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Conference Paper

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50th Congress of the European Regional Science Association: "Sustainable Regional Growth and Development in the Creative Knowledge Economy", 19-23 August 2010, Jönköping, Sweden

Provided in Cooperation with:

European Regional Science Association (ERSA)

Suggested Citation: Olsson, Michael (2010) : How robust are estimated country welfare? An investigation of welfare ranking, based on indices calculated using random weights., 50th Congress of the European Regional Science Association: "Sustainable Regional Growth and Development in the Creative Knowledge Economy", 19-23 August 2010, Jönköping, Sweden, European Regional Science Association (ERSA), Louvain-la-Neuve

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How robust are estimated country welfare? An investigation of welfare ranking, based on indices calculated using random weights.

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Abstract: We measure country welfare by an index number calculated from welfare components like GDP per capita and expected length of life. We rank countries from high to low welfare using such estimated welfare indices. In such calculation the chosen welfare components, the procedures used to normalize them, and the weight structure are important. Changing the components, the normalization procedures, or the weight structure change the welfare indices and it may in the next step also alter the rank order. In this paper, I present information about the importance of the weight structure, and the normalization procedure, taking the welfare components as given. I draw one million random weight structures and from the welfare indices that follow I extract the rank order for each structure. The result of this procedure is a rank order distribution for each country. In this paper, I present the rank order distribution for some countries. For example: According to the human development index Sweden is ranked at 7th place. With random weights Sweden is ranked as high as fifth place in 5.1 per cent of the cases, and as low as 14th in 13.6 per cent of the cases, with the mean rank equal to 9.8.

Key Words: Welfare, components, weights, index, ranking, distribution

1 Introduction

We use indices almost everywhere; we analyze for example entrepreneurial activity with the Global Entrepreneur Index, GEI (Bosma et al, 2008), economic freedom with the economic freedom of the world index (Gwartney et al, 2009), welfare with the Human Development

Index (UNDP, 2009). We also use indices to estimate school quality and hospital quality. There are projects in Sweden that tries to estimate the strain each patient cause to health care providers¹. The idea is that with such information resources can be distributed among health care providers in a fairer way. We rank decision making units (like countries, hospitals, schools and so on) with such indices. We use indices to rank the performance of foreign aid organizations. Even athletes are ranked, for example boxers and chess players, in a similar manner.

It is interesting to think of the welfare concept. We want to estimate the level of welfare at the country level, and we also want to relate the level in a country to the level in other countries. A measure of overall welfare is calculated by weighting normalized welfare components. With a specific set of weights each country receives a specific estimated welfare index. Based on these welfare indices countries are ranked from high to low welfare. Each country has a set of production factors, which they use to produce welfare. The Swedish Institute for Growth Policy Studies (ITPS) published an investigation of this production function (Hagén et al). In that work, the political controversy over the fact that Sweden was ranked 2nd (UNDP, 2002) in welfare and 17th (OECD, 2002a) in economic activity was central. The political opposition focused on the studies that ranked Sweden at 17th place, while the Social democrats naturally used the studies that ranked Sweden as number two.

It is obvious that the ranking is affected by the measure you choose to use. In the ITPS investigation the weight structure was questioned to some extent. It is not possible to find the correct weights, since they do not exist. From this the idea of random weights was borne. For each set of weights the rank of a country is calculated. By repeating this procedure one million times I get a distribution of the rank for each country, rather than a specific rank. The purpose with this paper is to present this distribution for some European countries and some other countries. The conclusion is that the rank is affected by the weights, the chosen components, and the procedure to normalize the components. Perhaps it is trivial, but the point is nevertheless that this has to be remembered. Moreover, the consequences are not the same for all countries. Olsson (2004) is an investigation of country welfare using random weights.

¹ The Swedish term for this is vårdtyngd.

2 Indices to measure country level welfare and data

Assume that we have n components, N , that we want to include in an aggregate index measure. Each of the components are given a weight, so that ω_i is the weight given to component i , and the sum of all weights is equal to one, $\sum_{i=1}^n \omega_i = 1$. Then, the index measure is equal to the weighted average of the components, $I = \sum_{i=1}^n \omega_i N_i$.

