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## Efficient Pricing of Drinking Water

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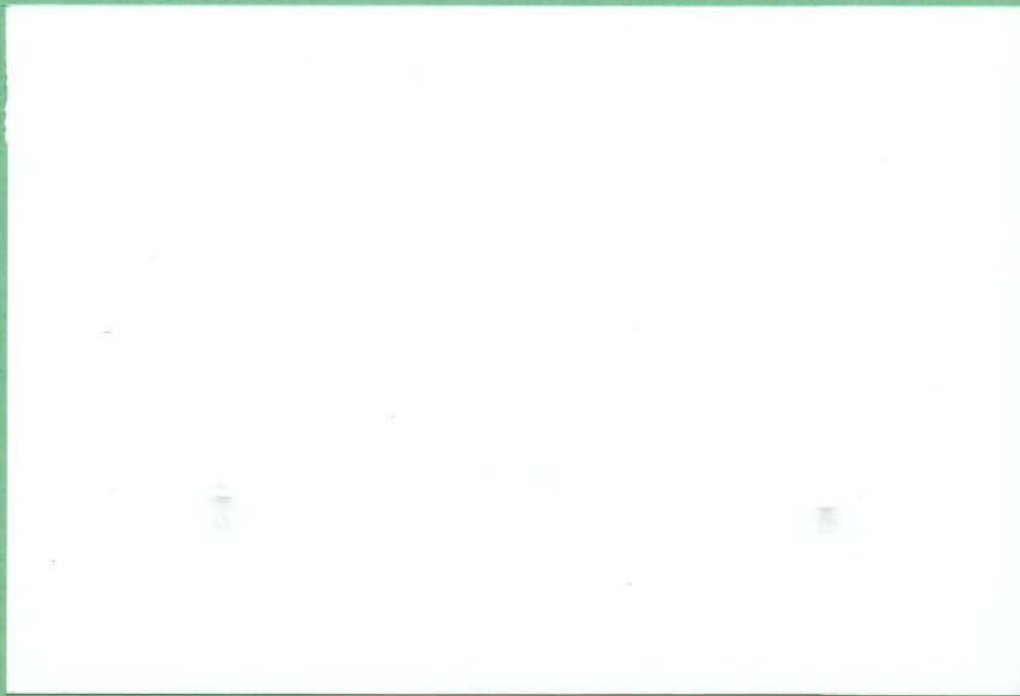
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# EFFICIENT PRICING OF DRINKING WATER

Rüdiger Pethig and Klaus Fiedler

Discussion Paper No. 18 – 91

## ABSTRACT:

Generation of drinking water from polluted raw water requires <sup>to reduce</sup> water pollution by overall emissions control, by technical raw water purification and/or by installing water protection areas. To achieve efficiency the purification effort must be equalised at all of these three margins. In a total equilibrium model the efficient price of drinking water is shown to exceed the marginal operating cost of water works by a mark-up for raw water quality and by another mark-up for the economic rent of raw water. Conditions are elaborated under which raw water does or does not carry a rent.

# EFFICIENT PRICING OF DRINKING WATER

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## 1. The problem

In many countries it is a growing concern that drinking water has to be generated from polluted raw water even when this raw water is taken from water protection areas. The conflict over raw water quality arises, of course, because the water works incur increasing costs for cleaning polluted raw water while the polluters from industry and/or agriculture (with its heavy use of herbicides, pesticides and manure) would have to bear additional costs if they were forced to reduce water pollution. Suppose water pollution outside of water protection areas is so severe that raw water used for the generation of drinking water must exclusively be taken from these areas. Then their size not only defines an upper bound for drinking water generation, but also influences the raw water quality because the larger is the body of water in that area the higher is its quality resulting from any given pollution spillover.

In this paper we are not concerned with arid zones where water is scarce irrespective of its quality. We rather think of European countries like Germany in which water is not scarce if its quantity is considered independently of its quality. Does this mean that efficiency requires the price of drinking water to equal its marginal production costs in the water works? Or is it true that high quality raw water from water protection areas is a scarce resource in the sense that it carries an economic rent (Brösse 1980, Pfaffenberger and Scheele 1990)? If raw water should turn out to be scarce, efficiency would require to set the

price of drinking water above marginal production cost. In the present paper we aim at answering that question.

Conceptually, high quality drinking water can be achieved by raising the activity level at any of the following three margins: overall emission reduction, intensification of technical raw water purification, and extension of water protection areas. The first of these margins is to reduce raw water pollution by decreasing the flow of pollutants altogether thus reducing spillovers into the raw water resource; secondly, the purification effort in the water works can be increased in order to compensate for decreasing raw water quality; the third margin is to reduce the pollution of raw water by enlarging the water protection area thus diminishing the effects of pollution spillovers from outside the protected area.

