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## The State as Social Insurer, Labour Supply and Investments in Human Capital

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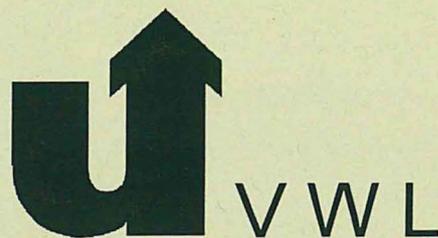
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**Volkswirtschaftliche  
Diskussionsbeiträge**



**The State as Social Insurer, Labour Supply and  
Investments in Human Capital**

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# The State as Social Insurer, Labour Supply and Investments in Human Capital

by

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**Abstract.** In this paper the welfare state is considered as insurance device. Redistributive taxation reduces the variance of life-time risk. Behind a veil of ignorance with regard to future position in society, agents decide in a  $(\mu, \sigma)$ -approach about labour supply and investment effort in human capital. Assuming DARA and see-through of agents, that means that they take the government's actions into account, it is shown that labour supply rises in dependence of increasing tax rates when investments are exogenous, and that investments rise in dependence of increasing tax rates when labour supply is exogenous. If investments and labour supply are free eligible, the adjustment of labour supply to fiscal tax changes is ambiguous. Redistributive taxation induces in any case individuals to invest successively more in human capital. Furthermore, fiscal policy effects on national income, income inequality and welfare are studied, and the constitutional chosen redistribution scheme is determined.

Keywords: insurance, redistribution, labour, human capital

JEL Classification No. H 21, H 23, I 22

## 1 Introduction

Redistributive taxation and welfare state are keywords in political and economic discussions. They are beneath contempt in times of high unemployment and high labour disincentives. This paper traces the question if labour disincentives are caused by the social state.

The economic profession has been aware that there are two justifications for redistributive taxation. The first one relies on the assumption that the society or a social planner decides on taxation so as to maximize a social welfare function. This approach was developed by Mirrlees [1971] and Sheshinski [1972] in the context of optimal income taxation. In the second income taxation plays the role of social insurance and the government plays the role of an insurance company<sup>1</sup> in the presence of risk aversion. Risk averse individuals might favour a progressive income taxation which reduces the uncertainty about disposable income. In both approaches taxation redistributes income from rich to poor people.

In this paper redistributive taxation is looked upon insurance activity. This idea traces back to Friedman [1953] and Buchanan and Tullock [1962]. Citizens are assumed to be identical and behind the Rawlsian [1971] veil of ignorance. They are in the broadest sense uncertain about their future position in society and future incomes, and they are strictly speaking uncertain about wage rates. The life-time income can be influenced by labour supply and investments in human and physical capital. The pleasant of this approach is that votes on redistribution schemes behind the veil of ignorance are of one voice. This is implied by the assumption of identical individuals. Decisions at this point of time can be viewed as constitutional decisions. The function for evaluating the redistribution schemes is the representative individual's utility function.

Eaton and Rosen [1980] were probably the first to find out that under uncertainty about the wage rate the change of labour supply to tax rate changes is ambiguous, and that the on the constitutional plain chosen taxation and lumpsum transfers are inefficient in the sense of Pareto<sup>2</sup>. Lumpsum transfers are in Eaton and Rosen's paper independent of the individual's behaviour. Thus lumpsum transfers are compatible with non-seeing-through the government's actions. Similar results followed by Koskela [1987] and Mazur [1989]. Lundholm [1992] showed that taxation is efficient for conditional lumpsum transfers. Conditional lumpsum

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<sup>1</sup> It is assumed that private insurance companies are not able to insure life-time risks. For reasons and an explanation of this assumption, and a discussion of related literature compare Sinn [1995, 1996].

<sup>2</sup> The pareto-efficient allocation has the following characterization: The representative individual prefers it to all other allocations.

transfers are equivalent to our see-through assumption. The essays of Sinn [1995, 1996] answer the question, how redistributive taxation affects investments in human capital. He states that taxation crowds out self-insurance activity which is another term for investments in human or physical capital.

The purpose of our paper is to combine the approaches of Eaton and Rosen [1980] and Sinn [1995]. We restrict the considerations to conditional lumpsum transfers. It is shown that the results in a pure labour supply model contradict empirical evidence found in Hausman [1985], and that in a pure investment model there is an incentive to invest in human capital with increasing tax rates and there are no disincentives like in Sinn [1995]. In the combined model the changes of labour supply are ambiguous, so that empirical behaviour is possible. Furthermore, there is still an incentive to invest.

The present model is constructed in the scale and location parameter methodology originally due to Tobin [1958], Meyer [1987] and Sinn [1983, 1989]. Agents maximize expected utility and the only restriction imposed on stochastic variables is to belong to the same linear distribution class. This approach has the advantage to explain national income and inequality movements in a much easier way than in an expected utility framework without this restriction.

The remainder of the paper is organized as follow. The analytical model is described in section 2. The result of optimal labour supply with exogenous investments and optimal investments with exogenous labour supply are represented and discussed in section 3 and 4. In addition, the constitutional chosen redistribution scheme is determined. Section 5 studies the effects of taxation on simultaneous decisions about labour supply and investments. Some brief conclusions are drawn in section 6.

## 2 The model

Individuals are supposed to be ex ante identical. The risk occurs in form of the stochastic wage rate  $W \geq 0$  which is determined by the random state of nature. The realizations of the wage rate ex post depend on illness, abilities, e.g. the talent to play tennis, or external events like the time or the place of birth. The proceeds from labour  $\beta(e) \cdot W \cdot \ell - e$  consist of a (gross-)labour

income  $\beta(e) \cdot W \cdot \ell$  less the professional expenses  $e$ . The representative agent has two possibilities to influence her proceeds from labour. On the one hand she can, varying the labour efforts  $\ell$ , increase or decrease the (gross-)labour income, and on the other hand she has the possibility to vary her investments in human or physical capital  $e$ . A rise of investments improves on the one side by the efficiency function  $\beta(e)$  the (gross-)labour income, but increases on the other side the professional expenses. The government plays the role of a social insurer, collects tax liabilities  $T$ , and provides transfers  $p$ . The individual's stochastic post-tax income is<sup>3</sup>

$$(1) \quad Y(e, \ell; \alpha, p, \tau) = \beta(e) \cdot W \cdot \ell - e - T(e, \ell; \alpha, \tau) + p.$$

$e$  and  $\ell$  are the decision variables of the representative consumer, and  $\alpha$ ,  $\tau$  and  $p$  are the decision variables of the government. The random variable  $W$  is exogenously given. Let the efficiency function  $\beta(e)$  be twice continuously differentiable and satisfy

$$(2) \quad \beta(e) \geq 0, \quad \beta_e > 0, \quad \beta_{ee} \leq 0 \quad \forall e \in [0, \bar{e}].$$

