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Does Zipf's Law Hold for Primate Cities? Some Evidence from a Discriminant Analysis of World Countries

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Abstract

Using data for some 177 worldwide countries, we investigate the degree to which the largest cities of countries 'outstand' in the national context, by leading in population size their nearest 'competitors.' According to our findings, high development levels are likely to reduce the gap in population sizes between the first and second city of the country, while the first city being the national capital is likely to widen the gap between it and its 'nearest neighbor' in the national city-size hierarchy.

Keywords: Main cities of countries; population size; 'frontrunner' approach; geographic and development attributes.

Introduction

Any country of the world has a city that dominates the national urban system. In some cases, such cities are only fractionally larger than their 'nearest neighbors' in the national city-size hierarchies. However, more often, they surpass the population size of their nearest 'competitors' manifold. Irrespectively of their actual population sizes, which may vary by country, these places are absolute winners in the national competitions for potential investors and migrants. As a result, better understanding of the factors behind their success may contribute to more informed development policies, aimed at encouraging urban growth wherever this objective is desirable.

As in the case of a successful marathon runner, who can estimate his leading position in the race in relation to his nearest competitor, a similar 'frontrunner' approach can be used to investigate the leading status of cities in the national urban systems. In particular, in the present study, we use this approach to by analyze the gap in the population sizes between the first and the second largest cities for a relatively large ensemble of 177 countries across the globe.

Several clarifications are important. By referring to a 'main' city, urban scientists and geographers often understand any large city above a certain population threshold, e.g., 500,000 or 1,000,000 residents. In this sense, a country (especially a large one) may have *several* cities of this type, with the very notion of main city being functional. In particular, a city of the same size may be considered 'main' in a small country like

Holland or Norway, and a medium-size city, or even a large town, in a more populous nation, such as India or China.

The definition of the main city used throughout this paper is different: by using this term we refer to a populated place that is *absolutely largest in the national urban system*. Each country is thus supposed to have *only one city* of this type, with its population size varying, being potentially smaller in small countries and larger in large ones.

In our view, this approach has an advantage: by keeping the city's leading status fixed, and measuring the *ratio between its population size and the size of its nearest competitor*, we can investigate the effects of various factors, including population size, local development levels, and geographic location, on the degree of primacy the main city enjoys in the national city-size hierarchy.¹

Two specific questions we attempt to answer in this paper are as follows: *Are there any distinctive patterns in the distribution of the values of the ratio measuring the size of the population gap between the first and second largest cities of countries? What are the main factors influencing this ratio in different countries across the globe?*

The rest of the paper is organized as follows: It starts with a brief discussion of the general patterns of city-size distribution worldwide and development attributes which may influence it. Discussion of the results of the study concludes the analysis.

General patterns of the city-size distribution

Since the pioneering works of Zipf (1941, 1949), research on city size distributions (CSDs) has been un-subsidizing. According to Zipf himself, who proposed the universality of his famous law, if all cities of a particular country are sorted by decreasing sizes and ranked accordingly, the size S_R of the city of rank R is given to:

$$S_R = S_M / R, \quad (1)$$

where S_M is the size of the largest city with rank equal to 1.

However, Zipf was aware that *not* all systems of cities obey his law and considered three different types of settlement sub-systems. The first case is in full conformity with his Law, the second one is given to:

$$S_R = S_M / R^k, \quad (2)$$

where k is an exponent different from 1, and the third case includes all other situations. Zipf called the first case "homogeneous" and two others - "inhomogeneous" (Zipf, 1941).

As shown by numerous empirical studies carried out to date (see *inter alia* Laherrere and Sornette, 1998), (2) is a good approximation for many countries (with k varying between 0.7 and 1.3), while other functional relationships between S and R have been also reported in the CSD literature.

However, in articles published on the topic, it is seldom noted that the largest cities of the national CSDs, do not normally follow the Zipf law, generally obeyed by the rest of the ensemble. Instead there seems to be a unique case of division of the ensemble into two parts: the "primate cities" and the rest. For example, in the case of the two major European countries – France and Germany (see Fig. 1), -- the largest cities of these

¹ The leading position of individual cities in the national urban systems is not necessarily fixed, with some places potentially exhibiting upward or downward mobility over time. However, we restrict our analysis to the recent distributional patterns only, as further detailed in the methodology section, mainly due to restrictions on availability and comparability of country-level data.

countries (with ranks between 1 and 3, $\text{Ln}(\text{Rank})=0\div 1$, marked by red circles on the diagrams), clearly ‘depart’ from the trend line formed by the rest of the ensemble.