These three activities form the process of raw water purification in a broader sense. Efficiency of this process requires to equalise the purification effort at all three margins. Placed in that perspective, the demonstration of raw water scarcity requires to show that in an efficient allocation the size of water protection areas is a binding constraint for the supply of drinking water. This issue cannot be studied in a partial equilibrium approach because it involves economic interdependence of demand and supply of all factors and commodities in an essential way. This is particularly evident from the last of the three margins which establishes a linkage between the supply of raw water quality and quantity; the demand of raw water depends, in turn, on relative prices and on consumer preferences.

As outlined in Pethig (1989a), a general equilibrium model is constructed with drinking water produced from raw water with the help of labor whose productivity increases with raw water quality. A conventional consumption good is produced with the inputs land and labor. Along with its production harmful waste products are generated and then discharged into the environment adversely affecting the quality of raw water in the water protection area. Raw water quality is assumed to depend on both the size of the water protection area and on that amount of assimilative services of the raw water resource which is not used for

waste assimilation. So far little is known about the precise nature of this "ecological production function". The analysis proceeds, therefore, with two plausible, but markedly different assumptions: linear homogeneity and homogeneity of degree zero.

After the description of the model Section 3 investigates the characteristics of efficient shadow prices. Assimilative services of the raw water resource and (hence) raw water quality turn out to be always scarce. Therefore the price of drinking water must not only reflect average operating costs of the water work, but must get a mark-up for raw water quality. Another mark-up is due if raw water carries a positive rent. It is shown that efficiency may but need not necessarily go along with raw water scarcity. The remaining part of the paper concentrates on specifying conditions for raw water scarcity.

The first basic but straightforward result is independent of how the ecological production function is specified: raw water turns out to be always scarce if industrial and/or agricultural pollution does not spill over at all into the water protection area. If this rent were ignored in the process of price formation of drinking water, one would expect water protection areas to be excessively (or inefficiently) large. Casual observation indicates, however, that the complete absence of pollution spillovers is of limited empirical relevance only. Therefore we proceed by assuming that pollution spillovers do occur.

Suppose first, the production function of raw water quality is linear homogeneous. Consider a general competitive equilibrium with a complete set of — partly fictitious — markets for labor, land, assimilative services, raw water, raw water quality, drinking water, and the consumption good. Under a additional restrictions on the production functions (Cobb—Douglas) the comparative—static analysis yields the following results: if the initial situation is a competitive equilibrium in which raw water is abundant then raw water becomes less abundant — and possibly eventually scarce — with increasing demand for drinking water, with increasing labor endowment ('economic growth') and with decreasing land endowment. These insights can be used to identify certain conditions on endowments, technolo-

gies and preferences under which the efficient price of raw water is, in fact, positive.

If the production function of raw water quality is homogeneous of degree zero the efficient allocation cannot be decentralised by (shadow) prices, i.e. no general competitive equilibrium exists. To obtain further insights raw water quality is assumed to be "managed" by a public enterprise that supplies raw water quality costlessly but efficiently. The scarcity of raw water is studied in this framework along similar lines as in case of the previously applied assumption of linear homogeneous raw water quality production. In particular, for similar restrictions on production functions, conditions are identified under which raw water carries an economic rent or is abundant. It is rather surprising that the sharply different hypotheses on ecological technology don't lead to profound changes of the comparative-statics results regarding raw water abundance as compared to the results reported in the preceding paragraph.

## 2. The basic model

Consider an economy producing the end products **drinking water** ( $w$ ) and a **consumption good** ( $y$ ). The consumption good is produced with the conventional factors **land** ( $b_y$ ) and **labor** ( $a_y$ ), but along with its production waste is generated as an undesired by-product.  $m_y$  denotes the amount of waste that is (i) produced and unabated and (ii) then discharged into the environment outside the water protection areas. The production of the consumption good is modeled by the concave and linear homogeneous production function  $\bar{Y}: D_y \rightarrow \mathbb{R}_+$ , where

$$(1) \quad y = \bar{Y}(a_y, b_y, m_y),$$

+   +   +

where  $\bar{Y}(0,0,0) = 0$  and  $D_y := \{(a_y, b_y, m_y) \in \mathbb{R}_+^3 \mid m_y \leq \Phi(a_y, b_y)\}$ , and where the function

$\Phi : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$  is convex and strict monotonically increasing<sup>1</sup>. To simplify the analysis the production function  $\bar{Y}$  from (1) is assumed to be weakly separable (Berndt and Christian-  
sen 1973, p. 404) in the sense that

$$(1') \quad y = \bar{Y}(a_y, b_y, e_y/\mu) = Y[a_y, H(b_h, e_h)]$$

where  $b_h = b_y$ ,  $e_h = e_y$ , and where  $H$  is linear homogeneous. For the production of drinking water the intermediate good **raw water** is needed in sufficient quantity and appropriate purity (**raw water quality**). In our model a **raw water enterprise** is supposed to provide three outputs: the quantity of raw water, the quality of raw water, and assimilative services. Denote by  $b_q$  the amount of land (**water protection area**) owned by this firm for the purpose of protecting from pollution the water resource located on or underneath this land (**raw water resource**). By assumption, water from outside the protected area is too polluted to be used for the generation of drinking water. The availability of raw water is therefore constrained by the size of the raw water resource. Suppose, for simplicity, that the maximum supply (per period) of raw water,  $r$ , is proportional to the water protection area:<sup>2</sup>

$$(2) \quad r = \rho b_q \quad \text{with } \rho > 0.$$

Being the owner of land  $b_q$  the raw water enterprise prohibits, of course, the direct discharge of pollutants into the raw water resource, but it cannot prevent the amount

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<sup>1</sup> For details of this formalisation see Pethig (1979).