The citizens tax liability is

$$(3) \quad T(e, \ell; \alpha, \tau) = \tau \cdot (W \cdot \beta(e) \cdot \ell - \alpha \cdot e)$$

where  $\tau \in [0, 1]$  is the tax rate and  $\alpha \in [0, 1]$  specifies which part of  $e$  is accepted as professional expenses. For  $\alpha = 1$  the proceeds from labour are taxed, because the professional expenses are fully deductible. For  $\alpha = 0$  (gross-)labour income is taxed. Professional expenses are not deductible. In equilibrium the fiscal budget constraint is determined by<sup>4</sup>

$$(4) \quad p = E[T(e, \ell; \alpha, \tau)].$$

In order to decide about labour effort and investment the agent has to take into account that  $Y$  is a random variable. The risk averse agent uses as location parameter the mean income  $\mu$  and as scale parameter the standard deviation  $\sigma$  for the valuation of different random variables. If the society is sufficiently large, and if the distribution of  $W$  is for all agents the same and independent of each other,  $Y$  can be interpreted due to the law of large numbers as the ex post realized income distribution. The probability of a realization  $y$  is identical to the relative cummulation of this realization. The parameters  $\mu$  and  $\sigma$  then measure the mean and the standard deviation of the ex post realized income distribution.  $\mu$  can be interpreted as the

<sup>3</sup> Stochastic variables are in capital, deterministic variables in small letters.

<sup>4</sup>  $E$  is the expectation operator.

national income per capita or the average income and  $\sigma$  as a measure for the income inequality.

$\mu$  and  $\sigma$  are specified as<sup>5</sup>

$$(5) \quad \begin{aligned} \mu &:= \mu(e, \ell; \alpha, p, \tau) := E[Y(e, \ell; \alpha, p, \tau)] \\ &= E[W] \cdot \beta(e) \cdot \ell - e - \tau \cdot (E(W) \cdot \beta(e) \cdot \ell - \alpha \cdot e) + p, \end{aligned}$$

$$(6) \quad \sigma := \sigma(e, \ell; \tau) := R[Y(e, \ell; \alpha, p, \tau)] = \beta(e) \cdot R[W] \cdot \ell \cdot (1 - \tau)$$

with

$$(7) \quad p := p(e, \ell; \alpha, \tau) = \tau \cdot E(W) \cdot \beta(e) \cdot \ell - \alpha \cdot e.$$

The definition of  $Y(e, \ell; \alpha, p, \tau)$  from (1) implies that the probability distributions of  $Y(e, \ell; \alpha, p, \tau)$  belong for a given distribution of  $W$  for all investments and labour efforts to the same linear distribution class. The equations (5) and (6) contain the policy parameters  $\alpha$ ,  $\tau$  and  $p$ . In the introduction we mentioned that the agent is seeing-through the government's movements. An implication of this assumption is that the individual takes the fiscal budget constraint into account.  $\mu$  then becomes

$$(8) \quad \mu = E[W] \cdot \beta(e) \cdot \ell - e.$$

As a result of the elimination of the policy parameter  $p$  the average income  $\mu$  is independent of  $\alpha$ .

Up till now labour supply and investments in human capital are the agent's decision variables. For the further analysis it is useful to introduce the pre-tax standard deviation per working capacity  $\sigma_G$  (in the following short pre-tax standard deviation). By analogy with Sinn [1995] we treat the pre-tax standard deviation  $\sigma_G$  instead of investments  $e$  as one of the choice variables.  $\sigma_G$  is defined as

$$(9) \quad \sigma_G := \sigma_G(e) := \sigma(e, \ell = 1, \tau = 0) = \beta(e) \cdot R[W].$$

Replacing (9) in (8) and (6) we obtain<sup>6</sup>

$$(10) \quad \mu = \frac{E[W]}{R[W]} \cdot \sigma_G \cdot \ell - \beta^{-1}\left(\frac{\sigma_G}{R[W]}\right) =: \bar{\mu}(\sigma_G, \ell),$$

$$(11) \quad \sigma = (1 - \tau) \cdot \sigma_G \cdot \ell.$$

The set of all  $(\mu, \sigma)$ -combinations satisfying (10) and (11) is called opportunity set.

<sup>5</sup>  $R$  is the symbol for the standard deviation.

<sup>6</sup>  $\beta^{-1}$  is the inverse image of the function  $\beta^{-1}$ .

The last step of rearrangements which is helpful for the following geometrical explanations and interpretations is to combine (10) and (11) which enables us to write  $\mu$  as

$$(12) \quad \mu = \frac{E[W]}{R[W]} \cdot \frac{\sigma}{(1-\tau)} - \beta^{-1} \left( \frac{\sigma}{R[W] \cdot (1-\tau) \cdot \ell} \right) =: M(\sigma(\sigma_G, \ell, \tau), \ell, \tau).$$

Before we start with the tax-political analysis we characterize for reference the laissez-faire, a situation without taxes and transfers.

**Definition 1:**

- (1)  $\mu = M(\sigma(\sigma_G, \ell, \tau = 0), \ell, \tau = 0)$  is called **investment-labour-function**.
- (2)  $\mu = M(\sigma(\sigma_G, \ell = \text{const.}, \tau = 0), \ell = \text{const.}, \tau = 0)$  is called **investment-function**.
- (3)  $\mu = M(\sigma(\sigma_G = \text{const.}, \ell, \tau = 0), \ell, \tau = 0)$  is called **labour-function**.

The investment-function and the labour-function are special cases of the investment-labour-function.

The description of the model is completed by introducing the agent's utility function  $u(y, \ell)$ . Individuals maximize expected utility. If all stochastic variables belong to the same linear distribution class, any von Neumann-Morgenstern utility function can be represented in  $(\mu, \sigma, \ell)$ -preferences. Tobin [1958], Meyer [1987] and Sinn [1983, 1989] show the transformation into the  $(\mu, \sigma)$ -space which now has to be extended to the  $(\mu, \sigma, \ell)$ -space. The individual's utility function is assumed to be given by

$$(13) \quad u(y, \ell) = \bar{u}(y) + \tilde{u}(\ell)$$

with

$$(A1) \quad u_y(y, \ell) = \bar{u}'(y) > 0, \quad u_{yy}(y, \ell) = \bar{u}''(y) < 0 \quad \forall y,$$

$$(A2) \quad u_\ell(y, \ell) = \tilde{u}'(\ell) < 0, \quad u_{\ell\ell}(y, \ell) = \tilde{u}''(\ell) < 0 \quad \forall \ell.$$

Additive separability is a simplification for technical reasons. (A1) is the standard risk aversion assumption. Utility rises in incomes with decreasing marginal utility. In addition, the agent is supposed to be DARA which implies for the absolute Arrow-Pratt measure  $(r(y) := -\bar{u}''(y) / \bar{u}'(y)) \quad r'(y) < 0$ . (A2) formulates the influence of labour on utility. The

utility function is decreasing and strictly concave in labour. Then the expected utility can be written, if the compared random variables  $Y$  belong to the same linear distribution class, as<sup>7</sup>

$$(14) \quad E(u(Y, \ell)) = E(u(\mu + \sigma \cdot Z, \ell)) = \int_{-b}^b u(\mu + \sigma \cdot z, \ell) \cdot dF(z) =: U(\mu, \sigma, \ell).$$