<<< Figure 1 about here >>>

Factors influencing the leading status of cities

In an early paper, the American geographer Mark Jefferson (1939) termed the largest cities of nations the ‘primate cities,’ attributing their success to sheer population size that becomes a strong pull factor. This simple explanation is, in fact, supported by more recent urban growth theories. Thus, according to the ‘neo-cultural’ growth approach, which emphasizes the role of ‘second-nature’ factors, such as cultural diversity, human capital, innovation, and creativity (see *inter alia* Florida 2002; Glaeser and Saiz, 2003; Markusen, 2006; Storper and Scott, 2009; Florida and Mellander, 2009), most successful cities of nations are those which capitalize on these ‘human capital’ factors, irrespectively of their location, history or physical attributes. Similar explanation is advocated by the ‘endogenous growth theory,’ which postulates that cities develop because of the diversity of the production factors they host, and that urban success (or lack thereof) has little to do with processes or circumstances outside the cities themselves (Jacobs 1969; Henderson 1974; Henderson *et al.* 1995; Glaeser *et al.* 1992).

In contrast, location is a fundamental concept underlying most early studies in urban geography, which emphasized the role of transport costs, distance ‘friction,’ commuting limits, and geographic barriers to trade (von Thünen, 1826; Christaller, 1933; Lösch, 1938; Isard, 1956; Beckmann, 1968; Duranton, 1999). According to this concept, cities succeed because of the advantages of their location, such as access to infrastructures, local natural resources, climatic advantages and spatial relationships with other urban places in the region (see *inter alia* Portnov and Schwartz, 2008; 2009). However, in recent urban debate, the effect of location on urban performance more often emerges as neither obvious nor straightforward. Thus, the proponents of a ‘non-spatial’ approach to urban development deny that individual urban locations may have any ‘natural growth advantages.’ According to them, as people of similar backgrounds, incomes and environmental preferences ‘flock together,’ location differences emerge (Gotlieb 1996; Hess, 2004).

The worldview behind this approach is that societal processes of repulsion and attraction, and individual drives to utility maximization are, rather than physical location per se, the real causes of urban success. Furthermore, major forces shaping economies in since the last half of the twentieth century – such as, globalization, the dominance of service industries, information technologies, innovation, network societies, etc., – are deemed by many to be *aspatial*, i.e. not strongly linked to particular places or regions, and tend to affect development worldwide (Sassen 1997; 2000). That point of view is essentially shared by the ‘new economic geography’, which assumes that concentration and de-concentration forces generate multiple equilibria, which may exist simultaneously in several *loci* of geographic space (Krugman 1993; 1995; Fujita et al. 2001; Fujita and Mori, 1995, 2005; Behrens and Thisse, 2006).

According to a recent study by Portnov and Schwartz (2009), the appearance that these ‘new-age’ forces ‘dwarfed’ traditional location attributes (such as distances and transport costs), whose role in development used to be crucial, may be misleading. In their view, this impression may be due to the fact that no location attribute, taken alone, may be sufficient to affect the socio-economic performance of a populated place. Only an

accumulation of location attributes, from which no essential ingredient is missing (favorable environmental conditions, physical infrastructures, employment and cultural opportunities in a wider region, etc.) may 'seal' the future performance of an urban place, by exposing it to a 'package' of location advantages. Viewing urban location as a 'package of attributes,' whose individual components can either enhance or detract from each other, is, in their view, key to understanding how urban location works.

Research methodology

Data sources

Data for the present analysis were drawn from two main sources:

- a) The geographic location and population data on the main cities of countries were drawn from the *Geonames* Database (2008), which covers some 83,000 populated places worldwide;
- b) The development attributes of countries were obtained from the geo-referenced database, maintained by ESRI (2007) and containing several development indicators of world-wide counties, such as land area, population size, per capita income levels, etc.

