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#### **Working Paper**

A Macroeconomic Growth Model of Competing Regions

Volkswirtschaftliche Diskussionsbeiträge, No. 90-00

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Suggested Citation: Buhr, Walter (2000): A Macroeconomic Growth Model of Competing Regions, Volkswirtschaftliche Diskussionsbeiträge, No. 90-00, Universität Siegen, Fakultät III, Wirtschaftswissenschaften, Wirtschaftsinformatik und Wirtschaftsrecht, Siegen

This Version is available at: https://hdl.handle.net/10419/118768

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## Volkswirtschaftliche Diskussionsbeiträge



A Macroeconomic Growth Model of Competing Regions

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Discussion Paper No. 90-00

ISSN 1433-058x

UNIVERSITÄT GESAMTHOCHSCHULE SIEGEN FACHBEREICH WIRTSCHAFTSWISSENSCHAFTEN

# A Macroeconomic Growth Model of Competing Regions

Walter Buhr\*

#### I. Characteristics of the Model

The Problem. In the growth context, this paper concentrates on the supplydemand determination of regional equilibrium incomes in regional goods markets as a framework for discussing the implications of competition among regions. Prices at the regional and national level are fixed so that all variables are of real magnitudes; there are no money markets. Regional factor stocks (private and public capital, labor force in the form of human capital) and regional demand set up the barriers to economic growth (cf. generally Barro, Sala-i-Martin (1995)). The analysis is restricted to two regions embedded in the State. The supply side of the model is represented by different regional production functions which generate regional factor demand. Demand-side determination of equilibrium regional incomes arises from the definitions of national accounting enriched by basic behavioral relationships. Substantial consideration is given to the equalization of regional supply and demand in diverging cases (e.g., excess demand in one of the two regions). Changes in factor stocks (differential equations) maintain the dynamics of the model. Regional competition is expressed by variations of regional and state parameters. Their numerical influences on growth will be dealt with in Bobzin (2000).

We assume there are two regions R1 and R2 comprising the State. Figure 1 describes essential linkages: ① Public resources  $F_{iS}^{pu}$  (i=1,2) are transferred from the public sector of region i to the State. ② These resources  $F^{pu}$  are redistributed in the form  $F_{Si}^{pu}$  to the regions. ③ Total private exports  $Ex_i^{pr}$  (i=1,2) of region i include goods exports  $Z_i^{pr}$  and transfers of private investment  $F_{ij}^{pr}$  (e. g.,  $F_{12}^{pr}$  meaning transfers from R1 to R2). The link ④ correspondingly sets up total private imports of region i,  $Im_i^{pr}$ . ⑤ Migration  $L_{12}$  from R1 to R2 refers to educated and raw labor,  $L_{12}^{edu}$  and  $L_{12}^{raw}$ , respectively.

Competition. Economic growth is taken as the result of competing intraregional and interregional activities. Accordingly, the actors of competition are the households, the firms and the public sectors of the regions, on the one hand, and the

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The basic idea for the present approach stems from a paper by Buhr (1995), which has been changed in some essential aspects.

<sup>&</sup>lt;sup>2</sup> For a multi-area approach cf. Treyz, Rickman, Shao (1992).

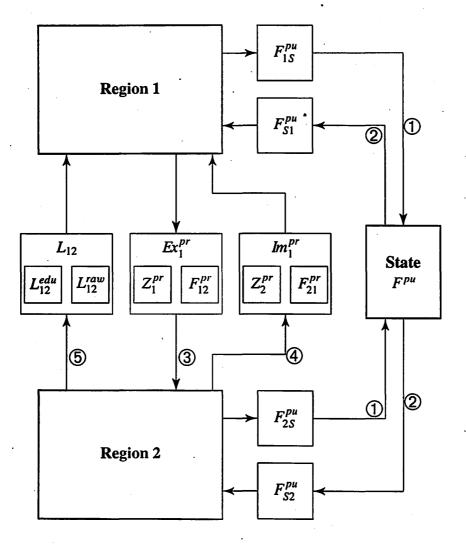


Figure 1: Linkages of the model

State, on the other hand. The instruments of competition appear in the form of parameters under the control of different institutional agents of economic activities. For example:

- the marginal rate of consumption (households of a region),
- production parameters (firms of a region),
- tax rate (public sector of a region)

and

• parameter of distributing state funds among the regions.

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Processes of regional competition such as interactions between economic agents of the same region or of other regions are not discussed in this model. Here the basis is laid to follow up numerically the immediate effects and the final results or regional competition.

#### II. Supply Side of the Model

**Regional Production.** The analysis is based on nested CES production functions, e. g. for R1 output  $X_1$  is a function of the stock of human capital  $H_1$ , the private capital stock  $K_1$  and the stock of infrastructure capital  $B_1$ .

$$X_{1} = f_{1}(H_{1}, K_{1}, B_{1})$$

$$= A_{1} \left[ \alpha_{1}^{H} (a_{1}H_{1})^{\rho_{1}} + \alpha_{1}^{K} (k_{1}K_{1})^{\rho_{1}} + \alpha_{1}^{B} (b_{1}B_{1})^{\rho_{1}} \right]^{1/\rho_{1}}$$

with  $\alpha_1^H + \alpha_1^K + \alpha_1^B = 1$  and  $0 \le \alpha_1^H$ ,  $\alpha_1^K$ ,  $\alpha_1^B \le 1$ . We assume<sup>3</sup>

$$a_1 = a_{10} + a_{11}t + a_{12}B_1^S + a_{13}K_1^S$$
  

$$k_1 = k_{10} + k_{11}t + k_{12}B_1^S + k_{13}K_1^S$$
  

$$b_1 = b_{10} + b_{11}t,$$

all parameters being positive, except  $k_{13} < 0$ .

For  $\rho_1 \to 0$  (i. e., the elasticity of substitution  $\sigma_1 = 1/(1 - \rho_1) = 1$ ), the CES function becomes a linear-homogeneous Cobb-Douglas production function

$$X_1 = A_1(a_1H_1)^{\alpha_1^H}(k_1K_1)^{\alpha_1^K}(b_1B_1)^{\alpha_1^H}$$

The constants  $\alpha_1^H$ ,  $\alpha_1^K$ ,  $\alpha_1^B$  here are the factor shares in output. For  $\rho_1 \to -\infty$  (i. e.,  $\sigma_1 = 0$ ) the CES function approaches a linear-limitational production function

$$X_1 = A_1 \min \{a_1 H_1, k_1 K_1, b_1 B_1\}$$

In this case the coefficients  $a_1$ ,  $k_1$ ,  $b_1$  are the average productivities of the inputs. Human capital  $H_i$  is aggregated by a linear-homogeneous CES function of educated and raw labor,  $L_i^{edu}$  and  $L_i^{raw}$ , respectively (i = 1, 2). For R1 we get

$$H_{1} = g_{1}(L_{1}^{edu}, L_{1}^{raw})$$

$$= A_{1}^{H} \left[ \varphi_{1}(a_{1}^{edu}L_{1}^{edu})^{\omega_{1}} + (1 - \varphi_{1})(a_{1}^{raw}L_{1}^{raw})^{\omega_{1}} \right]^{1/\omega_{1}},$$

Index S refers to supply, index D to demand.

where  $0 \le \varphi_1 \le 1$ .