$U(\mu, \sigma, \ell)$  is called the  $(\mu, \sigma, \ell)$ -preference function. An indifference function in the  $(\mu, \sigma, \ell)$ -space is determined by  $U(\mu, \sigma, \ell) = \text{const.}$ , and we yield the properties of the indifference function by total differentiation of  $U(\mu, \sigma, \ell) = \text{const.}$  The additive separability of the utility function implies that we can apply the results of Meyer [1987] and Sinn [1983, 1989], and thus assuming (A1) and DARA the partial derivatives of the  $(\mu, \sigma, \ell)$ -preference function have the signs  $U_{\mu} > 0$ ,  $U_{\sigma} < 0$ ,  $U_{\mu\mu} < 0$ ,  $U_{\mu\sigma} = U_{\sigma\mu} > 0$ ,  $U_{\sigma\sigma} < 0$ . From (A2) follows  $U_{\ell} < 0$ ,  $U_{\ell\ell} < 0$ , and the cross derivatives are  $U_{\ell\mu} = U_{\mu\ell} = 0$ ,  $U_{\ell\sigma} = U_{\sigma\ell} = 0$ . Consider now for the time being two-dimensional spaces. One of the variables  $\mu$ ,  $\sigma$  or  $\ell$  is held constant, respectively. The properties of the two-dimensional indifference functions are:

(I1) The indifference curve slope in the  $(\mu, \sigma)$ -space is given by

$$i(\mu, \sigma) := \frac{d\mu}{d\sigma} = -\frac{U_{\mu}(\mu, \sigma)}{U_{\sigma}(\mu, \sigma)}.$$

The indifference function enters the ordinate perpendicularly ( $i(\mu, 0) = 0$ ), is increasing ( $i(\mu, \sigma) > 0$ ), strictly convex ( $\frac{d^2\mu}{d\sigma^2} > 0$ ), and the partial derivatives are ( $i_{\mu}(\mu, \sigma) < 0$ ) and ( $i_{\sigma}(\mu, \sigma) > 0$ ).

(I2) The  $(\mu, \ell)$ -indifference function (indifference curve slope  $i(\mu, \ell) := \frac{d\mu}{d\ell} = -\frac{U_{\ell}(\mu, \ell)}{U_{\mu}(\mu, \ell)}$ )

is increasing ( $i(\mu, \ell) > 0$ ), strictly convex ( $\frac{d^2\mu}{d\ell^2} > 0$ ), and the partial derivatives are

( $i_{\mu}(\mu, \ell) > 0$ ) and ( $i_{\ell}(\mu, \ell) > 0$ ).

<sup>7</sup>  $Z$  is the standardized random variable of  $Y$ , and  $b$  and  $-b$  are the lower and upper bounds of integration of  $Z$ .

(I3) Last but not least in the  $(\sigma, \ell)$ -space we have: The indifference function (indifference curve slope  $i(\sigma, \ell) := \frac{d\sigma}{d\ell} = -\frac{U_\ell(\sigma, \ell)}{U_\sigma(\sigma, \ell)}$ ) is decreasing ( $i(\sigma, \ell) < 0$ ), strictly concave

$\left(\frac{d^2\sigma}{d\ell^2} < 0\right)$ , and the partial derivatives are ( $i_\ell(\sigma, \ell) < 0$ ) and ( $i_\sigma(\sigma, \ell) > 0$ ).

All three indifference curves joined together provide the indifference mountains in figure 1. Higher mountains lead to higher utilities.

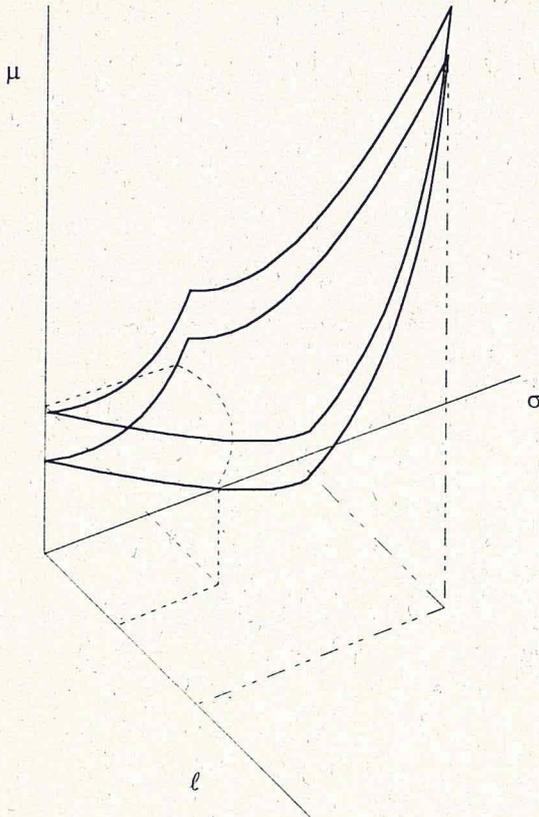


Figure 1: Indifference mountains in the  $(\mu, \sigma, \ell)$ -space

### 3 Labour supply

This section presents the decisions about labour supply for exogenous investments in human capital. The representative agent maximizes her utility under the restriction that the  $(\mu, \sigma)$ -combination is in the opportunity set which is in this case determined by the labour-redistribution-function.

**Definition 2:**

$\mu = M(\sigma(\sigma_G = \text{konst.}, \ell, \tau), \ell, \tau) =: M(\sigma(\ell, \tau), \ell, \tau)$  is called **labour-redistribution-function** for  $\tau > 0$ .

Formally, the representative agent's utility optimization problem is<sup>8</sup>

$$(15) \quad \max_{\ell} U(\mu, \sigma, \ell) \quad \text{s. t.} \quad \mu = \bar{\mu}(\ell), \sigma = (1 - \tau) \cdot \sigma_G \cdot \ell.$$

with the necessary condition<sup>9</sup>

$$(16) \quad i(\bar{\mu}(\ell), (1 - \tau) \cdot \sigma_G) \cdot (1 - \tau) \cdot \sigma_G + i(\bar{\mu}(\ell), \ell) = \bar{\mu}_{\ell}(\ell).$$