Statistical analysis

The analysis was carried out in two phases. First, we started with a general histogram analysis of the ratio between the population sizes of the main cities of countries. Next, the ratio between the size of the first largest city and the size of the second largest city of each country, called hereon the *B*-ratio was used to measure of the degree to which the main city 'outstands' in its national context. This analysis is performed for 177 countries across the world, for which complete and comparable development data were available, using discriminant analysis tools. The discriminant analysis is a multivariate statistical technique, which attempts to determine the best linear combination of explanatory variables (called "predictors") to differentiate between groups of cases into which the dependent variable is subdivided. An advantage of the discriminant analysis technique, compared to e.g., regression analysis, is that makes it possible to reveal some *common characteristics* of the cases forming subgroups into which the values of the dependent variable are split (Cacoullous, 1973; Huberty and Olejnik, 2006).

Explanatory variables

In order to explain why some countries may exhibit large *B*-ratios, while in other countries these ratios may be medium or small, the following predictors (or "explanatory variables") were used:

- Population size of a country (ln), '000 residents;
- Country's land area (ln), km²;
- Per capita GDP (ln), \$US;
- Percent urban population;
- Capital city (a dichotomous variable which takes on value 1, if the 1st city of a country is national capital, and 0 otherwise),
- Landlocked country (1 for landlocked countries and 0 for countries with sea or ocean access).

Each of these predictors may be important in its own right, as the following brief comments demonstrate. Thus, for instance, large and populous countries are likely to

exhibit, *ceteris paribus*, smaller *B*-ratios due to a fact that in such countries several urban centers may develop simultaneously to satisfy demand for employment and services of local residents in different, often remote regions. NYC and LA in the USA may be good examples of this trend. Concurrently, in small countries, such a demand can be “serviced” by a single large population centre, while other places may be considerably smaller in size. The country’s wealth and percent urbanization may work in the same direction: as a country becomes wealthier and its settlement system matures, more cities are likely to succeed alongside with the largest one. To distinguish between urbanization patterns on different continents, we also included four additional binary variables for America, Asia, Africa and Europe, which took on value 1 if a country is located on a given continent and 0 otherwise.

The inclusion of these variables in the analysis thus makes the variable set (though restricted, due to data availability, to a relatively small number of explanatory variables) fairly parsimonious. The essential descriptive statistics of the research variables are reported in Appendix 1.

Research results

Histogram analysis

Figure 2 shows the histogram of the *B*-ratio [(second city size) / (first city size)] for the 177 world countries in our sample.² One may remark on two things. First, the values of the ratio between the population size of the second and first cities (*B*-ratio) vary broadly: from 0.05 (5%) for Liberia to 0.90-0.94 (90-94%) for Lichtenstein and Kuwait. The second observation is that the histogram lacks any definite shape. In particular, It has several peaks and exhibits several minima for $B \approx 0.2\div 0.3$ and $0.6\div 0.7$ (see Fig. 2), - which could be indicative of a possible division of the countries into several sub-groups.

<<< Figure 2 about here >>>

This observation is confirmed by a more formal analysis of the *B*-ratio distribution, carried out BY the Jenks’ natural breaks method. This classification method determines natural groupings inherent in the data and helps to identify break points by picking the class breaks that group similar values and maximize the differences between classes. As a result, the observations are divided into classes (groups) with relatively big jumps in the data values (ESRI, 2007). The Jenks analysis generally confirms the impression we got from the visual examination of the *B*-ratio histogram: the countries under study indeed appear have two distinctive distributional breaks: $B=0.6$ and $B=0.3$.

The first group is composed of 35 countries, in which primate cities have similar sizes (B -ratio > 0.6). The second group ($0.6 > B$ -ratio > 0.3) consists of 63 countries, which can be defined as countries with the *B*-ratio expected under the Zipf Law ($k = 0.7 \div 1.3$). Lastly, the third group includes 79 countries for which the second and first cities are substantially different in size and the [2nd city size/1st city size] is small (B -ratio < 0.3). Notably, in the 1st group (B -ratio > 0.6) there are only four European countries: Cyprus, Lithuania, Montenegro, and the Netherlands. Most of the European countries are in the 2nd and 3rd group ($B < 0.6$). With the exception of Canada, Australia and New

² Our analysis covers all the countries of the world in which both of the two primate cities are larger than 1,000 residents.