As long as  $L_1^{edu}$  and  $L_1^{raw}$  are not perfect substitutes,  $\omega_1 < 1$  must apply. In addition

$$\begin{split} a_1^{edu} &= a_{10}^{edu} + a_{11}^{edu} B_1^{edu} + a_{12}^{edu} K_1^{dev} \\ a_1^{raw} &= a_{10}^{raw} + a_{11}^{raw} B_1^{edu}, \end{split}$$

where  $B_i^{edu}$  represents the level of education of region i and  $K_i^{dev}$  the pool of knowledge for research and development (R&D) of region i (i = 1, 2). It is assumed that the outputs  $X_i$  of the regions will be produced at minimum cost at every point of time. Let us turn to the general production case.

Taking again R1 as the reference region, the cost minimum rental rate of human capital is given by

$$w_1 \coloneqq (w_1^{edu} + \zeta_1 w_1^{raw})/\psi_1$$

where

$$\zeta_{1} \coloneqq \left(\frac{w_{1}^{raw}\varphi_{1}(a_{1}^{edu})^{\omega_{1}}}{w_{1}^{edu}(1-\varphi_{1})(a_{1}^{raw})^{\omega_{1}}}\right)^{\frac{1}{\omega_{1}-1}} \\
\psi_{1} \coloneqq A_{1}^{H} \left[\varphi_{1}(a_{1}^{edu})^{\omega_{1}} + (1-\varphi_{1})(a_{1}^{raw})^{\omega_{1}}\zeta_{1}^{\omega_{1}}\right]^{1/\omega_{1}}.$$

Thus  $w_1$  is linear-homogeneous in the two wage rates  $w_1^{edu}$  and  $w_1^{raw}$  (the function  $\zeta_1$  and therefore  $\psi_1$  are homogeneous of degree 0 in the wage rates).

Factor Demand. The demand for the factors of production in R1 may be expressed as

$$\begin{pmatrix} H_1^D \\ K_1^D \\ B_1^D \end{pmatrix} = \begin{pmatrix} a_1^{-\beta_1} (w_1/\alpha_1^H)^{\beta_1-1} \\ k_1^{-\beta_1} (r_1/\alpha_1^K)^{\beta_1-1} \\ b_1^{-\beta_1} (r_1^*/\alpha_1^B)^{\beta_1-1} \end{pmatrix} \frac{X_1}{A_1} Z_1^{-1/\rho_1}.$$

where  $(r_1 = \text{rental rate of private capital in R1, } r_1^* = \text{rental rate of infrastructure capital in R1})$ 

$$\beta_{1} \coloneqq \rho_{1}/(\rho_{1}-1)$$

$$Z_{1} \coloneqq (\alpha_{1}^{H})^{1-\beta_{1}} \left(\frac{w_{1}}{a_{1}}\right)^{\beta_{1}} + (\alpha_{1}^{K})^{1-\beta_{1}} \left(\frac{r_{1}}{k_{1}}\right)^{\beta_{1}} + (\alpha_{1}^{B})^{1-\beta_{1}} \left(\frac{r_{1}^{*}}{b_{1}}\right)^{\beta_{1}}$$

Moreover,

$$L_1^{edu.D} = H_1^D/\psi_1$$

$$L_1^{raw.D} = \zeta_1 H_1^D/\psi_1$$

Provided both kinds of labor are payed by their monetary marginal productivity, total payment to human capital in R1 amounts to  $(P_1 = \text{price level of R1})$ 

$$w_{1}H_{1}^{D} = P_{1} \frac{\partial X_{1}}{\partial H_{1}} \frac{\partial H_{1}}{\partial L_{1}^{edu}} L_{1}^{edu,D} + P_{1} \frac{\partial X_{1}}{\partial H_{1}} \frac{\partial H_{1}}{\partial L_{1}^{raw}} L_{1}^{raw,D}$$
$$= w_{1}^{edu} L_{1}^{edu,D} + w_{1}^{raw} L_{1}^{raw,D}$$

The consideration of the linear-homogeneous formation of human capital, the adding up theorem and the regional price level lead to the cost function of R1

$$c_1(w_1, r_1, r_1^*, X_1) = \frac{X_1}{A_1} Z_1^{1/\beta_1} = P_1 X_1$$
  
or  $Z_1^{1/\beta_1}/A_1 = P_1$ 

Factor Prices. The implied rental rate of human capital turns out to be

$$w_1 = a_1 \left(\alpha_1^H\right)^{\frac{\beta_1 - 1}{\beta_1}} \left[ (P_1 A_1)^{\beta_1} - (\alpha_1^K)^{1 - \beta_1} \left(\frac{r_1}{k_1}\right)^{\beta_1} - (\alpha_1^B)^{1 - \beta_1} \left(\frac{r_1^*}{b_1}\right)^{\beta_1} \right]^{1/\beta_1}$$

Wage rate of educated labor is defined as  $w_1^{edu}$ :

(a) Suppose that the time path of  $w_1^{raw}$  is determined by

$$w_1^{raw}(t) = w_1^{raw}(0) + \delta_1^{raw} t,$$

then the calculation of  $w_1^{edu}$  from  $w_1^{raw}$  and  $w_1$  amounts to

$$w_1^{edu} = \left(\frac{\pi_1^{\frac{\omega_1}{1-\omega_1}} \varphi_1(a_1^{edu})^{\omega_1}}{1 - \pi_1^{\frac{\omega_1}{1-\omega_1}} (1 - \varphi_1)(a_1^{raw})^{\omega_1} \pi_2^{\omega_1}}\right)^{\frac{1-\omega_1}{\omega_1}}$$

where 
$$\pi_1 \coloneqq w_1 A_1^H \varphi_1 (a_1^{edu})^{\omega_1}$$
  
and  $\pi_2 \coloneqq \left(\frac{w_1^{raw} \varphi_1 (a_1^{edu})^{\omega_1}}{(1 - \varphi_1)(a_1^{raw})^{\omega_1}}\right)^{\frac{1}{\omega_1 - 1}}$ 

