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Seemingly Unrelated Regressions with Spatial Effects. An Application to the Case of the European Regional Employment

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Seemingly Unrelated Regressions with Spatial Effects. An Application to the Case of the European Regional Employment

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Abstract

The seemingly unrelated regressions (SUR) equations are a traditional multivariate econometric formulation employed in very different fields including, obviously, spatial analysis. The basis of the approach is very well known due to the initial works of Zellner (1962), Theil (1971), Malinvaud (1970), Schmidt (1976) and Dwivedi and Srivastava (1978). Almost every econometric textbook includes a discussion of the SUR methodology, which is available in most popular econometric software packages. It hardly requires any further justification. However, in a spatial setting, this discussion is almost the same as described by Anselin (1988). It should be acknowledged that the current situation is not very satisfactory, as stated by LeGallo and Dall'Erba (2006).

In this paper, we address the case of a SUR model that involves spatial effects, under the configuration of a given number of equations, G , a finite number of cross-sections, T , and a large number of spatial units, R . The problem that we pose is testing for the presence of spatial effects, as in Mur and López (2009), and to select the most adequate spatial model for the data, as in Mur et al. (2010). Following these papers, we also assume a maximum-likelihood framework that facilitates the obtaining of simple Lagrange Multipliers, with good behaviour in small-sized samples.

We present an application of these techniques to the case of the Spanish wages, at NUTS II level and disaggregated by sectors of activity, for the period 1998 to 2009. We specify a flexible SUR model where each equation corresponds to a sector of activity, allowing for a different spatial autocorrelation parameter for each equation. Flexibility is gained combining the spatial SUR specification with panel data technique, in order to take into account any spatial heterogeneity underlying in the data, as well as the effect of any relevant unobservable variable. This latter option has been selected by the data and also offers the expected magnitude and sign of all the estimated parameters.

1. Introduction

The Seemingly Unrelated Regressions (SUR) equations are a traditional multivariate econometric formulation employed in very different fields including, obviously, spatial analysis. The basis of the approach is very well known due to the initial works of Zellner (1962), Theil (1971), Malinvaud (1970), Schmidt (1976) and Dwivedi and Srivastava (1978). Almost every econometric textbook includes a discussion of the SUR methodology, which is available in most popular econometric software packages. It hardly requires any further justification. However, in a spatial setting, this discussion is almost the same as described by Anselin (1988). This author proposed the term of spatial SUR for models that '*consists of an equation for each time period which is estimated for a cross-section of spatial units*'. That is, these models deal with a set of R spatial units observed through a total of T time period (RT observations). In each of these equations (cross-sections, in fact), some spatial elements may be introduced in either the form of mechanisms of intra-equation spatial error autocorrelation, or a spatially lagged dependent variable or both. Rey and Montouri (1999), Fingleton (2001, 2007), Egger and Pfaffermayr (2004), Moscone et al. (2007), Lauridsen et al. (2008) or LeGallo and Chasco (2008) have used this technique on different occasions.

The original design of Anselin's spatial SUR models has recently further developed by Mur and Lopez (2009). These authors have tested for the presence of spatial effects in the SUR specification and confront the question of identifying the type of spatial process most adequate to the data. As usual for SUR model, the spatial SUR specification is justified if there is a significant correlation between the error terms of the different time equation. That is, if the temporal correlation matrix is not diagonal. Otherwise, the spatial SUR models simplifies in a set of (unrelated) cross-sections.

In the same lines, Mur and López (2010) extend the original design of SUR models for accounting for more than one equation. That is, they deal with a set of R spatial units, observed through a total of T time period (RT observations), but not only regarding one equation, as before, but G equations. In this new context, they developed a set of Lagrange Multiplier tests to be used within a strategy of model selection. From an empirical point of view, specification developed in Mur and López (2010) is really interesting, since it lets developing many economic problems partially treated so far. For instance, in the case of demand functions, we could use Anselin's original spatial SUR model to analyze the evolution in time of the demand for an aggregated products (food, for instance) for a set of

spatial units. However, specification by Mur and López (2010) let analyse data on evolution of the demand of those spatial units for several types of food, which is the base of the traditional SUR specification in demand analysis (Deaton and Muellbauer, 1980; Bewley, 1986; Bewley and Young, 1987). The same justification applies to several other macro-economic analyses. For instance, in the case of labour analyses, variables such as wages, productivities or employment/unemployment rates are commonly analyzed for the different economic activity sector, but through the estimation of independent regressions (Brülhart and Mathys, 2008; Aláez et al., 2003). In our opinion, all these types of analyses could benefit from previous methodological proposal.

