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## Uncertainty, Capacity Costs and Competition in the Electric Power Industry

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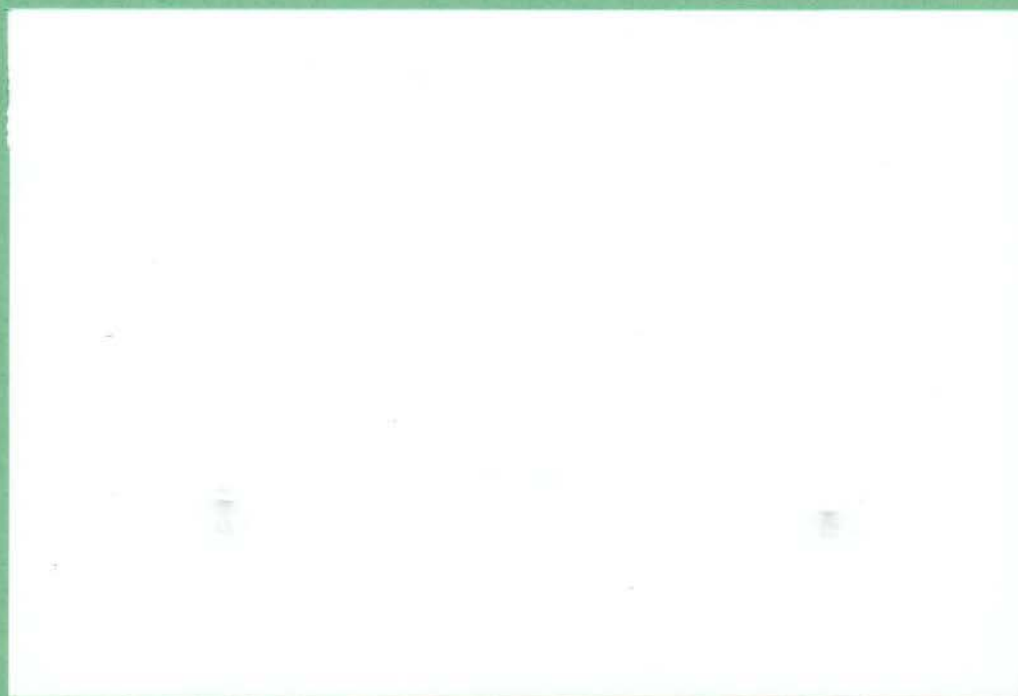
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# Uncertainty, Capacity Costs and Competition in the Electric Power Industry

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**Abstract:** Electric power is a commodity with a number of peculiarities that are often conjured when calling for shelter from competition in the electricity sector. Among those special features is the alleged difficulty or even failure of the market process to maintain security of provision in view of severe uncertainties such as volatile demand, uncertain costs of primary energy and the threat of production breakdowns. It is sometimes argued that coping with these difficulties and simultaneously facing high capacity building costs requires fully integrated utilities and cooperation among them.

The paper scrutinizes a simple parametric model explicitly allowing for uncertainty with respect to demand and production breakdowns. It aims at investigating how different institutional frameworks cope with these difficulties of power provision when capacity costs are accounted for. The point of departure are isolated regional utilities that are fully integrated profit-maximizing monopolies. It is then shown that cooperation among them tends to be beneficial for both the utilities and the consumers by taking advantage of pooling different states of the world. If wholesale trade is introduced among utilities, when these are price takers at the wholesale market but monopolistic price setters at home, it is shown that the performance is the same as in case of cooperative joint profit maximization.

Comparing perfect competition with cooperation shows that the both cope with uncertainties and capacity costs equally well where the former avoids, of course, the monopolistic deadweight loss. All these arguments are elaborated for the cases of demand uncertainty and breakdown uncertainty with surprising similarities in the principal results.

# Uncertainty, Capacity Costs and Competition in the Electric Power Industry <sup>1</sup>

Günter Knieps and Rüdiger Pethig

## 1. Introduction

Electric power is a commodity with a number of special properties: the 'transport' from generation to distribution occurs through transmission lines which have, like generation units, limited capacity; as a commodity, power cannot be stored, not even for seconds; any excess supply of power is immediately lost and any excess demand (with an attempt to realise it) leads to zero supply in the total power system because the transmission of power breaks down completely. Moreover, power generation requires large sunk investments, which have to be carried out before actual (random) demand for electricity is known.

As discussed at length in the literature all these peculiarities provide a rationale for vertical integration in the power system, i.e. for the emergence of utilities whose business encompasses all three segments: **power generation** (producers), (high-voltage) **transmission**, and **distribution**<sup>2</sup>. It is widely acknowledged that both transmission and distribution are natural monopolies which may or may not be operated by disintegrated firms, however. On the other hand, economics of scale do not seem to play a major role (any more) at the level of power generation. Empirical studies have shown that economies of scale in power generation are exhausted at such a level so that electricity can be produced efficiently by a large number of firms, e.g. Christensen and Greene (1976), Joskow and Schmalensee (1983), Helden and van Muyksen (1981). Hence it is the segment of power generation that is the most promising candidate for both disintegration and competition in the power system.

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<sup>1</sup> This paper has been written while Rüdiger Pethig was Visiting Professor at the Rijksuniversiteit Groningen. Helpful comments on an earlier version of this paper by Wolfgang Pfaffenberger are gratefully acknowledged.

<sup>2</sup> This discussion is based on the more general literature on vertical integration, e.g. Klein, Crawford and Alchian (1978), Kleindorfer and Knieps (1982), Williamson (1975, 1979).

Recent institutional changes in the power system of the US and European countries provide some evidence for the potential (and the limits?) of competitive markets for power. In the USA after the passing of the Public Utility Regulatory Policy Act (PURPA) of 1987 the regional electricity companies (public utilities) have to offer standard contracts to cogenerators and small producers of electricity using regenerative sources of energy (so-called 'qualifying facilities') (e.g. Bolle 1990, 1991). However, since small power producers had to be paid an attractively high rate (based on the avoided costs of public utilities), the huge amount of power available from cogenerators and small producers created great concern (e.g. Summerton and Bradshaw 1991). In the Netherlands the new electricity law from 1987 allows horizontal collaboration between different electricity producers, which jointly deal with newly founded distribution companies (e.g. Ministerie van Economische Zaken 1987). The question remains to what extent such collaboration introduces elements of competition in the Dutch electricity market. Moreover, possibilities for exchange of electricity between member countries of the European Community are currently discussed quite controversially. Thus, Finon (1990) argues that, in vertically integrated industries, the advantages of open access could be offset by the difficulties of combining competition with the obligation to supply, and by the disadvantages of new and complex regulation that it will be necessary to implement.

The present paper aims at investigating the potential of competition in the electric power system in a stylised, conceptional way in order to enlighten the current, strongly policy oriented controversy. Opponents to competition in the power sector content that competition fails to guarantee security of provision in view of substantive uncertainties with respect to demand, production costs and production breakdowns. Section 2 deals with demand uncertainty, the rather parallel case of cost uncertainty is considered in Appendix B, whereas Section 3 deals with uncertain production breakdowns under different institutional frameworks. This enables us to tackle two often stated basic arguments against the break-up of production and distributions which we consider the most significant ones. The principal question is to what extent markets can coordinate production and distribution under demand and break down uncertainty. First, we compare the vertical integrated monopoly with the cooperative solution, where no external coordination problem between regional monopolies, arise. Subsequently, competitive wholesale trade is introduced, where electricity customers still do not have the possibility to choose between different electricity producers. Finally, competitive retail markets are considered.

To concentrate on the market for power generation any frictions that may arise in the

areas of transmission and distribution (capacity bottleneck, monopoly elements etc.) are neglected. Throughout this paper we therefore assume that transmission of power is costless and unrestricted by limited capacity of the grid. Before production of electricity can take place sunk investments have to be made. Whereas the actual level of production may depend on the realization of random demand, sunk investment costs cannot be changed afterwards. Investment decisions and the decisions of power supply (depending on the state of the world) both are based (in the ex ante world) on the best available information (with the probability distribution of the relevant random variable representing common knowledge).

The underlying assumption is that there exists a complete set of contingent markets. Technological progress with respect to information technology is assumed to be strong enough, to exclude any "fine tuning" frictions (e.g. Walker 1986). The basic question is, whether – at least within this highly stylized framework – , the traditional vertical integration solutions can be dominated by market oriented institutional configurations.

## 2. Demand uncertainty under different institutional frameworks

### 2.1 The fully integrated regional monopoly

Suppose, a fully integrated electricity utility plans to build its power generation plant whose life time is assumed to be one time period<sup>3</sup>. For that whole period it faces the linear demand

$$(1) \quad p_j = a_j - \frac{b}{2} x_j \quad (a_j, b > 0).$$

Costs are linear in output and capacity, respectively.  $x_j$  is the quantity of power demanded and  $p_j$  the price of power. Denote by  $c$  the capacity cost per unit of production capacity  $y_{jc}$  and by  $v$  the operating cost per unit of actual output  $y_j$  ( $y_j \leq y_{jc}$ ). Hence total cost is

$$(2) \quad v y_j + c y_{jc}$$

in our simple one-period framework. Market clearing requires

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<sup>3</sup> Constraining the model to one period simplifies the analysis considerably but does not reduce its generality. The introduction of contingent future markets is possible but prevents the derivation of simple and unambiguous results.

$$(3) \quad x_j = y_j.$$

Suppose now, the demand parameter  $a_j$  is a random variable which assumes the value  $a_{j1}$  with probability  $w_{j1} \in [0, 1]$  and  $a_{j2}$  with probability  $w_{j2} = 1 - w_{j1}$  after the plants are built. Moreover, denote by  $y_{js}$  the actual output in state of demand  $s$ . Obviously,  $y_{jc} = \max(y_{jr}, y_{js})$  holds (with  $r, s = 1, 2; r \neq s$ ) and hence the monopolist's expected profit is

$$(4) \quad \sum_h w_{jh} \cdot (a_{jh} y_{jh} - \frac{b}{2} y_{jh}^2 - v y_{jh}) - c \cdot \max(y_{jr}, y_{js}).$$

For characterising the outcome of each state of demand one would have to know the value of  $\max(y_{jr}, y_{js})$  when (4) is maximised. Assume first that the profit maximising monopoly output satisfies  $y_{jr} > y_{js}$  for  $r, s = 1, 2; r \neq s$ . Then maximising (4) immediately yields

$$(5) \quad \begin{aligned} p_{js} &= \frac{a_{js} + v}{2} \quad \text{and} \quad y_{js} = \frac{a_{js} - v}{b}, \\ p_{jr} &= \frac{a_{jr} + v}{2} + \frac{c}{2w_{jr}} \quad \text{and} \quad y_{jr} = \frac{a_{jr} - v}{b} - \frac{c}{bw_{jr}}. \end{aligned}$$

Since  $y_{jr} > y_{js}$  had been presupposed, (5) represents the monopoly allocation if and only if

$$(5') \quad a_{jr} - a_{js} > \frac{c}{w_{jr}} \quad (r, s = 1, 2; r \neq s).$$

According to the condition (5') the choice of capacity output and the (ex ante) decision to choose actual output below or at capacity depends on the relative size of the demand parameters as well as on capacity costs and the probabilities with which the possible states of demand are realised.

