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## Demographic change, growth and agglomeration

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# Demographic change, growth and agglomeration

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*Preliminary version*

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## Abstract

This article presents a framework within which the effects of demographic change on both agglomeration and growth of economic activities can be analyzed. I introduce an overlapping generation structure into a New Economic Geography model with endogenous growth due to learning spillovers and focus on the effects of demographic structures on long-run equilibrium outcomes and stability properties. First, lifetime uncertainty is shown to decrease long-run economic growth perspectives. In doing so, it also mitigates the pro-growth effects of agglomeration resulting from the localized nature of learning externalities. Second, the turnover of generations acts as a dispersion force whose anti-agglomerative effects are, however, dampened by the growth-linked circular causality being present as long as interregional knowledge spillovers are not perfect. Finally, lifetime uncertainty also reduces the possibility that agglomeration is the result of a self-fulfilling prophecy.

**JEL classification:** F43; O33; J10; R11

**Keywords:** Agglomeration; Endogenous Growth; Demographic Change; Knowledge Spillovers

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# 1 Introduction

Recently, there has been wide interest in the “economics” of population aging (see e.g. *The Economist* (2009)). Demographic change has crucial consequences for economic behavior and development along various dimensions, ranging from such diverse aspects as the determination of aggregate labor productivity levels to retirement issues. In particular, it highlights that consumption decisions and incentives to invest in future growth prospects vary over the life-cycle. The latter has important implications for long-run economic growth perspectives. The former, on the other hand, should be decisive for the location of economic activity if one takes into account the mutual dependence between the spatial distribution of production and demand developments as emphasized in the New Economic Geography (NEG) literature (see e.g. Baldwin et al. (2003) for an overview). Both growth and agglomeration processes are, however, themselves interlinked. It is thus necessary not only to investigate the isolated effects of age-dependent heterogeneity on these two issues but to also consider whether and how lifetime uncertainty impacts upon the linkage between growth and the spatial distribution of economic activity.

The relationship between growth and agglomeration processes has been studied extensively in recent years. By claiming that “agglomeration can be thought as the territorial counterpart of economic growth” Fujita and Thisse (2002) emphasize that the emergence of concentration of economic activity is traditionally associated with modern economic growth. The positive link between growth and spatial agglomeration is mainly attributed to the fact that technological spillovers, being the engines of endogenous growth, are localized. Consequently, being close to innovation clusters should have positive effects on productivity and growth perspectives. Such considerations have led to the development of integrated frameworks that combine (endogenous) growth features with NEG models to study the joint process of creation and location of economic activity (see e.g. Martin and Ottaviano (1999), Martin and Ottaviano (2001), Baldwin and Forslid (2000) or Baldwin et al. (2001)).

By introducing endogenous capital in his constructed capital model Baldwin (1999) was the first one allowing for growth features in a NEG framework. The absence of capital mobility in his exogenous growth model implies a demand-linked circular causality fostering agglomeration. Higher

capital accumulation increases income and expenditures in the respective region which raises capital rental rates further and thus fosters capital accumulation even more. Baldwin et al. (2001) extended this framework by additionally allowing for learning externalities in the capital creation sector. In their endogenous two-region growth model both the above demand-linked agglomeration force and a growth-linked circular causality strengthen concentration of economic activity. The latter crucially depends on the localized nature of spillovers. As long as spillovers are not fully globalized, spatial concentration of capital in one region implies a lower cost of capital creation and thus speeds up accumulation relative to the other region. Growth itself can thus lead to catastrophic agglomeration of economic activity. Moreover, with localized learning spillovers, agglomeration also affects long-run growth perspectives. In particular, spatial agglomeration is conducive for growth since it decreases costs in the capital accumulation sector.

The traditional line of investigation of all these NEG approaches to growth focuses, however, on the joint consequences of increased economic integration on growth and agglomeration processes while ignoring any potential effect of demographic change. In particular, despite the fact that all these models are intrinsically dynamic, the impact of an economy's demographic structure, and in particular of life-cycle decisions, on consumption and saving patterns, which themselves play a crucial role for agglomeration forces, is completely ignored. Grafeneder-Weissteiner and Prettnner (2009) have clearly revealed that this limited perspective misses important mechanism that are fundamental for the location of productive factors. They show that incorporating an overlapping generation structure and lifetime uncertainty into the constructed capital of Baldwin (1999) introduces an additional dispersion force that considerably reduces the possibility of agglomeration processes. Prettnner (2009), on the other hand, provides evidence for a positive effect of life expectancy on long-run economic growth in an endogenous growth framework in the spirit of Romer (1990).

This paper merges both strands of analysis by generalizing Baldwin et al. (2001)'s NEG model with learning spillovers to allow for an overlapping generation structure with individuals that face a positive probability of death and differ with respect to age. In doing so, the main emphasis is twofold. First, the impact of lifetime uncertainty on long-run growth perspectives is investigated with a view to evaluating the pro-growth effect of spatial

concentration in a setting accounting for demographic structures. Second, the stability properties of the symmetric equilibrium with respect to varying mortality rates are analyzed. Here, attention is also paid to the impact of lifetime uncertainty on history-versus-expectations considerations.<sup>1</sup>

What I show is that, consistent with Grafeneder-Weissteiner and Prettner (2009), the turnover of generations acts as a dispersion force that dampens the pro-agglomerative growth-linked circular causality being present as long as interregional learning spillovers are not fully perfect. Moreover, the model reveals that lifetime uncertainty has important implications for equilibrium selection. An increase in the mortality rate reduces the possibility that expectations rather than history, represented by initial conditions, are decisive with respect to the question in which region agglomeration might take place. Finally, comparing the negative effect of lifetime uncertainty on equilibrium growth rates in the symmetric and core-periphery outcome, shows that, in sharp contrast to existing NEG growth models with localized knowledge spillovers, spatial agglomeration is not necessarily conducive to growth in a setting accounting for demographic structures.

The remainder of this paper is structured into four sections. The following section 2 presents the model framework and derives the dynamic system describing the evolution of the economy. Section 3 characterizes the long-run equilibria and investigates the impact of lifetime uncertainty on equilibrium growth rates. The joint effect of demography and spillovers on the stability properties of the symmetric equilibrium is analyzed in section 4, which also focuses on the role of mortality for history-versus-expectations considerations. Finally, section 5 contains concluding remarks and indicates further lines of research.

## 2 The model

This section describes how a notion of learning as in Baldwin et al. (2001) can be integrated into the generalized constructed capital model of Grafeneder-Weissteiner and Prettner (2009) to arrive at a NEG framework featuring both endogenous growth and demographic change.

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<sup>1</sup>This debate was initiated by Krugman (1991b) and deals with the question of equilibrium selection in a setting with multiple equilibria (see Baldwin (2001) for a nice overview).

## 2.1 Basic structure and underlying assumptions

Consider a world economy with two symmetric regions or countries, denoted by  $H$  for home and  $F$  for foreign<sup>2</sup>, with identical production technologies, trade costs, preferences of individuals, labor endowments and demographic structures. Each region has three economic sectors (agriculture, manufacturing and investment) with two immobile factors (labor  $L$  and capital  $K$ ) at their disposal.

### 2.1.1 Technology

The homogeneous agricultural good,  $z$ , is produced in a perfectly competitive market under constant returns to scale using labor as the only input with, by choice of units, an input coefficient of one. It can be traded between the two regions without any cost.

Manufacturing firms are modeled as in the monopolistic competition framework of Dixit and Stiglitz (1977) and thus supply horizontally differentiated varieties,  $m$ . A continuum of varieties  $i \in (0, V_H(t)]$  is produced at home, whereas a continuum of varieties  $j \in (0, V_F(t)]$  is manufactured in the foreign region. In contrast to the agricultural good, trade of manufactures involves iceberg transport costs such that  $\varphi \geq 1$  units of the differentiated good have to be shipped in order to sell one unit abroad (see e.g. Baldwin et al. (2003)). Each variety is produced with one unit of capital as fixed input and labor as the variable production factor where  $a_m$  represents the unit input coefficient for efficiency units of labor. Firms thus face an increasing returns to scale production technology with an associated cost function  $\pi(t) + w(t)a_m Y_m(i, t)$ , where  $\pi(t)$  is the capital rental rate representing the fixed cost,  $w(t)$  is the wage per efficiency unit of labor and  $Y_m(i, t)$  is total output of one manufacturing good producer.

