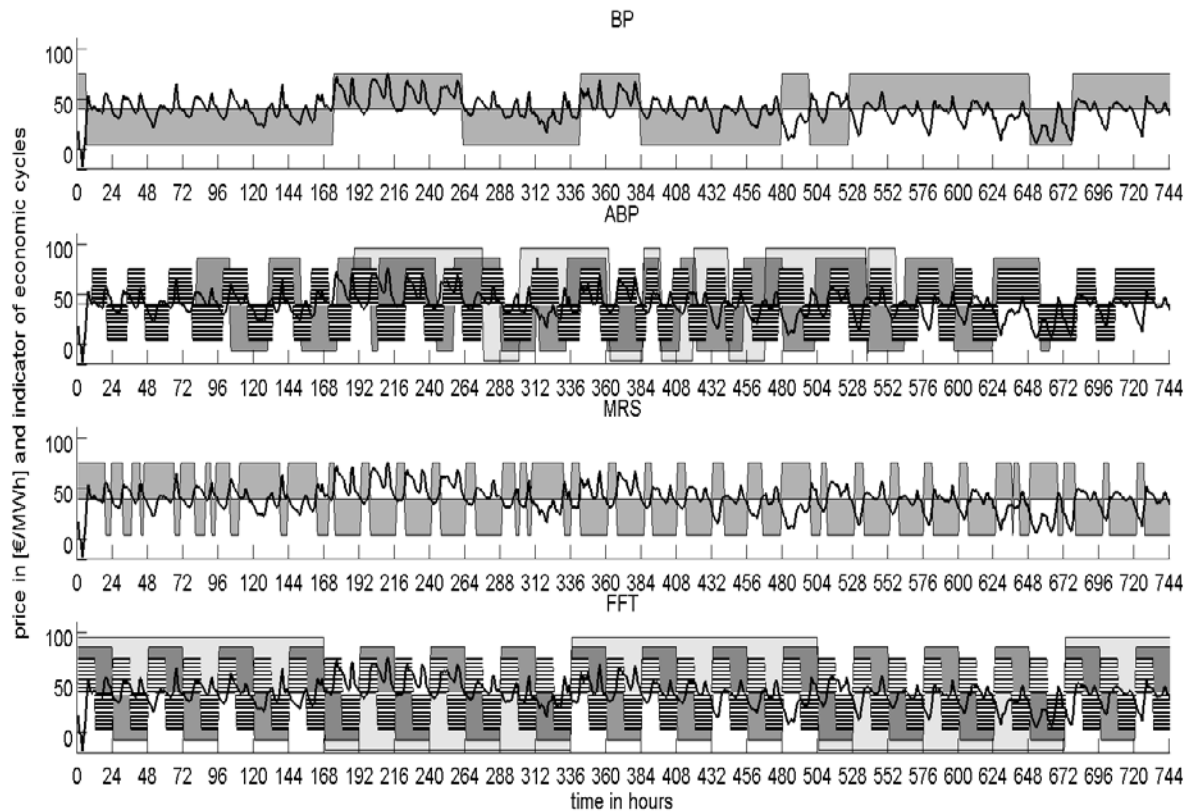
The dates of transitions between  $s_t = 0$  and  $s_t = 1$  are the “structural breaks” we use for the comparison.

### 4.3. Results and Comparison

#### *Microeconomic Time Series: Hourly Electricity Prices*

##### Overlaying and Time-Varying Economic Cycles

We apply the Bai and Perron (BP), adapted Bai and Perron (ABP), Markov Regime Switching (MRS) and Fast Fourier Transform (FFT) to electricity price data. For the ABP we use an equidistant grid of 10 time steps. The following diagrams illustrate the differences between the methods. The solid black line depicts the EEX electricity prices of March 2010 in all of the four diagrams. The rectangle areas indicate breaks and state changes – or economic cycles. They extend below or above the mean of the time series depicted by the fine horizontal line. Every time the indicator line crosses the mean line, the respective method finds a state switch, and the rectangle areas between the lines change their position from below to above and vice versa. The magnitude of the indicator line has no meaning; it is solely used to indicate breaks.



**Diagram 7** EEX day-ahead electricity prices, March 2010; identified economic cycles according to the different approaches (a) Bai and Perron endogenous structural break test, b) adapted Bai and Perron test, c) Markov Regime Switching, d) Fast Fourier Transform

The BP and MRS do not explicitly differentiate between different frequency levels of economic cycles. In other words, in a one-shot procedure they judge all cycles according to one single decision criterion – the reduction in residuals. These cycles will only be identified if and only if they are sufficiently important with respect to this criterion. In contrast, the ABP and FFT find a multiplicity of economic cycles because they decompose the time series vertically, i.e. according to overlaying cyclical information. Three partitions for exemplary nodes (window sizes) of the grid are depicted in the graphs.<sup>xv</sup>

