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## Time-Varying Devaluation Risk, Interest Rate Differentials and Exchange Rates in Target Zones: Empirical Evidence from the EMS

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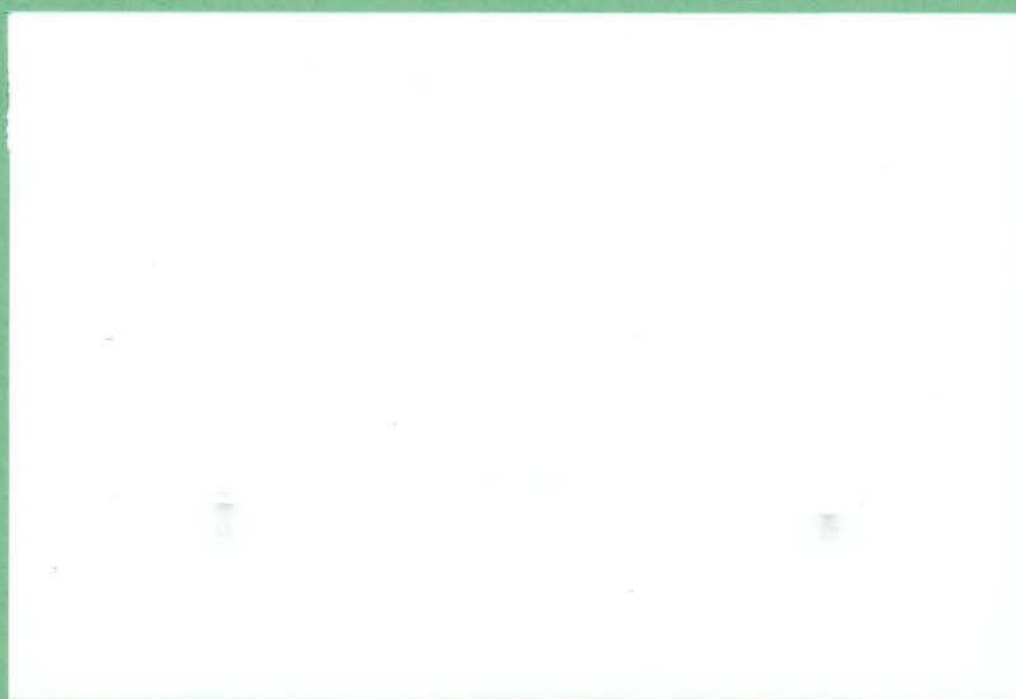
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## Time-Varying Devaluation Risk, Interest Rate Differentials and Exchange Rates in Target Zones: Empirical Evidence from the EMS

by

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### Abstract

Stylized empirical facts about the behaviour of exchange rates and interest rate differentials in real world target zone arrangements are at odds with the predictions of the simple (fully credible) target zone model. Incorporating time-varying devaluation risk in target zone models results in much richer data-generating structures and provides an interesting interpretation for the variability which standard target zone models leave unexplained. By using Bayesian time-varying parameter regression, the present paper shows that stochastic devaluation risk actually explains the EMS data quite well. Three key findings should be stressed: first, estimates of expected devaluation rates have recently declined significantly, but devaluation risks are not yet completely eliminated. Second, expected devaluation rates display 'hysteresis'. This contaminates with noise many of the relationships postulated by target zone models, but adjusting for expected devaluation rates frequently reveals almost noise-free relationships, which strongly supports the prediction from the theory. Finally, the estimates of expected devaluation rates suggest that some of the early EMS realignments were largely expected by the market.

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## NON-TECHNICAL SUMMARY

Stylized empirical facts about the behaviour of exchange rates and interest rate differentials in real world target zone arrangements, such as the European Monetary System (EMS), are typically at odds with the predictions of the simple (fully credible) target zone model. As a reaction to this disappointing empirical performance, augmented target zone models incorporating time-varying devaluation risks have recently been put forward in the literature. These models introduce stochastic fluctuations in expected devaluation rates as a second state variable (in addition to fundamentals) in order to introduce noise in the empirical relationship between interest rate differentials and exchange rates. Focusing on time-varying devaluation risk results in much richer data-generating structures and provides an interesting interpretation for the variability left unexplained by standard target zone models.

The present paper puts these models to the test and finds that stochastic devaluation risk explains the relationship between German mark exchange rates and the corresponding EMS interest rate differentials relative to Germany quite well. This is demonstrated in two steps. First, time-varying parameter regression is used to extract a measure of the unobservable expected devaluation rates from data on interest rate differentials and exchange rates. These estimates of expected devaluation rates are found to be less than perfectly correlated with the interest rate differential, and are seen to have declined significantly in recent years, indicating a transition of the EMS to a system of more credible exchange rate target zones. In a second step, it is shown that whilst no apparent relationship exists between actual interest rate differentials and the exchange rate's band position, as is commonly found in the literature, a clear and almost noise-free empirical relationship, matching the theory, can be derived after adjusting interest rate differentials for time-varying expected devaluation rates. This striking result provides strong support for this second generation of target zone models.

A number of additional features of the estimates also deserve mentioning: first, whilst the estimates of expected devaluation rates have recently declined significantly, devaluation risks are not yet completely eliminated. Second, the estimates of expected devaluation rates suggest that some of the early EMS realignments, in particular in the case of Italy, Belgium and Denmark, were largely expected by the market. For France, the Netherlands and Ireland, however, the results indicate that realignment expectations under speculative attacks account for only a small proportion of expected devaluation rates, implying that realignments of these currencies relative to the German mark were mainly unanticipated.

# Time-Varying Devaluation Risk, Interest Rate Differentials and Exchange Rates in Target Zones: Empirical Evidence from the EMS

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## 1. Introduction

Standard target zone models of exchange rate dynamics imply a deterministic, non-linear inverse relationship between exchange rate deviations from parity and interest rate differentials, as is derived by Svensson (1989,1991b) in his comprehensive theoretical treatment of the issue. The term structure of interest rate differentials is thereby endogenously determined via discrepancies between the expected maturity exchange rate and the instantaneous exchange rate, which in turn are both non-linear functions of the exogenous fundamentals. Consequently, the term structure of interest rate differentials also is a non-linear function of the fundamentals, and a target zone for the fundamentals implies both a target zone for exchange rates and interest rate differentials. Svensson (1991b) also shows that the relationship between the exchange rate and the interest rate differential is in principle non-linear, but becomes flatter and less non-linear for longer maturities.

Empirical evidence regarding the above hypothesis is provided for the unilateral Swedish exchange rate target zone in Svensson (1991b) for monthly data and in Söderlind and Lindberg (1991) for daily data. Svensson (1991b) estimates a *linearized* version of his model by regressing interest rate differentials of various maturity on the parity deviation of the exchange rate. The estimated slope coefficients do indeed exhibit the expected pattern of being smaller for longer

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\* I should like to thank Andrew Rose and Roel Beetsma for generously providing the daily interest rate and exchange rate data used in this paper.

maturities. However, the fit between theory and data is far from perfect, and a strongly serially correlated component is left unexplained in the relationship between exchange rates and interest rate differentials, as Bertola and Svensson (1991) note. Serious doubts about the empirical validity of the standard target zone model for the Swedish data are also raised in Söderlind and Lindberg (1991), who apply parametric methods to daily data and show that their results in most cases refute the standard target zone model.

For the EMS empirical evidence on the relationship between interest rate differentials and exchange rates is provided in Bertola and Caballero (1990) and in Flood, Rose and Mathieson (1990), albeit without Svensson's theoretical framework. Flood, Rose and Mathieson (1990) find no compelling evidence of the type of non-linearities implied by standard target zone models, and again the components left unexplained by non-linear models are highly serially correlated and, in many cases, so large as to raise serious doubts about the validity of target zone models, as Bertola and Svensson (1991) stress.

As a reaction to the disappointing empirical performance of the first generation of (fully credible) target zone models, augmented target zone models incorporating time-varying devaluation risks have recently been put forward by Bertola and Svensson (1991). Stochastic fluctuations in expected devaluation rates are modelled as a second state variable (in addition to fundamentals) in order to introduce noise in the empirical relationship between interest rate differentials and exchange rates. The authors use simulation experiments to show that this can account for some of the stylized empirical facts found in Flood, Rose and Mathieson (1990) for the EMS, but leave it to future research to carry out the empirical work. Some first empirical evidence based on this type of augmented target zone model is provided in Rose and Svensson (1991) and Svensson (1991d) for EMS exchange rates and interest rate differentials relative to Germany. Rose

and Svensson (1991) show that many of the features of the Bertola and Svensson (1991) model seem to be consistent with the data: adding fluctuating devaluation risk can reconcile some of the problems associated with early target zone models which have only a single forcing variable, the fundamentals process.

The present paper explores an alternative approach to evaluating the relevance of this second generation of target zone models which incorporate time-varying devaluation risks. Using time-varying parameter regression a measure of the unobservable expected devaluation rates is extracted from data on interest rate differentials and exchange rate parity deviations in the EMS. It is shown that whilst no apparent relationship exists between actual interest rate differentials and the exchange rate's band position, as is commonly found in the literature, a clear and almost noise-free empirical relationship, matching the theory, can be derived after adjusting interest rate differentials for time-varying expected devaluation rates, which in the fully credible target zone model are assumed to be zero. Furthermore, the estimates of expected devaluation rates, which are found to be less than perfectly correlated with the interest rate differential, are seen to have declined significantly in recent years, indicating a transition of the EMS to a system of more credible exchange rate target zones.

The remainder of paper is organized as follows: section 2 reviews the theoretical concepts and empirical implications of standard and augmented target zone models. Special emphasis is thereby placed on the relationship between finite term interest rate differentials and exchange rates in a target zone. The empirical approach to estimating this relationship by allowing for time-varying devaluation risk is outlined in section 3. Section 4 presents the empirical evidence obtained from applying this method to real world exchange rate data from the EMS. Section 5 concludes the paper with a summary of the main results and some suggestions for further research.

## 2. The Theory of Exchange Rate Target Zones

### 2.1. Exchange Rates and Exchange Rate Fundamentals in a Target Zone

The basic assumptions of standard target zone models, which originate in the work of Krugman (1990), may be characterized as follows:

- (a) the exchange rate  $e_t$  is driven by a stochastic forcing process  $f_t$ , referred to below as exchange rate fundamentals.<sup>1</sup> Actual exchange rates may, however, deviate from the fundamentals due to speculative bubbles  $E_t(de_t/dt)$ :

$$e_t = f_t + \frac{\alpha}{dt} E_t(de_t) ; \quad (1)$$

- (b) the fundamentals  $f_t$  typically include both variables with autonomous dynamics and variables under the direct control of the monetary authority, which is assumed to aim at maintaining a target zone for the fundamentals via foreign exchange intervention at pre-specified upper ( $\bar{f}$ ) and lower ( $\underline{f}$ ) bounds. This implies well-specified bounds ( $\bar{e}$ ,  $\underline{e}$ ) for the exchange rate, as will be demonstrated below;

- (c) in the absence of intervention the fundamentals  $f_t$  follow a continuous Brownian motion (or Wiener, or Wiener-Levy) process:

$$df_t = \eta dt + \sigma dz_t , \quad (2)$$

with instantaneous mean drift  $\eta$  and variance  $\sigma$ , where  $dz_t$  is the standard Wiener process ( $dz_t/dt \equiv w_t$ );

- (d) the observable process  $e_t$  is postulated to be a non-linear, twice continuously differentiable function  $x(\cdot)$  of the state  $f_t$ , which rules out irrational bubbles:

$$e_t = x(f_t) . \quad (3)$$

Using equations (1) to (3), Ito's lemma may be applied to obtain an expression for the expectations in (1):

$$\frac{1}{dt} E_t(de_t) = \eta x_f(f_t) + \frac{\sigma^2}{2} x_{ff}(f_t) . \quad (4)$$

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<sup>1</sup>Froot and Obstfeld (1989) show (in footnote 2) how equation (1) may be derived from a monetary model of exchange rates such as Mussa (1976). Miller and Weller (1988, 1989a,b) present an interpretation of this equation in terms of Dornbusch's (1976) overshooting model.



This results in a functional equation for the exchange rate:

$$x(f_t) = f_t + \eta x_f(f_t) + \frac{\sigma^2}{2} x_{ff}(f_t). \quad (5)$$

This second order differential equation has, as shown in Froot and Obstfeld (1989), the general stationary solution:

$$e_t = f_t + \alpha\eta + A_1 e^{\lambda_1 f_t} + A_2 e^{\lambda_2 f_t}, \quad (6a)$$

with

$$\lambda_1 = \frac{-\eta + \sqrt{\eta^2 + 2\sigma^2/\alpha}}{\sigma^2} > 0, \quad \lambda_2 = \frac{-\eta - \sqrt{\eta^2 + 2\sigma^2/\alpha}}{\sigma^2} < 0, \quad (6b)$$

where  $A_1$  and  $A_2$  are constants, determined by the boundary conditions  $x(\bar{f})$  [ $x(f)$ ] satisfied by the exchange rate  $e_t$  at the time of intervention:

$$A_1 = \frac{e^{\lambda_2 \underline{f}} - e^{\lambda_2 \bar{f}}}{\lambda_1 e^{\lambda_1 \bar{f} + \lambda_2 \underline{f}} - \lambda_1 e^{\lambda_1 \underline{f} + \lambda_2 \bar{f}}}, \quad A_2 = \frac{e^{\lambda_1 \bar{f}} - e^{\lambda_1 \underline{f}}}{\lambda_1 e^{\lambda_1 \bar{f} + \lambda_2 \underline{f}} - \lambda_1 e^{\lambda_1 \underline{f} + \lambda_2 \bar{f}}}. \quad (6c)$$

These 'smooth pasting' conditions, derived from equation (5) for  $x_f(\underline{f})=x_f(\bar{f})=0$ , ensure that  $x(f_t)$  is flat at the bounds of the fundamentals band and tangent to the boundaries of the implied exchange rate band in Krugman's model of infinitesimal marginal intervention. In economic terms, 'smooth pasting' ensures that the exchange rate is never expected to jump in response to intervention.<sup>2 3</sup> In Krugman's *perfectly credible* target zones 'smooth pasting' results for  $A_1 < 0$  and  $A_2 > 0$ , and the relationship between the exchange rate and its fundamentals has the well-known S-shape. This familiar S-shape of the exchange rate as a function of the fundamental is depicted in Graph 1. Note that as in Svensson (1990b) a zero fundamental drift ( $\mu = 0$ ),  $\sigma = 0.1$ ,  $\alpha = 3$  and a symmetrical fundamental band of  $\bar{f} = -\underline{f} = 0.094$ , which implies a symmetrical exchange rate band of  $\bar{e} = -\underline{e} = 0.015$ , has been used in all the graphs below.

<sup>2</sup>If such jumps were allowed for, risk neutral investors would face an arbitrage opportunity as fundamentals approach the point of intervention.

<sup>3</sup>Flood and Garber (1989) show that this no-jump requirement also provides boundary conditions for more general intervention policies, such as finite intervention strictly in the interior of the band.

## 2.2. The Term Structure of Interest Rate Differentials in a Target Zone.

The above inherent stabilization effect of a target zone for fundamentals on exchange rates also has important consequences for the behaviour of interest rate differentials, as shown in Svensson (1989, 1991b), on which the following exposition is based. Let  $i_t^*(\tau)$  denote an exogenous foreign nominal interest rate on a pure discount bond purchased at time  $t$  with term  $\tau$ , that is, maturing at time  $t+\tau$ ,  $\tau \geq 0$ . Further define  $i_t(f;\tau)$  as the nominal interest rate on a home currency pure discount bond, purchased at time  $t$  with the fundamental  $f_t$  equal to  $f$ , and maturing at time  $t+\tau$ ,  $\tau > 0$ . As shown in Svensson (1991b) the interest rate differential in an exchange rate target zone may then be derived (for  $t=0$ ) as:

$$\delta(f;t) \equiv i_t(f;\tau) - i_t^*(\tau) = \frac{E[e(f(\tau)) | f_0=f] - e(f)}{\tau}, \quad \tau > 0, \quad (7)$$

Determining (7) requires computing the expected exchange rate at maturity:

$$h(f;\tau) \equiv E[e(f(\tau)) | f_0=f], \quad (8)$$

which is a complicated non-linear heteroscedastic stochastic process with variable drift and instantaneous standard deviation. Svensson (1991b) shows that the function  $h(f,\tau)$  defined in (8) may be obtained as an analytical Fourier series solution<sup>4</sup> to the partial differential equation:

$$h(f;\tau) = \mu h_f(f;\tau) + \frac{\sigma^2}{2} h_{ff}(f;\tau), \quad \underline{f} \leq f \leq \bar{f}, \quad \tau \geq 0, \quad (9a)$$

with initial condition:

$$h(f;0) = e(f), \quad \underline{f} \leq f \leq \bar{f}, \quad (9b)$$

and derivative boundary or smooth pasting conditions:

$$h_f(\underline{f};\tau) = 0 \text{ and } h_f(\bar{f};\tau) = 0, \quad \tau \geq 0. \quad (9c)$$

The solution to the above parabolic partial differential equation, which is outlined in more detail in the Appendix, is illustrated in Graph 2, showing the term structure of expected maturity exchange rates as a non-linear function of the

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<sup>4</sup>Svensson (1991b) also derives a direct numerical solution by using the so-called explicit method described in Gerald and Wheatley (1989). Since both methods give the same result, the following exposition focuses exclusively on the analytical Fourier-series solution.

fundamentals. Two features of Graph 2 deserve attention: first, both the non-linearity and the positive slope of the expected exchange rate function are more pronounced at the short end of the maturity range, implying that the long-term expected exchange rate in a fully credible target zone is always close to the official parity (see upper small inset). Second, the expected maturity exchange rate's fluctuation band is strictly decreasing in the term (see lower small inset).