Zealand and the four abovementioned European countries, all the developed countries of the world are also in the 2nd and the 3rd group.

Discriminant analysis

As previously mentioned, in the present study, we used the discriminant analysis to determine *common attributes of countries*, which principal cities exhibit small, average and large ratios between population sizes of their largest cities. For this purpose, we split the countries in our sample into three sub-groups, defined by Jenks' natural break method (see the previous subsection on the histogram analysis):

- Group 1: B-ratio > 0.6 (1st and 2nd cities of a country are of similar size);
- Group 2: $0.6 > \text{B-ratio} > 0.3$ (average difference between 1st and 2nd cities generally expected under Zip's law), and
- Group 3: B-ratio < 0.3 (1st and 2nd cities of a country are substantially different in size).

In order not to overload the reader with unnecessary technical details, we shall report in this section, only general results, while moving all the numerical results and explanations into Appendices 1 & 2.

As the discriminant analysis indicates, the gap in the population size between the 1st and 2nd major cities of a country is largely attributed to the following five factors: development level (as measured by the national per capita GDP); percent urbanization; the capital status of the 1st city; the country's population size, and its land area (see Appendix 2).

In particular, it may be said that large (land-wise), poorly developed and sparsely populated countries in which the 1st city is the national capital are likely to exhibit large differences of the [2nd city size/1st city size] ratio. In contrast, in heavily populated and highly urbanized, high-income countries, 1st and 2nd cities tend to be of similar size.

The geographic location of countries also appears to play a role: "normal" B-ratios (i.e., generally corresponding to Zipf's law) are most likely to be found in Europe, while poorly developed and thinly urbanized countries located elsewhere are more likely to tend towards more "extreme" B-ratios.

Discussion

As development differentials between densely populated metropolitan areas and peripheral regions are increasing, overcoming inequalities in socio-economic development has become a key issue for urban and regional planners worldwide (Mera, 1995; Puga, 1999; Felsenstein and Portnov, 2005). In many sparsely populated peripheral regions, the inhabitants are often denied access to social amenities, which are available in denser populated regions (Portnov and Schwartz, 2009). As the population of a community increases, it crosses the threshold for higher-level services, and starts offering richer opportunities for employment, education and leisure. In this respect, knowledge about the factors that may affect the development of urban areas may have important policy implications. For instance, it may guide regional development policies aimed at enhancing urban growth in priority development areas.

According to the results of our analysis of 177 world countries, the values of the ratio between the population size of the second and the first largest city (B) of countries appears to vary broadly: from 0.05 for Liberia to 0.90-0.94 for Lichtenstein and Kuwait.

Our second observation is that the histogram of the B-ratio distribution exhibits two distinctive breaks – $B=0.6$ and $B=0.3$. The first group is composed of 35 countries, in which primate cities have similar sizes ($B\text{-ratio} > 0.6$). The second group ($0.6 > B\text{-ratio} > 0.3$) consists of 63 countries, which can be defined as countries with average B-ratios ($0.7 \div 1.3$). Lastly, the third group includes 79 countries for which the second and first cities are substantially different in size and the [2^{nd} city size/ 1^{st} city size] is small ($B\text{-ratio} < 0.3$).

In an attempt to understand what factors “drive” countries into one of these categories, the discriminant analysis was run. According to its results, countries with small-to-medium B-ratios tend to be *wealthier* and *more urbanized* than the rest of the ensemble. Concurrently, a country, in which its largest city functions as *the national capital*, and especially in *landlocked countries*, is likely to exhibit a large gap between the major city and its “nearest neighbor” in the national CSD.

The explanation for these trends may be fairly straightforward. The importance of concentration of the central power in the formation of cities is well known to urban scholars for centuries. Thus, in his *Al-Muqaddimah* (or Introduction to History), the XIVth century Arab scholar Ibn Khaldūn noted that all the major cities tended to be the *capital cities* of their respective nations, in which some sort of “growth causation” was set in motion: the concentration of power and wealth, attributed to taxes collected by king’s court, lead to the spatial concentration of wealth, elevated demand for goods and services, and their supply which readily followed. Artisans and craftsmen, traders and scientists settled near these *foci* of wealth and power, where scholars and intellectuals could enjoy the company of other scholars, and where artisans could learn from other artisans and thus improve their skills. More recently, this “urban causation” process was theorized by Weber (1921), and captured by more recent urban and regional development theories, ranging from Myrdal’s “cumulative causation” to “endogenous growth theory,” “learning cities” and “new economic geography.” Most of these theories emphasize the accumulation of knowledge and skills as well as the role of interpersonal contacts in the formation of cities.