In this study I discuss the measurement of country level of welfare. Given the chosen components and weights the country level of welfare is a number. Countries are compared using these welfare indices. Often the countries are ranked from high to low level of welfare. **Already at this moment it is clear that both the choice of components and the weight structure is important, since changing them may affect the rank order.**

We may want to include for example both the GDP per capita and the average length of life as components into the overall welfare index. The GDP per capita is a much bigger number than the average length of life. For this reason the components are normalized to the same scale. One way to transform a component into a number ranging from zero to 1 is to calculate the normalized component as $N_i = (C_i - \min\{C_i\}) / (\max\{C_i\} - \min\{C_i\})$. An alternative sometimes used to some component is to calculate the normalized component as $N_i = (\log(C_i) - \min\{\log(C_i)\}) / (\max\{\log(C_i)\} - \min\{\log(C_i)\})$. **It is clear that the procedure used to normalize the components is important, since it may affect the rank order.**

Despite all the concerns the country level of welfare is calculated and compared. I will now describe the procedure used by the UNDP. The UNDP Human Development Index is an index formed from four normalized welfare components. In Table 1, I have collected information about the welfare components. The normalized Swedish GDP per capita value is $(\log(36,712) - \log(100)) / (\log(40,000) - \log(100)) = 0.986$. The normalized Swedish length of life component value is $(80.8 - 25) / (85 - 25) = 0.930$. We calculate the overall welfare index

for Sweden to 0.963, by using the normalized component values and the corresponding component weight, $(1/3)0.986 + (1/3)0.930 + (1/6)0.943 + (1/6)0.990 = 0.963$. Such a calculation is performed for each country. Norway gets the highest overall welfare measure, 0.971. With this procedure Sweden is ranked as country number seven, after Norway, Australia, Iceland, Canada, Ireland, and the Netherlands. For details see table A2 in the appendix. This is based on the chosen 1) set of normalized components, 2) principle used to normalize components, and 3) weight structure. For additional details I refer you to UNDP (2009).

Table 1: Name, minimum, maximum, and the Swedish value, the Swedish normalized value, and the weight for each welfare component, 2007.

Name	Minimum value	Maximum value	Swedish value	Swedish normalized value	Weight
GDP/capita (PPP US\$)	100	40,000	36,712	0.986	1/3
Length of life (Years)	25	85	80.8	0.930	1/3
Enrolment to studies (%)	0	100	94.3	0.943	2/3
Literacy (%)	0	100	99	0.990	2/3

I have chosen to start with the countries ordered by their rank according to the Human Development Index. You find the data in table A1 in the appendix.

In table A1 in the appendix you find the value of each welfare component for the chosen set of countries. UNDP (2009) calculate normalized components using these values. In Table A2 in the appendix you have the normalized components for the chosen set of countries. In figure 1, I just want to illustrate the normalized components for three countries. I chose Norway to be one country, since they have the highest estimated HDI. I also chose Turkey, and the reason is that they have the lowest HDI, among the included countries. For obvious reasons I also included Sweden. We clearly see that Turkey will have a low level of welfare, since they have low value for all welfare components. Norway has a high HDI value, because they have a high value for each normalized component. The Swedish HDI is somewhat lower.