Given the behaviour of the the expected maturity exchange rate, it is straightforward to calculate the term structure of interest rate differentials for positive terms as:

$$\delta(f; \tau) = \frac{h(f; \tau) - e(f)}{\tau}, \tau > 0, \quad (10)$$

which is illustrated as a function of both the fundamental ( $f$ ) and the term ( $\tau$ ) in Graph 3. Note that  $\delta(f; \tau)$  is discontinuous in  $\tau$  at  $\tau=0$ : the instantaneous interest rate differential is decreasing in the fundamental, but it does not fulfill the smooth pasting conditions. For further details see Svensson (1989, 1991b). For positive and finite terms two properties of the interest rate differential in a target zone are worth noting: first, both the non-linearity and the negative slope of the interest rate differential as a function of the fundamentals are more pronounced at the short end of the maturity range (see upper small inset). Second, for a given fundamental band the interest rate differential's fluctuation band is strictly decreasing in the term (see lower small inset).

A further important theoretical result of Svensson (1990b) is the derived relationship between the term structure of interest rate differentials and actual exchange rates, which both are endogenously determined by the fundamentals. In order to derive this result, the fundamentals  $f$  and hence the expected maturity exchange rate in (10) must be solved as functions of the actual exchange rate:

$$\delta(f(e); \tau) = \frac{h(f(e); \tau) - e}{\tau}, \tau > 0, \quad (11)$$

with  $f(e)$  denoting the inverse of  $e(f)$ .<sup>5</sup> Graph 4 displays this functional relationship, which again is discontinuous in  $\tau$  at  $\tau=0$ . Both the instantaneous and finite term interest rate differentials are decreasing in the exchange rate, and the non-linearity as well as the negative slope of this function are again more pronounced at the short end of the maturity range (see upper small inset). As above, the interest rate differential's fluctuation band for a given exchange rate band is strictly decreasing in the term (see lower small inset).

To summarize, the standard model of a fully credible target zone implies a negative non-linear relationship between interest rate differentials and exchange rates. For positive terms the relationship is approximately linear, and all the more so for longer terms. Following Svensson (1989) it may therefore be approximated by the linearized equation:

$$\delta_t(\tau) = c(\tau) + b(\tau) e_t + \epsilon_t(\tau) . \quad (12)$$

According to theory, the coefficients  $b(\tau)$  are negative and increasing in term. Further, in the absence of any fundamental drift ( $\eta=0$ ), the constant  $c(\tau)$  should be zero for all  $\tau$  if there is no devaluation risk. As both interest rate differentials and exchange rates are observable at high frequencies, the above prediction of the standard target zone may easily be empirically tested. However, before doing so some recent results derived from augmented target zone models, which incorporate devaluation risk, should be discussed

### 2.3. Exchange Rate Target Zones and Devaluation Risks

Realignments, viewed here as a change of both the central parity and the upper and lower bounds of the band, may be introduced into the standard target zone model in a variety of ways. The treatment below follows Svensson (1989, 1990b) in

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<sup>5</sup>Iterating  $e_t$  in equi-distant steps and at each step numerically optimizing the right-hand side of equation (6a) yields the required values of  $f(e)$ , which then may be used to calculate  $h(f(e);t)$  from equation (9), as outlined in detail in the Appendix. Computing  $\delta(f(e);t)$  from equation (11) is then straightforward.

viewing realignments as re-occurring with some given constant probability, regardless of the exchange rate's position within the band. In particular, a realignment is modelled as a shift of magnitude  $g$  of the upper and lower bounds of the fundamentals band:

$$\underline{f}' = \underline{f} + gN, \quad \bar{f}' = \bar{f} + gN, \quad (13a)$$

and the same simultaneous shift  $g$  in the fundamentals themselves:

$$df_t = g dN + \eta dt + \sigma dz_t, \quad (13b)$$

where  $N$  is the number of realignments and  $dN$  is equal to unity with probability  $\nu dt$ , drawn from a Poisson distribution. As Svensson (1989, 1990b) shows, these devaluations, which leave the relative position of the fundamentals within the band unaffected, result in the following modified equations for the exchange rate:

$$e_t = x(f_t, N) = f_t + \alpha\eta + \alpha\nu g + A_1 e^{\lambda_1(f_t - gN)} + A_2 e^{\lambda_2(f_t - gN)}, \quad (14)$$

and the interest rate differential:

$$\delta(f, N; \tau) = \frac{h(f - gN; \tau) - \nu g \tau - e(f - gN)}{\tau}, \quad \tau > 0. \quad (15)$$

Note that even if no realignments have yet occurred (for  $N=0$ ), equations (14) and (15) differ from their counterparts (6a) and (10) in models of fully credible target zones by the inclusion of the terms  $\alpha\nu g$  and  $\nu g$  respectively. The implication of this for the term structure of interest rate differentials, as displayed in Graph 4, is that constant devaluation risk shifts all lines in Graph 4 for the term structure of interest rate differentials upwards by  $\nu g$ . For the above estimating equation (12) this implies that the intercept now depends on both the term  $\tau$  and the product of the probability intensity of a realignment ( $\nu$ ) and the expected size of the realignment ( $g$ ):

$$\delta_t(\tau) = c(\tau, \nu g) + b(\tau) e_t + \epsilon_t(\tau), \quad (16)$$

whilst the slope coefficient remains unaffected by this type of devaluation risks.

As noted in Bertola and Svensson (1991), the above constant devaluation risk is unlikely to result in empirically more successful target zone models. The authors therefore introduce stochastic devaluation risk  $g_t$  as a second source of time

variation in exchange rates and interest rate differentials. The time-varying state variable  $g_t$  is thereby assumed to follow a continuous Brownian motion process:

$$dg_t = \mu dt + s du_t, \quad (17)$$

with instantaneous mean drift  $\mu$  and variance  $s$ , where  $du_t$  is the standard Wiener process ( $du_t/dt \equiv v_t$ ).<sup>6</sup> Solving the model in the two state variables  $g_t$  and  $f_t$  then enables Bertola and Svensson (1991) to attribute much of the time variation in interest rate differentials, which standard target zone models leave unexplained, to the omission of the second state variable  $g_t$ .

An interesting result of Bertola and Svensson (1991) is that the negatively sloped instantaneous interest rate differential in Graph 4 fluctuates vertically as the expected rate of devaluation  $g_t$  changes over time, so that combined with the simultaneous fluctuations in the fundamentals  $f_t$  almost any pattern of exchange rate and interest rate differential observations may result. More specifically, as the variability of  $g_t$  relative to  $f_t$  increases, the correlation between the instantaneous interest rate differential and the exchange rate will be less negative, and may even become positive. For positive and finite terms matters are even more complicated, as one has to be more specific about what happens to the expected rate of devaluation in the future. According to equation (17) the expected devaluation rates  $g_t$  are driven by a Brownian motion process across realignment regimes, and thus interest rate differentials will be non-stationary. Alternatively, to obtain stationary interest rate differentials, some type of mean reverting properties (or re-setting at a realignment) of  $g_t$  is required. On purely theoretical grounds Bertola and Svensson (1991) prefer the latter alternative. Their simulations then show that for a low variability of the expected devaluation rates  $g_t$  relative to the fundamentals  $f_t$  the negative correlation between the term structure of interest rate differentials and exchange rates is maintained, but for relatively high degrees of

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<sup>6</sup>Bertola and Svensson (1991) further allow a non-zero correlation between the increments of the stochastic processes driving the expected realignment rates  $g_t$  and the fundamentals  $f_t$ , which is disregarded here for simplicity.

time-variability of expected devaluation rates  $g_t$  this negative correlation may vanish or even become positive. In both cases the realizations of interest rate differentials and exchange rates may be quite scattered in the  $(\delta, e)$ -plane.

The Bertola and Svensson (1991) model may be incorporated into the above linearized estimating equation by assuming a time-varying intercept  $c_t$  and a constant slope coefficient  $b$ :

$$\delta_t(\nu_t g_t, \sigma_g^2 / \sigma_f^2; \tau) = c_t(\tau, \nu_t g_t) + b(\tau, \sigma_g^2 / \sigma_f^2) e_t + \epsilon_t(\tau). \quad (18)$$

where  $b(\tau, \sigma_g^2 / \sigma_f^2)$  is negative and increasing in term for low degrees of variability of expected devaluations ( $\sigma_g^2$ ) relative to the fundamentals ( $\sigma_f^2$ ), and again the constant  $c_t(\tau)$  should largely reflect devaluation risks in the absence of fundamental drift ( $\eta=0$ ).

The empirical section below aims at quantifying the time-paths of expected devaluation rates implicit in EMS interest rate differentials by estimating the above augmented target zone models using time-varying parameter regression. As in the theoretical model of Bertola and Svensson (1991), this requires the specification of the future time-paths of the expected rate of devaluation  $g_t$ , as reflected in interest rate differentials by movements of  $c_t$ .

According to equation (17) the expected devaluation rates are driven by a Brownian motion process, the continuous time equivalent of a random walk (with drift), whilst the simulations of Bertola and Svensson (1991) use some mean-reverting process<sup>7</sup> in order to obtain stationary interest rate differentials. The empirical section below uses a flexible form, which allows expected devaluation rates and hence interest rate differentials to be a mixture of both stationary and non-stationary components, the composition of which may change

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<sup>7</sup>To obtain a stationary interest rate differential, Bertola and Svensson (1991) choose an ad-hoc mechanism which re-sets  $g_t$  to  $g_0$  at each realignment. However, a (regulated) mean-reverting process, such as the Ornstein-Uhlenbeck process used in Froot and Obstfeld (1989), may be preferable for modelling expected realignment risks implicit in stationary interest rate differentials.

over time. In particular, the expected devaluation rates  $c_t$  implied by interest rate differentials are modelled as a probability weighted average of a stationary (random) process:

$$c_t = \bar{c} + v_{1t}, \quad E(v_{1t})=0, E(v_{1t}v_{1t}')=\sigma_v^2, E(v_{1t}v_{1t-j}')=0 \forall j \neq 0, \quad (19a)$$

and a non-stationary (random walk) process:

$$c_t = c_{t-1} + u_{1t}, \quad E(u_{1t})=0, E(u_{1t}u_{1t}')=\sigma_u^2, E(u_{1t}u_{1t-j}')=0 \forall j \neq 0, \quad (19b)$$

which for a given probability  $\beta_t$  of the stationary process (and hence a probability  $1-\beta_t$  for the non-stationary process) results in the coefficient process:

$$\begin{aligned} c_t &= c_{t-1} - \beta_t(c_{t-1} - \bar{c}) + \beta_t v_{1t} + (1-\beta_t)u_{1t} \\ \implies c_t - c_{t-1} &= -\beta_t(c_{t-1} - \bar{c}) + w_t. \end{aligned} \quad (20)$$

This process has mean reverting properties as long as  $\beta_t$  takes intermediate values. The advantage of this set-up, which is outlined in more detail below, is that it allows the data to determine both the degree and the time-pattern of mean reversion of the expected devaluation rates implied by interest rate differentials, as this is likely to change both between EMS regimes and over time.



### 3. Estimating Time-Varying Devaluation Expectations in a Target Zone

To obtain a time-varying estimate of expected devaluation rates implied by interest rate differentials in a target zone, a learning algorithm, the so-called Bayesian multi-process Kalman filter of Harrison and Stevens (1971, 1976), is employed. For a more formal description of this method and for references to other applications in economics the discussion in Weber (1988) should be consulted.

The working of the algorithm may best be explained by transforming the linearized approximation of the above target zone model (18) for a given term  $\tau$  into its general state-space representation:

$$\delta_t = z_t' a_t + S_t v_t, \quad E(v_t) = 0, E(v_t v_t') = \sigma^2 H, E(v_t v_{t-j}') = 0 \quad \forall j \neq 0, \quad (21a)$$

$$a_t = T a_{t-1} + R u_t, \quad E(u_t) = 0, E(u_t u_t') = \sigma^2 Q, E(u_t u_{t-j}') = 0 \quad \forall j \neq 0, \quad (21b)$$

whereby the following specifications apply:

$$a_t = [c_t \quad b_t], \quad z_t = [1 \quad e_t], \quad v_t = [v_{1t} + \epsilon_t \quad v_{2t}], \quad u_t = [u_{1t} \quad u_{2t}], \quad (22)$$

$$S_t = z_t = [1 \quad e_t], \quad H = \begin{bmatrix} h_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} q_1 & 0 \\ 0 & 0 \end{bmatrix}.$$

For  $q_1=0$  and  $h_1=1$  this implies  $a_t = a_{t-1} = \dots = \bar{a}$ , which is equivalent to the model with constant expected devaluation rates, a variant of which is estimated in Svensson (1991b). For  $h_1=0$  and  $q_1=1$ , on the other hand, expected devaluation rates and hence the interest rate differentials are non-stationary and driven by a random walk.

The multi-process Kalman filter now requires both alternative process models to be set-up as sub-models of one hybrid model. The sub-model  $M_1^t$  with constant devaluation risk ( $h_1=1, q_1=0$ ) may thereby be viewed as the reference model, against which the alternative model  $M_2^t$  with time-varying devaluation risk has to be judged in terms of explaining those component of interest rate differentials that the standard target zone model leaves unexplained. Given these two alternative specifications of the variance-covariance matrices  $Q^i$  and  $H^i$  ( $i=1,2$ ), estimates of the unobservable state vectors  $a_t^i$  and their variance-covariance matrices  $\sigma^2 P_t^i$  may

then be extracted from the observable process  $\delta_t$  by using the Kalman filter. The possibility of stochastic process switching from stationary to non-stationary devaluation expectations, for example under speculative attacks, is explicitly taken into account here.

Assume that the observable interest rate differential process switches from model  $M_{t-1}^i$  ( $i=1,2$ ) in period  $t-1$  to model  $M_t^j$  ( $j \neq i$ ) in period  $t$ . Let us denote this process switching model by  $M_{t,t-1}^{ij}$ . The Kalman filter's prediction and update equations

$$\alpha_{t,t-1}^{ij} = T \alpha_{t-1,t-1}^i \quad (23a)$$

$$P_{t,t-1}^{ij} = T P_{t-1,t-1}^i T' + R Q^j R' \quad (23b)$$

$$\alpha_{t,t}^{ij} = \alpha_{t,t-1}^{ij} + K_{t,t}^{ij} \epsilon_t^{ij} \quad (23c)$$

$$P_{t,t}^{ij} = (I - K_{t,t}^{ij} z_t') P_{t,t-1}^{ij} (I - K_{t,t}^{ij} z_t')' + K_{t,t}^{ij} H^j K_{t,t}^{ij} \quad (23d)$$

$$\epsilon_t^{ij} = y_t - z_t' \alpha_{t,t-1}^{ij} \quad (23e)$$

$$K_{t,t}^{ij} = P_{t,t-1}^{ij} z_t (F_{t,t}^{ij})^{-1} \quad (23f)$$

$$F_{t,t}^{ij} = z_t' P_{t,t-1}^{ij} z_t + S^j H^j S \quad (23g)$$

may then be used to extract an estimate of the unobservable expected devaluation rates under two types of pure stochastic processes and two types of stochastic process switching, given a suitable initialization of the state vector ( $\alpha_{t_0}$ ) and its variance-covariance matrix ( $\sigma^2 P_{t_0}$ ). The empirical relevance of stochastic process switching may thereby be evaluated in probabilistic terms.

The probability distribution of the alternative process models is calculated and recursively updated in the Bayesian part of the multi-process Kalman filter by using Bayes' law. To illustrate this process, assume that each model  $M_t^i$  at each point in time has a prior probability  $E_{t-1} \pi_t^i$  as well as a posterior probability  $\pi_t^i$ , and that the probability of process switching  $M_{t,t-1}^{ij}$  is denoted by  $\pi_t^{ij}$ . According to Bayes' theorem, the conditional posterior probability  $\pi_t^{ij}$  of each model may then be

calculated as:

$$\begin{aligned} \pi_t^{ij} = & \text{PROB} \{ \delta_t \mid M_t^i, M_{t-1}^i, (\delta_{t-1}, \delta_{t-2}, \delta_{t-3}, \dots) \} \\ & * \text{PROB} \{ M_t^i \mid M_{t-1}^i, (\delta_{t-1}, \delta_{t-2}, \delta_{t-3}, \dots) \} \\ & * \text{PROB} \{ M_{t-1}^i \mid (\delta_{t-1}, \delta_{t-2}, \delta_{t-3}, \dots) \} \\ & / \text{PROB} \{ \delta_t \mid (\delta_{t-1}, \delta_{t-2}, \delta_{t-3}, \dots) \}, \end{aligned} \quad (24)$$

and can be formalized in terms of the Kalman filter from above as:

$$\pi_t^{ij} = k_t L_t^{ij} E_{t-1} \pi_t^i \pi_{t-1}^i, \quad (25)$$

with

$$L_t^{ij} = \left[ 2\pi \sigma^2 F_t^{ij} \right]^{-1/2} \exp \left[ -(\epsilon_t^{ij})^2 / 2\sigma^2 F_t^{ij} \right], \quad (26)$$

$$E_{t-1} \pi_t^i = \frac{\theta E_{t-2} \pi_{t-1}^i + \psi_{t-1}^i}{\sum_j (\theta E_{t-2} \pi_{t-1}^j + \psi_{t-1}^j)}, \quad \text{with } \theta=1, \quad (27)$$

$$\pi_{t-1}^i = \sum_j \pi_{t-1}^{ij}, \quad (28)$$

$$\psi_{t-1}^i = \sum_j \pi_{t-1}^{ij}, \quad (29)$$

The transformation of old prior ( $E_{t-2} \pi_{t-1}^i$ ) and posterior ( $\psi_{t-1}^i$ ) probabilities into new prior probabilities ( $E_{t-1} \pi_t^i$ ) for the subsequent period in equation (27) represents the Bayesian learning mechanism. This probability learning is largely determined by the relative likelihood of the individual models, as measured by the likelihood function (26) of each model.

