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Conference Paper Public Investment Regional Allocation: Evaluation of applicability of two methodologies

50th Congress of the European Regional Science Association: "Sustainable Regional Growth and Development in the Creative Knowledge Economy", 19-23 August 2010, Jönköping, Sweden

Provided in Cooperation with:

European Regional Science Association (ERSA)

Suggested Citation: Geraki, Martha; Polyzos, Serafeim; Krikeli, Olga (2010) : Public Investment Regional Allocation: Evaluation of applicability of two methodologies, 50th Congress of the European Regional Science Association: "Sustainable Regional Growth and Development in the Creative Knowledge Economy", 19-23 August 2010, Jönköping, Sweden, European Regional Science Association (ERSA), Louvain-la-Neuve

This Version is available at: https://hdl.handle.net/10419/119163

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<u>Public Investment Regional Allocation: Evaluation of</u> <u>Applicability of Existent Methodologies.</u>

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<u>Abstract</u>

Regional allocation of public investment has been considered of a great interest over the years. Regional policy uses investments as a basic developmental tool and it seeks for an effective allocation among the regions. In addition, national economic policy distributes investments aiming at both economic development of less developed regions and maximization of national product. Therefore, an effective allocation by the central government is of a great significance. In this paper, existent methodological approaches, distributing public investment in regions, will be critical reviewed. These models are found in the international bibliography. The features of these methodologies are described in a general context. The flaws and the possibilities of their application for real problem's solution are analyzed. Finally, it is discussed whether there can be an application of either of the examined models on the case of Greece.

1. Introduction

The problem of regional allocation of investment among a number of regions or among all regions in one nation has aroused the interest. The main issue is to allocate a budget among regions by maximizing some production function. This function for example could be the national income (Domazlicky ,1978). The purpose of this paper is to quote a number of existent methodologies leading to regional allocation of investment. Furthermore it is discussed whether these models can be applied on real problems and on the case of Greece.

At first, a simple model that deals with the allocation of investment in a two-region economy is presented (Rahman, 1963). The conditions followed in order to achieve this goal are as follows. First the basic aim of regional allocation is to maximize the rate of growth of national income through a certain time-period. Second any wide disparity is considered not to be brought by the process of economic growth. Last, there is difference in the two region productivity rate and the rate of saving. The presented model can be applied on closed economies.

Concentrating in the most productive region public investment doesn't necessary lead to the maximization of total national income growth rate. Each region has a different rate of saving. Having higher rate of saving doesn't mean that this region will be more productive or the opposite. The less productive region cannot claim investment allocation policy when the more productive one has the higher rate of saving as well. The condition changes when the less productive region has the higher saving rate. It is possible that this region can concentrate investment under some constraints. Investing in a less productive region will lead to an initial loss of income. This loss of income should be repaid by this region will be considered to be long enough for this repayment to happen (Rahman, 1963).

On the other hand a multi-criterion model takes into consideration multiple financial tools to achieve optimal regional allocation. These variables can be tax rate, transfer payments or public investment regional proportions. The aim is to maximize time-flow total income, optimize the employment rate, economic growth, equity, efficiency and full employment. Using these objectives the problem solved is more practical. But also is more complicated. The solution used for the simple-criterion models can not be used to solve such complicated problems. A method that can be used for solving multi-criterion models is genetic algorithm. The traditional investment distribution approaches based on negotiations cannot be used for solving this kind of complicated problems (Tian et al., 2007).