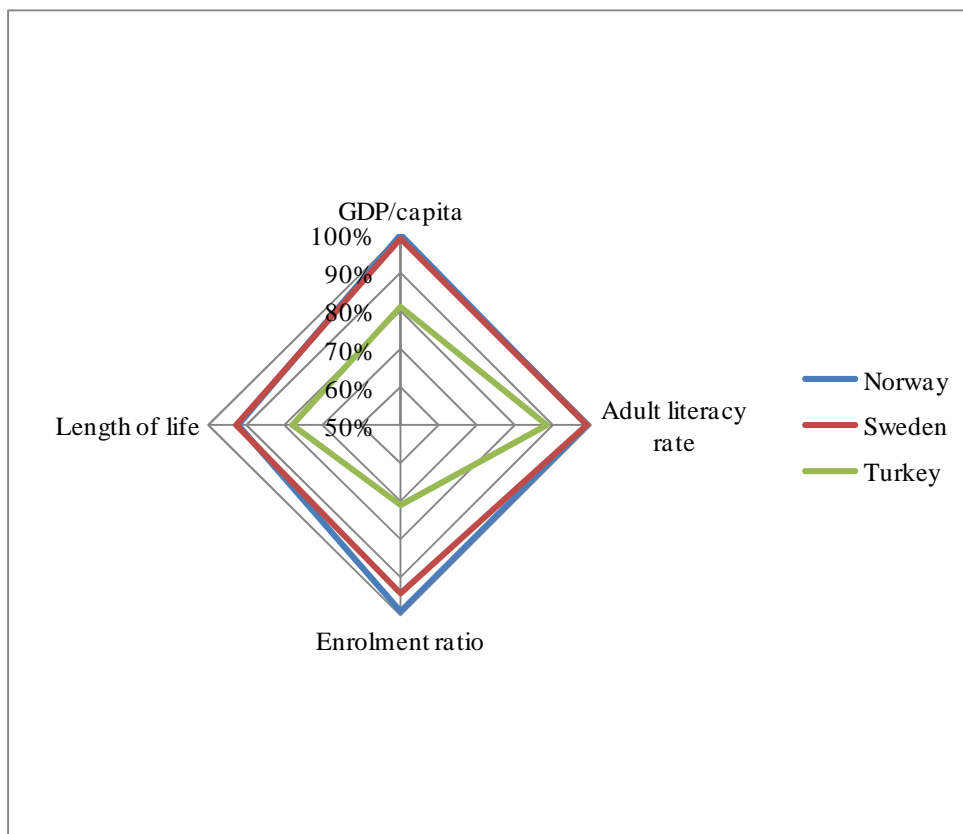


Figure 1: The four normalized welfare components for Norway, Sweden and Turkey. In the chosen set of countries, Norway has the highest HDI, and Turkey has the lowest HDI.

3 Modeling

In this paper, I use the same welfare components as UNDP (2009), and focus on the importance of the normalization procedure, and the effects of the weight structure. If we use the weights used by UNDP (2009) each country get a specific welfare index, and rank. In that case, Sweden is ranked as number seven. In the normalizing procedure I kept all minimum values at the level used by UNDP (2009). An alternative procedure would be to use the minimum for the countries included in the study. Such an approach was taken in Hagén et al (2003).

I pick a set of weights at random, and calculate the country welfare index using them instead of the one UNDP uses. With these welfare indices the countries are ranked from high to low welfare. In each repetition, each country gets a position in this list of countries. The country

with the highest welfare is ranked as number 1. By repeating this one million times I get one million such positions for each country. By altering the weights (randomly) I find that in some cases the welfare in a country is estimated to be relatively high, but in other cases, the welfare may be estimated to be relatively low. This paper will illustrate the rank distribution, and the effect on this rank distribution from changing the normalization procedure.

5 Results

Based on the human development index calculated by UNDP, Sweden is ranked as country number seven. In this study, I have repeated the welfare comparison using random weights one million times. I present the Swedish rank distribution in figure 2, and the corresponding cumulative rank distribution in figure 3. We observe that Sweden reach fifth place in 0.5 percent of the repetitions. Sweden does never reach positions one to four. The lowest position Sweden has in these repetitions is 14, which occurs in 13.6 per cent of the repetitions. The mean rank for Sweden is 9.8. I calculated the mean rank for all countries, and I ranked the countries using the mean rank. You find the rank order based on the mean rank in table 2. Sweden is ranked as country number nine, which differs from rank that follows from using the human development index. It is also so that the Swedish rank is distributed around the mean. It is interesting to see that the distribution has multiple peaks. This is a common feature for the rank distribution. Multiple peaks exist for most countries. This is related to the normalized welfare component values, which was illustrated by the diamond shapes in figure 1.

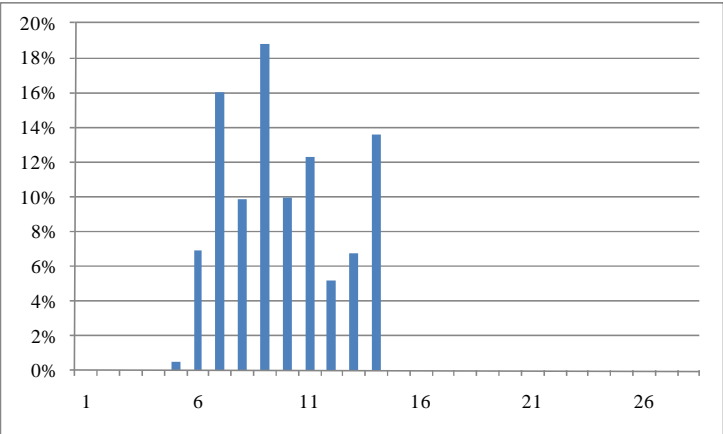


Figure 2: The rank distribution for Sweden.

