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Micro foundations for knowledge spillovers in spatial equilibrium models

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Abstract

Knowledge spillovers have recently been analysed by Barde (2009) and Østbye (2010) in a spatial general equilibrium framework. Both studies lack explicit micro foundations for the spillovers – spillovers just take place and depend on firm density. The models must therefore be seen as reduced forms that may potentially be consistent with several structural interpretations. In this paper, the aim is to go some way in offering plausible micro foundations. We consider two alternatives: knowledge dissemination through knowledge embodied in labour moving between firms following Combes and Duranton (2006) and knowledge creation and dissemination through research and development (R&D) following d’Asprémont and Jacquemin (1988).

Keywords: knowledge spillovers

1 Introduction

Knowledge spillovers have recently been analyzed theoretically in a spatial general equilibrium framework by Barde (2009) and Østbye (2010). Their models must be interpreted as reduced form representations since the microfoundation for knowledge creation and diffusion is deliberately left within the black box. In this paper we want to open the box and go some way towards analytical narratives explaining knowledge spillovers in spatial equilibrium models.

Knowledge spillovers have been offered as a possible important reason for agglomeration economies at least since Marshall (1890). The basic idea is that

knowledge spillovers obey Tobler’s first law of geography (Tobler, 1970): “everything is related to everything else, but near things are more related than distant things”. Micro foundations for local knowledge spillovers is touched upon in the broad survey by Duranton and Puga (2004) on micro foundations of urban agglomeration economies, but here we want to go further following two specific tracks.

One track is concentrating on knowledge dissemination through knowledge embodied in labour moving between firms. The analysis specifically tries to relate the reduced form model by Barde (2009) to the structural microeconomic model by Combes and Duranton (2006), henceforth CD, on labour pooling and labour poaching.

The other track is concentrating on knowledge creation and dissemination through research and development (R&D). The output from R&D is to various degrees not excludable (in particular the output from basic research) and therefore implies externalities or spillovers of benefits that are not captured by the producer. The spillovers may be involuntary leakages or voluntary exchange of useful technological information between firms (De Bondt 1996). More specifically we want to relate the reduced form model by Østbye (2010) to the vast non-spatial microeconomic literature on knowledge spillovers in the tradition from d’Asprémont and Jacquemin (1988), henceforth AJ.

The common aim across the two tracks is to bring possible inconsistencies between the micro level and the macro level out in the open and to discuss refinements that remove inconsistencies and avoid implausible implications.

The paper is organised in 4 main sections. Explicit R&D within the IO tradition is introduced in a Dixit-Stiglitz framework in Section 2. The labour pooling/ poaching story is analyzed in Section 3. Section 4 concludes.

2 Microeconomic R&D framework

At the micro level, the Industrial Organization (IO) literature has a long record of studying knowledge spillovers in a partial equilibrium framework. Although there has been progress during the last twenty years following AJ, the current state of affairs is well described by Amir et al. (2008) suggesting that “[t]he ultimate project in this area is to open up the black box of R&D and model the entire process, including spillovers, explicitly.” (p. 701) Despite the shortcomings, the IO literature with its emphasis on (stylized) firm behaviour seems to be the place to start when looking for microfoundations for knowledge spillovers through R&D within or outside a spatial context.

The framework used by AJ and many others, is a stage model with firms investing in cost reducing R&D subject to spillovers prior to meeting in the product market. The original model for a homogenous goods duopoly has been generalized to n firms and differentiated goods which is more relevant here (De Bond et al., 1992). The issues discussed have been related to different scenarios concerning cooperation or not in the R&D stage and different competition

intensities in the product market stage (see, e.g., Hinloopen 2000). How R&D affects the cost structure has been ignored, but in a spatial context this is a very important issue since fixed costs are necessary in order to make location choices non-trivial (Scotchmer and Thisse 1992). Furthermore, the IO models have been on strategic interaction, whereas the economic geography models typically abstract from strategic interaction and focus on entry and exit by firms and people through migration in a spatial environment with monopolistic competition based on Dixit and Stiglitz (1977). In the following subsections we will deal with the product market stage (2.1), the R&D stage (2.2), and the fixed and variable cost structure (2.3), translating the strategic IO environment to a non-strategic economic geography environment.

2.1 The product market

Consistent with the generalized AJ model, consider n Cournot competitors in a market with differentiated goods and linear demand. The profit of firm k net of (sunk) R&D investment costs is

$$\pi_k = (p_k - C(X_k)) q_k, C'(X_k) < 0, \quad (1)$$

where p_k is product price, q_k quantity, $C(X_k)$ variable unit cost, and X_k is known as effective R&D (Ruff 1969). The first order condition,

$$\partial \pi_k / \partial q_k = 0 \Rightarrow p_k (1 - \eta_k) = C(X_k), \quad (2)$$

where $\eta_k \equiv -(\partial p_k / \partial q_k)(q_k / p_k)$ is the (inverse) demand elasticity. Assuming linear (inverse) demand,

$$p_k = \alpha - q_k - \theta \sum_{j \neq k} q_j, \quad (3)$$

we obtain

$$\eta_k = q_k / (\alpha - q_k - \theta \sum_{j \neq k} q_j). \quad (4)$$

Substituting for η_k and for p_k from the demand function in the first order condition from (2) define the profit maximizing output conditional on effective R&D and the rivals' output (the reaction function in the product market stage).