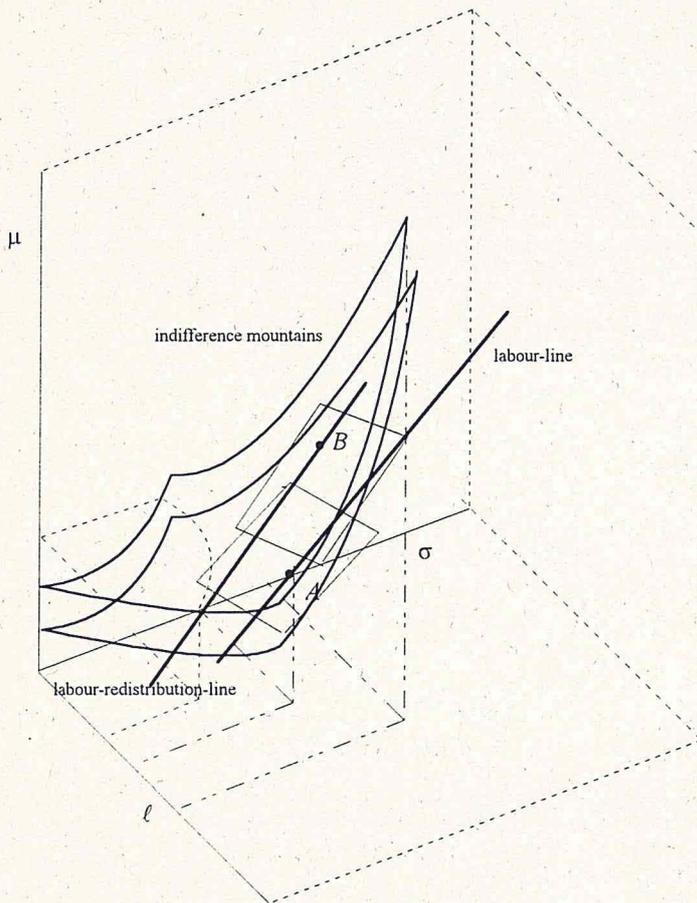


Figure 2: The optimal level of labour supply for given investments and tax rates

In the two-dimensional space the solution of a maximization problem under well-defined conditions is determined by the cut point of two curves where the tangents of the curves are identical. In the three-dimensional space the solution (16) is characterized by the condition that

<sup>8</sup> The function  $\bar{\mu}(\ell)$  is defined as  $\bar{\mu}(\ell) := \bar{\mu}(\sigma_G = \text{const.}, \ell)$ , see (10), and elucidates that investments and thus the pre-tax standard deviation are exogenous parameters.

<sup>9</sup> Second order conditions are derived in the appendix.

for the solution point the tangent of the labour-redistribution-line lies in the solution point's tangent plane of the indifference mountain. Figure 2 illustrates equation (16). The right straight line is the labour-line, and the left straight line is a labour-redistribution-line. The  $(\mu, \sigma)$ -combinations satisfying (16) are characterized as the points where the tangents of the labour-redistribution-line is in the tangent plane of the indifference mountain. Point *A* then is the laissez-faire solution ( $\tau = 0$ ) and point *B* represents a solution of (15) for a given  $\tau > 0$ .

Comparative static analysis of a marginal tax rate change of (16) yields the following result:

**Theorem 1:**

- (a) *Labour supply is an increasing function of the tax rate.*
- (b) *Average income is an increasing function of the tax rate.*
- (c) *Post-tax standard deviation falls (rises) with the tax rate if and only if*

$$-\ell \cdot [i_{\mu}(\mu, \ell) \cdot \bar{\mu}_{\ell}(\ell) + i_{\ell}(\mu, \ell)] + (1 - \tau) \cdot \sigma_G \cdot [i(\mu, \sigma) + \ell \cdot i_{\mu}(\mu, \sigma) \cdot \bar{\mu}_{\ell}(\ell)] < (>) 0.$$

- (d) *Utility is an increasing function of the tax rate.*

All proofs are given in the appendix.

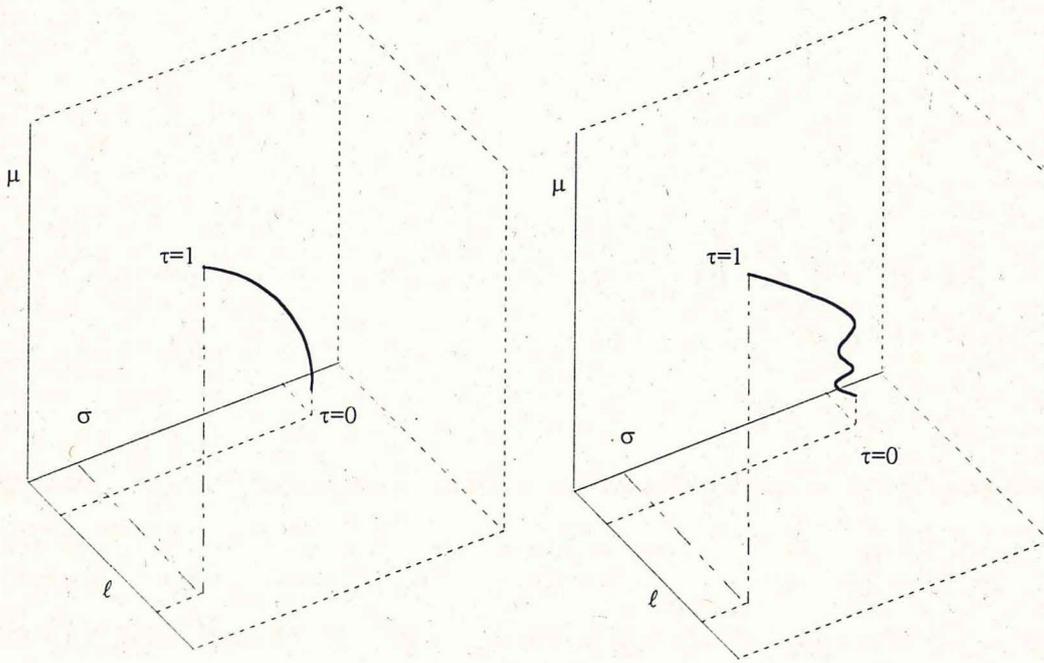


Figure 3: *Labour-equilibrium-lines for given investments*

Redistributive taxation raises labour supply. The immediate consequence as in theorem 1 shown is a growth of average income and therefore an improvement of the allocation. The change of inequality is ambiguous. For partial differentiation of  $\sigma$  to  $\tau$  we get

$$(17) \quad \frac{d\sigma}{d\tau} = -\sigma_G \cdot \ell + (1 - \tau) \cdot \sigma_G \cdot \frac{d\ell}{d\tau}.$$

The change of post-tax standard deviation is analytically decomposable into two terms. The first term is called insurance effect and indicates the change of post-tax standard deviation as a direct consequence of an increasing tax rate. This effect reduces post-tax inequality. The second term is the tax induced labour supply change which deteriorates the income distribution concerning equality. If the tax rate converges to one inequality is reduced definitely, otherwise the net effect is without further assumptions ambiguous.

Each point of the sketched lines in figure 3 satisfy (16) for constant  $\sigma_G$  and a given tax rate. The lines are called labour-equilibrium-lines and give information about the optimal  $(\mu, \sigma)$ -combination -optimal in the sense of satisfying (16)- for given investments in human capital in dependence of changing tax rates. In the left mapping inequality is an increasing function of the tax rate, whereas in the right mapping there are segments where the distribution moves more equally.

Theorem 1 gives a further message. Theorem 1(d) shows that  $\tau = 1$  is the tax rate the society chooses on the constitutional plain. The allocation resulting from  $\tau = 1$  is pareto-optimal which is achieved by conditional lumpsum transfers or seeing-through, respectively, compare with Lundholm [1992].

Empirical examinations concerned to labour supply, for a survey of these examinations see Hausman [1985], found out that labour supply should fall with tax rates. This contradicts our result<sup>10</sup> in a pure labour supply model, but we will demonstrate in section 5 the possibility of empirical behaviour in a combined model.

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<sup>10</sup> See Theorem 1a.