Given the individual state estimates  $M_t^i$  from the Kalman filter and their respective probabilities  $\pi_t^i$  from the Bayesian part of the algorithm, it is now possible to condense the estimates for the mean and variance of the state as:

$$\alpha_t^i = \sum_j \pi_t^{ij} \alpha_t^{ij} / \pi_t^i, \quad (30a)$$

$$P_t^i = \sum_j \pi_t^{ij} \{ P_t^{ij} + [(\alpha_t^{ij} - \alpha_t^i)(\alpha_t^{ij} - \alpha_t^i)] \} / \pi_t^i, \quad (30b)$$

where  $\pi_t^i = \sum_j \pi_t^{ij}$  holds. The inclusion of the term  $[(\alpha_t^{ij} - \alpha_t^i)(\alpha_t^{ij} - \alpha_t^i)]$  in addition to the individual estimates  $P_t^{ij}$  in equation (21b) is justified by the fact that a large dispersion of the point estimates around their average should reduce confidence in the precision of the average point estimate.

To summarize, the multi-process Kalman filter regression model allows an estimate of the unobservable expected devaluation rates implicit in interest rate differentials to be obtained from a linearized structural target zone model. The algorithm assumes devaluation risks both to vary over time and to switch between stationary and non-stationary fluctuations, say in periods of speculative attacks. The degree of time-variability in expected devaluation rates is estimated recursively by searching for that mix (or probability weighted average) of stationary and non-stationary parameter variation which best explains the observable interest rate differential data: under the reference model of purely constant expected devaluation rates the algorithm reduces to recursive least squares, whilst under the alternative model it reduces to the pure random walk parameter regression model. Finally, the intermediate case of mixed transitory and permanent parameter variation has mean reverting properties and is close in spirit to the time-varying regression model with return-to-normality coefficients.

An important by-product of the multi-process Kalman filter estimates of expected devaluation rates is that, under certain assumptions, it allows speculative attacks on currencies to be identified. Speculative attacks occur when a realignment is expected to take place within the near future, and this is typically reflected by an extreme rise in interest rate differential, which again drops to normal levels as the realignment takes place. Under speculative attacks the interest rate differential should therefore exhibit large transitory outliers, and this should be reflected in large jumps of the probability of the model with transitory parameter variation. The proportion of the interest rate differential due to speculative attacks may thus be approximated by the transitory component of the estimates of expected devaluation rates, and this may be used as a crude measure of the extent to which actual realignments were expected by the market before they actually took place.

#### 4. Empirical Evidence for the EMS

Before discussing the details of the estimates of time-varying devaluation risk in the EMS, it is important to mention that during the sample period (79/03/13 to 90/08/28) twelve EMS realignments took place, whereby both the frequency and the size of these EMS realignments have declined over time (See Table 1). This fact is typically interpreted as an indication that the EMS has recently become a system of more credible exchange rate target zones, and this should be reflected in a decline of the estimates of devaluation risks.

At a purely descriptive level, the postulated deterministic relationship between interest rate differentials and (the inverse of) the exchange rate's band position appears not to be supported by the data. This may easily be shown by plotting the time-paths of the relative band positions of the exchange rates of the original EMS countries *vis-a-vis* Germany against the corresponding interest rate differentials between Euro-market bills of twelve month (Box 1a) and one month (Box 1b) maturity: EMS realignments, that is, jumps in the central parity, typically coincide with jumps in the relative band position of the exchange rate, but these are only occasionally mirrored by corresponding jumps in the interest rate differentials. This rules out any simple deterministic relationship between both variables.

More formally, the inadequacy of a constant coefficient relationship between interest rate differentials and exchange rate parity deviations may also be illustrated by referring to the time-series properties of these series. Table 2a shows that the twelve month interest rate differentials appear to be non-stationary time series. Only for the Dutch-German case can a unit-root in the level of interest rate differentials be rejected at the one percent level for the overall period, whereas all interest rate differentials are stationary in first differences. A similar result holds for the one month interest rate differentials in Table 2b, but here the existence of a unit root in the level of interest rate differentials is rejected more frequently, most

noticeably in all cases for the overall period. Similarly, the parity deviations of exchange rates in Table 2c all appear to be stationary in the overall period, but are in some cases not even difference stationary in the sub-periods. Hence, these time series seem to frequently exhibit stochastic process switching between stationary and non-stationary movements at different points in time,<sup>8</sup> and any constant coefficient relationship is unlikely to capture this feature of the data.<sup>9</sup>

#### 4.1. Estimates of Expected Rates of Devaluation

Augmented target zone models focus on time-varying expected devaluation rates in their attempt to explain the time-varying wedge between interest rate differentials and the exchange rate's band position, as displayed in Box 1a and Box 1b. The analysis below aims at extracting some estimates of these perceived devaluation rates from EMS interest rate differentials. The estimates of expected devaluation rates are displayed in Figures 1a,b to 6a,b together with the estimates of the parity deviation response coefficients, the probability of transitory parameter variation and the standardized prediction errors.

As in the paper by Rose and Svensson (1991), the expected rate of devaluation in the French-German interest rate differentials, displayed in the upper left quadrants of Figures 1a and 1b, are highly correlated with the interest rate differential, especially when the interest rate differential takes extreme values in the first half of the EMS period. The estimates of expected devaluation rates in Figure 1b for the one month German-French interest rate differential are thereby almost indistinguishable from the corresponding estimates in Rose and Svensson (1991), despite being derived using a completely different approach: as in the Rose and Svensson (1991) paper, zero or even negative expected rates of devaluation are

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<sup>8</sup>See Weber (1991b) for more detailed empirical evidence on this issue.

<sup>9</sup>Note that the time-varying parameter regression method proposed above is especially designed to deal with this type of stochastic process switching between stationary and non-stationary random movements.

found for the second half of the EMS period, and expected devaluation rates are generally more variable than the relatively stable interest rate differentials. Moreover, the inclusion of time-varying devaluation risks largely reduces both the size and the degree of serial correlation in the components of interest rate differentials which are left unexplained by standard fully credible target zone models, as is indicated by the plots of the residuals in the lower left quadrants of Figures 1a and 1b. An additional finding of the present paper is that the parity deviation response coefficients, shown in the upper left quadrants of Figure 1a and 1b, have the postulated negative signs throughout the sample period and are significantly different from zero for the post-1982 period. Comparing the absolute values of the parity response coefficients in Figures 1a and 1b also reveals that they are larger at the shorter end of the maturity range, as is postulated in the theoretical model of Svensson (1991b). Movements in the parity deviation response coefficients further appear to be correlated with the exchange rate, as they exhibit permanent jumps at each of the devaluation dates. The increasing negative correlation between the interest rate differential of a given maturity and the band position of the exchange rate may thereby be taken as an indication of an increased credibility of the DM/FF exchange rate target zone. This statement is justified on the basis of the simulation results in Bertola and Svensson (1991), who show that reduced devaluation risk implies *ceteris paribus* steeper slope coefficients. A last important finding is that the probability of transitory parameter variation, as given in the lower left quadrant of Figures 1a and 1b, has frequently taken non-zero values in the pre-1983 EMS period, and that this feature has vanished recently. This also suggests that the DM/FF target zone has recently become more credible due to the absence of speculative attacks, which are viewed as a large rise in the expected rate of depreciation and which typically result in a jump of the probability of transitory parameter variation.

The results for Italy, Belgium, Denmark and Ireland in Figures 2a,b to 6a,b are close in spirit to those of France. In each case significant negative slope estimates are obtained for both maturities in the post-1983 EMS period, and again the estimated slope coefficients are typically larger (in absolute terms) at the shorter end of the maturity range. The only major difference to the case of France is found for the Netherlands in Figures 3a and 3b, where the parity deviations of the exchange rate have no significant impact on the twelve month interest rate differential, but display a marginally significant positive correlation with the one month interest rate differential. Such positive correlations may arise in the Bertola and Svensson (1991) model if the variance of expected devaluation rates relative to that of the fundamentals is high. Since both the level and the variance of expected DM/Hfl devaluation rates are the lowest amongst all EMS countries, this points towards extremely tightly controlled fundamentals as being the major explanation for the positive slope estimates. Note that this explanation is consistent with the bipolar view of the EMS put forward in Weber (1991a), where Germany and the Netherlands are viewed as a 'hard currency' bloc or small 'DM-zone' within the EMS.

#### **4.2. Expected Realignments and Actual Realignments**

As mentioned above, stochastic process switching in expected devaluation rates was found to have played an important role, at least in the early EMS period. It was mentioned that large transitory movements of expected devaluation rates typically characterize a speculative attack, and result in upward jumps of the probability of the model of transitory parameter variation, whilst this probability is close to zero in periods of tranquility. This distinction between transitory and permanent components of expected devaluation rates may now be used to derive a rough measure of the degree to which a realignment is expected by market participants prior to its occurrence. Boxes 2a and 2b display the expected



devaluation rates and their transitory components, which are interpreted here as a measure realignment expectations under a speculative attack.

In the case of France the results indicate that realignment expectations under speculative attacks account for only a small proportion of expected devaluation rates (except perhaps in October 1981, June 1982 and March 1983), whilst for Italy most of the pre-1983 devaluation expectations prior to official parity changes may be explained by speculation about an impending realignment. Similar conclusions emerge for Belgium and Denmark in the first half of the EMS period. However, in the case of Ireland and the Netherlands only a very small proportion of devaluation expectations are accounted for by speculative attacks, suggesting that realignments of these currencies relative to the German mark were mainly unanticipated by the market.

#### **4.3. Expected Devaluation Rates and the Empirical Fit of Target Zone Models**

A final point to be considered in this paper regards the 'value-added' of augmented compared to standard target zone models. The above empirical estimates, and the results derived in Rose and Svensson (1991) and Svensson (1991d), suggest that allowing for time-varying devaluation risk in target zone models is an essential and non-trivial extension of standard target zone models if the predictions of the theory are to be confronted with real world data from the EMS. The advantage of the Bertola and Svensson (1991) model over standard target zone models may be demonstrated by referring to Figures 8 to 12, which compare scatterplots of the standard (left) and the devaluation expectations augmented (right) relationships between the interest rate differential and the exchange rate's band position for deposits of one year (top) and one month (bottom) maturity respectively. Scatterplots similar to those on the left hand side of Figures 7 to 12 are examined in Bertola and Caballero (1989a), Flood, Rose and Mathieson (1990) and Bartolini and Bodnar (1991) for the EMS in search of a stable downward sloping, non-linear

(s-shaped) relationship. Needless to say, most authors find no clear patterns in the data, as Flood, Rose and Mathieson (1990) point out. Augmented target zone models of the Bertola and Svensson (1991) type, which predict such a negatively sloped non-linear relationship only between devaluation expectations adjusted interest rate differentials (actual differentials minus the expected rates of devaluation) and the exchange rate's band position, on the other hand appear to fit the data very well, as is suggested by the scatterplots on the left-hand side of Figures 7 to 12.

Three key features of the scatterplots deserve further discussion: first, the EMS data appear to strongly support the conjecture of Svensson (1991b,d) that the negative correlation between interest rate differentials and the exchange rate's band positions may be approximately linear for typical parameter values of the underlying target zone model. The similarity between the predictions of the theory (see upper right inset in Figure 4) and the actual relationships is most striking in the German-French case in Figure 7, which instead of a single negatively sloped line displays several such negatively sloped lines, with steeper slopes emerging in the later EMS regimes. The increasing negative correlation between interest rate differentials and the exchange rate's band position for a given maturity may thereby best be explained by the increased credibility (=lower devaluation risk) of the DM/FF target zone: Bertola and Svensson (1991) show that lower devaluation risk increases the slope and reduces the dispersion of the correlation between interest rate differentials and the exchange rate's band position, and both these features of the correlations are apparent from the left-hand side of Figure 7. Second, the strongest indication of any non-linearities in the correlations between interest rate differentials and the parity deviations of exchange rates are shown in Figure 9 for the German-Dutch case. This apparent non-linearity has exactly the type of slope postulated by the theory (see upper small inset of Graph 4). The non-linearity may further explain the failure of the linearized estimates for the

DM/Hfl target zone in picking up a significant negative correlation between the augmented interest rate differential and the exchange rate's band position in Figures 3a and 3b. Third, whilst positive correlations between augmented interest rate differentials and exchange rates are the exception, they nevertheless do exist in the German–Dutch and German–Belgian cases in Figures 9 and 10. However, Figures 3a and 3b, as well as Figures 4a and 4b, suggest that linear estimates of such positive correlations are generally not significantly different from zero.

Summarizing, one may state that the evidence presented in Figures 7 to 12 provides strong empirical support for augmented target zone models, as put forward by Bertola and Svensson (1991). Adjusting for devaluation expectations extracts a low-noise inverse relationship between (augmented) interest rate differentials and exchange rate parity deviations from generally very noisy and diffuse correlations in the original data. In most cases, these correlations found for the augmented target zone model are fairly linear. This result is consistent with the finding of Rose and Svensson (1991) that for the DM/FF target zone the non-linearities picked up by the inclusion of quadratic and cubic terms are at best marginally significant. However, the results of the present paper also indicate that in the case of the Netherlands a non-linear relationship is pointed out by the data. The type of non-linearity found in the data exactly resembles the curvature suggested by the theory if the DM/Hfl exchange rate is weak in the band, but is almost linear if the DM/Hfl rate is strong in the band. Taken at face value, this finding supports the conjecture of Bartolini and Bodnar (1991) that real world target zones may possess asymmetrical credibility.

## 5. Conclusions and Suggestions for Further Research

Augmented target zone models incorporating time-varying devaluation risks have been shown by Bertola and Svensson (1991) to be an important and non-trivial extension of the standard target zone model, since they can account for many of the empirical regularities found in real world target zones. The present paper puts these models to the test and finds that they match the EMS data quite well: once expected devaluation rates are taken into account by, for example, subtracting them from the interest rate differentials, the relationship between the remaining components of these interest rate differentials and exchange rates are consistent with the predictions of standard (fully credible) target zone models. This striking finding is unlikely to be the result of a special feature of the time-varying parameter regression estimation procedure adopted in the present paper, since Rose and Svensson (1991) and Svensson (1991d), who by using the same data but a vastly different empirical approach, nevertheless derive very similar estimates of expected realignment rates. The authors should therefore also detect similar, almost deterministic inverse correlations between their devaluation expectations augmented interest rate differentials and the exchange rate.

With respect to future empirical work, two fields of promising research should be mentioned: first, the time-varying parameter regression model proposed in the present paper may easily be extended to include non-linear elements in order to estimate target zone models more adequately. Following Rose and Svensson (1991) and including the exchange rate in a linear, squared and cubic fashion in the time-varying parameter regression is only one possibility. More complex non-linear estimation procedures based on the extended Kalman filter may also be applied fruitfully, as suggested in Weber (1991b). Second, the implications of target zone models for the relationship between expected future exchange rates (forward rates) and actual exchange rates (spot rates) may be evaluated in a fashion similar to the one discussed in the present paper.

Appendix

**Svensson's Analytical Solution for the Interest Rate Differential in a Target Zone**

As shown in Svensson (1991b) the interest rate differential in an exchange rate target zone may be derived as:

$$r(f; \tau) \equiv i(f, t; \tau) - i^*(t; \tau) = \frac{E[e(f(\tau)) | f(0) = f] - e(f)}{\tau}, \quad \tau > 0, \quad (A1)$$

Determining (A1) requires computing the expected exchange rate at maturity:

$$h(f; \tau) \equiv E[e(f(\tau)) | f(0) = f]. \quad (A2)$$

which is a complicated non-linear heteroscedastic stochastic process with variable drift and instantaneous standard deviation. Svensson (1991b) shows that the function  $h(f, \tau)$  defined in (A2) is the solution to the partial differential equation

$$h_t(f; \tau) = \mu h_f(f; \tau) + \frac{\sigma^2}{2} h_{ff}(f; \tau), \quad \underline{f} \leq f \leq \bar{f}, \quad t \geq 0, \quad (A3a)$$

with initial condition

$$h_t(f; 0) = e(f), \quad \underline{f} \leq f \leq \bar{f}, \quad (A3b)$$

and derivative boundary or smooth pasting conditions

$$h_f(\underline{f}; \tau) = 0 \text{ and } h_f(\bar{f}; \tau) = 0, \quad \tau \geq 0. \quad (A3c)$$

Using  $a = (\bar{f} - \underline{f})/\pi$ ,  $x = (f - \underline{f})/a$  and  $\theta = 2\mu/\sigma^2$  the analytical Fourier-series solution to (A3) can be written as:

$$h_t(f; \tau) = \sum_{n=0}^{\infty} c_n y_n(f) \exp(-\lambda_n \tau), \quad \underline{f} \leq f \leq \bar{f}, \quad \tau \geq 0, \quad (A4a)$$

with  $y_0(f) = 1,$  (A4b)

$$y_n(f) = \exp[-\theta(f - \underline{f})/2] \{2n \cos[n(f - \underline{f})/a] + \theta a \sin[n(f - \underline{f})/a]\}, \quad n \geq 1, \quad (A4c)$$

$$c_0 = \int_{\underline{f}}^{\bar{f}} e(f) df \frac{1}{\bar{f} - \underline{f}}, \quad \text{for } \mu = 0, \quad (A4d)$$

$$c_0 = \int_{\underline{f}}^{\bar{f}} \exp(\theta f) e(f) df \frac{\theta}{\exp(\theta \bar{f}) - \exp(\theta \underline{f})}, \quad \text{for } \mu \neq 0, \quad (A4e)$$

$$c_n = \int_{\underline{f}}^{\bar{f}} \exp[\theta(f - \underline{f})] e(f) y_n(f) df \frac{1}{4(\bar{f} - \underline{f})\lambda_n a^2 / \sigma^2}, \quad n \geq 1, \quad (A4f)$$

$$\lambda_0 = 0, \quad (A4g)$$

$$\lambda_n = (n^2/a^2 + \theta^2/4)\sigma^2/2 > 0, \quad n \geq 1. \quad (A4h)$$

Note that the expected maturity exchange rate  $h(f;\tau)$  in equation (A4a) involves the summation of infinitely many terms, and in practise this analytical solution has to be computed numerically as a summation of a truncated series (in Graphs 2 to 4 the expected maturity exchange rate  $h(f;\tau)$  was truncated at  $n=9$ ).