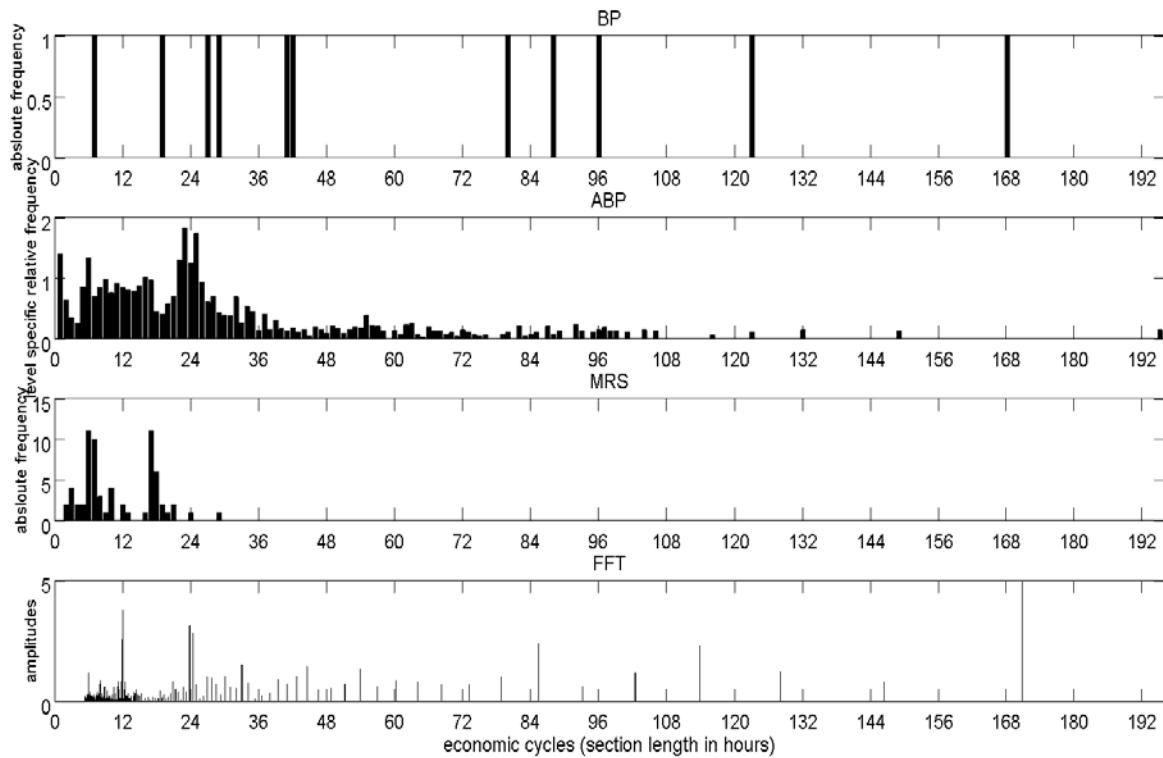
It is evident that the BP and the MRS search for level shifts in the time series. Whereas the BP reduces the overall squared residuals of the time series by adding additional breaks and inserting horizontal regression lines with variable intercepts, the MRS uses two (or more) states over the entire time series with identical intercepts in the respective states. Therefore, the identified states are similar for BP and MRS during large parts of the time series, such as the first time interval until about hour 170 and the interval from 530 until the end. Nevertheless, according to the MRS, many higher frequency interruptions of the respective current state are identified. Higher-frequency jumps, which are relatively large with respect to the intercept's difference of the two identified states, lead to this result. Moreover, the identified sections between breaks are asymmetric. This uncovers the short-term, time-varying characteristics of the time series. In contrast, the recursive logic and the test statistic, building on the overall sum of squared residuals of the BP, interrupt the search for breaks earlier and have higher degrees of freedom regarding the choice of intercepts.<sup>xvi</sup> However, both the BP and the MRS find time-varying economic cycles.

The ABP and FFT decompose dominant economic cycles of the time series explicitly on different frequency levels. The ABP identifies slow cycles as depicted by the pale rectangle areas in the background of the diagram. This economic cycle corresponds to a cycle of three and four days. The medium frequency is a little faster and finds daily cycles as well as some 1.5-day cycles. Both of these frequencies are similar to the ones identified by the classical BP. The highest frequency level depicts 12- and 18-hour cycles as well as a shorter cycle of only a few hours (6 to 8 hours). These faster economic cycles are similar to the cycles found with the MRS. It should be noticed that the cycles found are of varying speed over the time series. This

is evident when the higher frequency interval at about hour 200 is compared to the lower frequency interval at about hour 300. The FFT is not able to identify cycles of varying length. The three FFT-economic cycles are absolutely regular according to the circular signal transformation. The cycles found on different frequency levels by signal decomposition correspond to regular weekly, 24-hour and 12-hour cycles.

#### Distribution and Representativeness of Identified Economic Cycles

The following diagrams depict the cumulative distributions of the section lengths between the respective break dates for BP, MRS and ABP. In contrast, the FFT diagram depicts the coefficients of equation  $H(f) = \sum_{t=1}^N h(t) \omega_N^{(t-1)(f-1)}$ , which indicate the relevance of the respective frequency of the periodic cycles in explaining the time series. All four approaches, therefore, analyze time intervals with a frequency notion.



**Diagram 8** Distribution of frequencies of identified economic cycles in EEX electricity prices; a) Bai and Perron endogenous structural break test, b) adapted Bai and Perron test, c) Markov Regime Switching, d) Fast Fourier Transform

The BP diagram shows the distribution of the section lengths between the breaks determined by the applied BP endogenous structural break test. Section lengths are equally weighted because each of them occurs only once. Different clusters are observable with some cycles around 24, 42 and 80 to 96 hours as well as some single cycles at about six, 120 and 168 hours. We can, thus, roughly detect 24-hour, 48-hour, 96-hour sections and one 120-hour and one 168-hour section, sections that, for some cases, deviate from the typical 24-hour cycle. These deviations are often about six hours long, indicating nighttime off-peak sections or daytime peak sections of a short duration.

The distribution of the ABP with its successively applied rolling break tests on different frequency levels, exhibits much more vertical variation. Peaks at the cycles of six to seven, 16 to 17 and 22 to 24 hours are more clearly identifiable regarding the short-term economic cycles. Medium-term cycles are discovered at approximately 30 and 54 hours with some longer cycles at approximately 72, 84 and 96 hours. Finally, two recognizable long-run cycles are identified at 150 and 196 hours. The identified long-term cycles are relatively insignificant in amplitude. The application of the ABP in this form, i.e. weighting each filtered version of the time series equally before cumulating respective breaks, ignores the vertical variation of each filtered version (its minimum-maximum price spread). With respect to their contribution to the explanation of the time series' overall price variation however, these long-term cycles are more important because their minimum-maximum spread is relatively large compared to that of higher frequency levels (using shorter moving average filters). This is demonstrated by an application of the alternative amplitude based weighting of respective frequency levels in section 5. Another noticeable aspect is the wide, more uniform variation of cycle distribution found between six to seven and 16 to 17 hours. This additional information is useful to understand that short-term cycles do not strictly follow the asymmetric distribution characterized by the two peaks at six to seven and 16 to 17 hours.

The MRS diagram shows an interesting pattern. Only section lengths up to about one day are detected. The dominant lengths are, again, six to seven and 17 to 18 hours. The MRS completely fails to determine lower frequency cycles because of the magnitudes of the price movements on the different frequencies. In other words, were the amplitudes of cyclical

movements larger for longer cycles (larger moving average filters), the MRS would find these economic cycles instead and ignore the short-term cycles identified here.