2. Simple-criteria growth model

The problem can be defined assuming a country with a two region closed economy (Rahman,1963). The national income of this country is equal to the sum of the income of the two regions:

$$z_t = x_t + y_t \tag{2.1}$$

where z,x and y stand for the national income, income of region A and income for region B respectively. The consumption of each region, C_A and C_B respectively, is assumed to depend on current income and is equal to:

$$C_{A_t} = c_1 \cdot x_t$$

$$C_{B_t} = c_2 \cdot y_t$$
(2.2)

where c_1 and c_2 are the rates of consumption of each region. Moreover investment is assumed to have a "gestation lag" of one year for each region and is equal to for each region A and B:

$$I_{A_{t}} = k_{1}(x_{t+1} - x_{t})$$

$$I_{B_{t}} = k_{2}(y_{t+1} - y_{t})$$
(2.3)

where k_1 and k_2 are the familiar incremental capital/output ratios for each region. Let's denote s_1 and s_2 the rates of saving in regions A and B respectively, which are equal to:

$$s_1 = 1 - c_1 s_2 = 1 - c_2$$
(2.4)

By incorporating equations 2.2, 2.3 and 2.4 in 2.1, the following result derives:

$$k_1(x_{t+1} - x_t) + k_2(y_{t+1} - y_t) = s_1 x_t + s_2 y_t$$
(2.5)

A number of constraints can be imposed (Rahman ,1963). First, the coefficients $s_1 s_2$, k_1 and k_2 are assumed to be positive. Moreover it is assumed that the incremental capital/output ratio of region A, k_1 , is less than the one of region B, k_2 ; in other words $k_1 < k_2$. That means that region A is more productive than region B and therefore less investment is required to get a given increase of income in region A than in region B. The constraints imposed are the following two:

(i) The "non-disinvestment constraint". Total investment must be equal to total saving available in the economy in the year concerned. In other words, total investment is limited to total saving and therefore it is not possible to have any net consumption of capital or disinvestment in any region. The previous stated can be expressed as follows:

$$\begin{aligned} x_{t+1} &\ge x_t \\ y_{t+1} &\ge y_t \end{aligned} \tag{2.6}$$

(ii) The "political constraints". *The regional income disparity cannot exceed a certain political tolerance limit in either direction*. This regional income disparity is measured by the ratio of the two regional incomes. The previous stated can be expressed as follows:

$$\frac{y_{t+1}}{x_{t+1}} \ge r_1
\frac{x_{t+1}}{y_{t+1}} \ge r_2
0 < r_1 < 1
0 < r_2 < 1$$
(2.7)

where r_1 and r_2 are the mentioned political tolerance limits. For example if r_1 =0.75 then region B's income cannot fall under 75% of region A's.

The problem is to maximize the equation (2.1) subject to conditions (2.5), (2.6) and (2.7). Time t ranges between 0 and T-1 or in mathematical terms 0 < t < T - 1, and x_0 and y_0 are the initial conditions. The solution to the problem consists of a number of propositions, which all have the same previously described initial conditions (Rahman,1963). The method of proof that can be used consists of an application of Belman's Principle of Optimality (Rahman,1963). According to Bellman(1959) *if certain initial decisions are taken the remaining decisions must be optimal with respect to the state resulting from the initial decisions in order for the whole set of decisions to be optimal.*

A number of solutions of the problem can be categorized in two types, A and B. The optimum program 'Type A' indicates that the more productive region A is continuously favored during the planning period. This means that the ratio s_1/k_1 will be greater than the ratio s_2/k_2 or in mathematical terms $s_2/k_2 < s_1/k_1$. The less productive region B is unable to present a high enough saving rate s_1 in order to balance the higher productivity of region A. In order to reach the optimal program, investment is required every year to be concentrated in the more productive region A. This state should always take into consideration the constraints of the model (Rahman,1963).

The optimum program 'Type B' indicates that in the first years of the planning period the less productive region B is favored. For the rest of the years the policy changes so that the more productive region is favored. This means, in mathematical terms, that $s_2/k_2 > s_1/k_1$ must be valid and that the planning period should be long enough in order to overcome the loss of income deriving from investing in the less productive region B, with the contribution of the higher saving rate that region B has. The time needed to overcome the loss of income can be named as "period or recovery". The planning period must be greater than the period of recovery. Given the condition mentioned above $(s_2/k_2 \gg s_1/k_1)$ and regarding the previous statement, a Type B program emerges. If the period of recovery is one year and the planning horizon is greater than that, a Type B problem emerges. This means that the less productive region B is favored for the first T-1 years. If the planning period is not greater than the recovery period, then the optimum program will be Type A. But failing to follow Type B program can be characterized as "shortsightedness", (Rahman,1963).