One important characteristic of the spatial SUR models considered so far is the existence of a limited heterogeneity among the individuals. The solution in literature to the cases that it would necessary to introduce more flexibility, allowing for some individual heterogeneity, has consisted on specifying a spatial panel data model (Anselin et al, 2007, Baltagi, 2008, Elhorst, 2003, 2005, 2008).

Although both direction of researches, spatial SUR model and panel data models, are perfectly well defined and useful, we can guess some further developed when combining both of them. In this sense, in this paper we propose how to combine both types of methodological approaches in order to better explain a set of data. More precisely, in this paper we explain the structure of wage/per hour and worker in the five main economic activity sectors (agriculture, energy and manufacturing, construction, market services and non-market services). As explicative variables, we use figures on each sector productivity and unemployment rate for total economy figures. The SUR specification lets us jointly treat the wages of the different sectors, and by this way, we take into account the possible correlation of the error terms of the equations. Furthermore, making use of panel data techniques we are considering the spatial heterogeneity of data or, in other words, taking account the effect of certain unobservable variables that are not considered in the analysis (as it could be the human capital).

The paper is developed as follows. In the following section, we analyse the methodological basis of the paper. The third section is devoted to the data and the presentation of the main results. Finally, the last section offers the main conclusion of this paper as well as the future line of research.

2. Methodology

2.1 The different alternative models

i) The SUR-SARAR model

Let's start with the definition of the SUR-SARAR model defined in Mur and López (2010):

$$\begin{aligned}
 \left. \begin{aligned}
 y_{gt} &= \lambda_g \mathbf{W} y_{gt} + x_{gt} \beta_g + u_{gt} & \Rightarrow \mathbf{A}_g y_{gt} &= x_{gt} \beta_g + u_{gt} \\
 u_{gt} &= \rho_g \mathbf{W} u_{gt} + \varepsilon_{gt} & \Rightarrow \mathbf{B}_g u_{gt} &= \varepsilon_{gt}
 \end{aligned} \right\} \\
 E[\varepsilon_{gt}] &= 0 & E[\varepsilon_{gt} \varepsilon_{ht}'] &= \sigma_{gh} I_R \\
 \mathbf{A}_g &= I_R - \lambda_g \mathbf{W} & \mathbf{B}_g &= I_R - \rho_g \mathbf{W} \\
 g &= 1, 2, \dots, G; & t &= 1, 2, \dots, T
 \end{aligned} \tag{1}$$

Where y_{gt} , u_{gt} and ε_{gt} are $(R \times 1)$ vectors, x_{gt} is a matrix of exogenous variables of order $(R \times k_g)$, β_g is a vector of parameters of order $(k_g \times 1)$, λ_g and ρ_g are two scalars, I_R is the identity matrix of order $(R \times R)$ and \mathbf{W} is the known weighting matrices of order $(R \times R)$.

In order to considerate the unobserved heterogeneity, a fixed effect SUR-SARMA model can be defined as follows:

$$\begin{aligned}
 \left. \begin{aligned}
 y_{gt} &= \lambda_g \mathbf{W} y_{gt} + x_{gt} \beta_g + \alpha_g + u_{gt} & \Rightarrow \mathbf{A}_g y_{gt} &= x_{gt} \beta_g + \alpha_g + u_{gt} \\
 u_{gt} &= \rho_g \mathbf{W} u_{gt} + \varepsilon_{gt} & \Rightarrow \mathbf{B}_g u_{gt} &= \varepsilon_{gt}
 \end{aligned} \right\} \\
 \alpha_g &= [\alpha_{g1}, \alpha_{g2}, \dots, \alpha_{gR}]'
 \end{aligned} \tag{2}$$

Where the new term α_g capture the spatial unit specific effects that vary from equations but are invariant in time, and it is equivalent to introduce a dummy variable for each spatial unit in each equation. As said before, they take into account the heterogeneity among spatial units. In fact, these terms represent the effect of space-specific time-invariant unobserved omitted variables, difficult to measure or hard to obtain.

The easier way to estimate the fixed effect SUR-SARAR model gathered in (2) comes from applying maximum likelihood techniques designed for the SUR-SARAR model, but considering all variables of the model in deviation from its average over time (demeaning), following the expression:

$$\tilde{z}_{gtr} = z_{gtr} - \frac{\sum_{t=1}^T z_{gtr}}{T} \quad (g = 1, 2, \dots, G; t = 1, 2, \dots, T; r = 1, 2, \dots, R) \quad (3)$$

for $z = y, x$,

ML technique will offer estimation for the slope coefficient β (without the intercept), but not for the intercept and the variables α_g . However, they can be recovered thereafter (Baltagi 2005).

