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Aggregate Economic Growth with Subsistence Consumption

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Aggregate Economic Growth with Subsistence Consumption*

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Abstract: Four stylised facts of economic growth in DCs are set up initially. Despite its obvious simplicity the linear growth model with subsistence consumption is able to reproduce two of them: a rise in the saving rate along with per capita income as well as ß-divergence. The speed of convergence shows extraordinarily low values at early stages of economic development. Hence, the big diversity in growth rates can partly be explained to represent transitional phenomena. However, if international symmetry with respect to preferences and technology is supposed, the possible range of different growth rates is restricted. An extension of the basic model allows a more satisfactory explanation of the big diversity in growth experiences and additionally reproduces the hump-shaped relation between the growth rate and the level of per capita income.

Keywords: Divergence, Growth in DCs, Speed of Convergence, Subsistence Consumption
JEL-Classification: O1, O4

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1 Introduction

"The central problem in the theory of economic development is to understand the process by which a community which was previously saving 4 or 5 per cent of its national income or less, converts itself into an economy where voluntary saving is running at about 12 to 15 per cent of national income or more." Lewis (1954, p. 155)

"The development process is one of transition." Gersovitz (1988, p. 383)

If one takes Lewis (1954) and Gersovitz (1988) literally and is interested in an explanation of the process of economic growth and development in terms of growth theory, one is led to ask what class of growth models is consistent with these two statements. In addition, it is reasonable to ask what class of growth models is able to reproduce the main stylised facts of (aggregate) economic growth primarily applying to the lower range of per capita income:[1]

(1) A big diversity in growth rates of per capita income including zero and even negative growth,[2]

(2) a positive correlation between the saving rate and per capita income,[3] and

(3) a positive correlation between the growth rate and the level of per capita income, that is *ß-divergence*.[4]

(4) More specifically, many authors report a *hump-shaped relation* between the growth rate and the level of per capita income.[5]

The theory of economic growth reveals two elementary forces of growth. These consist in an increase in total factor productivity (TFP) as a result of innovation, imitation, adoption, and adaptation on the one hand and the accumulation of reproducible inputs (physical as well as

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[1] The general stylised facts of economic growth were formulated by Kaldor (1961). Romer (1989) adds five other prominent features of the data. Especially for DCs there are empirical regularities applying to the structure of the economy which are not considered in this paper. For this, see Kuznets (1973) and Wichmann (1997, chapter 3).

[2] This corresponds to Kaldor's (1961) sixth fact; see Romer (1989, p. 63) for a discussion. For the case of low-income countries this empirical regularity is particularly marked as Lucas (1988) observes: "*Within the poor countries (low and middle income) there is enormous variability*" Lucas (1988, p. 4).

[3] The empirical evidence in favour of a positive correlation between the saving rate and the level of per capita income is overwhelming. See Thirlwall (1974, chapter 7) for a review of the older and Reichel (1993) for a review of the more recent literature. In addition, see Rebelo (1992) and Ogaki, Ostry, and Reinhart (1996).

[4] For empirical evidence, see Baumol (1986), Zind (1991), and Pritchett (1996).

[5] For empirical evidence, see Baumol, Blackman, and Wolff (1989, pp. 301) Dollar (1992, section 2) and Easterly (1994, pp. 546-548).

human capital) on the other hand.[6] Especially in the long run, the accumulation of physical and human capital needs to be financed by internal saving.[7] Within the development literature it is stressed that saving in the case of developing countries (DCs) is determined by the *willingness to save* as well as the *ability to save* [e.g., Hemmer (1988, pp. 150)]. The usual constant-intertemporal-elasticity-of-substitution (CIES) formulation of preferences assumes that the entire income is disposable for saving. That is, these models abstract from the requirement of a minimum consumption level in order to sustain life. However, the requirement of subsistence consumption undoubtedly restricts the possibilities to substitute consumption intertemporally and, hence, the ability to save at least for the lower range of per capita income. Several questions arise which are of fundamental importance: Does the requirement of subsistence consumption influence the process of growth beyond this threshold? If so, how long does it take for the influence of subsistence consumption on growth to vanish? How does the requirement of subsistence consumption interact with other essential mechanisms of growth? The paper in hand seeks to answer these questions systematically within the context of simple endogenous growth models with Stone-Geary preferences. In addition, it will be shown that these models provide a potential explanation of the stylised facts listed above. It is assumed that the economy under study is symmetric to the rest of the world with respect to preferences and technology. The aim of this methodological assumption is to rule out explanations of differences in growth experiences that are based solely on the existence of cross-country differences in preferences and technology.[8]

In order to address specific theoretical as well as empirical questions, Stone-Geary preferences have occasionally been applied within the growth and development literature. This paragraph presents a comprehensive but concise survey of the different approaches: Nelson (1956) uses the notion of subsistence consumption to establish two essential elements of his low-level-equilibrium-trap model. These consist in the hypothesis concerning capital formation and population growth. Christiano (1989) uses a neoclassical model with

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[6] "Ideas and objects" in the words of Romer (1993) or according to Klenow and Rodríguez-Clare (1997, p. 611) "A versus K". The discussion of the East Asian Miracle largely centred around the question whether this "miracle" was primarily input driven or primarily caused by increases in the TFP. Young (1995) supports the view that an increase in inputs was the dominant force behind the economic progress observed for this group of countries.

[7] As far as specific consumption activities (nutrition, health efforts, and education) are productive, the accumulation of human capital does not necessarily require the renunciation of consumption; for this see Steger (1998). In the following this aspect is ignored.

preferences explicitly accounting for the requirement of subsistence consumption in which a trend is introduced in subsistence consumption to interpret the growth of Japan in the post-war period. According to Rebelo (1992), a "broad class of endogenous growth models" is inconsistent with cross-country diversity in growth rates in the face of international capital markets. As a solution to this theoretical problem, Rebelo suggests a linear growth model with Stone-Geary preferences. King and Rebelo (1993) study the transitional dynamics of the neoclassical growth model with CIES preferences and several modifications of this model by means of numerical solutions. One modification consists in the neoclassical model with Stone-Geary preferences. Easterly (1994) uses a constant-elasticity-of-substitution (CES) production function with the Jones-Manuelli property and Stone-Geary preferences to discuss the threshold effects of different policy measures on the long-run growth rates. Ben-David (1994) applies a neoclassical exogenous growth framework extended by subsistence consumption to demonstrate the possibility of multiple balanced-growth equilibria. Similarly, Sarel (1994) studies a neoclassical growth model with an increasing intertemporal-elasticity-of-substitution (IES) function employing Stone-Geary preferences. Azariadis (1996) formulates a modified overlapping generations model with an endogenously determined subsistence level of consumption. Within this framework, it is shown that a poverty trap results if the fraction of subsistence consumption in GNP is sufficiently sensitive to past income whenever that income is within a critical range.[9] Finally, Ogaki, Ostry, and Reinhart (1996) show that the saving rate and its sensitivity to the interest rate is much higher in middle-income developing countries than in low-income developing countries in a linear growth model in which the IES rises with the level of wealth.[10] Despite its widespread use, the requirement of subsistence consumption has not yet been systematically applied to provide a potential explanation of the process of aggregate growth in DCs which is the aim of the paper in hand.

The paper is organised as follows: Section 2 concisely discusses the concept of subsistence and its empirical importance. In section 3 a linear growth model with Stone-Geary preferences

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[8] As Rebelo (1992) points out: "*This type of symmetry is conventional in modern macroeconomic theory but it is foreign to the development economics tradition. Most of the models used in development economics emphasize the unique features of underdeveloped countries.*" Rebelo (1992, p. 7).

[9] This is one of three "impatience traps" which are demonstrated within the overlapping generations framework [Azariadis (1996, pp. 460)].

[10] Ogaki, Ostry, and Reinhart (1996) only analyse balanced-growth equilibria though the main implications of the model concern the transition process to an asymptotic growth equilibrium [Ogaki, Ostry, and Reinhart (1996, pp. 60/61)].

is analysed comprehensively. The quantitative convergence implications are investigated in addition to the qualitative convergence implication. In section 4 the basic model is extended by diminishing marginal returns to capital as well as by the general meaning of policy-induced distortions. Section 5 summarises and concludes with some final considerations.

2 The subsistence level of consumption

Subsistence is a widely-used concept with varying meanings and definitions. Sharif (1986) comprehensively surveys the concept, its importance in the context of different theories, and its measurement. Subsistence as a mode of production can be distinguished from subsistence as a mode of consumption. The first is usually defined as (mostly agricultural) production for home-consumption, while the latter denotes a standard of living that allows for the satisfaction of the minimum (physical and probably mental) basic needs of life. Three methods of the determination of the subsistence level of consumption can be distinguished: (a) The direct observation of the provision of subsistence consumption levels by the society (subsistence as a social norm), (b) the scientific estimation of the physiological and mental requirements for sustaining life [Stigler (1945), World Bank (1990)], and (c) the determination of subsistence consumption according to declarations or economic behaviour of individuals.[11]

The interpretation of subsistence as a mode of consumption corresponds to the concept of the poverty line, which is used to identify that part of the population which is regarded as absolutely poor. The theory of (absolute) poverty measurement is extensively based on the concept of the poverty line.[12] In order to determine the poverty line, usually a list of goods and services necessary for subsistence is made. Subsequently, that income which is necessary to purchase this basket of goods and services is calculated [e.g., Atkinson (1987, p. 930)]. Accordingly, all people who dispose of an income which falls short of the poverty line are regarded as absolutely poor. The World Bank (1990) uses a consumption-based poverty line in order to define absolute poverty. The poverty line comprises two elements: (a) the expenditure necessary to realise a minimum standard of nutrition and other basic necessities and (b) a further amount reflecting the cost of participation in everyday life of society [World

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[11] Sharif (1986, p. 568) reports the results of an empirical study which estimates a value of 0.6 for the income elasticity of "declared subsistence" with respect to per capita income. This is an empirical indication for the possible endogenity of the subsistence level of consumption.

[12] Absolute poverty refers to material needs irrespective of distributional issues, while relative poverty refers to income inequality. The poverty line is the basis for most measures of income poverty like the head count index, the poverty gap, and the severity of poverty [e.g., Ravallion (1992)].

Bank (1990, p. 26)].[13] Beside this distinction, two poverty lines are employed in order to distinguish between the "poor" and the "extremely poor". The lower poverty line amounts to $275, while the upper poverty line amounts to $370 in 1985 PPP prices [World Bank (1990, p. 27)]. In addition, Ben-David (1994, p. 11) reports calculations from Stigler (1945), who estimates the subsistence level of consumption defined as the least-cost requirement for sustaining an individual's dietary needs as approximately $300 a year (in 1980 US-$).

The relevance of the obvious requirement of subsistence consumption for economic growth is straightforward. The requirement of subsistence consumption restricts the ability to save. Consequently, within the framework of all growth models which explain growth as the result of the accumulation of the reproducible factors, the restriction of the ability to save crucially influences the process of growth. Intuitively, this influence is greater the nearer an economy is located at subsistence. How appropriate or relevant is this concept? There are clearly other heavy burdens that can inhibit growth in these countries: Poor and deteriorating infrastructure, a bad institutional set-up, and macroeconomic instability. However, if a large part of the population is concerned with nothing else but staying alive, these other issues are of minor importance for the poorest economies [Ben-David (1994, pp. 10/11)].

In order to obtain an impression of the empirical relevance of subsistence consumption, the subsequent table 1 displays the proportional difference between per capita income and the subsistence level of consumption. The latter is first identified with the lower (proportional difference I) and second with the upper poverty line (proportional difference II) as used by the World Bank (1990). The third column shows that per capita income exceeds the lower poverty line for the group of low-income economies only marginally; the proportional difference I amounts to 0.14. Moreover, per capita income even falls short of the upper poverty line as shown in the fourth column; the proportional difference II amounts to -0.16 for the same group of economies. Subsistence consumption considerations seem to be of minor importance for the middle-income economies [lower-middle income economies: 0.80 (prop. diff. I) and 0.73 (prop. diff. II), upper-middle-income economies: 0.92 (prop. diff. I) and 0.89 (prop. diff. II)]. For the high-income economies the requirement of subsistence consumption is nearly irrelevant [0.98 (prop. diff. I and II)].

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[13] In addition, the World Bank (1990) stresses that the second component probably varies from society to society as well as with the level of per capita income [World Bank (1990, p. 27)].

