

Buhr, Walter; Wolff, Reiner

Working Paper

Partial versus Global Optimizations in Economic Dynamics: The Case of Recursive Programming

Volkswirtschaftliche Diskussionsbeiträge, No. 26-91

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Suggested Citation: Buhr, Walter; Wolff, Reiner (1991) : Partial versus Global Optimizations in Economic Dynamics: The Case of Recursive Programming, Volkswirtschaftliche Diskussionsbeiträge, No. 26-91, Universität Siegen, Fakultät III, Wirtschaftswissenschaften, Wirtschaftsinformatik und Wirtschaftsrecht, Siegen

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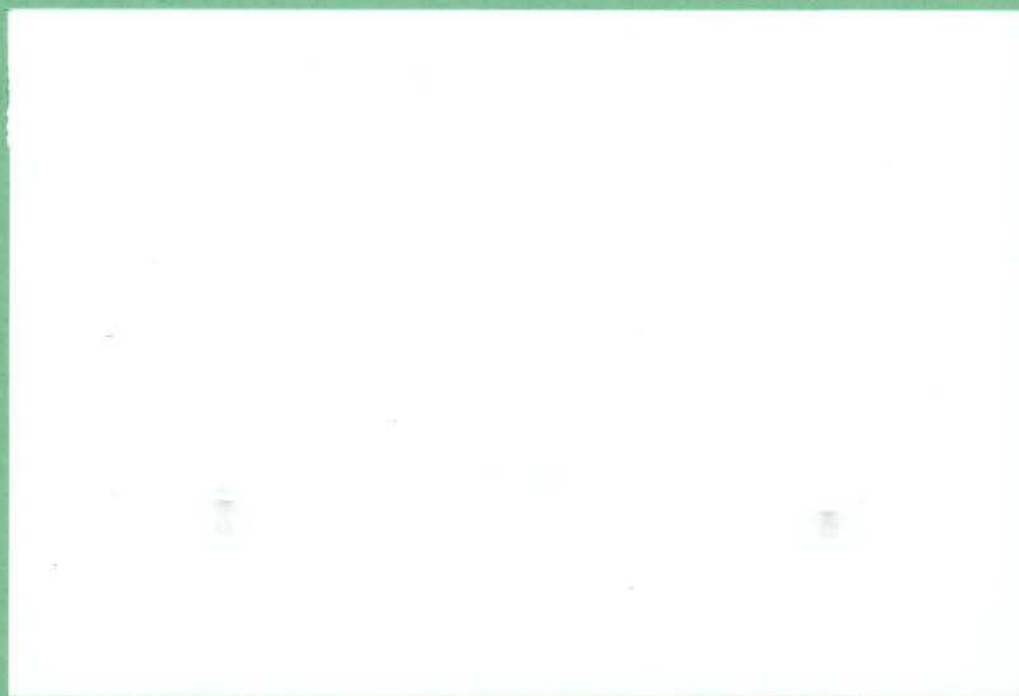
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**VOLKSWIRTSCHAFTLICHE
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**UNIVERSITÄT - GESAMTHOCHSCHULE - SIEGEN
FACHBEREICH WIRTSCHAFTSWISSENSCHAFTEN**

Partial versus Global Optimization in Economic Dynamics: The Case of Recursive Programming

*Walter Buhr and Reiner Wolff**

*Department of Economics
University of Siegen, Germany*

Discussion Paper No. 26-91

Paper Prepared in Honor of Rainer Thoss
on the Occasion of His 60th Birthday

Abstract

After having given a brief overview of important applications of recursive programming models which aim at partial optimization in economic dynamics, we present a reduced version of a type of recursive regional and sectoral allocation models suggested by Rainer Thoss. Then we discuss the question of what is implied by a Thoss partially optimizing model in terms of global intertemporal efficiency. With respect to this model we demonstrate that the many advantages of recursive programming concerning practical applicability come at the cost of possible dynamic allocation losses. We also indicate a line of future research to resolve this trade-off.

* This paper is for discussion purposes only. It must not be quoted without the authors' written consent.