When demeaning equation (2), we obtain a model that, in compact terms, can be expressed as follows:

$$\left. \begin{array}{l} \mathbf{A}\tilde{\mathbf{y}} = \tilde{\mathbf{X}}\boldsymbol{\beta} + \tilde{\mathbf{u}} \\ \mathbf{B}\tilde{\mathbf{u}} = \boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon} \sim \mathbf{N}(0, \boldsymbol{\Omega}) \end{array} \right\}$$

$$\tilde{\mathbf{y}} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \dots \\ \tilde{y}_T \end{bmatrix}_{TGR \times 1}; \quad \tilde{\mathbf{X}} = \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \dots \\ \tilde{X}_T \end{bmatrix}_{TGR \times k}; \quad \tilde{\mathbf{X}}_t = \begin{bmatrix} \tilde{x}_{1t} & 0 & \dots & 0 \\ 0 & \tilde{x}_{2t} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \tilde{x}_{Gt} \end{bmatrix}_{TGR \times k}; \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_G \end{bmatrix}_{k \times 1}; \quad \tilde{\mathbf{u}} = \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \dots \\ \tilde{u}_T \end{bmatrix}_{TGR \times 1}; \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_T \end{bmatrix}_{TGR \times 1} \quad (4)$$

Where $k = \sum_{g=1}^G k_g$; $\mathbf{A} = \mathbf{I}_T \otimes [\mathbf{I}_{GR} - \boldsymbol{\Lambda} \otimes \mathbf{W}]$ and $\mathbf{B} = \mathbf{I}_T \otimes [\mathbf{I}_{GR} - \boldsymbol{\Upsilon} \otimes \mathbf{W}]$, $\boldsymbol{\Lambda}$ (respectively $\boldsymbol{\Upsilon}$) is an $(G \times G)$ diagonal matrix with the parameters λ_g (respectively ρ_g) and \otimes is the Kronecker product. Moreover $\boldsymbol{\Omega} = \mathbf{I}_T \otimes \boldsymbol{\Sigma}_\varepsilon \otimes \mathbf{I}_R$ where $\boldsymbol{\Sigma}_\varepsilon = [\sigma_{\varepsilon ij}; i, j = 1, 2, \dots, G]$ is a $G \times G$ matrix. We assume normality in the error terms.

It is important to consider the followings remarks: i) We order the sampling information, first, temporarily; then, we sort each cross section by equation and, finally, by individuals; ii) we assume that the parameters of spatial dependence ($\lambda_g, \rho_g, g=1, 2, \dots, G$) are also constant in time but that may vary between the different equations.; and iii) the SUR effect is due to the fact that the same individual (the spatial unit) decides, simultaneously, about G different problems (equations).

The logarithm of the likelihood function of the SUR-SARAR of (4) is the following:

$$l(y; \theta) = -\frac{RTG}{2} \ln(2\pi) - \frac{RT}{2} \ln|\Sigma| + T \left[\sum_{g=1}^G \ln|\mathbf{B}_g| + \sum_{g=1}^G \ln|\mathbf{A}_g| \right] - \frac{(\tilde{\mathbf{A}}\tilde{\mathbf{y}} - \tilde{\mathbf{X}}\tilde{\beta})' \mathbf{B}' \Omega^{-1} \mathbf{B} (\tilde{\mathbf{A}}\tilde{\mathbf{y}} - \tilde{\mathbf{X}}\tilde{\beta})}{2} \quad (5)$$

where $\theta' = [\beta'; \lambda_1; \dots; \lambda_G; \rho_1; \dots; \rho_G; \sigma_{ij}]$ is the vector of parameters, of order $(k+2G+G(G+1)/2) \times 1$ and $\Omega^{-1} = \mathbf{I}_T \otimes \Sigma^{-1} \otimes \mathbf{I}_R$.

ii) The SUR-SLM model

The SUR-SLM (Spatial Lag Model) model is a particular case of the SARAR model, and it is obtained when $\rho_g = 0 \quad \forall g$. In compact terms, and for the fixed effect models, it can be expressed as follows:

$$\left. \begin{array}{l} \tilde{\mathbf{A}}\tilde{\mathbf{y}} = \tilde{\mathbf{X}}\tilde{\beta} + \varepsilon \\ \varepsilon \sim \mathbf{N}(0, \Omega) \end{array} \right\} \quad (6)$$

iii) The SUR-SEM model

The SUR-SEM (Spatial Error Model) model is obtained from the SARAR model, and it is obtained when $\lambda_g = 0 \quad \forall g$. In compact terms, and for the fixed effect models, it can be expressed as follows:

$$\left. \begin{array}{l} \tilde{\mathbf{y}} = \tilde{\mathbf{X}}\tilde{\beta} + \tilde{\mathbf{u}} \\ \mathbf{B}\tilde{\mathbf{u}} = \varepsilon \\ \varepsilon \sim \mathbf{N}(0, \Omega) \end{array} \right\} \quad (7)$$