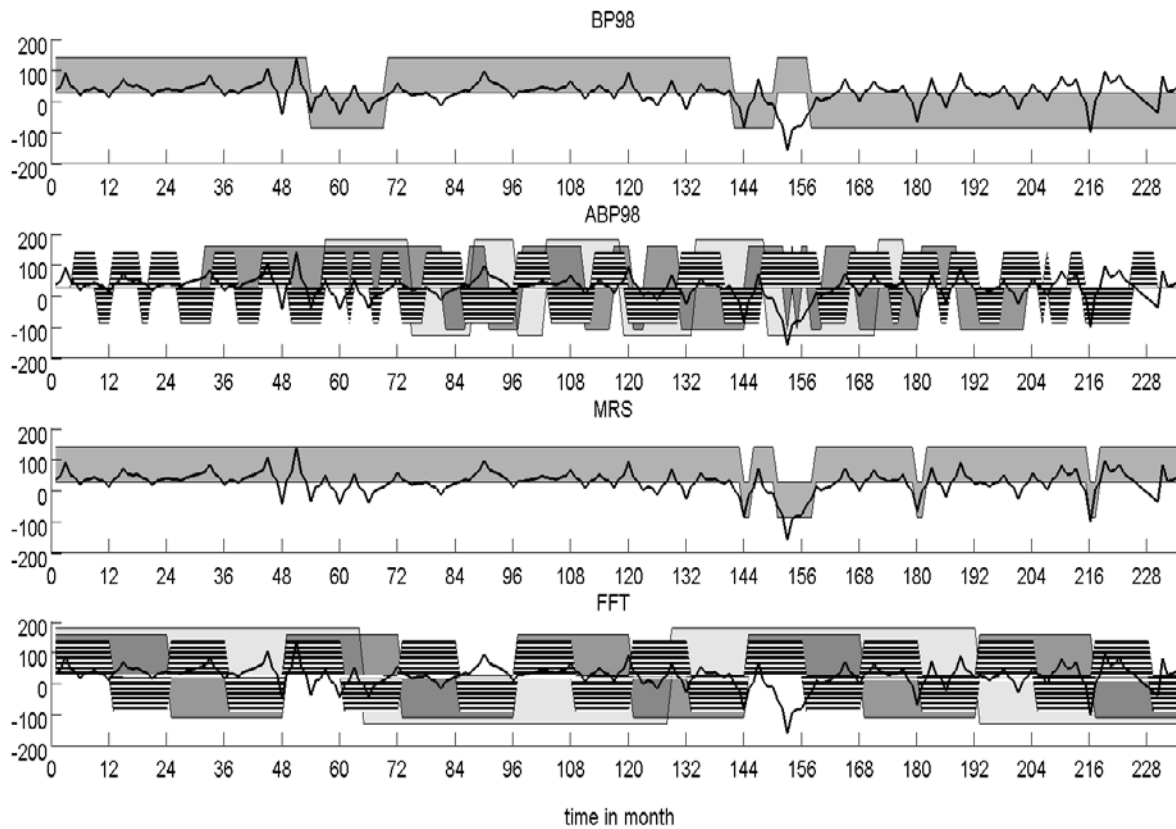
The last diagram describing the FFT results shows coefficients  $h(t)$  of the cyclical patterns determined for the time series. The requirement that the periodicity of the time series remains constant throughout the time series leads to the result that the shorter varying section lengths of six to seven and 17 to 18 hours determined by MRS and ABP are leveled out. Instead, the FFT offers 12- and 24-hour cycles. In contrast, it has no difficulty with non-varying lower frequency cycles (80 and 168 hours).

### *Macroeconomic Time Series: National Product Growth*

#### Overlaying and Time-Varying Economic Cycles

We now apply the BP, ABP, MRS and FFT to US product growth. For the ABP, we use an equidistant grid of five time steps. The following diagrams illustrate the differences between the methods. The diagrams are analogous to the microeconomic time series diagram 7. The solid black line depicts the US product growth in all of the four diagrams.





**Diagram 9** US product growth, January 1996 to July 2015; identified economic cycles according to the different approaches (a) Bai and Perron endogenous structural break test, b) adapted Bai and Perron test, c) Markov Regime Switching, d) Fast Fourier Transform

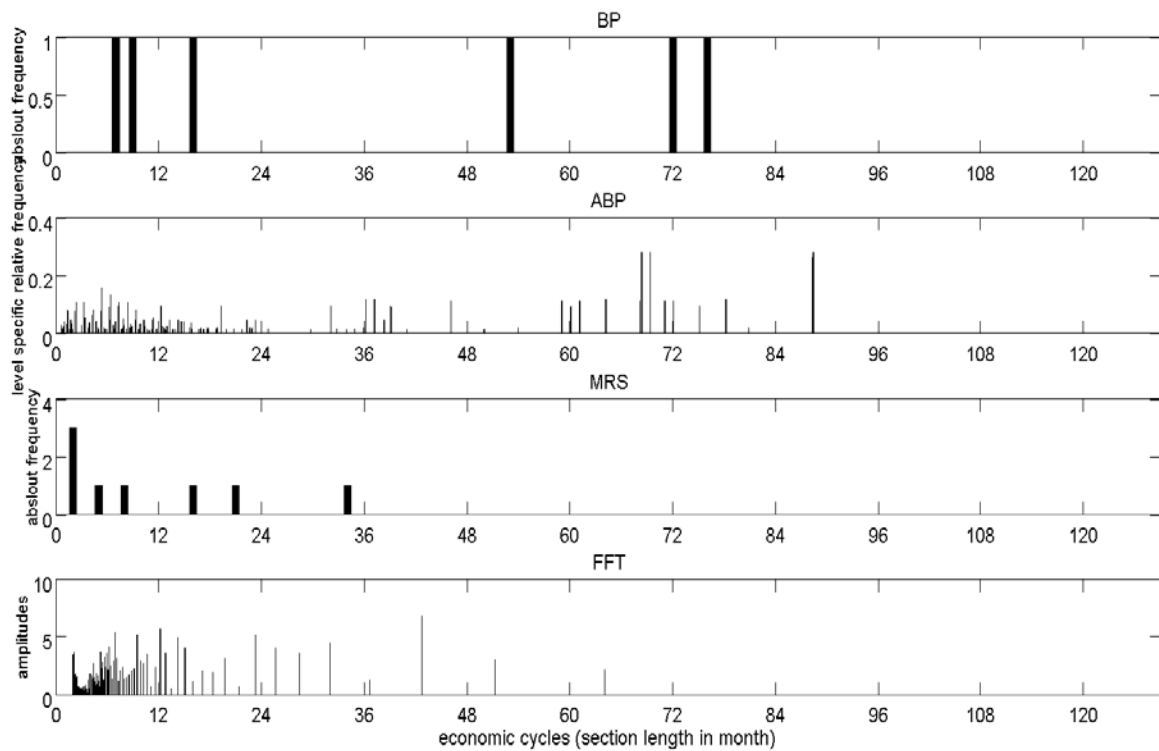
In general, the results are similar to the electricity price time series. ABP and FFT decompose dominant cycles of the time series explicitly on different frequency levels, which provides information on the different simultaneously overlaying cyclicalities. BP and MRS do not provide this information. The long ABP and FFT cycles correspond to lengths of about 50 to 60 months, whereas shorter cycles are found to be about 12 months for the FFT and range between six and 14 months for the ABP. Some longer cycles are found by the ABP, up to 72 months, whereas extremely short cycles can be as short as two to three months. These faster

cycles are similar to the cycles found with the MRS. The FFT is not able to identify varying cycles. The three FFT-economic cycles are absolutely regular according to the circular signal transformation.