I. Outline of Analysis

During the past decades much research has been done to improve our knowledge and understanding of the positive dynamic behavior of individual decision makers or groups of economic agents, respectively, operating over and through time. This paper is concerned with a particular type of a discrete-time dynamic decision making process, i.e. a process which can be cast in a recursive framework. The notion of a Recursive Decision System (RDS) is due to Day [1963] and has so far proved extremely useful to approach a great variety of rather different economic dynamic problems.

A typical economic model of an RDS will consist of two basic parts (see Day and Kennedy 1970a,1970b). The first of them may be called a 'decision operator' and will describe how a decision is derived in a given period of time t from some given amount of information available at that time. The second basic part of the model may be termed a 'feedback operator'. This component will provide a mechanism by which each choice of action will affect the decision maker's economic environment of period $t + 1$. It is important to note already at this point that the process of decision making is assumed short-horizon and myopic in that it is at least partially ignorant of feedback effects. As a result, time paths generated by an RDS will not in general possess intertemporal global optimum properties. They will rather describe a series of interrelated steps which are only partially optimizing in the sense of making a locally best decision. These statements are not in conflict with the fact that certain normative economic problems of optimum control and dynamic programming may exhibit globally optimal strategies which also have recursive structure (see the excellent monograph by Stokey and Lucas 1989).

As a first example of an RDS consider the familiar cobweb model where firms act so as to choose, along their supply curves, levels of output which will maximize profits given the current market price. What is not included in the making of these choices are price changes induced by deviations of market supply from market demand, both of which are unknown to the individual producer. The model of Cournot duopolistic competition is another, non-competitive example of this kind of behavior. These models bring up interesting theoretical questions regarding, e.g., the existence of stationary states of an RDS. We will not, however, pursue these questions any further and refer the reader to Day and Kennedy [1970a,1970b].

We will rather focus on a special class of recursive systems which we feel is most important from the viewpoint of applied economics. Within this class of systems choices are made according to constrained maximization in the form of a mathematical recursive programming problem. In Section II we will give a brief overview of prominent applications of recursive programming models. We will then in Section III present a reduced version of a type of regional and sectoral allocation models

suggested by Professor Rainer Thoss. These models were designed with respect to present-day German regional and sectoral economic policy and were first published in different articles in the late sixties and beginning seventies (see Thoss 1968/69, 1970, 1974). Our primary interest in the models lies in a question already posed and partially answered by Thoss: What is implied by his partially optimizing models in terms of global intertemporal efficiency? We will tackle this question in Section IV. A final Section V is devoted to a family of growth models which allow to maintain, to some degree, the flexibility of an RDS without sacrificing intertemporal efficiency.

II. Applications of Recursive Programming Models

Our idea of a Recursive Programming Model (RP model) is that of a sequence of programming problems, either linear or non-linear, the solution to each of which is related by a set of feedback functions to solutions to problems earlier in the sequence. Recursive programming models arise in a number of research areas. The following lines of investigation appear to be of particular significance (cf. Day 1973, 1979).

RP models are in use in *economic theory* to study the dynamic implications of short-term adaptive behavior in situations of economic disequilibrium. Examples of this kind of application of an RP model are, among others, Bayesian models of expected utility maximization as well as the Cournot duopoly and cobweb models already mentioned in Section I (see, e.g., Day and Tinney 1969). Various authors have constructed RP-type *computer simulation models* which are based on the assumed behavior of sub-optimizing agents. This breed of models may be used for policy formulation, most notably in the fields of industrial and agricultural development (a selection of contributions can be found, e.g., in Judge and Takayama 1973; also cf. Day and Heiden 1969, Day et al. 1970 and Day and Cigno 1980). Many *computational algorithms* developed to solve non-linear constrained optimization problems also proceed in the form of an RP model. Typical examples are iterative search techniques where the direction of search is obtained in each iteration from the solution to a programming subproblem which in turn is based on a penalty function (see M.C. Bartholomew-Biggs 1980 for a detailed discussion and for further references).