In contrast, the wealth of economy and large shares of urban population may work in the opposite direction, effectively minimizing the gap in the population sizes of primate cities: as a country becomes wealthier and its settlement system matures, more cities are likely to succeed alongside the “largest” one. It is thus unsurprising that the largest deviations from the Zipf law, as measured in this study by the gap in the size of the first and second cities, are observed in “low resource” countries – Liberia, Barbados, Eritrea, etc. The same process may take place on a regional scale as well: even in developed nations, there are sparsely developed regions in which major cities are few and surpass greatly in size their local “counterparts.”

If a country is landlocked, it may have substantial economic and development implications as well, partly because the role of port functions and associated industries on the formation of major cities is well known. A country with no such functions provides fewer opportunities for urban growth which may thus affect the gap in the size of its primate cities.

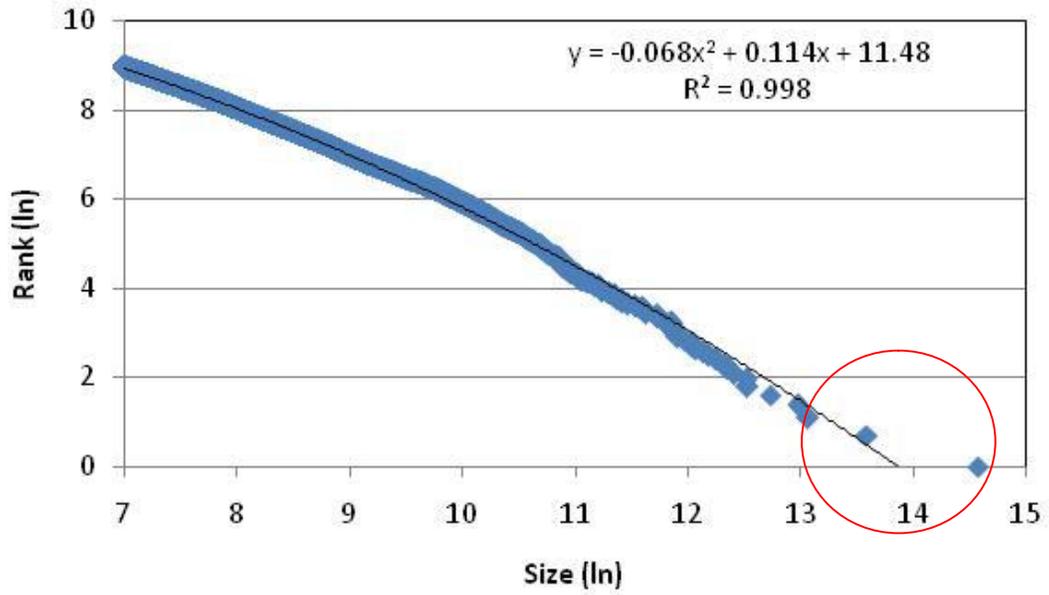
Finally we should note that the selection of observation units (either municipalities or built contiguities or metropolitan areas) is a critical consideration for analysis. Each of these units has its own advantages and disadvantages. Thus, a metropolitan area is

generally assumed to function as a whole and may be considered as an integrated labor market, formed by individual municipalities, connected by commuting flows. However, such large aggregate units (spreading in some cases up to 100-150 km from the central city) are likely to suffer from internal heterogeneity, stemming from the diversity of their population and uneven development patterns. Moreover, the definition of built contiguities and metropolitan areas differ by country, which is a serious constrain for a comparative analysis. In the present analysis, we opted for municipalities as primary units, as they constitute the smallest territorial units for which comparable population growth data for most world countries are available. Follow up studies may thus be needed to determine whether using built contiguity and metropolitan zones for the analysis may lead to different results.