The previous analysis can be further continued to find the conditions under which the "switch" at the terminal date occurs and the time this "switch" will or not happen (Takayama, 1967). The proposed method concerns a two-region economy. It can also be modified to a two-sector economy or be extended to an n-region or n-sector economy.

A two region economy is considered. Each region produces 'national income' Y_i (*i* = 1,2). Each capital/output ratio is fixed so that:

$$Y_i = \mathbf{b}_i \mathbf{K}_i, \ i = 1, 2.$$
 (2.8)

The savings from the whole economy are used as investment on these regions. In other words, national investment I equals national savings S. Defining K_i as the stock capital in region i and s_i the constant saving ratio in region i, it is valid that:

$$\dot{K}_1 + \dot{K}_2 = s_1 Y_1 + s_2 Y_2 \tag{2.9}$$

where \dot{K}_i is the regional increase in capital stock (Intriligator, 1964). Taking into consideration both functions (2.8) and (2.9) the result is

$$\dot{K}_1 + \dot{K}_2 = g_1 K_1 + g_2 K_2 \tag{2.10}$$

where g_i is the constant regional growth rate and is equal to $g_i = s_i b_i$, i = 1,2. Assuming that β is the allocation parameter:

$$\dot{K}_{1} = \beta(g_{1}K_{1} + g_{2}K_{2})$$

$$\dot{K}_{2} = (1 - \beta)(g_{1}K_{1} + g_{2}K_{2})$$
(2.11)

The initial capital stock for both regions is greater than zero and the values that the allocation parameter β can take range between zero and one:

$$K_{1}^{0} > 0$$

$$K_{2}^{0} > 0$$

$$0 \le \beta \le 1$$
(2.12)

The problem that needs to be solved is to maximize the allocation parameter $\beta(t)$ so as to maximize an objective function. The previous model's functions (Rahman, 1963) are used;

maximization of income at some known future terminal time T subject to 2.11 and 2.12 (Takayama, 1967):

maximize
$$Y(T) = Y_1(T) + Y_2(T) = b_1 K_1(T) + b_2 K_2(T)$$
 (2.13)

In order to solve the maximization problem of the objective function, the Maximum Principle is followed (Pontryagin et al., 1962) and the Hamiltonian is defined as follows:

$$H = p_1 \beta (g_1 K_1 + g_2 K_2) + (1 - \beta) p_2 (g_1 K_1 + g_2 K_2)$$
(2.14)

where p_i , i=1,2 are auxiliary variables that follow the conditions:

$$p_1(T) = b_1$$

$$p_2(T) = b_2$$
(2.15)

According to the Maximum Principle the control variables must be chosen so as to lead to the maximization of the Hamiltonian. The Hamiltonian system consists of equation 2.14 and of the following equations:

$$\dot{p}_i = -\frac{\partial H}{\partial K_i}, \qquad i = 1,2$$
 (2.16)

It is obtained for $p_1 > p_2$ then $\beta = 1$ and for $p_1 < p_2$ then $\beta = 0$. This means that the shadow price of investment, p_i , is higher in region 1 and that this region is chosen to invest. Then it can be written that:

$$\dot{p}_1 = -[\beta(p_1 - p_2) + p_2]g_1, \quad p_1(T) = b_1$$

$$\dot{p}_2 = -[\beta(p_1 - p_2) + p_2]g_2, \quad p_2(T) = b_2$$
(2.17)

Afterwards it can be obtained that (Takayama, 1967):

$$p_{1}(t) = \frac{g_{1}}{g_{2}} p_{2}(t) + \frac{b_{1} + b_{2}}{g_{2}} (s_{2} - s_{1}), \qquad p_{1}(T) = b_{1}, \quad p_{2}(T) = b_{2}$$

$$p_{1}(t) - p_{2}(t) = \frac{g_{1} - g_{2}}{g_{2}} (p_{2}(t) + \frac{b_{1}b_{2}}{g_{2}} (s_{2} - s_{1})$$
(2.18)

or

A number of cases are examined for the solution of the problem. In any case the question is whether this model can be applied or not. In order to give an answer it is necessary to answer the question if the output-capital ratio can be constant of it changes due to capital accumulation. This change might be avoided when labor is freely available. This is not possible because it is not easy to settle a mechanism that allocates employment of labor to regions that the capital/labor region remains constant (Takayama, 1967).