#### 4 Investments in human capital

In this section we assume labour supply is exogenously given, and the choice variable is now  $\sigma_G$ . We are in a pure investment model. The formal optimization problem of the representative agent is<sup>11</sup>

$$(18) \quad \max_{\sigma_G} U(\mu, \sigma, \ell) \quad \text{s.t.} \quad \mu = \bar{\mu}(\sigma_G), \sigma = (1 - \tau) \cdot \sigma_G \cdot \ell$$

with the first order condition

$$(19) \quad i(\mu, \sigma) = \frac{\bar{\mu}_{\sigma_G}(\sigma_G)}{(1 - \tau) \cdot \ell}$$

The LHS of (19) is the indifference curve slope in the  $(\mu, \sigma)$ -space and the RHS is the slope of the investment-redistribution-line. The solution of (18) is characterized by identical indifference curve and investment-redistribution-line slopes.

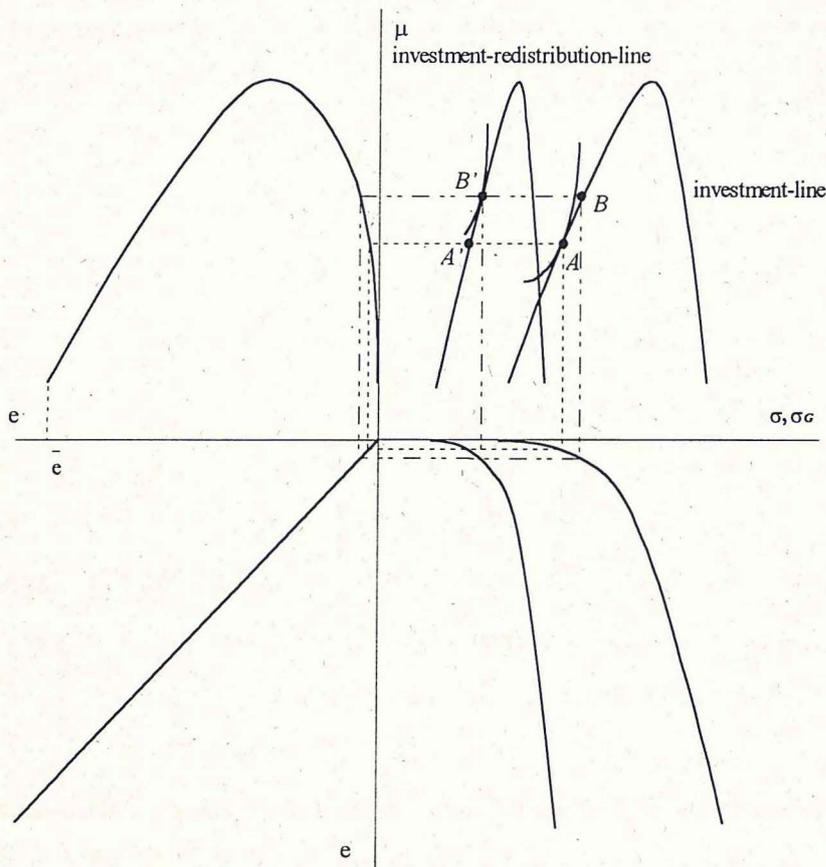


Figure 4: The optimal level of investments for given labour supply and tax rates

<sup>11</sup>  $\bar{\mu}(\sigma_G) := \bar{\mu}(\sigma_G, \ell = \text{const.})$  expresses that the labour supply is exogenous.

**Definition 3:**

$\mu = M(\sigma(\sigma_G, \ell = \text{const.}, \tau), \ell = \text{const.}, \tau) =: M(\sigma(\sigma_G, \tau), \tau)$  is called **investment-redistribution-function** for  $\tau > 0$ .

The above drawn figure provides an illustration of the investment and investment-redistribution-line. In the second quadrant equation (8) and in the third quadrant equation (6) for  $\tau = 0$  and  $\tau > 0$  are mapped. The resulting curvatures of the investment- and investment-redistribution-line follow from point to point transfer into the first quadrant. The first order condition is satisfied in  $A$  on the investment-line and in  $B'$  on the investment-redistribution-line.  $A$  is the solution-point in the special case of laissez-faire. Both  $A$  and  $B'$  determine optimal investments for given tax rates and exogenous labour supply.

The comparative static results are here:

**Theorem 2:**

- (a) *Pre-tax standard deviation is an increasing function of the tax rate.*
- (b) *Investment effort is an increasing function of the tax rate.*
- (c) *Average income is an increasing function of the tax rate.*
- (d) *Post-tax standard deviation falls (rises) with the tax rate if and only if*

$$\left[ (1 - \tau) \cdot \ell \cdot (i(\mu, \sigma) - \sigma_G \cdot i_\mu(\mu, \sigma) \cdot \bar{\mu}_{\sigma_G}(\sigma_G)) + \sigma_G \cdot \bar{\mu}_{\sigma_G \sigma_G}(\sigma_G) \right] < (>) 0.$$

- (e) *Utility is an increasing function of the tax rate.*

By analogy with the previous section redistributive taxation works as insurance and raises the endogenous variable, here the investments. In Sinn's [1995] essay self-insurance is another term for investments in human or physical capital. Based on a slightly varying model, taxation in the Sinn-model crowds out self-insurance activity. This statement is opposite to our theorem 2(b), although the effects on pre-tax standard deviation, inequality and national income are the same.

Figure 4 elucidates the changes. The insurance effect reducing the inequality is the movement from  $A$  to  $A'$ . The next step is the simultaneous growth of investments which causes the realization  $B'$ . The corresponding point  $B$  on the investment-line is connected with higher pre-tax inequality in comparison to the laissez-faire allocation. In  $B'$  the average income is higher and inequality is lower than in  $A$ . Theorem 2(d) shows that the effect on post-tax

inequality in general is again indetermined. Two possible curvatures of equilibrium-lines, all  $(\mu, \sigma)$ -combinations satisfying (19), are given in figure 5.

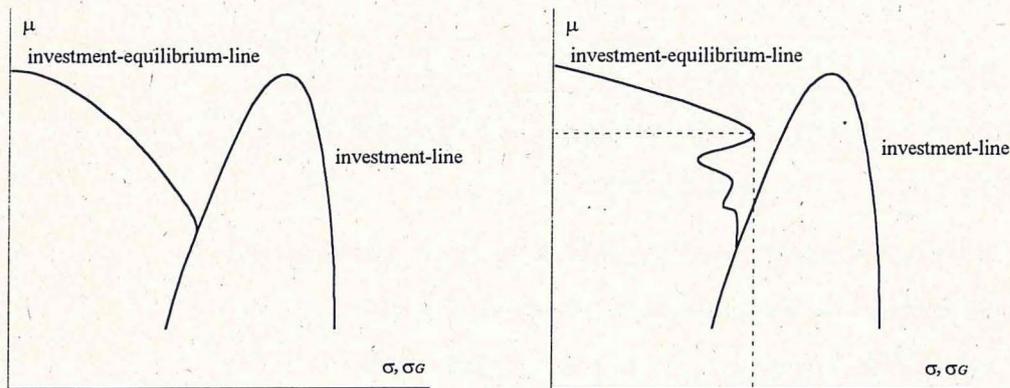


Figure 5: Investment-equilibrium-lines for given labour supply

## 5 Investments and labour supply

We now turn to the interesting question: What happens, if individuals choose investments and labour supply simultaneously. The maximization problem in this scenario is

$$(20) \quad \max_{\sigma_G, \ell} U(\mu, \sigma, \ell) \quad \text{s. t.} \quad \mu = \bar{\mu}(\sigma_G, \ell), \quad \sigma = (1 - \tau) \cdot \sigma_G \cdot \ell$$

with the first order conditions

$$(16) \quad i(\mu, \sigma) \cdot (1 - \tau) \cdot \sigma_G + i(\mu, \ell) = \bar{\mu}_\ell(\sigma_G, \ell) \quad \text{and}$$

$$(19) \quad i(\mu, \sigma) = \frac{\bar{\mu}_{\sigma_G}(\sigma_G, \ell)}{(1 - \tau) \cdot \ell}$$