(b) If we assume that the labor market perfectly adjusts the wage rates of educated and raw labor so that

$$\frac{w_1^{edu}}{w_1^{raw}} = \frac{\varphi_1(a_1^{edu})^{\omega_1}(L_1^{edu,S})^{\omega_1-1}}{(1-\varphi_1)(a_1^{raw})^{\omega_1}(L_1^{raw,S})^{\omega_1-1}}.$$

then the two wage rates may be calculated as follows. Setting

$$\zeta_1 = \frac{L_1^{raw.S}}{L_1^{edu.S}}$$

we have, considering that  $w_1 = P_1 \partial X_1 / \partial H_1$ 

$$\begin{split} w_1^{edu} &= w_1 \frac{\partial H_1^D}{\partial L_1^{edu}} \\ &= w_1 A_1^H \left[ \varphi_1 (a_1^{edu})^{\omega_1} + (1 - \varphi_1) (a_1^{raw} \zeta_1)^{\omega_1} \right]^{\frac{1 - \omega_1}{\omega_1}} \varphi_1 (a_1^{edu})^{\omega_1} \\ w_1^{raw} &= w_1 \frac{\partial H_1^D}{\partial L_1^{raw}} \\ &= w_1 A_1^H \left[ \varphi_1 (a_1^{edu})^{\omega_1} + (1 - \varphi_1) (a_1^{raw} \zeta_1)^{\omega_1} \right]^{\frac{1 - \omega_1}{\omega_1}} (1 - \varphi_1) (a_1^{raw})^{\omega_1} \zeta_1^{\omega_1 - 1} \end{split}$$

where all terms are known. This procedure implies that educated and raw labor will always have the same rate of unemployment as human capital.

The rental rates  $r_1$  and  $r_1^*$  are constants. Starting with an initial value of  $P_1$  the rental rate of human capital is determined. It will be changed by technical progress, i. e.  $a_1$ ,  $k_1$ , and  $b_1$  will vary over time.

**Special Aspects of Production.** Under modified production conditions factor demand and factor prices will change.

• Linear-homogeneous Cobb-Douglas production function: In this case factor demand is

$$\begin{pmatrix} H_1^D \\ K_1^D \\ B_1^D \end{pmatrix} = \begin{pmatrix} \alpha_1^H/w_1 \\ \alpha_1^K/r_1 \\ \alpha_1^B/r_1^* \end{pmatrix} \frac{\chi_1}{A_1} Z_1,$$

where the factor shares sum up to one:  $\alpha_1^H + \alpha_1^K + \alpha_1^B = 1$ . The rental rate of human capital is obtained as

$$w_{1} = \alpha_{1}^{H} a_{1} \left[ P_{1} A_{1} \left( \frac{r_{1}}{\alpha_{1}^{K} k_{1}} \right)^{-\alpha_{1}^{K}} \left( \frac{r_{1}^{*}}{\alpha_{1}^{B} b_{1}} \right)^{-\alpha_{1}^{B}} \right]^{1/\alpha_{1}^{H}}.$$

The wage rate  $w_1^{edu}$  must be calculated according to case a) of the CES function for a given wage rate  $w_1^{raw}$ .

• Linear-limitational production function: Here factor demand is

$$\begin{pmatrix} H_1^D \\ K_1^D \\ B_1^D \end{pmatrix} = \begin{pmatrix} 1/a_1 \\ 1/k_1 \\ 1/b_1 \end{pmatrix} \frac{\chi_1}{A_1}.$$

The rental rate of human capital turns out to be

$$w_1 = a_1 \left( 1 - \frac{r_1}{k_1} - \frac{r_1^*}{b_1} \right) P_1 A_1$$

with factor shares  $w_1/a_1$ ,  $r_1/k_1$ ,  $r_1^*/b_1$ .

The wage rate  $w_1^{edu}$  must again be determined according to case a) of the CES function for a given wage rate  $w_1^{raw}$ .

Restricted Factor Supply. The labor supply barriers are introduced by

$$L_1^{edu,D} \leq L_1^{edu,S}, L_1^{raw,D} \leq L_1^{raw,S}.$$

Maximum supply of human capital then is

$$H_1^S = \psi_1 \min\{L_1^{edu.S}, L_1^{raw.S}/\zeta_1\},$$

since, as given above,  $L_1^{edu} = H_1/\psi_1$  and  $L_1^{raw} = \zeta_1 H_1/\psi_1$ .

The demand for human, private, and public capital is also restricted by supply.

$$H_1^D \le H_1^S, K_1^D \le K_1^S, B_1^D \le B_1^S$$

Potential output ( $Z_1$  and  $w_1$  appropriately determined):

• CES case:

$$X_{1}^{pot} = A_{1} Z_{1}^{1/\rho_{1}} \min \left\{ \frac{H_{1}^{S} a_{1}^{\beta_{1}}}{(w_{1}/\alpha_{1}^{H})^{\beta_{1}-1}}, \frac{K_{1}^{S} k_{1}^{\beta_{1}}}{(r_{1}/\alpha_{1}^{K})^{\beta_{1}-1}}, \frac{B_{1}^{S} b_{1}^{\beta_{1}}}{(r_{1}^{*}/\alpha_{1}^{B})^{\beta_{1}-1}} \right\}$$

• Cobb-Douglas case:

$$X_{1}^{pot} = \frac{A_{1}}{Z_{1}} \min \left\{ \frac{w_{1}H_{1}^{S}}{\alpha_{1}^{H}}, \frac{r_{1}K_{1}^{S}}{\alpha_{1}^{K}}, \frac{r_{1}^{*}B_{1}^{S}}{\alpha_{1}^{B}} \right\}$$

• Linear-limitational production case:

$$X_1^{pol} = A_1 \min \{a_1 H_1^S, k_1 K_1^S, b_1 B_1^S\}$$

**Demand Barriers.** Introducing the Hahn-Negishi rule of transactions (cf. Hahn, Negishi (1962)) we get

$$X_1^{eq} \leq \min\left\{X_1^D, X_1^{pot}\right\}$$

Factor demand with respect to realized  $X_1^{eq}$  results from the above-mentioned relationships.

With reference to the given factor supply and the derived factor demand, we are now able to calculate the rates of factor stock idleness.