2.2 Model selection strategy

Among several alternatives, we will use the following strategy testing, which belong to what it is denoted as from a Specific to General strategy. Firstly, we test the null hypothesis of absence of spatial effects in the SUR model by testing the null of:

$$H_0: \lambda_g = \rho_g = 0 \quad (\forall g) \quad \text{vs} \quad H_A: \text{No } H_0 \quad (8)$$

The expression for testing this hypothesis is expressed in Mur and López (2010), it is denoted as $\mathbf{LM}_{\text{SARAR}}^{\text{SUR}}$ and asymptotically follows a $\chi^2(2G)$. At this stage, we have two possibilities. If the null hypothesis is not rejected, the selection process concludes and we select a SUR model without spatial effect [what can be called as SUR-SIM, SUR-Spatial Independent Model]. On the contrary, the procedure goes on in order to decide which spatial SUR model underlines the data.

Secondly, we have to select a model for explaining our data set. We can make use of the following statistics:

- a) For testing the null hypothesis of no spatial effects in the SLM model, that is, $H_0: \lambda_g = 0 (\forall g)$ vs $H_A: \text{No } H_0$, we can make use of the Lagrange Multiplier denoted by $\mathbf{LM}_{\text{SLM}}^{\text{SUR}}$, as well as its robust version to misspecification errors in the alternative denoted by $\mathbf{LM}_{\text{SLM}}^{*\text{SUR}}$. Both statistic are asymptotically distributed as $\chi^2(G)$.
- b) For testing the null hypothesis of no spatial effects in the SEM model, that is, $H_0: \rho_g = 0 (\forall g)$ vs $H_A: \text{No } H_0$, we can make use of the raw and robust Lagrange Multiplier statistics denoted by $\mathbf{LM}_{\text{SEM}}^{\text{SUR}}$ and $\mathbf{LM}_{\text{SEM}}^{*\text{SUR}}$, also asymptotically distributed as $\chi^2(G)$.

At this stage, and mainly trusting on robust statistics, four different situations can be considered:

Situation 1: $\mathbf{LM}_{\text{SEM}}^{*\text{SUR}}$ is significant but $\mathbf{LM}_{\text{SLM}}^{*\text{SUR}}$ is not statistically significant. The model appears to be a SEM. We confirm this identification by means of the corresponding likelihood ratio.

Situation 2: $\mathbf{LM}_{\text{SLM}}^{*\text{SUR}}$ is significant but $\mathbf{LM}_{\text{SEM}}^{*\text{SUR}}$ is not statistically significant. The model appears to be a SLM. As before, we confirm the selection by means of the corresponding likelihood ratio.

Situation 3: Both robust Multipliers reject their respective null hypotheses. In this case, we have to make use of the marginal Multipliers denoted by denoted by $\mathbf{LM}_{\text{SEM}}^{\text{SUR}}(\rho|\lambda)$ and $\mathbf{LM}_{\text{SEM}}^{\text{SUR}}(\rho|\lambda)$. The first one let testing the null of $\rho_g = 0 (\forall g)$ conditioned on the presence of the SLM parameters, while the second one let testing the null of $\rho_g = 0 (\forall g)$, conditioned on the presence of the SLM parameters. The strategy will be as follows:

- $\mathbf{LM}_{\text{SLM}}^{\text{SUR}}(\lambda/\rho)$ and $\mathbf{LM}_{\text{SEM}}^{\text{SUR}}(\rho/\lambda)$ are statistically significant; we select a SARAR model.
- Only $\mathbf{LM}_{\text{SEM}}^{\text{SUR}}(\rho/\lambda)$ is statistically significant; we select a SEM model.
- Only $\mathbf{LM}_{\text{SLM}}^{\text{SUR}}(\lambda/\rho)$ is statistically significant; we select a SLM model.

Situation 4: None of the two null hypotheses can be rejected with the robust Multipliers. We conclude that it is a SIM model.

In Figure 4 we present a sketch of this strategy to solve the problem of selecting the most appropriate specification for a SUR model.

(Insert Figure 4)

3. Data and estimation results

In the application that follows, we use data on wage/per worker and hour (measured in €), productivity (measured in €) and unemployment rate (in percentages), for each of the 19 Spanish regions (NUTS II administrative spatial unit in terms of Eurostat). The data for the three variables are gathered, for the period 1998 to 2009 (T=12), from the Cambridge Database. The spatial distribution of the variables in the five economic activity sectors is displayed in Figures 1 to 3.

(Insert Figures 1 to 3)

From figures, we can appreciate certain patten of spatial autocorrelation, however, we will test this issue in the following lines.