Per capita income in excess of subsistence

Group of countries[a] *(number of countries)*	*GNP per capita*[b]	*Proportional difference I*[c]	*Proportional difference II*[d]
Low-income economies (36)	320	0.14	-0.16
China and India (2)	340	0.19	-0.32
Other low-income (34)	280	0.02	-0.09
Lower-middle-income economies (34)	1,380	0.80	0.73
Upper-middle-income economies (14)	3,240	0.92	0.89
High-income economies (25)	17,080	0.98	0.98

Table 1: Per capita income in excess of subsistence, source: World Bank (1990, pp. 178).

a Country groups: Low-income economies: GNP per capita $\leq$ \$545, Lower-middle-income economies: \$545 < GNP per capita $\leq$ \$2,200, Upper-middle-income economies: \$2,200 < GNP per capita < \$6000 [World Bank (1990, p. x)].
b in 1988 Dollars.
c Proportional difference between GNP per capita and the lower poverty line.
d Proportional difference between GNP per capita and the upper poverty line.

The meaning of the requirement of subsistence consumption for intertemporal consumption decisions can be easily formalised by means of an intertemporal Stone-Geary utility function. The subsistence level of consumption ($\bar{c} > 0$) is interpreted as that amount of consumption which is a necessary prerequisite for sustaining life. Solely consumption which exceeds the subsistence level creates well-being, that is the instantaneous utility function is defined on the range $c \geq \bar{c}$:[14]

$$U[\{c(t)\}] = \int_0^\infty \frac{[c(t)-\bar{c}]^{1-\theta}-1}{1-\theta} \cdot e^{-(\rho-n)\cdot t} dt . \qquad (2.1)$$

The instantaneous utility function in (2.1) is twice continuously differentiable and strictly concave [$u'(c) > 0$, $u''(c) < 0$].[15] The intertemporal utility function assumes that consumption is additively separable and that the value of the utility functional converges to a finite value which requires $\rho > n$. The usual CIES function can be regarded as a special case of (2.1) with $\bar{c} = 0$, that is the requirement of subsistence consumption is ignored. This might approximately be justified in the case of developed countries. However, for developing

[14] The so-called Stone-Geary utility function is a generalisation of the Cobb-Douglas utility function. It was developed by Klein and Rubin (1947-48) and Samuelson (1947-48) and econometrically applied by Stone (1954) and Geary (1950).

countries subsistence consumption considerations are of major importance and should be taken into account within theoretical analyses. The instantaneous Stone-Geary utility function implies a constant elasticity of the marginal utility with respect to consumption in excess of subsistence which equals $\eta(c-\bar{c}) \equiv -\frac{u''(c-\bar{c})\cdot(c-\bar{c})}{u'(c-\bar{c})} = \theta$. However, with respect to consumption the elasticity of the marginal utility is $\eta(c) \equiv -\frac{u''(c)\cdot(c)}{u'(c)} = \frac{\theta \cdot c}{c-\bar{c}}$. Consequently, the IES for two immediate points in time, which equals the inverse of the elasticity of the marginal utility, reads as follows:[16]

$$\sigma(c) \equiv -\frac{u'(c)}{u''(c)\cdot c} = \frac{c-\bar{c}}{\theta \cdot c} \tag{2.2}$$

Several properties of the IES are noteworthy: (a) The IES is zero for consumption equal to the subsistence level of consumption, (b) the IES increases with the level of consumption $c(t)$, hence (2.1) does not belong to the class of CIES utility functions, and (c) the IES asymptotically converges to θ^{-1} as consumption per capita grows without bound. These properties make good economic sense. If income equals the subsistence level of consumption, the individual is simply unable to substitute consumption intertemporally and the IES equals zero. For an increasing income level the individual must first achieve subsistence consumption letting intertemporal considerations guide its decisions only for that portion of the budget which exceeds subsistence.

3 A linear growth model with subsistence consumption

3.1 The model

The fundamental importance of the requirement of subsistence consumption for the process of growth mainly relevant to low-income countries is analysed. For this a linear one-sector growth model with Stone-Geary preferences is employed. The linear growth model is linear in the sense that the time rate of change of the reproducible factor is linear in its level. More

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[15] In addition, the Stone-Geary utility function is non-homothetic and implies linear expenditure systems. It is consistent with linear Engel curves and an elasticity of consumption expenditures with respect to income which is smaller than one [e.g., Silberberg (1990, pp. 406) and Chung (1994, pp. 20)].

[16] Maußner and Klump (1996, pp. 120/121) show that the IES for two immediate point in time equals the inverse of the elasticity of marginal utility.

generally, in the case of models with more than one reproducible input, the time rate of change of each input is a constant-returns-to-scale function in the levels of input.[17] The linear growth model can be regarded to represent the basic version of a broad class of endogenous growth models including the Uzawa-Lucas model.[18] The notion of the requirement of subsistence consumption is formalised by the use of a Stone-Geary utility function. The economy considered is closed and the representative consumer-producer household is assumed to maximise its dynastic lifetime utility. The corresponding dynamic optimisation problem is a concave, infinite time problem of optimal control with a bounded control set:[19]

$$\max_{\{c(t)\}} \int_0^{\infty} \frac{[c(t)-\bar{c}]^{1-\theta}-1}{1-\theta} \cdot e^{-(\rho-n)\cdot t} dt,$$

$$\text{s. t.} \quad \dot{k}(t) = A \cdot k(t) - (\delta + n) \cdot k(t) - c(t),$$

$$k(0) = k_0,\ k(t) \geq 0,$$

$$\bar{c} \leq c(t) \leq A \cdot k(t). \tag{3.1}$$

The sole control variable is per capita consumption (c) and the sole state variable is per capita capital (k). Intertemporal preferences are described by the infinite sum of the discounted instantaneous utility, where $\bar{c}$ denotes the subsistence level of per capita consumption, θ a constant preference parameter, ρ the individual time preference rate, and n the constant growth rate of population, respectively. As described above, the instantaneous utility function is strictly concave and possesses a variable elasticity of the marginal utility with respect to consumption, that is a variable IES. Gross output (y) is a linear function of capital per capita. $y = A \cdot k$, where k denotes the stock of capital per capita and A a constant technology parameter. The absence of diminishing returns is crucial for the generation of endogenous growth and can be justified mainly by two interpretations: First, capital is thought

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[17] Romer (1989, pp. 104) notes that these models have a balanced-growth path along which the growth rates are constant.

[18] The linear growth model can be obtained from a Uzawa-Lucas economy that follows a balanced-growth path, has no externalities, and is characterised by identical technologies in the production of physical and human capital by aggregating both types of capital [Rebelo (1992, p. 7)]. Additionally, the linear model can be interpreted as the basic version of an endogenous growth model which emphasises an increasing variety or quality of inputs [Pack (1994, p. 56)].

[19] For a presentation of optimal control theory with bounded control sets see Kamien and Schwartz (1990, section 10). Especially for optimal control theory with state-dependent inequality constraints on the control variable see Feichtinger and Hartl (1986, chapter 6).

to exhibit positive spill-over effects [Romer (1986)]. Second, capital can be interpreted broadly to include human capital as well as physical capital [Rebelo (1991)].

In order to derive the first-order conditions for an optimal solution, the Lagrangian and current-value Hamiltonian for the dynamic problem (3.1) are set up (the time index is suppressed in the following):

$$L(c,k,\lambda,\nu_1,\nu_2) = H(c,k,\lambda) + \nu_1 \cdot [A \cdot k - c] + \nu_2 \cdot (c - \bar{c}) \tag{3.2}$$

$$H(c,k,\lambda) = u(c - \bar{c}) + \lambda \cdot [A \cdot k - (\delta + n) \cdot k - c] \tag{3.3}$$

The application of the maximum principle leads to the first-order conditions, where ν_1 and ν_2 denote the dynamic Lagrangian multipliers associated with each of the inequality constraints stated in (3.1):[20]

$$\frac{\partial L}{\partial \lambda} = \dot{k} = A \cdot k - (\delta + n) \cdot k - c, \tag{3.4}$$

$$\dot{\lambda} = \lambda \cdot (\rho - n) - \frac{\partial L}{\partial k} = \lambda \cdot [\rho + \delta - A] - \nu_1 \cdot A, \tag{3.5}$$

$$\frac{\partial L}{\partial c} = (c - \bar{c})^{-\theta} - \lambda - \nu_1 + \nu_2 = 0, \tag{3.6}$$

$$\nu_1 \geq 0, \qquad \nu_1 \cdot [A \cdot k - c] = 0, \tag{3.7}$$

$$\nu_2 \geq 0, \qquad \nu_2 \cdot (c - \bar{c}) = 0. \tag{3.8}$$

The first-order conditions (3.6), (3.7), and (3.9) indicate the basic possibility of corner and interior solutions: (a) A corner solution with the optimal choice of the control variable reaching its lower bound requires $(c - \bar{c})^{-\theta} \leq \lambda$. With respect to (3.5) it is clear that this corner solution will result whenever $A - \delta - \rho < 0$. (b) The optimal control only reaches its upper bound if $(c - \bar{c})^{-\theta} \geq \lambda$. As long as the marginal utility exceeds the current-value shadow price of capital per capita, an increase of consumption to the upper limit of the control set is rational. (c) An interior solution is obtained for $(c - \bar{c})^{-\theta} = \lambda$, that is along the optimal path the marginal utility equals the current-value shadow price of capital per capita.

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[20] Because the Hamiltonian is concave in the control and the state, the necessary conditions are also sufficient for a maximum. In addition to the first-order conditions, an optimal trajectory must satisfy the transversality condition: $\lim_{t \to \infty} e^{-(\rho - n) \cdot t} \cdot \lambda(t) \cdot k(t) = 0$.

The dynamics of consumption per capita in the case of interior solutions can be obtained by differentiating $(c-\bar{c})^{-\theta}=\lambda$ with respect to time, subsequently dividing the result by the original relation, eliminating the shadow price using equation (3.5), and noting that ν_1 is zero for interior solutions:

$$\frac{\dot{c}}{c}=\frac{c-\bar{c}}{\theta\cdot c}\cdot(A-\delta-\rho). \tag{3.9}$$

Equation (3.9) is the *Keynes-Ramsey rule* of optimal consumption for the case of a linear technology and Stone-Geary preferences. The first term on the right-hand side represents the variable IES. The IES reflects the ability *as well as* the willingness to substitute consumption over time.[21] The value of the IES is dependent on the proportional difference between per capita consumption and the subsistence level of consumption. With a rising level of consumption per capita, the IES increases and the individuals choose a steeper consumption path. In the case of unbounded growth, the IES eventually converges to a constant value θ^{-1}.

The differential equation (3.9) together with the equation of motion of capital per capita stated in (3.1) constitute the canonical equations of the dynamic problem. It represents an autonomous differential equation system that governs the dynamics of the economy for interior solutions.[22] An analytical solution to this linear first-order differential equation system can be easily found (see appendix A1):

$$c(t)=\bar{c}+\left(c(0)-\bar{c}\right)\cdot e^{\theta^{-1}(A-\delta-\rho)\cdot t}, \tag{3.10}$$

$$k(t)=\bar{k}+\left(k(0)-\bar{k}\right)\cdot e^{\theta^{-1}(A-\delta-\rho)\cdot t} \qquad \text{with} \qquad \bar{k}\equiv\frac{\bar{c}}{(A-\delta-n)} \tag{3.11}$$

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[21] Within the frame of the CIES-preference formulation the IES is usually interpreted to merely reflect the willingness to substitute consumption intertemporally. Again this turns out to represent a special case with $\bar{c}=0$.

[22] The terminology is as follows: The differential equation system consisting in the control and the state variable is usually called the canonical system, while the differential equation system consisting in the state and the costate variable is called the Hamiltonian system of the dynamic problem [e.g., Gandolfo (1996, p. 382) and Romer (1989, p. 82)].

3.2 Implications

3.2.1 Dynamic equilibria

The solutions (3.10) and (3.11) immediately indicate that there are, in principle, three possible dynamic equilibria: Two steady states with consumption and capital per capita being constant and one asymptotic balanced-growth equilibrium with consumption and capital per capita asymptotically growing with constant rates of growth.