In the perfectly competitive investment (or innovation) sector, capital is produced using labor as the only input with an input coefficient of  $a_i(t)$ . Capital is viewed here as new knowledge embedded in an interregionally immobile manufacturing facility. Wages in this sector are being paid out of

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<sup>2</sup>If further distinction is needed, foreign variables are moreover indicated by an asterisk. In particular, the superscript  $F$  denotes that a good was *produced* in the foreign region, whereas the asterisk indicates that it is *consumed* in the foreign region. In what follows, emphasis will be on the home region. The corresponding expressions for the foreign region can be derived by symmetry.

the individuals' savings. To endogenize long-run growth in this framework, a sector-wide learning curve is modeled by assuming that the marginal cost of producing new capital,  $G(t) \equiv a_i(t)w(t)$ , declines as the sector's cumulative output rises.<sup>3</sup> Specifically,

$$a_i(t) = \frac{1}{K(t) + \eta K^*(t)}, \quad (1)$$

where  $K(t)$  and  $K^*(t)$  is the home respectively foreign capital stock and  $0 \leq \eta \leq 1$  determines the degree of internationalization of learning effects<sup>4</sup> with  $\eta = 0$  denoting purely localized knowledge spillovers and  $\eta = 1$  corresponding to the case of global learning effects. As long as  $\eta < 1$ , the costs of producing new capital units in each region thus depend on the interregional distribution of capital. The foreign technology is isomorphic with  $a_i^*(t) = \frac{1}{K^*(t) + \eta K(t)}$ . Following Romer (1990), there is no capital depreciation.

### 2.1.2 Demographic structure and preferences

As far as the demographic structure of our model economy is concerned, this paper closely follows Grafeneder-Weissteiner and Prettnner (2009) by adopting Blanchard (1985)'s overlapping generation framework. At each point in time,  $\tau \in [0, \infty)$ , a large cohort consisting of new individuals is born. These newborns receive no bequests and thus start their lives without any wealth. The size of this cohort is  $N(\tau, \tau) = \mu N(\tau)$ , where  $\mu > 0$  is the constant birth rate and  $N(\tau) \equiv \int_{-\infty}^{\tau} N(t_0, \tau) dt_0$  is total population at time  $\tau$  with  $N(t_0, \tau)$  denoting the size of the cohort born at  $t_0$  for any given point in time  $\tau$ .<sup>5</sup> Consequently, cohorts can be distinguished by the birth date  $t_0$  of their members. Since there is no heterogeneity between members of the same cohort, each cohort can be described by one representative individual, who inelastically supplies his efficiency units of labor on the

<sup>3</sup>Romer (1990) e.g. rationalizes this assumption by referring to the non-rival nature of knowledge.

<sup>4</sup>New capital units can be thus viewed as having two distinct components. On the one hand, a new capital unit represents private knowledge of how to produce a new variety, which can be sold in the form of a patent to a manufacturing firm. In this sense capital is interregionally immobile. On the other hand, however, it also contains public knowledge since it makes it easier to produce further capital units (imperfectly mobile spillover component).

<sup>5</sup>In what follows the first time index of a variable will refer to the birth date, whereas the second will indicate a certain point in time.

labor market with perfect mobility across sectors but immobility between regions. Each individual faces lifetime uncertainty, i.e. its time of death is stochastic with an exponential probability density function. In particular, the instantaneous probability of death of each individual is also given by the age independent parameter  $\mu$ . This implies that population size is constant and can be normalized to one.<sup>6</sup> Finally, as in Yaari (1965), a perfect life-insurance company offers actuarial notes, which can be bought or sold by each individual and are canceled upon the individual's death.

The overlapping generation structure implies that the overall economy does not feature *one* single representative individual. It is thus necessary to aggregate over the cohorts to arrive at aggregate variables, e.g. aggregate capital stock of the economy at a certain point in time  $t$  is defined as

$$K(t) \equiv \int_{-\infty}^t k(t_0, t) N(t_0, t) dt_0, \quad (2)$$

where  $k(t_0, t)$  represents the individual capital stock.

Preferences over the agricultural good and a CES composite of the manufacturing varieties are Cobb-Douglas. In particular, expected lifetime utility of a representative individual of cohort  $t_0$  at time  $t_0$ <sup>7</sup> is given by

$$U(t_0, t_0) = \int_{t_0}^{\infty} e^{-(\rho+\mu)(\tau-t_0)} \ln \left[ (c_z(t_0, \tau))^{1-\xi} (c_m^{agg}(t_0, \tau))^{\xi} \right] d\tau, \quad (3)$$

where  $\rho > 0$  is the pure rate of time preference,  $0 < \xi < 1$  is the manufacturing share of consumption and

$$c_m^{agg}(t_0, \tau) \equiv \left[ \int_0^{V_H(\tau)} (c_m^H(i, t_0, \tau))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F(\tau)} (c_m^F(j, t_0, \tau))^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

represents consumption of the CES composite with  $\sigma > 1$  denoting the

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<sup>6</sup>From now on, we will refer to  $\mu$  as the mortality rate. Note, however, that  $\mu$  equivalently represents the birth rate, e.g. demographic change as captured by variations in  $\mu$  means that both the mortality and the birth rate change by the same amount such that population size remains constant. This also implies that we restrict attention to changes in the population age structure (lower  $\mu$  implies population aging) while neglecting variations in the population growth rate due to demographic change. In particular, emphasis is put on comparing a situation that fully ignores demographic structures, i.e. where  $\mu = 0$ , to one that allows for them by considering nonzero mortality rates.

<sup>7</sup>Equation (3) can be easily derived by calculating expected lifetime utility where the date of death is a random variable with an exponential probability density function parameterized by a constant instantaneous mortality rate  $\mu$ .



elasticity of substitution between varieties.

### 2.1.3 Capital accumulation

As outlined in section 2.1.1, individual savings, defined as income minus consumption expenditures, are converted into capital in the investment sector. The wealth constraint of a representative individual can thus be written as

$$\dot{k}(t_0, \tau) = \frac{w(\tau)l + \pi(\tau)k(t_0, \tau) - e(t_0, \tau)}{G(\tau)} + \mu k(t_0, \tau), \quad (4)$$

where  $l$  refers to the efficiency units of labor an individual supplies and  $e(t_0, \tau)$  are individual total expenditures for consumption defined as

$$e(t_0, \tau) \equiv p_z(\tau)c_z(t_0, \tau) + \int_0^{V_H(\tau)} p_m^H(i, \tau)c_m^H(i, t_0, \tau)di + \int_0^{V_F(\tau)} p_{m,\varphi}^F(j, \tau)c_m^F(j, t_0, \tau)dj.$$

Here  $p_z(\tau)$  is the price of the agricultural good,  $p_m^H(i, \tau)$  the price of a manufactured variety produced at home and  $p_{m,\varphi}^F(j, \tau)$  the price of a manufactured variety produced abroad with the subscript  $\varphi$  indicating the dependence on transport costs.

The particular law of motion for capital given in equation (4) is based on Yaari (1965)'s full insurance result implying that all individuals only hold their wealth in the form of actuarial notes, i.e. each individual itself first converts its savings into capital and then leaves it to the insurance company. Therefore, the market rate of return on capital,  $\frac{\pi(\tau)}{G(\tau)}$ , has to be augmented by  $\mu$  to obtain the fair rate on actuarial notes (see Yaari (1965)).

Using the demographic assumptions described in section 2.1.2 to substitute out  $N(t_0, t)$  in equation (2), then differentiating with respect to  $t$  and substituting for  $\dot{k}(t_0, t)$  from equation (4) and for  $a_i(t)$  from equation (1) yields the aggregate law of motion of capital<sup>8</sup>

$$\dot{K}(t) = [\pi(t)K(t) + w(t)L - E(t)] \frac{K(t) + \eta K^*(t)}{w(t)}, \quad (5)$$

where aggregate consumption expenditures,  $E(t)$ , are analogously defined

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<sup>8</sup>For details of the aggregation procedure see Grafeneder-Weissteiner and Prettnner (2009).

to the aggregate capital stock in equation (2).<sup>9</sup> As outlined in Grafeneder-Weissteiner and Prettnner (2009), the aggregate capital accumulation equation does not feature the mortality rate  $\mu$  anymore. This is in sharp contrast to the law of motion for individual capital and captures the fact that  $\mu K(t)$  does not represent aggregate capital accumulation but is a transfer - via the life insurance company - from individuals who died to those who survived. Similarly, the corresponding law of motion for the foreign capital stock is given by<sup>10</sup>

$$\dot{K}^*(t) = [\pi^*(t)K^*(t) + w^*(t)L - E^*(t)] \frac{K^*(t) + \eta K(t)}{w^*(t)}. \quad (6)$$

## 2.2 Short-run equilibrium

Analogously to Baldwin et al. (2001), consumers maximize utility, firms maximize profits and all goods and factor markets clear for given levels of  $K(t)$  and  $K(t)^*$  in the short-run equilibrium.