Suppose now, the profit maximising output is such that  $y_{js} = y_{jr}$  for  $r, s = 1, 2$ . This case occurs if and only if the inequality (5') does not hold. Then the maximisation of (4) gives us the solution (for  $r, s = 1, 2; r \neq s$ ):

$$(6) \quad y_{js} = \frac{\sum_h w_{jh} a_{jh} - (v + c)}{b} \quad \text{and} \quad p_{js} = \frac{a_{js} + w_{jr}(a_{js} - a_{jr}) + v + c}{2}.$$

The solution (5) and (6) can be conveniently illustrated by the upper part of Figure 1 (Its lower part refers to Section 2.3 and will be discussed later). This figure shows that the output chosen in state of demand  $s$ , i.e.  $y_{js}$ , will be full capacity output  $y_{jc}$ , if  $a_{jr} - a_{js} \leq c/w_{js}$ . Otherwise, the firm produces below capacity ( $y_{jr} = y_{jc} > y_{js}$ ) in state of demand  $s$  which is ex ante profit maximising when the demand parameter  $a_j$  in state of demand  $r$  is sufficiently greater than in state of demand  $s$ .

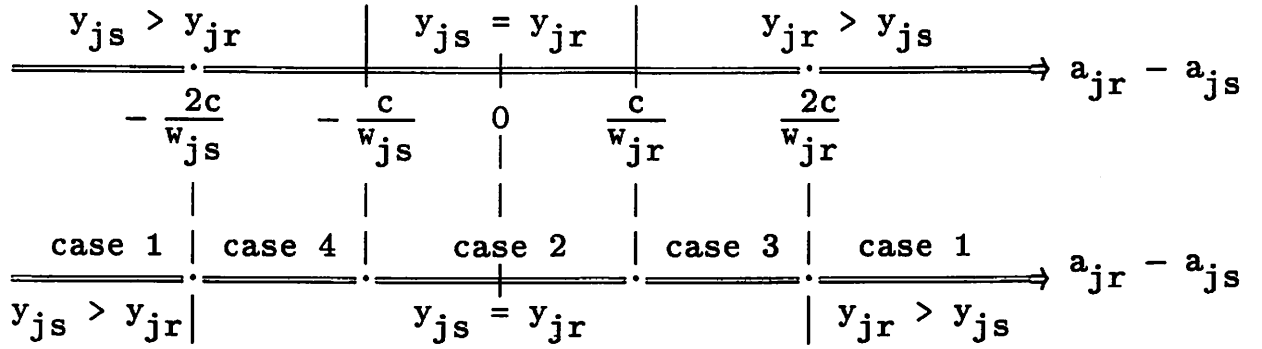


Figure 1: Output, capacity, and demand uncertainty

To be more specific, suppose condition (5') holds and state  $s$  materialises. Then it is profit maximising for the monopolist to produce less than capacity output, and in this case it is even conceivable that the monopolist accrues a loss. Low or negative profit must not be taken as evidence, however, for incorrect ex ante decision making on production and capacity output. Since the entrepreneur faces a lottery in his ex ante decision making, he knows very well that the outcome might turn out to be unfavorable for him. The capacity he builds up in view of this risk may be "excessive" from the ex post perspective, but ex ante it was optimal for the monopolist to take that risk because he also faced the prospect of an outstanding profit provided that the state of high demand (rather than that of low demand) would have emerged. To hold reserve capacity for the uncertain case of high demand may be efficient within the ex ante world.

## 2.2 Cooperation among regional monopolies

Each utility  $j$  ( $j = 1, 2$ ), satisfies (1) and (2). To keep the analysis simple, the parameters



b, v and c are assumed to be the same for both utilities. Then total demand is given by<sup>4</sup>

$$(7) \quad p = \frac{a_1 + a_2}{2} - \frac{b}{4}x$$

Suppose that demand is uncertain in each regional market. To be more specific let for  $j = 1, 2$  the demand parameter  $a_j$  (in market  $j$ ) attain the value  $a_{j1}$  with probability  $w_{j1}$  and  $a_{j2}$  with probability  $w_{j2} = 1 - w_{j1}$ . Then there exist four states of total demand as characterised in Table 1. The parameters  $\alpha_s$  specify the term  $(a_1 + a_2)/2$  from (7) for all feasible demand constellations  $s = 1, \dots, 4$ . State  $s$  occurs with probability  $\omega_s$ ,  $\sum_s \omega_s = 1$ . When regional demand uncertainties are independent one would have e.g.  $\omega_3 = w_{i2} \cdot w_{j1}$ .

For the purpose of promoting our central argument it is appropriate to simplify the model even further by assuming that the regional demand of only one utility is subject to demand uncertainty. More specifically, set  $w_{i1} = 1$  (hence  $w_{i2} = 0$ ) and  $w_{j1} \in (0, 1)$ . As a consequence, one obtains  $\omega_1 = w_{j1}$ ,  $\omega_2 = w_{j2}$ , and  $\omega_3 = \omega_4 = 0$ . For convenience of notation we write  $w_{js} = w_s$  for  $s = 1, 2$  in the following.

With these simplifications it is straightforward to define the expected joint profit by

$$(8) \quad w_r \left[ \frac{a_i + a_{jr}}{2} \cdot y_r - \frac{b}{4} y_r^2 - v y_r \right] + w_s \left[ \frac{a_i + a_{js}}{2} \cdot y_s - \frac{b}{4} y_s^2 - v y_s \right] - c \cdot \max(y_r, y_s).$$

The maximisation of (8) can be calculated in a similar way as in the previous section. Suppose first the profit maximising joint output satisfies  $y_r > y_s$  with  $r, s = 1, 2; r \neq s$ . Then maximising (8) yields

$$(9) \quad y_s = \frac{2}{b} \left[ \frac{a_i + a_{js}}{2} - v \right] \quad \text{and} \quad y_r = \frac{2}{b} \left[ \frac{a_i + a_{jr}}{2} - v - \frac{c}{w_r} \right].$$

The equations (9) form the solution if and only if

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<sup>4</sup> This demand function emerges from the regional demand functions (1) in an intermediate range of quantities demanded. More specifically, (7) is valid on the domain  $[x_{\min}, x_{\max}]$ , where  $x_{\max} := 2(a_1 + a_2)/b$  and  $x_{\min} := 2 \cdot [\max(a_1, a_2) - \min(a_1, a_2)]/b$ . For  $x \in [0, x_{\min}]$  the total demand function is  $p = \max(a_1, a_2) - bx/2$ . In the following it is assumed that the market solution(s) are found in the domain  $[x_{\min}, x_{\max}]$ .

$$(9') \quad a_{jr} - a_{js} > \frac{2c}{w_r} \quad (r, s = 1, 2; r \neq s).$$

If the inequality (9') does not hold, the profit maximising joint output is the same in both states of demand:

$$(10) \quad y_1 = y_2 = \frac{a_i + w_1 a_{j1} + w_j a_{j2} - 2(c + v)}{b}.$$

### 2.3 Cooperation versus isolated monopolies

Qualitatively, the solution (9) and (10) is very similar to the solution (5) and (6) of the isolated monopoly. Its characterisation in the lower part of Figure 1 demonstrates an interesting difference: In isolation it is only optimal for the utility  $j$  to produce at full capacity in the state of low demand, if the difference between low and high demand is small. In case of cooperation, full capacity output in the state of low demand remains optimal also for some broader difference between low and high demand because electricity can also be "exported" to the second electricity company. Technically speaking, the interval in which full capacity output is optimal in the state  $s$  of low demand is extended from the interval  $[0, \frac{c}{w_r}]$  to  $[0, \frac{2c}{w_r}]$ .

