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The Conditions for Ecological Sustainable Development in the Context of a Double-Limited Selfpurification Model of an Aggregate Water Recourse

Volkswirtschaftliche Diskussionsbeiträge, No. 50-94

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Suggested Citation: Fiedler, Klaus (1994): The Conditions for Ecological Sustainable Development in the Context of a Double-Limited Selfpurification Model of an Aggregate Water Recourse, Volkswirtschaftliche Diskussionsbeiträge, No. 50-94, Universität Siegen, Fakultät III, Wirtschaftswissenschaften, Wirtschaftsinformatik und Wirtschaftsrecht, Siegen

This Version is available at: https://hdl.handle.net/10419/118743

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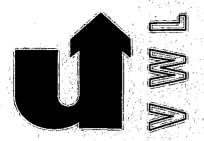
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VOLKSWIRTSCHAFTLICHE DISKUSSIONSBEITRÄGE



The Conditions for Ecological Sustainable Development in the Context of a Double-Limited Selfpurification Model of an Aggregate Water Resource

Klaus Fiedler

Diskussionsbeitrag Nr. 50 - 94

UNIVERSITÄT - GESAMTHOCHSCHULE - SIEGEN FACHBEREICH WIRTSCHAFTSWISSENSCHAFTEN

The Conditions for Ecological Sustainable Development in the Context of a Double-Limited Selfpurification Model of an Aggregate Water Resource

Klaus Fiedler

Abstract: Based on the classical biological Monod-Model a so called double-limited self-purification model working in an aggregate water resource is constructed. From this model we derive a stationary selfpurification function and three conditions for ecological sustainable development.

1. Introduction

In the economic literature (e. g. Barbier, 1987; Serafy, 1992; Tisdell, 1993) there are several definitions and / or interpretations of sustainable development. The following working definition of this key word has been given by Pearce and Turner (1990): "It involves maximising the net benefits of economic development, subject to maintaining the (assimilative)² services and quality of natural resources over time." Starting out from this definition we elabourate a set of conditions to maintain stationary assimilative services (e. g. the existence of a stationary selfpurification function) in an aggregate water resource continually polluted by a single degradable pollutant. There is not only a great variety of definitions of sustainable development in the economic literature but also a diverse spectrum of hypotheses about stationary selfpurification processes in water resources (Fiedler, 1994).

For that reason we have to construct at first a selfpurification model based on the mass

University—GH—Siegen, Fachbereich 5, Hölderlinstr 3, 57076 Siegen (Germany). An earlier version has been presented at the 5th EAERE Annual Conference, University College Dublin held in June 22—24, 1994. I would like to thank Rüdiger Pethig (University—GH—Siegen, Department of Public Economics (Volkswirtschaftslehre IV) and Naomi Zeitouni (University of Haifa in Israel, Natural Resources & Environmental Research Centre) for valuable comments and stimulating discussions. Furtheron this paper has been profiting from the friendly and stimulating atmosphere at the Department of Public Economics (Volkswirtschaftslehre IV) [e. g. my colleagues Hildegard Müller and Daniel Weinbrenner]. At last but not at least I have to thank my wife Petra Ullmann—Fiedler for critically reading this text. However remaining errors are of my own responsibility.

²The adjective "assimilative" has been added by the author himself.

conservation principle and on the classical Monod-Model (Monod, 1942). Monod pointed out that microorganisms use pollutants as food source.³ The Monod-Model is based on the enzyme kinetics of Michaelis and Menten (1913). Being well supported by experimental results (Thomas Jr., 1972; Jones, 1978; Andrews, 1978) it has found the natural science's approval (e. g. Ohgaki and Wantawin, 1989; James, 1993). Since the Monod-Model has been used only to reflect the assimilation of pollutant stocks we have to supplement it by adding a waste flow. Furtheron aerobic selfpurification processes are of higher economic relevance.⁴ For that reason we consider aerobic microorganisms and take oxygen limitation of selfpurification into account (Ohgaki and Wantawin, 1989). Since in our selfpurification model both the stock of pollutant and the stock of oxygen work as a limiting factor⁵ on bacterial growth we call it double-limited following Bader (1978) and Ohgaki and Wantawin (1989). At last one characteristic of pollutants used as food source is the so called pollutant inhibition occuring in selfpurification processes (Andrews, 1978; Hartmann, 1983; Ohgaki and Wantawin, 1989).