### 2.2.1 Utility maximization

The representative individual of cohort  $t_0$  chooses at each instant  $\tau > t_0$  consumption of the agricultural good,  $c_z(t_0, \tau)$ , consumption of varieties produced at home,  $c_m^H(i, t_0, \tau)$ , and consumption of varieties produced abroad,  $c_m^F(j, t_0, \tau)$  to maximize expected lifetime utility given in equation (3) subject to the wealth constraint (4). This optimization problem can be solved by applying a three stage procedure.<sup>11</sup> In the first stage the dynamic saving-consumption decision is analyzed which yields the individual Euler equation

$$\frac{\dot{e}(t_0, \tau)}{e(t_0, \tau)} = \frac{\pi(\tau)}{G(\tau)} - \rho + \frac{\dot{G}(\tau)}{G(\tau)}. \quad (7)$$

Note that to each individual the labor input coefficient  $a_i(\tau)$  and thus the marginal cost of investment in new capital units,  $G(\tau) \equiv a_i(\tau)w(\tau)$ , is a parameter. This means that the saving decision's impact on the aggregate capital stock and thus on  $a_i(\tau)$  is not taken into account, i.e. no internalization of the knowledge spillover takes place.

<sup>9</sup>The aggregate efficiency units of labor  $L$  are equal to the individual supply  $l$  since population size is normalized to one.

<sup>10</sup>Note that  $L = L^*$  due to symmetry between regions.

<sup>11</sup>For details see again Grafeneder-Weissteiner and Prettnner (2009).

To arrive at the law of motion for aggregate consumption expenditures  $E(t)$  it is necessary to again “sum” over all cohorts.<sup>12</sup> This yields the “aggregate Euler equation” of the economy

$$\frac{\dot{E}(t)}{E(t)} = -\mu(\rho + \mu)G(t)\frac{K(t)}{E(t)} + \frac{\dot{e}(t_0, t)}{e(t_0, t)}, \quad (8)$$

where individual expenditure growth,  $\frac{\dot{e}(t_0, \tau)}{e(t_0, \tau)}$ , is given in equation (7). As described in detail in Grafeneder-Weissteiner and Prettnner (2009), the difference between individual and aggregate savings behavior is captured by a correction term representing the distributional effects due to the turnover of generations. In particular, aggregate expenditure growth falls short of individual growth as wealthy old individuals with high expenditure levels are continually replaced by newborns with no capital holdings and thus low expenditure levels. Analogously, the corresponding law of motion for the foreign region is given by<sup>13</sup>

$$\frac{\dot{E}^*(t)}{E^*(t)} = -\mu(\rho + \mu)G^*(t)\frac{K^*(t)}{E^*(t)} + \frac{\dot{e}^*(t_0, t)}{e^*(t_0, t)}. \quad (9)$$

Stage two and three of the individual optimization problem finally deal with the static consumption allocation between the CES composite and the agricultural good as well as the allocation of consumption to each of the manufactured varieties. Altogether this leads to the following demand functions for the agricultural good and for each of the manufactured varieties

$$c_z(t_0, \tau) = \frac{(1 - \xi)e(t_0, \tau)}{p_z(\tau)}, \quad (10)$$

$$c_m^H(i, t_0, \tau) = \frac{\xi e(t_0, \tau)(p_m^H(i, \tau))^{-\sigma}}{\left[ \int_0^{V_H(\tau)} (p_m^H(i, \tau))^{1-\sigma} di + \int_0^{V_F(\tau)} (p_{m,\varphi}^F(j, \tau))^{1-\sigma} dj \right]}, \quad (11)$$

$$c_m^F(j, t_0, \tau) = \frac{\xi e(t_0, \tau)(p_{m,\varphi}^F(j, \tau))^{-\sigma}}{\left[ \int_0^{V_H(\tau)} (p_m^H(i, \tau))^{1-\sigma} di + \int_0^{V_F(\tau)} (p_{m,\varphi}^F(j, \tau))^{1-\sigma} dj \right]}. \quad (12)$$

### 2.2.2 Profit maximization

Marginal cost pricing in the perfectly competitive agricultural sector and perfect labor mobility across sectors implies that the equilibrium wage rate

<sup>12</sup>For details see again Grafeneder-Weissteiner and Prettnner (2009).

<sup>13</sup>Note that  $\mu = \mu^*$  and  $\rho = \rho^*$  due to symmetry between regions.

in the economy is pinned down by the price of the agricultural good. As free trade between home and foreign equalizes this price, wages also equalize between the two regions as long as each of them produces some agricultural output which will be assumed from now on.<sup>14</sup> Finally, choosing the agricultural good as numeraire leads to

$$w(t) = w^*(t) = 1. \quad (13)$$

Manufacturing firm's profit maximization yields the familiar rule that prices are equal to a constant markup over marginal costs<sup>15</sup>

$$p_m^H(i, t) = \frac{\sigma}{\sigma - 1} w(t) a_m, \quad (14)$$

$$p_{m, \varphi}^F(i, t) = p_m^H(i, t) \varphi. \quad (15)$$

Mill pricing is optimal, i.e. the only difference between prices in the two regions is due to transport costs (see e.g. Baldwin et al. (2003)).

Free entry into the manufacturing sector drives pure profits down to zero which implies that the capital rental rate must equal the operating profit of each manufacturing firm. Using optimal prices given in equations (14) and (15) together with equations (11) and (12) and redefining global quantities and regional share variables<sup>16</sup> gives operating profits and thus capital rental rates as<sup>17</sup>

$$\begin{aligned} \pi &= Bias \left( \frac{\xi E^W}{\sigma K^W} \right); \quad Bias \equiv \left( \frac{\theta_E}{\theta_K + \phi(1 - \theta_K)} + \frac{(1 - \theta_E)\phi}{\phi\theta_K + 1 - \theta_K} \right) \quad (16) \\ \pi^* &= Bias^* \left( \frac{\xi E^W}{\sigma K^W} \right); \quad Bias^* \equiv \left( \frac{1 - \theta_E}{1 - \theta_K + \phi\theta_K} + \frac{\theta_E\phi}{\phi(1 - \theta_K) + \theta_K} \right), \quad (17) \end{aligned}$$

where  $\phi \equiv \varphi^{1-\sigma}$  is a measure of openness between the two regions with  $\phi = 0$  indicating prohibitive trade barriers and  $\phi = 1$  free trade. World expenditures are defined as  $E^W \equiv E + E^*$  and the world capital stock as

<sup>14</sup>See Baldwin (1999) for details on this assumption.

<sup>15</sup>For details of the derivations see again Grafeneder-Weissteiner and Prettner (2009).

<sup>16</sup>In particular, note that the number of varieties in the home region  $V_H(t)$  is equal to the capital stock at home  $K(t)$  as one variety exactly requires one unit of capital as fixed input (analogously  $K^*(t) \equiv V_F(t)$ ).

<sup>17</sup>Time arguments are ignored from now on. Note, moreover, that  $\xi = \xi^*$  and  $\sigma = \sigma^*$  due to symmetry between regions. For further details of the derivations see again Grafeneder-Weissteiner and Prettner (2009).

$K^W \equiv K + K^*$  with  $\theta_K$  and  $\theta_E$  being the respective home shares of these quantities, i.e.  $\theta_K \equiv \frac{K}{K^W}$  and  $\theta_E \equiv \frac{E}{E^W}$ . The terms labeled *Bias* and *Bias\** can be interpreted as the bias in national sales, i.e. *Bias* measures the extent to which a home variety's sales ( $\sigma\pi$ ) differ from the world average sales per variety ( $\frac{\xi E^W}{K^W}$ ).