Turning to the analogue for the monopolistic competitor, the first order condition is identical in a formal sense, but demand within the Dixit- Stiglitz framework is not linear. Clearly, linearity in the AJ framework is adopted for convenience and has no substantial justification. Indeed, we may argue that linear demand should be avoided when the number of firms is arbitrary since the calibration of parameters must be adjusted to the number of goods in order to ensure non-negative prices.¹

¹Apart from calibration, there are other problems with the standard linear demand function for differentiated goods. Some of these may be mitigated using the alternative proposed by Shubik and Levitan (1980). See also the discussion in Motta (2004), p. 568.

With the two-tier structure in Østbye (2010), where the monopolistic competitors sell intermediates to a downstream constant returns to scale industry, the demand elasticity, $\epsilon_k \equiv 1/\eta_k$, is given by

$$\epsilon_k = \sigma + (1 - \sigma - \vartheta)p_k q_k / (pq). \quad (5)$$

Intermediates enter downstream production as a CES composite factor of production q and the corresponding CES price index p ,

$$q = \left[\sum_k q_k^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad p = \left[\sum_k p_k^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad (6)$$

with σ as the elasticity of substitution between any varieties. An elasticity close to 1 implies nearly perfect substitutes whereas a high value means diversity is important (see, e.g. Rivera-Batiz 1988).² The parameter ϑ in (5) represents the constant cost share of all intermediates in downstream costs, and $p_k q_k / (pq)$ is the cost share of variety k within the costs of intermediates. Letting the number of varieties grow, the cost share of variety k goes to zero and ϵ_k to σ . Assuming n to be sufficiently large to substitute $1/\sigma$ for η_k in (2), we abstract from strategic interaction in the product market stage.

2.2 The R&D stage

Unit variable production costs in the AJ framework are specified to depend on effective R&D by simply assuming $C(X_k) = \tilde{A} - X_k$, where \tilde{A} is constant unit cost with no R&D. In the AJ partial equilibrium framework constant input prices are subsumed in the constant \tilde{A} . In anticipation of the general equilibrium environment, we assume \tilde{A} to be proportional to the wage rate, v , consistent with the technology in the monopolistic competition industry where output is proportional to labor input, writing $\tilde{A} \equiv Av$. By implication,

$$C(X_k) = Av - X_k. \quad (7)$$

The net benefit to firm k of conducting R&D is given by

$$X_k q_k - \Gamma(x_k), \quad (8)$$

where x_k is own R&D and the R&D cost function $\Gamma(x_k)$ is specified as quadratic,

$$\Gamma(x_k) = \gamma x_k^2 / 2, \quad (9)$$

with $\gamma > 0$. Effective R&D is specified as

$$X_k = x_k + \beta \sum_{j \neq k} x_j, \quad (10)$$

²Following Østbye (2010), discrete summation is used instead of integration over a continuum. By implication, the number of intermediates is assumed to be sufficient for the integer constraint not to be binding.

with β as a spillover parameter contained in the interval $[0, 1]$. There is no substantial reason behind these specifications beyond convenience and tradition. On the contrary, criticisms have been raised against both the cost function as well as the additive effective R&D function.

Amir et al. (2008) argue that the simple quadratic R&D cost function is inconsistent with the plausible criterion that any total R&D investment level should not generate more cost reductions if allocated to n labs rather than spent all in one lab (Amir et al. 2008, p.697). They demonstrate that no positive spillover parameter is consistent with the criterion within the AJ framework, and suggest some modifications to the R&D cost function to avoid this. One alternative, that we are going to adopt here, is to let $\Gamma'(0)$ be positive by assuming $\delta > 0$ and

$$\Gamma(x_k) = \delta x_k + \gamma x_k^2/2, \quad (11)$$

Kamien and Zang (2000) argue that own R&D is necessary to build absorptive capacity in order to benefit from spillovers. The specification of effective R&D in AJ implies that spillovers rain down on the firm "like manna from heaven" (Kamien and Zang, 2000, p. 957) regardless of own R&D efforts. A simple alternative specification of effective R&D that accomodates this criticism is

$$X_k \equiv x_k \left(\sum x_j^\alpha \right)^\beta, \quad (12)$$

where summation is over all j and convexity is restricted by assuming α and β to be contained in the interval $(0, 1)$, since, as we will see, $\alpha\beta < 1$ is necessary for a finite R&D equilibrium level to exist. This alternative implies that own R&D is essential for effective R&D.