<sup>2</sup>  $r$  is that amount of raw water which can be withdrawn without any reduction of the water table. It is implicitly assumed in this paper that the upper bound  $r$  for water withdrawal is always observed (due to effective enforcement) – in contrast to empirical evidence in some regions. The impact of relaxing this assumption is studied in Pethig (1988) and Linde (1988). In a spacial economy – as in the real world – one encounters the additional possibility of local drinking water excess demands implying local raw water scarcity (Brösse 1980). In Germany this mismatching of demand and supply is balanced by large-scale compound systems for interregional raw water delivery. In the present paper this issue is ignored.



$$(3) \quad e_y = \mu m_y, \quad \text{with } \mu \in [0,1),$$

of pollutants from spilling over – or leaking – into the restricted zone. For any given  $b_q$  and  $q$  the amount

$$(4) \quad e_y = E(b_q, q)$$

of emissions implies an "ecological equilibrium" in the sense that if the amount  $E(b_q, q)$  spills over into the raw water resource period by period, then the raw water quality does not change in time<sup>3</sup>. Function  $E$  is increasing in  $b_q$  and decreasing in  $q$  on some non-empty interval  $(q_0, q_m]$ . More specifically, suppose  $E$  has a unique maximum with  $q_0 = \arg \max E(b_q, q)$  for all  $b_q$  and let

$$(5) \quad e := \max_q E(b_q, q) = \eta b_q \quad \text{with } \eta > 0.$$

Clearly,  $e = \eta b_q$  can be interpreted as the **supply of assimilative services** or as the assimilative potential of the raw water resource. Correspondingly, the maximal emission flow sustainable in the long run is  $\eta b_q$ . In what follows we disregard all (inefficient) raw water qualities  $q < q_0$ .

The assimilative capacity can be either used for waste assimilation ( $e_y$ ) or for natural improvements of the raw water quality ( $e_q$ ), where  $e_q$  is defined as that amount of assimilative services which is not used for waste assimilation, i.e.

$$(6) \quad e_q + e_y = \eta b_q.$$

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<sup>3</sup>It is beyond the scope of the present paper to provide a dynamic analysis of ecological processes. Appendix I offers some additional information of how our "static" model makes use of stationary ecological states and comparative dynamics.

In view of (5) and (6) stationary ecological states are described by

$$(7) \quad \eta b_q - e_q - E(b_q, q) = 0.$$

Equation (7) is an implicit ecological production function for raw water quality, sustainable in the long run, by means of the productive factors  $e_q$  and  $b_q$ . For  $q$  in the interval  $[q_0, q_m]$  the implicit function (7) can be rewritten in explicit form as

$$(8) \quad q = Q(b_q, e_q),$$

where the properties of  $Q$  are determined by the properties of  $E$ . Unfortunately, the "ecological" information about function  $E$  appears to be incomplete (Fiedler 1989). For that reason we proceed by considering two alternative additional specifications of function  $E$ :

**Assumption A:**  $Q$  satisfies  $Q_b, Q_e > 0$ , and  $Q$  is concave and linear homogeneous<sup>4</sup>.

**Assumption B:** The function  $E$  takes the special form

$$(9) \quad E(b_q, q) = b_q \cdot F(q),$$

where  $F$  satisfies  $F(q_0) = F(q_m) = 0$  and is strictly concave on  $[q_0, q_m]$ .

To see the implication of (9) for function  $Q$  observe first that  $q_0 = \arg \max F(q)$  and  $\max_q E(b_q, q) = F(q_0)b_q = \eta b_q$ . Moreover, if  $E$  from (4) is specified by (9) and if  $q$  is restricted to the interval  $[q_0, q_m]$ , then the ecological relationship (7) can be transformed into a production function  $Q: D_q \rightarrow [q_0, q_m]$  such that

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<sup>4</sup> It is shown in Appendix II that these properties are compatible with the assumptions on  $E$  which have already been introduced and with equation (7) — approximately, at least. Observe also that  $Q$  is linear homogeneous only until its maximum value  $q_m$  is attained. For larger input combinations the function is flat at the level  $q = q_m$ .

$$(10) \quad Q(b_q, e_q) = F^{-1}\left(\eta \cdot \frac{e_q}{b_q}\right)$$

and  $D_q := \{(b_q, e_q) \mid b_q F(q_o) \geq \eta b_q \geq e_q \geq 0, e_q \geq 0, b_q > 0\}$ . As demonstrated in Appendix II, this function  $Q$  is increasing in  $e_q$ , decreasing in  $b_q$  and non-concave.