- The first condition for sustainable ecological development our model implies is that the continual waste flow has to be less than or equal to the assimilative capacity of the aggregate water resource. Of cource this view is not a novel in economic literature (e. g. Barbier and Markandya, 1990, Pearce and Turner, 1990) but it has not been interpretated from the ecological point of view.
- The second condition requires the stock of pollutant neither to be too low nor too high.
- Following the third condition the oxygen saturation level has to be sufficiently high to sustain life of the aerobic microorganisms.

2. The Monod-Model

Fundamental models considering bacterial growth and pollutant assimilation are often

³The Monod-Model is a biological selfpurification model. Biological selfpurification processes are the most important ones compared to the chemical and physical processes (Habeck-Tropfke, 1980; Uhlmann, 1988).

⁴At first the aerobic processes go off more quickly over time than the anaerobic ones (e. g. Hartmann, 1988). And at the second a high oxygen stock indicates a high water quality while a zero oxygen level is going along with low water quality not being of economic interest.

⁵A factor works as a limiting factor on any process if and only if this process doese not go off without this factor.

based on the classical Monod-Model (Monod, 1942; James, 1993) expressed by the following two coupled homogeneous differential equations:

(2.1)
$$\dot{\mathbf{s}} := \frac{\mathrm{d}\mathbf{s}}{\mathrm{d}\mathbf{t}} = -\mathbf{M}_0(\mathbf{s}) \cdot \mathbf{x}$$

(2.2)
$$\dot{\mathbf{x}} := \frac{\mathbf{d}\mathbf{x}}{\mathbf{d}\mathbf{t}} = \gamma \cdot \mathbf{M}_0(\mathbf{s}) \cdot \mathbf{x} - \mathbf{k}_d \cdot \mathbf{x}$$

In the Monod-Model a well-mixed and therefore a homogeneous water resource is assumed (James, 1993). Furtheron the temperature and the pressure insight the aggregate watersource is assumed to be constant over time. The specific growth rate $M_0(s)$ [1 / period] of the microorganism stock⁶ is defined as $M_0(s) := \frac{\mu_0 \cdot s}{k_m + s}$, μ_0 [1 / period] symbolises the maximal growth rate, km [mass] is the Michaelis-Menten-Constant and s [mass / period] represents the assimilation rate of the single degradable pollutant s into the mass of the microorganism stock x.7 The net rate of change of microbial mass x [mass / period] in equation (2.2) is equal to the growth rate $\gamma \cdot M_0(s) \cdot x$ [mass / period] minus the decay rate k_d·x [mass / period], where k_d [1 / period] represents a decay parameter. The decay rate represents altogether death, cell decomposition and consumption as food source by higher terrestrial organisms (Thomas Jr., 1972). The consumption by higher land organisms causes the biomass x probably to get out of the water resource. This fact will be analysed later in the context of water resource eutrophication. In the following we set for simplicity the yield coefficient γ [biomass / pollutant mass] equal to one. At last oberve that the pollutant s works as a limiting factor on the microbial growth and on the assimilation rate. We precise the limiting factor by the following way: $M_0(s) = \dot{s} = \dot{x} = 0 \iff s = 0$.

3. Construction of a Double-Limited Selfpurification Model Based on the Monod-Model

Since water pollution is a continual pollution problem we have to supplement the Monod-Model by considering a waste emission rate e [mass / period] entering the aggregate water resource directly. Furtheron we focus our attention on aerobic self-purification processes being of higher intererst for economic analysis than the anaerobic ones and assume the stock of mikroorganisms to be aerobic. Therefore we have to take

⁶The term for M₀(s) has been derived from the enzyme kinetics of Michaelis and Menten (1913) for example by Morris (1972), Thomas Jr. (1972) and Murray (1989).