The functional form selected to explain our variable has been an inverse relationship between wage and productivity but a lineal relation between wages and unemployment rate. That is while y_{gt} represents the wage in economic sector g in period t and x_{gt} is a vector composed by the inverse of productivity in sector g and the unemployment rate in economy, both in period t ($\frac{1}{\text{Productivity}_{gt}}$, unemployment rate $_{gt}$).

We estimate a SUR without considering spatial heterogeneity. Results are gathered in Table 1. The first block of results in Table 1 shows the estimated parameters of a non-spatial SUR model (SUR-SIM). The use of these estimated parameters together with the score function and information matrices of the spatial SUR model let us calculating the corresponding Lagrange Multiplier tests to test for any type of spatial autocorrelation. As shown in Table 1, non-spatial SUR model suffer from misspecification, but it is not clear the alternative model, since all set of LM tests are significant. As explained above, since both robust Multipliers reject their respective null hypotheses, we have to make use of the marginal Multipliers, $\mathbf{LM}_{SEM}^{SUR}(\rho|\lambda)$ and

$LM_{SEM}^{SUR}(\rho|\lambda)$. In our case, since only the $LM_{SEM}^{SUR}(\rho/\lambda)$ is statistically significant we obtain empirical evidence in favor of the SEM model.

(Insert Table 1)

Analogous results are obtained if we derive the same types of models extended by also considering the spatial heterogeneity of data. Results are shown in Table 2. As deduced from displayed results, the SUR-SEM model is also the specification that outperforms the rest of the models.

(Insert Table 2)

Finally, the selection among the two set of SUR-SEM models can be carried out through a Likelihood Ratio (LR) tests for comparing SUR-SEM models from Tables 1 and 2. The consequence LR test takes a value of 1137.2, which is higher than the corresponding critical value at the 5% level of significance, $\chi_{0.05}^2(95) = 118.75$. That is, the fixed effect SUR-SEM model appears to be the best option.

The sign of the estimated parameters are as expected. Productivity positively affects wages in all economic sectors, although the effect is not homogeneous among them. As shown in Table 2, the highest effect of productivity on wages corresponds to the market and non-market service sectors. As regards, the unemployment rate effect, we observed in Table 2 that although the sign is positive, as expected, it is not significant at the 5% level of significant. This can be due to the fact that we are considering the unemployment rate of total economy while it would be better to introduce this variable desegregated by the activity sectors (as well as the productivity variable). Finally, the parameter of spatial autocorrelation appears to be stronger in the cases of services and construction sectors.

4. Concluding remarks

In this paper we have made use of recent methodological advances in the field of spatial SUR models. The proper testing and selection between alternative spatial SUR models was an important gap in the literature so far. We complete proposed SUR models by the capture the remaining spatial heterogeneity of the data, making use of what we called the fixed effect SUR models. As far as we know, this is the first empirical application on this direction. Results indicate that fixed effect SUR spatial models are the preferred specifications for explaining the determination of wage in the

different economic sector in Spain. The sign and magnitude of estimated parameters are as expected.

Further development of this research would be directed thought the improvement of the data base used, since for instance it would be more advisable to use unemployment rate in each economic sector as explicative variable for the respective regression (instead of the total economic unemployment rate used in this analyses). The unavailability of other interested variables, such as human capital, has been minimized through the use of fixed effect models. However, a better set of data would be of interest in the very short term.

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Figure 1. Wage per worker and hour in the different sectors in 2009 (€)