#### III. Demand-Side Determination of Equilibrium Regional Incomes

**Regional Accounting.** In order to be able to determine equilibrium regional incomes on the demand side we apply the principles of national accounting to the regions and the State. Each region is characterized by a private sector and a public sector (cf. Figure 2) whose retained incomes are  $Y_i^{pr.r}$  and  $Y_i^{pu.r}$ , respectively (i = 1, 2). The outputs  $X_i$  of the private sectors are equal to total factor payments, that is,  $w_i H_i^D + r_i K_i^D + r_i^* B_i^D$ . The private sectors must raise direct taxes  $T_i$  and interest payments for the use of infrastructure capital supplied by the public

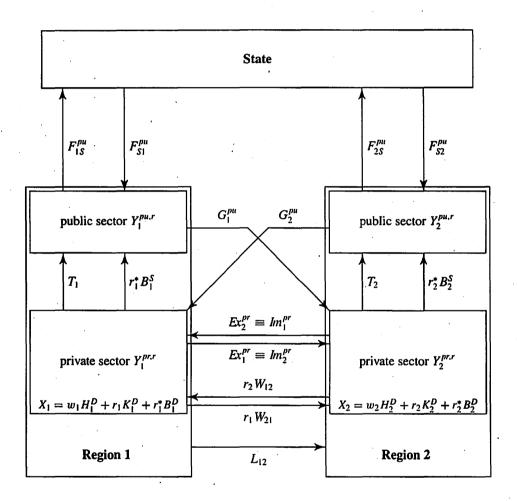


Figure 2: Private and public sectors of R1 and R2 and State

sectors,  $r_i^* B_i^S$ . Observe that there is no capital depreciation in both sectors of the regions. In addition, we assume balanced budgets for the public sectors of the regions and the State. An important interrelationship between the regions is formed by interregional interest payments on private assets held in the other region, for example  $r_1 W_{21}$  refers to the interest payments of R1 on assets owned by R2 in R1 ( $r_i$ = rate of interest in region i). The symbol  $G_i^{pu}$  indicates public investment subsidies of region i granted to investors of the other region. All other relationships shown in Figure 2 have been discussed before.

For example, for R1 we get the following basic relationships of accounting. R1's income:

$$Y_1 := X_1 + r_2 W_{12} - r_1 W_{21}$$

and factor distribution of R1's output:

$$X_1 := w_1 H_1^D + r_1 K_1^D + r_1^* B_1^D$$

where

$$w_1 H_1^D = w_1^{edu} L_1^{edu,D} + w_1^{raw} L_1^{raw,D}$$
 (total payment to R1's human capital)

R1's retained private income:

$$Y_1^{pr,r} := Y_1 - r_1^* B_1^S - T_1 = (1 - t_1)(Y_1 - r_1^* B_1^S)$$
$$= C_1^{pr} + I_1^{pr,dev} + S_1^{pr}$$

R1's retained public income:

$$Y_1^{pu,r} := T_1 + r_1^* B_1^S + F_{S1}^{pu} - F_{1S}^{pu} = (1 - \tau_1)(T_1 + r_1^* B_1^S) + F_{S1}^{pu}$$
$$= C_1^{pu} + S_1^{pu} + G_1^{pu}$$

**Behavioral Relationships.** For R1 we introduce the following important behavioral relationships. Note that the variables and parameters of R1 not yet explained are  $C_1^{pr}$  for private consumption,  $C_1^{pu}$  for public consumption,  $S_1^{pr}$  for private savings,  $S_1^{pu}$  for public savings,  $I_1^{pr,dev}$  for private expenditures on research and development (R&D),  $t_1$  for the tax rate,  $\tau_1$  for the state share in the revenue of R1's public sector.

Private consumption function:

$$C_1^{pr} = c_1^{pr} Y_1^{pr,r}, \quad 0 < c_1^{pr} < 1$$

function of private expenditures on investment and research and development (R&D):

$$\begin{split} \tilde{I}_{1}^{pr} &= I_{1}^{pr} + I_{1}^{pr,dev} = \frac{u_{1}}{r_{1}} Y_{1}^{pr,r} + u_{1}^{aut} K_{1}^{S}, \quad u_{1} > 0, u_{1}^{aut} > 0 \\ \text{where} \quad I_{1}^{pr,dev} &= \varepsilon_{1}^{dev} \tilde{I}_{1}^{pr} \quad \text{with} \quad \varepsilon_{1}^{dev} = \varepsilon_{10}^{dev} + \varepsilon_{11}^{dev} H_{1}^{S} < 1 \end{split}$$

private investment function:  $I_1^{pr} = (1 - \varepsilon_1^{dev}) \tilde{I}_1^{pr}$  export function:

$$Ex_1^{pr} = Z_1^{pr} + F_{12}^{pr}$$
 with  $Z_1^{pr} = i_{12} \frac{r_2}{r_1} Y_2^{pr,r}$ ,  $i_{12} > 0$ 

import function:

$$Im_1^{pr} = Z_2^{pr} + F_{21}^{pr}$$
 with  $Z_2^{pr} = i_{21} \frac{r_1}{r_2} Y_1^{pr,r}$ ,  $i_{21} > 0$ 

public consumption function:

$$C_1^{pu} = c_1^{pu} Y_1^{pu,r}, \quad 0 < c_1^{pu} < 1$$

tax function:

$$T_1 = t_1(Y_1 - r_1^* B_1^S), \quad 0 < t_1 < 1$$

It is assumed that the public sector requires the payment of  $r_1^*B_1^S$  instead of  $r_1^*B_1^D$  (provided  $B_1^D < B_1^S$ ), so that the difference  $r_1^*(B_1^S - B_1^D)$  has the effect of an additional tax.

Public investment is residually determined.

Public investment:

$$I_1^{pu} = (1 - c_1^{pu})Y_1^{pu,r} + F_{1S}^{pu} - F_{S1}^{pu}$$

public expenditure on education:

$$I_1^{pu,edu} = \varepsilon_1^{edu} I_1^{pu} \quad \text{with} \quad \varepsilon_1^{edu} = \varepsilon_{10}^{edu} + \varepsilon_{11}^{edu} Y_1^{pr,r} + \varepsilon_{12}^{edu} K_1^{dev} + \varepsilon_{13}^{edu} H_1^S < 1$$

public investment subsidies:

$$G_1^{pu} = h_1 I_1^{pu}$$
  $(h_1 > 0 \text{ small enough to guarantee } S_1^{pu} \ge 0).$ 

Investments interregionally initiated by public investment subsidies are

$$F_{21}^{pr}=v_{21}G_1^{pu}, \quad v_{21}\geq 0,$$

$$F_{12}^{pr} = v_{12}G_2^{pu}, \quad v_{12} \geq 0.$$

State revenue is by definition

$$F^{pu} := F_{1S}^{pu} + F_{2S}^{pu}$$
 with  $F_{1S}^{pu} = \tau_1 (T_1 + r_1^* B_1^S)$ ,  $0 < \tau_1 < 1$ , for R1, for example.

State expenditures amount to

$$F_{S1}^{pu} = \nu F^{pu}$$
 and  $F_{S2}^{pu} = (1 - \nu) F^{pu}$ ,  $0 \le \nu \le 1$ .