⁷The assimilation rate can also be interpretated as the selfpurification rate.

into consideration that aerobic selfpurification processes are limited by the oxygen stock. For that reason we couple up an additional differential equation describing the time evolution of the oxygen stock. The most of pollutants are going along with the so called pollutant inhibition when being used as food source by microorganisms in water resources (e. g. Andrews, 1978; Ohgaki and Wantawin, 1989; Bever, Stein and Teichmann, 1993). This fact requires to substitute the specific growth rate $M_0(s)$ by the term M(s) := $\frac{\mu \cdot s}{k_m + s + k_p s^2}$ [1 / mass-period], where k_p [1 / mass] symbolizes an inhibition constant⁸ and μ [1 / mass-period] is a constant. Taking these additional considerations into account our selfpurification model has the following form:

$$(2.3) \qquad \dot{s} = e - M(s) \cdot x \cdot o$$

$$\dot{\mathbf{x}} = \mathbf{M}(\mathbf{s}) \cdot \mathbf{x} \cdot \mathbf{o} - \mathbf{k}_{\mathbf{d}} \cdot \mathbf{x}$$

(2.5)
$$\dot{o} := \frac{do}{dt} = k_0(o_s - o) - M(s) \cdot x \cdot o$$

In equation (2.5) k_0 [1 / period] represents the reaeration constant and o_s [mass] is the oxygen saturation constant. The net rate of change of the oxygen stock o [mass / period] is equal to the reaeration rate $k_0(o_s - o)$ minus the rate of oxygen consumption $M(s) \cdot x \cdot o$. The former term should be also interpretated as the selfpurification rate and as the microbial growth rate starting out from the equations (2.3) and (2.4). Observe that both the stock of pollutant and the stock of oxygen work as limiting factors on bacterial growth. Therefore we call our selfpurification model following Bader (1978) and Ohgaki and Wantawin (1989) double—limited.

The double-limited selfpurification model can be even better understood by introducing the following investigations:

(2.6)
$$\dot{s} \geqslant 0 \Leftrightarrow e \geqslant M(s) \cdot x \cdot o$$

The stock of pollutant increases / remains constant / decreases over time if and only if the pollution rate e (continual waste flow from the economic sector) is greater than / equal to / less than the selfpurification rate $M(s) \cdot x \cdot o$.

⁸The term M(s) has been derived from the generalised enzym kinetics of Michaelis and Menten by (Morris 1972).

(2.7)
$$\dot{x} \geq 0 \iff M(s) \cdot x \cdot o \geq k_d \cdot x$$

The biomass increases / remains constant / decreases over time if and only if the microbial growth rate $M(s) \cdot x \cdot o$ is greater than / equal to / less than the decay rate $k_d \cdot x$ of the stock of microorganisms.

(2.8)
$$\dot{o} \geq 0 \iff k_0(o_s - o) \geq M(s) \cdot x \cdot o$$

The stock of oxygen increases / remains constant / decreases over time if and only if the reaeration rate $k_0(o_s - o)$ is greater than / equal to / less than the degradation rate $M(s) \cdot x \cdot o$ of the oxygen stock.

Considering these investigations an eutrophication [that is the accumulation of the biomass x in the water resource over time (James, 1993)] does not take place if and only if

$$\min[e, k_0(o_s - o)] \le k_d \cdot x$$
 $\Rightarrow \dot{x} \le 0$

This expression prescribes that the smaller amount of both the pollutant flow and the reaeration rate must not exceed the rate of microorganism mass leaving the water resource through consumption by succeeding higher land organisms. Observe that the term $\min[e, k_0(o_s - o)]$ makes the double-limitation of this model precise.