In order to assess the allocative impact of cooperation, we now compare the outcome of joint profit maximisation with that of isolated monopoly provision of power by **both** utilities. In the absence of demand uncertainty utility  $i$ 's monopoly output is clearly

$$(11) \quad y_i = \frac{a_i - v - c}{b}.$$

As shown in Figure 1 both the cooperative output and utility  $j$ 's output as an isolated monopoly depend on specific parameter constellations. Therefore a comparison between cooperation and isolated monopolies must be carried out successively for corresponding parameter regions. It is straightforward from Figure 1 that four different cases must be distinguished:

$$\text{Case 1:} \quad (a_{jr} - a_{js}) \in (-\infty, -\frac{2c}{w_r}) \quad \text{or} \quad (a_{jr} - a_{js}) \in (\frac{2c}{w_r}, \infty).$$

These conditions are equivalent to (9'), and (9') implies (5'). Therefore (5) and (11)

determine the aggregate production capacity of the isolated monopolies as

$$y_c^{iso} = \frac{a_i - v - c}{b} + \frac{a_{jr} - v - c/w_r}{b} = \frac{a_i + a_{jr} - 2v}{b} - \frac{c(1 + w_r)}{b w_r}.$$

On the other hand, (9) yields  $y_c^{co} = y_r$  (with superscript co for cooperation and with subscript c for capacity) or  $y_c^{co} = \frac{a_i + a_{jr} - 2v}{b} - \frac{2c}{b w_r}$ . Hence one obtains  $y_c^{iso} - y_c^{co} = \frac{c w_r}{b w_r} > 0$ . Suppose now that state of demand s occurs under the conditions of case 1. It is then easy to calculate

$$y_s^{iso} - y_s^{co} = -\frac{c}{b} < 0.$$

If (9') does not hold, the following additional cases have to be distinguished:

**Case 2:**  $(a_{jr} - a_{js}) \in \left[-\frac{c}{w_s}, \frac{c}{w_r}\right]$ .

Under this condition, utility j's capacity output is given by (6). Therefore the aggregate capacity of the isolated monopolies equals (10), i.e.  $y_c^{iso} = y_c^{co}$ . In all states of demand the aggregate output is the same, and cooperation does not make a difference to isolated monopolies.

**Case 3:**  $(a_{jr} - a_{js}) \in \left(\frac{c}{w_r}, \frac{2c}{w_r}\right)$ .

In this case utility j's capacity output is  $y_{jr}$  from (5) when j is an isolated monopoly.

$$\begin{aligned} \text{Hence } y_c^{iso} - y_c^{co} &= \frac{1}{b} \cdot \left\{ \left[ a_i + a_{jr} - 2v - \frac{c(1+w_r)}{w_r} \right] - \left[ a_i + w_r a_{jr} + w_s a_{js} - 2(v+c) \right] \right\} \\ &= \frac{w_s}{b} \cdot \left[ (a_{jr} - a_{js}) - \frac{c}{w_r} \right]. \text{ Note that } (a_{jr} - a_{js}) > c/w_r \text{ by presupposition.} \end{aligned}$$

Consequently,  $y_c^{iso} - y_c^{co} > 0$ . If state of demand s occurs under the conditions of case 3

$$\text{the difference in output is } y_s^{iso} - y_s^{co} = \frac{w_r}{b} \cdot \left[ \frac{c}{w_r} - (a_{jr} - a_{js}) \right] < 0.$$

**Case 4:**  $(a_{jr} - a_{js}) \in (-\frac{2c}{w_s}, -\frac{c}{w_s})$ .

Applying the same arguments as in the preceding paragraph it is straightforward to show that all results are the same as those derived in case 3 excepts for an interchange of the states of demand  $r$  and  $s$ . This time, capacity output is produced in state of demand  $s$  with

$$y_s^{iso} - y_s^{co} = y_c^{iso} - y_c^{co} = \frac{w_r}{b} \left[ \frac{c}{w_r} - (a_{jr} - a_{js}) \right] > 0 \text{ and}$$

$$y_r^{iso} - y_r^{co} = \frac{w_s}{b} \left[ (a_{jr} - a_{js}) - \frac{c}{w_r} \right] < 0.$$

In summary, cooperation among monopolistic utilities tends to reduce aggregate capacity. In contrast to the case of isolated monopolies this requires coordination of (sunk) investment decisions (as well as other relevant parameters). Whenever aggregate capacity output is strictly smaller in case of cooperation than in isolation, the cooperative low-demand joint output exceeds that of isolated production. Therefore cooperation tends to smooth aggregate output fluctuations. Except for case 2 (in which the outcomes are identical) monopolistic cooperation makes both firms better off since otherwise joint profit maximisation would have yielded the same allocation as isolated profit maximisation. Thus cooperation is a (partial) substitute for ex ante planned reserve capacity in order to deal with demand uncertainty. From the point of view of insurance theory, cooperation reduces the costs of demand uncertainty and creates the well-known pooling advantages.

The welfare implications of cooperation are not so easy to assess in an explicit manner. Observe, however, that the expected total output is the same in isolation and cooperation (in all cases 1 – 4!). If we consider the sum of expected consumer and producer surplus the relevant welfare measure, then cooperation turns out to be welfare improving because the same expected total output is produced with the same operating costs in both scenarios whereas the total capacity costs of the isolated monopolies exceed those under cooperation.

## 2.4 Competitive wholesale trade and monopolistic regional markets

Consider two utilities characterised by (1) and (2) and maintain the assumption of the previous section that the regional demand of utility  $i$  is deterministic whereas utility  $j$  faces two states of demand with probabilities  $w_{jr} \in (0, 1)$  and  $w_{js} = 1 - w_{jr}$  ( $s, r = 1, 2$ ;

$s \neq r$ ). Denote by  $z_{js} \in \mathbb{R}$  the quantity demanded ( $z_{js} > 0$ ) or supplied ( $z_{js} < 0$ ) of utility  $j$  on the wholesale market for power at state of demand  $s$ . The price of power in that market is  $q_s$ , and both firms are assumed to be price takers in the wholesale market. On the other hand, they are assumed to be monopolists in their regional (retail) markets.

Suppose first, **both firms are profit-maximising monopolists** in their respective home market. Then a monopolistic equilibrium of the total power sector with competitive wholesale market is constituted by a set of vectors  $\{p_{js}^0, x_{js}^0, y_{js}^0, z_{js}^0\}_{s,j=1,2}$  and  $\{q_1^0, q_2^0\}$  such that

$$(12a) \quad x_{js}^0 = y_{js}^0 + z_{js}^0 \quad \text{and} \quad p_{js}^0 = a_{js} - \frac{b}{2} x_{js}^0 \quad (s, j = 1, 2);$$

$$(12b) \quad \text{for } j = 1, 2 \text{ the vector } (y_{js}^0, z_{js}^0)_{s=1,2} \text{ maximises the expected profit}$$

$$\sum_s w_s \cdot [(a_{js} - \frac{b}{2} y_{js} - \frac{b}{2} z_{js})(y_{js} + z_{js}) - v y_{js} - q_s^0 z_{js}] - c \cdot \max(y_{j1}, y_{j2})$$

$$(12c) \quad \sum_j z_{js}^0 = 0 \quad (s = 1, 2).$$

In order to characterise the equilibrium allocation consider the profit maximising calculus of firm  $j$  as given in (12b). Suppose its solution is such that  $y_{jr} > y_{js} > 0$ . Then  $q_r = v + c/w_r$  and  $q_s = v$ , because if the wholesale prices were greater than that, the firms' profits would be unbounded. Conversely, in case of the reversed inequality sign the firms would not generate any power but rather acquire on the wholesale market all the power they need. Due to these arguments it must also be true that  $y_{ir} > y_{is} > 0$  if and only if  $y_{jr} > y_{js} > 0$ . Therefore  $q_r = v + c/w_r$  and  $q_s = v$  are the equilibrium prices and

$$(13a) \quad x_{js} = \frac{a_{js} - v}{b} \quad \text{and} \quad x_{jr} = \frac{a_{jr} - v}{b} - \frac{c}{bw_r} \quad \text{for } j = 1, 2$$

are the equilibrium demands. Owing to (12a) and (12c) one has  $x_{is} + x_{js} = y_{is} + y_{js}$  for  $s = 1, 2$ . Hence  $y_{jr} > y_{js}$  for  $j = 1, 2$  if and only if (9') holds. Otherwise the equilibrium demands are

$$(13b) \quad x_{is} = x_{js} = \frac{a_i + w_1 a_{j1} + w_2 a_{j2} - 2(c + v)}{b} \quad \text{for } s = 1, 2.$$

In summary, the competitive wholesale market leads to the same quantities of total demand, output and capacity, respectively, as the joint profit maximising cooperative solution. Note, however, that on the firm levels, trade and production flows are indeterminate, because production costs have been assumed to be linear and the same across the firms. For example, if the solution implies  $y_{jr} > y_{js}$  for  $j, r, s = 1, 2$  and  $r \neq s$ , then it is true that

$$(13c) \quad y_{js} + z_{js} = \frac{a_{js} - v}{b} \quad \text{and} \quad y_{jr} + z_{jr} = \frac{a_{jr} - v}{b} - \frac{c}{bw_r}.$$

These findings are summarised as follows:

- (i) The introduction of a competitive wholesale market into a power system of regional monopolies leads to the same allocation as under explicit joint profit maximisation.
- (ii) Since the distribution of trade and production flows is not uniquely determined at equilibrium prices, it is not entirely clear how equilibrium quantities emerge by "decentralised market forces". Since expensive capital investments are at stake one may be reluctant to recommend costly trial and error procedures. This dilemma may provide an argument in favor of complementing the price mechanism by an effort of coordinating capacity decisions similarly as in the case of joint profit maximisation. However, since this indetermination is a consequence of the simplifying model assumption of linear costs this necessity of coordination may not be overstated.

## 2.5 Perfect competition in the power market

Rather than introducing a large number of utilities to motivate perfectly competitive behavior, suppose for convenience of exposition that two regional utilities seek to maximise their profit as price takers. Then a perfectly competitive equilibrium is constituted by the vectors  $\{p_1^*, p_2^*\}$  and  $\{x_{js}^*, y_{js}^*\}_{s,j=1,2}$  such that

$$(14a) \quad x_{js}^* = \frac{2a_{js}}{b} - \frac{2p_j^*}{b};$$

$$(14b) \quad (y_{j1}^*, y_{j2}^*) \text{ maximises } \sum_s w_s \cdot [p_{js}^* y_{js} - v y_{js}] - c \cdot \max(y_{j1}, y_{j2}) \quad \text{for } j = 1, 2;$$

$$(14c) \quad \sum_j x_{js}^* = \sum_j y_{js}^*.$$

This definition of perfect competition in the power sector presupposes that interregional transmission of power is costless and unrestricted by limited capacity of the grid. In particular, not only transportation between producers is costless but also consumers can choose between different producers with additional costless transportation requirements. It implies that demanders of power are free to choose their utility and that each utility can deliver power to each customer irrespective of her location. Consequently Jevons' Law of Indifference applies:  $p_{js} = p_s$  for all  $j$ .