The first steady state is the *subsistence steady state* with consumption per capita equal to the level of subsistence consumption ($\overline{c}$) and capital per capita equal to the level of subsistence capital ($\overline{k}$). The net saving rate per capita is zero and the output produced in each period just suffices to cover subsistence consumption as well as gross investment in order to replace a reduction in the stock of capital per capita resulting from depreciation and population growth. The subsistence steady state can be the result of two constellations: (a) The net marginal product of the reproducible factors falls short of the time preference rate. In this case, the *subsistence steady state* is a (saddle-point) stable steady-state equilibrium and meets the criteria of a *poverty trap* or *low-level-development trap.*[23] The individuals are simply not willing to postpone consumption because the net marginal product is low relative to the time preference rate. Irrespective of the initial conditions, the subsistence steady state is the optimal long-run solution of the dynamic problem (3.1). (b) Provided that the initial endowment with capital per capita equals the subsistence level of capital per capita, the economy must persist within this low-level-development equilibrium.[24] However, whenever the net marginal product of capital exceeds the time preference rate, this low-level-development equilibrium is unstable. Hence, it does not represent a poverty trap. Even a marginal increase in the endowment of productive resources per capita due to an external resource transfer resulting from running into debt or development aid induces sustained growth. However, within this model it is impossible to catch up with rich countries and the process of convergence to the asymptotic balanced-growth path might be extraordinarily long (see section 3.3).

The second steady state might be considered to represent merely a theoretical possibility which is not very probable. If the net marginal product of capital just equals the time

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[23] A poverty trap or low-level-development trap requires that the economy possesses multiple dynamic equilibria, the lowest of these which is stable is called a poverty trap [Azariadis (1996, pp. 450/451)].

preference rate, the economy displays zero growth with consumption and capital probably above subsistence. An external resource transfer would increase the level of consumption and capital per capita but cannot stimulate sustained growth. This type of equilibrium with zero growth and consumption above subsistence will be worked out in a more plausible way in section 4.2.1.

The third possibility is represented by an asymptotic balanced-growth equilibrium. Whenever the net marginal product of capital exceeds the time preference rate and the economy starts with an initial stock of capital above the subsistence level of capital, the economy pursues unbounded growth. The solutions (3.10) and (3.11) immediately show that the rates of growth of consumption and capital per capita converge towards their common asymptotic balanced-growth-equilibrium value:

$$\lim_{t\to\infty}\frac{\dot{c}}{c}=\lim_{t\to\infty}\frac{\dot{k}}{k}=\theta^{-1}\cdot(A-\delta-\rho). \tag{3.12}$$

In addition, the solutions (3.10) and (3.11) allow the formulation of the stable arm of the saddle-path leading towards the asymptotic balanced-growth equilibrium or – depending on the constellation of parameters – to the subsistence steady state. This stable arm of the saddle-path is called the policy function and shows the optimal choice of the control variable as a function of the state variable:[25]

$$c=z\cdot(k-\bar{k})+\bar{c}, \qquad \text{where} \qquad z\equiv A-\delta-n-\theta^{-1}\cdot(A-\delta-\rho)>0. \tag{3.13}$$

Given an initial condition for capital per capita, the policy function immediately shows the corresponding value of consumption per capita so that all necessary first-order conditions (including the transversality condition) are satisfied. In order to give a clear interpretation for all possible dynamic equilibria, the policy function is slightly reworded to:

$$c=(A-\delta-n)\cdot k-\theta^{-1}(A-\delta-\rho)\cdot(k-\bar{k}). \tag{3.14}$$

Equation (3.14) illustrates that consumption is chosen in such a way that capital per capita is constant, $\dot{k}=(A-\delta-n)\cdot k-c=0$, whenever the capital stock equals its subsistence

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[24] This low initial endowment with productive resources might be the result of a war or a natural catastrophe, for example.

[25] For a description of the concept "policy function" see Barro and Sala-i-Martin (1995, p. 76).

level or the net marginal product of capital equals the time preference rate. Figure 1 shows the phase diagram of the linear growth model with Stone-Geary preferences, which illustrates the possibilities of multiple dynamic equilibria depending on the constellation of parameters.

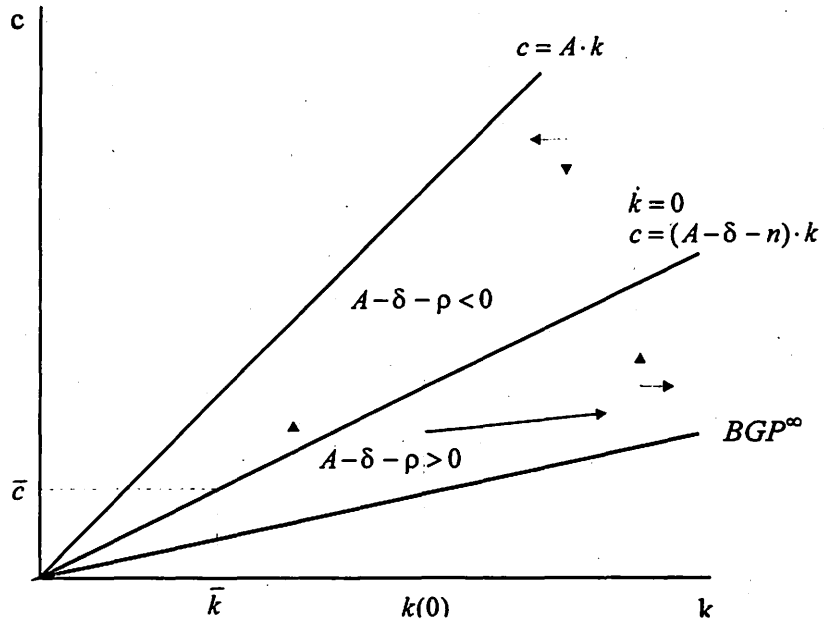

Figure 1: Phase diagram of the linear growth model with Stone-Geary preferences.

The three rays starting from the origin represent the production function, the $\dot{k} = 0$-locus, and the asymptotic balanced-growth path (BGP^{∞}), respectively. Starting with an initial stock of capital per capita above the subsistence level [$k(0) > \bar{k}$], figure 1 shows in accordance with the policy function (3.14) three possibilities: (a) For $A - \delta - \rho < 0$ consumption is chosen above the $\dot{k} = 0$-locus and, consequently, the stable saddle-path leads the economy to the subsistence steady state ($\bar{c}$, $\bar{k}$). (b) For $A - \delta - \rho = 0$ consumption is chosen to be located on the $\dot{k} = 0$-locus and the economy rests at the initial point [$c(0)$, $k(0)$]. (c) For $A - \delta - \rho > 0$ consumption is chosen below the $\dot{k} = 0$-locus and the starting point is located on the stable arm of the saddle-path which converges towards the BGP^{∞}.

3.2.2 Transitional dynamics

In what follows, the transition process to an asymptotic balanced-growth equilibrium is investigated. In this case, the solutions (3.10) and (3.11) show that the growth rates of consumption and capital per capita monotonically increase and eventually converge to their

asymptotic balanced-growth values. The growth rate of per capita income varies positively with the distance between the current and the balanced-growth-equilibrium state of the economy. Consequently, the model illustrates a fundamental mechanism of (conditional) ß-divergence: In the case of growth, the saving rate increases causing the growth rate of income per capita to rise as well.

Using the policy function (3.13), the net saving rate defined as the relation between net investment and net output reads as follows:

$$s = \frac{A-\delta-\rho}{\theta\cdot(A-\delta-n)}\cdot\frac{k-\bar{k}}{k}. \tag{3.15}$$

It should be noted explicitly that a value of the net saving rate equal to zero implies a constant stock of capital per capita. That is, the amount of gross output not consumed (gross investment) just suffices to replace depreciation ($\delta\cdot k$) and to enlarge the stock of capital in accordance with population growth ($n\cdot k$). The preceding equation shows that the net saving rate is zero whenever the stock of capital equals its subsistence level. As far as the initial capital stock exceeds its subsistence level [$k(0)>\bar{k}$], equation (3.15) demonstrates that the saving rate is positive whenever $A-\delta-\rho>0$ and negative whenever $A-\delta-\rho<0$. In both cases, the saving rate monotonically converges to its dynamic-equilibrium value which reads $s^*=\frac{A-\delta-\rho}{\theta\cdot(A-\delta-n)}$ for $A-\delta-\rho>0$ and $s^*=0$ for $A-\delta-\rho<0$.

Growth theory (especially endogenous growth theory) as well as traditional development theory assigns a dominant role to internal saving and investment for the process of economic growth and development. Within the latter, the saving rate is considered to be determined by the *ability to save* as well as by the *willingness to save* [e.g., Hemmer (1988, pp. 150)]. These two concepts can be assigned directly to the determinants of the saving rate as expressed in (3.15). The ability to save (which is also called "economic or investible surplus") at any moment in time can be defined as the proportional difference between the current and the subsistence level of income. Within the framework of the linear growth model with subsistence consumption, the ability to save is represented by the term $(k-\bar{k})/k$. It is near zero for k near $\bar{k}$ and converges to one as k approaches infinity. In principle, approximately the entire output is disposable for saving as capital per capita grows without bound. The willingness to save, which is the second determinant of the per capita saving rate, is

determined by the preference and technology parameters of the model as well as the growth rate of population and is represented by the term $\frac{A-\delta-\rho}{\theta\cdot(A-\delta-n)}$. The subsequent figure 2 shows the time paths of the saving rate for different starting values of the stock of capital per capita in relation to its subsistence level, that is for different initial abilities to save $a \equiv \frac{k(0)-\bar{k}}{k(0)}$, where the following set of parameters is employed: $A = 0.1$, $\theta = 3$, $\delta = 0.02$, $\rho = 0.01$, $n = 0.03$, $\bar{c} = 2$.[26]

In the case of developed economies, a is sufficiently close to one, so that the analysis of the requirement of subsistence consumption is nearly irrelevant for this group of countries. However, in the case of low-income countries the consideration of subsistence consumption might significantly influence the process of growth. The initial ability to save as expressed by a determines the saving rate chosen at the initial point in time and the subsequent time path, while the willingness to save determines the asymptotic value of the saving rate.

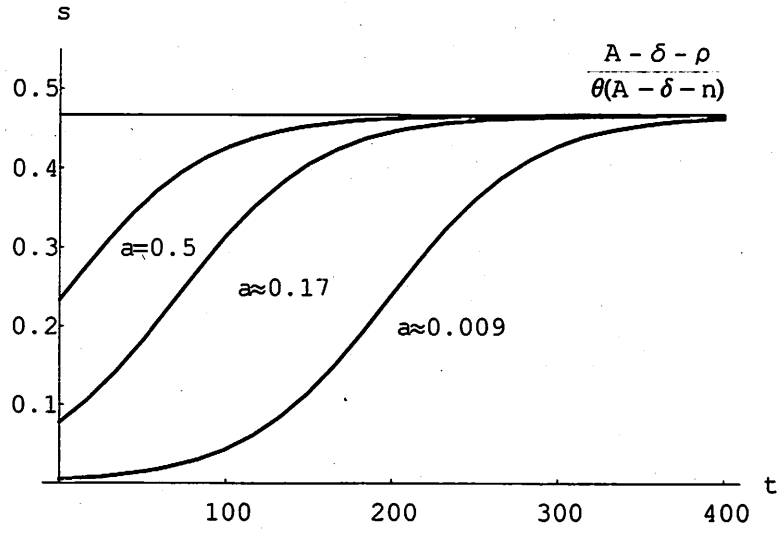

Figure 2: Time paths of the saving rate.

[26] The parameter values are consistent with usual calibrations; see table 2 in appendix A3. Most of these sets of parameters represent calibrations of the neoclassical model and, therefore, the steady-state value of the marginal product of capital is implied by the other parameter values. In the case of the linear growth model, this value must be determined by the choice of A. Kaldor (1961, p. 178) reports a rate of profit on capital for the UK (1870-1914) of 10.5 %.

The rise in the saving and the investment rate causes the growth rate of capital per capita to increase as well. The time path of the growth rate of capital per capita, which equals the growth rate of income per capita, monotonically increases and converges towards its asymptotic balanced-growth-equilibrium value as illustrated in figure 3.