### 2.2.3 The evolution of the economy

Combining the intermediate results of sections 2.2.1 and 2.2.2 yields a system of three differential equations in  $E$ ,  $E^*$  and  $\theta_K$  that fully describes the evolution of the economy. By substituting for  $a_i$  from equation (1) and for  $\dot{K}$  and  $\dot{K}^*$  from equations (5) and (6) and by imposing the equilibrium wage rate from equation (13) as well as the capital rental rates from equations (16) and (17), the aggregate Euler equations given in (8) and (9) can be rewritten as

$$\begin{aligned} \frac{\dot{E}}{E} &= -\mu(\rho + \mu)\frac{\theta_K}{AE} - (1 + \eta\frac{A^*}{A})L + (E + \eta\frac{A^*}{A}E^*) - \rho \\ &\quad + \frac{\xi}{\sigma}(E + E^*) \left[ (A - \theta_K)Bias - \eta\frac{A^*}{A}(1 - \theta_K)Bias^* \right], \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\dot{E}^*}{E^*} &= -\mu(\rho + \mu)\frac{1 - \theta_K}{A^*E^*} - (1 + \eta\frac{A}{A^*})L + (E^* + \eta\frac{A}{A^*}E) - \rho \\ &\quad + \frac{\xi}{\sigma}(E + E^*) \left[ (A^* - (1 - \theta_K))Bias^* - \eta\frac{A}{A^*}\theta_K Bias \right], \end{aligned} \quad (19)$$

where  $A \equiv \theta_K + \eta(1 - \theta_K)$  such that  $a_i = \frac{1}{K^WA}$  and analogously for  $A^*$ .

The law of motion of  $\theta_K$  is obtained by differentiating the definition of this share variable with respect to time and then substituting for  $\dot{K}$  and  $\dot{K}^*$  from equations (5) and (6) which yields

$$\begin{aligned} \dot{\theta}_K &= \theta_K(1 - \theta_K) \left( \frac{\dot{K}}{K} - \frac{\dot{K}^*}{K^*} \right) \\ &= (1 - \theta_K)A \left[ L + \frac{\sigma}{\xi}(E + E^*)\theta_K Bias - E \right] \\ &\quad - \theta_K A^* \left[ L + \frac{\sigma}{\xi}(E + E^*)(1 - \theta_K)Bias^* - E^* \right], \end{aligned} \quad (20)$$

where again the equilibrium wage rate from equation (13) as well as the capital rental rate from equation (16) is imposed.

The remaining sections 3 and 4 will analyze this three-dimensional dy-

dynamic system (18), (19) and (20) more thoroughly to identify the channels via which the demographic parameter  $\mu$  impacts upon both the steady-state growth rates and the stability properties of the symmetric equilibrium. Similarly to Grafeneder-Weissteiner and Prettnner (2009), the incorporation of overlapping generations and lifetime uncertainty affects the system only via the turnover correction terms in the aggregate Euler equations. This already emphasizes the central role the generational turnover will play for the relationship between demographic change and agglomeration tendencies. Note, moreover, that setting  $\mu = 0$ , i.e. considering the case of an infinitely lived representative agent, yields the laws of motion obtained by Baldwin et al. (2001). The framework developed in this paper thus nests their model, which fully ignores demographic structures, as a special case.

### 3 Long-run equilibrium

A long-run equilibrium is characterized by the steady-state values  $\bar{E}$ ,  $\bar{E}^*$  and  $\bar{\theta}_K$  for which  $\dot{E} = \dot{E}^* = \dot{\theta}_K = 0$ . Due to the nonlinearities arising from the turnover term in the aggregate Euler equations, one cannot solve for all equilibria analytically. Numerical investigations, however, reveal an equilibrium pattern similar to Baldwin et al. (2001). Before presenting these results, subsection 3.1 analytically characterizes the symmetric interior equilibrium as well as the core-periphery outcome.<sup>18</sup>

#### 3.1 Lifetime uncertainty in the symmetric and core-periphery equilibrium - Is agglomeration pro-growth?

Inserting the symmetric outcome with  $\theta_K = 0.5$  and  $E = E^*$  into the three-dimensional system reveals that it is indeed a steady state with the equilibrium level of expenditures given by<sup>19</sup>

$$\bar{E}_{sym} = \bar{E}_{sym}^* = \frac{L(1 + \eta) + \rho + \sqrt{(L(1 + \eta) + \rho)^2 + 4\mu(\mu + \rho)}}{2(1 + \eta)} \quad (21)$$

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<sup>18</sup>These and most other results were derived with Mathematica. The corresponding files are available from the author upon request.

<sup>19</sup>Solving the system for the symmetric equilibrium value of expenditures in fact yielded two solutions. Attention is restricted to the economically meaningful one.

Equation (21) clearly shows that aggregate expenditures in the symmetric equilibrium increase in the mortality rate. This is fully consistent with Grafeneder-Weissteiner and Prettnner (2009), who, however, could show the positive dependence in their framework without spillovers only numerically.<sup>20</sup> The above finding moreover indicates that the mortality rate influences consumption expenditures primarily via its effect on discounting, i.e. a higher mortality rate increases expenditure levels of individuals. This “discount channel” dominates the “age structure based channel”. The latter captures the effect of the mortality rate on the age composition of the population and implies a negative dependence since a higher mortality rate increases the proportion of poor and young individuals with low expenditure levels to wealthy and old individuals with higher expenditure levels (see Grafeneder-Weissteiner and Prettnner (2009) for details).

The growth rate, defined as  $g \equiv \frac{\dot{K}}{K}$ , in the symmetric equilibrium can be obtained from equation (5) by using the equilibrium level of expenditures of equation (21) and simplifying. It is given by

$$\begin{aligned} \bar{g}_{sym} &= \bar{g}_{sym}^* = (1 + \eta) \left[ L - \frac{\sigma - \xi}{\sigma} \bar{E}_{sym} \right] \\ &= (1 + \eta) L - \frac{(\sigma - \xi)}{2\sigma} \left[ L(1 + \eta) + \rho + \sqrt{(L(1 + \eta) + \rho)^2 + 4\mu(\mu + \rho)} \right]. \end{aligned} \tag{22}$$

Investigating the dependence of this growth rate on the mortality rate immediately yields the following proposition.

**Proposition 1.** *The equilibrium growth rate in the symmetric equilibrium decreases in the mortality rate.*

*Proof.* The derivative of the growth rate with respect to mortality is

$$\frac{\partial \bar{g}_{sym}}{\partial \mu} = - \frac{(\sigma - \xi)(2\mu + \rho)}{\sigma \sqrt{(L(1 + \eta) + \rho)^2 + 4\mu(\mu + \rho)}} < 0,$$

since  $\sigma > \xi$ . □

This finding is fully consistent with Prettnner (2009), who investigates the consequences of varying mortality rates for growth perspectives in a

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<sup>20</sup>Surprisingly, the introduction of endogenous growth into their framework substantially simplifies the dynamic system describing the evolution of the economy and thus allows for more analytical results.

one-region endogenous growth model in the spirit of Romer (1990). The negative effect of the mortality rate on the growth rate works via the increased level of equilibrium expenditures (see equation (21)). If individuals face lifetime uncertainty, available resources are more heavily used for current consumption purposes than for investment in capital and thus new varieties. This is due to the fact that future is discounted more strongly and thus investment in future growth prospects becomes less important.

Similar findings also apply to the core-periphery outcome.<sup>21</sup> First, for a threshold value of the mortality rate of

$$\mu_{cp} = \frac{-\rho + \sqrt{\left(\frac{(L+\rho)(\sigma-\xi(\phi\eta-1))+2L\xi(1-\frac{\eta}{\phi})}{\sigma+\xi(\phi\eta-1)}\right)^2 - (L^2 + 2L\rho)}}{2} \quad (23)$$

accumulation of capital in only one region, i.e.  $\theta_K = 1$ , can be shown to be a steady state of the three-dimensional system<sup>22</sup> with associated expenditure levels

$$\bar{E}_{cp} = \frac{(L + \rho) + \sqrt{(L + \rho)^2 + 4\mu(\rho + \mu)}}{2}, \quad \bar{E}_{cp}^* = L. \quad (24)$$

Whereas equilibrium expenditures at home again increase in the mortality rate, the foreign expenditure level is of course independent of the mortality rate since, even with infinitely lived individuals, all available resources are immediately used for consumption purposes. The growth rate in the core-periphery equilibrium is finally obtained by combining equation (5) with the core-periphery expenditure levels of (24) which yields

$$\begin{aligned} \bar{g}_{cp} &= \frac{\sigma + \xi}{\sigma}L - \frac{\sigma - \xi}{\sigma}\bar{E}_{cp} \\ &= \frac{(3\xi + \sigma)L - (\sigma - \xi)(\rho + \sqrt{(L + \rho)^2 + 4\mu(\rho + \mu)})}{2\sigma}. \end{aligned} \quad (25)$$

Analogously to the symmetric equilibrium, the impact of lifetime uncertainty on this growth rate can be immediately summarized in the following proposition.