The solution turns out to be

$$(15) \quad y_{js} + z_{js} = \frac{2(a_{js} - v)}{b} \quad \text{and} \quad y_{jr} + z_{jr} = \frac{2(a_{jr} - v)}{b} - \frac{c}{bw_r} \quad \text{for } j = 1, 2.$$

As a consequence of (14) one obtains

$$(16) \quad y_{js} + y_{is} = 2 \cdot \frac{a_i + a_{js} - 2v}{b} \quad \text{and} \quad y_{jr} + y_{ir} = 2 \cdot \left[ \frac{a_i + a_{jr} - 2v}{b} - \frac{2c}{bw_r} \right]$$

if and only if (9') holds. Otherwise total (capacity) output is

$$(17) \quad y_1 = y_2 = 2 \cdot \frac{a_i + w_1 a_{j1} + w_2 a_{j2} - 2(c + v)}{b}.$$

Comparing (16) with (9) and (17) with (10) is easy, since the parameter constellation for which the output is below or at capacity is the same in both cases: For any parameter constellation and for each state of demand the perfectly competitive output is twice as large as in case of cooperation (because of linear demand). In both cases the relevant total demand function is given by (7). Hence the (deadweight) welfare loss of joint profit maximisation is given by the Harberger triangle. The competitive output is efficient, of course, implying that the competitive market handles demand uncertainty in an efficient way. In other words the amount of reserve capacity is optimized.

Observe also that the competitive outcome could also be achieved if transactions were restricted to the wholesale markets with both firms following the marginal cost pricing rule – e. g. as 'ideally' regulated public utilities. To see this modify the definition (12) of a monopolistic equilibrium of the total power sector with competitive wholesale market by substituting (12b) by

$$(12b') \quad \text{for } j = 1, 2 \text{ the vector } (y_{js}^0, z_{js}^0)_{s=1,2} \text{ maximises expected profit}$$

$$\sum_s w_s \cdot [p_{js}^0 (y_{js} + z_{js}) - v y_{js} - q_s^0 z_{js} - c \cdot \max(y_{j1}, y_{j2})].$$

It must be obviously true that  $p_{1s}^0 = p_{2s}^0 = q_s^0$  for  $s = 1, 2$  implying

$$\sum_j x_{js}^* = \sum_j x_{js}^0 = \sum_j y_{js}^0 + \sum_j z_{js}^0 = \sum_j y_{js}^0 = \sum_j y_{js}^*.$$

In other words, if all utilities would follow the marginal-cost-pricing rule, competitive wholesale trade would be a complete substitute for perfect competition in the power sector. If, however, utilities are regional profit maximising monopolists, competitive wholesale trade yields the same allocation as joint profit maximisation. As far as marginal-cost-pricing regulation is ineffective (as implied, e.g., by the so called "capture theory of regulation") wholesale trade cannot be regarded as a close or even perfect substitute for full-scale competition in the power sector.

### 3. Uncertainty with regard to production breakdowns under different institutional frameworks

#### 3.1 The fully integrated regional monopoly

We modify our model by considering demand and production costs as being known with certainty. But now it is uncertain whether the output (up to capacity) can be produced as planned because of random production breakdowns. Note that these breakdowns occur in addition to scheduled interruptions of power generation for regular maintenance and inspection (which are ignored in the following). For expository purposes consider the simple scenario that in case of a random breakdown the production of output is constrained by

$$(18) \quad y_j \leq \tau y_{jc}$$

with  $y_{jc}$  denoting capacity output of firm  $j$  and  $\tau \in [0, 1]$ . To fix our ideas we refer to the state of production breakdown as state 2 and to the state of undisturbed production as state 1. The former occurs with probability  $w_{j2}$  and the latter with  $w_{j1} = 1 - w_{j2}$ . Consider the problem



$$(19) \quad \text{Maximise } \sum_s w_{js} \cdot (a y_{js} - \frac{b}{2} y_{js}^2 - v_j y_{js}) - c y_{jc}$$

subject to the constraints  $y_{j1} \leq y_{jc}$  and  $y_{j2} \leq \tau y_{jc}$ .

For solving (19) we form the Lagrangean

$$(20) \quad L(y_{j1}, y_{j2}, y_{jc}, \lambda_1, \lambda_2) :=$$

$$= \sum_s w_{js} \cdot [(a - v_j) y_{js} - \frac{b}{2} y_{js}^2] - c y_{jc} + \lambda_1 (y_{jc} - y_{j1}) + \lambda_2 (\tau y_{jc} - y_{j2}).$$

If the solution satisfies  $y_{jc} > 0$ , the relevant Kuhn-Tucker conditions are

$$(21) \quad w_{js}(a - b y_{js} - v_j) - \lambda_s \leq 0 \quad \text{and} \quad [w_{js}(a - b y_{js} - v_j) - \lambda_s] \cdot y_{js} = 0 \quad \text{for } s = 1, 2;$$

$$(22) \quad \lambda_1 + \tau \lambda_2 = c;$$

$$(23) \quad y_{jc} - y_{j1} \geq 0 \quad \text{and} \quad \lambda_1 (y_{jc} - y_{j1}) = 0;$$

$$(24) \quad \tau y_{jc} - y_{j2} \geq 0 \quad \text{and} \quad \lambda_2 (\tau y_{jc} - y_{j2}) = 0.$$

Suppose  $\tau y_{jc} > y_{j2}$ . Then  $\lambda_2 = 0$  and  $y_{j2} \geq (a - v_j)/b$  because of (21). Moreover, (22) gives us  $\lambda_1 = c > 0$ . Hence  $y_{jc} = y_{j1} > 0$  and (21) yields

$$y_{j1} = \frac{a - v_j - c/w_{j1}}{b} < y_{j2}$$

contradicting the presupposition. Therefore  $\tau y_{jc} = y_{j2}$ . Suppose now that  $y_{jc} > y_{j1}$ . Then  $\lambda_1 = 0$  and  $\lambda_2 = c/\tau$ .  $y_{j1}$  is not feasible, since  $a > v_j$  by assumption. As a consequence of the above considerations the solution is

$$(25) \quad y_{j1} = \frac{a - v_j}{b} < y_{jc} \quad \text{and} \quad y_{j2} = \tau y_{jc} = \frac{a - v_j}{b} - \frac{c}{b\tau w_{j2}}$$

if and only if

$$(25') \quad \tau(1-\tau) w_{j2} > \frac{c}{a - v_j}.$$

Otherwise the solution is

$$(26) \quad y_{j1} = \frac{y_{j2}}{\tau} = y_{jc} = \frac{a - v_j}{b} \cdot \frac{w_{j1} + \tau w_{j2}}{w_{j1} + \tau^2 w_{j2}} - \frac{c}{b(w_{j1} + \tau^2 w_{j2})}.$$

Therefore, in analogy to the demand side, it is a profit maximizing strategy to keep reserve capacity in order to tackle the break-down problem.

### 3.2 Cooperation among regional monopolies

When two utilities as characterised in the previous section consider to jointly supply the total demand given by (7), we have to distinguish four states of production. But as in Section 2.2 we simplify the following analysis by setting  $w_{i2} = 0$ . Hence we set again  $\omega_1 = w_{j1} =: w_1$ ,  $\omega_2 = w_{j2} = w_2$  and  $\omega_3 = \omega_4 = 0$ . But with identical cost structures this assumption would clearly imply shutting down utility  $j$  altogether in an effort to maximise joint profit. In the following, this rather uninteresting corner solution will be avoided by allowing for  $v_i \neq v_j$ . Intuitively speaking, if  $v_i$  is much larger than  $v_j$ , it is optimal to use only utility  $j$  in spite of its positive breakdown probability. On the other hand, if the difference  $v_i - v_j$  is 'sufficiently' small, firm  $j$  will not be used at all for power generation<sup>5</sup>. Rather than exploring in detail the conditions under which joint supply is profit maximising we assume for the remainder of this section that both utilities build positive production capacities. A necessary condition for this situation to emerge is

$$(27) \quad v_i > v_j,$$

because firm  $i$  dominates firm  $j$  in our model if  $v_i \geq v_j$ . The simplifying assumptions  $c_i = c_j = c$  and  $a_i = a_j = a$  will be maintained, for convenience. With this parameter specification, cooperation among the utilities requires to

$$(28) \quad \text{Maximise } \sum_{s=1,2} w_s \cdot [a(y_{is} + a_{js}) - \frac{b}{4}(y_{is} + y_{js})^2 - v_i y_{is} - v_j y_{js}] - c(y_{ic} + y_{jc})$$

subject to the constraints  $y_{is} \leq y_{ic}$  ( $s = 1, 2$ ),  $y_{ji} \leq y_{jc}$  and  $y_{j2} \leq \tau y_{jc}$ .