As mentioned, the additive structure in the AJ framework appears to be a matter of convenience and tractability rather than substance. For our purpose, a multiplicative structure is preferable to the additive structure for the same reasons. We posit $C(X_k) = Av/X_k$ instead of $Av - X_k$, and net benefit $X_k q_k / \Gamma(x_k)$ instead of $X_k q_k - \Gamma(x_k)$. Summarizing, we write unit cost (including R&D costs) as

$$Av\Gamma(x_k)/X_k = Av(\delta + \gamma x_k/2) / \left(\sum x_j^\alpha \right)^\beta \equiv A\tilde{\Gamma}(x_k)/\tilde{X}_k, \quad (13)$$

It may be instructive to illustrate $\tilde{\Gamma}(x_k)$ and \tilde{X}_k under symmetric R&D ($x_k = x$ for all k) as done in Figure 1. Unit cost is minimized for $\tilde{\Gamma}' = \tilde{X}'$ where the vertical distance between \tilde{X} and $\tilde{\Gamma}$ is largest provided net benefits are positive (\tilde{X} above $\tilde{\Gamma}$), since the slope of \tilde{X} equals the slope of $\tilde{\Gamma}$, and \tilde{X} is more concave than (the linear) $\tilde{\Gamma}$.

Let us now proceed by looking at the solution to the firm's R&D decision problem in more detail.

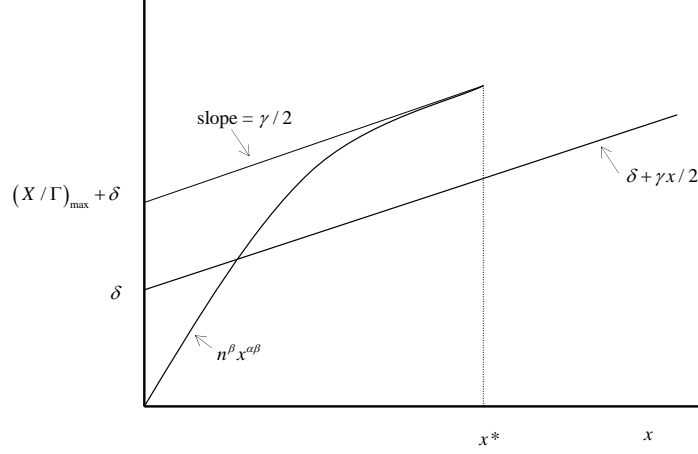


Figure 1: Symmetric R&D equilibrium

2.3 Profit maximizing R&D

In the AJ framework, the profit maximizing R&D levels are found by solving the stage game involving R&D after having solved the product market stage game conditional on R&D levels, using Kuhn's algorithm (backward induction). As we have seen, assuming monopolistic competition in the product market for sufficiently many firms, the problem to be solved in the product market stage is transformed from a complicated strategic to a simple non-strategic problem. The same applies to the R&D stage.

The firm must decide what R&D level maximizes net benefits. Formally, the problem for the firm to solve is

$$\max_{x_k} \left\{ \frac{X}{\Gamma} = \frac{\tilde{X}}{\tilde{\Gamma}} = \frac{(\sum x_j^\alpha)^\beta}{\delta + \gamma x_k/2} \right\}. \quad (14)$$

As in the product market, this is a strategic problem depending on what other firms do, as long as the number of rivals is small. The optimal R&D level must fulfill the condition

$$\partial \tilde{X}_k / \partial x_k = \beta \left(\sum x_j^\alpha \right)^{\beta-1} \alpha x_k^{\alpha-1} = \partial \tilde{\Gamma} / \partial x_k = \gamma/2. \quad (15)$$

Symmetry suggests we look for a symmetric equilibrium R&D level, x ,

$$\beta n^{\beta-1} x^{\alpha(\beta-1)} \alpha x^{\alpha-1} = \gamma/2 \Rightarrow x = [(2\alpha\beta/\gamma)n^{\beta-1}]^{1/(1-\alpha\beta)}. \quad (16)$$

Under symmetry, $\Gamma/X = (\delta + \gamma x/2) (n^\beta x^{\alpha\beta})$. Substituting for x , we obtain

$$\Gamma/X = \frac{\delta(\gamma/2\alpha\beta)^{\alpha\beta/(1-\alpha\beta)}}{n^{\beta(1-\alpha)/(1-\alpha\beta)}} + \frac{2\alpha\beta}{n}. \quad (17)$$

We observe that since $1 > \beta > \beta(1-\alpha)/(1-\alpha\beta)$, although both terms approach zero as n grows, the last term faster than the first. For sufficiently large n , the last term is negligible although the first is not (and more so the larger α is). For sufficiently many firms, not only may the demand elasticity be treated a constant in the product market stage, but the unit (variable) cost simplifies to

$$Av \frac{\delta(\gamma/2\alpha\beta)^{\alpha\beta/(1-\alpha\beta)}}{n^{\beta(1-\alpha)/(1-\alpha\beta)}} \equiv \varsigma_\theta v/n^\theta, \quad (18)$$

with the reduced form spillover parameter $\theta \equiv \beta(1-\alpha)/(1-\alpha\beta)$ contained in the interval $(0, 1)$ and $\varsigma_\theta \equiv A\delta(\gamma/2\alpha\beta)^{\alpha\beta/(1-\alpha\beta)}$ a positive constant. The right hand side of (18) is exactly the reduced form for unit variable costs used in Østbye (2010).