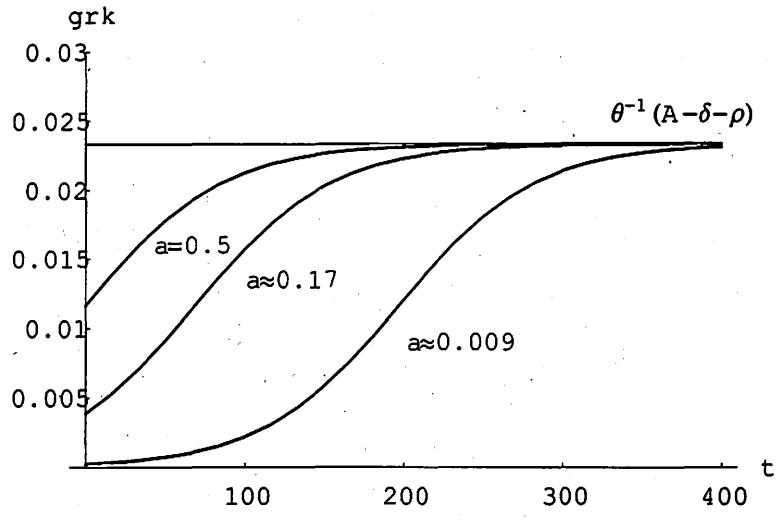

Figure 3: Time paths of the growth rate of capital per capita.

Obviously, the resulting time path of the growth rate of capital per capita crucially depends on the initial stock of capital per capita in relation to its subsistence level (a). The smaller this ratio, the longer it takes for the transition towards the asymptotic balanced-growth equilibrium. For $a \cong 0.009$, the growth rate of capital per capita requires nearly 200 years to eliminate half the distance between its initial and its balanced-growth-equilibrium value. The time span required for the adjustment process is extremely long, and the implied speed of convergence is correspondingly small. This will be investigated in more detail within the next section.

Despite its obvious simplicity, the linear growth model with subsistence consumption is extraordinarily useful because it is able to reproduce two of the stylised facts enumerated in the introduction, that is the positive correlation between the saving rate and the level of per capita income [stylised fact (2)] and ß-divergence [stylised fact (3)]. In addition, the model displays important properties which are in line with theoretical and empirical research: (a) As Rebelo (1992) has pointed out, the model is able to explain internationally divergent growth dynamics even in the presence of international capital markets. (b) The model avoids the

counterfactual implication of extraordinarily high interest rates at early stages of economic development which characterises most versions of the neoclassical model [King and Rebelo (1993)]. (c) The model predicts that the IES is low for low incomes per capita and increases thereafter, which is in line with empirical research on the IES [Giovannini (1985) and Ogaki, Ostry, and Reinhart (1996)]. However, the model clearly fails to reproduce the hump-shaped relation between the growth rate and the level of per capita income [stylised fact (4)]. In addition, provided that symmetry with respect to preference and technology parameters is supposed the model has difficulty in explaining the big diversity in growth rates for the group of low-income countries [stylised fact (1)].[27] More specifically, as far as divergent growth experiences are explained as transitional phenomena, the ability to explain these different growth rates essentially depends on the speed of convergence. This issue is investigated within the next section.

3.3 Convergence considerations

Within the context of convergence analyses as a branch of growth theory, two questions arise which are of fundamental interest for the process of growth and development: (a) Does the model imply convergence or divergence in the levels of per capita income between rich and poor economies? For poor countries to catch up with rich countries, it is necessary that the growth rate is higher compared to rich countries. This is the concept of *absolute ß-convergence*, which is specifically defined by a negative relation between the growth rate and the level of per capita income, that is poor countries grow faster on average than rich countries. If differences in the long-run growth rates are explicitly taken into account, the concept of *conditional ß-convergence* applies. Conditional ß-convergence requires that the growth rate of per capita income tends to be negatively related to the distance between the current and the balanced-growth-equilibrium state of the economy.[28] Accordingly, (conditional) *ß-divergence* requires a positive relation between the growth rate of per capita income and the distance between the current and the balanced-growth-equilibrium state of the economy on average. In this case, the difference in the levels of per capita income between

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[27] These shortcomings will be taken up in section 4.

[28] Conditional ß-convergence is a generic property of ordinary autonomous differential equations $\dot{x} = F(x)$ with a stationary equilibrium defined by $F(x^*) = 0$ and does not depend on the specific functional form of the differential equations [Koch (1997, pp. 2/3)]. Beside ß-convergence the concept of σ-convergence exists, which is defined as a reduction in the dispersion of per capita incomes over time. For the relations between the two concepts see Sala-i-Martin (1996).

rich and poor countries grows without bound.[29] (b) What does the model predict about the speed of convergence towards its (probably asymptotic) balanced-growth equilibrium? This second question is meaningful irrespective of the answer to the first question and concerns the quantitative implications of the transition process.[30] The speed of convergence provides important information about whether the emphasis of analysis should be placed on transitional or balanced-growth dynamics. In addition, beside the qualitative implication of ß-convergence or ß-divergence, the speed of convergence is an important quantitative property of each model which represents an empirically refutable hypothesis.[31]

The linear growth model with subsistence consumption implies *conditional ß-divergence*. The growth rate of per capita income increases with the level of per capita income conditioned on a (unique) balanced-growth equilibrium or – in the terminology of econometrics – after controlling for the influence of those variables which determine the long-run growth rate. Figure 4 plots the relation between the growth rate (gry) and the level of per capita income (y) based on the same set of parameters used within the preceding section with $a \cong 0.009$.

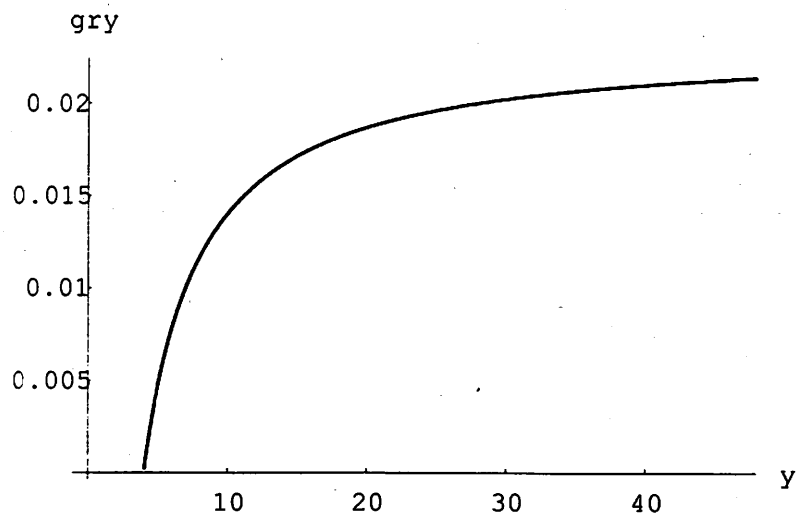

Figure 4: ß-divergence.

[29] Most growth models imply conditional ß-convergence like the Solow-Swan as well as the Ramsey-Cass-Koopmans model [e.g., Barro and Sala-i-Martin (1995, pp. 36 and pp. 80)]. An example for a growth model with ß-divergence is Romer (1986).

[30] A model might contain the qualitative implication of a positive relation between the growth rate and the level of per capita income (ß-divergence) during the transition to the balanced-growth equilibrium. At the same time it is meaningful to ask how fast it converges to its balanced-growth equilibrium.

[31] For example, Mankiw, Romer, and Weil (1992) demonstrate that the neoclassical growth model is quantitatively consistent with the empirical estimates of the speed of convergence if it is extended to explicitly include human capital. In addition, for this see Barro and Sala-i-Martin (1995, chapter 3) and Klenow and Rodríguez-Clare (1997, p. 604).

The reason for this divergence mechanism, which might in analogy to the neoclassical convergence mechanism be labelled as "*subsistence-divergence mechanism*", is simply that in the case of growth the ability to save rises continuously. Consequently, the saving and investment rates increase as well. Unlike the neoclassical model, the linear growth model displays a constant marginal and average product of capital. As a result, the productive contribution of the entire stock of the reproducible factors remains constant and an increase in the investment rate fully translates into an increase in the growth rate of output.

Empirical analyses use the following regression equation in order to test for conditional ß-convergence or conditional ß-divergence within a cross-sample of countries [Sala-i-Martin (1996, p. 1027)]:

$$\gamma_{i,t,t+T} = a_0 - a_1 \cdot \ln[x_{i,t}] + \mathbf{v}\mathbf{x}_{i,t} + \varepsilon_{i,t,t+T} \qquad \text{with} \qquad a_1 \equiv \frac{1-e^{-\lambda \cdot T}}{T}, \qquad (3.16)$$

where $\gamma_{i,t,t+T}$ denotes the average growth rate of per capita income of country i for the period $[t,T]$, a_0 a positive constant, $x_{i,t}$ the level of per capita income, $\mathbf{v}$ a vector of further regression coefficients, $\mathbf{x}$ a vector of conditioning or control variables which captures the determinants of the long-run growth rate, and ε an independently and identically distributed error term with mean zero, respectively.[32] Conditional ß-convergence applies if $a_1, \lambda > 0$ and conditional ß-divergence applies if $a_1, \lambda < 0$. If the linear growth model with subsistence consumption describes an important mechanism of growth, one should expect to find a negative a_1 and λ for the group of low-income countries.

In the next step consider the quantitative implications of the transition process. For this, the current state of the economy is expressed in terms of a variable $x(t)$ so that this variable monotonically converges to a stationary value denoted by x^*, which represents the balanced-growth-equilibrium state of the economy. The *instantaneous speed or rate of convergence* can reasonably be defined as (the negative of) that share of the distance between the current and the balanced-growth-equilibrium state of the economy which is eliminated during the current period [Romer (1996, p. 21) and Ortigueira and Santos (1997, p. 390)]:

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[32] The average growth rate can be considered to consist of two elements: The long-run growth rate is represented by $a_0 + \mathbf{v}\mathbf{x}$, while that part of growth which reflects transitional dynamics is shown by $a_1 \cdot \ln[x_{i,t}]$.

$$\lambda(t) \equiv -\frac{\dot{x}(t)}{x(t)-x^*}.$$ [33] (3.17)

The instantaneous speed of convergence shown in (3.17) is only constant if the underlying differential equation $\dot{x}(t) = F[x(t)]$ is linear. For non-linear differential equations, the speed of convergence varies with the distance $x(t)-x^*$.[34] Because the economy in question exhibits unbounded growth, the distance between the current and the balanced-growth-equilibrium state of the economy $[x(t)-x^*]$ cannot be expressed in terms of the original variables, that is the control (per capita consumption) or the state variable (per capita capital). This distance must rather be expressed in terms of variables which are functions of the original variables and, at least asymptotically, converge towards stationary values. This is true, for example, for the growth rate of capital per capita as well as the consumption-capital ratio.

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[33] The regression equation (3.16) is consistent with and can be derived from the definition of the speed of convergence as shown in (3.17). Assuming that the speed of convergence is constant, (3.17) can be expressed as a linear differential equation in the variable $x(t)-x^*$,

$$\dot{x}(t) = -\lambda \cdot [x(t)-x^*], \quad \text{(i)}$$

with the following solution:

$$x(t)-x^* = [x(0)-x^*]\cdot e^{-\lambda \cdot t}. \quad \text{(ii)}$$

Rearranging (ii) gives:

$$x(t)-x(0) = -(1-e^{-\lambda \cdot t})\cdot [x(0)-x^*]. \quad \text{(iii)}$$

Empirical analyses usually employ the logarithm of the variables under study. In the present case, $\ln(x)$ might denote the logarithm of per capita income. Accordingly, equation (iii) can be written as

$$\frac{\ln[x(t)]-\ln[x(0)]}{t} = -\frac{1-e^{-\lambda \cdot t}}{t}\cdot \ln[x(0)/x^*], \quad \text{(iv)}$$

which is the basis of the regression equation (3.16). In addition, (ii) implies the following definition of the speed of convergence, which is also frequently used [e.g., Romer (1996, p. 22)]:

$$\lambda = -\ln\left(\frac{x(t)-x^*}{x(0)-x^*}\right)/t. \quad \text{(v)}$$

However, (v) shows the average speed of convergence for the time interval $[0, t]$. In addition, (v) demonstrates the relation between the (average) speed of convergence and the half-life time ($t_{0.5}$):

$$t_{0.5} = -\ln(0.5)/\lambda. \quad \text{(vi)}$$

[34] Because the differential equations which describe the dynamics of the economy are mostly non-linear, the speed of convergence is usually calculated on the basis of a first-order Taylor series approximation around a stationary value and is, therefore, only valid within a "small neighbourhood" around the stationary value [Barro/Sala-i-Martin (1995, pp. 36/37) and Romer (1996, pp. 21/22)]. For a method which allows the calculation of the instantaneous rate of convergence without having the solution of the differential equation under study see Koch (1997, pp. 6).