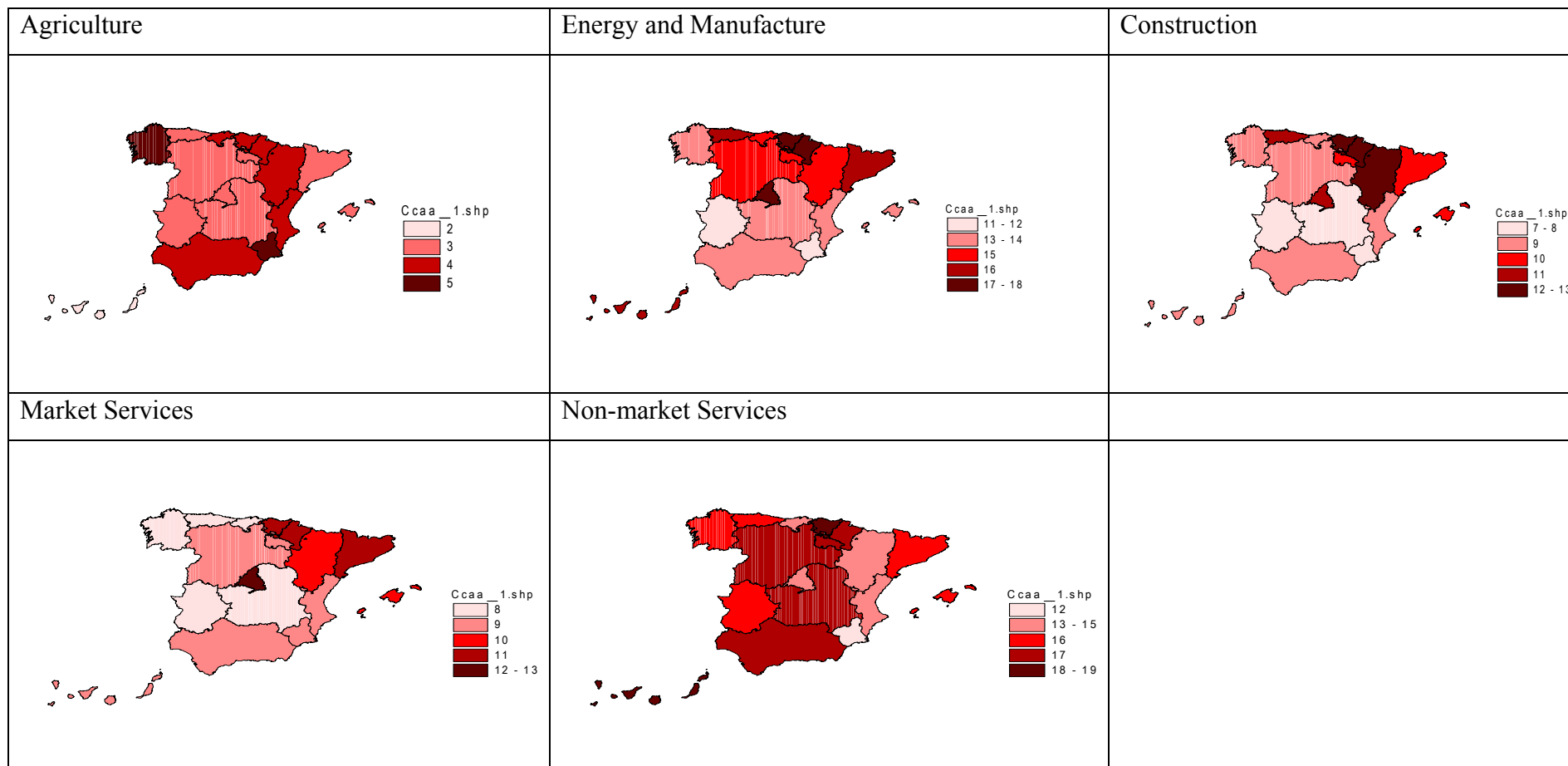


Figure 2. Productivity in the different sectors in 2009(miles of €)