2.4 Fixed costs

As mentioned in the introduction to Section 2, we cannot ignore fixed costs if the location choice is to remain non-trivial. Introducing fixed costs in the AJ framework is straightforward. The profit of firm k net of (sunk) R&D investment costs may be written

$$\pi_k = (p_k - C(X_k)) q_k - F(Y_k); C', F' < 0, \quad (19)$$

where $F(Y_k)$ are fixed costs that depend on effective fixed costs reducing R&D, Y_k . Introducing fixed costs have no consequence for the product market stage. In the R&D stage, we now need to specify the R&D cost function pertaining to the new type of R&D as well as the effective R&D function and how effective R&D and R&D costs affect total costs. The obvious approach is to assume the same kind of structure for both types of R&D. By analogy, fixed costs (not unit fixed costs), may be written as

$$Fv \frac{\delta(\gamma/2\alpha\beta)^{\alpha\beta/(1-\alpha\beta)}}{n^{\beta(1-\alpha)/(1-\alpha\beta)}} \equiv \varsigma_\phi v/n^\phi, \quad (20)$$

with the reduced form spillover parameter $\phi \equiv \beta(1-\alpha)/(1-\alpha\beta)$ contained in the interval $(0, 1)$ and $\varsigma_\phi \equiv F\delta(\gamma/2\alpha\beta)^{\alpha\beta/(1-\alpha\beta)}$ positive. There is a slight abuse of notation here, since the parameters δ , γ , α , and β must be interpreted as generic and may take different values in the expressions for unit variable costs and fixed costs. Unsurprisingly, the right hand side of (20) is the reduced form for fixed costs used in Østbye (2010).

2.5 Implications

In Østbye (2010), the firm density in the industry affects setup and operating costs. There is no explicit modelling of R&D. An exogenous shift in spillover

parameters lowers costs and allows more firms in equilibrium. But the effect is stronger in the core with more firms at the outset, than in the periphery. Hence, the spatial distribution is changed as well, leading to a reinforcement of the leading role played by the core.

In this Section, we have made explicit how R&D may affect costs so that the reduced form relationship between firm density and costs is vindicated. We have seen that a finite R&D equilibrium level necessitates $\alpha\beta < 1$ which is true for arbitrary positive $\alpha < 1$ if positive $\beta < 1$, and vice versa. Assuming $\alpha, \beta < 1$ for both types of spillovers implies that the reduced form spillover parameters $\theta, \phi < 1$. Hence, $\phi < 1$, that is shown in Østbye (2010) to be necessary and sufficient in order to avoid the "black-hole" (where all activity located in the core is the only general equilibrium) with fixed cost spillovers only, follows from the existence of a finite fixed cost reducing R&D equilibrium in the structural model and does not have to be imposed ad hoc. More generally, to avoid the "black-hole" under spillovers pertaining to fixed costs and variable costs, both θ and ϕ must be restricted (more for ϕ than θ). Finite R&D equilibria must exist for both types of R&D in the structural model for this to happen. Hence, existence of finite R&D equilibria is necessary (although not sufficient) when we allow for both types of spillovers.

Another important message to take back to the general equilibrium model is that the reduced form parameters ς_ϕ and ς_θ depend on α and β , as do θ and ϕ . Hence, comparative statics for changes in θ and ϕ , treating ς_ϕ and ς_θ as constant is not innocuous. Inference is not necessarily invalidated, but comparative statics predictions must be interpreted as conditional on the adjustment of the R&D cost parameters δ and γ so that ς_ϕ and ς_θ are kept constant. It may be worthwhile to redo comparative statics in terms of structural parameters to fully understand what is going on.

3 Microeconomic labour market framework

3.1 Extending the Combes and Duranton model

The contribution of the CD approach is to show how knowledge embodied in workers can provide the support for spillovers between firms and affect the spatial distribution of activity. While they show that poaching is not neutral from a location point of view, the framework analyses the strategic interaction of a duopoly, making it difficult to reconcile it with the monopolistic approach of New Economic Geography models. This is generalization to an n -firm framework is suggested in their conclusion, not only because of its intrinsic desirability, but also because the change in market power might modify the strength of the poaching interactions. A second extension suggested by CD is to integrate the R&D intensity decision within firm procurement on the factor market, as an initial stage prior to poaching from competitors. The aim of this section is therefore to extend the CD framework along these two lines. This will be shown to affect the predictions of the model; therefore it is important to briefly

reiterate its properties beforehand. The profit of a firm in their framework is given by:

$$\pi = pq - (\Lambda + c(\lambda^P) q) w - \int_0^{\theta^P} \omega^P(\theta) d\theta - \int_0^{\theta^R} \omega^R(\theta) d\theta \quad (21)$$

where the fixed requirement Λ is the exogenous number of strategic workers hired in the first stage to build up the firm-specific knowledge. $c(\lambda^P)$ is the marginal cost of production, which is a negative function of the number of workers poached from the competitor. Finally, the integral sums correspond to the additional strategic costs of bidding for poached and retained workers on the factor market. Workers are discriminated with respect to their type θ , which characterises the cost/benefit of poaching that particular worker.