Shorter time-varying cycles are detected by BP, ABP and MRS after the 2008 (month 156 and subsequent months) crisis. However, after the shock in year 2000 (month 60 and subsequent months), only the ABP detects the turbulence characterized by a series of shorter cycles during the subsequent months. The BP test does detect a different, longer period of product growth level shift. The MRS does not detect any of the shorter cycles because of the relatively low variation of product growth compared to later shifts in the time series, i.e. the 2008 crisis and its aftermath.

#### Distribution and Representativeness of Identified Economic Cycles

The following diagrams depict, analogous to diagram 7, the cumulative distributions of the section lengths between the respective break dates for BP, MRS and ABP.



**Diagram 10** Distribution of frequencies of identified economic cycles in US product growth; a) Bai and Perron endogenous structural break test, b) adapted Bai and Perron test, c) Markov Regime Switching, d) Fast Fourier Transform

As above, the BP diagram shows the distribution of the section lengths between the BP breaks. Section lengths are equally weighted because each of them occurs only once.

The distribution of the ABP with its successively applied rolling tests on different frequency levels exhibits significant vertical variation. More clearly, peaks at the cycles of about three and six to nine and 12 months are identifiable regarding the shorter-term seasonal cycles. Longer-term business cycles are identified to range regularly between 36 and 72 months. Some exceptions occur at 24, 48 and 90 months.

The MRS fails to detect long-term cycles of more than 36 months. In contrast, it finds the most extreme, short-term dips of two months to the most regularly occurring cycle of the time series. Shorter cycles of five and eight months and then medium-term cycles of 16, 20 and 34 months are found.

The last diagram shows the FFT coefficients of the cyclical patterns. The FFT detects seasonal six, nine and 12 month cycles as they occur more irregularly than in the electricity price time series. Furthermore, very short-term shock dips of about two to three months are identified. The business cycles, however, are characterized by shorter lengths of about 24 to 48 months, compared to the ABP.

## **5. Discussion**

### **5.1. ABP, BP, FFT, and MRS**

The ABP identifies time-varying economic cycles at different frequency levels or, in other words, at different filtered versions of the original time series. However, the ABP does not seem to give a very clear picture of which section lengths are representative, especially concerning very short-term cycles. One might wish to obtain more clearly peaking cycles as, for example, is the case for the FFT. This is not a shortcoming. Instead, this result is one of the envisaged goals of using the ABP approach, which tries to extract more differentiated information about a time series' properties. This argument is best explained by taking a look at the short-term cycles. The density of the frequencies of ABP economic cycles is high over the entire interval from hours six to seven up to hours 16 to 17. This demonstrates first that large parts of the variation of the entire time series are due to cycles of these lengths and

second that there is not a single dominant cycle at one of these short-term frequencies. On the other hand, there is still exact information on the distribution of peaks (at hours 6, 16 and 17 and later at 22/24) and deviations from these peaks.<sup>xvii</sup> Similarities and differences in comparison to the other methods are discussed in the following to highlight the main characteristics and their consequences for the respective results obtained.

The BP determines time-varying characteristics and finds homogeneous periods though the time series length influences the partition. This becomes clear from a simple example. A sine function of length  $2\pi$  will result in several breaks at  $\pi/2$ . Extending this series to  $50\pi$  will not set an additional break to the minimum number of two because there is no significant reduction in the overall variation of the time series.<sup>xviii</sup> In consequence, the number of breaks found in a time series depends on the ratio of the series' length to its variation. The BP is constructed in a way that makes it difficult to find breaks in homogeneously recurring, periodic movements (e.g. sinusoidal functions). It is better suited to situations where shifts are the dominant cause of structural heterogeneity. Furthermore, it does not explicitly address different frequency levels. It only finds longer cycles if their amplitudes are sufficiently large. The ABP, in contrast, may find breaks on different frequency levels by its use of rolling regressions. There is also no signal about the importance of a certain economic cycle. Weighting on the basis of, for example, the minimum-maximum spreads of a time series on each respective frequency level or filtered version of the time series is possible and will provide further information on the significance of the contribution of the respective frequency level or average cycle length to the overall time series variance. This will be discussed later in this section.

Similarly, the FFT has the capability of determining different economic cycle levels and assigns weights to the frequencies but fails in separating the time series into intervals of time-varying cycles on a given frequency level (see section 4). This is better performed by the BP, the ABP and the MRS. The FFT builds on cyclical frequencies (e.g. combinations of sinusoidal functions), which makes it difficult to extract regularly but asymmetrically varying processes such as in the 6/18 (MRS), six to seven and 16 to 17 hours (ABP), as compared to the 12 and 24 hours for the FFT example above. It is further impossible to date breaks exactly. Unlike the other methods, it does not use break dates to determine frequencies but rather directly fits the frequencies to the time series and determines coefficients  $h(t)$  as amplitudes.