In this case it is important to point out that the optimization of regional allocation of investment depends very much on the planner's choice of the objective. For example choosing to maximize the income the result is the increase of output/capital ratio. On the other

hand, maximizing the per capita consumption leads to high growth regions (Intriligator, 1964).

The growing rate of *national income is not necessarily maximized by concentrating investment in the most productive region of country* (Rahman 1963), taking into consideration that the saving rates of the two regions are not the same. Moreover it is for the country to examine whether the less productive region can offer higher saving rates or internal rates of growth than the most productive region in order to overcome the rates of the most productive region in the first years of the investment program. A high saving rate does not indicate that the productivity of the region will be high as well. That is because saving depends on a number of factors besides the income. These factors that can lead to higher rate of saving can be the social habits, the institutions and, *in a controlled economy, the political ability of the central authority to squeeze saving out of the region* (Rahman, 1963).

3. Multi-Criterion growth model

The main goal of a government when dealing with the allocation of investment is to find the optimum policy to achieve multiple objectives. Economic growth, full employment, equity can be the government's multiple goals. The initial public investment models previously described can evolve to more practical resulting models. Using large number of criteria the result is more efficient.

New objective functions are proposed in order to accomplish a relevant result. Time flow is introduced in the objective of final total income. The combination of time flow total income maximization and of total income gives a new objective that explains in a better way total welfare. So the total welfare objective can be written as follows (Tian et al.,2007):

$$MaxW = \eta \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} \omega_{i} \xi_{ij} Y_{ij}(T) + (1-\eta) \int_{T_{o}}^{T} e^{-\mu(t-T_{o})} (\sum_{i=1}^{n} \sum_{j=1}^{m} \omega_{i} \xi_{ij} Y_{ij}(t)) dt$$
(3.1)

where ω_i is the weight of region i, ξ_{ij} is the weight of sector j of region i, Y_{ij} is the income of region i of region j, μ is the exponential discounting factor.

Maximization of employment rate is important for the regional development. The employment objective is described as follows (Tian et al.,2007):

$$MaxP = \frac{\sum_{i=1}^{n} L_{i}(t)}{\sum_{i=1}^{n} N_{i}(t)}$$
(3.2)

subject to
$$\frac{L_i(t)}{N_i(t)} \ge B$$
 and $0 < B < 0$ (3.3)

where L_i is the labor in region i N_i the population of region I and B is a lower limit of regional employment rate in order to achieve moderate employment rate and equity between regions.

The third objective formulating the model is about the cross-region income per capita gap minimization (Tian et al.,2007).

$$MaxE = (-1)\sum_{k,\nu=1}^{n} \int_{T_{o}}^{T} |Y_{k}(t)/N_{k}(t) - Y_{\nu}(t)/N_{\nu}(t)| dt$$
(3.4)

subject to

$$I(t) = K'(t) + \gamma \cdot K(t)$$
(3.5)

$$L_{ij}(t) = \lambda_{ij} \cdot I_{ij} + C_{ij} \qquad \lambda_{ij}, C_{ij} > 0$$
(3.6)

$$K'(t) = r(t) \sum_{i=1}^{n} [(1 - \sum_{k=1}^{n} b_{ij}) z_i \sum_{j=1}^{m} \varphi_{ij} Y_{ij}(t)] + (1 - r(t)) \cdot \sum_{i=1}^{n} [(1 - \sum_{k=1}^{n} a_{ij}) s_i \sum_{j=1}^{m} \phi_{ij} Y_{ij}(t)] - \gamma \sum_{i=1}^{n} \sum_{j=1}^{m} K_{ij}(t)$$
(3.7)