**Regional Equilibrium Incomes.** By combining the above given relationships we obtain a system of simultaneous equations to calculate the goods market equilibrium incomes  $Y_1$  and  $Y_2$  of the regions:

$$I_1^{pr} = S_1^{pr} - Z_1^{pr} - F_{12}^{pr} + Z_2^{pr} + r_1 W_{21} - r_2 W_{12} - G_2^{pu}$$

$$I_2^{pr} = S_2^{pr} - Z_2^{pr} - F_{21}^{pr} + Z_1^{pr} + r_2 W_{12} - r_1 W_{21} - G_1^{pu}$$

or

$$Y_1 = f_1(Y_2)$$
  
 $Y_2 = f_2(Y_1)$ 

as regional demand functions which intersect at  $(Y_1^D, Y_2^D)$ , cf. Figure 3, for example.

#### IV. Equalization of Supply and Demand

Lack of Demand. Under regular circumstances regional supply and demand will not be equal, so that we must discuss the problem of adjustment for different cases (cf. Sneesens (1984, p. 190); Ramser, Stadler (1997, pp. 46–49)).

In the case of regional demand shortages, the regional demand functions are simultaneously fulfilled under the condition  $Y_i^D \leq Y_i^{pol}$ . Excess capacities (looking here only at R1) are eliminated by reducing potential output  $X_1$  to  $X_1^{eq}$  so that

$$X_1^{eq} + r_2 W_{12} - r_1 W_{21} = Y_1^D < Y_1^{pol} = X_1 + r_2 W_{12} - r_1 W_{21}$$

Consequently, all three factors of production are unemployed and realized output  $X_1^{eq}$  implies

$$Y_1^D = Y_1^{eq} = w_1 H_1^D + r_1 K_1^D + r_1^* B_1^D + r_2 W_{12} - r_1 W_{21}$$
where  $w_1 H_1^D = w_1^{edu} L_1^{edu,D} + w_1^{raw} L_1^{raw,D}$ 

Excess Demand in One of the Two Regions. Let us assume that the regional demand functions generate the following inequalities .

$$Y_1^D < Y_1^{pot}$$
  
 $Y_2^D > Y_2^{pot} = X_2 + r_1 W_{21} - r_2 W_{12} = Y_2^S = Y_2^{eq}$ 

Since  $Y_2^D$  cannot be realized, disposable incomes of R2 must be corrected downwards.

$$Y_2^{pr,r} = (1 - t_2)(Y_2^{eq} - r_2^* B_2^S)$$
  

$$Y_2^{pu,r} = (1 - \tau_2)[t_2 Y_2^{eq} + (1 - t_2)r_2^* B_2^S] + F_{52}^{pu}$$

With respect to R1 only exports  $Z_1^{pr}$  must be adjusted, since  $Y_2^{pr,r}$  of R2 has fallen. The revised  $Y_1^D$  can be determined by inserting  $Y_2^{eq}$  into  $Y_1 = f_1(Y_2)$ . For the lower  $Y_1^{eq} = f_1(Y_2^{eq})$  we get:

$$Y_1^{pr,r} = (1 - t_1)(Y_1^{eq} - r_1^* B_1^S)$$
  

$$Y_1^{pu,r} = (1 - \tau_1)[t_1 Y_1^{eq} + (1 - t_1)r_1^* B_1^S] + F_{S1}^{pu}$$

Since  $Y_1^{eq}$  continues to indicate a demand-side equilibrium for R1, residually determined private investment  $I_1^{pr}$  according to the goods market equilibrium of R1 must be equal to the private investment demand shown in R1's gross savings and investment account.

Now in R2, income  $Y_2$  falls if we introduce a reduced  $Y_1$  ( $Y_1^D o Y_1^{eq}$ ) into  $Y_2 = f_2(Y_1)$ . Suppose for this revised case  $Y_2^{pol} < Y_2^{D.actual} = f_2(f_1(Y_2^{eq}))$ , then the residually determined private investment  $I_2^{pr}$  according to the goods market equilibrium of R2 is smaller by the amount  $\Gamma$  than the private investment demand coming up in R2's gross savings and investment account:  $\Gamma = I_2^{pr,D} - I_2^{pr} > 0$ . It can be shown that excess demand is created in the private sector.

$$Y_2^{pr,r} = C_2^{pr} + \tilde{I}_2^{pr} + F_{12}^{pr} + F_{12}^{pr} + G_1^{pu} - \Delta F_1^n < C_2^{pr,D} + \tilde{I}_2^{pr,D} + F_{12}^{pr} + G_1^{pu} - \Delta F_1^n$$

where the sum of  $F_{12}^{pr} + G_1^{p\dot{u}}$  is determined by R1's public sector decision-making. Thus, it seems convincing to eliminate the balance  $\Gamma = I_2^{pr,D} - I_2^{pr}$  with reference to private consumption  $C_2^{pr}$  and private investment  $\tilde{I}_2^{pr} = I_2^{pr} + I_2^{pr,dev}$ .

$$\begin{split} C_2^{pr} &= C_2^{pr,D} - \frac{C_2^{pr,D}}{\tilde{I}_2^{pr,D} + C_2^{pr,D}} \, \Gamma = C_2^{pr,D} \left( 1 - \frac{\Gamma}{\tilde{I}_2^{pr,D} + C_2^{pr,D}} \right) \\ \tilde{I}_2^{pr} &= \tilde{I}_2^{pr,D} - \frac{\tilde{I}_2^{pr,D} + C_2^{pr,D}}{\tilde{I}_2^{pr,D} + C_2^{pr,D}} \, \Gamma = \tilde{I}_2^{pr,D} \left( 1 - \frac{\Gamma}{\tilde{I}_2^{pr,D} + C_2^{pr,D}} \right) \end{split}$$

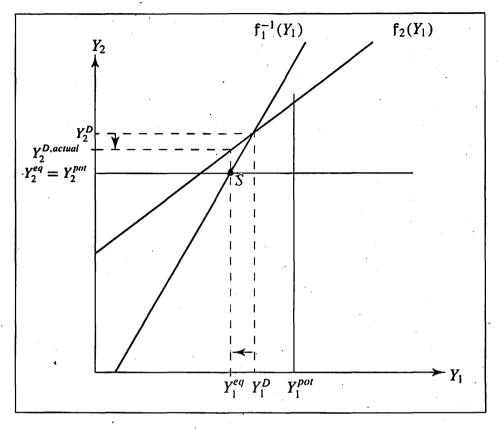


Figure 3: Excess demand in R2

By dividing both relationships we see that the proportion of private disposable income dedicated to  $C_2^{pr}$  and  $\tilde{I}_2^{pr}$  will be spent in such a way that  $C_2^{pr}/\tilde{I}_2^{pr}$  remains constant.