The overall profit maximization in the duopoly occurs over several stages, which allow the game to be solved by backward induction. In the first stage, each firm chooses a location and sets up the firm by hiring Λ workers. In the second stage, firms interact on the factor market by attempting to poach workers from each other. Finally, firms produce and interact strategically on the goods market. The shift to a monopolistic competition structure implies two major departures from this duopoly framework, which affect both the functional form of the spillovers and the way the model solves. Importantly, however, the general motivation and spirit of the model is maintained.

The first important change is that it becomes possible to merge the worker type continuum used by CD with the variety continuum of the standard monopolistic competition models. In the duopoly approach, within each firm θ is a measure of worker characteristics which allows the calculation of the integral sums for the cost of poaching and retaining workers in (profit equation). In the monopolistic framework, it will be assumed that the Λ strategic workers are of a firm all possess same firm-specific knowledge, making them identical from the point of view of potential poachers. This means that the variety continuum of monopolistic competition doubles up as the knowledge/worker type continuum: each firm produces a distinct variety and has distinct knowledge. When a firm poaches a worker from another firm, both the net amount of knowledge obtained and the cost of poaching can be measured by the distance between the two firms on the variety continuum. Furthermore, as illustrated in Figure 2, if one assumes that the continuum is arranged on a ring rather than a segment, i.e. the 0th and nth varieties are identical, the knowledge distance from competitors becomes symmetric for all firms. It will be assumed throughout that the knowledge received from poaching a worker is a positive function of the distance on this continuum, in other words firms benefit from diversity. This simplification is central to integrating the CD framework with standard monopolistic models.

The second aspect to be considered in the shift away from the duopoly framework is the change in the strategic interaction. On the goods markets, the market power of firms is reduced. For the specific case of the Dixit and Stiglitz (1977) framework, this aspect actually disappears altogether, as the

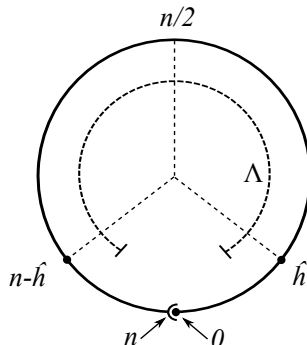


Figure 2: Knowledge distance and variety continuum

markup on marginal costs is typically constant. Neither the quantity of the producer nor its competitors affect the marginal revenue, as was the case in the framework of CD. The increase in the number of firms competing also affects the strategic interaction on the labour market. As outlined in the conclusion of CD, the increase in the number of varieties increases the opportunities for poaching, as more knowledge can potentially be extracted. Furthermore, the cost of poaching will also change: at the poaching stage workers in a given firm can now be targeted by more than one competitor, and workers within a firm will now also compete against each other for the bids received by poachers.

As a result of this, the analysis shifts from profit maximization to cost minimization, as the strategic interaction occurs only on the factor market, prior to production taking place. In some sense, the interaction between firms and workers and the resulting spillovers define the production technology that will be used, i.e. the fixed and variable requirements, and not the quantity to be produced in the end. The model of interaction on factor markets analysed below therefore develops a best-response cost-function that can be inserted at a later stage into the profit function for a standard NEG model. Finally, as the purpose of the extension is to integrate the CD within the monopolistic competition approach of New Economic Geography, the 1st stage no longer contains a location decision, and instead simply contains an endogenous R&D intensity. The location of firms is resolved as part of the long run adjustment to zero-profits of the NEG framework through firm entry or exit. In the discussion below, firms are assumed to interact only on their local labour markets.

In the following the notation will be kept similar to CD for ease of comparison. Given an exogenous and unknown quantity of output q , firms will attempt to minimize the following cost function:

$$C = (\Lambda + c(\Lambda, \lambda^P, \lambda^R) q) w + \int_0^{\lambda^P} \omega^P(h) dh + \int_0^{\lambda^R} \omega^R(h) dh \quad (22)$$

There are two differences with the CD cost structure in equation (21) for the duopoly. The first is the domain of integration for the cost of poaching and retaining, which is defined by the continuum of varieties, as mentioned above. The second is that the marginal cost of production now also depends on the total number of first stage workers Λ and the number of retained workers λ^R . These assumptions are required in order to allow for an endogenous R&D decision, as well as maintain an incentive for firms to retain workers. In the original framework, firms incur no productivity cost if a worker is poached and leaves. However, because the poached knowledge improves the productivity of the competitor, this does reduce the revenue of the firm through the strategic interaction on the goods market. Thus, even though the firm does not incur a production cost when a worker is poached, it does incur a strategic one, giving it an incentive to try and retain workers. As stated above, in the monopolistic setting the strategic interaction of the good markets vanishes, therefore there is no incentive to retain workers unless firms directly incur a higher cost production of production when a worker is poached.