$$Y(t) = \sum_{i=1}^{n} \sum_{j=1}^{m} A_{ij} K_{ij}(t)^{\alpha_{ij}} L_{ij}(t)^{\beta_{ij}}$$
(3.8)

where γ is the current capital stock depreciating constant rate, λ_{ij} is the labor investment ratio of sector j of region i, I_{ij} is the investment on sector j of region i, C_{ij} is the necessary simple labor of sector j of region i, K(t) is the capital stock, K'(t) = dK(t)/d(t), r is the income tax rate, a_{ij} and b_{ij} are the proportions of capital transfer loss between regions, z_i and s_i are the rates of savings of public and private sectors respectively of region i, φ_{ij} is the weight of public sector investment to sector j of region i, ϕ_{ij} is the weight of private sector investment to sector j of region i, A_{ij} is the contribution of technological innovation to output of sector j of region i and finally α_{ij} and β_{ij} are the increase of output that will happen when the capital and simple labor respectively will increase 1%.

The described investment allocation model is maximizing all three equations (3.1), (3.2) and (3.4) subject to the constraints (3.3), (3.5), (3.6), (3.7) and (3.8).

For solving this optimal investment problem a powerful stochastic technique is applied: Genetic algorithm. The previous described model can be transformed in order to make the procedure applicable. Resolving the constraint (3.5) and combining its result with (3.6) and (3.8):

$$K(t) = e^{-\gamma t} \left(\int I(t) e^{\gamma t} dt + \widetilde{C} \right)$$
(3.9)

$$Y_{ij}(t) = A_{ij} \left(e^{-\gamma t} \left(\int I_{ij}(t) e^{\gamma t} dt + \widetilde{C} \right) \right)^{a_{ij}} \left(\lambda_{ij} I_{ij} + C_{ij} \right)^{\beta_{ij}}$$
(3.10)

Incorporating some of the constrains and the above equations into the objective functions the model is rewritten (Tian et al.,2007):

$$MaxE = (-1)\sum_{k,\nu=1}^{n} \int_{0}^{T} \frac{\sum_{j=1}^{m} A_{kj} (e^{-\gamma t} (\int I_{kj}(t) e^{\gamma t} dt + \widetilde{C}))^{\alpha} (\lambda_{kj} I_{kj} + C_{kj})^{\beta}}{N_{k}(t)} - \frac{\sum_{j=1}^{m} A_{\nu j} (e^{-\gamma t} (\int I_{\nu j}(t) e^{\gamma t} dt + \widetilde{C}))^{\alpha} (\lambda_{\nu j} I_{\nu j} + C_{\nu j})^{\beta}}{N_{\nu}(t)} dt$$
(3.11)

$$MaxW = \eta \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} \omega_{i} \xi_{ij} A_{ij} (e^{-\gamma t} (\int I_{ij}(t) e^{\gamma t} dt + \widetilde{C}))^{a_{ij}} (\lambda_{ij} I_{ij} + C_{ij})^{\beta_{ij}} + (1 - \eta) \int_{T_{0}}^{T} e^{-\mu(t - T_{0})} (\sum_{i=1}^{n} \sum_{j=1}^{m} \omega_{i} \xi_{ij} A_{ij} (e^{-\gamma t} (\int I_{ij}(t) e^{\gamma t} dt + \widetilde{C}))^{a_{ij}} (\lambda_{ij} I_{ij} + C_{ij})^{\beta_{ij}}) dt$$
(3.12)

$$MaxP = \frac{\int_{T_o}^{T} \sum_{i=1}^{n} \sum_{j=1}^{m} (\lambda_{ij} \cdot I_{ij} + C_{ij}) dt}{\int_{T_o}^{T} \sum_{k=1}^{n} N_k(t) dt}$$
(3.13)

where $I_i(t)$ is the investment process and is described as follows:

$$I_{ij}(t) = \begin{cases} \widetilde{I}_{ij} , t = T_0 \\ (\phi_{ij}I - \widetilde{I}_{ij})/(T - T_0), t > T_0 \end{cases}$$
(3.14)

where \tilde{I}_{ij} is the initial investment. The rest is invested gradually and is described in (3.14). The problem now can be solved by maximizing (3.12), (3.13) and (3.14) subject to (3.3) and (3.7). In other words the aim is to solve the proportion of investment in sector j of region i to the total investment. This formula can be solved with genetic algorithm. The key to the application of genetic algorithm is to encode and decode the solutions into chromosomes. There is a possibility of premature convergence of the algorithm so it should be carefully considered to relax the constraints.

The results that arise from the application of genetic algorithm are thought to be superior to other methods' results. Using this method to solve the optimization problem is possible to take into consideration multiple criteria. The optimization problem of allocation of public investment is very complicated. A large number of criteria must be taken into consideration in order to come to a conclusion. Regional economic growth, equity per capita and employment rate should all be considered in the solution of this problem. More over this method allows the solution of multi-criterion problems. This multi-criterion problem can be transformed into a single objective programming model and make the procedure of finding an solution effortless. Thus, applying this method the planner should take into account the possibility of premature convergence and introduce methods not to permit this.

4. The case of Greece

The question to answer is whether these models could be applied on the case of Greece. Following, estimation will be made whether these models could have a practical application. Following, applicability of the models described is discussed taking into consideration the variables used, the constraints and the derived solution.

In the first model the objective function is about maximizing the total regional income. The income of each region is the sum of consumption and investment. The constraints that have to be fulfilled take into consideration total investment, total savings and the political tolerance limit. In this simple model the planner needs to be aware of these variables. In the second model the objective functions include the following variables: the income of each sector of each region, the population of each region, the time-flow total income and the labor. Moreover, the constraints include the variables: the rates of savings of public sector and of private sector, the contribution of technological innovation, the capital stock and the investment of public sector and of private sector.

Greece is considered to be a developed country. The Human Development Index in 2007 was high, as well as the quality-of-life index in 2005. Some of the main industries that developed through the years are tourism, shipping, industrial products, food and tobacco processing, chemicals, metal products and mining. The main problems that Greek economy faces are the high rate of unemployment, bureaucracy, corruption and tax evasion. The global competitiveness is low compared to the other countries of the European Union. Economic growth since 2009 is diminishing. The ratio of loans to saving is over 100% during the first months of 2010. This shows that a trend of over-lending exists. Moreover it is important to point out that according to the poll published by the Groningen Growth & Development

Centre, Greek workers, between 1995 and 2005, worked the most hours per year compared to the other European countries (1900 hours/year followed by the Spanish workers with 1800 hours/year).

Regarding the simple-criterion model, there could be an application on Greece but the results wouldn't be efficient and applicable. The political tolerance limit should be carefully considered because of the political and economical corruption. Labor is not taken into consideration and what's more regional inequality and disparity strongly exists. This model could be extended to more than two regions. But yet the sectors in each region are not examined. Greek regions are supported financial not only by the public sector but also by the private sector. This model doesn't include this kind of variables. So in an application of low significance or in order to have a first estimation of the solution to the problem of regional allocation the planner could use the simple-criterion model.

The multi-criterion model seems to be more appropriate for the case of Greece. It seems that is not fully relevant but doesn't have the deficiencies that the first one has. Technological innovation is taken into account. This variable doesn't fit well to the Greek case. Labor, population and the contribution of public and private sector participate in this model. These are factors that affect the growth of a Greek region. It is likely that this model will give more relevant results than the first one.

5. Conclusions

Existent methodologies are presented and their flaws are pointed out. The simple structure model described can be easy applied to a double region economy and can be extended to an economy with more regions. The multi-criterion model is more complicated. Thus it is more practical. This kind of problems can be solved with genetic algorithm. The case of Greece indicates that it is more suitable to use a multi-criterion model to solve an investment regional allocation problem.

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