Essential elements of these three research areas generating RP models have been brought together by Thoss [1968/69, 1970, 1974] to create a loose, highly flexible and consistent planning framework for the formulation of a spatial standard-guide for an economy. This framework was deliberately designed for the purpose of deriving both empirically meaningful and practically useful regional and sectoral public policies for the Federal Republic of Germany. It thus greatly supported Thoss's various government consulting efforts for many years. The policies proposed aim at the

maximization of the gross domestic product as their major flexible objective. The Thoss regional and sectoral RP model is therefore oriented at the long-run balance of interregional and intersectoral factor productivities in pursuit of optimum economic growth. A similar approach has been suggested by Thoss and Agnew [1976] for the coordination of regional and sectoral public policies in the German state of Hesse. In what follows, we will present in Section III a simplified version of the first variant of the Thoss RP model which appeared in Thoss [1968/69,1970]. We will subsequently turn to the question of the intertemporal efficiency of factor allocation.

III. A Simplified Version of the Thoss RP Model

The Thoss [1968/69,1970] regional and sectoral RP model may be characterized in short as a sequence of linear programming problems directed towards a step by step maximization of the German gross domestic product and linked to one another by gradual quantity changes of the factors labor, capital and land. The entire country is sub-divided into a total of 100 regions with three sectors of production per region. These sectors include agriculture (sector 1), manufacturing, trade and transportation (sector 2) and government and other services (sector 3). Income and employment in sector 3 are assumed, for reasons of measurement of output, to depend on income and employment in the private sectors 1 and 2 on which we will now concentrate. Outputs of these two sectors are determined by Cobb-Douglas and CES-type production functions which have been estimated from regional cross-section data. As a result, sectoral production functions are taken as identical across all regions. Therefore, we will drop the regional component of the model. This further simplification is not to deny the significance of the chosen form of regional disaggregation. In fact, private factor supply and public infrastructure will in general differ from region to region even with all sectoral production functions being the same. However, we believe that the question of partial versus intertemporal global efficiency of dynamic factor allocation can already be dealt with in the context of a one-region two-sector version of the Thoss model. For the same reason we do not consider land use an explicit variable of our dynamic reduced system.

Our notation is as follows (see Thoss 1968/69,1970). We write as L_t and A_t the amounts of labor inputs available in sectors 1 and 2 at the beginning of some given period of time t . Variables C_t and K_t stand for the sectoral initial supplies of private capital in period t . There shall exist sectoral production functions in the form of

$$(1) \quad U_t = F(L_t, C_t), \quad V_t = G(A_t, K_t, t)$$

with exogenous (Hicks-neutral) technical progress prevailing in sector 2. $F(\cdot)$ and $G(\cdot)$ are assumed to be quasi-concave and twice continuously differentiable in inputs L_t and C_t and inputs A_t and K_t , respectively. $G(\cdot)$ is also supposed to be

continuously differentiable in t . Outputs are measured at constant prices.

The problem in each period t is to maximize by choice of changes of sectoral input quantities the increase of total sectoral output. Total output changes are always computed along a first-order Taylor series approximation to the sum $U_t + V_t$:

$$(2) \quad \max_{\Delta L_t, \dots, \Delta K_t} \frac{\partial F}{\partial L_t} \Delta L_t + \frac{\partial F}{\partial C_t} \Delta C_t + \frac{\partial G}{\partial A_t} \Delta A_t + \frac{\partial G}{\partial K_t} \Delta K_t.$$

Admissible choices of re-allocations $\Delta L_t, \dots, \Delta K_t$ are constrained by the requirement that the total change of labor demands L_t and A_t must not exceed the given change of the domestic labor force R_t in period t :

$$(3) \quad \Delta L_t + \Delta A_t \leq \Delta R_t.$$

Furthermore, investments ΔC_t and ΔK_t must be selected so as to be consistent with a fixed investment ratio g :

$$(4) \quad \Delta C_t + \Delta K_t = g(U_t + \Delta U_t + V_t + \Delta V_t),$$

where ΔU_t and ΔV_t are again approximated by linear functions of ΔL_t and ΔC_t and of ΔA_t , ΔK_t and Δt , respectively. Finally, since optimum adjustments in reality often cannot be accomplished within a single time period, so-called flexibility constraints impose lower and upper bounds upon changes of single input quantities. There are three such constraints, e.g., with respect to agricultural employment and concerning the stocks of private capital of our region:

$$(5) \quad |\Delta L_t| \leq 0.005 L_t, \text{ if } \Delta L_t < 0, \quad \Delta C_t \geq 0.05 C_t, \quad \Delta K_t \leq 0.20 K_t.$$

Investments ΔC_t and ΔK_t are also subject to non-negativity constraints whereas both ΔL_t and ΔA_t are allowed to take positive as well as negative values.