As already mentioned, the MRS is also capable of finding not only average but also time-varying cycles, such as the peak/off-peak relation in the above example. However, the MRS has some well-known drawbacks (cf. section 2). In this regard, it can be seen as a static approach. It assumes the number of states as exogenously given. In contrast to the other methods, the MRS cannot, therefore, detect more varying cycles than assumed ex ante. Furthermore, in this rudimentary version of the MRS, these states remain fixed over time and refer to characteristics of the entire time series, such as the series' mean. As it does not consider different frequency levels, it only finds state changes when values reach a state defined as relative to the full amplitude over the time series. Therefore, only economic cycles on the frequency level with the maximum amplitude determine the states. In addition, the MRS only separates between different levels of states (e.g. high and low in the 2-state case). This implies that, unlike the BP and ABP, this method does not search for homogeneous periods. In contrast, sufficient changes with respect to the maximum amplitude in the time

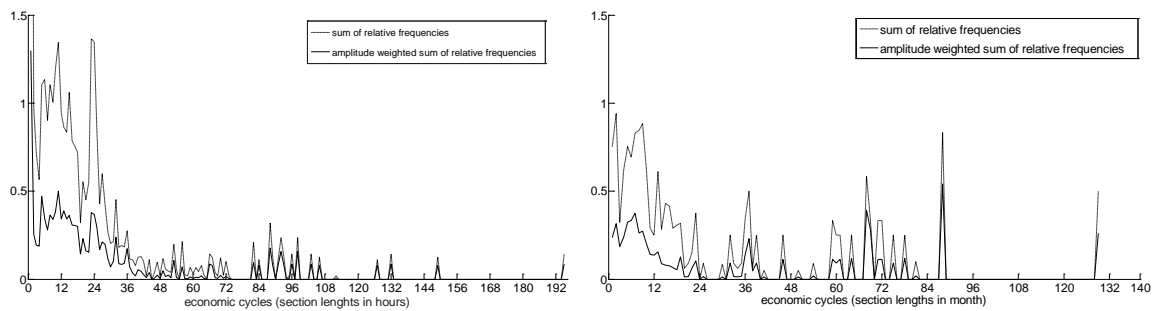
series will cause a state switch. Hence, once values are in a certain state, the MRS ignores small local variation. In its basic version, the MRS – and also the FFT – is a static approach, which assumes constant characteristics of the data generating process over the whole time series. It has to be mentioned that time-varying MRS versions, which attenuate this problem by reducing the reference for the state definition by referring to the local maximum amplitude, do exist. However, to our knowledge, it is impossible to address simultaneously overlaying cycles. Furthermore, the time-varying MRS makes necessary the definition of the varying parameters (e.g. the length of the local time interval for reference), which remain constant over time as well.

## **5.2. The ABP Approach – Significance of Overlaying and Time-Varying Economic Cycles**

The methods compared are suited to different degrees to time varying and overlying analyses of economic cycles.

The FFT is constructed for the decomposition of the overlaying cycles of a time series. It is not suited to identify time-varying properties. In contrast, BP and MRS are methods well suited to finding the time-varying properties of a time series, ignoring the simultaneous existence of its overlaying cycles. The ABP is suited to identifying overlaying and time-varying economic cycles at the same time. However, due to the involved degrees of freedom in the analysis, it does not necessarily perform better than either BP or MRS – given a certain frequency level of analysis, i.e. given a certain average cycle length to be analyzed – or than the FFT, absent time-varying cycles.

As mentioned earlier in this section, the ABP can be enhanced according to the goals of an investigation. The contribution of the respective cycles on each frequency level to the overall variance of a time series, for example, can be considered by additional weighting of the cumulated cycle length distributions of detected cycles on each frequency level by the maximum-minimum spreads of corresponding filtered time series values. This is, then, more similar to the FFT coefficients. The following diagram contains a comparison between unweighted and amplitude weighted frequency levels, i.e. the different filtered versions of the EEX electricity price (lhs) and US product growth (rhs) time series.



**Diagram 11** Distribution of frequencies of economic cycles in EEX electricity prices (lhs) and US product growth (rhs) by the adapted Bai and Perron test (ABP); solid lines represent unweighted and dotted lines amplitude weighted (corresponding maximum-minimum spread) cumulated cycle length distributions

Amplitudes on each frequency level, i.e. the corresponding versions of the filtered time series, are measured as the maximum-minimum spread of values of the respective filtered time series. Weights are then derived by calculating ratios with reference to the maximum-minimum spread of the unfiltered time series. In this application, the highest frequency levels for both time series dispose of the largest amplitudes. This is easily seen from the time series diagram (see diagram 5). Therefore, these frequencies increase most in value or, in other words, gain most significance with regard to the relative frequency distributions of cycle



lengths. The other cycle lengths' relative frequencies also increase, whereas, for the EEX electricity prices cycle, lengths between 84 and 96 hours gain most and, for the US product growth cycle, lengths of about 36 and between 60 and 90 months gain most.

Thereby, different weighting procedures can help to identify economic cycles according to the researcher's interest. Still, other weightings could be chosen, such as the average amplitudes of the windows over an entire time series or the standard deviation, depending on one's taste for robustness or representativeness.

### **5.3. The ABP Approach – Possible Variations**

The ABP approach arbitrarily uses certain methods at different stages. As an alternative to the simple rolling averaging approach, other extraction methods could be used, such as band-pass filters (cf. Baxter and King 1999). It should be noticed that the band-pass filter application would only replace the moving average filtering of our analyses, maintaining all other steps of the analysis (rolling BP test application, cumulating break dates, clustering to determine definite breaks and analyzing distributions of determined cycle lengths). However, in our view, this would only alter the results to a minor degree.

The other methods of comparison might be extended to achieve more differentiated results similar to the ABP. For example, instead of the BP structural break test, one could use the MRS on the different frequency levels after filtering. Alternatively, the FFT could be decomposed along the time domain. Once the different dominant frequency levels are derived, subsections would have to be analyzed separately to obtain time-varying analyses of

the economic cycles over the entire time series. However, these analyses are beyond the scope of this article.

## **6. Conclusions**

This article adapts the endogenous structural break test by Bai and Perron (1998) (BP) and successively applies it in rolling regressions in combination with a filtering of a time series. This approach offers richer information than classical BP, Markov Regime Switching (MRS) or Fast Fourier Transform (FFT) applications to a time series. The presented adaptation of the Bai and Perron test (ABP) disaggregates the time series with regard to both time-varying cycles, similar to the MRS and BP, and overlaying cycles, similar to the FFT. This allows determining time-varying and overlaying cycles without having to form strong assumptions on data generating processes *ex ante*. This can improve empirical analysis, especially in cases when there is uncertainty about the economic model driving the behavior of market participants or about fundamental market forces. Examples in this regard are the relative importance of hourly vs. weekly effects in electricity prices and their cycle variation over time or the relative importance of business vs. seasonal cycles in product growth (cf. Gabaix 2011, Beaudry and Portier 2014). This information can serve to disentangle effects on different overlaying cycle levels to e.g. separate time-varying behavioral supply side impacts on prices, which may occur on different frequency levels than demand side effects. This can be the case e.g. in electricity wholesaling as analyzed in this article where much strategic conduct takes place on an hourly basis, but demand side effects vary with e.g. working week and week-end intervals. The identification of time-varying behavior is also of interest in markets of high

frequency interaction, such as Internet markets, or markets with cyclical regularities or asymmetric cost pass-throughs, which alter over time (Peltzman 2000). In practical microeconomic work, it can be used for antitrust policy as, for example, in collusion detection (Abrantes-Metz et al. 2006, Harrington and Chen 2006) or in the important field of market delineation, which is becoming more prominent again in view of a more (short-term) behavior based, “more economic approach”.