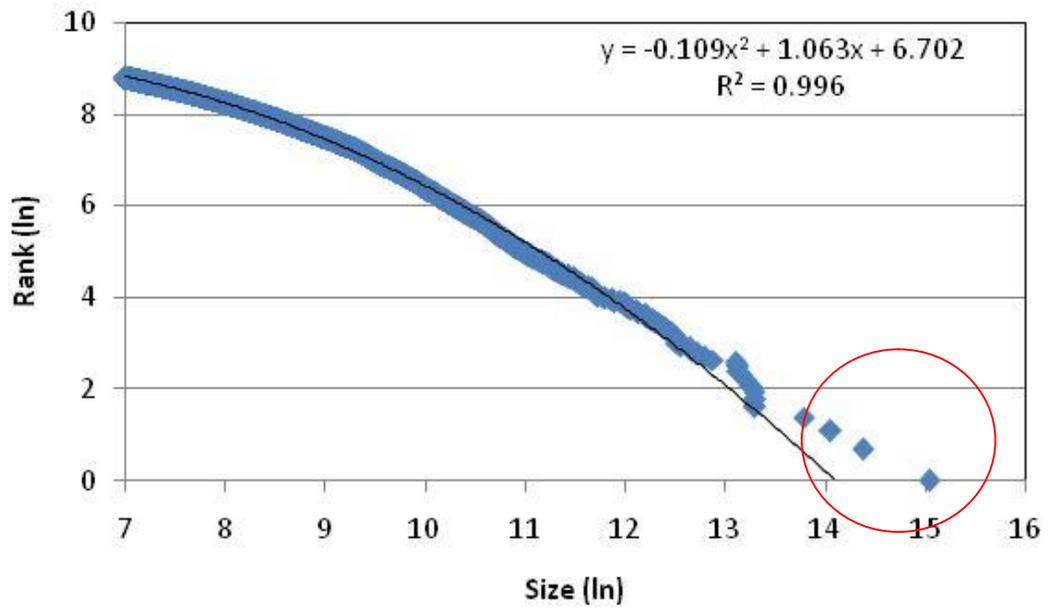
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A



B

Figure 1: Rank-size distribution of settlements with 1,000+ residents in France (A) and Germany (B)

Note: Diagrammed using data from the Geonames database (2008)