The above reduced version of the Thoss RP model is a mathematical constrained maximization problem with respect to quantity changes $\Delta L_t, \dots, \Delta K_t$ that can be brought into a standard linear programming format. We assume that there will exist a unique optimum solution $\Delta L_t^*, \dots, \Delta K_t^*$ which depends on initial endowments at the beginning of period t as well as on exogenous labor force growth and technical progress. This solution along with

$$(6) \quad \begin{aligned} L_{t+1} &:= L_t + \Delta L_t^*, & C_{t+1} &:= C_t + \Delta C_t^*, \\ A_{t+1} &:= A_t + \Delta A_t^*, & K_{t+1} &:= K_t + \Delta K_t^*, \end{aligned}$$

and a given change ΔR_{t+1} will set the stage for the next programming problem in the sequence which is to be solved in period $t+1$. Iterating this process will then generate the path of choice of allocation and growth of the economy over time. In

the long run this path will possess the property that marginal productivities of labor and capital are at balance across the sectors such that no improvements in terms of total outputs can be achieved by changing the current factor allocation. None of the flexibility constraints will then be effective any more.

At this point we would like to stress that Thoss's original model includes a substantial number of additional equations and inequalities which we have chosen not to report in our reduced model version for ease of presentation. They are related, e.g., to the public sector of the economy, most notably with regard to the provision of infrastructure facilities. They also deal with various spatial aspects of economic activity such as land use, migration and transportation. Additional model relationships handle questions of regional and sectoral distribution of incomes. The interested reader is referred to Thoss [1968/69,1970] for details.

IV. Dynamic Properties of the Model Solution

We now turn to the problem of intertemporal global allocative efficiency. This problem has already been addressed by Thoss in the context of a continuous-time neoclassical model of intertemporal output maximization (see Thoss 1968/69,1970, 515-526 and 113-120, respectively). This model deviates from (1)-(5) in that it is formulated in terms of absolute levels of variables and is oriented at long-run growth such that constraints (5) can be neglected. It also explicitly assumes a situation of full-employment of inputs and exponential growth of the domestic labor force at a constant rate n . Maximization is carried out over some finite interval $[0, T]$ starting with initial stocks C_0 and K_0 . We thus obtain the following calculus of variations problem. Note that a dot ' $\dot{}$ ' is meant to indicate total differentiation with respect to time. Notation is otherwise straightforward.

$$(7) \quad \begin{aligned} & \max_{L(t), \dots, K(t)} \int_0^T (F(L, C) + G(A, K, t)) dt \\ & \text{subject to } \dot{C} + \dot{K} = g(F(L, C) + G(A, K, t)) \quad \text{and} \quad L + A = R_0 e^{nt} \\ & \text{with initial conditions } C(0) = C_0, \quad K(0) = K_0. \end{aligned}$$

Associated with (7) is the Lagrangian functional

$$(8) \quad \int_0^T \{F(L, C) + G(A, K, t) + \lambda [\dot{C} + \dot{K} - g(F(L, C) + G(A, K, t))] + \omega (L + A - R_0 e^{nt})\} dt =: \int_0^T Z(L, A, C, K, \dot{C}, \dot{K}, \lambda, \omega, t) dt$$

and the set of Euler differential equations

$$\begin{aligned}
(9) \quad & \frac{\partial Z}{\partial L} - \frac{d}{dt} \left[\frac{\partial Z}{\partial \dot{L}} \right] = (1 - \lambda g) \frac{\partial F}{\partial L} + \omega = 0, \\
& \frac{\partial Z}{\partial A} - \frac{d}{dt} \left[\frac{\partial Z}{\partial \dot{A}} \right] = (1 - \lambda g) \frac{\partial G}{\partial A} + \omega = 0, \\
& \frac{\partial Z}{\partial C} - \frac{d}{dt} \left[\frac{\partial Z}{\partial \dot{C}} \right] = (1 - \lambda g) \frac{\partial F}{\partial C} - \dot{\lambda} = 0, \\
& \frac{\partial Z}{\partial K} - \frac{d}{dt} \left[\frac{\partial Z}{\partial \dot{K}} \right] = (1 - \lambda g) \frac{\partial G}{\partial K} - \dot{\lambda} = 0.
\end{aligned}$$