The solution of (20) is determined by (16) and (19). Firstly, the tangent of a labour-redistribution-line's point has to be in the tangent plane of the corresponding indifference mountain, and secondly, the slope of the  $(\mu, \sigma)$ -indifference curve has to be conform to the slope of the investment-redistribution-line.

The comparative static analysis of a marginal tax rate change yields in this scenario:

### Theorem 3:

Let the condition  $H := A_{\sigma_G} \cdot B_\ell - B_{\sigma_G} \cdot A_\ell > 0$  with

$$A_\ell = i(\mu, \sigma) \cdot (1 - \tau) + (1 - \tau) \cdot \ell \cdot [i_\mu(\mu, \sigma) \cdot \bar{\mu}_\ell(\sigma_G, \ell) + i_\sigma(\mu, \sigma) \cdot (1 - \tau) \cdot \sigma_G] - \bar{\mu}_{\sigma_G \ell}(\sigma_G, \ell),$$

$$A_{\sigma_G} = (1 - \tau) \cdot \ell \cdot [i_\mu(\mu, \sigma) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) + i_\sigma(\mu, \sigma) \cdot (1 - \tau) \cdot \ell] - \bar{\mu}_{\sigma_G \sigma_G}(\sigma_G, \ell),$$

$$B_\ell = (1 - \tau) \cdot \sigma_G \cdot [i_\mu(\mu, \sigma) \cdot \bar{\mu}_\ell(\sigma_G, \ell) + i_\sigma(\mu, \sigma) \cdot (1 - \tau) \cdot \sigma_G] + i_\mu(\mu, \ell) \cdot \bar{\mu}_\ell(\sigma_G, \ell) + i_\ell(\mu, \ell),$$

$$B_{\sigma_G} = i(\mu, \sigma) \cdot (1 - \tau) + (1 - \tau) \cdot \sigma_G \cdot [i_\mu(\mu, \sigma) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) + i_\sigma(\mu, \sigma) \cdot (1 - \tau) \cdot \ell] + i_\mu(\mu, \ell) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) - \bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) \quad \text{be satisfied, then:}$$

(a) *Pre-tax standard deviation is an increasing function of the tax rate.*

(b) *Investment effort is an increasing function of the tax rate.*

(c) *Labour supply falls (rises) with the tax rate if and only if*

$$(21) \quad \bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) - i(\mu, \sigma) \cdot (1 - \tau) - \bar{\mu}_{\sigma_G \sigma_G}(\sigma_G, \ell) \cdot \frac{\sigma_G}{\ell} - i_\mu(\mu, \ell) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) < (>) 0.$$

(d) *Average income is an increasing function of the tax rate.*

(e) *Post-tax standard deviation falls (rises) with the tax rate if and only if*

$$\begin{aligned} & -\sigma_G \cdot \ell + \frac{A_\tau}{H} \cdot (1 - \tau) \cdot \left[ \ell \cdot i_\ell(\mu, \ell) + \sigma_G \cdot (\bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) - i(\mu, \sigma) \cdot (1 - \tau)) \right. \\ & + \ell \cdot i_\mu(\mu, \ell) \cdot \bar{\mu}_\ell(\sigma_G, \ell) + \sigma_G \cdot \bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) - \sigma_G \cdot i(\mu, \sigma) \cdot (1 - \tau) \\ & \left. - \frac{\sigma_G^2}{\ell} \cdot \bar{\mu}_{\sigma_G \sigma_G}(\sigma_G, \ell) - \sigma_G \cdot i_\mu(\mu, \ell) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) \right] < (>) 0, \quad \text{with} \end{aligned}$$

$$A_\tau = -i(\mu, \sigma) \cdot \ell - (1 - \tau) \cdot \sigma_G \cdot \ell^2 \cdot i_\sigma(\mu, \sigma).$$

(f) *Utility is an increasing function of the tax rate.*

The assumption  $H > 0$ , see theorem 3, is the second order condition of (20) and implies that the solution of (20) is a maximum. In theorem 3(a) and 3(b) we find the same results like in theorem 2(a) and 2(b). Labour supply can fall with the tax rate, see theorem 3(c) which is in contrast to theorem 1(a). That is the reason why that in empirical studies established behaviour now may be theoretically supported. Consider to this end the special case  $\beta_{ee} = 0$  which implies  $\bar{\mu}_{\sigma_G \sigma_G}(\sigma_G, \ell) = 0$ . Observe that  $\bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) \cdot \sigma_G = \bar{\mu}_\ell(\sigma_G, \ell)$ . Inserting (16) into (21) then yields

$$(22) \quad \text{sign}\left(\frac{d\ell}{d\tau}\right) = \text{sign}\left(\frac{i(\mu, \ell)}{\sigma_G} - i_\mu(\mu, \ell) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell)\right).$$

If the first term on the LHS of (22) is in the amount smaller than the second term, then labour supply falls with the tax rate. Converges  $\tau$  to one we yield in all cases a clear statement. The sign of the labour supply change as a result of a marginal change of tax rate is due to

$$\bar{\mu}_{\sigma_G}(\sigma_G, \ell) = 0 \text{ from (16): } \text{sign}\left(\frac{d\ell}{d\tau}\right) = \text{sign}\left(-\bar{\mu}_{\sigma_G\sigma_G}(\sigma_G, \ell) \cdot \frac{\sigma_G}{\ell} + \bar{\mu}_{\ell\sigma_G}(\sigma_G, \ell)\right) > 0.$$

Despite a possible labour supply reduction, investment growth predominates labour supply changes with the success to increase national income. The distribution of the society depends analytically on an insurance effect  $(-\sigma_G \cdot \ell)$ , an investment effect  $\left((1-\tau) \cdot \ell \cdot \frac{d\sigma_G}{d\tau}\right)$  and a labour supply effect  $\left((1-\tau) \cdot \sigma_G \cdot \frac{d\ell}{d\tau}\right)$ . Implicit differentiation of the standard deviation yields:

$$(23) \quad \frac{d\sigma}{d\tau} = (-\sigma_G \cdot \ell) + (1-\tau) \cdot \left[ \ell \cdot \frac{d\sigma_G}{d\tau} + \sigma_G \cdot \frac{d\ell}{d\tau} \right].$$

Summing up the sign of the labour and investment effect results in an effect, of which sign is again ambiguous. Taking the insurance effect into account it turns out the same result unless  $\tau$  is in the near of one. Then distribution moves definitely more equally. Theorem 3(f) shows that in this scenario the utility maximizing tax rate is one, too.