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<sup>21</sup>Here only the  $\theta_K = 1$  case is considered. Using symmetry between the regions, analogous results can be shown to hold for  $\theta_K = 0$ .

<sup>22</sup>Following Baldwin et al. (2001), who investigate the equilibria of the model for varying trade cost levels instead of mortality rates, one can thus conclude that for all  $\mu \leq \mu_{cp}$  the core-periphery outcome  $\theta_K = 1$  represents a long-run equilibrium. This follows from taking into account the boundary condition  $0 \leq \theta_K \leq 1$ .



**Proposition 2.** *The equilibrium growth rate in the core-periphery equilibrium decreases in the mortality rate.*

*Proof.* The derivative of the growth rate with respect to mortality is

$$\frac{\partial \bar{g}_{cp}}{\partial \mu} = -\frac{(\sigma - \xi)(2\mu + \rho)}{\sigma \sqrt{(L + \rho)^2 + 4\mu(\rho + \mu)}} < 0,$$

since  $\sigma > \xi$ . □

As in the symmetric equilibrium, a positive mortality rate decreases the equilibrium growth rate via its positive impact upon equilibrium *home* expenditures.

Although the mortality rate has the same qualitative effect on the growth rates in the symmetric and core-periphery equilibrium, its quantitative impact differs. This has crucial implications for the growth rate differential between the symmetric and the core-periphery equilibrium.<sup>23</sup> Baldwin et al. (2001) show that in their model with infinitely lived individuals the core-periphery growth rate exceeds the growth rate in the symmetric equilibrium as long as spillovers are localized, i.e.  $\eta < 1$ . In the case of global spillovers the growth rates equalize which is intuitive since there is no spillover gain from agglomeration of innovative activity. Agglomeration is thus pro-growth unless interregional learning externalities are perfect. Surprisingly, the introduction of overlapping generation structures and lifetime uncertainty qualifies this finding. In particular, for the case of global spillovers, i.e.  $\eta = 1$ , the growth rate differential is given by

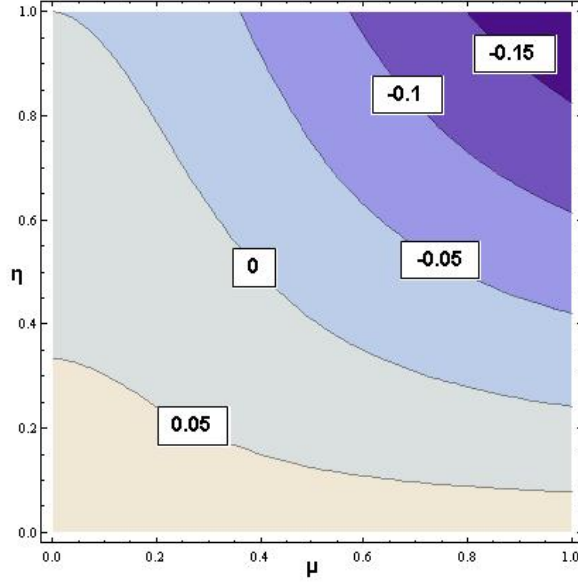
$$\begin{aligned} \bar{g}_{cp} - \bar{g}_{sym}|_{\eta=1} &= \frac{\sigma - \xi}{2\sigma} \sqrt{(2L + \rho)^2 + 4\mu(\rho + \mu)} - L \\ &\quad - \frac{\sigma - \xi}{2\sigma} \sqrt{(L + \rho)^2 + 4\mu(\rho + \mu)}. \end{aligned} \quad (26)$$

It is easily verified that  $\bar{g}_{cp} - \bar{g}_{sym}|_{\eta=1}$  for  $\mu = 0$  and that this differential decreases in the mortality rate. Consequently, with lifetime uncertainty, the core-periphery outcome even features a smaller growth rate than the symmetric equilibrium for the case of fully globalized spillovers. The growth advantage of the symmetric equilibrium resulting from the possibility of

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<sup>23</sup>Note that in both equilibria steady-state growth in real income is equalized across regions. This is due to the fact that real income growth is driven by the steady decrease in the price index along the growth path which itself results from the growing number of worldwide varieties  $K^W$  (see Baldwin et al. (2001)).

Figure 1: The growth rate differential  $\bar{g}_{cp} - \bar{g}_{sym}$  for varying  $\mu$  and  $\eta$



death is, however, dampened the more spillovers are localized. This follows from noting that the growth rate in the symmetric equilibrium is smaller the more spillovers are localized, i.e. the lower is  $\eta$ , whereas the rate in the core-periphery outcome is independent of the spillover parameter. Figure 1 illustrates these findings by plotting the level curves of the growth rate differential  $\bar{g}_{cp} - \bar{g}_{sym}$  for varying mortality rates and degrees of interregional spillovers given the parameter values  $\rho = 0.015$ ,  $\xi = 0.3$ ,  $\sigma = 4$ ,  $L = 1$ . In sharp contrast to standard NEG growth models (see e.g. Baldwin and Martin (2004) for an overview), figure 1 nicely shows that even in the case of localized spillovers the symmetric equilibrium's growth rate can exceed the core-periphery's one, e.g. for  $\eta = 0.9$ , mortality rates above 0.127 would imply a negative differential. These observations are summarized in the following proposition.

**Proposition 3.** *The growth rate differential  $\bar{g}_{cp} - \bar{g}_{sym}$  decreases in the mortality rate with the decrease being dampened the more spillovers are localized.*

*Proof.*

$$\frac{\partial(\bar{g}_{cp} - \bar{g}_{sym})}{\partial\mu} = \frac{(\sigma - \xi)(2\mu + \rho)}{\sigma} \times \left( \frac{1}{\sqrt{(L(1 + \eta) + \rho)^2 + 4\mu(\rho + \mu)}} - \frac{1}{\sqrt{(L + \rho)^2 + 4\mu(\rho + \mu)}} \right) \leq 0,$$

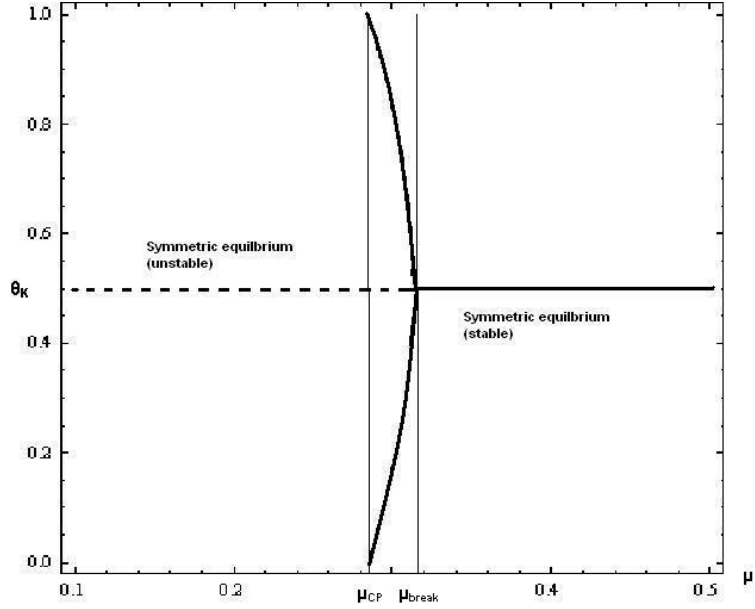
since  $\eta \geq 0$  makes the term in brackets nonpositive and  $\sigma > \xi$ . Clearly, the above derivative decreases in  $\eta$  which proves the second part of the proposition.  $\square$

In a setting with lifetime uncertainty, spatial agglomeration is thus not necessarily conducive to growth. The intuition for this result becomes clear when looking more thoroughly at the impact of lifetime uncertainty on the growth rates in the symmetric and core-periphery outcome. In general, when spillovers are localized, agglomeration of innovative activity makes innovation cheaper and thus results in higher growth rates. A positive mortality rate, however, countervails this pro-growth effect of spatial agglomeration. In particular, it has a less negative effect on the growth rate in the symmetric equilibrium than in the core-periphery equilibrium if spillovers are not purely local. This can be easily seen by comparing the derivatives of the growth rates with respect to the mortality rate in the proofs of propositions 1 and 2 which, as the proof of proposition 3 also shows, only equalize for  $\eta = 0$ . Intuitively, in the symmetric equilibrium with some knowledge spillovers from the other region, the decrease in saving incentives due to a higher mortality rate is not as unfavorable for the economy's growth rate as in the core-periphery equilibrium with full concentration of innovative activity. The reason is that spillovers from the other region increase productivity in the innovation sector such that it is *ceteris paribus* more attractive to use resources for investment purposes than for current consumption. Since this effect is, however, only present in the symmetric equilibrium, a positive mortality rate decreases equilibrium expenditures less in the symmetric than in the core-periphery equilibrium and thus has a lower negative effect on the growth rate.