Drinking water is produced by the **water work** with the help of the production function

$$(11) \quad w = \min \left[ \underset{+}{r_w}, \underset{+}{W}(\underset{+}{a_w}, \underset{+}{q_w}) \right],$$

where  $a_w$  represents the labor input,  $r_w$  the quantity and  $q_w$  the quality of raw water. The minimum condition in (11) reflects the obvious limitation of drinking water production by the availability of the raw water input. If raw water is abundant, the (unconstrained) function  $W$  is the production function of drinking water.  $W$  is assumed to be increasing in both arguments, concave and linear homogeneous. This specification of  $W$  is chosen for analytical convenience. It implies that if the factor "raw water quality" is unprized and if the price of drinking water is to be efficient then the water work's revenues must exceed its labor costs. This conclusion would not necessarily hold if  $W$  were of the type  $W(a_w, q_w) = a_w \bar{W}(q_w)$  with  $\bar{W}$  increasing and concave. Even though such a function might be considered a realistic hypothesis for long-term drinking water production it is obvious that its non-concavity would render the analysis considerably more complex.

The description of the supply side is completed by listing all resource constraints:

$$(12) \quad \left\{ \begin{array}{ll} q_w \leq q & a_y + a_w \leq a_o \\ w_d \leq w & b_y + b_q \leq b_o \\ y_d \leq y & e_y + e_q \leq \eta b_q \\ & r_w \leq r \end{array} \right.$$

The inequalities (12) conveniently summarise the commodity space of the model. There are two conventional productive factors: labor and land; two end products: drinking water and

good Y; in addition, there are three natural resources or "environmental goods": (the quantity of) raw water, raw water quality, and assimilative services of the raw water resource.<sup>5</sup>

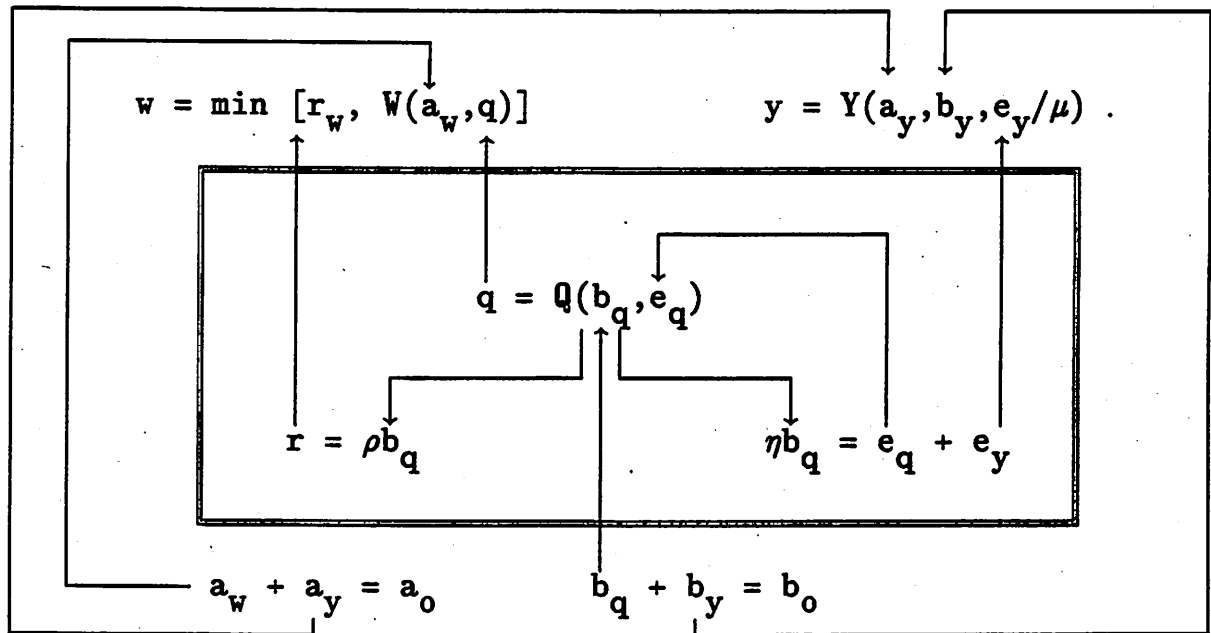


Table 1: The supply side of the economy

Table 1 offers a compressed view of the economy's supply-side interdependence: It shows how the two primary factors, land and labor, ultimately produce the two end products, drinking water and the consumption good Y. The intermediate part of the model, in Table 1 marked by the double-lined box, represents its "ecological core", where three intermediate outputs, namely raw water, assimilative services and raw water quality, are produced with the help of the inputs land (water protection area) and assimilative services.