Then again

$$I_2^{pr} = \varepsilon_2^{dev} \tilde{I}_2^{pr}$$
 and  $I_2^{pr,dev} = (1 - \varepsilon_2^{dev}) \tilde{I}_2^{pr}$ 

The solution (point S in Figure 3) is:

$$Y_1^{eq} < Y_1^{pot}, \quad Y_2^{eq} = Y_2^{pot}$$

meaning for R1: all three factors of production are unemployed,

for R2: at least one factor is fully employed in the framework of regional demand-supply equilibria. Observe:

1. The described approach can only be applied, if the slope of the line  $f_2(Y_1)$  is smaller than that of  $f_1^{-1}(Y_1)$ .

2. The corresponding problem  $Y_1^D > Y_1^{pot}$ ,  $Y_2^D < Y_2^{pot}$  can be solved analo-

Excess Demand in Both Regions. If  $Y_1^D > Y_1^{pot}$ ,  $Y_2^D > Y_2^{pot}$ , then calculate

$$Y_1^D > Y_1^{D,actual} = f_1(Y_2^{pot})$$
  

$$Y_2^D > Y_2^{D,actual} = f_2(Y_1^{pot})$$

according to the goods market equilibrium relationships of R1 and R2.

Now three cases are possible, subject to  $Y_i^{D,actual} \ge Y_i^{pot}$  (i = 1, 2) (The slopes of the curves exclude the possibility that  $Y_i^{D,actual} < Y_i^{pot}$  in both regions at the

Case 1: In both regions we have  $Y_i^{D,actual} \ge Y_i^{pot}$  (cf. Figure 4) so that each region produces at full capacity. The resulting balances  $\Gamma_i = I_i^{pr,D} - I_i^{pr}$  will be eliminated as described before.

The solution (point S in Figure 4) is:

$$Y_i^{eq} = Y_i^{pot}$$

meaning that in both regions at least one factor of production is fully employed in

the framework of regional demand-supply equilibria. Case 2: Given  $Y_1^{D,actual} \geq Y_1^{pot}$  and  $Y_2^{D,actual} < Y_2^{pot}$ , income  $Y_1^{D,actual}$  must be corrected downwards. Substitute  $Y_2^{pot}$  by  $f_2(Y_1^{pot}) = Y_2^{D,actual}$ :

$$\begin{aligned} Y_1^{eq} &= Y_1^{pol} < \mathfrak{f}_1 \big( \mathfrak{f}_2 (Y_1^{pol}) \big) = \tilde{Y}_1^{D,actual} & \Longrightarrow \Gamma_1 = I_1^{pr,D} - I_1^{pr} > 0 \\ Y_2^{eq} &= \mathfrak{f}_2 (Y_1^{pol}) = Y_2^{D,actual} < Y_2^{pol} & \Longrightarrow \Gamma_2 = 0 \end{aligned}$$

Case 3:  $Y_2^{D,actual} \ge Y_2^{pot}$  and  $Y_1^{D,actual} < Y_1^{pot}$ . Here

$$Y_1^{eq} = f_1(Y_2^{pot}) < Y_1^{pot} \qquad \Longrightarrow \Gamma_1 = 0$$

$$Y_2^{eq} = Y_2^{pot} < f_2(f_1(Y_2^{pot})) = \tilde{Y}_2^{D,actual} \qquad \Longrightarrow \Gamma_2 = I_2^{pr,D} - I_2^{pr} > 0$$

Calculate for both regions on the basis of  $Y_i^{eq}$ 

$$Y_i^{pr,r} = (1 - t_i)(Y_i^{eq} - r_i^* B_i^S)$$
  

$$Y_i^{pu,r} = (1 - \tau_i)[t_i Y_i^{eq} + (1 - t_i)r_i^* B_i^S] + F_{Si}^{pu}$$

where  $F_{Si}^{pu}$  is a portion of

$$F^{pu} = \tau_1[t_1Y_1^{eq} + (1-t_1)r_1^*B_1^S] + \tau_2[t_2Y_2^{eq} + (1-t_2)r_2^*B_2^S]$$

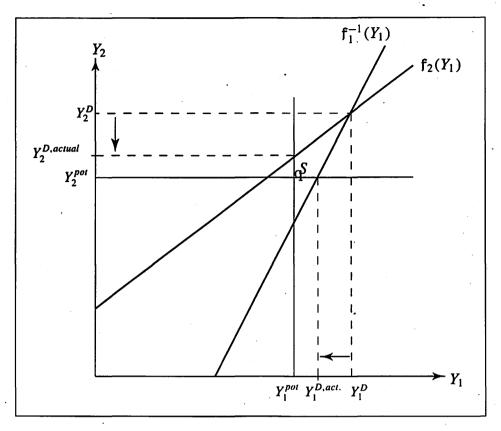


Figure 4: Excess demand in R1 and R2

#### V. Dynamics of the Model

A system of differential equations describes the dynamics of the model. These equations refer to

(a) changes of private and public capital stocks:

$$\begin{split} \dot{K}_{1}^{S} &= I_{1}^{pr} + F_{21}^{pr}, \\ \dot{K}_{2}^{S} &= I_{2}^{pr} + F_{12}^{pr}, \\ \dot{B}_{i}^{S} &= I_{i}^{pu} + F_{Si}^{pu} - F_{iS}^{pu}, \quad (i = 1, 2); \end{split}$$

(b) changes in the pool of knowledge for research and development and in the level of education:

$$\dot{K}_{i}^{dev} = I_{i}^{pr,dev}, \quad (i = 1, 2),$$
 $\dot{B}_{i}^{edu} = I_{i}^{pu,edu}, \quad (i = 1, 2),$ 

the initial values being

$$K_i^{dev} = \eta_i^{pr,dev} K_i^S, \quad (i = 1, 2),$$
  
 $B_i^{edu} = \eta_i^{pu,edu} B_i^S, \quad (i = 1, 2);$ 

(c) changes in the labor force:

$$\dot{L}_1^j = n_1^j L_1^{j,S} - L_{12}^j, \quad j \in \{raw, edu\},$$
  
 $\dot{L}_2^j = n_2^j L_2^{j,S} + L_{12}^j, \quad j \in \{raw, edu\},$ 

where total migration is  $L_{12} = L_{12}^{edu} + L_{12}^{raw}$ ,

$$\begin{split} L_{12}^{edu} &= L_{12}^{edu,A} + L_{12}^{edu,B} + L_{12}^{edu,C}, \\ L_{12}^{raw} &= L_{12}^{raw,A} + L_{12}^{raw,B}; \end{split}$$

the first incentive for migration  $(j \in \{edu, raw\})$  is captured by

$$L_{12}^{j,A} = \begin{cases} e_{A1}^{j}(w_{2}^{j} - w_{1}^{j})L_{1}^{j,S}, & \text{if } w_{1}^{j} < w_{2}^{j}, \\ e_{A2}^{j}(w_{2}^{j} - w_{1}^{j})L_{2}^{j,S}, & \text{if } w_{1}^{j} \ge w_{2}^{j}, \end{cases}$$