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Tab. 1: EMS Realignments, ECU Weight Revisions and Major Council Decisions

1979	Mar. 13	exchange rate mechanism (ERM) starts to operate; initial currency weights in ECU currency basket: DM 32.0%, FF 19.0%, UKL 15.0%, Lit 10.2%, Hfl 10.1%, BF 8.5%, Dkr 2.7%, Dra 1.3%, IrL 1.2%
	Sep. 24	1. realignment (DM +2%, Dkr -2.9%)
	Nov. 30	2. realignment (Dkr -4.8%)
1981	Mar. 22	3. realignment (Lit -6%)
	Oct. 5	4. realignment (DM +5.5%, FF -3%, Lit -3%, Hfl +3.5%)
1982	Feb. 22	5. realignment (BF -8.5%, Dkr -3%)
	June 14	6. realignment (DM +4.25%, FF -5.75%, Lit -2.75%, Hfl +4.25%)
1983	Mar. 21	7. realignment (DM +5.5%, FF -2.5%, Lit -2-5%, Hfl +3.5%, BF +1.5%, Dkr +2.5%, IrL -2.5%)
1984	Sep. 17	revision of currency weights in ECU currency basket (DM 32.0%, FF 19.0%, UKL 15.0%, Lit 10.2%, Hfl 10.1%, BF 8.5%, Dkr 2.7%, Dra 1.3%, IrL 1.2%)
1985	July 22	8. realignment (DM +2%, FF +2%, LIT -6%, Hfl +2%, BL +2%, Dkr +2%, IRL +2%)
1986	Apr. 7	9. realignment (DM +3%, FF -3%, Hfl +3%, Bf +1%, Dkr +1%)
	Aug. 4	10. realignment (IRL -8%)
1987	Jan.12	11. realignment (DM +3%, Hfl +3%, Bf +2%, Dkr +2%)
	Sep. 12	Basle-Nyborg Agreement of the Committee of Central Bank Governors to strengthen the ERM; measures include a wider use of fluctuation bands
1989	June 19	Spain enters the exchange rate mechanism of the EMS with a wide fluctuation margin of $\pm 6\%$
	Sep. 21	revision of currency weights in ECU currency basket (DM 30.1%, FF 19.0%, UKL 13.0%, LIT 10.15%, Hfl 9.4%, Bf 7.9%, PES 5.3%, Dkr 2.45%, IRL 1.1%, DRA 0.8%, ESC 0.8%)
1990	Jan. 8	12. realignment (LIT - 3.7%), narrowing of band to $\pm 2.25\%$
	Oct. 8	United Kingdom enters the exchange rate mechanism of the EMS with a wide fluctuation margin of $\pm 6\%$

Sources: OECD *Economic Surveys: Germany, France, Italy, Netherlands, Belgium and Luxembourg, Denmark, Ireland*, various issues, Commission of the European Communities *The EMS: Ten Years of Progress in European Monetary Co-operation*, and Ungerer *et al.* (1986).

Notes: At realignments + (-) indicates a revaluation (devaluation) in % against those currencies whose bilateral parities remained unchanged, except for the two general realignments (March 1983, July 1985), for which the percentages from the official communiqué are shown.

**Table 2a: Augmented Dickey-Fuller Unit-Root Test for Twelve Month German Interest Rate Differentials, Daily Data, Five Lags of Differenced Data and Constant Included.**

	France	Italy	Holland	Belgium	Denmark	Ireland
EMS	/+++	/+++	***/+	/+++	/+++	/+++
Regime 1	/+++	/+++	/+++	/+++	/+++	na/na
Regime 2	/+++	***/+	/+++	/+	/+++	na/na
Regime 3	/+++	/+++	/+++	/+++	/+++	na/na
Regime 4	/+++	/+++	/+++	/+++	/+++	na/na
Regime 5	/+++	***/+	/+++	***/+	/+++	/+++
Regime 6	/+++	/+++	/+++	/+++	/+++	/+++
Regime 7	/+++	/+++	/+++	/+++	/+++	/+++
Regime 8	**/+	/+++	*/+	***/+	***/+	/+++
Regime 9	/+++	/++	/+++	/+++	/+++	/+++
Regime 10	**/+	/+++	/+++	*/+	/+++	/++
Regime 11	/+++	/+++	/+++	*/+	**/+	/+++
Regime 12	/+++	/+++	/+++	/+++	/+++	/+++
Regime 13	/+++	/+++	/+++	/+++	/++	/++

Key: The signs \*\*\*/+ indicate the significance of the Augmented Dickey-Fuller test for the rejection of a unit root in the levels/first differences of the interest rate differentials relative to Germany at the ten (\*/+), five (\*\*/++) and one (\*\*\*/+ ) percent levels.

**Table 2b: Augmented Dickey-Fuller Unit-Root Test for One Month German Interest Rate Differentials, Daily Data, Five Lags of Differenced Data and Constant Included.**

	France	Italy	Holland	Belgium	Denmark	Ireland
EMS	***/+	***/+	***/+	***/+	***/+	***/+
Regime 1	/+++	/+++	*/+	/+++	na/na	na/na
Regime 2	/++	/++	/+++	/++	na/na	na/na
Regime 3	/+++	*/+	/+++	/+++	na/na	na/na
Regime 4	/+++	/+++	/+++	**/+	na/na	na/na
Regime 5	**/+	***/+	/+++	*/+	/+++	/+++
Regime 6	/+	/+++	/+++	/+	/+	*/+
Regime 7	/+++	/+++	/+++	/+++	*/+	/+++
Regime 8	***/+	***/+	***/+	**/+	***/+	***/+
Regime 9	*/+	/+++	/+++	/+++	/+++	/+++
Regime 10	/+++	**/+	/++	*/+	/++	***/+
Regime 11	/+++	/+++	/+++	**/+	***/+	/+++
Regime 12	/+++	***/+	/+++	/+++	/+++	/+++
Regime 13	/+++	/+++	/+++	/+++	/+++	/+++

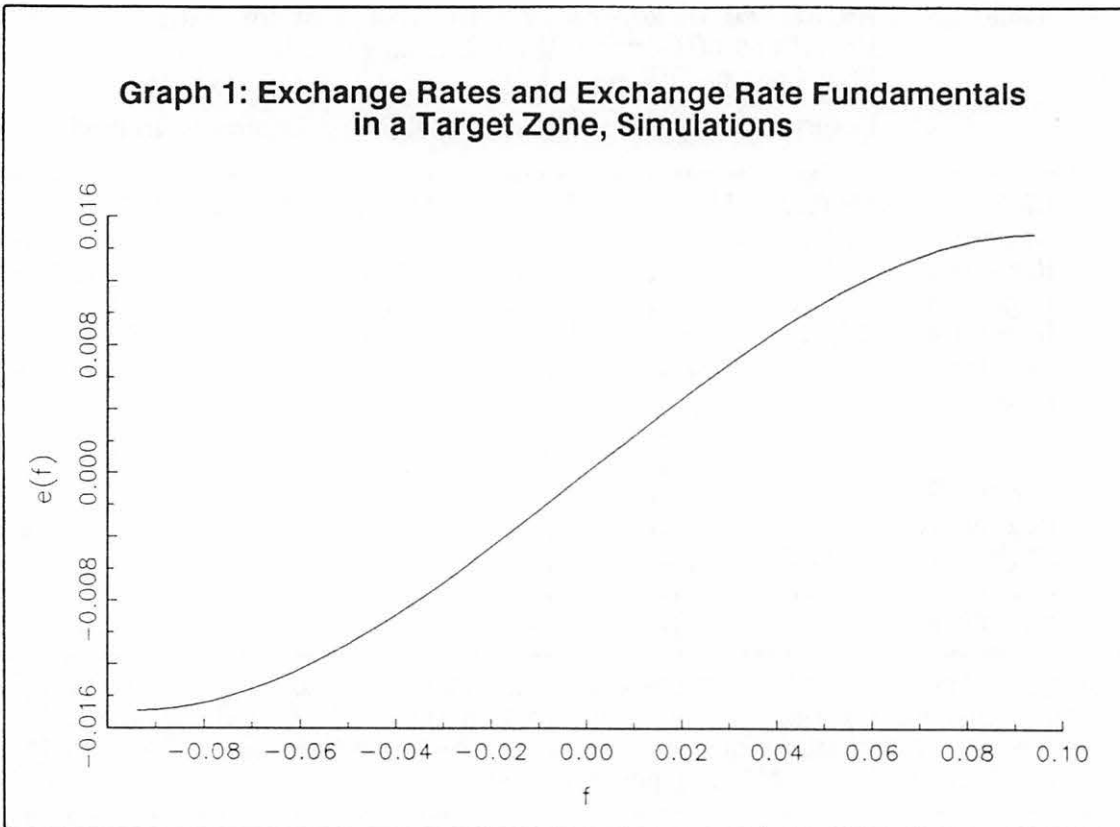
Key: The signs \*\*\*/+ indicate the significance of the Augmented Dickey-Fuller test for the rejection of a unit root in the levels/first differences of the interest rate differentials relative to Germany at the ten (\*/+), five (\*\*/++) and one (\*\*\*/+ ) percent levels.

Table 2c: Augmented Dickey-Fuller Unit-Root Test for Parity Deviations of German Mark Exchange Rates, Daily Data, Five Lags of Differenced Data and Constant Included,

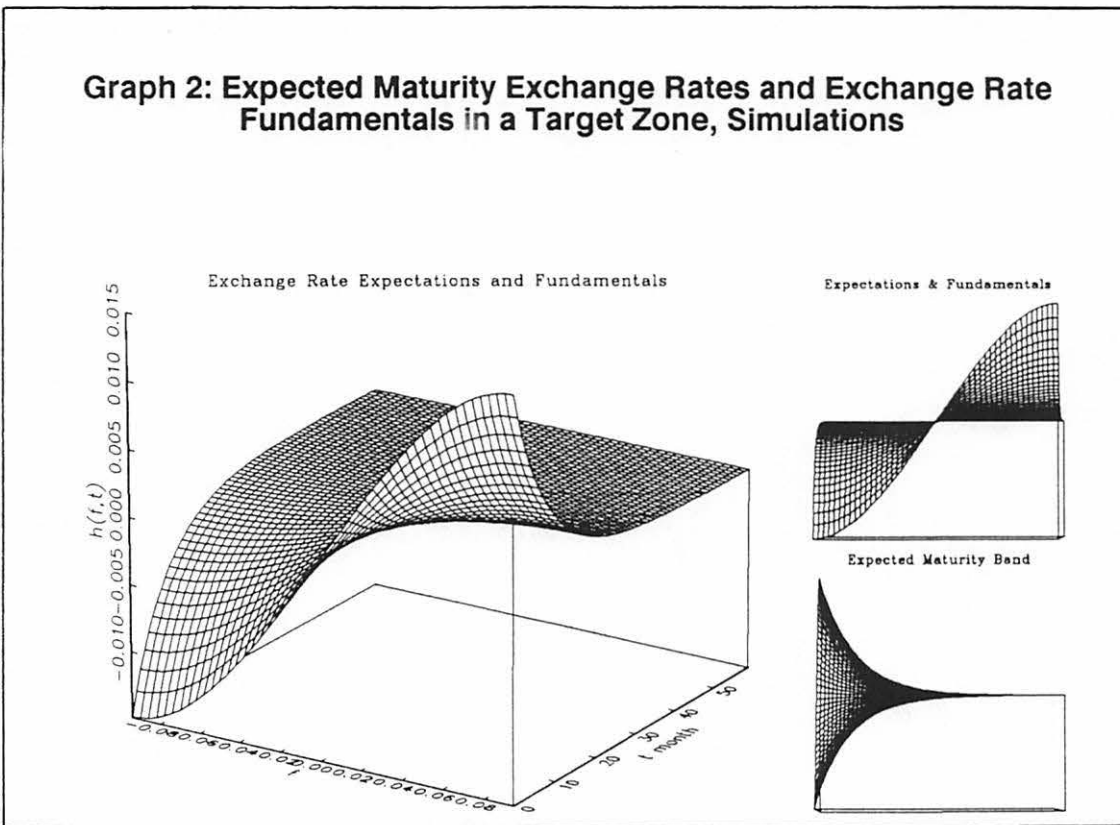
	France	Italy	Holland	Belgium	Denmark	Ireland
EMS	***/+	***/+	***/+	***/+	***/+	***/+
Regime 1	/+++	/+++	/+++	/+++	/+	/++
Regime 2	/+++	/+++	/+++	/+++	/+++	/+++
Regime 3	/+++	/+++	/+++	/+++	/+++	/+++
Regime 4	*/+	/+++	**/+	/+++	/+++	**/+
Regime 5	/+++	/+++	/+++	/+++	/+++	/+++
Regime 6						
Regime 7	/+++	/+++	/+++	/+++	/+++	/+++
Regime 8	/+++	/+++	/+++	/+++	/+++	/+++
Regime 9		/+++	/+++		/+++	
Regime 10	/+	/+++	/++	/+++	/++	/++
Regime 11	**/+	/+++	/++			/++
Regime 12	/+++	/+++	/+++	/+++	/+++	/+++
Regime 13	/+	/+++	/+++	/+++	/++	/+++

Key: The signs \*\*\*/+ indicate the significance of the Augmented Dickey-Fuller test for the rejection of a unit root in the levels/first differences of the parity deviations of exchange rates *vis-a-vis* Germany at the ten (\*/+), five (\*\*/++) and one (\*\*\*/+) percent levels.

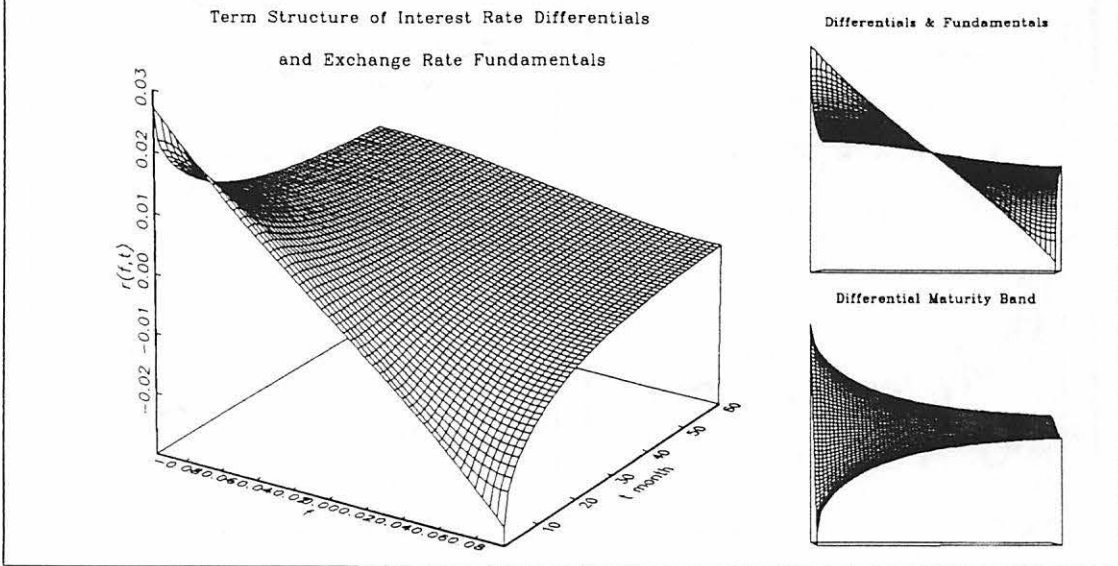
**Graph 1: Exchange Rates and Exchange Rate Fundamentals in a Target Zone, Simulations**



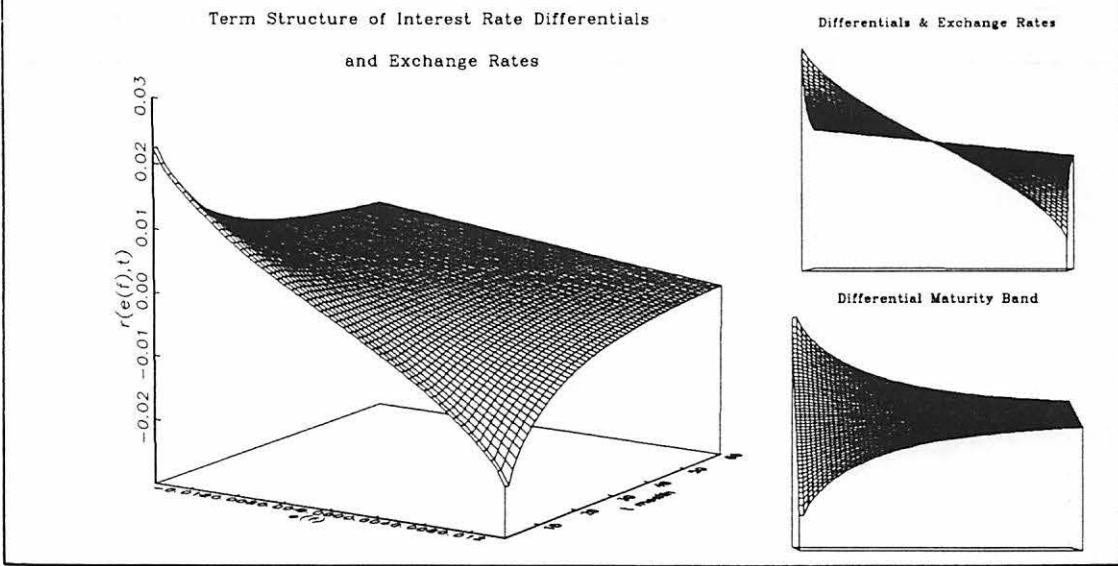
**Graph 2: Expected Maturity Exchange Rates and Exchange Rate Fundamentals in a Target Zone, Simulations**