In the long run, $\dot{s} = \dot{x} = \dot{o} = 0$, the stationary reaeration rate $e = k_0(o_s - o) \ge 0$, the specific growth rate M(s) and the stationary oxygen demand function

(2.9)
$$o = O(s) := \frac{k_d}{M(s)} \le o_s$$

determine the stationary selfpurification function of the model:

(2.10)
$$e = S(s) := k_0 \cdot o_s - k_0 \cdot \frac{k_d}{M(s)} \ge 0$$

In (2.10) the relation $S(s) \ge 0$ results from (2.9). The ecological interpretation of the stationary selfpurification function can be given as follows: The continual pollution rate e is assimilated into biomass in each point of time by the aerobic microorganisms within

the water resource. This stationary selfpurification process runs in such a way that all the stock of pollutant, the stock of microorganisms and the stock of oxygen simultaneously remains constant over time. Figure 1 illustrates the geometrical derivation of the function S(s):

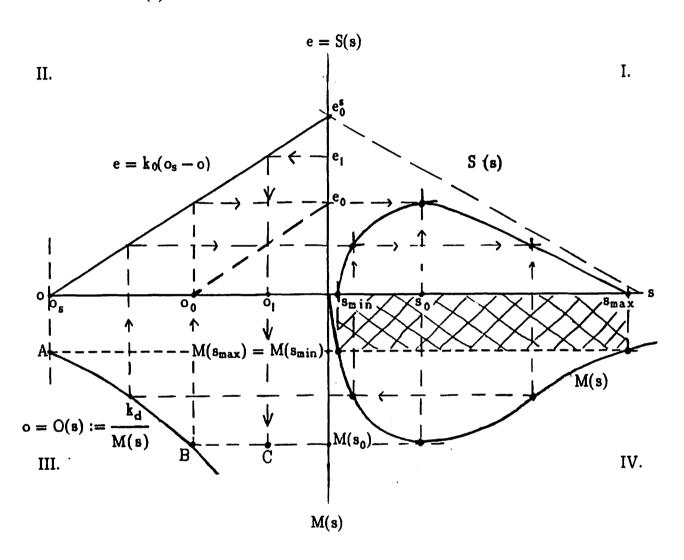


Figure 1: Geometrical derivation of the stationary selfpurification function

The selfpurification function depicted in the first quadrant of figure 1 has the following analytical properties: S(s) is concave shaped, $S_{ss} = -\frac{2k_dk_0k_m}{\mu \cdot s^3} < 0$, hits the zero locus at the points $s_{min} := \frac{\alpha}{2} - \left[\frac{\alpha^2}{4} - \frac{k_m}{k_p}\right]^{\frac{1}{2}} > 0$ and $s_{max} := \frac{\alpha}{2} + \left[\frac{\alpha^2}{4} - \frac{k_m}{k_p}\right]^{\frac{1}{2}} > s_{min}$ with α defined as $\alpha := \frac{\mu o_s - k_d}{k_d k_p}$, reaches its global maximum at $s_0 := \left[\frac{k_m}{k_p}\right]^{\frac{1}{2}}$ with $s_0 \in [s_{min}, s_{max}]$ and has the assimilative capacity $e_0 = S(s_0) := k_0 o_s - k_0 \frac{k_d}{M(s_0)} = \max{[S(s)]}$. The term $o_0 := \frac{k_d}{M(s_0)} = O(s_0) = \min{[O(s)]} > 0$ has to be interpretated as the absolute minimum of the stationary oxygen stock (point B in figure 1) enough to sustain a stationary microbial life. Furtheron the following properties of our stationary selfpurification

function have to be proved:

- I. $e_0 \ge 0 \Leftrightarrow o_s \ge o_0$: This property results immediately by employing the definition $o_0 := \frac{k_d}{M(s_0)}$ in $e_0 = k_0 o_s k_0 \frac{k_d}{M(s_0)}$. Thus we obtain $e_0 = k_0 (o_s o_0) \ge 0 \Leftrightarrow o_s \ge o_0$.
- II. $\frac{\alpha^2}{4} \frac{k_m}{k_p} \ge 0 \iff o_s \ge o_0$: With the help of the definition of α and of o_0 this assertion results after some rearrangements of terms.⁹
- III. $s_{min} > 0$: Using the definition of s_{min} and of α the relation $k_m/k_p > 0$ results. This is true since $k_m > 0$ and $k_p > 0$. \square
- IV. $s_{min} < s_0 < s_{max} \Leftrightarrow o_s \ge o_0$: Employing the definitions of s_{min} , s_0 , s_{max} and α we obtain the following two relations after some transformations: $s_{min} < s_0 \Leftrightarrow o_s \ge o_0$ and $s_0 < s_{max} \Leftrightarrow o_s \ge o_0$. \square

4. Conditions for sustainable ecological development

We are now in the position to cite a set of sustainability conditions for the long run development of our aggregate water resource.