We are interested in that subset of parameter values, of course for which the operation of

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<sup>5</sup> To exclude globally dominated technologies is a common assumption in the peak-load pricing literature (e.g. Crew-Kleindorfer 1976).

both utilities is joint profit maximising. Therefore we assume in the following that the solution of (28) exhibits  $y_{jc} > 0$  and hence  $(\partial L / \partial y_{jc}) = 0$  for  $j = 1, 2$ . Under the additional constraint (26) the solution of (28) is shown in the Appendix to have the properties  $y_{j2} = \tau y_{jc}$ ,  $y_{j1} = y_{jc}$  and  $y_{i2} = y_{ic}$  whenever<sup>6</sup>

$$(29) \quad v_i - v_j \neq \frac{(1-\tau) c}{w_2 \tau}.$$

It is also shown in the Appendix A that under the constraint (26) the solution of (28) is

$$(30) \quad (y_{i1} + y_{j1}, y_{i2} + y_{j2}) = \begin{cases} (y_{1a}, y_{2a}) \\ (y_{1b}, y_{2b}) \end{cases} \text{ if and only if } v_i - v_j \begin{cases} > \\ < \end{cases} \frac{(1-\tau) c}{w_1 + \tau w_2}$$

where

$$(31) \quad y_{1a} := y_{i1} + y_{j1} = \frac{2}{b} \left[ a - v_j - \frac{(1-\tau) c - \tau w_2 (v_i - v_j)}{w_1} \right] = y_{jc},$$

$$(32) \quad y_{2a} := y_{i2} + y_{j2} = \frac{2}{b} \left[ a - v_i - \frac{c}{w_2} \right] = y_{ic} + \tau y_{jc},$$

$$(33) \quad y_{1b} := y_{i1} + y_{j1} = \frac{2}{b} \left[ a - v_j - \frac{(1-\tau) c - \tau (v_i - v_j)}{(1-\tau) w_1} \right] = y_{ic} + y_{jc},$$

$$(34) \quad y_{2b} := y_{i2} + y_{j2} = \frac{2}{b} \left[ a - v_j - \frac{v_i - v_j}{(1-\tau) w_2} \right] = y_{ic} + \tau y_{jc}.$$

Observe that the equations (31) to (34) also uniquely determine the utilities' capacity output  $y_{ic}$  and  $y_{jc}$ . This result does not generalise, however, because with more than one utility of each type one would face a multiplicity problem as discussed in Section 2.2.

### 3.3 Cooperation versus isolated monopolies

Similar to the case of uncertain demand the cooperative solution (if it is different from the separate regional monopoly case) results into an economic advantage. In the following the effects on the relevant parameters (capacity etc.) shall be considered. In order to compare the allocation in isolated monopoly with that of cooperation, suppose

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<sup>6</sup> The equality  $v_i - v_j = (1-\tau)c/\tau w_2$  which is excluded by (29) is a special parameter configuration that can be readily neglected because it is "non-generic".

that (26) holds and that

$$(v_i - v_j) > \frac{(1-\tau) c}{w_1 + \tau w_2}.$$

Then the solution of (28) is given by (31) and (32). From these two equations it follows that

$$(35) \quad y_c^{co} = y_{ic} + y_{jc} = y_{2a} + (1-\tau) y_{1a} = \\ = \frac{2}{b} \left[ (2-\tau) a - \frac{w_1 - \tau(1-\tau) w_2}{w_1} \cdot v_i - \frac{(w_1 + w_2 \tau)(1-\tau)}{w_1} \cdot v_j - \frac{w_1 + (1-\tau)^2 w_2}{w_1 w_2} \cdot c \right].$$

On the other hand, in isolation the outputs  $y_{j1}$  and  $y_{j2}$  are given by (25). Obviously, one also has  $y_{i1} = y_{i2} = \frac{a - v_i - c}{b} = y_{ic}$ . Therefore

$$(36) \quad y_c^{iso} = \frac{1}{b} \left[ \frac{1+\tau}{\tau} \cdot a - v_i - \frac{v_j}{\tau} - \frac{1 + \tau^2 w_2}{\tau^2 w_2} \cdot c \right].$$

Closer inspection of (35) and (36) shows that the difference  $y_c^{iso} - y_c^{co}$  can be positive or negative depending on parameter values. Nevertheless, if it yields a different solution than the isolated monopolies, cooperation reduces the total costs of electricity supply. Since profit maximizing with respect to the regional monopolies does not change total surplus increases as a consequence of cooperation.

### 3.4 Competitive wholesale trade and monopolistic regional markets

If both utilities are price takers in the wholesale market but monopolists in their respective "domestic" markets, then the firms  $i$  and  $j$ , respectively, solve the Lagrangean

$$(37) \quad \Sigma_s w_s [(a_i - \frac{b}{2} y_{is} - \frac{b}{2} z_{is})(y_{is} + z_{is}) - v_i y_{is} - q_s z_{is}] + \lambda_{i1}(y_{ic} - y_{i1}) + \lambda_{i2}(y_{ic} - y_{i2})$$

$$(38) \quad \Sigma_s w_s [(a_j - \frac{b}{2} y_{js} - \frac{b}{2} z_{js})(y_{js} + z_{js}) - v_j y_{js} - q_s z_{js}] + \lambda_{j1}(y_{jc} - y_{j1}) + \lambda_{j2}(\tau y_{jc} - y_{j2}).$$

It is shown in Appendix B1 that if  $v_i - v_j \neq \frac{(1-\tau)c}{w_2 \tau}$  (condition (29)) holds, the monopolistic equilibrium of the total power sector with competitive wholesale market (defined in analogy to (12)) satisfies the three properties<sup>7</sup>

$$(i) \tau y_{jc} = y_{j2}, \quad (ii) y_{jc} = y_{j1}, \quad \text{and} \quad (iii) y_{ic} = y_{i2}.$$

The next step is to substitute the above equations (i) – (iii) in (37) and (38), respectively, and assume that condition (29) holds. The resulting Lagrangeans

$$(39) \quad F^i(\cdot) = w_1 \cdot \left[ a y_{i1} + a z_{i1} - \frac{b}{2} y_{i1}^2 - b y_{i1} z_{i1} - \frac{2}{5} z_{i1}^2 - v_i y_{i1} - q_1 z_{i1} \right] + \\ + w_2 \cdot \left[ a y_{ic} + a z_{i2} - \frac{b}{2} y_{ic}^2 - b y_{ic} z_{i2} - \frac{b}{2} z_{i2}^2 - v_i y_{ic} - q_2 z_{i2} \right] - c y_{ic} + \lambda (y_{ic} - y_{i1}),$$

$$(40) \quad F^j(\cdot) = w_1 \cdot \left[ a y_{jc} + a z_{j1} - \frac{b}{2} y_{jc}^2 - b y_{jc} z_{j1} - \frac{b}{2} z_{j1}^2 - v_j y_{jc} - q_1 z_{j1} \right] + \\ + w_2 \cdot \left[ a \tau y_{ic} + a z_{j2} - \frac{b}{2} y_{jc}^2 \tau^2 - b y_{jc} \tau z_{j2} - \frac{b}{2} z_{j2}^2 - v_j \tau y_{jc} - q_2 z_{j2} \right] - c y_{jc}.$$

are discussed in Appendix B2, and the solution turns out to be given by (30) – (34). In other words, when regional monopolists introduce perfectly competitive wholesale trade, one obtains the same allocation as in case of joint profit maximisation.

### 3.5 Perfect competition in the power market

Suppose now that both utilities are price takers not only in the wholesale market but also in their respective domestic markets, where prices are denoted  $p_{js}$  ( $j,s=1,2$ ). Then the following Lagrangeans have to be solved:

$$(41) \quad \sum_s w_s \cdot [p_{is}(y_{is} + z_{is}) - v_i y_{is} - q_s z_{is}] - c y_{ic} + \sum_s \mu_{is}(y_{ic} - y_{is});$$

$$(42) \quad \sum_s w_s \cdot [p_{js}(y_{js} + z_{js}) - v_j y_{js} - q_s z_{js}] - c y_{js} + \lambda_{j1} (y_{jc} - y_{j1}) + \mu_{j2} (\tau y_{jc} - y_{j2}).$$

An obvious implication is  $p_{is} = p_{js} = q_s$  for  $s = 1, 2$ , i.e. the global power market is perfectly competitive. The next step is to check which constellations of outputs are compatible with market equilibrium. For this purpose denote by  $p_s^i$  the market price in

<sup>7</sup> Recall that these three properties are the same as those characterising the joint profit maximising allocation. See also the Claims 1 – 3 in Appendix A.

state  $s$  which is required to prevail in order to support a specific output vector  $(y_{i1}, y_{i2})$  of utility  $i$ . Clearly, if  $p_s^i \neq p_s^j$  ( $s=1$  or  $s=2$ ) for a given configuration of outputs of both utilities, this configuration cannot constitute an equilibrium allocation. The information from solving (50) and (51) is compiled in Table 1.

output configurations	$y_{i1} < y_{ic}$ $y_{i2} = y_{ic}$	$y_{i1} = y_{ic}$ $y_{i2} < y_{ic}$	$y_{i1} = y_{ic}$ $y_{i2} = y_{ic}$
$y_{j2} < y_{jc}$ $y_{j2} = \tau y_{ic}$	$p_1^i \leq v_i$ $p_1^j = v_j > p_1^i$	$p_1^i = v_i + \frac{c}{w_1}$ $p_1^j \leq v_j < p_1^i$	$p_1^i = v_i + \frac{\mu_{i1}}{w_1} \geq v_i$ $p_1^j \leq v_j < p_1^i$
$y_{j1} = y_{jc}$ $y_{i2} < \tau y_{jc}$	$p_2^i = v_i + \frac{c}{w_2}$ $p_2^j \leq v_j < p_2^i$	$p_2^i = v_i$ $p_2^j = v_j < p_2^i$	$p_2^i = v_i + \frac{\mu_{i1}}{w_1} \geq v_i$ $p_2^j \leq v_j < p_2^i$
$y_{j1} = y_{jc}$ $y_{j2} = \tau y_{jc}$	feasible	$p_1^i = v_i + \frac{c}{w_1}$ $p_1^j = v_j + \frac{\mu_{j1}}{w_1} < p_1^i$	feasible

**Table 1: Feasible equilibrium output configurations**

The calculation of most entries of Table 1 is straightforward. Some comments on the less obvious cases is offered in Appendix B3.

Table 1 thus demonstrates that the equilibrium allocation must satisfy  $y_{j1} = y_{jc}$ ,  $y_{j2} = \tau y_{jc}$ ,  $y_{i2} = y_{ic}$  and  $y_{i1} \in [0, y_{ic}]$  – exactly as in the case of cooperation and in case of the competitive wholesale market with regional monopolists. Moreover, the feasible constellations of equilibrium outputs (Table 1) turn out to be given by (30) with the important difference, that for  $s = 1, 2$  and  $e = a, b$  the terms  $y_{se}$  are substituted by  $2y_{se}$ .