These equations imply as a necessary condition for *intertemporal* efficiency that the marginal productivities of labor and capital must coincide across sectors 1 and 2:

$$(10) \quad \frac{\partial F}{\partial L} = \frac{\partial G}{\partial A}, \quad \frac{\partial F}{\partial C} = \frac{\partial G}{\partial K}.$$

Thus we may conclude that the sequence of partially optimizing recursive programs (2)-(5) will gradually approach a path of factor allocation which is *globally* optimal. This statement represents Thoss's main result.

Although formally appealing, this result depends on the assumption that the economy's investment ratio g is exogenous and constant. The following thoughts will make clear that this assumption is indeed vital. It cannot easily be forsaken in favor of a flexible investment ratio which is to be determined endogenously in pursuit of intertemporal output maximization. This is due to the fact that investment outputs \dot{C} and \dot{K} tend to increase the productive capacity of future periods while consumption outputs will not. The output maximizing investment ratio will for that reason always amount to unity which cannot be considered a meaningful investment policy as it involves zero consumption. Therefore, one would have to impose upon g an upper bound less than unity which is essentially the approach taken by Thoss.

Another criterion of choice of an optimum investment ratio is to maximize the stream of real consumption output rather than real total output. This alternative means to replace the target functional in (7) by

$$(11) \quad \max_{L(t), \dots, K(t), g(t)} \int_0^T (1 - g)(F(L, C) + G(A, K, t)) dt,$$

in which case the economy's investment ratio g serves as a control variable in the dynamic programming sense. Then from the Maximum Principle by Pontryagin it follows that this ratio will possess, along an optimum path, a property typical of a so-called bang-bang control: All output will be used for capital accumulation during a first part of the interval $[0, T]$. It will be entirely available for consumption in the second part of that interval (see, e.g., Shell 1969 who was among the first to

study this type of economic maximization problems; also see Wolff 1981). Formally speaking, there will exist a time period $t^* \in (0, T)$ such that

$$(12) \quad g(t) = \begin{cases} 1, & \text{if } t \in [0, t^*); \\ \text{arbitrary}, & \text{if } t = t^*; \\ 0, & \text{if } t \in (t^*, T]. \end{cases}$$

Therefore, we are again confronted with the problem of bounding $g(t)$ from above in order to prevent zero consumption.

There is another basic question inherent in the maximization of consumption *outputs*, i.e. the question of measurement of economic welfare. One may argue from a utilitarian viewpoint that the economy should seek to maximize over $[0, T]$ a discounted stream of future *utilities* of consumption. Suppose that society has agreed on a social welfare function $U : R_+ \mapsto R$ in each period of time. This function shall be increasing, concave and twice continuously differentiable. We denote by $\delta (\geq 0)$ a constant social discount rate to be applied to utilities of future periods. Then we are faced with the following Ramsey-type variational model of intertemporal utility maximization:

$$(13) \quad \begin{aligned} & \max_{L(t), \dots, K(t)} \int_0^T e^{-\delta t} U(F(L, C) + G(A, K, t) - \dot{C} - \dot{K}) dt \\ & \text{subject to } L + A = R_0 e^{\delta t} \quad \text{and} \quad C(0) = C_0, \quad K(0) = K_0. \end{aligned}$$