The sequence of stages in the cost-minimisation decision mirrors those of the original CD framework: in the first stage firms hire Λ strategic workers and accumulate their knowledge, thus modifying *ex ante* their marginal requirements. In the second stage, firms target workers in competing firms in an attempt to poach them and reduce production costs through the knowledge acquired. Finally, in the third stage the workers in each firm compete for the bids, the equilibrium poaching rent is found and successfully poached workers change firms. At all stages output q is assumed to be given, and the final cost function becomes the best-response for that level of production. As for the duopoly framework, the stages can be solved by backward induction.

3.2 Solving the extended model

Stage 3: Workers compete for the bids

Let $\{\omega^P(h)\}$ be the set of bids received by the Λ workers of a target firm h^* .³ The number of bids received is determined in stage 2, and will depend on the distance between the poaching firm and the target firm, and therefore on the amount of net knowledge the poacher receives if successful. Because the strategic workers within a firm possess the same knowledge, poachers are indifferent to which of the workers they hire. Therefore, workers can use reverse bidding against each other to try and obtain for themselves the contract with the highest bid, offered by the $(n/2)^{\text{th}}$ variety. The result of this reverse auction is that the strategic rents paid to poached workers equalise at the level of lowest bid the Λ workers have received, up to a term $\gamma(h)$ which is the cost of transferring the worker across firms. Two solutions are possible, depending on which side of the market is short:

³It is assumed here that $\omega^P(h) \geq \omega^R(h^*)$, i.e. the marginal benefit of a worker to the poacher is always higher than the marginal benefit to its own firm.

$$\begin{cases} \omega^P(h) = \omega^R(h^*) + \gamma(h) \forall h & \text{if } \lambda^R(h^*) > 0 \\ \omega^P(h) = \min\{\omega^P(h)\} + \gamma(h) \forall h & \text{if } \lambda^R(h^*) = 0 \end{cases} \quad (23)$$

If the firm is able to retain $\lambda^R(h^*) > 0$ workers at the strategic rent $\omega^R(h^*)$, i.e. the market is short on bids, these workers will ensure that leaving the firm are not paid more than $\omega^R(h^*)$: should a poached worker try to raise his premium above that level, any of the λ^R retained workers can immediately undercut him. If on the other hand the market is short on workers, firms will receive more bids than there are workers available, and all workers will be poached. Again, should a poached worker try to raise his premium above the lowest bid received, any worker can undercut him. Competition between workers implies that poaching firms will be able to extract surplus from the factor market, as it will end up paying poached workers below its willingness to pay. This is similar to the result of CD, however this now occurs through reverse bidding of workers rather than direct strategic interaction.

Stage 2: Firms submit their initial bids on the labour market

In the second stage firms bid for the workers of competitors, which determines the number of poached and retained workers. As in CD, this is done by determining the willingness to pay of a firm for workers, obtained by taking the first order condition of the cost function with respect to poached and retained types.

$$\begin{cases} \left. \frac{\partial C}{\partial h} \right|_{h \in \lambda^P} = qw \frac{\partial c(\Lambda, \lambda^P, \lambda^R)}{\partial h} \bigg|_{h \in \lambda^P} + \omega^P(h) = 0 \\ \left. \frac{\partial C}{\partial h} \right|_{h \in \lambda^R} = qw \frac{\partial c(\Lambda, \lambda^P, \lambda^R)}{\partial h} \bigg|_{h \in \lambda^R} + \omega^R(h) = 0 \end{cases} \quad (24)$$

This confirms that the willingness to pay, which determines the initial bid of a firm, is equal to the marginal cost reduction brought by a worker, both for retained and poached workers. Given the result of the previous stage, the critical distance \hat{h} which defines the point at which will not bid for workers is when the cost reduction brought by a poached worker is equal to the marginal cost reduction of a retained worker up to the transfer cost term $\gamma(h)$. Equating these two terms allows the critical distance to be found, and therefore the number of workers that will be poached and retained. As for stage 3, there are two solutions depending on which side of the market is short.

$$\begin{cases} \text{if } \Lambda > n - 2\hat{h} & \text{if } \Lambda < n - 2\hat{h} \\ \left\{ \begin{array}{l} \lambda^P = n - 2\hat{h} \\ \lambda^R = \Lambda - \lambda^P \end{array} \right. & \left\{ \begin{array}{l} \lambda^P = \Lambda \\ \lambda^R = 0 \end{array} \right. \end{cases} \quad (25)$$