The ABP might also be of use in e.g. business cycle analysis. Here, it is important to disentangle the characteristics of a time series at different, a priori unknown and possibly dynamically varying, cycle lengths. Thereby, the ABP complements methods using exogenous, constant cycle definitions, such as MRS, and avoids their inappropriate features for the analysis of overlaying and time-varying cycles as described in Chauvet and Hamilton (2005).

This approach might be extended in various dimensions, such as offering the possibility to cope with possibly cointegrated explanatory variables (Kejriwal and Perron 2008, Bataa et al. 2013). Moreover, a more complete picture with regard to variations of this adaptation approach, such as the use of different filtering techniques described in section 5, would be desirable.

## **References**

Abrantes-Metz, R.M., Froeb, L.M., Geweke, J., Taylor, C.T. (2006), A variance screen for collusion, *International Journal of Industrial Organization*, vol. 24(3), 467–486

- Altuğ, S., Tan, B., Gencer, G. (2013), Cyclical dynamics of industrial production and employment: Markov chain-based estimates and test. *Journal of Economic Dynamics and Control*, vol. 36(10), 1534–1550
- Bai, J., Perron, P. (1998), Estimating and testing linear models with multiple structural changes, *Econometrica*, vol. 66(1), 47–78
- Bai, J., Perron, P. (2003), Computation and analysis of multiple structural change models, *Journal of Applied Econometrics*, vol. 18(1), 1–22
- Bataa, E., Osborn, D.R., Sensier, M., van Dijk, D. (2013), Structural breaks in the international dynamics of inflation, *Review of Economics and Statistics*, vol. 95(2), 646–659
- Baxter, M., King, R.G. (1999), Measuring business cycles: approximate band-pass filters for economic time series, *Review of Economics and Statistics*, vol. 81(4), 575–593
- Beaudry, P., Portier, F. (2014), News-driven business cycles: insights and challenges, *Journal of Economic Literature*, vol. (52), 993–1074
- Borgy, V., Clerc, L., Renne, J.-P. (2014), Measuring aggregate risk: can we robustly identify asset-price boom-bust cycles? *Journal of Banking & Finance*, vol. (46), 132–150
- Bry, G., Boschan, C. (1971), *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*. National Bureau of Economic Research, New York.
- Burns, A.F., Mitchell, W.A. (1946), *Measuring Business Cycles*. National Bureau of Economic Research, New York.
- Canova, F., Alain Schlaepfer, A. (2015), Has the Euro-Mediterranean partnership affected Mediterranean business cycles? *Journal of Applied Econometrics*, vol. (30), 241–262

- Chauvet, M., Hamilton, J.D. (2005), Dating business cycle turning points, in “Nonlinear time series analysis of business cycles;” ed. Van Dijk, Milas, and Rothman, Elsevier’s Contributions to Economic Analysis Series, 1–54
- Chauvet, M., Piger, J. (2008), A comparison of the real-time performance of business cycle dating methods, *Journal of Business & Economic Statistics*, vol. (26), 42–49
- Christiano, L.J., Fitzgerald, T.J. (2003), The band pass filter, *International Economic Review*, vol. 44(2), 435–465
- Claessens, S., Kose, M.A., Terrones, M.E. (2012), How do business and financial cycles interact? *Journal of International Economics*, vol. (87), 178–190
- Corts, K.S. (1999), Conduct parameters and the measurement of market power, *Journal of Econometrics* (88), 227–250
- Fama, E.F., MacBeth, J.D. (1973), Risk, return, and equilibrium: empirical tests, *Journal of Political Economy*, vol. 81(3), 607–636
- Foster, D.P., Nelson, D.B. (1996), Continuous record asymptotics for rolling sample variance estimators, *Econometrica*, vol. 64(1), 139–174
- Gabaix, X. (2011), The granular origins of aggregate fluctuations, *Econometrica*, vol. (79), 733–742
- Haltiwanger, J., Harrington, J.E. Jr. (1991), The impact of cyclical demand movements on collusive behavior, *RAND Journal of Economics*, vol. (22), 89–106
- Hamilton, J.D. (2003), Comment on “A Comparison of Two Business Cycle Dating Methods”, *Journal of Economic Dynamics and Control*, vol. (27), 1691–693
- Harding, D., Pagan, A.R. (2002), Dissecting the cycle: a methodological investigation. *Journal of Monetary Economics*, vol. (49), 365–381

- Harding, D., Pagan, A.R. (2003), A comparison of two business cycle dating methods, *Journal of Economic Dynamics and Control*, vol. (27), 1681–1690
- Harding, D., Pagan, A.R. (2006), Synchronization of cycles, *Journal of Econometrics*, vol. (132), 59–79
- Harrington, J.E. Jr., Chen, J. (2006), Cartel pricing dynamics with cost variability and endogenous buyer detection, *International Journal of Industrial Organization*, vol. 24(6), 1185–1212
- Hartigan, J.A., Wong, M.A. (1979), Algorithm AS 136: a K-means clustering algorithm, *Journal of the Royal Statistical Society, Series C (Applied Statistics)*, vol. (28), 100–108
- Harvey, A.C., Trimbur T.M. (2003) General model-based filters for extracting cycles and trends in economic time-series, *Review of Economics and Statistics*, vol. 85(2), 244–255
- Hayek, F.A. von (1945), The use of knowledge in society, *American Economic Review*, vol. 35(4), 519–530
- Hodrick, R.J., Prescott, E.C. (1997), Postwar U.S. business cycles: an empirical investigation, *Journal of Money, Credit and Banking*, vol. 29(1), 1–16
- Kejriwal, M., Perron, P. (2008), The limit distribution of the estimates in cointegrated regression models with multiple structural changes, *Journal of Econometrics*, vol. 146(1), 59–73
- Kim, I.-M. (1997), Detecting the number of structural breaks, *Economics Letters*, vol. (57), 145–148
- Kose, M.A., Otrok, C., Prasad, E. (2012), Global business cycles: convergence or decoupling? *International Economic Review*, vol. (53), 511–538
- Maskin, E., Tirole, J. (1988), A theory of dynamic oligopoly II: price competition, kinked demand curves, and Edgeworth cycles, *Econometrica*, vol. (56), 571–599