## 4. Conclusions

Electric power is a commodity with a number of peculiarities that are often conjured when calling for shelter from competition in the electricity sector. Among those are the impossibility to store power and the difficulty or even failure of the market process to maintain security of provision in view of severe uncertainties such as volatile demand, uncertain costs of primary energy and unpredictable production breakdowns. Opponents of competition in the power sector argue that coping with these difficulties and simultaneously facing high capacity building costs requires fully integrated utilities and cooperation among them. The present paper sets up a simple parametric model of the power sector in order to analyze institutional alternatives to vertical integration.

In Section 2 demand uncertainty under different institutional configurations has been analyzed. It is shown that competition among monopolistic utilities tends to reduce aggregate capacity and turns out to be welfare improving. The introduction of a competitive wholesale market into a power system of regional monopolies leads to the same allocation as under joint profit maximization. Moreover, if all utilities would follow the marginal-cost-pricing rule, competitive wholesale trade would be a complete substitute for perfect competition in the power system.

In Section 3 uncertainty with regard to production breakdowns is analyzed under different institutional configurations. Although cooperation among monopolistic utilities turns out to be welfare improving it may either increase or decrease aggregate capacity. Similar to the case of demand uncertainty, when regional monopolists introduce perfectly competitive wholesale trade one obtains the same allocation as in case of joint profit maximization. Moreover, the equilibrium allocation must satisfy the same conditions as in case of cooperation and in case of competitive wholesale trade among regional monopolists.

Although decentralized market decisions may play an important role in the electricity power industry within European countries in the future, the limits of our highly stylized approach should be kept in mind. Firstly, capacity bottlenecks within the grid and the costs of electricity transportation reduce the potential benefits of cooperation and electricity trade. Efficient access charges and non-discriminatory access rules to the electricity grids have to be designed in order to reduce these barriers to trade. Secondly, strategic behavior and oligopoly models may add more realistic elements to the modelling of the electricity sector, at the price of more complex and subtle model formulations (e.g. Grossman 1981, Klemperer and Meyer 1989). However, the advantage

of our parametric approach has been to allow an explicit comparison of institutional configurations.

## Appendix A: Cooperation under breakdown uncertainty

Associate to the maximisation problem (32) the Lagrangean

$$(A1) \quad L(\cdot) = \sum_s w_s \cdot [a(y_{is} + y_{js}) - \frac{b}{4}(y_{is} + y_{js})^2 - v_i y_{is} - v_j y_{js}] - c(y_{ic} + y_{jc}) + \\ + \lambda_{i1}(y_{ic} - y_{i1}) + \lambda_{i2}(y_{ic} - y_{i2}) + \lambda_{j1}(y_{jc} - y_{j1}) + \lambda_{j2}(\tau y_{jc} - y_{j2}).$$

The pertinent Kuhn–Tucker conditions are

$$(A2) \quad \frac{\partial L}{\partial z} \leq 0 \quad \text{and} \quad z \cdot \frac{\partial L}{\partial z} = 0 \quad \text{for } z = y_{i1}, y_{i2}, y_{j1}, y_{j2}, y_{ic}, y_{jc}.$$

$$(A3) \quad \frac{\partial L}{\partial \lambda_{js}} \geq 0 \quad \text{and} \quad \lambda_{hs} \cdot \frac{\partial L}{\partial \lambda_{js}} = 0 \quad \text{for } j = 1, 2; s = 1, 2..$$

All solution values are non-negative. The following analysis is restricted to situations of joint supply of both utilities as characterised by conditions

$$y_{ic} > 0 \text{ and } y_{jc} > 0 \text{ and therefore } \frac{\partial L}{\partial y_{ic}} = \frac{\partial L}{\partial y_{jc}} = 0.$$

**Claim 1:**  $\tau y_{jc} = y_{j2}$ .

**Proof:** Suppose the contrary:  $\tau y_{jc} > y_{j2} \geq 0$ . Then  $\lambda_{j2} = 0$ , and (A2) yields  $y_{i2} + y_{j2} \geq \frac{2(a - v_j)}{b}$ . Therefore

$$\frac{\partial L}{\partial y_{i2}} = w_2 \cdot [a - \frac{b}{2}(y_{i2} + y_{j2}) - v_i] - \lambda_{i2} \leq w_2 \cdot [a - \frac{b}{2} \frac{2(a - v_j)}{b} - v_i] - \lambda_{i2} = \\ = w_j(v_j - v_i) - \lambda_{i2} < 0.$$

Using (A2) again, the last inequality gives us  $y_{i2} = 0$ . Owing to (A3)  $y_{i2} = 0$  implies  $\lambda_{i2} = 0$ . For  $a > v_j$  it is true that

$$\frac{\partial L}{\partial y_{j2}} \bigg|_{\substack{y_{i2}=0 \\ y_{j2}=0}} = w_2 [a - \frac{b}{2}(y_{i2} + y_{j2}) - v_j] > 0.$$



Therefore  $y_{j2} > 0$ . More specifically (A2) and (A3) imply

$$(A4) \quad y_{ic} + \tau y_{jc} \geq y_{i2} + y_{j2} = y_{j2} = \frac{2(a - v_j)}{b}.$$

From  $\frac{\partial L}{\partial y_{ic}} = \frac{\partial L}{\partial y_{jc}} = 0$  and  $\lambda_{i2} = \lambda_{j2} = 0$  we conclude  $\lambda_{i1} = \lambda_{j1} = c > 0$  which in turn means that  $y_{jc} = y_{j1}$  for  $j = 1, 2$ . Hence

$$(A5) \quad y_{i1} + y_{j1} = y_{ic} + y_{jc} = \frac{2(a - v_i)}{b} - \frac{2c}{b w_1}.$$

The incompatibility of (A4) and (A5) proves the Claim 1.

**Claim 2:**  $y_{jc} = y_{j1}$  if and only if  $v_i - v_j \neq \frac{c(1-\tau)}{w_2 \tau}$ .

**Proof:** Suppose the contrary:  $y_{jc} > y_{j1}$ . Then  $\lambda_{j1} = 0$  and (A2) yields

$$y_{i1} + y_{j1} \geq \frac{2(a - v_j)}{b}.$$

Therefore  $\frac{\partial L}{\partial y_{i1}} = w_1 [a - \frac{b}{2}(y_{i1} + y_{j1}) - v_i] - \lambda_{i1} \leq w_1 (v_j - v_i) - \lambda_{i1} < 0$ .

In view of (A2) and  $\frac{\partial L}{\partial y_{i1}} < 0$ ,  $y_{i1}$  must take the value  $y_{i1} = 0$ . Consequently  $\lambda_{i1} = 0$

which implies  $\lambda_{i2} = c > 0$ . (A2) also yields

$$(A7) \quad y_{i2} + y_{j2} = \frac{2(a - v_i)}{b} - \frac{2c}{b w_2}.$$

Substitute (A7) in (A2):

$$(A8) \quad \frac{\partial L}{\partial y_{j2}} = w_2 [a - \frac{b}{2} \cdot \frac{2}{b} (a - v_i - \frac{c}{w_2}) - v_j] - \lambda_{j2} = w_2 (v_i - v_j) + c - \lambda_{j2} = 0.$$

Recall that  $\lambda_{j1} = 0$  specifies  $\lambda_{j2} = c/\tau$  and therefore (A8) is satisfied, if and only if  $v_i - v_j \neq c(1-\tau)/w_2\tau$ . Hence Claim 2 is proved.

**Claim 3:**  $y_{ic} = y_{i2}$  if and only if  $v_i - v_j \neq \frac{c(1-\tau)}{w_2\tau}$  holds.

**Proof:** Suppose the contrary:  $y_{ic} > y_{i2}$ . Then  $\lambda_{i2} = 0$  and  $c = \lambda_{i1} > 0$ . Hence  $y_{ic} = y_{i1}$ . From Claim 2 we know that  $y_{jc} = y_{j1}$ . Therefore (A2) yields  $y_{i1} + y_{j1} = y_{ic} + y_{jc} = \frac{2(a-v_i)}{b} - \frac{2c}{bw_1}$ . On the other hand, (A2) and (A3) also yield  $y_{ic} + \tau y_{jc} > y_{i2} + y_{j2} \geq \frac{2(a-v_i)}{b}$ . This contradiction proves the Claim 3.

In view of the Claims 1–3 the Lagrangean (A1) can be rewritten as

$$(A9) \quad L(\cdot) = w_1 \cdot [a(y_{i1} + y_{jc}) - \frac{b}{4}(y_{i1} + y_{jc})^2 - v_i y_{i1} - v_j y_{jc}] - c(y_{ic} + y_{jc}) + \\ + w_2 \cdot [a(y_{ic} + \tau y_{jc}) - \frac{b}{4}(y_{ic} + \tau y_{jc})^2 - v_i y_{ic} - v_j \tau y_{jc}] + \lambda_{i1}(y_{ic} - y_{i1}).$$

Among the Kuhn–Tucker conditions are

$$(A10) \quad \frac{\partial L}{\partial y_{i1}} = w_1 \cdot [a - \frac{b}{2}(y_{i1} + y_{jc}) - v_i] - \lambda_{i1} \leq 0,$$

$$(A11) \quad \frac{\partial L}{\partial y_{jc}} = w_1 \cdot [a - \frac{b}{2}(y_{i1} + y_{jc}) - v_j] + w_2 \tau [a - \frac{b}{2}(y_{ic} + \tau y_{jc}) - v_j] - c = 0,$$

$$(A12) \quad \frac{\partial L}{\partial y_{ic}} = w_2 \cdot [a - \frac{b}{2}(y_{ic} + \tau y_{jc}) - v_i] - c + \lambda_{i1} = 0.$$

Three cases have to be distinguished: (a)  $\lambda_{i1} = 0$  and  $y_{i1} \in (0, y_{ic})$ ; (b)  $\lambda_{i1} = 0$  and  $y_{i1} = 0$ ; and (c)  $\lambda_{i1} > 0$  (and hence  $y_{i1} = y_{ic}$ ). The following solutions to these cases are found with the help of (A10) – (A12), after some rearrangement of terms:

**Case a:**

$$(A13) \quad y_{i1} + y_{j1} = \frac{2}{b} \left[ a - v_i \right] < y_{ic} + y_{jc},$$

$$y_{i2} + y_{j2} = \frac{2}{b} \left[ a - v_i - \frac{c}{w_2} \right] = y_{ic} + \tau y_{jc},$$

under the condition  $v_i - v_j = \frac{c}{w_1 + \tau w_2}$ .