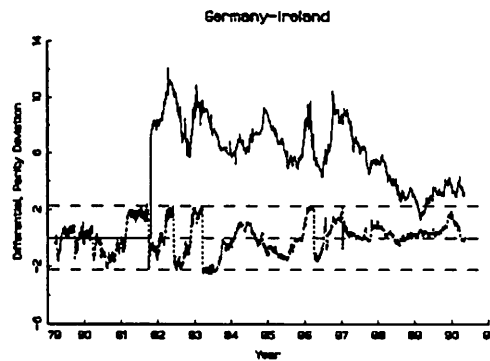
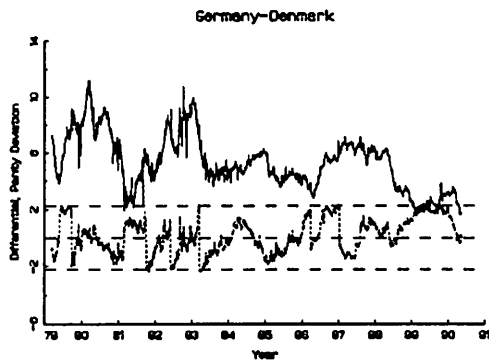
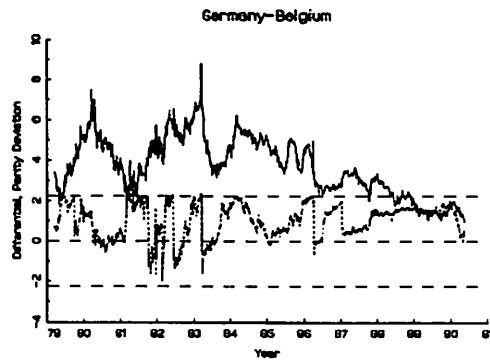
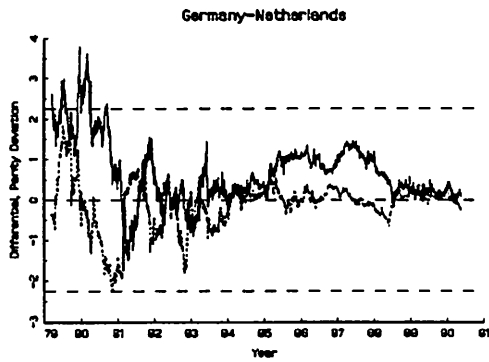
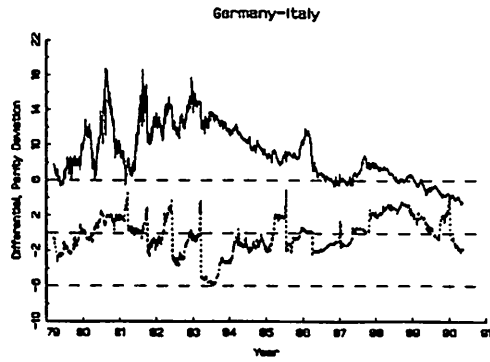
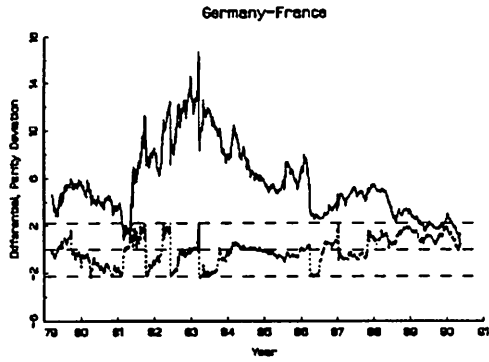
**Graph 3: Term Structure of Interest Rate Differentials and Exchange Rate Fundamentals in a Target Zone, Simulations**



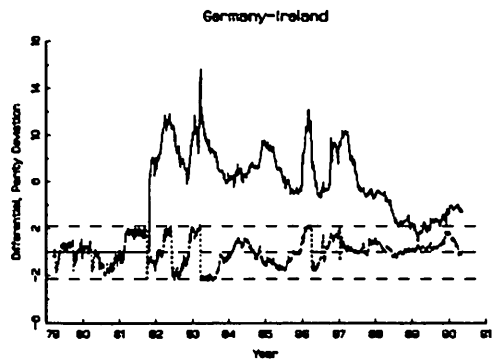
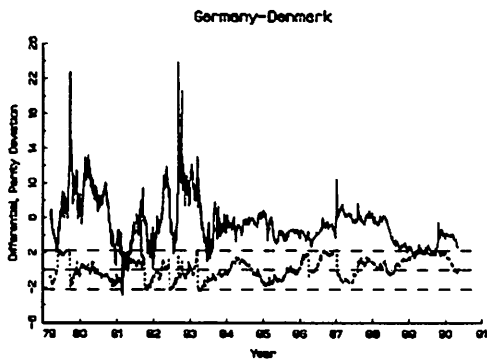
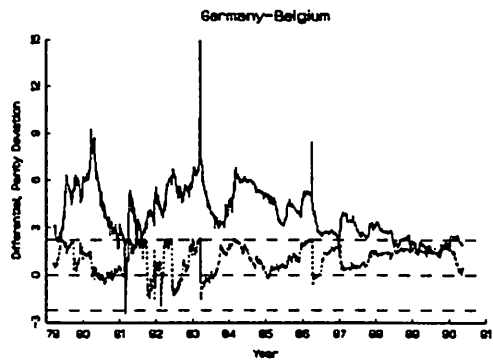
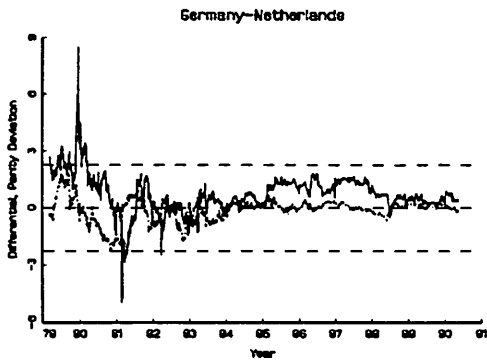
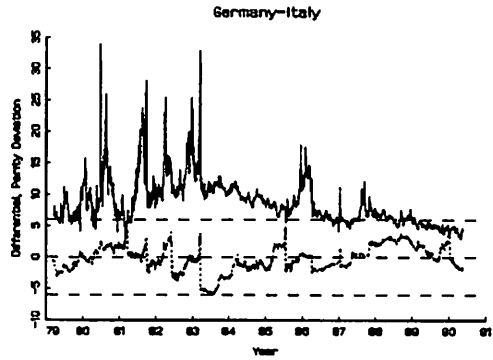
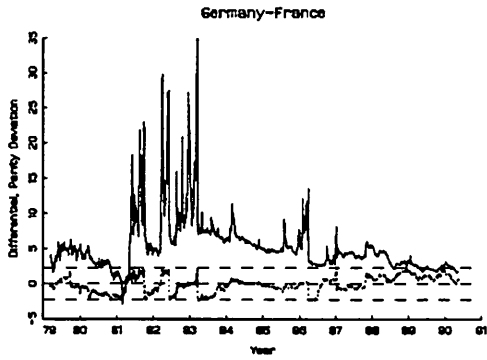
**Graph 4: Term Structure of Interest Rate Differentials and Actual Exchange Rates in a Target Zone, Simulations**



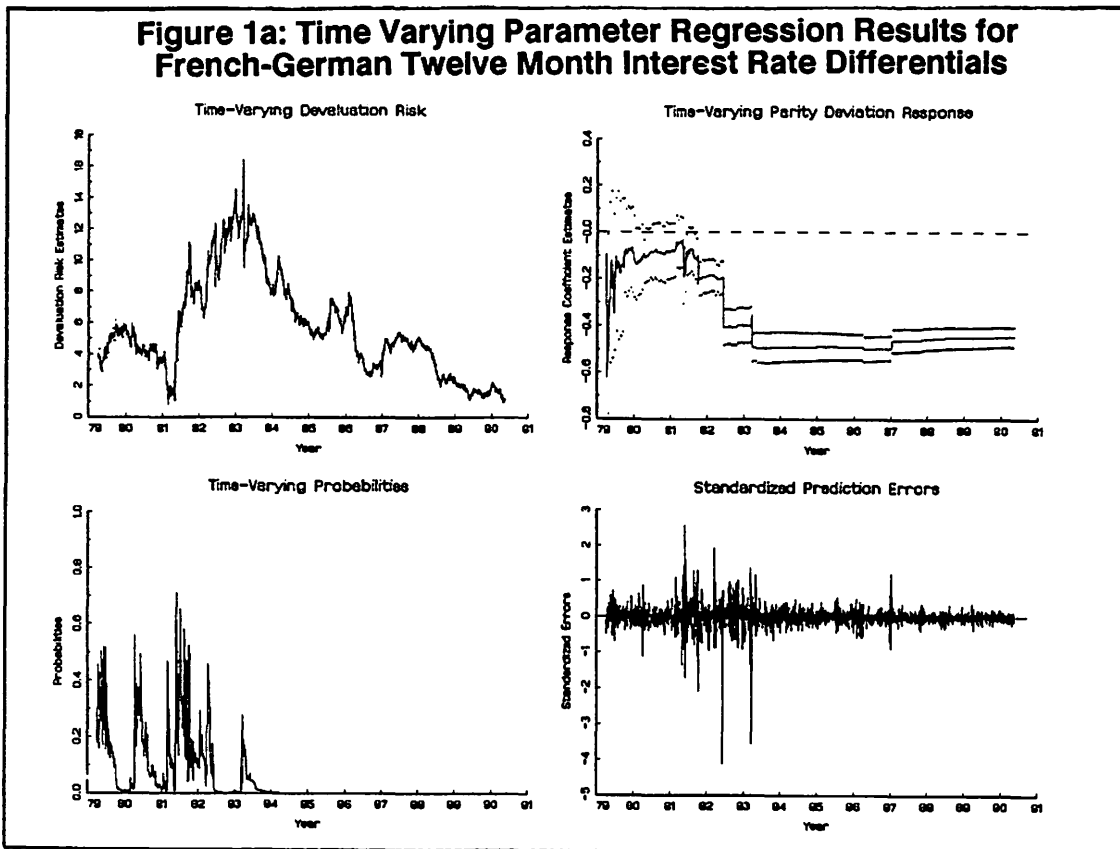
### Box 1a: EMS Parity Deviations of German Mark Exchange Rates and Corresponding Twelve Month Interest Rate Differentials



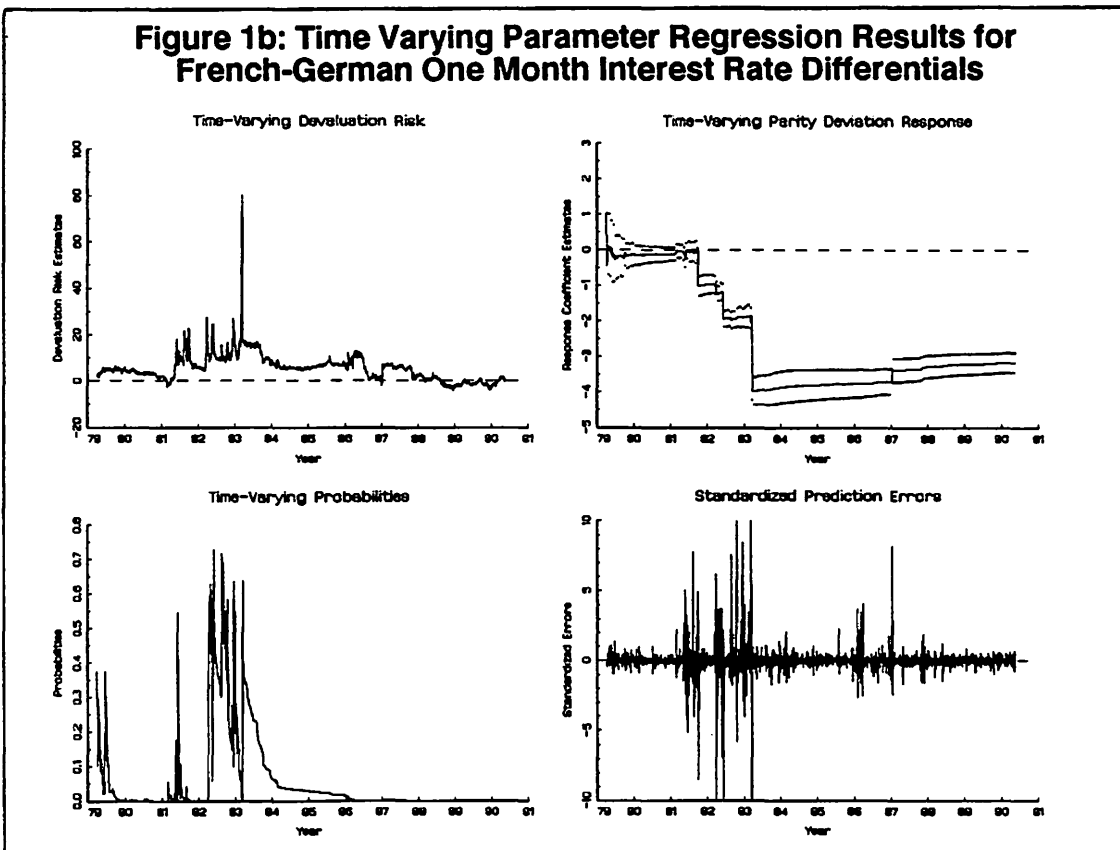
### Box 1b: EMS Parity Deviations of German Mark Exchange Rates and Corresponding One Month Interest Rate Differentials