Proposition: In the context of our selfpurification model a sustainable ecological development is possible if and only if a positive stationary selfpurification function e = S(s) > 0 exists within the water resource.

This proposition can easily be related to the following conditions of sustainable ecological development given by Pearce and Turner (1990) and Pearce, Barbier and Markandya (1990): Maximising the net benefits of economic development

- 1. must not decrease the natural capital stock over time, and
- 2. have to maintain the assimilative services and the quality of natural resources over time.

⁹In geometrical terms the case $\frac{\alpha^2}{4} - \frac{k_m}{k_p} = 0 \Leftrightarrow o_s = o_0$ means that the function S(s) degenerates to one single point in which the three parameters s_{min} , s_0 and s_{max} coincidate.

Our sustainable condition is equivalent to these two conditions if we identify the stationary selfpurification function with a natural capital stock giving perpetual assimilative services to any economic sector. The existence of a stationary selfpurification function is bound to the following three conditions which have to be satisfied simultaneously:

Condition 1: $e \le e_0$

The pollution rate must not exceed the assimilative capacity within the water resource. This condition has been given by Pearce and Turner (1990) but it has not yet been interpretated from the ecological point of view. To interpret the first condition in the context of our selfpurification model we assume that this condition is violated (e. g. by $e_1 > e_0$ in figure 1). In this case the stationary stock of oxygen is not sufficient to sustain the life of the aerobic microorganisms in the long run. Therefore the selfpurification process will come to a standstill and the stock of pollutant will increase over time. This mechanism calls for closer inspection: The stationary reaeration rate is given by $e = k_0(o_s - o)$ implying

(2.11)
$$o = o_s - \frac{e}{k_0}$$

For the case $e = e_0$ equation (2.11) leads to (point B in figure 1)

(2.12)
$$o = o_s - \frac{e_0}{k_0} = \frac{k_d}{M(s_0)} =: o_0 > 0.$$

Employing the assumption $e_1 > e_0$ in (2.12) exhibits the following relation:

(2.13)
$$o = o_s - \frac{e_1}{k_0} =: o_1 < o_0 = o_s - \frac{e_0}{k_0}$$

Further on $o_1 < o_0 = \frac{k_d}{M(s_0)}$ implies $\dot{x} < 0$ (point C in figure 1) by considering (2.7). A decreasing stock of microorganisms leads to x = 0 in the long run, consequently $M(s) \cdot x \cdot o = 0$ is valid and therefore using (2.6) $\dot{s} = e_1 > 0$ results. \Box

Condition 2: $s_{min} < s < s_{max}$

This condition is immediately obtained by solving the relation e = S(s) > 0 for the variable s. If the second condition is violated the microorganisms will have not enough

food to sustain life for $s < s_{min}$ and will be poisoned if $s > s_{max}$ is valid. Consequently the selfpurification process stops and the stock of pollutant increases over time. This fact should be proved: The relations $s < s_{min}$ and $s > s_{max}$ imply $M(s) \le M(s_{min}) = M(s_{max})$. From (2.8) we obtain the stationary stock of microorganisms

(2.14)
$$x = \frac{k_0 (o_s - o)}{M(s) \cdot o} \ge 0.$$

Furtheron (2.9) implies:

$$(2.15a) o = \frac{k_d}{M(s)}$$

(2.15b)
$$o_s = \frac{k_d}{M(s_{min})} = \frac{k_d}{M(s_{max})} \qquad \forall x \ge 0$$

Employing these equations into (2.14) yields

(2.16)
$$x = k_0 \cdot \frac{M(s) - M(s_{\min})}{M(s) \cdot M(s_{\min})} \ge 0 \Leftrightarrow M(s) \ge M(s_{\min}).$$

According to (2.16) the stationary stock of microorganisms x is greater than / equal to zero if and only if the specific growth rate M(s) is greater than / equal to the mimimal specific growth rate $M(s_{\min})$. Consequently in the case of $s < s_{\min}$ and $s > s_{\max}$ implying $M(s) \le M(s_{\min})$ the microorganisms become extinct. Therefore in the long run x = 0 is valid and thus $\dot{s} = e > 0$.