### 3.2 Interior asymmetric equilibria

As already mentioned, deriving analytical expressions for all steady states is too cumbersome. It is, however, possible to reduce the system of three

Figure 2: Steady-state values of  $\theta_K$  as a function of  $\mu$



equilibrium equations in three variables, i.e.  $\dot{E} = \dot{E}^* = \dot{\theta}_K = 0$ , to one equation in  $\theta_K$ , whose roots represent all long-run equilibria. Numerically investigating this equation reveals that for a small range of mortality rates, in particular as soon as the threshold value  $\mu_{cp}$  is passed, the core-periphery equilibria turn into interior asymmetric equilibria. The following figure 2 illustrates these equilibrium characteristics by plotting the steady states of the system as a function of  $\mu$ .<sup>24</sup> It clearly indicates that there exist, at least for a small range of mortality rates, interior equilibria with an unequal distribution of capital across regions which are symmetric around the persistent steady state  $\theta_K = 0.5$ . Similar to Baldwin et al. (2001), figure 2 is highly reminiscent of a pitchfork bifurcation. The following section will provide additional evidence for the existence of a supercritical pitchfork bifurcation by illustrating that the symmetric equilibrium loses its stability as soon as the two interior asymmetric equilibria appear.

<sup>24</sup>Note that for the parameter values used, i.e.  $\rho = 0.015$ ,  $\xi = 0.3$ ,  $\sigma = 4$ ,  $L = 1$ ,  $\eta = 0.5$  and  $\phi = 0.9$ ,  $\mu_{cp} = 0.285455$ .

## 4 The joint effect of demography and spillovers on agglomeration

Figure 2 shows the existence of long-run equilibria that are characterized by an unequal distribution of capital. Whether such an agglomeration of economic activity takes place crucially depends on the stability properties of the symmetric equilibrium. If it turns out that this steady state is unstable, any slight perturbation of the symmetric outcome triggers agglomeration processes that might result in either the asymmetric interior or the core-periphery equilibria.

In general, NEG models focus on the role of changing trade costs for the emergence of spatial structures, i.e. they show how economic integration lowering the costs of trading goods leads to concentration of economic activity. Other features of the economy could, however, be similarly decisive for agglomeration processes. Grafeneder-Weissteiner and Prettnner (2009) have shown that the adoption of an overlapping generation structure with lifetime uncertainty, i.e. increasing  $\mu$  above zero, considerably reduces the possibility of the symmetric equilibrium to be unstable in the constructed capital framework of Baldwin (1999). The main question to be investigated in this section is whether an analogous finding also holds in a setting with endogenous growth due to learning spillovers. In particular, it is of interest how demographic structures interact with the additional growth-linked circular causality resulting from the incorporation of knowledge spillovers in the investment sector. Finally, it is worth investigating the role of lifetime uncertainty in the selection among the asymmetric equilibria, i.e. how it influences the importance of initial conditions relative to expectations in choosing the region where agglomeration takes place.

### 4.1 Formal stability analysis

The stability properties of the symmetric long-run equilibrium are analyzed by following the classical approach (see e.g. the appendix on mathematical methods in Barro and Sala-i-Martin (2004)) of linearizing the non-linear dynamic system (18), (19) and (20)<sup>25</sup> around the symmetric equilibrium and then by evaluating the eigenvalues of the corresponding  $3 \times 3$  Jacobian

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<sup>25</sup>Equations (18) and (19) were multiplied by  $E$  or  $E^*$  to obtain  $\dot{E}$  and  $\dot{E}^*$ .

matrix

$$\begin{pmatrix} j_{11} & j_{12} & j_{13} \\ j_{12} & j_{11} & -j_{13} \\ j_{31} & -j_{31} & j_{33}, \end{pmatrix} \quad (27)$$

whose entries are given in appendix A. Solving the characteristic equation yields the following three eigenvalues

$$eig1 = \sqrt{L^2(\eta + 1)^2 + 2L\rho(\eta + 1) + (2\mu + \rho)^2}, \quad (28)$$

$$eig2 = \frac{1}{2(\eta + 1)(\phi + 1)^2\sigma}(r - \sqrt{rad}), \quad (29)$$

$$eig3 = \frac{1}{2(\eta + 1)(\phi + 1)^2\sigma}(r + \sqrt{rad}), \quad (30)$$

where

$$\begin{aligned} r &\equiv (-2L\eta + eig1)((\eta + 1)(\phi + 1)^2\sigma) - 2\phi(\eta\phi - 1)Q\xi, \\ rad &\equiv \left( ((\eta - 1)eig1 + 2\eta\rho)\sigma(\phi + 1)^2 + 2(\phi - \eta)Q\xi \right)^2 \\ &\quad - 8\eta(\eta + 1)(\phi + 1)^3\sigma((\phi + 1)\sigma + (\phi - 1)\xi) \times \\ &\quad \times \left( \frac{Q \left( -L\eta^2 + L + \frac{1}{2}Q \left( \eta - \frac{2(\eta\phi^2 - 2\phi + \eta)\xi}{(\phi + 1)^2\sigma} - 1 \right) \right)}{2(\eta + 1)} - \mu(\mu + \rho) \right), \end{aligned}$$

with the parameter cluster  $Q \equiv L(\eta + 1) + eig1 + \rho$ . The signs and nature of these eigenvalues fully characterize the system's local dynamics around the symmetric equilibrium. Since there are two jump variables  $E$  and  $E^*$ , stability requires at least one eigenvalue to be negative. In particular, saddle path stability prevails if one out of the three eigenvalues is negative.

First it is easily established that eigenvalue 1 is always real and positive for all possible parameter values.<sup>26</sup> As far as the remaining two eigenvalues are concerned things turn out to be more complicated. Checking  $rad$  for various parameter specifications shows that it changes sign, i.e. one must differentiate between the case where eigenvalues 2 and 3 are real and the case where they are complex. In both cases stability properties crucially depend on the sign of  $r$ . Since  $(-2L\eta + eig1) > 0$  and  $(\eta\phi - 1) \leq 0$   $r$  is unambiguously positive for all possible parameter ranges.

<sup>26</sup>Recall the parameter ranges  $\sigma > 1$ ,  $\rho > 0$ ,  $\mu > 0$ ,  $0 < \xi < 1$  and  $0 \leq \phi \leq 1$ ,  $0 \leq \eta \leq 1$  and  $L > 0$  which also imply that  $Q > 0$ .

## 4.2 The case of real eigenvalues - The opposing stability effects of demography and spillovers

With real eigenvalues,  $r > 0$  immediately implies that eigenvalue 3 is also positive resulting in lemma 1.

**Lemma 1.** *For the case of real eigenvalues, i.e.  $\text{rad} > 0$ , eigenvalue 2 is decisive for the local stability properties of the symmetric equilibrium. A positive eigenvalue 2 implies instability, a negative one saddle path stability.*

*Proof.* See above arguing.  $\square$

Lemma 1 implies that changes in the mortality rate can only influence the stability properties of the symmetric equilibrium via eigenvalue 2. Numerically investigating this eigenvalue immediately results in the following proposition.

**Proposition 4.** *The possibility of agglomeration crucially hinges on the mortality rate.*

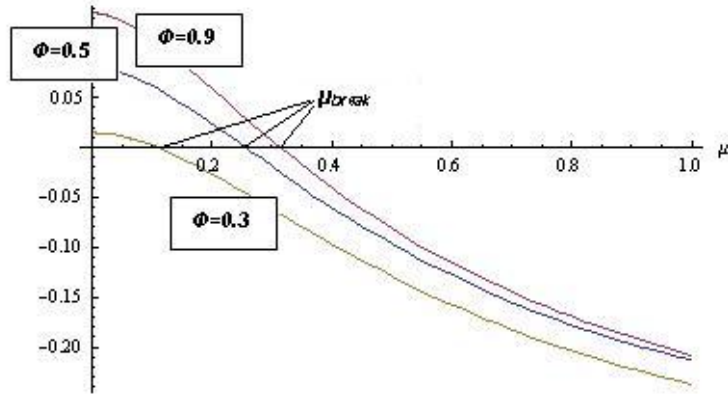
*Proof.* Figure 3<sup>27</sup>, which plots eigenvalue 2 as a function of the mortality rate for an intermediate level of interregional spillovers  $\eta = 0.5$  and for three different levels of economic integration, clearly reveals that eigenvalue 2 switches sign depending on the mortality rate.  $\square$

Proposition 4 verifies the crucial importance of demographic structures for agglomeration processes found by Grafeneder-Weissteiner and Prettnner (2009) also for a setting that additionally allows for endogenous growth due to learning spillovers. Figure 3, however, does not only show this decisive role of the mortality rate but also reconfirms the stabilizing effect of introducing overlapping generations and lifetime uncertainty. Consistent with Grafeneder-Weissteiner and Prettnner (2009), eigenvalue 2 decreases in the mortality rate, i.e. only for sufficiently low mortality rates agglomeration processes may set in. In their framework without knowledge spillovers, this stabilizing effect even implied that for plausible parameter values instability could never occur. In particular, they showed that for mortality rates corresponding to life expectancies of less than approximately 3500 years<sup>28</sup> the

<sup>27</sup>Figure 3 is again plotted for  $\rho = 0.015$ ,  $\xi = 0.3$ ,  $\sigma = 4$  and  $L = 1$ .