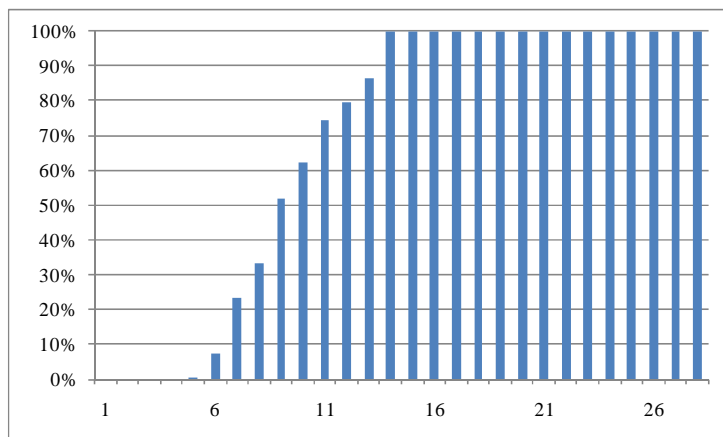


Figure 3: The cumulative rank distribution for Sweden.

I use 28 countries in this study, and each country has a rank distribution. I calculated the mean rank for each country and sorted the countries in that order. In other words, I rank the countries in order based on their mean rank. This order is not the same as the order one get using the human development index. This was made clear earlier, when it was shown that the rank order based on mean rank for Sweden is nine, while Sweden is ranked as number 7 based on the human development index. In table 2 you find the rank order based on the human development index and the mean rank for all 28 countries. I illustrate the rank distribution for all countries in figure 4. I have separated the countries into four groups. In the first group I have the seven countries that were ranked as being 1-7. In the second group, I have collected the countries ranked as 8-14. In the third group, I have the countries ranked as 15-21. In the fourth group I have the seven countries with ranks 22-28. The rank order based on mean rank differs from the rank order based on the human development index, for most countries. It is also the case that most countries have rank order distributed around the mean. Turkey deviates from this, since Turkey always ends up as the country with the lowest level of welfare, independent of weight structure. This is explained by the fact that Turkey has a low value for all welfare components compared to all other countries in this study.