**Figure 1a: Time Varying Parameter Regression Results for French-German Twelve Month Interest Rate Differentials**

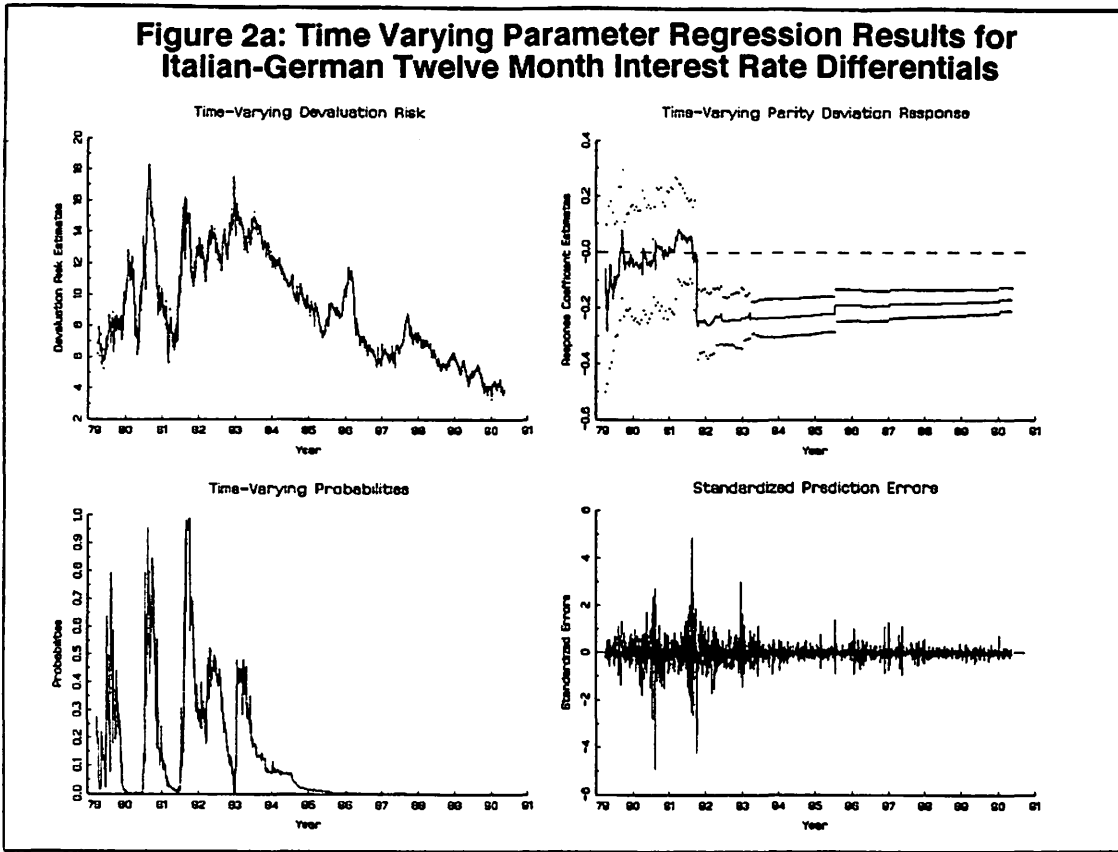


**Figure 1b: Time Varying Parameter Regression Results for French-German One Month Interest Rate Differentials**

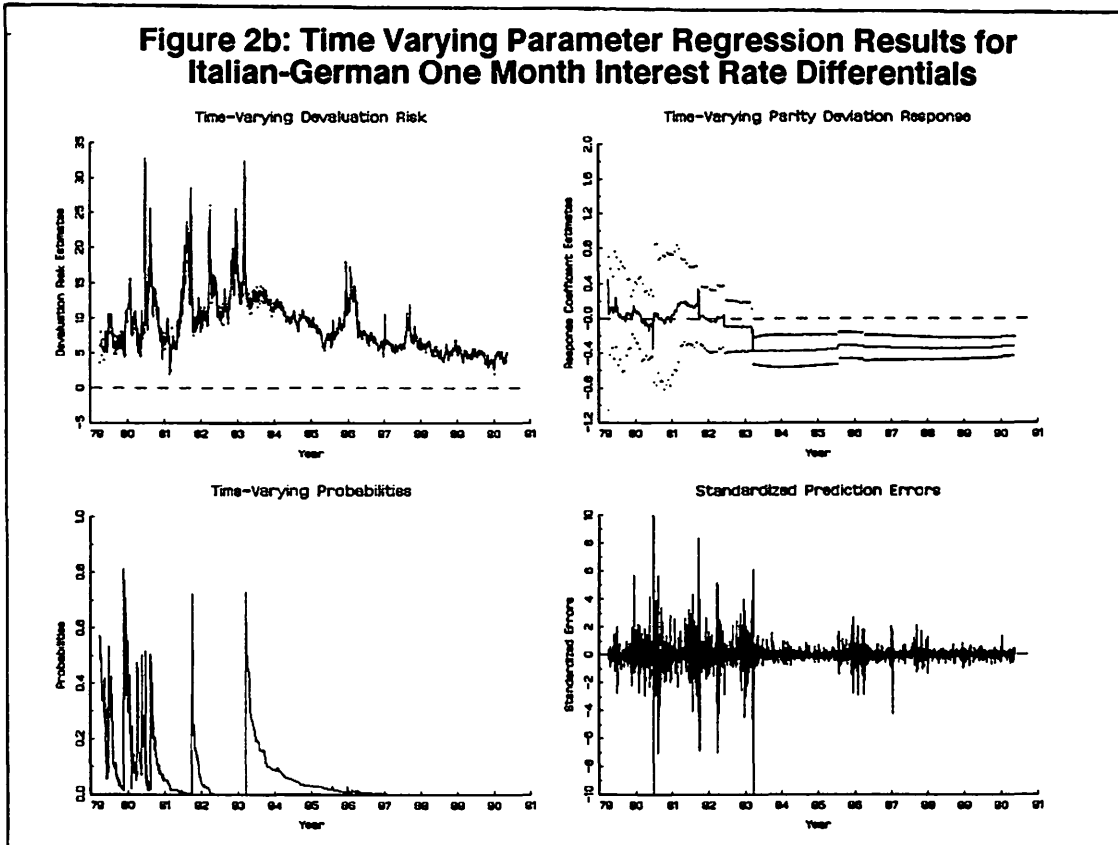




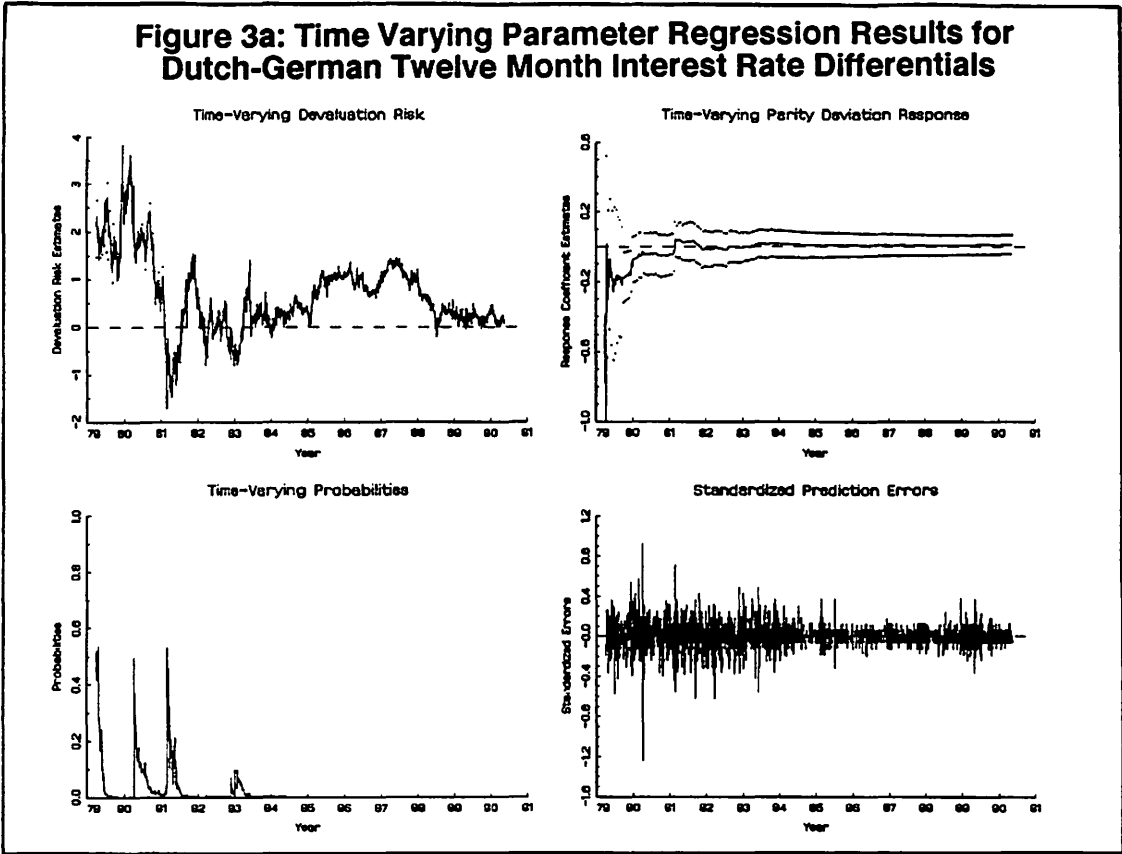
**Figure 2a: Time Varying Parameter Regression Results for Italian-German Twelve Month Interest Rate Differentials**



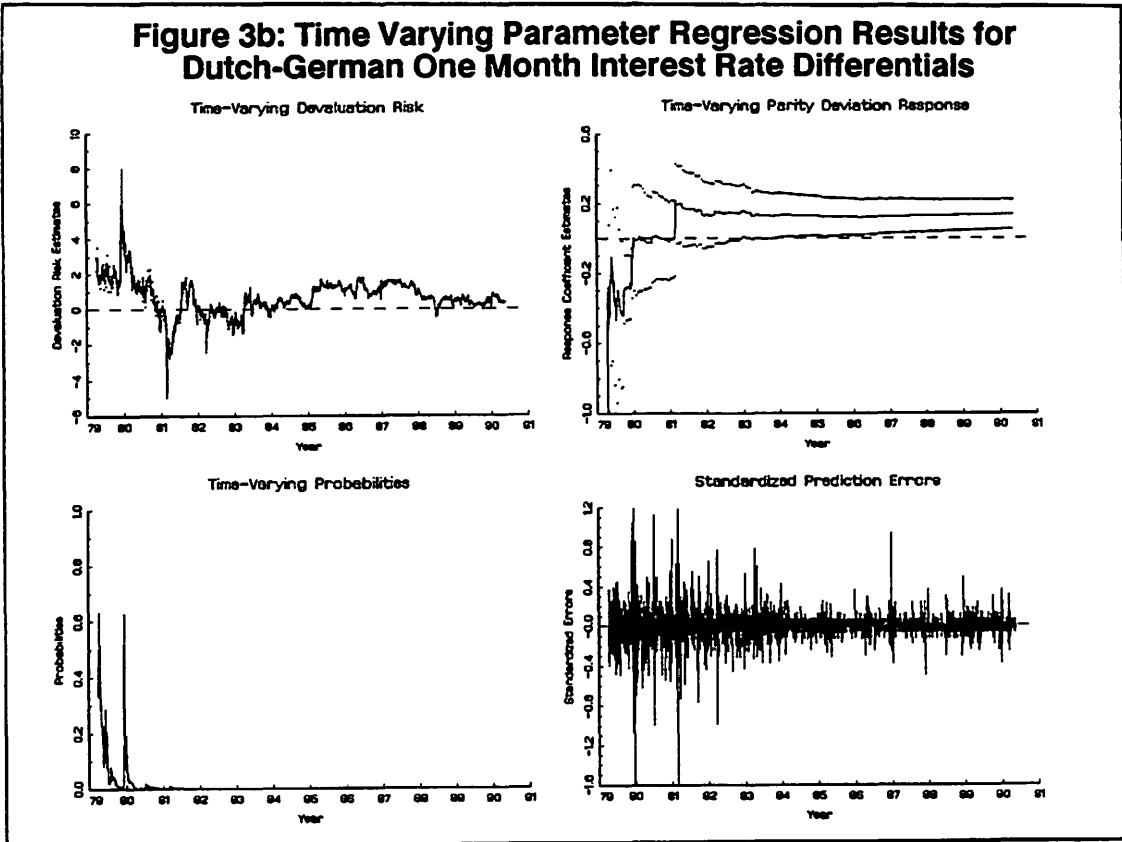
**Figure 2b: Time Varying Parameter Regression Results for Italian-German One Month Interest Rate Differentials**



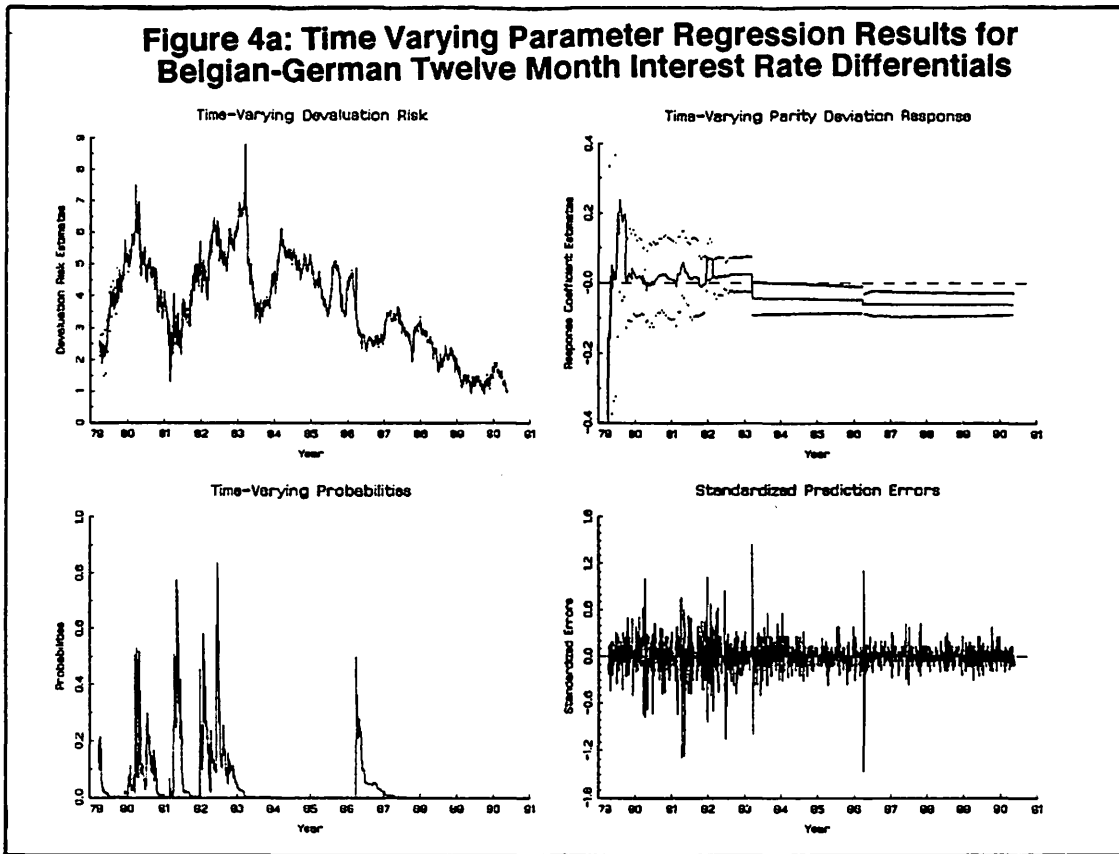
**Figure 3a: Time Varying Parameter Regression Results for Dutch-German Twelve Month Interest Rate Differentials**



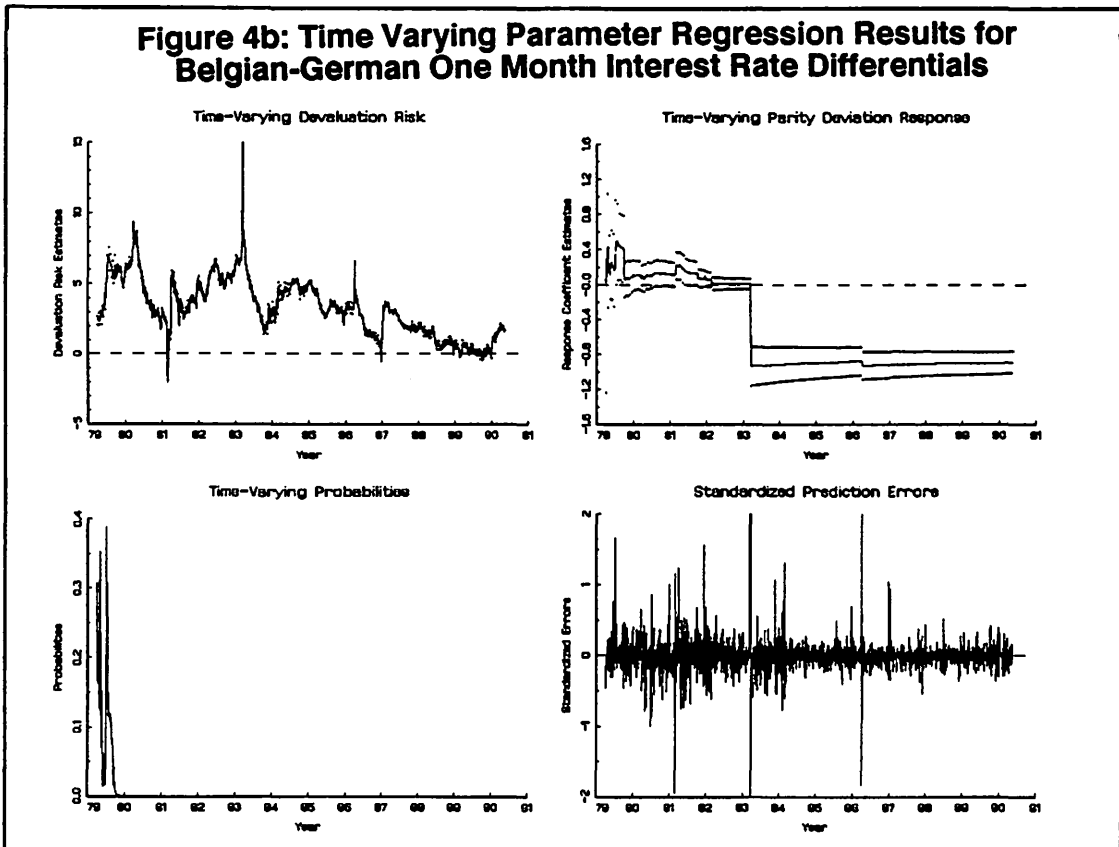
**Figure 3b: Time Varying Parameter Regression Results for Dutch-German One Month Interest Rate Differentials**



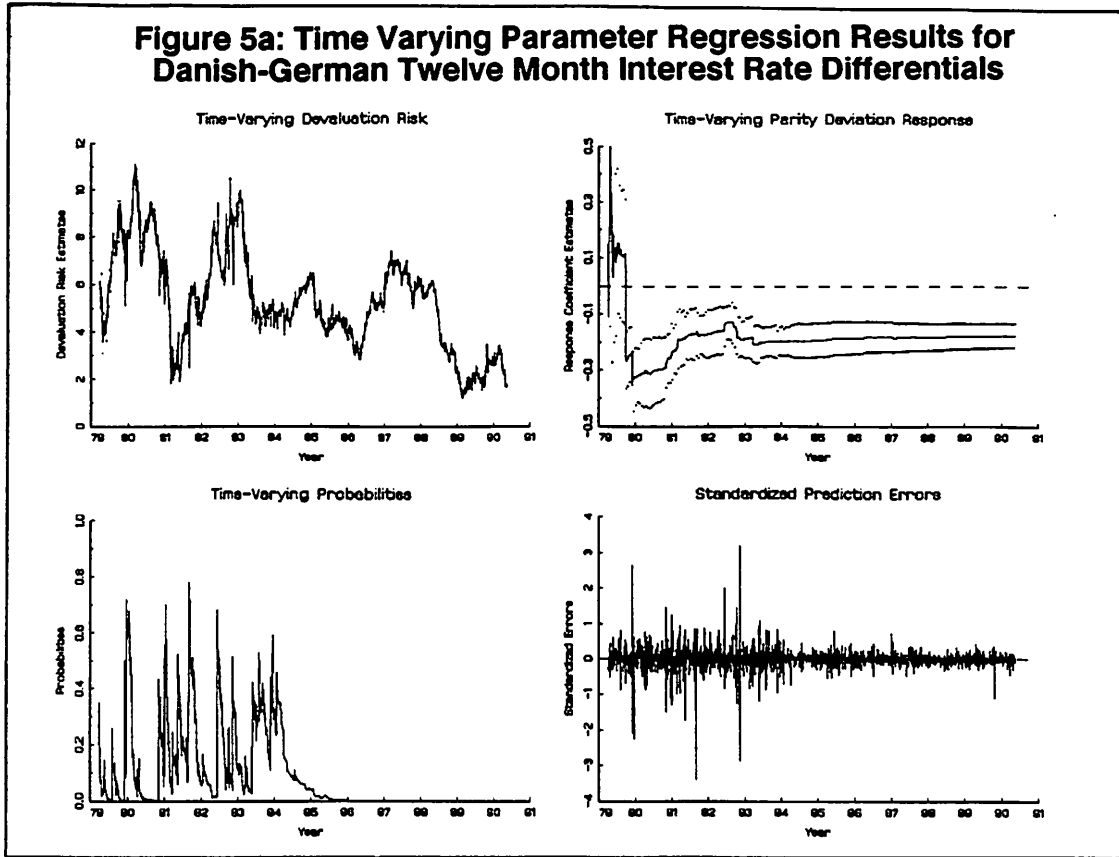
**Figure 4a: Time Varying Parameter Regression Results for Belgian-German Twelve Month Interest Rate Differentials**