the second incentive  $(j \in \{edu, raw\})$  by

$$L_{12}^{j,B} = \begin{cases} e_{B1}^{j} \left( \ell(L_{1}^{j}) - \ell(L_{2}^{j}) \right), & \text{if } \ell(L_{1}^{j}) > \ell(L_{2}^{j}), \\ e_{B2}^{j} \left( \ell(L_{1}^{j}) - \ell(L_{2}^{j}) \right), & \text{if } \ell(L_{1}^{j}) \leq \ell(L_{2}^{j}), \end{cases}$$

where

$$\ell(L_i^j) := \frac{L_i^{j,S} - L_i^{j,D}}{L_i^{j,S}}$$
  $i = 1, 2; j \in \{edu, raw\},$ 

and the third incentive specified only for educated labor (cf. Palivos, Wang (1996)) by

$$L_{12}^{edu,C} = \begin{cases} e^{edu} \left( \frac{H_2^S}{L_2^{edu,S}} - \frac{H_1^S}{L_1^{edu,S}} \right) L_1^{edu,S}, & \text{if } \frac{H_1^S}{L_1^{edu,S}} < \frac{H_2^S}{L_2^{edu,S}} \\ e^{edu} \left( \frac{H_2^S}{L_2^{edu,S}} - \frac{H_1^S}{L_1^{edu,S}} \right) L_2^{edu,S}, & \text{if } \frac{H_1^S}{L_1^{edu,S}} \ge \frac{H_2^S}{L_2^{edu,S}}; \end{cases}$$

all migration parameters e are assumed to be positive;

(d) changes in assets:

$$\dot{W}_{12} = G_2^{pu} + F_{12}^{pr} + Z_1^{pr} - Z_2^{pr} + r_2 W_{12} - r_1 W_{21}, 
\dot{W}_{21} = G_1^{pu} + F_{21}^{pr}.$$

#### VI. Competition among Regions

Regional competition is expressed by changes of the regional and state parameters. They are (i = 1, 2)

technical parameters:

$$A_{i}, A_{i}^{H}, a_{i} (a_{i0}, a_{i1}, a_{i2}, a_{i3}), a_{i}^{edu}(a_{i0}^{edu}, a_{i1}^{edu}, a_{i2}^{edu}), a_{i}^{raw} (a_{i0}^{raw}, a_{i1}^{raw}), b_{i} (b_{i0}, b_{i1}), k_{i} (k_{i0}, k_{i1}, k_{i2}, k_{i3}), \alpha_{i}^{B}, \alpha_{i}^{H}, \alpha_{i}^{K}, \eta_{i}^{pr,dev}, \eta_{i}^{pu,edu}, \rho_{i}, \varphi_{i}, \omega_{i}$$
 private (behavioral) parameters:

$$c_{i}^{pr}$$
,  $e_{Ai}^{edu}$ ,  $e_{Bi}^{edu}$ ,  $e_{Ci}^{edu}$ ,  $e_{Ai}^{raw}$ ,  $e_{Bi}^{raw}$ ,  $n_{i}^{edu}$ ,  $n_{i}^{raw}$ ,  $i_{12}(i_{21})$ ,  $u_{i}$ ,  $u_{i}^{aut}$ ,  $v_{12}(v_{21})$ ,  $\varepsilon_{i}^{dev}(\varepsilon_{i0}^{dev}$ ,  $\varepsilon_{i1}^{dev})$ 

market parameters:

$$r_i, r_i^*, \delta_i^{raw}$$

regional public parameters:

$$c_i^{pu}$$
,  $h_i$ ,  $t_i$ ,  $\varepsilon_i^{edu}$  ( $\varepsilon_{i0}^{edu}$ ,  $\varepsilon_{i1}^{edu}$ ,  $\varepsilon_{i2}^{edu}$ ,  $\varepsilon_{i3}^{edu}$ ) state parameters:

 $\nu$ ,  $\tau_i$ 

The influences of these parameters on regional and national growth are of central interest in the present context. The numerical and empirical implications of this topic will be dealt with in Bobzin (2000).

#### VII. Concluding Remarks

The basic advantage of the presented approach to modelling regional economic growth is that the static part of the model may be alternatively formulated with respect to different aspects, such as the emphasis on economic phenomena considered for investigation or the chosen degree of disaggregation and thus the size and details of the approach. The dynamic part of this growth model is maintained by changing factor stocks. Thus, regional economic growth is the result of motivated and adjusted economic activities, with an important adjustment mechanism being regional competition.

### VIII. List of Symbols

(Index D refe	ers to demand, index S to supply; $i = 1, 2$ )
$A_i$	absolute level of production of region $i$ (parameter)
$A_i^H$	absolute level of human capital in region $i$ (parameter)
$B_i$	stock of infrastructure capital in region i
B <sub>i</sub> <sup>edu</sup> C <sub>i</sub> <sup>pr</sup> C <sub>i</sub> c <sub>i</sub> Ex <sub>i</sub> <sup>pr</sup>	level of education in region i
$C_i^{pr}$	private consumption in region i
$C_i^{pu}$	public consumption in region i
$Ex_i^{pr}$	total private exports of region $i$ (including capital exports to
•	the other region)
$\Delta F_1^n$	net investment of R1 in R2
$F_{12}^{pr}(F_{21}^{pr})$	transfer of private investment from R1 to R2 (from R2 to R1)
$F^{pu}$	total state revenue
$F_{iS}^{pu}$	public resources transferred from the public sector of region $i$
	to the State
$F^{pu}_{Si}$	public resources transferred from the State to region i
$F^{pu}_{Si}$ $G^{pu}_i$	public investment subsidies of region i
$H_i$	stock of human capital in region i
$H_i \  ilde{I}_i^{pr}$	private expenditures on investment and on research and
	development (R&D) in region i
$I_i^{pr}$	private investment of region i
$I_i^{pr} \ I_i^{pr,dev}$	private expenditures on research and development (R&D)
	of region i
$I_i^{pu}$	public investment in region i
$I_i^{pu}$ $I_i^{pu,edu}$	public expenditure on education in region $i$
lm <sub>i</sub> <sup>pr</sup>	total private imports of region $i$ (including capital imports
	from the other region)
$K_i \ K_i^{dev}$	private capital stock of region i
$K_i^{dev}$	pool of knowledge for research and development (R&D)
	in region i
$L_i$	labor force of region i
$L_i^{edu}$	educated labor of region i
L <sub>i</sub> L <sup>edu</sup> L <sup>raw</sup>	raw labor of region i
$L_{12}$	migration from R1 to R2
$L^{edu}_{12}$	educated workers migrating from R1 to R2