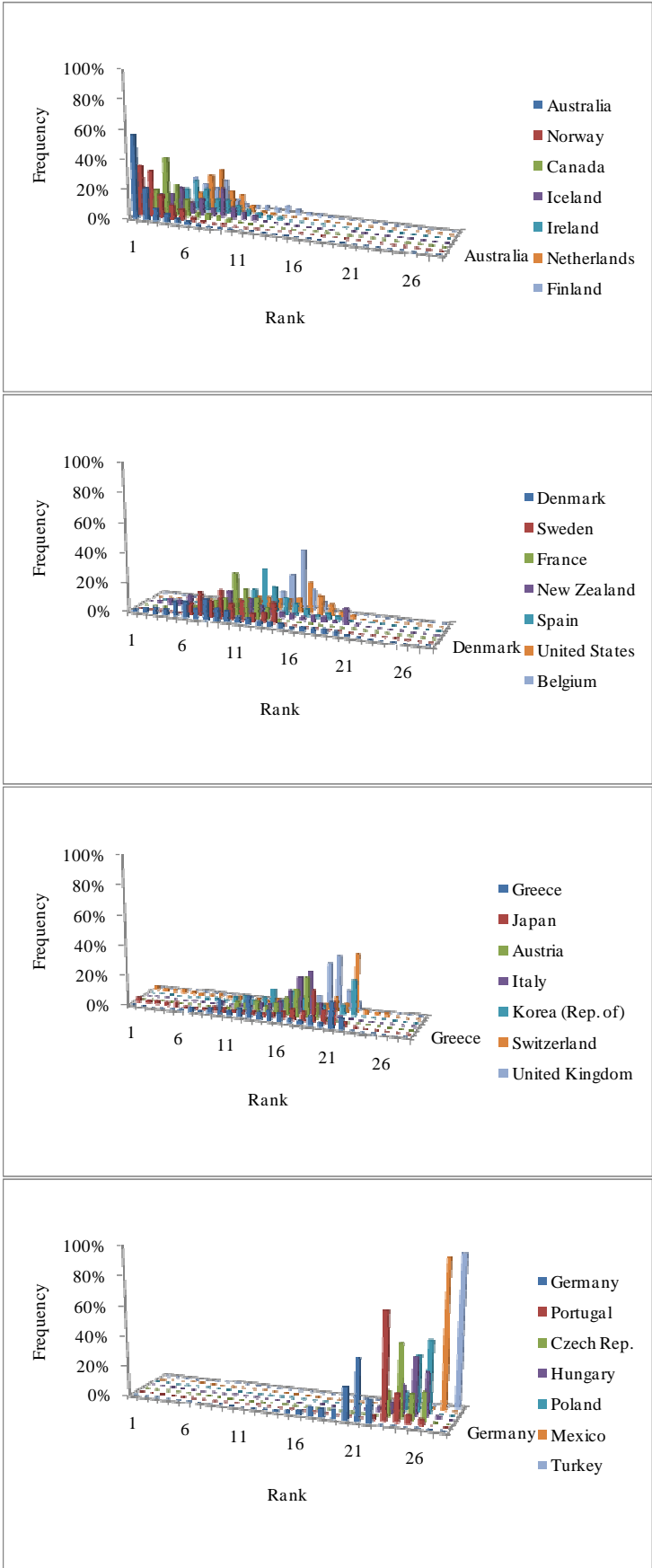


Figure 4: The rank distribution for all countries included in this study.

It is difficult to argue that there is an optimal weight structure to use. Nevertheless, in the calculation of the human development index a specific weight structure is used. The weight structure used in calculation of the human development index ranks countries in one way. This rank order is not the same if other weights are used. It is even the case that the rank based on mean rank in a repeated calculation using random weights differs from the rank order using the human development index. In the first column in table 2 I present the rank according to the human development index. In the second column I present the rank of the mean rank using random weights. You find the same information in figure 5. We observe that the difference between the rank based on mean rank and the rank based on the human development index vary among countries.

Table 2: Information about the rank based on the human development index and the rank based on mean rank.

Country	HDI	Rank of mean rank order
Norway	1	2
Australia	2	1
Iceland	3	4
Canada	4	3
Ireland	5	5
Netherlands	6	6
Sweden	7	9
France	8	10
Switzerland	9	20
Japan	10	16
Finland	11	7
United States	12	13
Austria	13	17
Spain	14	12
Denmark	15	8
Belgium	16	14
Italy	17	18
New Zealand	18	11
United Kingdom	19	21
Germany	20	22
Greece	21	15

Korea (Republic of)	22	19
Portugal	23	23
Czech Republic	24	24
Poland	25	26
Hungary	26	25
Mexico	27	27
Turkey	28	28

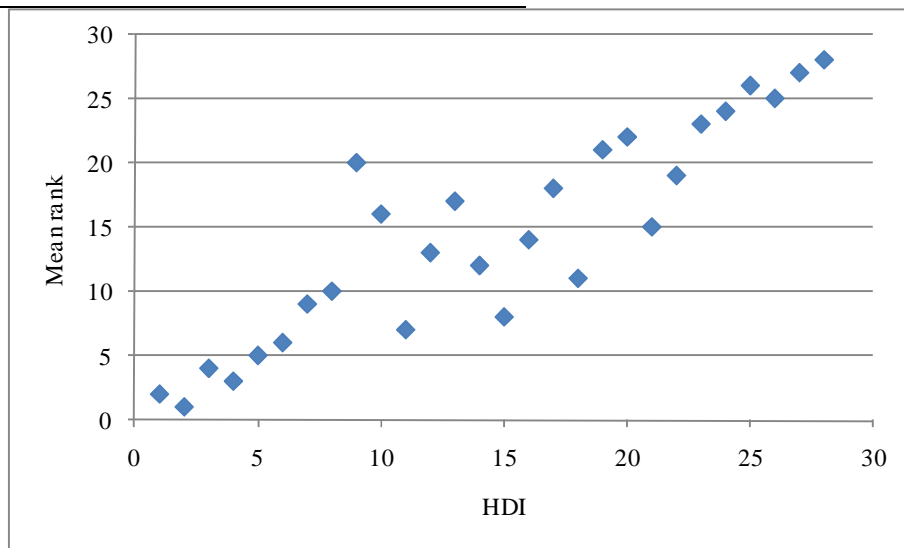


Figure 5: The rank based on mean rank versus the rank based on the human development index.