- Neveu, A.R. (2013), Fiscal policy and business cycle characteristics in a heterogeneous agent macro model. *Journal of Economic Behavior & Organization*, vol. (92), 224–240
- Noel, M.D. (2007), Edgeworth price cycles, cost-based pricing, and sticky pricing in retail gasoline markets, *Review of Economics and Statistics*, vol. (89), 324–334
- Officer, R.R. (1973), The variability of the market factor of the New York Stock Exchange, *Journal of Business*, vol. (46), 434–453
- Olsina, F., Weber, C. (2009), Stochastic simulation of spot power prices by spectral representation, *IEEE Transactions on Power Systems*, vol. (24), 1710–1719
- Peltzman, S. (2000), Prices rise faster than they fall, *Journal of Political Economy*, vol. (108), 466–502
- Preuß, P., Puchstein, R., Dette, H. (2013), Detection of multiple structural breaks in multivariate time series. SFB 823 Discussion Paper Nr. 33/2013, University of Bochum. URL: [https://eldorado.tu-dortmund.de/bitstream/2003/30598/1/DP\\_3313\\_SFB823\\_Preu%C3%9F\\_Puchstein\\_Dette.pdf](https://eldorado.tu-dortmund.de/bitstream/2003/30598/1/DP_3313_SFB823_Preu%C3%9F_Puchstein_Dette.pdf)
- Rotemberg, J., Saloner, G. (1986), A supergame-theoretic model of price wars during booms, *American Economic Review*, vol. (76), 390–407
- Rouseeuw, P.J. (1987), Silhouettes: a graphical aid to the interpretation and validation of cluster analysis, *Journal of Computational and Applied Mathematics*, vol. (20), 53–65
- Tibshirani, R., Walther, G., Hastie, T. (2001), Estimating the number of clusters in a data set via the gap statistic, *Journal of the Royal Statistical Society, Series B (Statistical Methodology)*, vol. (63), 411–423
- Verlinda, J.A. (2008), Do rockets rise faster and feathers fall slower in an atmosphere of local market power? Evidence from the retail gasoline market, *Journal of Industrial Economics*, vol. (56), 581–612

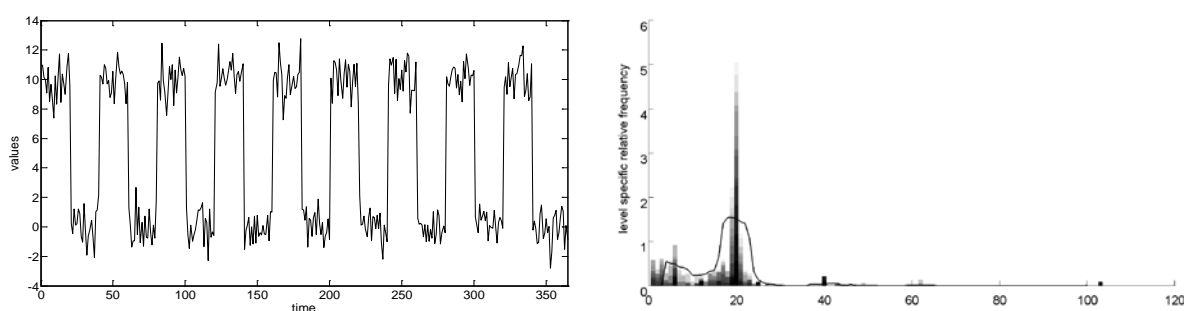
Wang, Z. (2009), (Mixed) strategy in oligopoly pricing: evidence from gasoline price cycles before and under a timing regulation, *Journal of Political Economy*, vol. (117), 987–1030

Zarnowitz, V., Ozyildirim, A. (2006), Time series decomposition and measurement of business cycles, trends and growth cycles. *Journal of Monetary Economics*, vol. (53), 1717–1739

Zhou, Y., Wan, A.T.K., Xie, S., Wanga, X. (2010), Wavelet analysis of change-points in a non-parametric regression with heteroscedastic variance, *Journal of Econometrics*, vol. (159), 183–201

## 7. Online Appendix

For a better understanding of our method, the adapted Bai and Perron test for structural breaks (ABP), we apply the method to three simulated time series. Simulating the time series gives the advantage that we know the cycles in these time series exactly and can see how the ABP will identify these cycles. The first simulated time series is called binary jump. Here, we construct a time series that jumps every 20 time steps between zero and 10. Within the 20 time steps, we modeled some white noise to avoid computational errors. Diagram 12 shows the simulated time series and the cycles identified with the ABP.

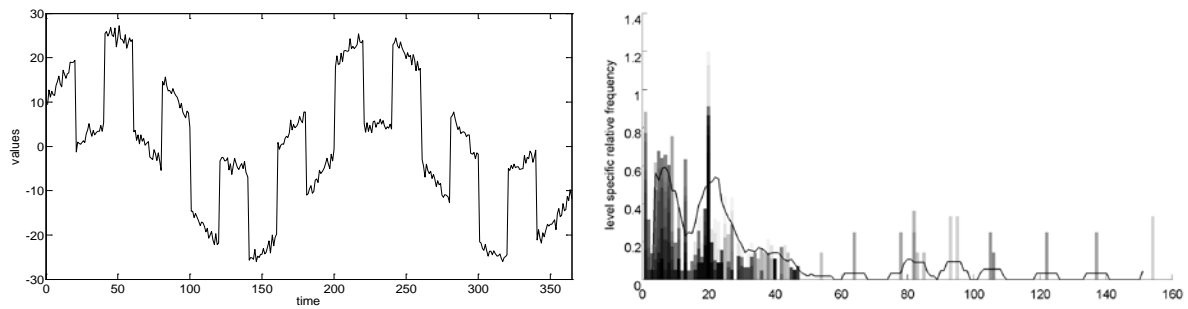


**Diagram 12** a) Values of the simulated time series binary jump (lhs), b) Distribution of frequencies weighted relative to the level-specific maximum frequency of the occurrence of breaks (rhs)



It is obvious that the 20 time step cycle is the major cycle in the results of the ABP. Additionally, we have some small cycles resulting from white noise within the 20 days. Other cycles have no significant occurrence.