Replacing $\lambda^P(\Lambda, n)$ and $\lambda^R(\Lambda, n)$, as well as the values for $\omega^P(h)$ and $\omega^R(h)$ into (22) gives the following cost function:

$$C = (\Lambda + (c(\Lambda, n) + c_2(\Lambda, n))q)w \quad (26)$$

Where c_2 , which is the overall strategic cost of poaching, depends on the strategic rents determined in stage 3:

$$\begin{cases} c_2(\Lambda, n) = qw\Lambda \times \frac{\partial c(\Lambda, n)}{\partial h} \Big|_{h \in \lambda^R} & \text{if } \lambda^R > 0 \\ c_2(\Lambda, n) = qw\Lambda \times \min \left(\frac{\partial c(\Lambda, n)}{\partial h} \Big|_{h \in \lambda^P} \right) & \text{if } \lambda^R = 0 \end{cases} \quad (27)$$

Stage 1: Firms setup and choose R&D intensity

In the first stage, firms choose the number of strategic workers that in order to minimise costs. It is important to point out that at this point that each firm takes as given the poaching behaviour of stage 2. This gives the following first-order condition, where intuitively the increase in the fixed and strategic labour requirements must be balanced out by the gain in the variable requirement.

$$\frac{\partial C}{\partial \Lambda} = 1 + \frac{\partial c(\Lambda, n)}{\partial \Lambda} q + \frac{\partial c_2(\Lambda, n)}{\partial \Lambda} q = 0 \quad (28)$$

3.3 Example: Additive spillovers

As one can see from the previous discussion, in order to obtain more specific results for stages 1 and 2, a functional form has to be provided for the marginal cost requirement $c(\Lambda, \lambda^P, \lambda^R)$. In order to provide an example, we choose the following additive functional form, similar to that chosen by CD, where the two integral sums show the reductions in marginal requirements brought respectively by retained and poached workers.

$$c(\Lambda, \lambda^P, \lambda^R) = \bar{c}(\Lambda) - \int_0^{\lambda^R} adh - \int_0^{\lambda^P} bhdh \quad (29)$$

The structure of the cost reductions in the integral sums is illustrated in Figure 3. As a simplification, it is assumed that the marginal cost reductions are linear functions of the distance. While this departs somewhat from the circular structure of Figure 2, the general implication remain the same. The bh marginal reduction brought by poached workers is net of the transfer cost, which is assumed to be $\gamma(h) = a \quad \forall h$ as a further simplification.

The stage 2 result is given by:

$$\begin{cases} \frac{\partial C}{\partial h} \Big|_{h \in \lambda^P} = qwbh + \omega^P(h) = 0 \\ \frac{\partial C}{\partial h} \Big|_{h \in \lambda^R} = qwa + \omega^R(h) = 0 \end{cases} \quad (30)$$

As explained above, the stage 3 result is that in the presence of retained workers $\omega^P(h) = \omega^R(h^*)$. Replacing the first order conditions above one can solve for the critical distance, $\hat{h} = a/b$. One then immediately obtains the volumes of workers poached and retained using (25). Replacing these in the

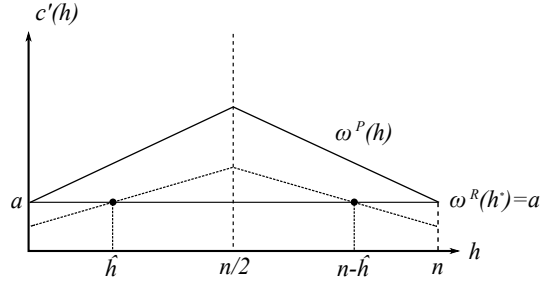


Figure 3: Marginal productivity on the distance continuum

cost function (22), one obtains the stage 2 results. Two results are possible, depending on whether firms manage to retain workers. Both cases are illustrated in Figure 4. The first case occurs if $\lambda^R > 0$ and $\omega^R(h) = a$:

$$C = \left(\Lambda + \left(\bar{c}(\Lambda) - \frac{b}{2} \left(\frac{n}{2} - \hat{h} \right)^2 \right) q \right) w \quad (31)$$

The second case occurs if firms are unable to retain any workers and $\lambda^R = 0$. One can see that in this case, the lowest bid accepted by a poached worker is issued from the firm at distance $(n - \Lambda)/2$. Taking into account the higher strategic cost in the integral sum, the cost function becomes:

$$C = \left(\Lambda + \left(\bar{c}(\Lambda) - \frac{b}{2} \left(\frac{\Lambda}{2} \right)^2 \right) q \right) w \quad (32)$$

This is in fact the same functional form as equation (31), allowing for the fact that the number of poached workers is Λ and not $n - 2\hat{h}$. Finally, the first stage result is given by the first order condition of (31) and (32) with respect to the fixed requirement Λ . In this simple example, one can see that there is no interaction between the *ex ante* variable requirement Λ and the spillovers, leading to a straightforward result, which is identical in both cases. Assuming for instance that the variable requirement is inversely related to the initial R&D outlay, $\bar{c}(\Lambda) = \bar{c}\Lambda^{-1}$, one obtains for the both cases:⁴

$$\frac{\partial C}{\partial \Lambda} = 1 - \frac{\bar{c}}{\Lambda^2} q = 0$$

$$\Lambda = \sqrt{\bar{c}q} \quad (33)$$

⁴For the case of equation (32), one does not differentiate the Λ^2 term. This is because the term corresponds to productivity gains resulting from poaching from competitors. Hence, unilaterally increasing or reducing the fixed requirement of a firm keeping requirements of competitors constant will not affect this term.