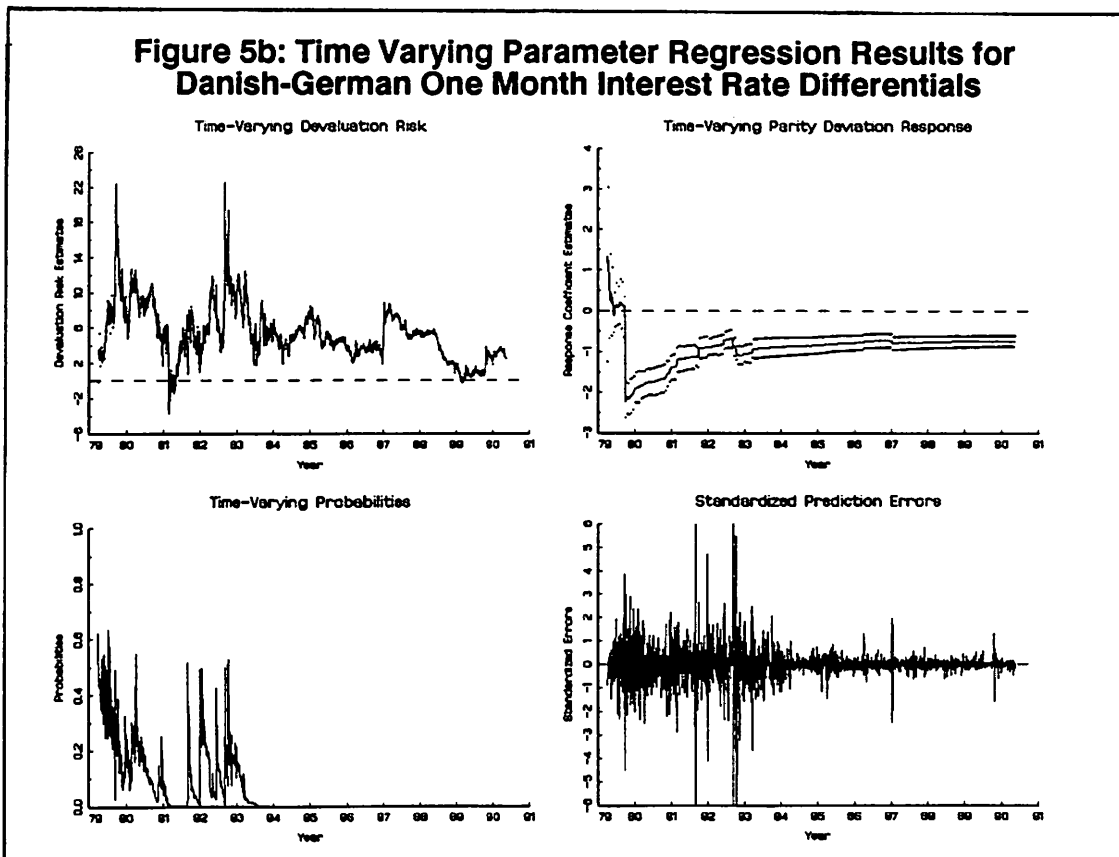
**Figure 4b: Time Varying Parameter Regression Results for Belgian-German One Month Interest Rate Differentials**



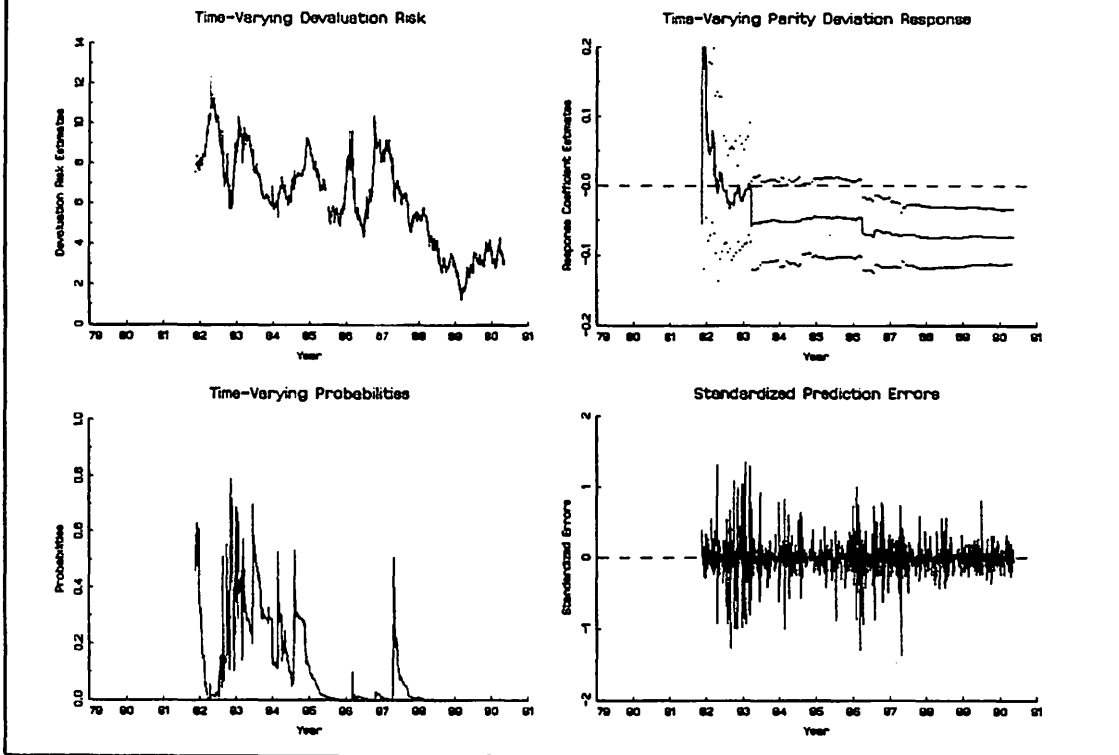
**Figure 5a: Time Varying Parameter Regression Results for Danish-German Twelve Month Interest Rate Differentials**



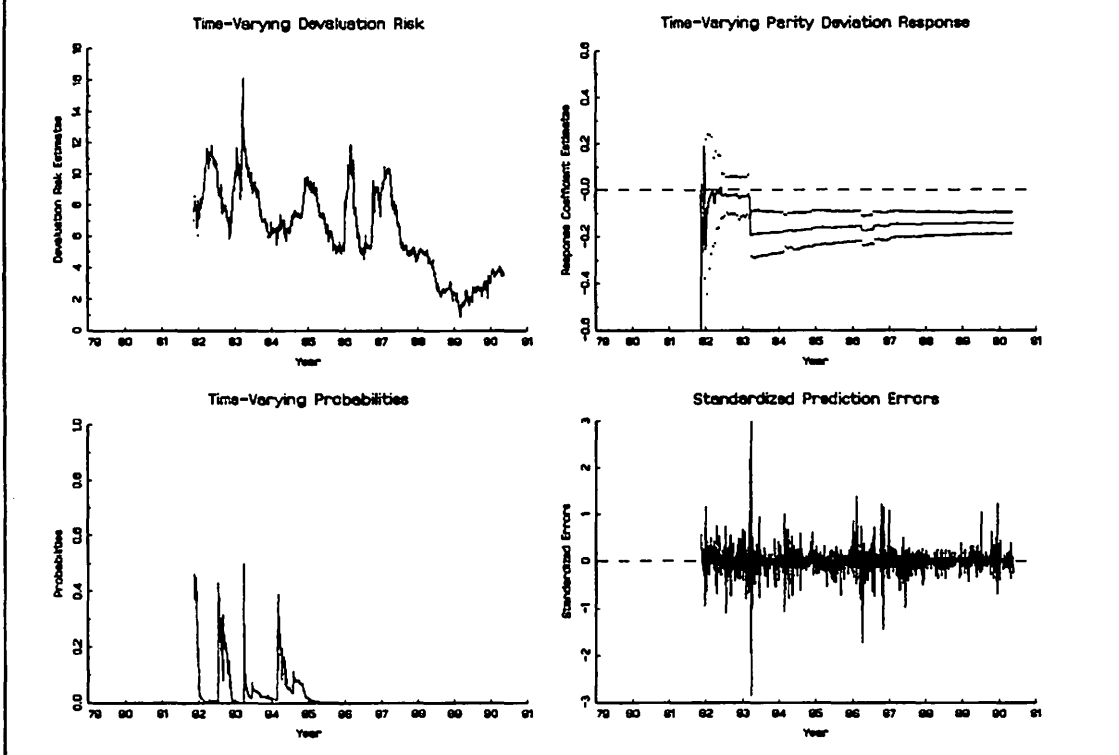
**Figure 5b: Time Varying Parameter Regression Results for Danish-German One Month Interest Rate Differentials**



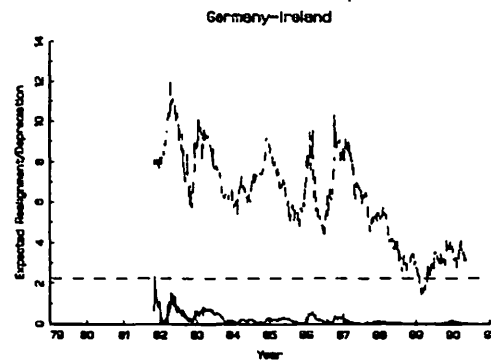
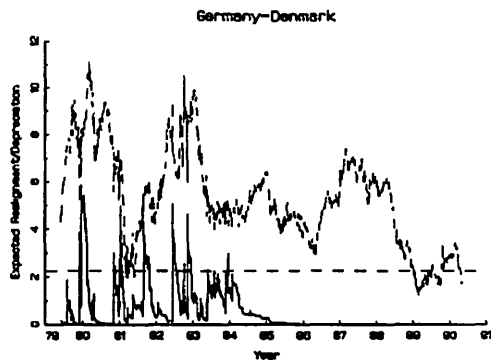
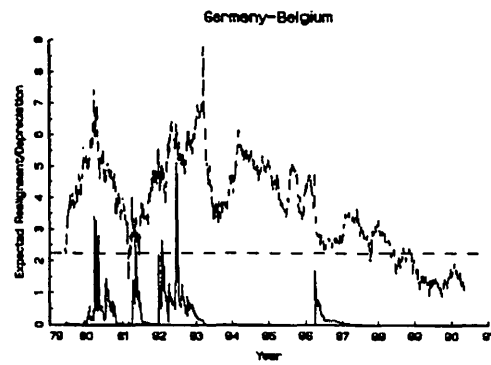
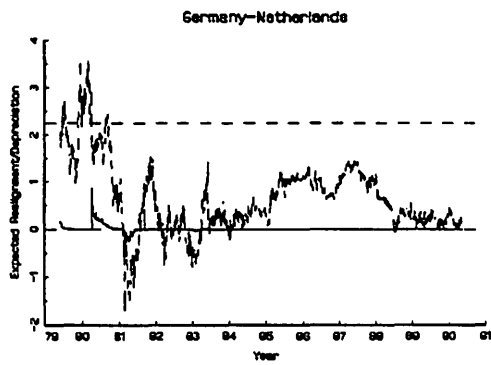
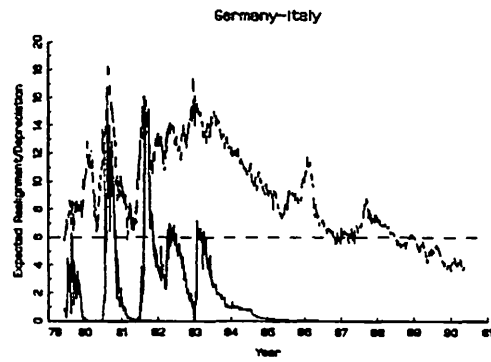
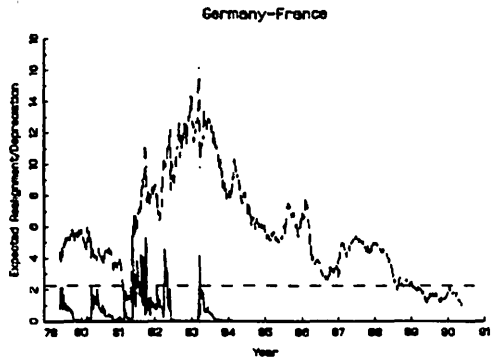
**Figure 6a: Time Varying Parameter Regression Results for Irish-German Twelve Month Interest Rate Differentials**



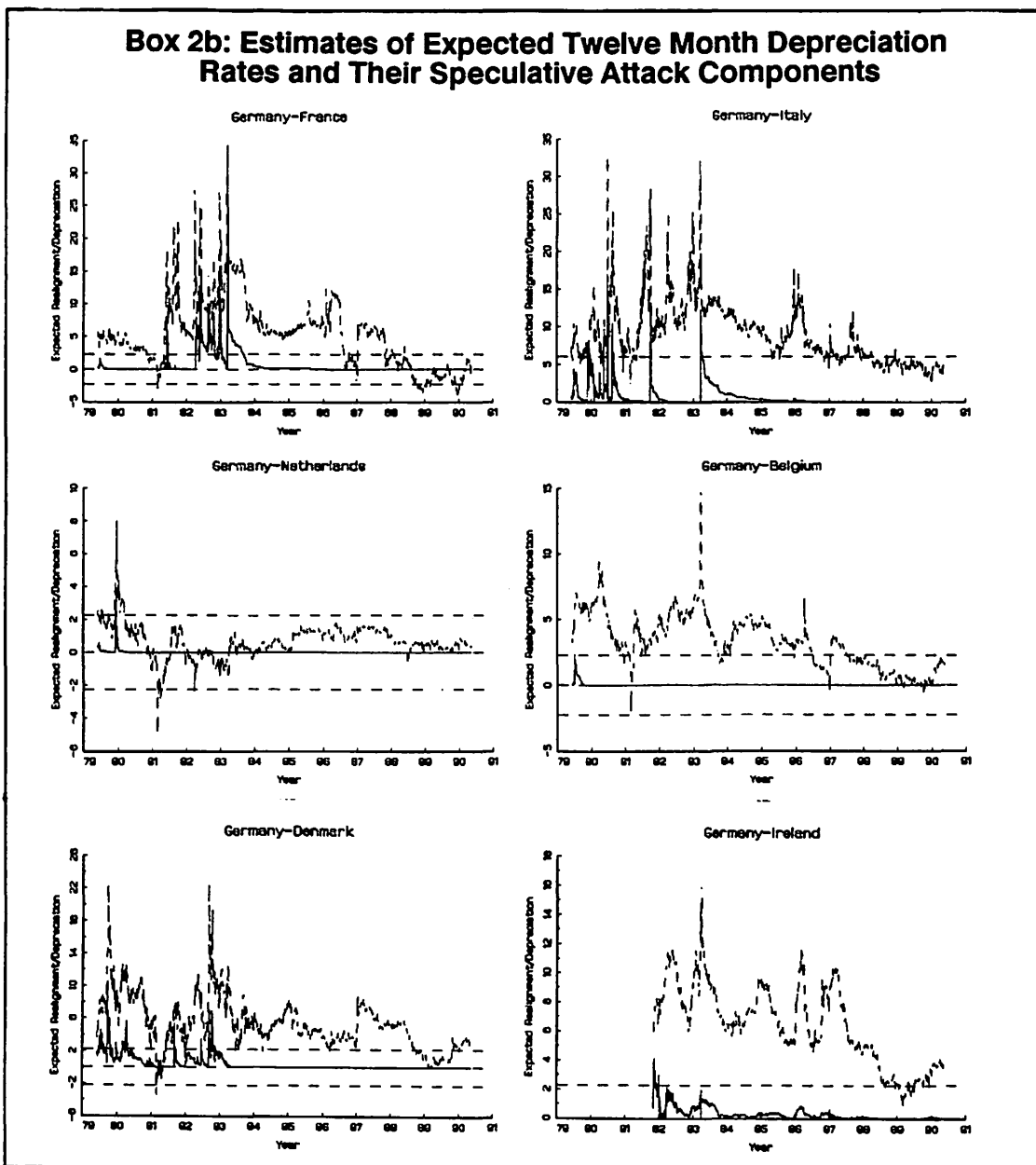
**Figure 6b: Time Varying Parameter Regression Results for Irish-German One Month Interest Rate Differentials**