The instantaneous speed of convergence for the linear growth model with Stone-Geary preferences is expressed on the basis of the consumption-capital ratio. For this, both the c/k-ratio and the logarithm of the c/k-ratio are employed. Both variables monotonically move towards their stationary point, which reads $A-\delta-n-\theta^{-1}(A-\delta-\rho)$ in the case of the c/k-ratio and $\ln\left(A-\delta-n-\theta^{-1}(A-\delta-\rho)\right)$ in the case of the logarithm of the c/k-ratio. The rate of convergence calculated on the basis of the c/k-ratio will show the instantaneous as well as the *true* rate of convergence. This rate of convergence is "true" in the sense that it describes the actual speed of convergence of the economy expressed in original variables.[35] However, empirical estimates on the speed of convergence are usually based on logarithmic variables. Therefore, the logarithm of the c/k-ratio is additionally employed in order to obtain a perfect comparability between the theoretically implied and the empirically estimated values of the speed of convergence.

The subsequent figure 5 plots the time path of the instantaneous speed of convergence both based on the c/k-ratio $[\lambda(c/k)]$ and based on the logarithm of the c/k-ratio $[\lambda(\ln(c/k))]$, where the following set of parameters is employed: $A=0.1$, $\theta=3$, $\delta=0.02$, $\rho=0.01$, $n=0.03$, $\bar{c}=2$, $a\cong 0.009$.[36] As one would expect, the speed of convergence based on the logarithm of the c/k-ratio is higher compared to the speed of convergence based on the original c/k-ratio. However, the qualitative behaviour of both rates of convergence is identical and the quantitative difference does not seem to be substantial.

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[35] Ortigueira and Santos (1997, p. 383) note that generically along a stable path to the balanced-growth equilibrium all endogenous variables must converge with the same speed towards the balanced-growth equilibrium.

[36] A positive value of the speed of convergence for the time series of c/k and $\ln(c/k)$ does not contradict the qualitative implication of conditional ß-divergence as noted above. The analytical expressions which underlie the instantaneous rates of convergence as shown in figure 5 are presented in appendix A2.

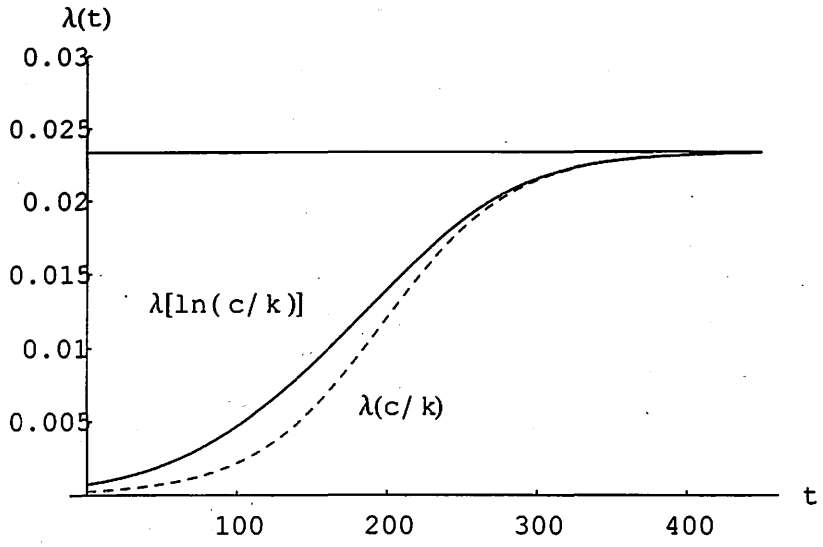

Figure 5: The instantaneous speed of convergence.

The asymptotic value of the speed of convergence shown in figure 5 is consistent with most empirical estimates on the speed of convergence which lie between two and three percent.[37] However, figure 5 demonstrates that the speed of convergence locally around the asymptotic balanced-growth equilibrium (the asymptotic value is about 2.3 %) is not a good estimate for the global converging behaviour.[38] At early stages of economic development, the speed of convergence is extraordinarily low (below 0.1 % per year at the initial point in time) and increases only slightly. As a result, the time span required for the transition process towards the asymptotic balanced-growth equilibrium is extremely long. As demonstrated above, the saving rate and the growth rate of capital per capita require a very long time span for the transition towards their balanced-growth-equilibrium values with half-life times of about 200 years. Therefore, the big diversity in growth rates [stylised fact (1)] can partly be explained as representing a transition phenomenon. However, if international symmetry with respect to preferences and technology is supposed, the range of possible growth rate

[37] The values of the empirical estimates are positive indicating (conditional) ß-convergence. See Barro (1991), Barro and Sala-i-Martin (1992), and Mankiw, Romer, and Weil (1992).

[38] In contrast, Ortigueira and Santos (1997) demonstrate that the local speed of convergence around the dynamic equilibrium is a good estimate for the global convergence behaviour within a wide range of capital per capita in the case of the neoclassical and the Uzawa-Lucas model [Ortigueira and Santos (1997, pp. 390/391)].

differences is restricted.[39] The economy converges asymptotically with a constant speed of convergence towards its asymptotic balanced-growth equilibrium, that is the speed of convergence is asymptotically constant either.

The subsequent figure 6 shows the time paths of the instantaneous speed of convergence based on the c/k-ratio for alternative parametrisations of the marginal product of capital (A) and the elasticity of the marginal utility (θ).

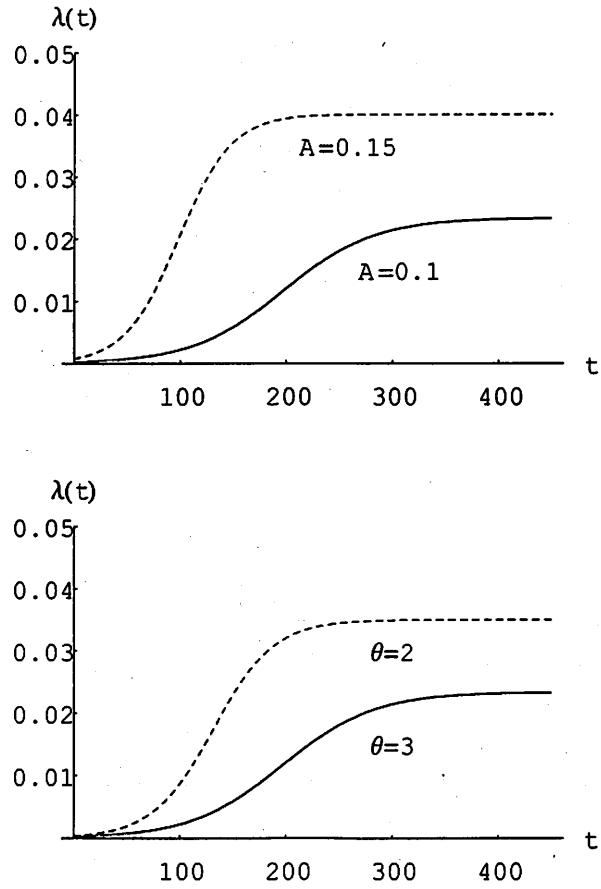

Figure 6: Instantaneous speed of convergence – comparative dynamics.

In general, each variation of parameters which increases the growth rate of income per capita results in a reduction of the time span required for the transition process. As a result, the speed of convergence increases. According to the first graph in figure 6, the speed of convergence increases with the (marginal and average) productivity of capital. A rise in the

[39] This can be illustrated as follows. Let us define a DC as an economy with a per capita income of units of y^* or less. Then the range of possible growth rates of per capita income which can be explained by the model in hand varies from 0 to $gry(y^*)$.

(marginal) rate of return to capital increases the willingness to save. In addition, a rise in the average product of capital increases the productive contributions of capital to output growth and reduces the time span required for the adjustment process to the balanced-growth equilibrium. The second graph displays a negative relation between the speed of convergence and the elasticity of the marginal utility. A falling elasticity of the marginal utility (a rising IES) increases the willingness to save. The saving rate decreases causing the growth rate of per capita income to decrease as well. As a result, growth increases, the time span for transition shortens, and the speed of convergence rises.

4 Extensions of the basic linear growth model

The linear growth model discussed in the previous section is able to reproduce the positive correlation between the saving rate and the level of per capita income [stylised fact (2)] as well as ß-divergence [stylised fact (3)]. However, supposing that the symmetry assumption applies, the model has difficulty in explaining the big diversity in growth rates [stylised fact (1)] and it clearly fails to reproduce the hump-shaped relation between the growth rate and the level of per capita income [stylised fact (4)]. Consequently, the basic model will be extended in order to provide a potential explanation for the remaining empirical regularities.

4.1 The model

The basic linear growth model with Stone-Geary preferences is extended in two directions. First, the presence of fixed factors such as land or (raw) labour is important at low incomes but becomes negligible as income grows without bound. As a result, the production function is allowed to exhibit diminishing returns to the reproducible factor, which eventually converge to a lower bound (Jones-Manuelli technology) [Jones and Manuelli (1990), Easterly, King, Levine, and Rebelo (1992, p. 13), and Easterly (1994, pp. 525)]. Second, in order to capture the influence of national government policy, a general index reflecting several distortions is incorporated into the production function. The aggregate production function of the private sector for gross output "net of distortions" is assumed to read as follows:

$$y(t) = (1-\tau)\cdot\left(A\cdot k(t) + B\cdot k(t)^{\alpha}\right), \tag{4.1}$$

where $y(t)$ denotes gross output per capita at time t; A, B, and α (with $0<\alpha<1$) denote constant technology parameters. The distortions summarised in τ (with $0\le\tau<1$) reduce the marginal product of the reproducible factors and can be considered to result mainly

from bad government policies which are frequently found especially in DCs. The extent of distortions is considered to vary internationally and results from income or investment taxes [Barro (1990), Jones and Manuelli (1990), Rebelo (1991, 1992)],[40] inflation taxes [Easterly, King, Levine, and Rebelo (1992), Easterly (1994)], bad institutional set-ups, which reduce the volume of transactions as a result of a suboptimal level of transaction costs [North (1987) and Rebelo (1992)], sectoral distortions due to unequal taxation of different sectors, dual exchange rate systems, and sectorally discriminating tariffs and import quotas [Easterly, King, Levine, and Rebelo (1992), Rebelo (1992), and Easterly (1994)]. The production function (4.1) exhibits the Jones-Manuelli property, that is the marginal product of the reproducible factors decreases, however, converging to a positive constant as capital grows without bound: $\lim_{k\to\infty} \frac{\partial y}{\partial k} = (1-\tau)\cdot A > 0$ [Jones and Manuelli (1990, pp. 1016) and Jones and Manuelli (1997, pp. 81)].[41]

The model with subsistence consumption and a Jones-Manuelli technology including a general index of distortions reads as follows:

$$\max_{\{c(t)\}} \int_0^\infty \frac{[c(t)-\bar{c}]^{1-\theta}-1}{1-\theta}\cdot e^{-(\rho-n)\cdot t}\,dt,$$

$$\text{s. t.} \quad \dot{k}(t) = (1-\tau)\cdot\left(A\cdot k(t) + B\cdot k(t)^{\alpha}\right) - (\delta+n)\cdot k(t) - c(t),$$

$$k(0) = k_0,\ k(t) \geq 0,$$

$$\bar{c} \leq c(t) \leq (1-\tau)\cdot\left(A\cdot k(t) + B\cdot k(t)^{\alpha}\right). \qquad (4.1)$$

---

[40] As far as taxes reduce the rate of return to reproducible inputs they lower the rate of growth. However, in order to determine the overall effects on growth and welfare the use of tax revenues is crucial. If tax revenues are "thrown away", growth and welfare decrease. If tax revenues finance public consumption, and public consumption enters the utility function, growth decreases but welfare does not necessarily fall [Rebelo (1992, pp. 10/11)]. If tax revenues finance public goods which are inputs in production, the effect on growth and welfare is unequivocal [Barro (1990)].

[41] The Jones-Manuelli property also applies for a CES function with two inputs and an elasticity of substitution greater than one [Easterly (1994, p. 529) and Jones and Manuelli (1997, p. 82)].