<sup>28</sup>Since the probability of death during each year equals  $\mu$ , average life expectancy is  $\frac{1}{\mu}$ .

Figure 3: Eigenvalue 2 as a function of  $\mu$



symmetric equilibrium was stable for all levels of economic integration.<sup>29</sup> This is in sharp contrast to the present setting with endogenous growth due to learning effects. Indeed, figure 3 clearly indicates that agglomeration is still a fully possible outcome. It shows that for a rather low level of trade openness  $\phi = 0.3$ , the critical mortality rate  $\mu_{break}$  below which eigenvalue 2 is positive and thus the symmetric equilibrium unstable<sup>30</sup> is 0.109. This corresponds to a life expectancy of only 9 years, which illustrates that, in contrast to Grafeneder-Weissteiner and Prettnner (2009), a plausible choice of parameter values does not eliminate the possibility of agglomeration of economic activity even though we allow for nonzero mortality rates. Summarizing, the findings so far imply that, although the introduction of overlapping generations still acts as a dispersion force in a NEG model with endogenous growth, its impacts are, in general, not strong enough to prevent regions from unequal development.

The intuition for this result is simple. It is based on the existence of two countervailing effects which would not be present in a setting without learn-

<sup>29</sup>This holds for their most plausible choice of parameter values, i.e.  $\rho = 0.015$ ,  $\delta = 0.05$ ,  $\xi = 0.3$  and  $\sigma = 4$ . Setting  $\delta = 0$  as in this setting only insignificantly reduces this life expectancy threshold value.

<sup>30</sup>The analytical expression for  $\mu_{break}$  is too unwieldy to report. The Mathematica file deriving it is available upon request.



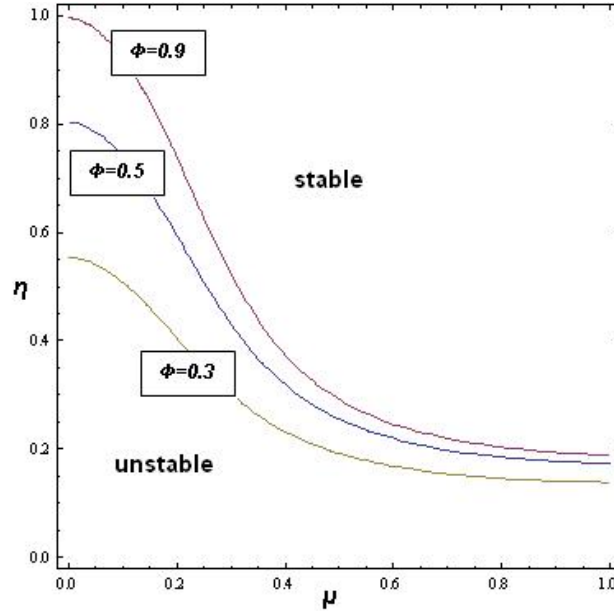
ing spillovers and ignoring demographic structures. In particular, whether agglomeration takes place in this model economy depends on the relative strength of four distinct agglomeration or dispersion forces. Each of them captures how an exogenous increase of the capital share impacts upon the rate of capital accumulation. If it raises it, agglomeration of capital takes place since a circular causality sets in. Otherwise, the decreased capital accumulation rate acts as a self-correcting force promoting dispersion rather than concentration of economic activity.

First, there are two forces that are neither linked to the demographic structure nor to the spillover specification. These are the standard anti-agglomerative local competition effect and the pro-agglomerative demand-linked circular causality first introduced by Baldwin (1999). The latter shows that a higher share of capital in one region increases expenditures and thus operating profits, i.e. capital rental rates, which speeds up capital accumulation. The former, on the other hand, captures the negative impact of agglomeration of capital, i.e. firms, on capital rental rates due to more severe competition. Both of these forces are, however, not the channels via which the mortality rate on the one hand and learning spillovers on the other hand impact upon the stability properties of the symmetric equilibrium.

The particular channel via which demography influences agglomeration processes has first been identified by Grafeneder-Weissteiner and Prettner (2009) as the anti-agglomerative turnover effect. An exogenous rise in the home capital share increases wealth and thus expenditure levels of individuals being currently alive in the home region relative to foreign-based individuals. The negative distributional effects on aggregate expenditures resulting from death, i.e. the replacement of these individuals by newborns whose consumption expenditures are lower since they have zero wealth levels, are thus more pronounced in the home region. This, in turn, decreases the home expenditure share and therefore relative profitability and the relative capital rental rate.

Finally, the impact of learning spillovers on the stability properties of the symmetric equilibrium are captured by the growth-linked circular causality introduced by Martin and Ottaviano (1999). If learning spillovers are localized, a higher share of capital in the home region lowers the marginal cost of producing new capital relative to the foreign region (see equation 1) and thus strengthens capital accumulation. Endogenous growth is thus a power-

Figure 4: Symmetric equilibrium stability map



ful destabilizing force that mitigates the stabilizing effect of the turnover of generations. The strength of this growth-linked circular causality crucially depends on the degree of localization of spillovers. As can be seen in figure 4<sup>31</sup>, which plots the zero level curve of eigenvalue 2, i.e. the dividing line between stability and instability, for three different levels of trade openness, agglomeration is fostered the more learning externalities are localized. In particular, with no learning spillovers across regions, i.e.  $\eta = 0$ , eigenvalue 2 is positive for all possible parameter ranges and the system is always unstable. Figure 4, however, also illustrates the stabilizing effect of increased interregional spillovers. For a given mortality rate of e.g. 0.0125 resulting in a life expectancy of 80 years and intermediate trade openness levels  $\phi = 0.5$ , fostering interregional knowledge spillovers above a level of about 0.8 could still prevent regions from unequal development. The strength of this stabilizing effect of knowledge spillovers becomes clear if one considers the case of globalized learning effects, i.e.  $\eta = 1$ . In this case the symmetric equilibrium is stable for all levels of trade costs as long as individuals face a life expectancy of less than approximately 3600 years, i.e. as in Grafeneder-

<sup>31</sup>Figure 4 is again plotted for  $\rho = 0.015$ ,  $\xi = 0.3$ ,  $\sigma = 4$  and  $L = 1$ .

Weissteiner and Prettner (2009) agglomeration of economic activity does not occur for plausible parameter values.

Last but not least, the stability findings of this section also provide additional evidence that the dynamic system undergoes a supercritical pitchfork bifurcation as the mortality rate crosses a certain threshold value. In particular, for the parameter values of figure 2, the critical mortality rate  $\mu_{break}$  at which the symmetric equilibrium loses stability is 0.313. As can be seen from figure 2, this value exactly coincides with the threshold mortality rate where the two interior asymmetric steady states, that finally turn into the core-periphery equilibria, show up. Following Baldwin (2001), one can thus conclude that as soon as the mortality rate crosses  $\mu_{break}$  from above, the symmetric steady state loses its stability to the two appearing neighboring asymmetric interior steady states. The question which out of these two equilibria will then be reached, i.e. in which region agglomeration will take place, is briefly addressed in the next subsection.

### 4.3 The case of complex eigenvalues - Lifetime uncertainty and the history-versus-expectations debate

The analysis so far has shown that nonzero mortality rates foster a more equal distribution of economic activity due to the turnover of generations. With complex eigenvalues, i.e.  $rad < 0$ , the symmetric equilibrium is, however, always unstable since  $r$  is unambiguously positive for all possible parameter ranges. As will become clear, the mortality rate nonetheless plays a decisive role for the dynamics of the system by influencing the dividing line between the case of monotone divergence, that occurs for a positive real eigenvalue 2, and the case of diverging oscillations resulting from complex eigenvalues.