Now it remains to specify the demand side of the model. Since the main focus of the pre-

<sup>5</sup> Observe that raw water and assimilative services are essentially private goods while raw water quality is a public good. However, this publicness does not cause allocative difficulties in the subsequent analysis because our "aggregate" model assumes one single supplier and one single demander in the market for raw water quality. In a disaggregated model one would have to introduce a "Lindahl market" for raw water quality.

sent paper is on allocative efficiency rather than on distributional issues the demand side is kept as simple as possible: There are  $n$  consumers all of whom have identical, strictly quasi-concave and homothetic preferences represented by

$$(13) \quad u = U(y_d/n, w_d/n).$$

### 3. Efficient shadow prices

To characterise an efficient allocation consider the problem of maximising the representative consumer's utility (13) subject to (1'), (2), (3), (8), (11) and (12). The associated Lagrangean is

$$(14) \quad L = U(y_d/n, w_d/n) + \lambda_y[Y(a_y, h_y) - y] + \lambda_w[W(a_w, q_w) - w] + \lambda_h[H(b_h, e_h) - h] \\ + \lambda_q[Q(b_q, e_q) - q] + \lambda_r(r_w - w) + \lambda_\rho(\rho b_q - r) + p_y(y - y_d) + p_w(w - w_d) + \\ p_q(q - q_w) + p_h(h - h_y) + p_b(b_o - b_h - b_q) + p_e(\eta b_q - e_h - e_q) + p_a(a_o - a_y - \\ a_w) + p_r(r - r_w).$$

In case of assumption A the Kuhn-Tucker conditions are necessary and sufficient for an efficient allocation. Under assumption B they are only necessary. When the attention is restricted to interior solutions it is easy to show that the shadow prices  $p_v$  for  $v = a, b, e, h, q, w, y$  are positive. But raw water may be either scarce ( $p_r > 0$ ) or a free good ( $p_r = 0$ ). More specifically,  $p_r > 0$  implies  $r = r_w = w = W(a_w, q_w)$  and  $p_r = 0$  implies  $r \geq r_w \geq w = W(a_w, q_w)$ . To see as to why raw water may or may not be scarce observe that  $r_w > 0$  implies  $p_r = \lambda_r$ , and that  $w \geq w_d > 0$  implies  $p_w = \lambda_w + p_r = U_w/n > 0$ . Moreover  $p_a > 0$  and  $W_a > 0$  implies  $\lambda_w > 0$ , which can be shown to assure  $w = W(a_w, q_w)$  and  $r_w \geq W(a_w, q_w)$ . But  $\lambda_w > 0$  is still compatible with  $r > r_w$ . Only if  $\lambda_r = p_r > 0$  it follows that



$w = r_w = W(a_w, q_w)$ . To sum up, in an interior solution the efficient price of drinking water is

$$p_w = \lambda_w + p_r \geq \lambda_w > 0,$$

where  $p_r \geq 0$  is the scarcity price of raw water and where  $\lambda_w$  is the marginal-cost price of generating drinking water from raw water in the water work. In other words,  $\lambda_w$  is the efficient price of drinking water in case of abundant raw water. To illustrate this relationship we write the water work's profit as

$$(15) \quad g_w = p_w \{\min[r_w, W(a_w, q_w)]\} - p_a a_w - p_q q_w - p_r r_w.$$

Recall that  $r_w = W(a_w, q_w)$  for  $p_r > 0$  and  $r_w \geq W(a_w, q_w)$  for  $p_r = 0$ . Therefore it is possible to rewrite (15) as

$$(15') \quad g_w = p_w^n \cdot W(a_w, q_w) - p_a a_w - p_q q_w$$

with  $p_w^n := \lambda_w = p_w - p_r > 0$  being the **producer price of drinking water**. Since the Kuhn-Tucker conditions imply marginal-cost pricing in the water work, linear homogeneity of  $W$  requires

$$(16) \quad p_w = p_a(a_w/w) + p_q(q_w/w) + p_r.$$

Hence, the **efficient consumer price of drinking water** must exceed the operating cost of the water work per unit of drinking water, i.e.  $p_a(a_w/w)$ , by a mark-up for raw water quality,  $p_q(q_w/w)$ , and by the raw water rent  $p_r$  per unit of water. To see how the mark-up for raw water quality is determined, consider the profit of the raw water enterprise:

$$(17) \quad g_q = p_q Q(b_q, e_q) + p_e \eta b_q + p_r \rho b_q - p_b b_q - p_e e_q,$$

which stems from selling raw water quality and raw water to the water work and from selling assimilative services to sector Y. Costs are incurred from buying land ( $b_q$ ) and assimilative services ( $e_q$ ). Obviously, (17) is easily transformed to read

$$(17') \quad \xi_q = G^q(b_q, e_q) := p_q Q(b_q, e_q) - p_b^n \cdot b_q - p_e \cdot e_q$$

with  $p_b^n := p_b - \eta p_e - \rho p_r$  being the raw water producer's effective (or net) price for the water protection area. It is smaller than the price for land because of the revenue generated by the two joint products of the water protection area: assimilative services and raw water. The Kuhn–Tucker conditions imply that for  $b_q$  (and therefore  $r$  and  $w$ ) to be positive in an efficient allocation  $p_b^n$  must be positive under assumption A, but negative under assumption B. It is interesting to note that (17') looks as if the raw water enterprise is simply a producer of raw water quality. But it is evident from (17) that this enterprise does indeed produce the two outputs raw water and assimilative services in addition to raw water quality.