## 6 Concluding Remarks

Our goal has been to reexamine labour supply and investments in human capital in an uncertainty framework where individuals take the government budget constraint into account. An important result is that labour supply and investments are increasing functions of tax rates in pure models, whereas in the combined model the individual's behaviour concerning labour supply is less clear. Hence we draw the conclusion that investments in human capital are more appropriate to take precautions against life-time risk than labour supply. Under this perspective empirical labour supply behaviour may be theoretically supported for taxes less than 100 %. Another result of our paper is to have shown that a modification of the starting-point equation leads to opposite results with regard to investments in comparison to the Sinn [1995] essay. In

all cases the welfare state has a justification by rising national income, sometimes equality and most important utility and welfare.

Conditionality or seeing-through the government's actions is from the ex ante perspective plausible, but it is ex post problematical. It is required that there exists a kind of ideal insurance market where the government monitors individual's actions and announces for each labour-investment-strategy a fair insurance premia. That is a quite difficult venture. Therefore a research agenda is to determine taxation effects when agents are not seeing-through the government, and to check if there is still an redistribution paradoxon as found by Sinn [1995]. There are many open questions and this leaves a lot of space for further research.

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## Appendix

Before proving theorems 1-3 consider the maximization problem (20).

$$\max_{\sigma_G, \ell} U(\mu, \sigma, \ell) \quad \text{s. t.:} \quad \mu = \bar{\mu}(\sigma_G, \ell), \sigma = (1 - \tau) \cdot \sigma_G \cdot \ell.$$

The first order conditions are

$$U_\mu \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) + U_\sigma \cdot (1 - \tau) \cdot \ell = 0, \quad \text{respectively}$$

$$(16) \quad A := i(\mu, \sigma) \cdot (1 - \tau) \cdot \ell - \bar{\mu}_{\sigma_G}(\sigma_G, \ell) = 0 \quad \text{and}$$

$$U_\mu \cdot \bar{\mu}_\ell(\sigma_G, \ell) + U_\sigma \cdot (1 - \tau) \cdot \sigma_G + U_\ell = 0, \quad \text{respectively}$$

$$(19) \quad B := i(\mu, \sigma) \cdot (1 - \tau) \cdot \sigma_G + i(\mu, \ell) - \bar{\mu}_\ell(\sigma_G, \ell) = 0.$$

Observe that at the definition of  $A$  and  $B$  the original first order conditions are multiplied with  $(-1)$ .

The partial derivatives of  $A$  and  $B$  are:

$$A_\tau = -i(\mu, \sigma) \cdot \ell - (1 - \tau) \cdot \sigma_G \cdot \ell^2 \cdot i_\sigma(\mu, \sigma),$$

$$A_\ell = i(\mu, \sigma) \cdot (1 - \tau) + (1 - \tau) \cdot \ell \cdot [i_\mu(\mu, \sigma) \cdot \bar{\mu}_\ell(\sigma_G, \ell) + i_\sigma(\mu, \sigma) \cdot (1 - \tau) \cdot \sigma_G] - \bar{\mu}_{\sigma_G \ell}(\sigma_G, \ell),$$

$$A_{\sigma_G} = (1 - \tau) \cdot \ell \cdot [i_\mu(\mu, \sigma) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) + i_\sigma(\mu, \sigma) \cdot (1 - \tau) \cdot \ell] - \bar{\mu}_{\sigma_G \sigma_G}(\sigma_G, \ell),$$

$$B_\tau = -i(\mu, \sigma) \cdot \sigma_G - (1 - \tau) \cdot \sigma_G^2 \cdot \ell \cdot i_\sigma(\mu, \sigma),$$

$$B_\ell = (1-\tau) \cdot \sigma_G \cdot [i_\mu(\mu, \sigma) \cdot \bar{\mu}_\ell(\sigma_G, \ell) + i_\sigma(\mu, \sigma) \cdot (1-\tau) \cdot \sigma_G] + i_\mu(\mu, \ell) \cdot \bar{\mu}_\ell(\sigma_G, \ell) + i_\ell(\mu, \ell),$$

$$B_{\sigma_G} = i(\mu, \sigma) \cdot (1-\tau) + (1-\tau) \cdot \sigma_G \cdot [i_\mu(\mu, \sigma) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) + i_\sigma(\mu, \sigma) \cdot (1-\tau) \cdot \ell] + i_\mu(\mu, \ell) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) - \bar{\mu}_{\ell\sigma_G}(\sigma_G, \ell).$$

The assumption  $H := A_{\sigma_G} \cdot B_\ell - B_{\sigma_G} \cdot A_\ell > 0$ , and  $-A_{\sigma_G} < 0$ ,  $-B_\ell < 0$  ensure that the Hessian matrix  $\begin{bmatrix} -A_{\sigma_G} & -A_\ell \\ -B_{\sigma_G} & -B_\ell \end{bmatrix}$  is negative-definite. Thus the second-order condition is satisfied.

### Proof of theorem 1:

$$(a) \quad \frac{d\ell}{d\tau} = -\frac{B_\tau}{B_\ell} > 0.$$

$$(b) \quad \frac{d\mu}{d\tau} = \bar{\mu}_\ell(\ell) \cdot \frac{d\ell}{d\tau} = k \cdot \sigma_G \cdot \frac{d\ell}{d\tau} > 0.$$

$$(c) \quad \frac{d\sigma}{d\tau} = (1-\tau) \cdot \sigma_G \cdot \frac{d\ell}{d\tau} - \sigma_G \cdot \ell = \frac{-(1-\tau) \cdot \sigma_G \cdot B_\tau - \sigma_G \cdot \ell \cdot B_\ell}{B_\ell},$$

$$\frac{d\sigma}{d\tau} = \sigma_G \cdot \frac{(1-\tau) \cdot \sigma_G \cdot [i(\mu, \sigma) + \ell \cdot i_\mu(\mu, \sigma) \cdot \bar{\mu}_\ell(\ell)] - \ell \cdot [i_\mu(\mu, \ell) \cdot \bar{\mu}_\ell(\ell) + i_\ell(\mu, \ell)]}{B_\ell}.$$

$$(d) \quad \frac{dU}{d\tau} = -U_\sigma \cdot \sigma_G \cdot \ell > 0. \quad \square$$