In the base case, I used the logarithmic version to normalize GDP. With this approach each country gets a rank distribution (Figure 4). I have complemented this approach with one when GDP is normalized in itself, in the same way as the other components. This changes the rank order distribution, and in figure 6 I illustrate the effect for Sweden.

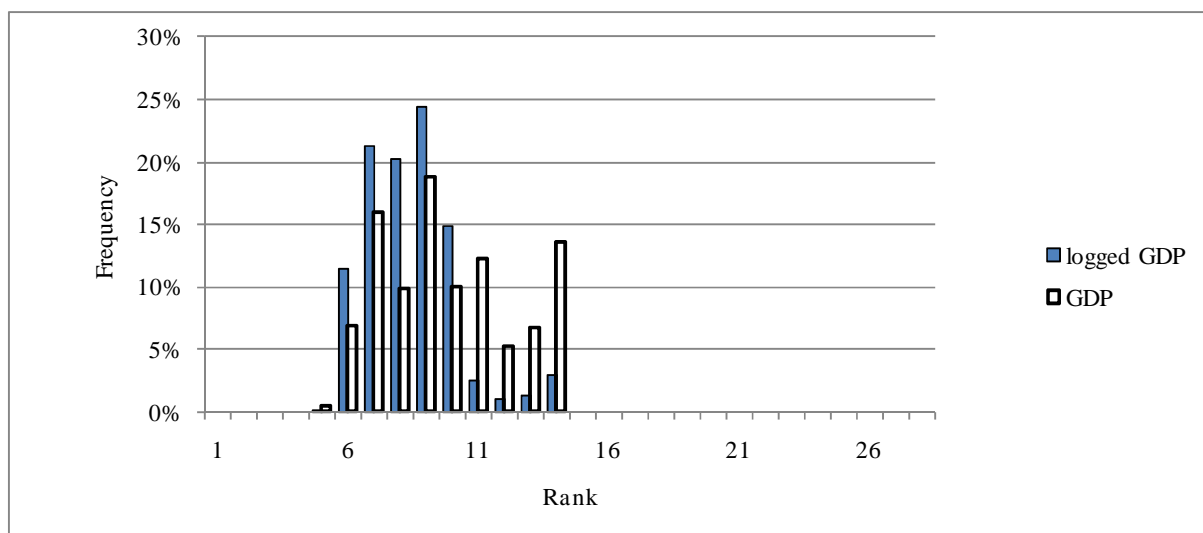


Figure 6: The Swedish rank distribution, using either logged GDP or GDP.

6 Final comments

In the analysis, I kept the minimum values in calculation of the normalized component values at the level used by UNDP in their calculation of the human development index. An alternative would be to use the minimum value for any country included in this study. This would alter the distribution, for sure. This alternate approach was used in Hagén et al (2003). This is something I could add to this paper.

Before the previous Swedish parliament election, the Social democrats claimed that Swedish welfare was high, while the opposition claimed that Sweden was ranked below many other countries. If you rule, you have an interest to show that the policy leads to high welfare. The opposition has incentives to show the opposite. In the last election, the Social democrats lost. This is very interesting, since now we can observe this clearly. The opposition this time was the ruling party last time. Now, the Social democrats most likely pick a welfare index that indicates that the Swedish level of welfare is low.

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Appendix 1: Data

In this appendix you find the data I used in this study. You can also find them online at:

<http://hdr.undp.org/en/reports/global/hdr2009/>, extracted 2010-02-09.

Table A1: GDP per capita, adult literacy, education enrolment, and length of life per country. Sorted by HDI, 2007.