The current-value Hamiltonian for the dynamic optimisation problem (4.1) reads accordingly; only interior solutions are considered and the time index is omitted:

$$H(c,k,\lambda)=\frac{(c-\bar{c})^{1-\theta}-1}{1-\theta}+\lambda\cdot\left[(1-\tau)\cdot\left(A\cdot k+B\cdot k^{\alpha}\right)-(\delta+n)\cdot k-c\right], \qquad (4.2)$$

and the necessary first-order conditions are[42]

$$\frac{\partial H}{\partial \lambda}=\dot{k}=(1-\tau)\cdot\left(A\cdot k+B\cdot k^{\alpha}\right)-(\delta+n)\cdot k-c, \qquad (4.3)$$

$$\dot{\lambda}=\lambda\cdot(\rho-n)-\frac{\partial H}{\partial k}=\lambda\cdot\left[\rho+\delta-(1-\tau)\cdot\left(A-\alpha\cdot B\cdot k^{\alpha-1}\right)\right], \qquad (4.4)$$

$$\frac{\partial H}{\partial c}=(c-\bar{c})^{-\theta}-\lambda=0. \qquad (4.5)$$

Differentiating (4.5) with respect to time, subsequently dividing by the original relation, and eliminating the shadow price using (4.4) yields the optimal growth rate of consumption per capita:

$$\frac{\dot{c}}{c}=\frac{c-\bar{c}}{\theta\cdot c}\cdot\left[(1-\tau)\cdot\left(A+\alpha\cdot B\cdot k^{\alpha-1}\right)-\delta-\rho\right]. \qquad (4.6)$$

The preceding equation shows the Keynes-Ramsey rule of optimal consumption. As in the case of the model in the previous section, the first part on the right-hand side represents the IES, while the second part on the right-hand side shows the difference between net marginal product of capital and the time preference rate. Equation (4.6) indicates that there are multiple dynamic equilibria which will be investigated within the next section. In addition, in the case of growth there are two opposing forces affecting the evolution of the growth rate of per capita consumption. Therefore, it is unequivocal a priori whether the growth rate increases or decreases. The simulation results presented in section 4.2.2 illustrate the transitional dynamics at least for a specific set of parameter values.

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[42] Because the Hamiltonian is concave in the control and the state, the necessary conditions are also sufficient for a maximum. In addition to the first-order conditions, an optimal trajectory must satisfy the transversality condition: $\lim_{t\to\infty} e^{-(\rho-n)t}\cdot\lambda(t)\cdot k(t)=0\cdot$

4.2 Implications

4.2.1 Dynamic equilibria

In order to discuss the implications of the model, the canonical equations of the dynamic problem (4.1) are considered. This autonomous differential equation system governs the dynamics of the economy in the case of interior solutions:

$$\dot{c} = (c - \bar{c}) \cdot \theta^{-1} \cdot \left[(1 - \tau) \cdot \left(A + \alpha \cdot B \cdot k^{\alpha - 1} \right) - \delta - \rho \right], \tag{4.7}$$

$$\dot{k} = (1 - \tau) \cdot \left(A \cdot k + B \cdot k^{\alpha} \right) - (\delta + n) \cdot k - c. \tag{4.8}$$

There are three dynamic equilibria defined by constant growth rates. Specifically, the model displays two steady states with stationary values of consumption and capital per capita and one balanced-growth equilibrium with unbounded growth. With respect to (4.7) it is clear that there are three possibilities for the long-run evolution of consumption per capita: (a) zero growth with consumption at subsistence [$\dot{c} = 0$, $c^* = \bar{c}$], (b) zero growth with consumption above subsistence [$\dot{c} = 0$, $c^* > \bar{c}$], and (c) unbounded growth [$c \to \infty$]. All three possibilities will be discussed in more detail.

The subsistence equilibrium with zero growth and consumption at subsistence can be the result of two constellations: (a) the initial consumption equals its subsistence level [$c(0) = \bar{c}$], the ability to save is zero, and the economy rests in a low-level equilibrium, not necessarily stable. (b) The private net marginal product of capital evaluated at the subsistence capital stock falls short of the time preference rate, that is $(1 - \tau) \cdot \left(A + \alpha \cdot B \cdot \bar{k}^{\alpha - 1} \right) - \delta - \rho < 0$. As a result of the diminishing marginal returns to capital, the validity of the preceding inequality evaluated at the subsistence level of capital implies that this inequality holds for each initial stock of capital above subsistence. In this case, individuals are simply not willing to postpone consumption. The individuals will consider it optimal to reduce the stock of capital per capita and accept a decreasing level of consumption eventually converging to the subsistence equilibrium ($\bar{c}$, $\bar{k}$), which represents a poverty trap in this case. The steady-state value of capital per capita is implicitly defined by

$(1-\tau)\cdot\left(A\cdot\bar{k}+B\cdot\bar{k}^{\alpha}\right)-(\delta+n)\cdot\bar{k}-\bar{c}=0$.[43] This unique steady state is saddle-point stable as is shown in appendix A4.

The second steady state with consumption above subsistence applies whenever the private net marginal product of the capital exceeds the time preference rate at the initial point in time, that is $(1-\tau)\cdot\left(A+\alpha\cdot B\cdot k(0)^{\alpha-1}\right)-\delta-\rho>0$ with $k(0)>\bar{k}$. In this case, the individuals are able and willing to substitute consumption intertemporally and the growth rate of consumption is positive. However, as the economy grows, the private marginal product of capital eventually converges to a constant. This asymptotic marginal product of capital might be too low in order to guarantee unbounded growth: $(1-\tau)\cdot A-\delta-\rho\leq 0$. Eventually the golden rule will apply, that is the private net marginal product of capital equals the time preference rate and the economy converges to the golden rule steady state with $k^{*}=\left(\frac{\delta+\rho-(1-\tau)\cdot A}{(1-\tau)\cdot\alpha\cdot B}\right)^{\frac{1}{\alpha-1}}$, $c^{*}=\left[(1-\tau)\cdot A-(\delta+n)\right]\cdot k^{*}+B\cdot(k^{*})^{\alpha}$. Again, this steady state is saddle-point stable as shown in appendix A4.

The balanced-growth equilibrium applies whenever the economy starts with sufficient productive resources above the subsistence level and, more importantly, if the growth condition is initially and asymptotically valid: $(1-\tau)\cdot A-\delta-\rho>0$. In this case, the economy pursues unbounded growth. From (4.7) one can see that the asymptotic growth rate of consumption per capita reads: $\lim_{t\to\infty}\frac{\dot{c}}{c}=\theta^{-1}\cdot\left[(1-\tau)\cdot A-\delta-\rho\right]$. In addition, equation (4.8) indicates that consumption and capital per capita must expand asymptotically at the same rate if the growth rate of capital per capita should converge asymptotically to a constant value:[44]

$$\lim_{t\to\infty}\frac{\dot{c}}{c}=\lim_{t\to\infty}\frac{\dot{k}}{k}=\theta^{-1}\cdot\left[(1-\tau)\cdot A-\delta-\rho\right]. \tag{4.9}$$

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[43] This equation might have two solutions for $\bar{k}$; one less than the golden-rule value for k and one greater. In this case, the lower value is the relevant one [Easterly (1994, p. 531)].

[44] See Barro and Sala-i-Martin (1995) for a similar reasoning within the context of the Jones-Manuelli model with CIES preferences [Barro and Sala-i-Martin (1995, pp. 161)].

Within the c/k-plane, the optimal trajectory converges towards its asymptotic balanced-growth path (BGP^{∞}), which is a linear ray starting from the origin. This indicates an asymptotically constant relation between consumption and capital per capita.

Provided that the economy starts above the subsistence steady state, the resulting long-run equilibrium is independent from the initial condition. However, the lower bound of the private rate of return to the reproducible inputs in relation to the time preference rate crucially determines whether the economy stagnates or pursues unbounded growth. As has been shown in the literature, the private rate of return to the reproducible inputs is primarily affected by government policies, which were summarised by the index of overall distortions. Consequently, international variations in detrimental government policies which reduce the marginal product of the reproducible factors represent a potential explanation of internationally diverging growth experiences. More specifically, within the former model A denotes the internationally uniform productivity of the reproducible inputs. Within the present framework this productivity is represented by $(1-\tau)\cdot A$, where τ denotes the extent of general distortions, which is allowed to vary internationally. Therefore, the incorporation of an index of distortions into the production function enables a meaningful explanation of different long-run as well as different transitional growth rates while assuming symmetry with respect to preferences and technology between the economy under study and the rest of the world [Easterly, King, Levine, and Rebelo (1992), Rebelo (1992), and Easterly (1994)].

4.2.2 Transitional dynamics: simulation results

In order to illustrate the dynamics of the Jones-Manuelli model with subsistence consumption, the transition process in the case of unbounded growth is simulated. This signifies that the system of differential equations (4.7) and (4.8) is approximated numerically, supposing that the growth condition, $(1-\tau)\cdot A-\delta-\rho>0$, holds. This is done by using the subroutine NDSolve of Mathematica. The growth model under study belongs to the class of growth models which do not possess a balanced-growth equilibrium. However, it possesses an asymptotic balanced-growth path which describes the asymptotic properties of long-run

growth.[45] The following set of parameters is employed: $\tau = 0.1$, $A = 0.1$, $B = 0.1$, $\alpha = 0.8$, $\delta = 0.02$, $n = 0.03$, $\theta = 1$, $\rho = 0.05$, and $\bar{c} = 2$.[46]

The main difference between the linear growth model with Stone-Geary preferences and the Jones-Manuelli model with Stone-Geary preferences consists in the possibility to reproduce *non-monotonic dynamics of growth rates*. Heuristically, the equation for the growth rate of consumption per capita (4.6) indicates that there are two opposing forces governing the dynamics of the growth rates. At early stages of economic development, the IES is low reflecting a low ability to save due to the requirement of subsistence considerations. As the economy develops, the saving rate and the growth rate of output increase (*subsistence-divergence mechanism*). On the other hand, the Jones-Manuelli technology (4.1) exhibits a falling marginal product of capital. As a result, the *neoclassical convergence mechanism* applies and reduces both the incentives to save as well as the average contribution of capital to output growth causing the growth rate of per capita income to fall. In principle, for specific parametrisations the interaction of both forces can produce a non-monotonic pattern of the growth rate dynamics.

The subsequent figure 7 shows the time path of the growth rate of per capita income (gry), the relation between the growth rate of per capita income and the logarithm of per capita income ($\ln y$), the time paths of the intertemporal elasticity of substitution (IES), the marginal product of capital (MPC), the saving rate (s), and the consumption-capital ratio (c/k), respectively. Several observations should be emphasised.

The growth rate of per capita income displays non-monotonic dynamics; specifically a hump-shaped relation is observed. Figure 7 (a) shows the time-path of the growth rate of per capita income, while figure 7 (b) shows the relation between the growth rate of per capita income and the logarithm of per capita income. Growth initially accelerates reaching a maximum growth rate and subsequently decelerates. In other words, the model implies

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[45] Other models with an asymptotic balanced-growth path are the linear growth model with Stone-Geary preferences (see the preceding section) and the Jones-Manuelli model [Jones and Manuelli (1990)]. Another model with this property consists in a linear growth model with productive consumption [Steger (1998)].