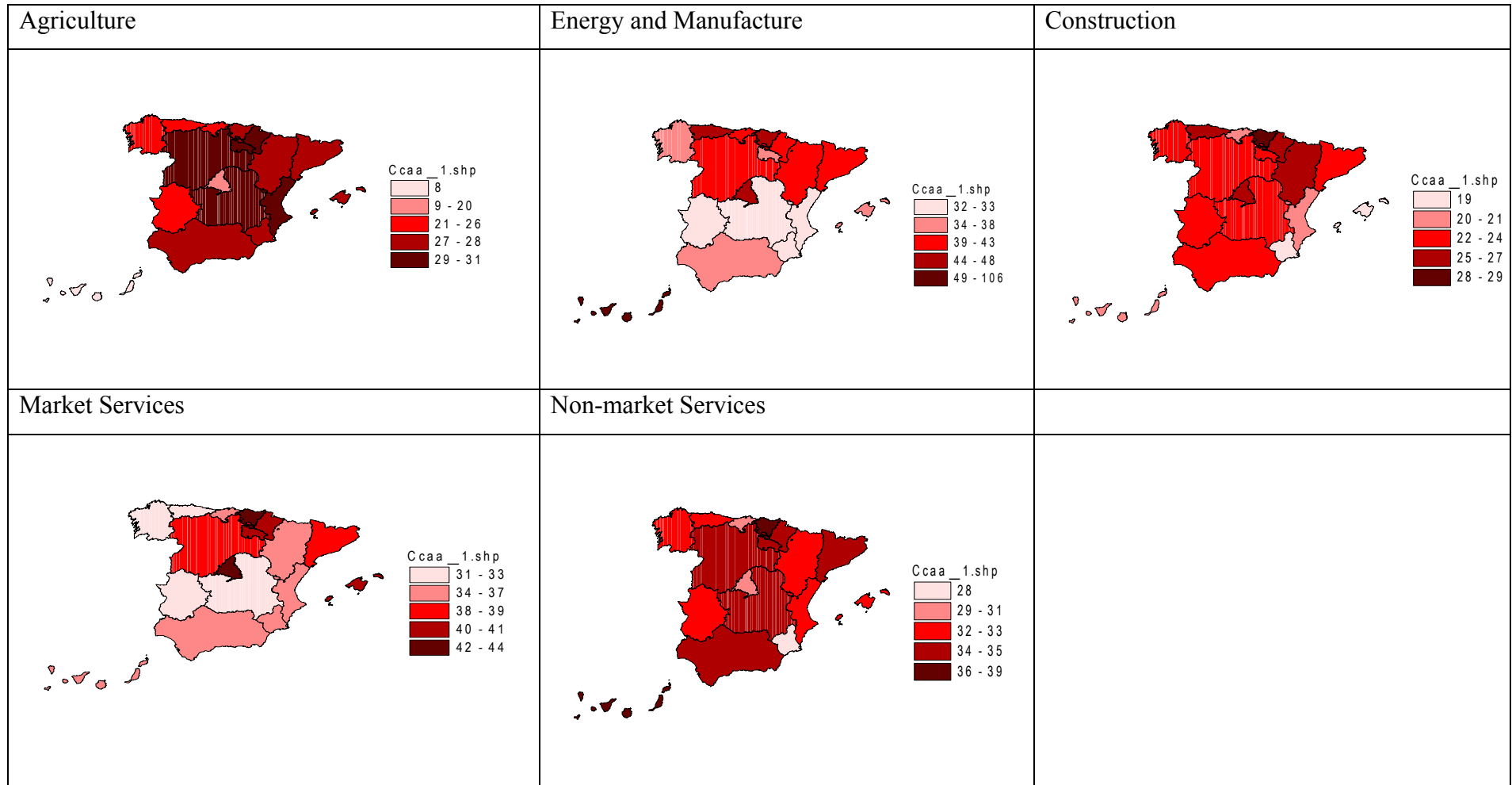


Figure 3. Unemployment rate in global economy in 2009

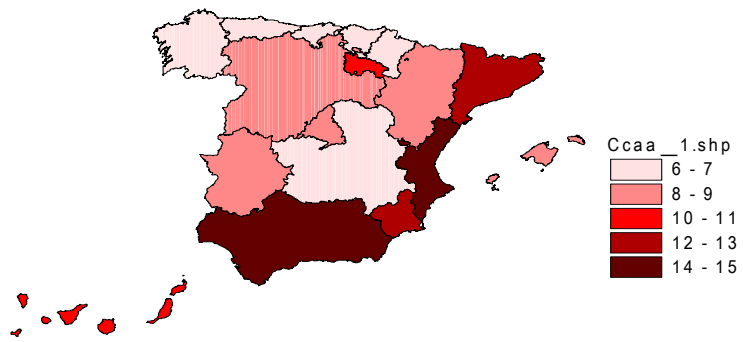
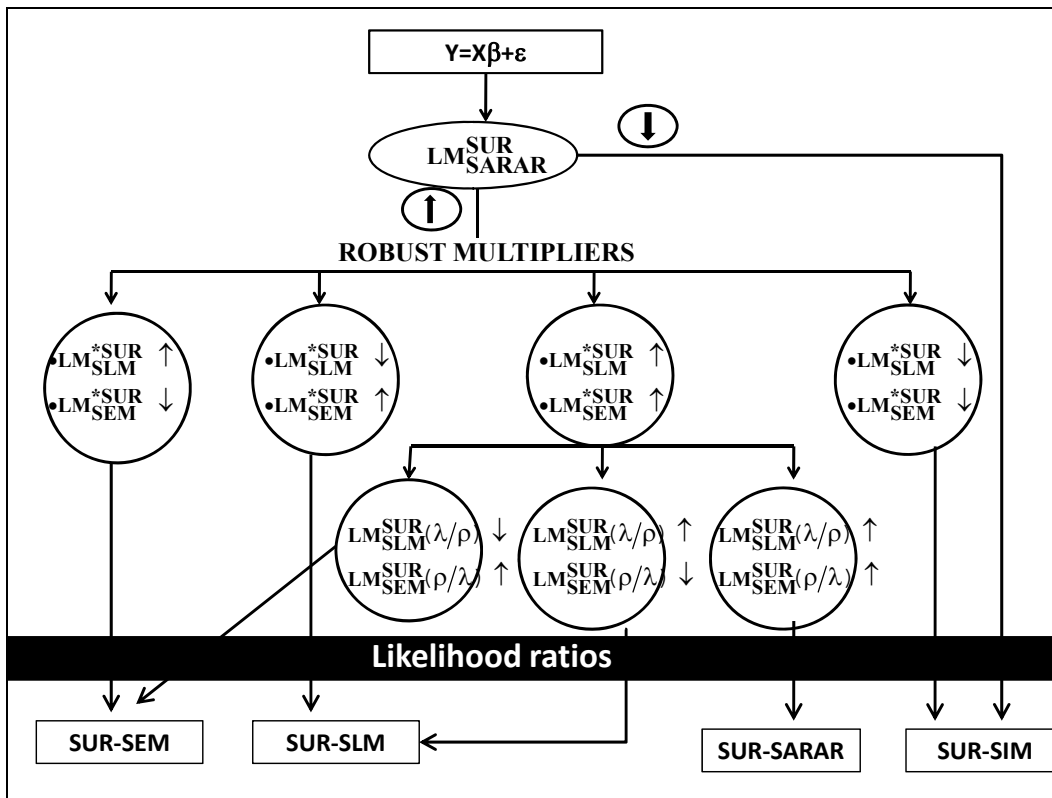