The second simulated time series is called sinusoidal jump. Here, we construct a time series that jumps every 20 time steps along a sinusoid function with a period of 180 time steps. Within the 20 time steps, we again modeled some white noise. Diagram 13 shows the simulated time series and the cycles identified with the ABP.

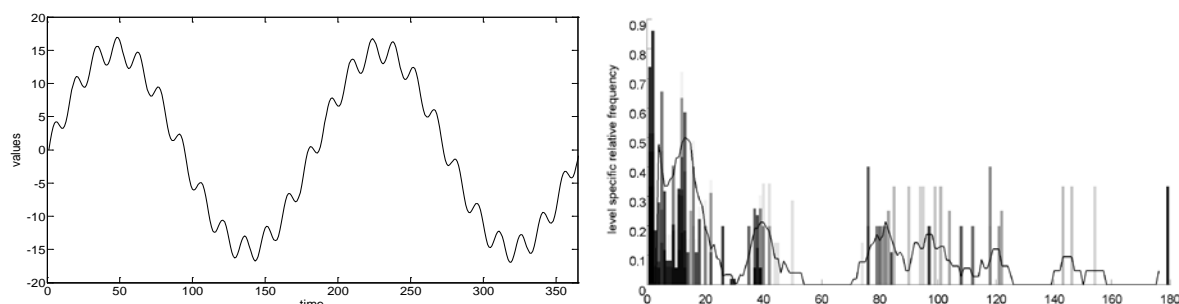


**Diagram 13** a) Values of the simulated time series sinusoidal jump (lhs), b) Distribution of frequencies weighted relative to the level-specific maximum frequency of the occurrence of breaks (rhs)

We can again see a major cycle at 20 time steps resulting from the jump. Additionally, we have a cycle of 10 time steps. This results from the white noise within the 20 days in combination with strong increasing or decreasing phases of the sinusoid. Then, the ABP will split up the 20 time steps with white noise in the middle. Another cycle with significant occurrence is around 45 time steps. This is the distance of the inflection point of the sinusoid function ( $180/4=45$ ).

The third simulated time series is called two sinusoids. Here, we construct a time series with two overlaying sinusoidal functions. The first one is with a period of 180 time steps. The

second one is with a shorter period of 14 time steps. Additionally, the shorter sinusoid has a smaller amplitude. Diagram 14 shows the simulated time series and the cycles identified with the ABP.



**Diagram 14** a) Values of the simulated time series for two sinusoids (lhs), b) Distribution of frequencies weighted relative to the level-specific maximum frequency of the occurrence of breaks (rhs)

We can see a major cycle of 14 time steps resulting from the shorter sinusoid. In addition, we have a cycle around 45 time steps. This again results from the distance of the inflection point of the longer sinusoid function. We can also see a significant occurrence of cycles around 90 time steps. This is half of the period length of the longer sinusoid function.

## Endnotes

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<sup>i</sup> See Bai and Perron (1998) and Chauvet and Hamilton (2005).

<sup>ii</sup> Low-pass, high-pass, and band-pass filtering are related techniques, which can be used to isolate and extract cyclical characteristics. E.g. Baxter and King (1999) propose a frequency domain filter and compare it to other methods in their seminal article to isolate business cycles. Further approaches are e.g. Hodrick and Prescott (1997) or Christiano and Fitzgerald (2003). Harvey and Trimbur (2003) propose a generalized model-based filter and, similar to Baxter and King, apply it to a simple macroeconomic time series (US investment).

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- <sup>iii</sup> Cf. Peltzman (2000). Beaudry and Portier (2014) study under which circumstances decentralized information can translate into cyclicalities and especially business cycles. See also Gabaix (2011) for a study on electricity prices.
- <sup>iv</sup> MRS approaches in microeconomics are used to identify certain patterns, such as positively skewed distributions (over time) of prices in asymmetric pricing. See e.g. Noel (2007). These asymmetric pricing models consist of asymmetric cost pass-through models (when input price or demand changes) and Edgeworth cycle models. See Haltiwanger and Harrington (1991), Rotemberg and Saloner (1986), and Maskin and Tirole (1988).
- <sup>v</sup> Recent application of non-parametric techniques is found, for example, in Neveu (2013) in the fiscal policy area; Claessens et al. (2012) compare financial and business cycles, and Canova and Schlaepfer (2015) study the business cycle convergence of Mediterranean countries.
- <sup>vi</sup> Cf. applications such as Borgy et al.'s (2014) investigation of housing and stock prices or Olsina and Weber's (2009) analysis of an electricity price time series.
- <sup>vii</sup> See for example Zhou et al.'s (2010) use of wavelet analysis to determine break points or Preuß et al.'s (2013) attempt to separate different sections for which spectral analysis offers insights into the variance characteristics of a time series.
- <sup>viii</sup> Cf. for example, Baxter and King (1999) choosing business cycle lengths of six to 32 quarters.
- <sup>ix</sup> The structural change model corresponds to the structural change model in (1) for the original time series instead of for the averaged time series, respectively  $T_0 = 1$  and  $T_{m+1} = T$ .

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<sup>x</sup> These patterns can be – in the most simple case – the mean characteristics of the partitioned series but can also be given by variances, the coefficient of variations or similar measures.

<sup>xi</sup> The test “endogenously decides” upon the number of breaks according to its test statistics.

<sup>xii</sup> The covariates  $x_t$  are not filtered by moving averages because the impact of their exact events or value changes shall be measured. The alternative of filtering covariates will measure the impact of a wider interval of covariate changes on the dependent variable, including leads and lags.

<sup>xiii</sup> See Hartigan (1975) and Hartigan and Wong (1979).

<sup>xiv</sup> We will discuss alternative weightings such as amplitude based weighting considering the maximum-minimum distance of each filtered time series’ version in section 5.2.

<sup>xv</sup> The complete mapping of all of the cycles on the different frequency levels is restricted to the following diagram for the sake of clarity.

<sup>xvi</sup> Regarding the BP, the positions of the rectangles relative to the time series’ mean does not indicate a “high” or “low” state because intercepts of the horizontal regression lines between the break dates are allowed to vary. Positions of the rectangles only indicate a “high” or “low” state for the MRS.

<sup>xvii</sup> Alternative weights and their impact on the identification of cycles are discussed later in the article.

<sup>xviii</sup> This can also be shown analytically by analysing the BP test statistic, assuming a symmetric sinusoidal time series.