Starting from the Lagrangian functional

$$(14) \quad \begin{aligned} & \int_0^T \{ e^{-\delta t} U(F(L, C) + G(A, K, t) - \dot{C} - \dot{K}) + \omega (L + A - R_0 e^{\delta t}) \} dt \\ & =: \int_0^T \tilde{Z}(L, A, C, K, \dot{C}, \dot{K}, \omega, t) dt, \end{aligned}$$

we obtain the Euler first-order conditions:

$$(15) \quad \begin{aligned} \frac{\partial \tilde{Z}}{\partial L} - \frac{d}{dt} \left[\frac{\partial \tilde{Z}}{\partial \dot{L}} \right] &= e^{-\delta t} U' \frac{\partial F}{\partial L} + \omega = 0, \\ \frac{\partial \tilde{Z}}{\partial A} - \frac{d}{dt} \left[\frac{\partial \tilde{Z}}{\partial \dot{A}} \right] &= e^{-\delta t} U' \frac{\partial G}{\partial A} + \omega = 0, \\ \frac{\partial \tilde{Z}}{\partial C} - \frac{d}{dt} \left[\frac{\partial \tilde{Z}}{\partial \dot{C}} \right] &= e^{-\delta t} U' \frac{\partial F}{\partial C} - e^{-\delta t} (\delta U' - \dot{U}') = 0, \\ \frac{\partial \tilde{Z}}{\partial K} - \frac{d}{dt} \left[\frac{\partial \tilde{Z}}{\partial \dot{K}} \right] &= e^{-\delta t} U' \frac{\partial G}{\partial K} - e^{-\delta t} (\delta U' - \dot{U}') = 0. \end{aligned}$$

The latter two of these relationships are known as the Ramsey rule

$$(16) \quad \frac{\partial F}{\partial C} = -(\frac{\dot{U}'}{U'} - \delta), \quad \frac{\partial G}{\partial K} = -(\frac{\dot{U}'}{U'} - \delta).$$

We conclude again from (15) and (16) that a solution to (13) must satisfy (10). Balancing marginal productivities across sectors is therefore indeed in the context of the Thoss RP model a necessary condition for an intertemporal global optimum of allocation and growth. This condition may even conform in theory to a constant optimum investment ratio. A golden rule path of capital expansion is an important example in case. But, given (13), such a path is rather unlikely to exist as Thoss does not assume constant returns to scale in production. Nor does he assume Harrod-neutrality of technical progress. The economy's optimum investment ratio will thus in general come out as non-constant and will, therefore, change over time.

However, our conclusion is subject to a number of rather strong assumptions. They include the assumed existence of a broadly accepted social evaluation of current and future consumption. Another important aspect, at least if viewed from (13), is that there has to be complete knowledge of future technology over the entire time interval $[0, T]$. This means that any type of technological change not foreseen in $t = 0$ may render the computed time path $g(t)$ non-optimal. For these reasons, it appears to us that it makes good sense to suggest along with Thoss that the economy's investment ratio be treated as exogenous to his model and that the numerical value of this ratio be taken from past empirical observations.

V. Conclusion

With reference to the Thoss RP model we have argued in this paper that the method of recursive programming can be considered a very flexible and powerful tool of economic policy in the fields of regional and sectoral planning. In this respect, Thoss has substantially contributed to the formulation of practically relevant economic policies in the Federal Republic of Germany. However, we have also demonstrated that the many advantages of recursive programming in terms of practical applicability come at the cost of possible dynamic allocation losses (also cf., e.g., Day 1973, 330-331). It certainly would be most desirable from an economic policy point of view to proceed so as to maintain the advantages of short-horizon decision making without having to sacrifice intertemporal efficiency.

An important attempt into this direction has been made in a paper by Buhl, Eichhorn and Gleißner [1982] on optimal investment policies in a certain class of optimum growth models (also see Buhl 1983, 1984 and Buhl and Siedersleben 1984). In their paper Buhl, Eichhorn and Gleißner establish the result that an economy's optimum investment ratio will be independent in some sense of future technology and will even be independent of the planning horizon T if the economy exhibits a vintage production structure with finite capital longevity. Economic policy makers may thus do with limited information about the future. Also the well-known problem of

intergenerational justice in long-term optimum growth is greatly reduced. It seems that empirical applications of this breed of models in the tradition of the Thoss regional and sectoral recursive programming model have not yet been formulated. We consider this a challenge for future research.

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