In case of assumption A and in view of the linear homogeneity of  $Q$  the price of raw water quality is specified by

$$(18) \quad p_q = p_b^n (b_q/q) + p_e (e_q/q).$$

It consists of the sum of the net price for the water protection area and the price for assimilative services used by the raw water enterprise. If assumption B holds, the Kuhn Tucker conditions also require marginal–cost pricing by the raw water producer, but  $Q_b/Q_e = p_b^n/p_e$  is equal to  $p_b^n b_q + p_e e_q = 0$  so that  $G^q(b_q, e_q) := p_q Q(b_q, e_q) > 0$  by (17'). In this case the price for raw water quality cannot be readily split up in its factor cost components.

The information that  $p_r$  can but need not be positive is not entirely satisfactory, because we do not know whether there is any specification of the model exhibiting an efficient allocation in which the price of raw water is positive. Additional insight is readily obtained, however, for the limiting case in which no pollution spillover occurs at all into the water protection area: *Under this assumption raw water turns out to be always scarce.*

To see this observe first that  $\mu = 0$  means to substitute the resource constraint  $\eta b_q \geq e_h + e_q$  by  $\eta b_q \geq e_q$  and to substitute  $h = H(b_h, e_h)$  by a linear function of the type  $h = \beta b_h$ . Incorporate these changes into equation (14) and consider the resulting marginal conditions. In particular, combine the marginal condition  $\lambda_q Q_b + \lambda_\rho \rho + p_e \eta = p_b$  with  $p_e = \lambda_q Q_e$  and  $p_r = \lambda_\rho$  to obtain  $\lambda_q (Q_b + Q_e \eta) + p_r \rho = p_b$ . In addition,  $e_q = \eta b_q$  implies  $Q_b + Q_e \eta = 0$  under both assumptions A and B so that  $p_b = p_r \rho$ . From  $p_b = p_y Y_b$ ,  $p_y = U_y/n > 0$  and  $Y_b > 0$  it follows that  $p_b = p_r \rho > 0$ .

In view of the structure of our model this result clearly confirms one's intuition. Recall that the raw water supply is proportional to the size of the water protection area the extension of which is costly because land has positive opportunity costs. Therefore the scarce resource land would be wasted if raw water were in excess supply. The scarcity of land makes high quality raw water scarce with its price matching the marginal opportunity costs of the water protection area.

It is interesting to observe that in the absence of pollution spillovers the price for both assimilative services and raw water quality is still positive. But the net cost of assimilative services is zero in the raw water enterprise, because in view of (17'),  $p_b = p_r \rho$  and  $e_q = \eta b_q$  one has

$$g_q = p_q Q(b_q, e_q) - (p_b - p_e \eta - p_r \rho) \cdot b_q - p_e \cdot \eta b_q = p_q Q(b_q, e_q) > 0.$$

Though it is important to know how the price of raw water is affected when  $\mu$  tends to

zero,  $\mu > 0$  is, of course, the *raison d'être* of the problem under consideration. Is there a reason to believe that the scarcity result for  $\mu = 0$  could be reversed in case of  $\mu > 0$ ? To see the logic of this possibility suppose for simplicity the flow of pollutants,  $e_h$ , is constant. Then raw water quality varies inversely with the size of the water protection area. Suppose, moreover, that it is technically infeasible (or prohibitively costly) to generate drinking water from raw water whose quality is below some given threshold value. In this somewhat pathological scenario the threshold value for raw water quality determines a minimum size of the water protection area which may induce a raw water supply in excess of drinking water demand. In the following section we wish to provide more information about this issue by taking account of the two different assumptions on the production of raw water.

## 4. The scarcity of raw water

### 4.1 Linear homogeneous production of raw water quality

Assume in this section that function  $Q$  satisfies assumption A. Moreover, let the property rights for all environmental goods be exclusively assigned to and costlessly enforced by the raw water enterprise so that there is a competitive market for each of the seven commodities with prices  $p_a$ ,  $p_b$ ,  $p_e$ ,  $p_q$ ,  $p_r$ ,  $p_w$ , and  $p_y$ . Since the model is well behaved in the Arrow-Debreu sense, a general competitive equilibrium exists and is Pareto efficient. In what follows our attention focuses on the question as to whether raw water is a scarce natural resource when pollution spills over into the water protection area.