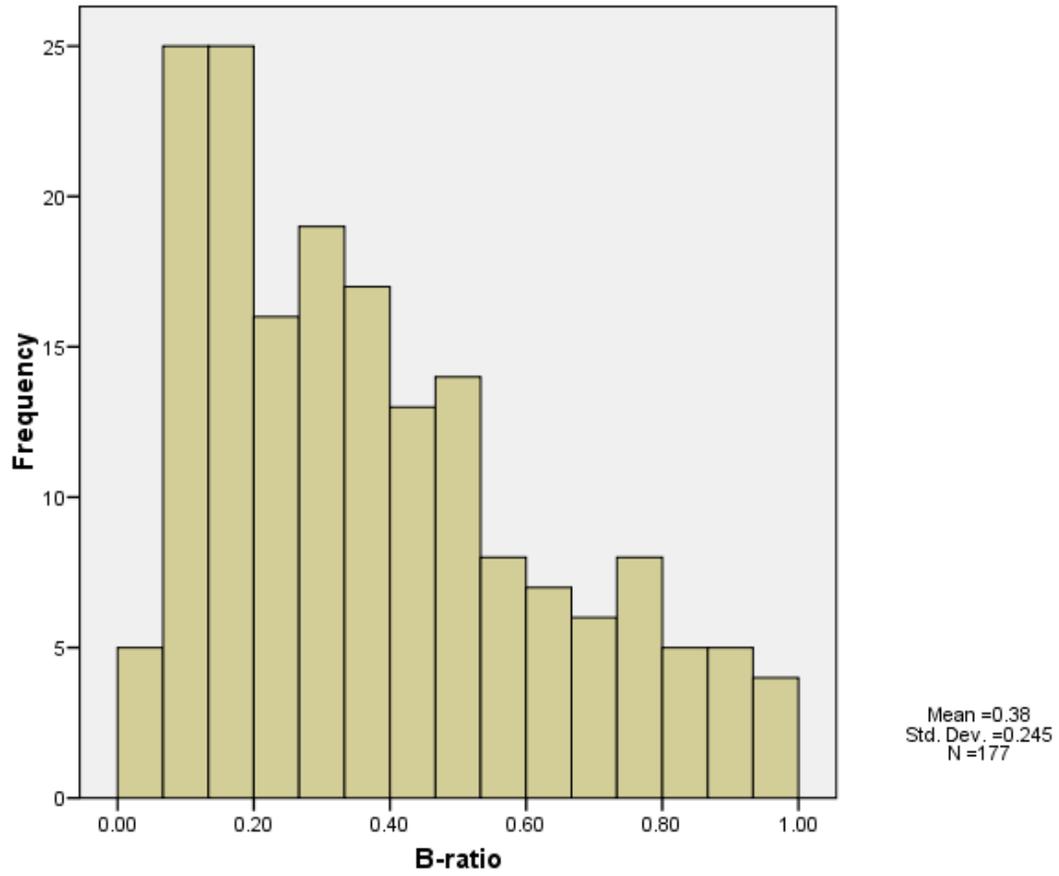


Figure 2: Histogram of world countries according to their [second city size]/[first city size] ratio

Appendix 1

General description of the discriminant function and research variables

A. Discriminant function

In the discriminant analysis, the linear combinations of predictors separating between individual values of the dependent variable are called “discriminant functions” and are defined as follows:

$$d_{ik} = b_{0k} + b_{1k}x_{i1} + \dots + b_{pk}x_{ip}, \quad (3)$$

where d_{ik} is the value of the k_{th} discriminant function for the i_{th} case, p is the number of predictors, b_{jk} is the value of the j_{th} coefficient of the k_{th} function, and x_{ij} is the value of the i_{th} case of the j_{th} predictor (SPSS, 2009).

Since our dependent variable has three different levels (that is, groups of countries with small, medium and large difference between 1st and 2nd cities), we thus expect to have two different discriminant functions, differentiating between Group 1 and 2 and between Group 2 and 3.

B. Research variables

Variable	Description	Range of values (min/max)	Comment
B-ratio	Dependent variable	1 (Group 1) 2 (Group 2) 3 (Group 3)	2 nd and 1 st cities are of similar size (B-ratio > 0.6) Medium difference in the population size of the 2 nd and 1 st cities (0.6 > B-ratio > 0.3) Large difference in the population size of the 2 nd and 1 st cities (B-ratio < 0.3).
Population size of a country (ln), '000 residents	Predictor	8.83/20.97	
Country's land area (ln), km ²	-“-	2.94/16.64	
Per capita GDP (ln), \$US	-“-	6.14/10.37	
Percent urban population	-“-	6.16/100.00	
Capital city	-“-	0/1	1=1 st city is the national capital; 0=not capital
Landlocked	-“-	0/1	1=country is landlocked; 0=sea/ocean access
Continent	-“-	0/1	Location in either America, Asia, Africa or Europe

APPENDIX 2

Numerical results and explanations

Table 1 reports canonical discriminant function (CDF) estimates. In this table, standardized coefficients with large absolute values (two left columns) correspond to variables with greater discriminating ability. That is, a variable with large standardized coefficients perform better in explaining why a particular country belongs to e.g., Group 1 (1st and 2nd cities are of similar size) and to, say, Group 3 (1st and 2nd cities of substantially different size).

Table 1: Summary of canonical discriminant functions (CDFs)

Variable	Standardized Coefficients (SC)		Structure Matrix Coefficients (SMC)	
	Function 1	Function 2	Function 1	Function 2
Population, '000 (ln)	-0.447	0.823	-0.035	0.668*
Land area, km ² (ln)	0.478	-0.144	0.078	0.467*
GDP per capita, \$US (ln)	-0.377	0.291	-0.404*	0.386
Percent urban	-0.280	0.099	-0.338*	0.319
Capital city	0.771	0.186	0.788*	0.134
Landlocked	0.098	-0.084	0.312*	-0.073
America	0.261	-0.530	-0.148	-0.486*
Asia	0.280	-0.102	0.014	0.137*
Africa	0.219	-0.147	0.145	-0.148*
Europe	0.587	0.135	0.110	0.570*
% of Variance	73.9	26.1		
χ^2	69.5	19.5		
Sig.	<0.001	0.021		

* Marks a variable's largest absolute correlation with one of the CDFs.