### Proof of theorem 2:

$$(a) \quad \frac{d\sigma_G}{d\tau} = -\frac{A_\tau}{A_{\sigma_G}} > 0.$$

$$(b) \quad \frac{de}{d\tau} = \beta_{\sigma_G}^{-1}(\sigma_G / R(W)) \cdot \frac{d\sigma_G}{d\tau} \cdot \frac{1}{R(W)} > 0.$$

$$(c) \quad \frac{d\mu}{d\tau} = \bar{\mu}_{\sigma_G}(\sigma_G) \cdot \frac{d\sigma_G}{d\tau} > 0, \text{ because the first order condition (19) implies } \bar{\mu}_{\sigma_G}(\sigma_G) > 0.$$

$$(d) \quad \frac{d\sigma}{d\tau} = -\sigma_G \cdot \ell + (1-\tau) \cdot \ell \cdot \frac{d\sigma_G}{d\tau} = \frac{-\sigma_G \cdot \ell \cdot A_{\sigma_G} - (1-\tau) \cdot \ell \cdot A_\tau}{A_{\sigma_G}},$$

$$\frac{d\sigma}{d\tau} = \ell \cdot \frac{(1-\tau) \cdot \ell \cdot [i(\mu, \sigma) - \sigma_G \cdot i_\mu(\mu, \sigma) \cdot \bar{\mu}_{\sigma_G}(\sigma_G)] + \sigma_G \cdot \bar{\mu}_{\sigma_G\sigma_G}(\sigma_G)}{A_{\sigma_G}}.$$

$$(e) \quad \frac{dU}{d\tau} = -U_\sigma \cdot \sigma_G \cdot \ell > 0. \quad \square$$

### Proof of theorem 3:

Total differentiation of  $A$  and  $B$  yields the following equations in matrix notation:

$$\begin{bmatrix} A_{\sigma_G} & A_\ell \\ B_{\sigma_G} & B_\ell \end{bmatrix} \cdot \begin{bmatrix} d\sigma_G \\ dl \end{bmatrix} = \begin{bmatrix} -A_\tau \cdot d\tau \\ -B_\tau \cdot d\tau \end{bmatrix}.$$

$$(a) \quad \frac{d\sigma_G}{d\tau} = \frac{-A_\tau \cdot B_\ell + B_\tau \cdot A_\ell}{A_{\sigma_G} \cdot B_\ell - B_{\sigma_G} \cdot A_\ell} = \frac{-A_\tau \cdot B_\ell + B_\tau \cdot A_\ell}{H} = \frac{A_\tau}{H} \cdot \left[ -B_\ell + A_\ell \cdot \frac{\sigma_G}{\ell} \right] \text{ due to}$$

$$A_\tau \cdot \frac{\sigma_G}{\ell} = B_\tau.$$

$$\frac{d\sigma_G}{d\tau} = \frac{A_\tau}{H} \left[ i_\mu(\mu, \ell) \cdot \bar{\mu}_\ell(\sigma_G, \ell) + i_\ell(\mu, \ell) + \frac{\sigma_G}{\ell} \cdot (\bar{\mu}_{\sigma_G \ell}(\sigma_G, \ell) - i(\mu, \sigma) \cdot (1 - \tau)) \right].$$

Addition of  $\bar{\mu}_{\sigma_G \ell}(\sigma_G, \ell)$  to (16) we obtain

$$\begin{aligned} & \bar{\mu}_{\sigma_G \ell}(\sigma_G, \ell) - i(\mu, \sigma) \cdot (1 - \tau) \\ &= \bar{\mu}_{\sigma_G \ell}(\sigma_G, \ell) - \frac{\bar{\mu}_{\sigma_G}(\sigma_G, \ell)}{\ell} = \beta_{\sigma_G}^{-1}(\sigma_G / R(W)) \cdot \frac{1}{R(W) \cdot \ell} > 0 \end{aligned}$$

and it results  $\frac{d\sigma_G}{d\tau} > 0$ .

$$(b) \quad \frac{de}{d\tau} = \beta_{\sigma_G}^{-1}(\sigma_G / R(W)) \cdot \frac{d\sigma_G}{d\tau} \cdot \frac{1}{R(W)} > 0.$$

$$(c) \quad \frac{dl}{d\tau} = \frac{-A_{\sigma_G} \cdot B_\tau + B_{\sigma_G} \cdot A_\tau}{H} = \frac{A_\tau}{H} \cdot \left[ B_{\sigma_G} - A_{\sigma_G} \cdot \frac{\sigma_G}{\ell} \right],$$

$$\frac{dl}{d\tau} = \frac{A_\tau}{H} \cdot \left[ \bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) - i(\mu, \sigma) \cdot (1 - \tau) - \frac{\sigma_G}{\ell} \bar{\mu}_{\sigma_G \sigma_G}(\sigma_G, \ell) - i_\mu(\mu, \ell) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) \right]$$

with  $\bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) - i(\mu, \sigma) \cdot (1 - \tau) - \frac{\sigma_G}{\ell} \bar{\mu}_{\sigma_G \sigma_G}(\sigma_G, \ell) > 0$  and

$$-i_\mu(\mu, \ell) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) < 0.$$

$$(d) \quad \frac{d\mu}{d\tau} = \bar{\mu}_{\sigma_G}(\sigma_G, \ell) \cdot \frac{d\sigma_G}{d\tau} + \bar{\mu}_\ell(\sigma_G, \ell) \cdot \frac{dl}{d\tau} > 0,$$

$$\begin{aligned} \frac{d\mu}{d\tau} &= \frac{A_\tau}{H} \cdot \left[ \bar{\mu}_{\sigma_G}(\sigma_G, \ell) \cdot \left[ i_\ell(\mu, \ell) + \frac{\sigma_G}{\ell} \cdot (\bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) - i(\mu, \sigma) \cdot (1 - \tau)) \right] \right. \\ &\quad \left. + \bar{\mu}_\ell(\sigma_G, \ell) \cdot \left[ \bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) - i(\mu, \sigma) \cdot (1 - \tau) - \frac{\sigma_G}{\ell} \bar{\mu}_{\sigma_G \sigma_G}(\sigma_G, \ell) \right] \right] > 0. \end{aligned}$$

$$(e) \quad \frac{d\sigma}{d\tau} = -\sigma_G \cdot \ell + (1 - \tau) \cdot \ell \cdot \frac{d\sigma_G}{d\tau} + (1 - \tau) \cdot \sigma_G \cdot \frac{dl}{d\tau},$$

$$\begin{aligned} \frac{d\sigma}{d\tau} &= -\sigma_G \cdot \ell + \frac{A_\tau}{H} \cdot (1 - \tau) \cdot \left[ \ell \cdot i_\mu(\mu, \ell) \cdot \bar{\mu}_\ell(\sigma_G, \ell) + \ell \cdot i_\ell(\mu, \ell) \right. \\ &\quad \left. + \sigma_G \cdot (\bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) - i(\mu, \sigma) \cdot (1 - \tau)) + \sigma_G \cdot \bar{\mu}_{\ell \sigma_G}(\sigma_G, \ell) - \sigma_G \cdot i(\mu, \sigma) \cdot (1 - \tau) \right. \\ &\quad \left. - \frac{\sigma_G^2}{\ell} \cdot \bar{\mu}_{\sigma_G \sigma_G}(\sigma_G, \ell) - \sigma_G \cdot i_\mu(\mu, \ell) \cdot \bar{\mu}_{\sigma_G}(\sigma_G, \ell) \right]. \end{aligned}$$

$$(f) \quad \frac{dU}{d\tau} = -U_\sigma \cdot \sigma_G \cdot \ell > 0. \quad \square$$

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