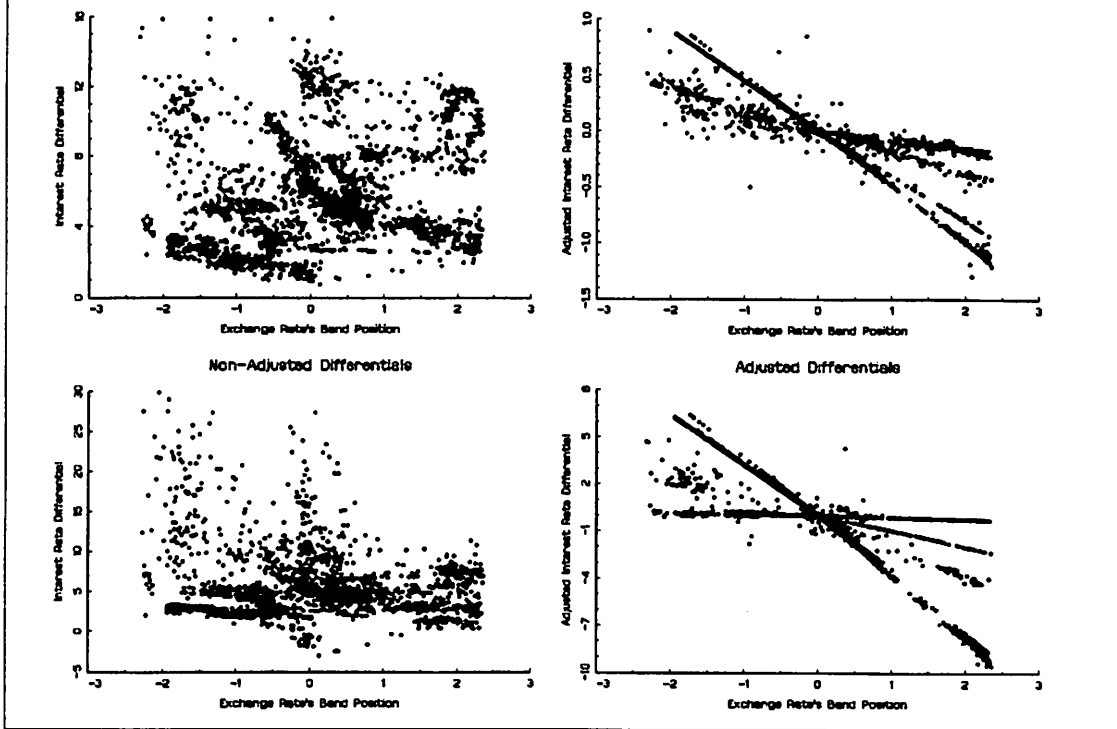
### Box 2a: Estimates of Expected One Month Depreciation Rates and Their Speculative Attack Components



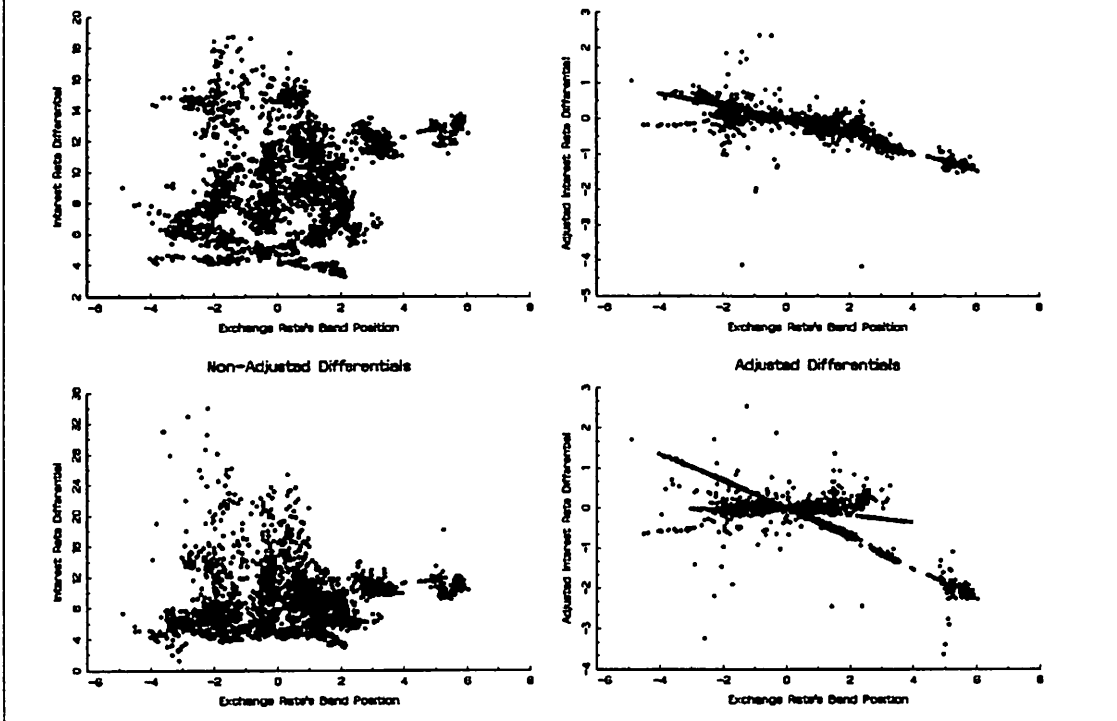
### Box 2b: Estimates of Expected Twelve Month Depreciation Rates and Their Speculative Attack Components



**Figure 7: Standard versus Augmented Target Zone Models, French-German Twelve (Top) and One (Bottom) Month Differentials**

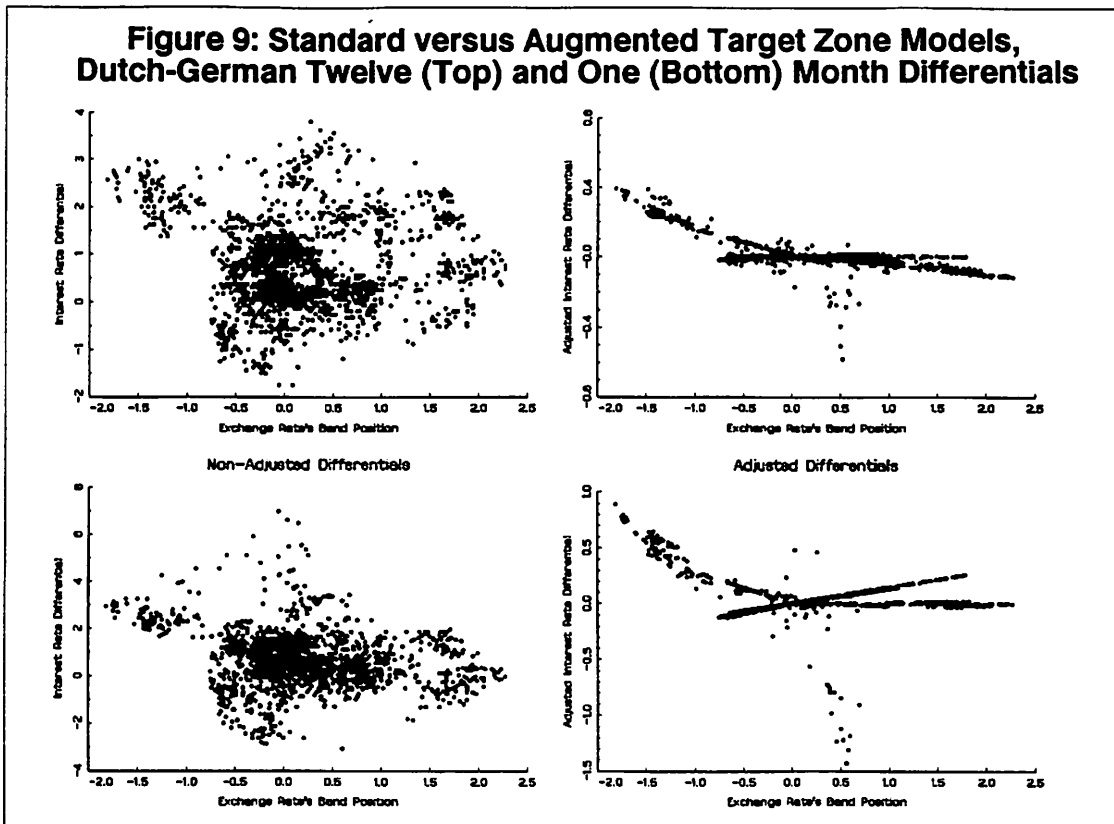


**Figure 8: Standard versus Augmented Target Zone Models, Italian-German Twelve (Top) and One (Bottom) Month Differentials**

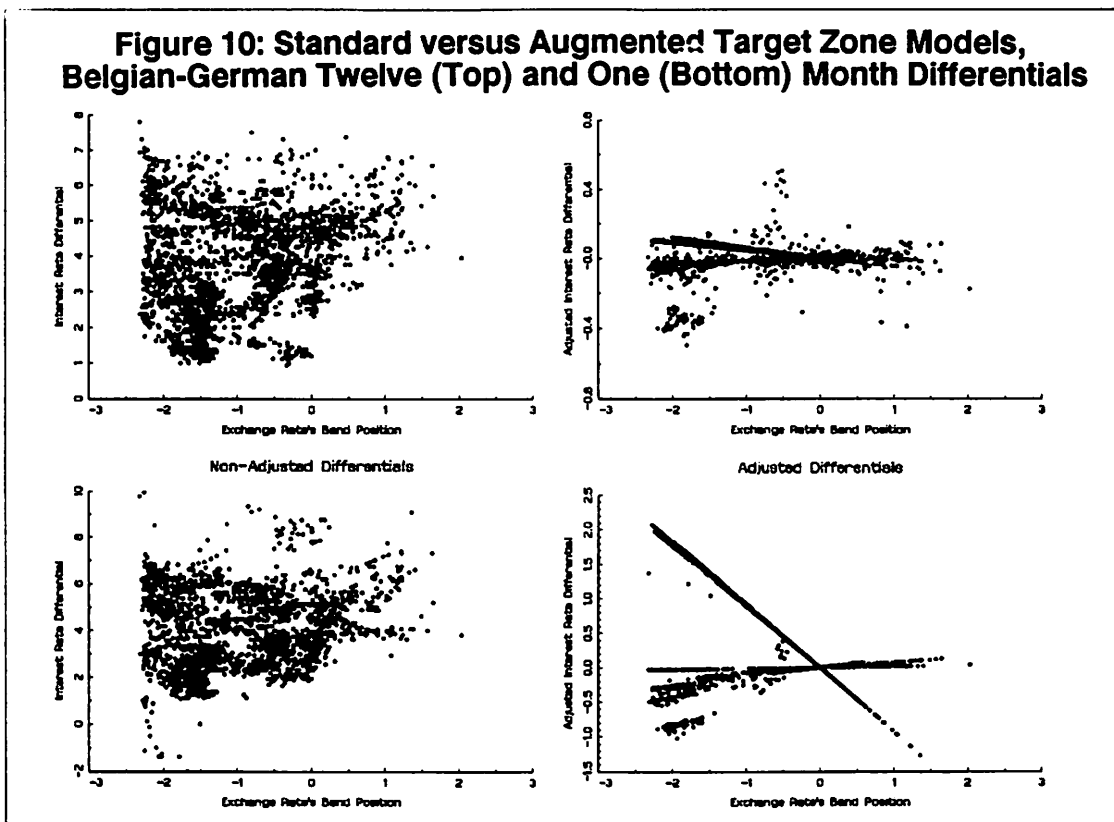




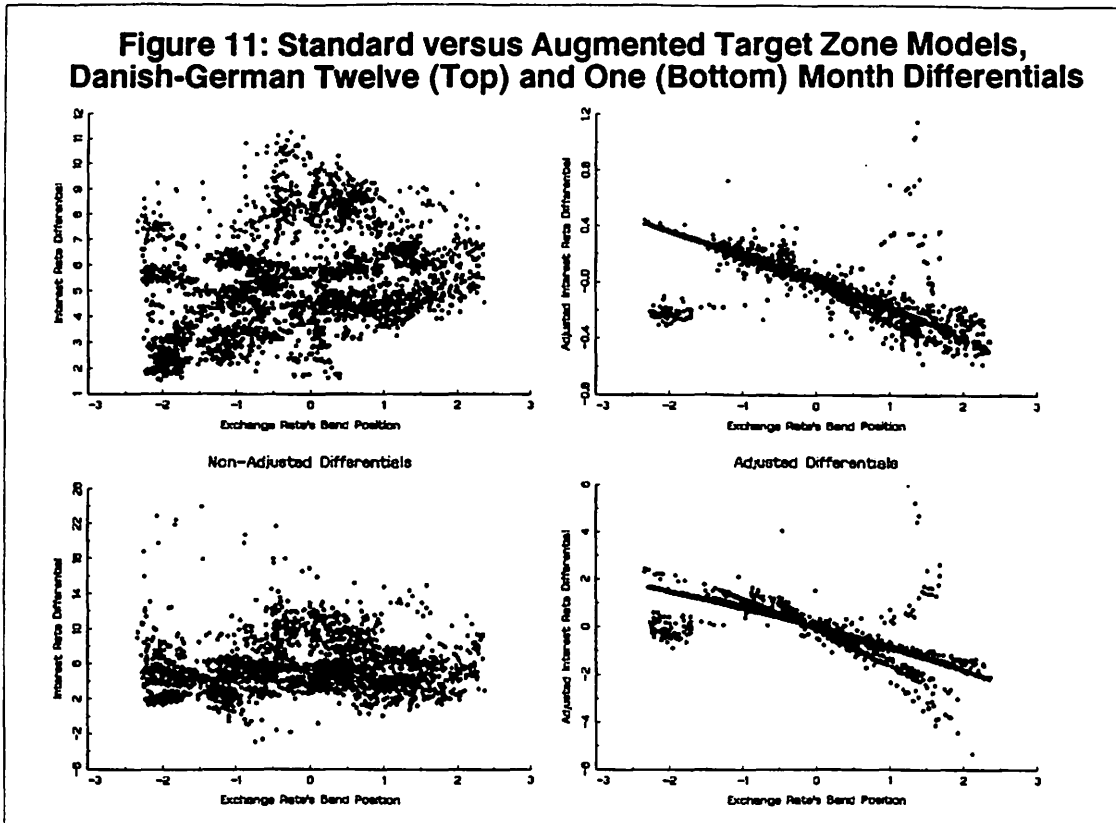
**Figure 9: Standard versus Augmented Target Zone Models, Dutch-German Twelve (Top) and One (Bottom) Month Differentials**



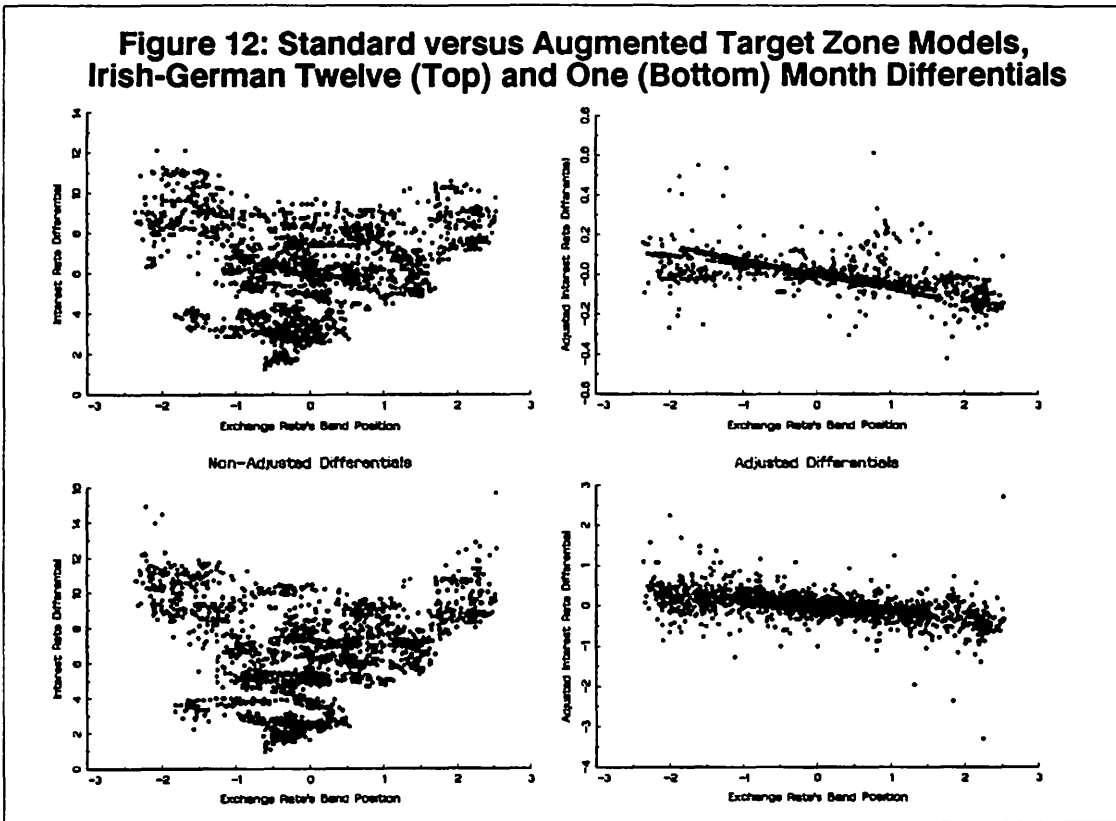
**Figure 10: Standard versus Augmented Target Zone Models, Belgian-German Twelve (Top) and One (Bottom) Month Differentials**



**Figure 11: Standard versus Augmented Target Zone Models, Danish-German Twelve (Top) and One (Bottom) Month Differentials**



**Figure 12: Standard versus Augmented Target Zone Models, Irish-German Twelve (Top) and One (Bottom) Month Differentials**



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