[46] The initial conditions are expressed by the parameter $a \cong 0.00004$ and were determined as follows: First, an arbitrarily but sufficiently high value of capital per capita is selected. The value of consumption per capita is chosen to be located on the asymptotic balanced-growth path. Second, starting from this point the dynamic system is solved backwards; the resulting trajectory runs into the equilibrium point ($\bar{c}$, $\bar{k}$). Third, the starting point of the forward solution is chosen to be located on this trajectory slightly above ($\bar{c}$, $\bar{k}$). The initial values of the backward solution are sufficiently high in the following sense: Multiplying these values by two and following the same procedure does not alter the time paths as shown in figure 7.

conditional ß-divergence for the lower range of income and *conditional ß-convergence* for the higher range of income. Eventually, the growth rate converges towards its asymptotic balanced-growth-equilibrium value. The Jones-Manuelli model with subsistence consumption is, therefore, able to reproduce an important stylised fact of economic growth [stylised fact (4)].[47]

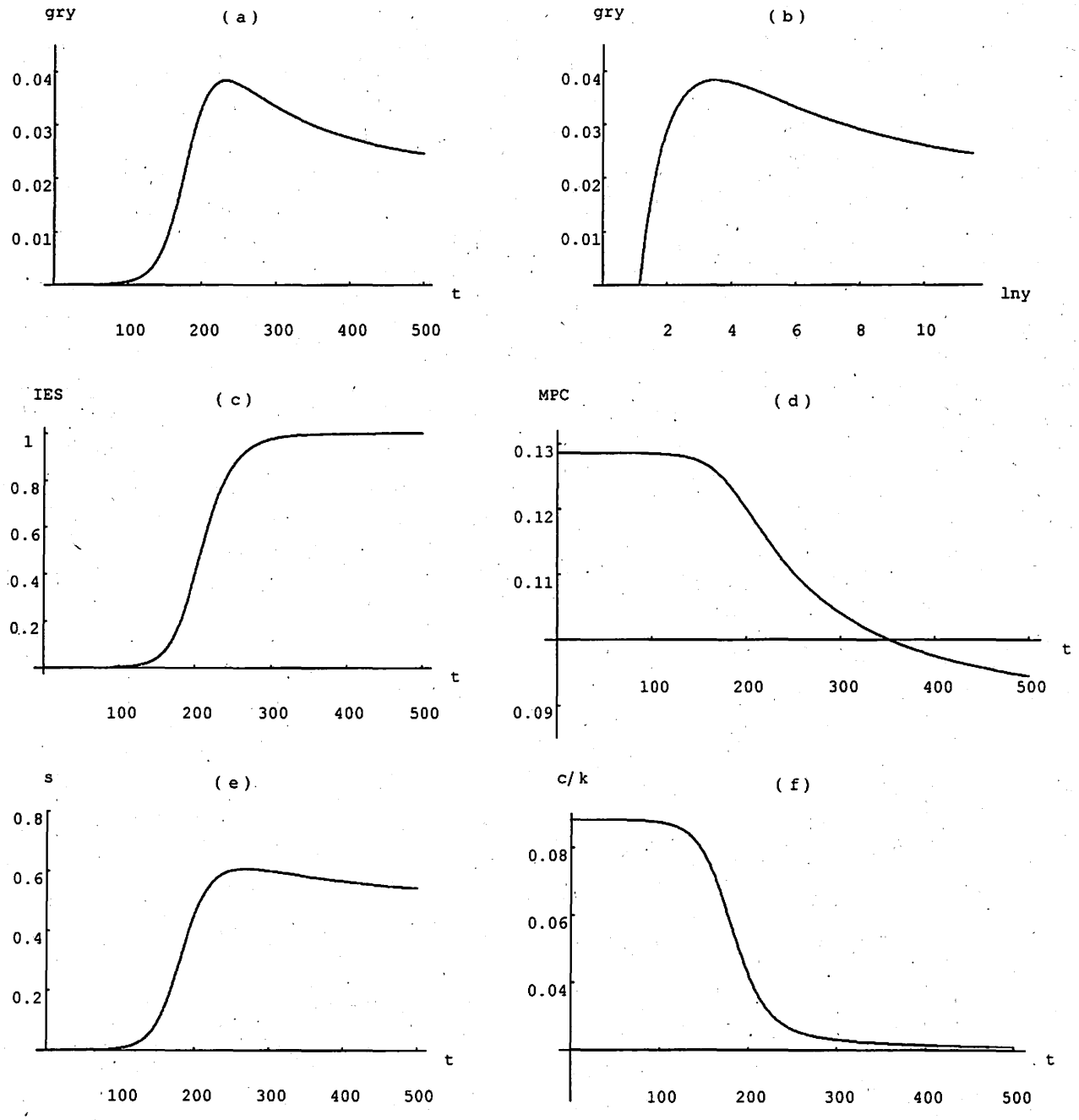

Figure 7: Time paths for the Jones-Manuelli model with subsistence consumption.

[47] It should be noted explicitly that this result essentially hinges on the parameter values employed. If, for example, the technology parameter α is reduced sufficiently, the marginal product of capital decreases more rapidly. As a result, a monotonic rise in the growth rate of per capita income results.

This pattern of evolution is the result of two opposing forces as can be illustrated on the basis of the simulation results: First, at early stages of economic development the IES increases [figure 7 (c)] reflecting an increase in the ability to save. As a result, the saving rate rises initially [figure 7 (e)], causing the growth rate of per capita income to rise as well (*subsistence-divergence mechanism*). Second, the marginal product of capital falls with an increase in the level of capital per capita [figure 7 (d)]. Economically, this bears two implications: (a) A fall in the rate of return to the factors that can be accumulated reduces the incentives to save.[48] (b) Along with the marginal product the average product of capital decreases as well. Consequently, the productive contribution of the whole stock of capital to output growth is reduced and the growth rate of output per capita falls. Both mechanisms together [(a) and (b)] constitute the *neoclassical convergence mechanism*. In the present case, at early stages of economic development the divergence-mechanism dominates the neoclassical convergence mechanism, while the reverse is true for later stages of development.

The c/k-ratio falls monotonically and converges towards its asymptotic balanced-growth-equilibrium value [figure (f)]. Therefore, the c/k-ratio is used to give a rough estimate of the *average speed of convergence*. Analogous to the definition of the instantaneous speed of convergence used in the previous section, the average speed of convergence at any time t can be defined as (the negative of) the average share of the difference between the current and the balanced-growth-equilibrium state of the economy which is eliminated during the time interval $[t,\ t+\upsilon]$, that is $\lambda_a[t,t+\upsilon] \equiv -\dfrac{[x(t+\upsilon)-x(t)]/\upsilon}{x(t)-x^*}$. The average speed of convergence for $t=0$ and $\upsilon = 200$ measured on the basis of the c/k-ratio accordingly reads $\lambda_a[0,200] = -\dfrac{[x(200)-x(0)]/200}{x(0)-x^*} \cong 0.0034$. That is, during the first 200 years on average 0.34 % of the distance between the initial and the balanced-growth-equilibrium state of the economy gets eliminated per year. As in the model of the previous section, this value is extraordinarily low and the implied time span required for the transition to the balanced-growth equilibrium is extremely long.[49]

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[48] The interaction of this mechanism together with the increase of the saving rate as a result of a rise in the ability to save explains the slight overshooting of the saving rate [figure 7 (e)].

[49] The possibility of a non-monotonic relation between the growth rate and the level of per capita income casts some doubt on the usual procedure to estimate the quantitative aspect of the convergence process, that is the speed of convergence.

At last, the following observation is worth noting. The previous figures show that the saving rate [figure 7 (e)] and the growth rate of per capita income [figure 7 (a)] seem to be nearly constant during the first 100 years; in fact they increase very slowly. Empirically, this pattern of dynamics could be interpreted as representing a balanced-growth equilibrium with low saving and low growth. However, within the frame of the present model this phenomenon represents a transitional phenomenon. Following this early stage of economic development, saving and growth increase significantly. This pattern of dynamics is consistent with several theoretical approaches as well as empirical observations on the stages of economic development. As an example, Reynolds (1983) comprehensively describes the growth experiences of the "third world" over the time period from 1850 to 1980. He stresses the distinction between extensive and intensive growth.[50] According to Reynolds, most countries experienced a turning point, that is extensive turned into intensive growth. Within the frame of the present model, this "turning point" occurs after about 100 years as figure 7 (a) indicates. Therefore, the growth model under study is able to explain this transition from extensive to intensive growth endogenously.[51]

5 Summary and conclusion

The requirement of subsistence consumption unambiguously affects the process of economic growth. It clearly restricts the ability to save not only for levels of per capita income at or slightly above subsistence. The intertemporal elasticity of substitution, which reflects both the ability and the willingness to save, increases with the level of per capita consumption and asymptotically converges to a constant. As a result, the requirement of subsistence consumption causes the growth rate of all the original variables to increase. It therefore represents an important mechanism of ß-divergence, which might be called *subsistence-divergence mechanism*.[52] For realistic and widely employed parameter values, the speed of convergence is exceptionally low at early stages of economic development and the time span required for the transition towards the asymptotic balanced-growth equilibrium is correspondingly long.

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[50] Extensive growth is defined as population growth with per capita income being constant, while intensive growth is defined as population growth with a rise in per capita income [Reynolds (1983, p. 943)].

[51] Reynolds' (1983) concept of "turning point" resembles the take-off period originated by Rostow (1956).

[52] The growth dynamics of the real world are simultaneously affected by the different mechanisms of convergence and divergence. Several authors have proposed to separate, at least theoretically, the different mechanisms of convergence [Barro and Sala-i-Martin (1992, p. 247) and de la Fuente (1997, p. 69)].

Despite its obvious simplicity the linear growth model with subsistence consumption is able to reproduce two of the stylised facts enumerated in the introduction, which primarily apply to the lower range of per capita income. The model implies a rise in the saving rate along with the level of per capita income [stylised fact (2)] as well as ß-divergence [stylised fact (3)]. On account of the extraordinarily low values of the speed of convergence at early stages of economic development, different growth rates can partly be explained to represent transitional phenomena [stylised fact (1)]. However, if international symmetry with respect to preferences and technology is supposed, the possible range of growth rates is restricted. Hence, the linear growth model with subsistence consumption has some difficulty in explaining the big diversity in growth experiences observable for the group of developing countries. In addition, the model clearly fails to reproduce the hump-shaped relation between the level and the growth rate of per capita income [stylised fact (4)].

An extension of the basic model by a general index of policy-induced distortions, which is sensibly allowed to vary internationally, permits a more satisfactory explanation of stylised fact (1). The incorporation of a general index of distortions enhances the possible range of long-run growth rates while maintaining symmetry with respect to preferences and technology. The big diversity in growth rates is explained to result from both transitional dynamics as well as different balanced-growth equilibria. The extended model shows two steady states and one asymptotic balanced-growth equilibrium. The selection of the dynamic equilibrium crucially depends on the relation between the marginal product of capital net of depreciation as well as "net of distortions" on the one hand and the time preference rate on the other. In the case of unbounded growth one observes policy continuity, that is the growth rate falls steadily as the extent of distortions increases [Easterly (1994, p. 532) and Jones and Manuelli (1997, pp. 86/87)]. Altogether, policy-induced distortions are important in understanding the big diversity in growth rates [stylised fact (1)].

The model is further extended by diminishing marginal returns to the factors that can be accumulated in order to reproduce the remaining stylised fact (4). The interaction between the subsistence-divergence mechanism and the neoclassical convergence mechanism produces an acceleration subsequently followed by a deceleration of growth, that is the hump-shaped relation between the growth rate and the level of per capita income [stylised fact (4)].

6 Appendix

A1: Analytical solution of the linear growth model with subsistence consumption

The first-order conditions of the dynamic problem (3.1) lead to the linear system of differential equations in the control and the state variable:

$$\dot{c}(t) = c(t) \cdot \theta^{-1}(A - \delta - \rho) - \bar{c} \cdot \theta^{-1}(A - \delta - \rho), \tag{6.1}$$

$$\dot{k}(t) = A \cdot k(t) - (\delta + n) \cdot k(t) - c(t). \tag{6.2}$$

Because of the linear technology the differential equation system can be solved one for one. The solution to (6.1) is:

$$c(t) = \bar{c} + b \cdot e^{\theta^{-1}(A-\delta-\rho)\cdot t}, \tag{6.3}$$

where b is a constant of integration which must be chosen optimally in order to satisfy the transversality condition. The linear first-order differential equation in k accordingly reads:

$$\dot{k}(t) = (A - \delta - \rho) \cdot k - \bar{c} - b \cdot e^{\theta^{-1}(A-\delta-\rho)\cdot t}. \tag{6.4}$$

The solution to the homogenous part of (6.4) is:

$$k(t) = k(0) \cdot e^{(A-\delta-n)\cdot t}, \tag{6.5}$$

while the solution to the non-homogenous part $[\tilde{k}(t)]$ can be found as follows [e.g., Gandolfo (1996, pp. 160)]:

$$\tilde{k}(t) = -e^{(A-\delta-n)\cdot t} \cdot \int [\bar{c} + b \cdot e^{\theta^{-1}(A-\delta-\rho)\cdot t}] \cdot e^{-(A-\delta-n)\cdot t} dt, \tag{6.6}$$

and integration finally yields,

$$\tilde{k}(t) = \frac{\bar{c}}{(A - \delta - n)} - \frac{b}{\theta^{-1}(A - \delta - \rho) - (A - \delta - n)} \cdot e^{\theta^{-1}(A-\delta-\rho)\cdot t} - \alpha_0 \cdot e^{(A-\delta-n)\cdot t}$$

$$\text{with} \quad \alpha_0 \equiv \alpha_1 + \alpha_2, \tag{6.7}$$

where α_1 and α_2 are some arbitrary constants of integration. According to the superposition principle, the solution to (6.4) is as follows:

$$k(t) = [k(0) - \alpha_0] \cdot e^{(A-\delta-n)\cdot t} + \frac{\bar{c}}{(A-\delta-n)} - \frac{b}{\theta^{-1}(A-\delta-\rho)-(A-\delta-n)} \cdot e^{\theta^{-1}(A-\delta-\rho)\cdot t} \tag{6.8}$$