To that end consider an initial state of competitive equilibrium of the economy in which raw water is abundant. In what follows we want to investigate how the excess supply of raw water responds to shifts in the endowment of land and labor and to shifts of consumers' preferences. Successive increases in the endowment of labor can be interpreted as

mimicking economic growth with its increasing pressure on the use of natural resources. Or it may serve to compare two countries differing exclusively by the size of their labor force. The latter interpretation is also applicable for shifts in the endowment of land. Shifts in consumer preferences are appropriate to model differences in demand side pressure on water resources interpreted either as a particular country's changing preferences or as cross-country comparison. The simplest way to formalise such a demand shift is to specify the utility function (13) by

$$(19) \quad u = \min [w, \gamma \cdot y] \text{ with } \gamma > 0.$$

Moreover, to obtain informative results we further specify the production functions  $W$ ,  $Y$ ,  $H$ , and  $Q$  to be Cobb–Douglas. For simplicity we also set  $(a_w W_a/w) = (a_y Y_a/y)$ . Under these restrictions, Table 2a summarises the comparative statics for exogenous changes in  $a_o$ ,  $b_o$  and  $\gamma$  as elaborated in Appendix III. This table demonstrates a special feature of our model: exogenous shocks of land or labor leave unaffected the relative prices  $\pi_{eb}$  and  $\pi_{qh}$ . Moreover, shocks in the labor endowment have no impact on the allocation of land and assimilative services so that the supply of all "intermediate goods" are unaffected by such shocks.



shock → response ↓	$\hat{a}_0$	$\hat{b}_0$	$\hat{\gamma}$
$\hat{a}_w$	+	-	?
$\hat{a}_y$	+	?	?
$\hat{b}_h$	0	+	-
$\hat{b}_q = \hat{e} = \hat{r}$	0	+	+
$\hat{e}_h$	0	+	+*
$\hat{e}_q$	0	+	?
$\hat{h}$	0	+	-
$\hat{q}$	0	+	+
$\hat{w}$	+	?	+
$\hat{y}$	+	?	-
$\hat{\pi}_{wa}$	+	-	+
$\hat{\pi}_{eb}$	0	0	-*
$\hat{\pi}_{ha}$	+	-	+
$\hat{\pi}_{yw}$	0	0	-
$\hat{\pi}_{qa}$	+	-	+
$\hat{\pi}_{qh}$	0	0	-
$\hat{r} - \hat{w}$	-	+	?

$\hat{a}_0$	$\hat{b}_0$	$\hat{\gamma}$
+	+	+
+	-	-
0	+	0
0	+	0
-*	?	-
+*	+	+
-	+	-
+*	+	+
+	+	+
+	+	-
+	?	?
+*	+	+
+	-	?
?	?	+
+	?	?
+*	+	+
-	+	-

Table 2a (\*:  $e_q/b_q > \frac{1-\delta}{1-\alpha}(e_h/b_h)$ )Table 2b (\*:  $\beta > \xi$ )

Table 2: Abundant raw water and comparative statics

It follows that if (13) takes the form (19) and if the initial general competitive equilibrium is characterised by excess supply of raw water at zero price, then

*raw water becomes less abundant — and possibly eventually scarce — with increasing increasing labor endowment  $a_o$  and with decreasing land endowment  $b_o$ .*

As Table 2a (last row, last column) shows, the effect on raw water excess supply of an increase in the demand parameter  $\gamma$  is not clear-cut. It is save to argue, however, that with increasing  $\gamma$  the difference  $\hat{r} - \hat{w}$  tends to diminish. Observe that  $\rho b_o$  is the maximum possible supply of raw water and that  $W(a_o, q_m)$  is the maximum possible supply of drinking water under the proviso that there is no shortage of raw water. Obviously,  $W(a_o, q_m) > \rho b_o$  is a sufficient condition for a tendency of raw water excess supply to diminish with successive increases of  $\gamma$ . In fact, under the condition  $W(a_o, q_m) > \rho b_o$  there is always a  $\bar{\gamma} > 0$  such that raw water is scarce for any  $\gamma > \bar{\gamma}$ .

Similar considerations hold with respect to changes in labor and land endowments. To be more specific, suppose that  $b_o$  and  $\gamma$  are fixed. Denote the associated input quantities by  $b_q(b_o, \gamma)$ ,  $b_h(b_o, \gamma)$ ,  $e_q(b_o, \gamma)$ ,  $e_h(b_o, \gamma)$ , yielding the outputs  $h(b_o, \gamma) := H[b_h(b_o, \gamma), e_h(b_o, \gamma)]$  and  $q(b_o, \gamma) := Q[b_q(b_o, \gamma), e_q(b_o, \gamma)]$ . Now specify the production functions  $Y$  and  $W$  as  $y = a_y^\theta \cdot h^{1-\theta}$  and  $w = a_w^\theta \cdot q^{1-\theta}$ . With the help of  $a_o = a_w + a_y$  the transformation function is

$$y = \left( a_o - w^{\frac{1}{\theta}} \cdot [q(b_o, \gamma)]^{\frac{\theta-1}{\theta}} \right)^\theta \cdot [h(b_o, \gamma)]^{1-\theta}.$$

The demand condition  $w = \gamma \cdot y$  from (19) allows to calculate  $w = c(b_o, \gamma) \cdot a_o^\theta$  with

$$c(b_o, \gamma) := \frac{\gamma [h(b_o, \gamma)]^{1-\theta}}{1 + \gamma [h(b_o, \gamma)] / [b(b_o, \gamma)]^{1-\theta}}$$

We are now in the position to state: For any  $b_o$  and  $\gamma$  there is a labor endowment  $a_o(b_o, \gamma) := [\rho \cdot b_q(b_o, \gamma)/c(b_o, \gamma)]^{1/\theta} > 0$  such that raw water carries a positive economic rent, if and only if  $a_o > a_o(b_o, \gamma)$ .