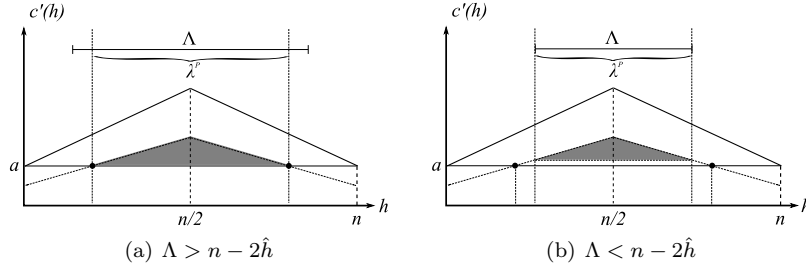


Figure 4: Poaching intensity and worker surplus extracted

4 Discussion

In this paper we have explored micro foundations for knowledge spillovers in spatial general equilibrium models. Two different strands have been followed.

Building on the partial equilibrium IO literature on R&D spillovers, effective R&D and explicit R&D investment have been introduced in a Dixit-Stiglitz (1977) framework with inter-regional linkages through horizontal trade and migration and intra-regional vertical trade. This structural model has been compared to a reduced form where R&D is not explicit and spillovers depend on firm density (Østbye, 2010). The analysis suggests that the standard IO framework must be modified in order to obtain an explicit general equilibrium solution. The most obvious candidates for change is effective R&D and the R&D cost function.

Instead of purely additive spillovers coming as "manna from heaven" we have considered spillovers where own and foreign R&D enter multiplicatively so that own R&D is essential for effective R&D as argued by Kamien and Zang (2000). In other words, the firm needs absorptive capacity obtained through own R&D in order to benefit from spillovers.

Instead of R&D costs that implies, implausibly, that any total R&D investment level generates more cost reductions if allocated to n labs rather than spent all in one lab, we should consider modifications to avoid this. Following Amir et al. (2008), we have slightly changed the standard cost function by assuming positive marginal cost even for zero R&D, which is sufficient to escape the implausible implication. Another alternative, not considered here, is to introduce fixed R&D costs.⁵

Through these two modifications of the standard structural IO model, that already have been suggested as improvements of the structural model in the literature, we have demonstrated that the general equilibrium model in (Østbye, 2010) is consistent with the structural model. We have also made visible relationships between structural and reduced form parameters that are useful in

⁵Fixed R&D costs could possibly replace fixed setup costs in production in order to preserve costly production fragmentation in the model, necessary to make location choices non-trivial according to the 'folk theorem of spatial economics'.

order to get a more comprehensive understanding of how comparative statics must be interpreted in the general equilibrium model and how the model may be modified in order to do comparative statics in terms of structural parameters. This is useful since a change in a structural parameter may affect several reduced form parameters, basically invalidating the *ceteris paribus* assumption underlying the reduced form comparative statics predictions.

The second strand is based on the framework by Combes and Duranton (2006) on labour pooling and poaching. We find that the strategic interaction between firms (through poaching and pooling) on the labour market can be carried across from a duopoly to monopolistic competition. This can be done by merging the worker continuum used by CD with the standard variety continuum of monopolistic competition workers. The main result in terms of specification is that for a sufficiently large number of firms, there are own cost reductions ($\partial mC/\partial n < 0$) such that agglomeration of firms in a region result in lower production costs. Barde (2010) identifies these as the critical condition for location decisions.

With additive spillovers of the form chosen by CD and explored here, the resulting functional form of the cost function obtained is nevertheless very different from the one assumed in Barde (2010). First of all, costs are not inversely related to the number of firms and secondly, decay of the spillovers over space is absent because firms are assumed to poach only within their region of location. However, it should be possible to reduce the difference between the Barde (2010) specification and the one obtained here without major changes to the framework of analysis. Further work could use multiplicative rather than additive spillovers in equation (29), and introduce a geographical segmentation, where firms can poach in foreign markets, but at an added cost which depends on a distance measure.

Recent work on oligopoly in general equilibrium suggest that some aspect of strategic interaction may be preserved in general equilibrium (see Neary, 2003). If strategic interaction could also here somehow be included without compromising tractability, this would certainly strengthen the bridge between partial equilibrium analysis of knowledge spillovers in the presence of strategic interaction and spatial general equilibrium analysis. We leave this open for future research.

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