Rank	Country	GDP/capita (PPP USD)	Adult literacy (%)	Education enrolment (%)	Length of life (Years)
1	Norway	40,000	99.0	98.6	80.5
2	Australia	34,923	99.0	100.0	81.4
3	Iceland	35,742	99.0	96.0	81.7
4	Canada	35,812	99.0	99.3	80.6
5	Ireland	40,000	99.0	97.6	79.7
6	Netherlands	38,694	99.0	97.5	79.8
7	Sweden	36,712	99.0	94.3	80.8
8	France	33,674	99.0	95.4	81.0
9	Switzerland	40,000	99.0	82.7	81.7
10	Japan	33,632	99.0	86.6	82.7
11	Finland	34,526	99.0	100.0	79.5
12	United States	40,000	99.0	92.4	79.1
13	Austria	37,370	99.0	90.5	79.9
14	Spain	31,560	97.9	96.5	80.7
15	Denmark	36,130	99.0	100.0	78.2
16	Belgium	34,935	99.0	94.3	79.5
17	Italy	30,353	98.9	91.8	81.1
18	New Zealand	27,336	99.0	100.0	80.1
19	United Kingdom	35,130	99.0	89.2	79.3
20	Germany	34,401	99.0	88.1	79.8
21	Greece	28,517	97.1	100.0	79.1
22	Korea (Rep. of)	24,801	99.0	98.5	79.2
23	Portugal	22,765	94.9	88.8	78.6
24	Czech Republic	24,144	99.0	83.4	76.4
25	Poland	15,987	99.3	87.7	75.5
26	Hungary	18,755	98.9	90.2	73.3
27	Mexico	14,104	92.8	80.2	76.0
28	Turkey	12,955	88.7	71.1	71.7

Table A2: Normalized welfare components: GDP per capita, adult literacy, education enrolment, and length of life. Sorted by HDI, 2007.

Rank	Country	HDI	GDP/capita	Adult literacy	Education enrolment	Length of life
1	Norway	0.971	1.000	0.990	0.986	0.925
2	Australia	0.970	0.977	0.990	1.000	0.940
3	Iceland	0.969	0.981	0.990	0.960	0.946
4	Canada	0.966	0.982	0.990	0.993	0.927
5	Ireland	0.965	1.000	0.990	0.976	0.911
6	Netherlands	0.964	0.994	0.990	0.975	0.914
7	Sweden	0.963	0.986	0.990	0.943	0.930
8	France	0.961	0.971	0.990	0.954	0.933
9	Switzerland	0.960	1.000	0.990	0.827	0.945
10	Japan	0.960	0.971	0.990	0.866	0.961
11	Finland	0.959	0.975	0.990	1.000	0.908
12	United States	0.956	1.000	0.990	0.924	0.902
13	Austria	0.955	0.989	0.990	0.905	0.915
14	Spain	0.955	0.960	0.979	0.965	0.929
15	Denmark	0.955	0.983	0.990	1.000	0.887
16	Belgium	0.953	0.977	0.990	0.943	0.908
17	Italy	0.951	0.954	0.989	0.918	0.935
18	New Zealand	0.950	0.936	0.990	1.000	0.919
19	United Kingdom	0.947	0.978	0.990	0.892	0.906
20	Germany	0.947	0.975	0.990	0.881	0.913
21	Greece	0.942	0.944	0.971	1.000	0.902
22	Korea (Republic of)	0.937	0.920	0.990	0.985	0.904
23	Portugal	0.909	0.906	0.949	0.888	0.893
24	Czech Republic	0.903	0.916	0.990	0.834	0.856
25	Poland	0.880	0.847	0.993	0.877	0.842
26	Hungary	0.879	0.874	0.989	0.902	0.805
27	Mexico	0.854	0.826	0.928	0.802	0.850
28	Turkey	0.806	0.812	0.887	0.711	0.779

Appendix 2: Program

```
filename='UsednonlogGDPcaseCountryData2007.xlsx';
Input=xlsread(filename, 'Sheet1');
for i=1:28;
    for j=1:28;
        SFR(i,j)=0;
    end;
end;
for k=1:10000;
    k
    for i=1:28;
        for j=1:28;
            FR(i,j)=0;
        end;
    end;
    Weigth=rand(10000,4);
    WelfareLevel=Weigth*Input';
    [S,O]=sort(WelfareLevel,2);
    for i=1:10000;
        for j=1:28;
            R(i,O(i,j))=28-j+1;
        end;
    end;
    for j=1:28;
        FR(:,j)=1/10000*hist(R(:,j), [1:28]);
    end;
    SFR=SFR+FR;
end;
SFR=1/10000*SFR;
surf(SFR); figure(gcf);
xlswrite('FR.xls',SFR);
```

Comment: It is much faster to run 10,000 repetition 10,000 times than all at once.