The transversality condition can be written in the following form [Barro and Sala-i-Martin (1995, p. 142)]:

$$\lim_{t\to\infty}[k(t)\cdot e^{-(A-\delta-n)\cdot t}] = 0 \tag{6.9}$$

and inserting (6.8) into the transversality condition gives:

$$\lim_{t\to\infty}\{[k(0)-\alpha_0] + e^{-(A-\delta-n)\cdot t}\cdot\frac{\bar{c}}{(A-\delta-n)} + \frac{b}{(A-\delta-n)-\theta^{-1}(A-\delta-\rho)}\cdot e^{-[(A-\delta-n)-\theta^{-1}(A-\delta-\rho)]\cdot t}\} = 0 \tag{6.10}$$

The finiteness of the utility integral in (3.1) requires $A-\delta-n-\theta^{-1}(A-\delta-\rho)>0$ [Barro and Sala-i-Martin (1995, pp. 142/143)]. Consequently, the third term inside the brackets in (6.10) asymptotically vanishes. The second term asymptotically vanishes as well if $A-\delta-n>0$. This parameter restriction inevitably follows in the case of growth, that is $A-\delta-\rho>0$. In the case of regress, $A-\delta-\rho<0$, it represents an additional restriction (which does not contradict any other parameter restriction). Consequently, the transversality condition requires $k(0)-\alpha_0$ to equal zero and the optimal solutions read as follows:

$$c(t) = \bar{c} + b\cdot e^{\theta^{-1}(A-\delta-\rho)\cdot t}, \tag{6.11}$$

$$k(t) = \frac{\bar{c}}{(A-\delta-n)} + \frac{b}{(A-\delta-n)-\theta^{-1}(A-\delta-\rho)}\cdot e^{\theta^{-1}(A-\delta-\rho)\cdot t} \tag{6.12}$$

According to (3.1) the initial value of capital per capita is predetermined. The corresponding value of the control variable can easily be determined by using the policy function, which immediately results from (6.11) and (6.12):

$$c(t) = z \cdot [k(t) - \bar{k}] + \bar{c}, \quad \text{where} \quad z \equiv A - \delta - n - \theta^{-1} \cdot (A - \delta - \rho) > 0. \tag{6.13}$$

Equation (6.13) together with (6.11) imply the following relation:

$$b = e^{-\theta^{-1}(A-\delta-\rho)\cdot t}[c(t) - \bar{c}] = e^{-\theta^{-1}(A-\delta-\rho)\cdot t} \cdot z \cdot [k(t) - \bar{k}]. \tag{6.14}$$

For each value of $k(t)$ there is a unique optimal choice of $c(t)$ which satisfies all first-order conditions (including the transversality condition); this is the content of the policy function. The asymptotic growth rate of $c(t)$ and $k(t)$ according to (6.11) and (6.12) is:

$$\lim_{t\to\infty} \frac{\dot{c}}{c} = \lim_{t\to\infty} \frac{\dot{k}}{k} = \theta^{-1} \cdot (A - \delta - \rho). \tag{6.15}$$

The preceding relation demonstrates that the optimal trajectory asymptotically approaches the BGP^{∞} provided that the transversality condition is fulfilled. Otherwise the solutions for $c(t)$ and $k(t)$ would be different from (6.11) and (6.12). This is compatible with the requirement that $c(0)$ must be chosen according to the policy function to be located on the stable arm of the saddle-path which converges towards the BGP^{∞}.

A2: The speed of convergence

The analytical expressions for the instantaneous speed of convergence according to the definition (3.17) based on both the c/k-ratio and the logarithm of the c/k-ratio are as follows:

$$\lambda(c/k) = -\frac{\dfrac{\left(\bar{k} + \dfrac{b}{z} \cdot e^{w\cdot t}\right) \cdot w \cdot b \cdot e^{w\cdot t} - \left(\bar{c} + b \cdot e^{w\cdot t}\right) \cdot \dfrac{w \cdot b}{z} \cdot e^{w\cdot t}}{\left(\bar{k} + \dfrac{b}{z} \cdot e^{w\cdot t}\right)^2}}{\dfrac{\bar{c} + b \cdot e^{w\cdot t}}{\bar{k} + \dfrac{b}{z} \cdot e^{w\cdot t}} - z} \tag{6.16}$$

$$\lambda[\ln(c/k)] = -\frac{\left(\bar{k}+\frac{b}{z}\cdot e^{w\cdot t}\right)\cdot w\cdot b\cdot e^{w\cdot t} - \left(\bar{c}+b\cdot e^{w\cdot t}\right)\cdot\frac{w\cdot b}{z}\cdot e^{w\cdot t}}{\left(\bar{k}+\frac{b}{z}\cdot e^{w\cdot t}\right)\cdot\left(\bar{c}+b\cdot e^{w\cdot t}\right)}}{\ln\left(\frac{\bar{c}+b\cdot e^{w\cdot t}}{\bar{k}+\frac{b}{z}\cdot e^{w\cdot t}}\right)-\ln(z)} \tag{6.17}$$

with $\bar{k} \equiv \frac{\bar{c}}{A-\delta-n}$, $w \equiv \theta^{-1}\cdot(A-\delta-\rho)$, and $z \equiv A-\delta-n-\theta^{-1}(A-\delta-\rho)$.

The preceding expressions show that the instantaneous speed of convergence depends on time, the technology and preference parameters, and initial conditions expressed in b.

A3: Sets of parameters

King and Rebelo (1993, p. 916); RCK model with technical progress.	$\theta = 1$ and $\theta = 10$, $\delta = 0.1$, $n = 0.014$, ($\alpha = 0.33$, $\alpha = 0.5$, and $\alpha = 0.9$)
Easterly (1994, p. 534); JM model with SG preferences	$\theta = 1$, $\delta = 0.05$, $\rho = 0.1$, $n = 0.02$ ($\tau = 0.2$)
Sarel (1994, p.4); RCK model with technical progress	$\theta = 1.5$, $\delta = 0.04$, $\rho = 0.02$, $n = 0$ ($x = 0.01$ and $\alpha = 0.4$)
Barro and Sala-i-Martin (1995, p. 83); RCK model with technical progress	$\theta = 3$, $\delta = 0.05$, $\rho = 0.02$, $n = 0.01$, ($x = 0.02$, $\alpha = 0.3$ and $\alpha = 0.75$)
Maußner and Klump (1996, p. 128); RCK model without technical progress	$\theta = 3$, $\delta + n = 0.05$, $\rho = 0.01$, ($\alpha = 0.33$ and $\alpha = 0.66$)
Ortigueira and Santos (1997, p. 390); RCK model without technical progress	$\theta = 1.5$, $\delta = 0.05$, $\rho = 0.05$, $n = 0.01$ ($x = 0.01$ and $\alpha = 0.4$)
Barro (1990, p. S110); linear growth model with two types of capital (private and public).	$\theta = 1$, $\delta = 0$, $\rho = 0.02$, $n = 0$, ($\alpha = 0.25$)

Table 2: Sets of parameters.

JM: Jones-Manuelli, RCK: Ramsey-Cass-Koopmans, SG: Stone-Geary. α denotes the capital share, δ the depreciation rate of capital, θ the elasticity of marginal utility, n the growth rate of population, x the rate of (exogenous) technical progress, and ρ the time preference rate, respectively.

A4: Stability

Linearising the differential equation system (4.7) and (4.8) by means of a Taylor approximation of first order around any stationary point $(\hat{c},\ \hat{k})$ gives:

$$\begin{aligned}\dot{c} \cong\ & \theta^{-1}\cdot\left[(1-\tau)\cdot\left(A+\alpha\cdot B\cdot\hat{k}^{\alpha-1}\right)-\delta-\rho\right]\cdot(c-\hat{c}) \\ & +(\hat{c}-\bar{c})\cdot\theta^{-1}\cdot(1-\tau)\cdot\left(A+\alpha\cdot(\alpha-1)\cdot B\cdot\hat{k}^{\alpha-2}\right)\cdot\left(k-\hat{k}\right)\end{aligned} \tag{6.18}$$

$$\dot{k} \cong -(c-\hat{c})+\left[(1-\tau)\cdot\left(A+\alpha\cdot B\cdot\hat{k}^{\alpha-1}\right)-(\delta+n)\right]\cdot\left(k-\hat{k}\right) \tag{6.19}$$

The Jacobian matrix accordingly reads:

$$\mathbf{J}=\begin{pmatrix}\theta^{-1}\left[(1-\tau)\cdot\left(A+\alpha B\hat{k}^{\alpha-1}\right)-\delta-\rho\right] & (\hat{c}-\bar{c})\cdot\theta^{-1}\cdot\left[(1-\tau)\cdot\alpha\cdot(\alpha-1)\cdot B\hat{k}^{\alpha-2}\right] \\ -1 & (1-\tau)\cdot\left(A+\alpha B\hat{k}^{\alpha-1}\right)-(\delta+n)\end{pmatrix} \tag{6.20}$$

and the Jacobian determinant is

$$\begin{aligned}|\mathbf{J}| = & \left\{\theta^{-1}\cdot\left[(1-\tau)\cdot\left(A+\alpha B\hat{k}^{\alpha-1}\right)-\delta-\rho\right]\right\}\cdot\left[(1-\tau)\cdot\left(A+\alpha B\hat{k}^{\alpha-1}\right)-(\delta+n)\right] \\ & +(\hat{c}-\bar{c})\cdot\theta^{-1}\left[(1-\tau)\cdot\alpha\cdot(\alpha-1)\cdot B\hat{k}^{\alpha-2}\right]\end{aligned} \tag{6.21}$$

In order to prove for local stability properties of the differential equation system (4.7) and (4.8), the Jacobian has to be evaluated at the two stationary points: $(\bar{c},\bar{k})$, (c^*,k^*), where $\bar{c}$ is a constant positive parameter, $\bar{k}$ is a positive constant too, defined implicitly by $(1-\tau)\cdot\left(A\cdot\bar{k}+B\cdot\bar{k}^{\alpha}\right)-(\delta+n)\cdot\bar{k}-\bar{c}=0$,[53] $c^*=\left[(1-\tau)\cdot A-(\delta+n)\right]\cdot k^*+B\cdot(k^*)^{\alpha}$, and $k^*=\left(\dfrac{\delta+\rho-(1-\tau)\cdot A}{(1-\tau)\cdot\alpha\cdot B}\right)^{\frac{1}{\alpha-1}}$.

[53] As has been noted already, this equation might have two solutions for $\bar{k}$; one less than the golden-rule value for k and one greater. In this case, the lower value is the relevant one [Easterly (1994, p. 531)].

The subsistence equilibrium $(\bar{c},\bar{k})$ requires $(1-\tau)\cdot\left(A+\alpha B\bar{k}^{\alpha-1}\right)-\delta-\rho<0$. This inequality together with the parameter restriction $\rho>n$ implies that $(1-\tau)\cdot\left(A+\alpha B\bar{k}^{\alpha-1}\right)-(\delta+n)>0$. In addition, because the last term in (6.21) equals zero for $\hat{c}=\bar{c}$, it follows that $|\mathbf{J}|<0$ and the subsistence steady state is (locally) saddle-point stable.

The steady state with consumption above subsistence (c^*,k^*), requires $(1-\tau)\cdot\left(A+\alpha B(k^*)^{\alpha-1}\right)-\delta-\rho=0$ and, therefore, the Jacobian determinant is left to $|\mathbf{J}|=(c^*-\bar{c})\cdot\theta^{-1}\left[(1-\tau)\cdot\alpha\cdot(\alpha-1)\cdot B(k^*)^{\alpha-2}\right]$. The Jacobian determinant is negative because $0<\alpha<1$. Consequently, this case implies that (c^*,k^*) is (locally) saddle-point stable either [e.g., Lorenz (1989, pp.16-21)].

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