FIGURE 4: A Stge algorithm to spatial SUR model selection* .



* \uparrow Reject Null Hypothesis ; \downarrow Non Reject Null Hypothesis

Table 1: Model estimation for explaining the wage/per hour in the different activity sectors, without considering spatial heterogeneity ^(a)

	Agriculture	Energy and Manufacturing	Construction	Market Services	Non-market Services
SUR MODEL WITHOUT SPATIAL EFFECT					
Intercept	0.0027*	0.0151*	0.0120*	0.0158*	0.0268*
$\frac{1}{\text{Productivity}}$	0.0001*	-0.1280*	-0.0543*	-0.2375*	-0.3732*
Unemployment rate	-0.0001	-0.0001*	-0.0001*	-0.0001*	-0.0002*
LM_{SARAR}^{SUR}	318.70*				
LM_{SLM}^{SUR}	164.25*				
LM_{SLM}^{*SUR}	88.64*				
LM_{SEM}^{SUR}	162.42*				
LM_{SEM}^{*SUR}	86.47*				
SUR – SLM MODEL					
Intercept	0.0017*	0.0107*	0.0052*	0.0098*	0.0170*
$\frac{1}{\text{Productivity}}$	-0.0041*	-0.0792*	-0.0349*	-0.2450*	-0.3572*
Unemployment rate	0.0000	-0.0001*	0.0000*	-0.0001*	0.0000*
λ	0.3759*	0.2557*	0.5985*	0.6563*	0.6297*
$LM_{SEM}^{SUR}(\rho \lambda)$	33.35*				
Log-ver	-6937.13				
SUR – SEM MODEL					
Intercept	0.0024*	0.0132*	0.0108*	0.0157*	0.0240*
$\frac{1}{\text{Productivity}}$	-0.0046*	-0.0801*	-0.0575*	-0.2930*	-0.3469*
Unemployment rate	0.0000	0.0000	0.0000	0.0000	0.0000
ρ	0.5589*	0.2679*	0.6555*	0.7297*	0.8683*
$LM_{SLM}^{SUR}(\lambda \rho)$	6.004				
Log-ver	-6925.53				

(a) An asterisk means that the corresponding null hypothesis is rejected at the 5% level of significance.

Table 2: Model estimation for explaining the wage/per hour in the different activity sectors, considering spatial heterogeneity ^(a)

	Agriculture	Energy and Manufacturing	Construction	Market Services	Non-market Services
SUR MODEL WITHOUT SPATIAL EFFECT					
$\frac{1}{\text{Productivity}}$	0.0126	-1.1211	0.0207	0.0616	-0.0365
Unemployment rate	0.0000	0.0000	-0.0000	-0.0000	0.0000
LM_{SARAR}^{SUR}	445.41*				
LM_{SLM}^{SUR}	430.15*				
LM_{SLM}^{*SUR}	49.70*				
LM_{SEM}^{SUR}	356.08*				
LM_{SEM}^{*SUR}	24.37*				
SUR – SLM MODEL					
$\frac{1}{\text{Productivity}}$	0.0111*	-0.8329*	0.0262*	0.0377*	-0.0096
Unemployment rate	0.0000	0.0000	0.0000	0.0000	0.0000*
λ	0.6854*	0.8195*	0.7128*	0.9097*	0.9372*
$LM_{SEM}^{SUR}(\rho \lambda)$	31.48*				
Log-ver	-6374.64				
SUR – SEM MODEL					
$\frac{1}{\text{Productivity}}$	0.0083*	-0.7361*	0.0260*	0.0314*	-0.0032
Unemployment rate	0.0000*	0.0000	0.0000	0.0000	0.0000*
ρ	0.7038*	0.8830*	0.7127*	0.9143*	0.9404*
$LM_{SLM}^{SUR}(\lambda \rho)$	10.37				
Log-ver	-6356.93				

(a) An asterisk means that the corresponding null hypothesis is rejected at the 5% level of significance.