**Case b:**

$$(A14) \quad y_{i1} + y_{j1} = y_{jc} = \frac{2}{b} \left[ a - v_j - \frac{c(1-\tau) - \tau w_2(v_i - v_j)}{w_1} \right] = y_{jc},$$

$y_{i2} + y_{j2}$  as in (A15),

if and only if  $v_i - v_j > \frac{c(1-\tau)}{w_1 + w_2 \tau}$ . Otherwise one obtains

**Case c:**

$$(A15) \quad y_{i1} + y_{j1} = \frac{2}{b} \left[ a - v_j - \frac{c(1-\tau) - \tau(v_i - v_j)}{(1-\tau)w_1} \right] = y_{ic} + y_{jc},$$

$$y_{i2} + y_{j2} = \frac{2}{b} \left[ a - v_j - \frac{v_i - v_j}{(1-\tau)w_2} \right] = y_{ic} + \tau y_{jc}.$$

## Appendix B: Wholesale trade under breakdown uncertainty

**B1:** If  $v_i - v_j \neq \frac{(1-\tau)c}{w_2 \tau}$  (condition (29)) holds, then the monopolistic equilibrium of the total power sector with competitive wholesale market satisfies the three properties

$$(i) \quad \tau y_{jc} = y_{j2}, \quad (ii) \quad y_{jc} = y_{j1}, \quad \text{and} \quad (iii) \quad y_{ic} = y_{i2}.$$

Ad (i): Suppose,  $\tau y_{jc} = y_{j2}$  does not hold. Then one obtains from (38)  $\lambda_{j2} = 0$ ,  $\lambda_{j1} = c$  and hence  $q_1 = v_j + c/w_1$  and  $q_2 = v_j$ . If  $y_{i2} > 0$ , (37) would require  $q_1 \geq v_i > v_j$ . So  $y_{i2} = 0$  and therefore  $y_{i1} = y_{ic} > 0$ . This implies  $q_1 = v_i + c/w_1$ . Contradiction.

Ad (ii): Suppose  $y_{jc} = y_{j1}$  does not hold. Then  $\lambda_{j1} = 0$ , and either  $y_{j1} = 0$  or  $y_{j1} > 0$ . Since  $a_j > v_j$  by assumption, the solution to (38) requires  $y_{j1} + z_{j1} > 0$ . Hence  $y_{j1} > 0$  and  $z_{j1} > 0$  (disregarding "marginal" boundary cases) and therefore  $q_1 = v_j$ . If  $y_{i1} <$

$y_{ic}$ , (37) implies  $q_1 \leq v_i$ . From  $v_i > v_j$  and  $q_1 = v_j$  one obtains  $q_1 < v_i$  and hence  $y_{i1} = 0$ . This, in turn, makes  $y_{i2} = y_{ic} > 0$  so that (37) yields  $q_2 = v_i + c/w_2$ . On the other hand,  $\lambda_{j1} = 0$  means  $\lambda_{j2} = c/\tau$  so that (38) renders  $q_2 = v_j + c/\tau w_2$ . By condition (29) these two values of  $q_2$  cannot occur simultaneously. Consider now the case  $y_{i1} = y_{ic} > 0$ . Under this condition (37) requires that  $q_1 = v_i + \lambda_{i1}/w_1$  with  $\lambda_{i1} \geq 0$ . but (38) requires  $q_1 = v_j < v_i$ . Contradiction.

Ad (iii): Suppose,  $y_{ic} = y_{i2}$  does not hold. Then  $y_{i1} = y_{ic} > 0$  and (37) yields  $q_{1a} := v_i + c/w_1$ .  $y_{j1} > 0$  also implies  $y_{j1} + z_{j1} = (a - v - \lambda_{j1}/w_1)/b = (a - q_{1b})/b$  or  $q_{1b} := v_j + \lambda_{j1}/w_1$ . But  $\lambda_{j1} = c - \lambda_{j2} \leq c$ . Hence  $q_{1a} > q_{1b}$ . Contradiction.

**B2:** The solution to the Lagrange functions (39) and (40) is given by (A13) – (A14).

Proof: Since the trade flows are not sign constrained the solutions of (39) and (40) satisfy  $(\partial F_j / \partial z_{js}) = 0$  for  $j, s = 1, 2$ . The relevant first-order conditions are

$$(B1) \quad (w_1 + \tau w_2)(a - v_j) = w_1 b (y_{jc} + z_{j1}) + \tau w_2 b (\tau y_{jc} + z_{j2}) + c,$$

$$(B2) \quad y_{jc} + z_{j1} = \frac{a - q_1}{b} \quad \text{and} \quad \tau y_{jc} + z_{j2} = \frac{a - q_2}{b},$$

$$(B3) \quad y_{i1} + z_{i1} \leq \frac{a - v_i}{b} - \frac{\lambda}{w_1 b} \quad \text{and} \quad y_{i1} \cdot \frac{\partial F_i}{\partial y_{i1}} = 0,$$

$$(B4) \quad y_{ic} + z_{i2} = \frac{a - v_i}{b} - \frac{c - \lambda}{w_2 b},$$

$$(B5) \quad y_{i1} + z_{i1} = \frac{a - q_1}{b} \quad \text{and} \quad y_{ic} + z_{i2} = \frac{a - q_2}{b}.$$

Three cases have to be distinguished: (a)  $\lambda = 0$  and  $y_{i1} \in (0, y_{ic})$ ; (b)  $\lambda = 0$  and  $y_{i1} = 0$ ; and (c)  $\lambda > 0$  (and hence  $y_{i1} = y_{ic} > 0$ ). In view of (B3) – (B5) one obtains for Case a

$$(B6) \quad y_{i1} + z_{i1} = \frac{a - v_i}{b} \quad \text{and} \quad y_{ic} + z_{i2} = \frac{a - v_i}{b} - \frac{c}{w_2 b} \quad \text{as well as}$$

$$(B7) \quad q_1 = v_i \quad \text{and} \quad q_2 = v_i + \frac{c}{w_2}.$$

Substitution of (B7) in (B2) yields

$$(B8) \quad y_{jc} + z_{j1} = \frac{a - v_i}{b} \quad \text{and} \quad \tau y_{jc} + z_{j2} = \frac{a - v_i}{b} - \frac{c}{w_2 b}.$$

Note that (B8) and (B1) are compatible only if (29) does not hold. Under this special condition, (B6) and (B8) yield total outputs (A13).

For Case b the equations (B3) – (B5) imply

$$(B9) \quad y_{i1} + z_{i1} \geq \frac{a - v_i}{b} \quad \text{and} \quad y_{ic} + z_{i2} = \frac{a - v_i}{b} - \frac{c}{w_2 b},$$

$$(B10) \quad q_1 \geq v_i \quad \text{and} \quad q_2 = v_i + \frac{c}{w_2}.$$

It follows immediately from these findings that  $y_{i2} + y_{j2} = y_{ic} + \tau y_{jc}$  is determined as in (A13). In order to solve for  $q_1$ , substitute (B5) and  $y_{ic} + z_{i2}$  from (B9) in (B1). This yields, after some rearrangements of terms,

$$(B11) \quad q_1 = v_i + \frac{(1-\tau)c - (w_1 + \tau w_2)(v_i - v_j)}{w_1} = v_j + \frac{(1-\tau)c - \tau w_2(v_i - v_j)}{w_1}.$$

Clearly,  $q_i \geq v_i$  as required by (B10), if and only if  $v_i - v_j \geq \frac{c(1-\tau)}{w_1 + \tau w_2}$ .

Combining (B11) and (B5) gives us

$$(B12) \quad y_{i1} + z_{i1} = \frac{1}{b} \left[ a - v_j - \frac{(1-\tau)c - \tau w_2(v_i - v_j)}{w_1} \right].$$

Hence  $y_{i1} + z_{i1} + y_{j1} + z_{j2} = y_{jc}$  is the same as in (A14).

Suppose finally, Case c holds. Then (B1) – (B5) can be turned into

$$(B13) \quad \sum_s w_s \cdot (q_s - v_i) = c \quad \text{and} \quad w_1 \cdot (q_1 - v_j) + \tau w_2 \cdot (q_2 - v_j) = c.$$

We solve these equations for  $q_1$  and  $q_2$ :

$$(B14) \quad q_2 = \frac{v_i - (w_1 + \tau w_2) v_j}{(1-\tau) w_2} = v_j + \frac{v_i - v_j}{(1-\tau) w_2} \quad \text{and}$$

$$q_1 = v_j + \frac{(1-\tau) c - \tau(v_i - v_j)}{(1-\tau) w_1}.$$

Substitute  $q_1$  and  $q_2$  from (B14) in (B2) and (B5) to get (A15). Clearly, these total outputs characterise the equilibrium if and only if  $v_i - v_j < \frac{c(1-\tau)}{w_1 + \tau w_2}$ . Note, however, that the equilibrium outputs of the firms are indeterminate if more than one firm of each type is in the market.

**B3: The derivation of various entries of Table 1.**

(i) 1st column and 1st row: The Kuhn–Tucker conditions lead to  $p_1^j \leq v_j$  rather than to  $p_1^j = v_j$  as written in the table. But suppose  $p_1^j < v_j$ . Then  $p_1^i < v_i$  owing to (28). This implies  $y_{i1} = y_{j1} = 0$ . But consumers would respond to the "equilibrium price"  $p_1^j = p_1^i < v_j$  with a positive demand because  $a > v_j$  by assumption. Hence the wholesale market is not cleared.