The structure matrix coefficients (the 3rd and 4th columns in Table 1) indicate simple correlations between the variables and the discriminant functions and can be used to assign meaningful labels to the discriminant functions.

As Table 1 shows, the first canonical discriminant function (Function 1) is the most statistically significant function ($\chi^2=69.5$; $P<0.001$) and explain most of the data variance (approx. 74%). It correlates with per capita GDP, percent of urban residents in a country, the capital city status, and inland location (see the variables marked by asterisks in Table 2). The second function (Function 2), which is somewhat weaker in its discriminant ability ($\chi^2=19.5$; $P=0.021$), correlates with a country's population size, land area and its regional location (either America, Europe, Asia or Africa). These functions can thus conditionally be labeled "development status" and "geography," respectively.

So what do these discriminant functions actually “discriminate” between? The graph known as the “territorial map” (see Figure 3) may be helpful in answering this

question. Numbers 1, 2 and 3 on this map represent individual countries falling into classification categories as detailed in Appendix 1 – either Group 1, 2 or 3 (see the research variable section). Concurrently, the X and Y axes feature values which the Canonical Discriminant Function (CDF1 and CDF2), reported in Table 1, can possibly take – from negative (the lower left corner of the diagram) to positive ones (the upper right corner). Each country is thus mapped in this two-dimensional space, according to the values of the above discriminant functions (see Table 1), generated according to the country’s development attributes – population size, land area, the continent of location, etc.

As Figure 3 shows, the clustering of countries belonging to B-ratio Groups 2 and 3 occurs where Canonical Discriminant Function 1 (featured on the X-axis) takes on *positive values*, while the clustering of countries belonging to B-ratio Groups 1 and 2 occurs where CDF1 values are *negative*. Since this function is positively correlated with capital city status, land area and negatively correlated with population size, per capita GDP and percent urban (see standardized coefficients in Table 1), it may be said that large (land-wise), poorly developed and sparsely populated countries, in which the 1st city functions as the national capital, are likely to exhibit large differences of the [2nd city size/1st city size] ratio. In contrast, in heavily populated and urbanized high-income countries, 1st and 2nd cities tend to be of similar size.

Similarly, the clustering of the 1 and 3 values occurs in the right-bottom part of the diagram corresponding to negative values of CDF2 (Axis Y). Since CDF2 positively correlates with population size, per capita GDP and location in Europe and negatively correlated with land area and location outside Europe (see Table 1: Standardized Coefficients), “normal” B-ratios (Group 2) are most likely to be found in heavily populated and rich countries of Europe, while poorly developed and thinly populated countries located elsewhere are more likely to tend towards “extreme” B-ratios (Groups 1 & 3).

Fisher’s analysis of classification function coefficients, which results are reported in Table 2, generally supports these conclusions, indicating that a country’s membership in Group 3 (extreme differences between the first and second cities) is strongly and positively associated with the first city’s status as the national capital (Capital city: Group 3 = 15.041 vs. Group 1 = 11.212) and negatively associated with percent urban population (Percent urban: Group 3=-0.134 vs. Group 1=-0.113).

Table 2: Fisher's Classification Function Coefficients

Variable	Group 1	Group 2	Group 3
Population, '000 (ln)	8.728	8.816	8.383
Land area, km ² (ln)	-4.012	-3.851	-3.700
GDP per capita, \$US (ln)	15.767	15.600	15.197
Percent urban	-0.113	-0.123	-0.134
Capital city	11.212	14.147	15.041
Landlocked	7.634	7.735	8.035
America	-1.035	-1.395	-0.021
Asia	-1.587	-1.077	-0.541
Africa	13.051	13.329	13.868
Europe	-17.511	-15.682	-15.109
(Constant)	-112.657	-116.422	-109.052
No of cases	35	63	79

*Group 1: B-ratio>0.6% (similar size); Group 2: 0.3< B-ratio <0.6% ("Zipf-expected" difference); Group 3: B-ratio <0.3 (substantially different size).