A similar condition is stated by Barbier and Markandya (1990). However according to the authors there exists only an upper limit s_{max} to the pollutant stock in such a way that environmental degradation destroys the natural selfpurification processes if the pollutant stock exceeds s_{max} .

Condition 3: $o_s > o_0$

This condition requires the oxygen saturation limit to be sufficiently high to sustain the microbial life, i. e. to exceed the threshold o₀. This should be proved: In the steady—state

¹⁰This equation results form substituting s_{min} and s_{max} into the specific growth function after some rearrangements. From the ecological point of view $M(s_{min}) = M(s_{max}) = \frac{k_d}{o_s} > 0$ is the minimal specific growth rate going along with a stationary stock of microorganisms (see figure 1). Observe that the effect of pollutant scarcity and abundance is symmetrical in the sense that $M(s_{min}) = M(s_{max})$ is valid.

the relation $0 \le S(s) = e = k_0(o_s - o) \iff o_s \ge o \ge o_0$ is valid. Consequently our third sustainability condition $e = S(s) > 0 \iff o_s > o_0$ is also valid.¹¹

With it we have according to $x = \frac{k_0 (o_s - o)}{M(s) \cdot o} \ge 0 \Leftrightarrow o_s \ge o \ge o_0$. In the case $o_s = o_0$ the oxygen saturation level is too low to sustain the microbial life in the long run. Hence the pollutant stock rises over time, for $x = \frac{k_0 (o_0 - o)}{M(s) \cdot o} = M(s) \cdot o \cdot x = 0$ and thus $\dot{s} = e > 0$. In geometrical terms the line representing the stationary reaeration in the second quadrant of figure 1 is shifted parallelly from the position $o_s e_o^s (o_s > o_0)$ to the position $o_0 e_o^s$. Hence the function S(s) degenerates to one single point $s = s_0 = s_{min} = s_{max}$ lying on the s—axis. In this point e = 0 is valid.

5. Concluding Remarks

A double—limited selfpurification model based on the classical Monod—Model implies the following three conditions for sustainable ecological development of an aggregate water resource: At first the continual waste flow has to be less than or equal to the assimilative capacity of the aggregate water resource. The second condition prescribes the stock of pollutant not to be too low nor too high. Following the third condition the oxygen saturation limit has to be sufficiently high to sustain life of the aerobic microorganisms.

In addition one can also construct triple—, quadruple—limited selfpurification models and so forth (Ohgaki and Wantawin 1989). Furtheron ovserve that the stationary selfpurification function e = S(s) is just shaped as the yield—effort curve belonging to the classical fishing models (Clark 1976) but has rather different ecological explanations. At last notice that the implications of the environmental economics' models (e. g. the shape of the transformation—frontier with pollutant versus waste flow or water quality versus

¹¹We have proved above that $e_o \ge 0 \Leftrightarrow o_s \ge o_o$. Observe that this assertion is a special case of the condition 3, since $e = e_o$ is a functual value of S(s) according to $e_o = \max[S(s)]$.

waste flow)¹² are depending significantilly on the shape of the function e = S(s). To our best knowledge this paper is the first one which both derives and explans a stationary selfpurification function from ecological building blocks.

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¹²According to Mäler (1974), Pethig (1976) and Fiedler (1994) it is convenient to use the linear transformation function $q = V(s) := q_{nat} - s$ with the definitions $q = V(s_{max}) = q_{nat} - s_{max} =: 0$ and $s_{max} := \frac{\alpha}{2} + \left[\frac{\alpha^2}{4} - \frac{k_m}{k_p}\right]^{1/2}$. The parameter $q_{nat} \ge q$ denotes the natural water quality. By carrying out the transformation one obtains from the stationary selfpurification function (2.10) the corresponding water quality regeneration function $e = E(q) := k_0 \cdot o_s - k_0 \cdot \frac{k_d}{M(q)}$. The water quality regeneration function is also shaped like the yield—effort curve implied by the fishing models.

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