(ii) 2nd column and 2nd row: The argument of the preceding paragraph applies to  $p_2^j = v_j$ .

(iii) 1st column and 3rd row: In this case, the Kuhn–Tucker conditions give us  $p_{i1} \leq v_i$ ,  $p_{j1} = v_j + \frac{c}{w_1}$  and therefore

$$(B15) \quad w_1(v_i - v_j) \geq \lambda_{j1}$$

One also obtains  $p_{i2} = v_i + \frac{c}{w_2}$ ,  $p_{j2} = v_j + \frac{c - \lambda_{j1}}{\tau w_2}$  and therefore

$$(B16) \quad \lambda_{j1} = (1-\tau) c - \tau w_2(v_i - v_j).$$

In view of (B15) and (B16) the output constellation characterised in the 2nd column and the 2nd row is feasible if and only if

$$v_i - v_j \geq \frac{(1-\tau) c}{w_1 + \tau w_2}.$$

The associated equilibrium prices are

$$(B17) \quad p_1 = v_j + \frac{c}{w_1} \quad \text{and} \quad p_2 = v_i + \frac{c}{w_2}.$$

(iv) 3rd column and 3rd row: Setting  $p_{is} = p_{js} = p_s$  for  $s = 1, 2$ , one obtains from the Kuhn–Tucker conditions the two equations

$$(B18) \quad w_1(p_1 - v_i) + w_2(p_2 - v_j) = c \quad \text{and} \quad w_1(p_1 - v_j) + \tau w_2(p_2 - v_i) = c.$$

These two equations determine the prices  $p_1$  and  $p_2$ :

$$(B19) \quad p_2 = v_j + \frac{v_i - v_j}{(1-\tau) w_2} \quad \text{and} \quad p_1 = \frac{c}{w_2} - \frac{w_1 + \tau w_2}{(1-\tau) w_1} \cdot (v_i - v_j).$$

Observe that  $p_2$  is positive because of condition (28). In view of (B19)  $p_1 \geq 0$  if and only if

$$v_i - v_j \leq \frac{(1-\tau) c}{w_1 + \tau w_2}.$$

## Appendix C: Cost uncertainty in different institutional frameworks

### 1. The fully integrated regional monopoly

Consider the model (1), (2) and (3) with the modification that the demand parameter  $a_j$  is certain (set  $a_j = a$ ) but that the operating cost per unit,  $v$ , is a random variable taking the value  $v_{js}$  with probability  $w_{js} \in (0, 1)$  ( $s = 1, 2$ ),  $\sum_s w_{js} = 1$ . The firm maximises expected profit

$$(C1) \quad \Pi^j(y_{j1}, y_{j2}) := \sum_s w_{js} \cdot (a y_{js} - \frac{b}{2} y_{js}^2 - v_{js} y_{js}) - c \cdot \max(y_{j1}, y_{j2})$$

Simple calculations along the lines of Section 2.1 yield the solution

$$(C2) \quad p_{js} = \frac{a + v_{js}}{2} \quad \text{and} \quad p_{jr} = \frac{a + v_{jr}}{2} + \frac{c}{2 w_{jr}},$$

$$y_{js} = \frac{a - v_{js}}{b} \quad \text{and} \quad y_{jr} = \frac{a - v_{jr}}{b} - \frac{c}{b w_{jr}},$$

with  $y_{jr} > y_{js}$ , if and only if

$$(C2') \quad v_{js} - v_{jr} > \frac{c}{w_{jr}} \quad (r, s = 1, 2; r \neq s).$$

If (C2') does not hold, the solution turns out to be

$$(C3) \quad y_{js} = \frac{a - \sum_h w_{jh} v_{jh} - c}{b} \quad \text{and} \quad p_{js} = \frac{a + \sum_h w_{jh} v_{jh} + c}{2} \quad \text{for } s = 1, 2.$$

## 2. Joint profit maximisation

Similar as in Section 2 we now consider two regional utilities with total demand given by equation (7). For simplicity, set  $(a_1 + a_2)/2 =: a$  and assume that utility  $i$  is not exposed to uncertain costs while utility  $j$  faces per unit operating costs  $v_{js}$  with probability  $w_{js} = w_s$  and  $v_{jr}$  with probability  $w_{jr} = w_r = 1 - w_s$ . Since the cases  $v_i < \min(v_{jr}, v_{js})$  and  $v_i > \max(v_{jr}, v_{js})$  would imply one active firm only, we will restrict attention to the domain of parameters satisfying

$$(C4) \quad v_{js} > v_i > v_{jr}.$$

Under this condition, the joint profit is given by

$$(C5) \quad \sum_s w_s \cdot \left[ \left( a - \frac{b}{4} (y_{js} + y_{is})(y_{js} + y_{is}) - v_{js} y_{js} - v_i y_{is} \right) - c \cdot \max(y_{j1}, y_{j2}) - c \cdot \max(y_{i1}, y_{i2}) \right].$$

First we want to show that under condition (C4) it is not possible in any state  $s = 1, 2$  ( $s \neq r$ ) that maximisation of (C5) yields both  $y_{js} > y_{jr}$  and  $y_{is} > y_{ir}$ . Suppose the contrary. Then the first-order conditions read for  $s = 1, 2$ :



$$a - \frac{b}{2}(y_{js} + y_{is}) - v_i - \frac{c}{w_s} = 0 \quad \text{and} \quad a - \frac{b}{2}(y_{js} + y_{is}) - v_{js} - \frac{c}{w_s} = 0,$$

implying  $v_i = v_{js}$ . Contradiction. It is also easy to show that under (C4) joint profit maximisation does not imply positive outputs the outputs of both utilities in state  $s$  unless  $v_i - v_{js} = c/w_{js}$  is satisfied. we ignore this "degenerate" parameter subspace by restricting the following analysis to the full-dimensional parameter space satisfying

$$(C4') \quad v_{js} - v_i > \frac{c}{w_s} \quad \text{and} \quad v_i - v_{jr} > \frac{c}{w_r}.$$

Then the maximisers of (C5) are<sup>8</sup>

$$(C6) \quad y_s = y_{js} = 2 \cdot \frac{a - v_{ir} - c/w_r}{b} \quad \text{and} \quad y_{is} = 0,$$

$$(C7) \quad y_r = y_{jr} = 2 \cdot \frac{a - v_{ir} - c/w_r}{b} \quad \text{and} \quad y_{ir} = 0.$$

Now we compare this result to the case of isolated provision of power by two fully integrated monopolies. Observe first that (C4) implies (C2'). Hence (C2) is the isolated performance of utility  $j$  while utility  $i$  provides the output (11) in each state of costs. It follows that

$$(C8) \quad y_s^{iso} = \frac{2a - v_i - v_{is} - c}{b} \quad \text{and} \quad y_r^{iso} = \frac{2a - v_i - v_{ir}}{b} - \frac{(1 + w_r)c}{b w_r}.$$

Comparing (C8) and (C6) yields  $y_s^{iso} - y_s^{co} = \frac{1}{b} [-(v_{js} - v_i) + \frac{2 - w_s}{w_s} c]$  which is greater than  $c/b > 0$  in view of (C4'). Similarly, the comparison of (C8) and (C7) gives  $y_r^{iso} - y_r^{co} < -c/b < 0$ . Since  $y_c^{co} = y_r^{co} + y_s^{co}$  and  $y_c^{iso} = y_r^{iso}$  it follows a fortiori that  $y_c^{iso} - y_c^{co} < 0$ .

To sum up, cooperation among monopolistic utilities changes the production and power provision dramatically. Under the conditions (C4') total capacity is extended in case of cooperation, and the relatively costly power plant does not produce at all. If the state of low costs [high costs] prevails, total output is larger [smaller] under joint profit maximisation than in isolation. Thus cooperation takes cost (dis-)advantages into

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<sup>8</sup> Nonnegativity of outputs requires  $w_{jr} \in [\frac{c}{a-v_{jr}}, \frac{a-v_i-c}{a-v_i}]$ .

account. Observe also that

$$w_s y_s^{\text{iso}} + w_r y_r^{\text{iso}} < w_s y_s^{\text{co}} + w_r y_r^{\text{co}}.$$

In that sense joint profit maximisation improves the provision of power.

### 3. Perfect competition

As price takers, the firms  $h = i, j$  maximise

$$(C9) \quad \sum_s w_s \cdot [p_s y_{hs} - v_{hs} y_{hs}] - c \cdot \max(y_{h1}, y_{h2})$$

for predetermined  $p_s$  ( $s = 1, 2$ ). Suppose condition (C4) holds and the maximisers satisfy  $y_{js} > y_{jr}$  and  $y_{is} > y_{ir}$  in any state  $s$ ,  $s \neq r$ . Then  $p_s < p_r$  from (7). Moreover, the first order conditions for profit maximisation yield

$$y_{is} > y_{ir} \Leftrightarrow p_s - p_r > \frac{c}{w_s} \quad \text{and} \quad y_{js} > y_{jr} \Leftrightarrow p_s - p_r > v_{js} - v_{jr} + \frac{c}{w_s}.$$

Contradiction. Hence it is not true that both utilities produce at full capacity in any single state of costs. Similar as in Section 3.2 one can also show that both firms have positive outputs in state of costs  $s$  and  $r$  only if the special conditions  $v_{js} - v_i = c/w_s$  and  $v_i - v_{jr} = c/w_r$  hold, respectively. Hence we restrict the subsequent analysis to the conditions (C4'). For this case the equilibrium allocation turns out to be, as expected,

$$(C10) \quad \begin{aligned} y_s = y_{is} &= 4 \cdot \frac{a - v_i - c/w_s}{b} \quad \text{and} \quad y_{js} = 0, \\ y_r = y_{jr} &= 4 \cdot \frac{a - v_{jr} - c/w_r}{b} \quad \text{and} \quad y_{ir} = 0. \end{aligned}$$

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