Krugman (1991a)<sup>32</sup> shows that in the first situation history, represented by initial conditions, is the crucial factor with respect to equilibrium selection, whereas in the latter self-fulfilling expectations might also be decisive. In particular, as long as all eigenvalues are real, agglomeration of capital will take place in the region with the initially larger share of capital. In the case of complex eigenvalues, on the other hand, equilibrium paths overlap such that there exists a range around the symmetric equilibrium, where a given

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<sup>32</sup>Baldwin (2001) gives a nice overview over the history-versus-expectations debate in NEG models initiated by Krugman (1991a).

initial distribution of capital corresponds to paths each leading to agglomeration in a different region and expectations determine which path is chosen. Since the parameters of the model determine whether there are complex or real eigenvalues it is perfectly possible to investigate the role of the mortality rate with respect to such history-versus-expectations considerations.

According to Baldwin (2001), a sufficient condition for there to be some overlap of saddle paths is that the eigenvalues of the Jacobian evaluated at the unstable equilibrium are complex, i.e.  $rad < 0$ . Checking the dependence of  $rad$  on  $\mu$  reveals that lifetime uncertainty strengthens the role of initial conditions in choosing among the multiple long-run equilibria. This is summarized in proposition 5.

**Proposition 5.** *The possibility of self-fulfilling expectations rather than initial conditions being decisive for equilibrium selection arises only for sufficiently low mortality rates.*

*Proof.* Using Mathematica it can be shown that  $\frac{\partial rad}{\partial \mu} \geq 0$  for all possible parameter ranges.<sup>33</sup> □

Intuitively, a higher mortality rate implies that individuals discount the future more heavily. This impatience means that they do not care too much about expected profitability of capital accumulation which itself depends on the future investment decisions of other individuals (recall that a region's relative attractiveness with respect to capital accumulation depends on its capital share). But this is exactly one main reason why expectations could be important. If everyone expects that investment will mainly take place in the home region, then this increases the attractiveness of also investing there independently of the current situation, i.e. independent of the relative investment profitability in the initial condition. With low patience such considerations about future investment returns lose, however, importance and it is rather the current relative returns that are decisive.

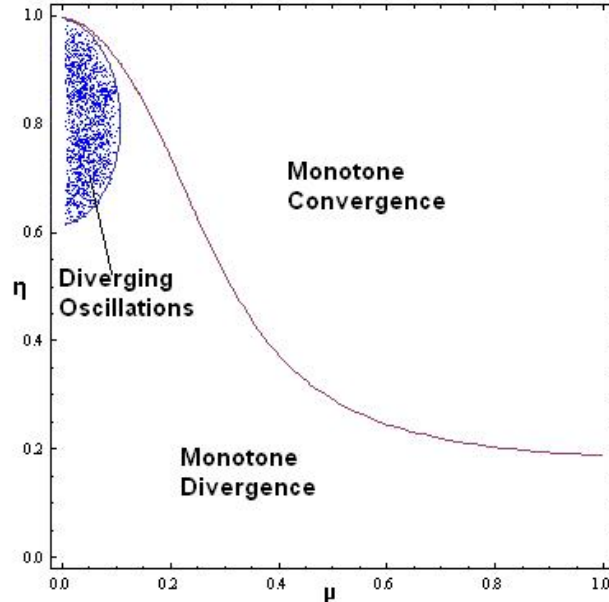
Finally, numerically investigating the parameter region within which  $rad < 0$ <sup>34</sup> more thoroughly reveals that the possibility of expectations to be decisive for equilibrium selection in our model is, in general, rather low. For

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<sup>33</sup>  $\frac{\partial rad}{\partial \mu}$  is too cumbersome to be revealing. The calculations are available upon request.

<sup>34</sup> Note that the threshold value  $\mu_{real}$ , obtained from setting  $rad = 0$ , below which there are complex eigenvalues, is lower than  $\mu_{break}$ , obtained from setting eigenvalue 2 equal to zero, i.e.  $r = \sqrt{rad}$ , since  $r > 0$  and  $rad$  has been shown to increase in  $\mu$ .

Figure 5: The subordinate role of expectations relative to initial conditions for equilibrium selection



the parameter values of figure 2 e.g., the eigenvalues are real for all levels of the mortality rate. Figure 5, which replicates, for varying mortality rates and spillovers, the dividing line between stability and instability of figure 4 and additionally shows the level curve where  $rad = 0$ , i.e. the dividing line between monotone divergence and diverging oscillations, indicates a similar conclusion. The area inside the instability region featuring diverging oscillations instead of monotonic divergence is relatively small. This again emphasizes the subordinate role of expectations in answering the question in which region agglomeration will take place.

## 5 Concluding remarks

By incorporating an overlapping generation structure and lifetime uncertainty into a NEG model that features endogenous growth through learning externalities, this paper has shed further light on whether and how demographic structures impact upon the spatial distribution of economic activity. In doing so, it moreover gives deeper insights on how lifetime uncertainty af-

fects the long-run equilibrium growth perspectives of the symmetric relative to the core-periphery outcome.

The main results are twofold. First, consistent with the findings in Grafeneder-Weissteiner and Prettnner (2009), nonzero mortality rates resulting in age-dependent heterogeneity of individuals support a more equal distribution of productive factors by introducing an additional dispersion force that countervails the agglomeration tendencies resulting from endogenous growth through knowledge spillovers. In particular, if interregional knowledge spillovers are sufficiently encouraged across regions, the turnover of generations can even prevent regions from unequal development. Moreover, the possibility of agglomeration being the result of a self-fulfilling prophecy is substantially reduced when considering individuals who face a positive probability of death. Second, lifetime uncertainty lowers both the symmetric equilibrium's as well as the core-periphery outcome's growth rate. As long as learning spillovers are not purely localized, this decrease is, however, more pronounced for the latter one. Thus, in sharp contrast to existing NEG growth models with localized knowledge spillovers, spatial concentration of economic activity is not necessarily conducive to growth if one takes into account demographic structures. This also implies that there might not be any trade-off between fostering an equal distribution of productive factors and high economic growth which would result from e.g. increased economic integration if agglomeration were unambiguously pro-growth.

Due to analytical tractability, the present framework still draws a very simplified picture of reality. The model could, however, be easily extended to investigate further relevant issues related to the interaction between agglomeration, growth and demography. What happens if regions are asymmetric with respect to mortality rates or the degree of interregional spillovers? What are the exact welfare implications of the results obtained so far? What are the main differences, in particular with respect to growth rates, between partial and full agglomeration equilibria, i.e. between the interior asymmetric and the core-periphery steady states? These are only few questions that could still be investigated more thoroughly within the model framework developed in this paper.

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## Appendix

### A Intermediate results for the stability analysis

The Jacobian matrix given in equation (27) has the following entries

$$\begin{aligned}
 J_{11} &= -\eta L - L - \rho + \frac{(\eta L + L + \rho + eig1) \left( \eta + \frac{\eta\xi - \eta\phi\xi}{\phi\sigma + \sigma} + 2 \right)}{2(\eta + 1)}, \\
 J_{12} &= \frac{\eta (\eta L + L + \rho + eig1) \left( \frac{(\phi-1)\xi}{(\phi+1)\sigma} + 1 \right)}{2(\eta + 1)}, \\
 J_{13} &= \frac{4\eta \left( \frac{(\eta L + L + \rho + eig1) \left( -L\eta^2 + L + \frac{1}{2}(\eta L + L + \rho + eig1) \left( \eta - \frac{2(\eta\phi^2 - 2\phi + \eta)\xi}{(\phi+1)^2\sigma} - 1 \right) \right)}{2(\eta+1)} - \mu(\mu + \rho) \right)}{(\eta + 1)^2}, \\
 J_{31} &= -\frac{(\eta + 1)((\phi + 1)\sigma + (\phi - 1)\xi)}{4(\phi + 1)\sigma}, \\
 J_{33} &= \frac{(\eta L + L + \rho + eig1) \left( \eta - \frac{(\eta\phi^2 - 2\phi + \eta)\xi}{(\phi+1)^2\sigma} \right)}{\eta + 1} - 2L\eta,
 \end{aligned}$$

where  $eig1 = \sqrt{L^2(\eta + 1)^2 + 2L\rho(\eta + 1) + (2\mu + \rho)^2}$ .



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