	,
$L_{12}^{edu,A}$	migrating educated workers oriented to wage rate
	differences between the regions
$L_{12}^{edu,B}$	migrating educated workers oriented to differences
	between the unemployment rates of the regions
$L_{12}^{edu,C}$	migrating educated workers oriented to differences in the
	relative regional supply of human capital
$L_{12}^{raw}$	raw laborers migrating from R1 to R2
$L_{12}^{raw} \ L_{12}^{raw,A}$	migrating raw laborers oriented to wage rate differences
12	between the regions
$L_{12}^{raw, B}$	migrating raw laborers oriented to differences between the
12	unemployment rates of the regions
$P_i$	price level of region i
$S_i^{pr}$	private savings of region i
$S_i^{pr}$ $S_i^{pu}$	public savings of region i
$\dot{T_i}$	direct tax revenue of region i
$W_{12}(W_{21})$	assets of R1 held in R2 (assets of R2 held in R1)
$X_i$	output of region i
$X_i^{eq}$	equilibrium output of region i
$X_i^{eq}$ $X_i^{pot}$ $Y_i$ $Y_i^{eq}$ $Y_i^{pot}$ $Y_i^{pot}$ $Y_i^{pr,r}$ $Y_i^{pu,r}$	potential output of region i
$Y_i$	income (net social product) of region i
$Y_i^{eq}$	equilibrium income of region i
$Y_i^{pot}$	potential income of region i
$Y_i^{pr,r}$	retained private income of region i
$Y_i^{pu,r}$	retained public income of region i
$Z_i^{pr}$	private goods exports of region i
$a_i$	production coefficient referring to human capital of region i
$a_{i0}$	autonomous production coefficient referring to human capital
	of region i
$a_{i1}$	technical progress dependent production coefficient referring
	to human capital of region i
$a_{i2}$	public capital dependent production coefficient referring to
	human capital of region i
$a_{i3}$	private capital dependent production coefficient referring to
	human capital of region i

$a_i^{edu}$	educated labor parameter of aggregating human capital of
·	region i
$a_{i0}^{edu}$	autonomous educated labor parameter of aggregating human
	capital of region i
$a_{i1}^{edu}$	educated labor parameter of aggregating human capital of
	region $i$ dependent on the level of education in region $i$
aedu 12	educated labor parameter of aggregating human capital of
	region $i$ dependent on the pool of knowledge for research and
	development in region i
$a_i^{raw}$	raw labor parameter of aggregating human capital of region $i$
$a_{i0}^{raw}$	autonomous raw labor parameter of aggregating human capital
	of region i
$a_{i1}^{raw}$	raw labor parameter of aggregating human capital of region i
	dependent on the level of education in region $i$
$b_i$	production coefficient referring to infrastructure capital of
	region i
$b_{i0}$	autonomous production coefficient referring to infrastructure
	capital of region i
$b_{i1}$	technical progress dependent production coefficient referring
	to infrastructure capital of region i
$c_i$	cost function of region i
C <sub>i</sub> <sup>pr</sup> C <sub>i</sub> <sup>pu</sup> e <sup>edu</sup>	marginal (average) private propensity to consume of region $i$
$c_i^{pu}$	marginal (average) public propensity to consume of region $i$
e <sup>edu</sup> Ai	coefficient of migrating educated workers of region i oriented
	to wage rate differences between the regions
$e_{Bi}^{edu}$	coefficient of migrating educated workers of region $i$ oriented
	to differences between the unemployment rates of the regions
e <sup>edu</sup> Ci	coefficient of migrating educated workers of region i oriented
	to differences in the relative regional supply of human capital
$e_{Ai}^{raw}$ .	coefficient of migrating raw laborers of region i oriented to
	wage rate differences between the regions
e Bi	coefficient of migrating raw laborers of region i oriented to
	differences between the unemployment rates of the regions
h <sub>i</sub>	quotient of public subsidies and public investment in region $i$
$i_{12}(i_{21})$	parameters of exports from R1 to R2 (from R2 to R1)

$k_i$	production coefficient referring to private capital of region $i$
$k_{i0}$	autonomous production coefficient referring to private capital of region <i>i</i>
$k_{i1}$	technical progress dependent production coefficient referring to private capital of region <i>i</i>
$k_{i2}$	public capital dependent production coefficient referring to
	private capital of region i
$k_{i3}$	private capital dependent production coefficient referring to private capital of region <i>i</i>
$\ell(L_i^j)$	rate of unemployment in region $i$ for category of labor $j$
•	$(j \in \{edu, raw\})$
$n_i^{edu}$	growth rate of educated natural labor force in region i
$n_i^{raw}$	growth rate of raw natural labor force in region i
$r_i$	rental rate of private capital in region i
$r_i^*$	rental rate of infrastructure capital in region i
ť	time
$t_i$ .	direct tax rate of region i
$u_i$	investment parameter referring to disposable private income
	in region i
u <sup>aut</sup>	investment parameter referring to capital supply of region i
$v_{12}(v_{21})$	parameter of private capital attracted from R1 to R2 (from R2 to R1)
$w_i$	rental rate of human capital in region i
$w_i^{edu}$	wage rate of educated labor in region i
$w_i^{raw}$	wage rate of raw labor in region i
$\alpha_i^B$	production coefficient referring to infrastructure capital in region i
$\alpha_i^H$	production coefficient referring to human capital in region i
$\alpha_i^K$	production coefficient referring to private capital in region i
$ \alpha_i^H $ $ \alpha_i^K $ $ \delta_i^{raw} $	variation rate in time of wage rate for raw labor in region $i$
$\varepsilon_i^{dev}$	quotient of private expenditures for research and development
	(R&D) and private expenditures on investment and on research
•	and development (R&D) in region $i$ (in short: research and
	development quotient)

autonomous research and development quotient of region i human capital dependent quotient of research and development in region i  $\varepsilon_i^{edu}$ quotient of public expenditures on education and public investment in region i (in short: quotient of education) autonomous quotient of education in region i $\varepsilon_{i1}^{edu}$ income dependent quotient of education in region i $\varepsilon_{i2}^{edu}$ quotient of education in region i dependent on the pool of knowledge for research and development in region i $\varepsilon_{B}^{edu}$ quotient of education in region i dependent on human capital of region i  $\eta_i^{\mathit{pr},\mathit{dev}}$ quotient of the pool of knowledge for research and development (R&D) and private capital stock in region i  $\eta_i^{pu,edu}$ quotient of the level of education and the public capital stock in region i parameter of the State for the distribution of revenues to R1 and R2 production parameter of region i  $\rho_i$ elasticity of factor substitution for region i ( $\sigma_i = 1/(1 - \rho_i)$ )  $\sigma_i$ state share in the revenue of the public sector of region i $\tau_i$ parameters of deriving human capital for region i  $\varphi_i, \omega_i$ 

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