The rationale of the above arguments for raw water scarcity are straightforward. Owing to the joint production property of the three goods raw water quality, assimilative services and raw water supply, the supply of raw water is not responsive to changes of the labor endowment as long as  $p_r = 0$ . In other words, if the labor endowment is sufficiently low then raw water is in excess supply. But economic growth (increasing  $a_o$ ) eventually turns raw water into a scarce resource.

To understand as to why the rent of raw water may be positive or zero in equilibrium recall that the raw water producer's decision to raise her demand for land simultaneously increases both the maximum supply of raw water and the supply of assimilative services of the raw water resource. If for given amounts of pollutants a large water protection area is needed to achieve a raw water quality appropriate for generating drinking water, then the amount of raw water available in the large water protection area might be in excess supply, i.e. a free good. Conversely, if raw water pollution is not a severe problem then raw water carries a scarcity price.

#### 4.2 Zero homogeneous production of raw water quality

Suppose now assumption B governs the generation of raw water quality. In the Appendix II function  $Q$  is shown to be non-concave under assumption B. An immediate implication is that conventional methods may fail to characterise Pareto-efficient allocations. Moreover, price-taking profit maximisation may but need not be defined. In fact, it is true that under assumption B the profit function  $G^q(b_q, e_q)$  from (14') does not exhibit a maximum on its domain  $D_q$  for any given set of prices  $(p_b, p_e, p_r) \in \mathbb{R}_+^3$ .

To show this suppose contrary to the assertion that for some given set of prices  $p_q$ ,  $p_b^n$  and  $p_e$  there exists a maximiser  $(b_q^*, e_q^*) \in D_q$  for the function  $G^q$  from (14'). Denote by  $c^* := p_b^n b_q^* + p_e e_q^*$  the profit-maximising production cost and define  $c^\lambda := p_b^n \lambda b_q^* + p_e \lambda e_q^*$  with  $\lambda > 0$ . Obviously, zero homogeneity of  $Q$  yields  $q^* = Q(b_q^*, e_q^*) = Q(\lambda b_q^*, \lambda e_q^*) = q^\lambda$  for all  $\lambda > 0$ . If  $c^* > 0$  then it is true that for every  $\lambda \in (0, 1)$  one obtains  $q^\lambda = q^*$  but  $c^\lambda > c^*$ . Hence  $(b_q^*, e_q^*)$  is not a maximiser of  $G^q$ . Similarly, if  $c^* > 0$  then for every  $\lambda > 1$  one also obtains  $q^\lambda = q^*$  but  $c^\lambda > c^*$ . Suppose finally that  $c^* = 0$ . Then the raw water producer can increase her land input by  $\Delta b > 0$  to the effect that  $c^1 := p_b^n(b_q^* + \Delta b) + p_e e_q^* < 0$  and  $q^1 := Q(b_q^* + \Delta b, e_q^*) \geq Q(b_q^*, e_q^*)$ . Moreover, she can multiply all inputs by the factor  $\lambda$  yielding the costs  $c^{1\lambda} := p_b^n \lambda(b_q^* + \Delta b) + p_e \lambda e_q^* < 0$  and the raw water quality  $q^{1\lambda} = q^1$  for all  $\lambda > 0$ . It follows that there is  $\lambda^0 \geq 1$ , such that  $p_q q^1 - c^{1\lambda} > p_q q^* - c^*$  for all  $\lambda \geq \lambda^0$ . This completes the proof of the proposition.

This argument implies that under assumption B the economy cannot attain a competitive equilibrium with a complete set of markets. In other words, it is not possible to decentralise Pareto efficient allocations by competitive prices. Alternative routes to proceed are to consider the raw water enterprise as a (regulated) private monopoly or as a public enterprise. Depending on the regulations imposed it is likely that the associated equilibrium yields a second best allocation in which prices are known to be unreliable indicators of scarcity.

In what follows we therefore aim at constructing an efficient regime of raw water management. For that purpose we first investigate the conditions for production efficiency and then proceed to establish rules for the (public) raw water enterprise which secure production efficiency. Consider the Lagrangean (14). An interior solution — assumed to imply allocative efficiency<sup>6</sup> — is characterised by the two marginal conditions

<sup>6</sup> Recall that function  $Q$  is not concave. Hence the Kuhn-Tucker conditions are not necessary and sufficient for solving the associated economic optimisation problem. We proceed, however, by assuming that an efficient interior solution exists.

$$(20) \quad \frac{H_b}{H_e} = -\frac{e_q}{b_q} + \eta + \rho \frac{p_r}{p_e} = \frac{p_b}{p_e} \quad \text{and}$$

$$(21) \quad \frac{W_q}{W_a} \frac{Q_e}{H_e} = \frac{Y_h}{Y_a} = \frac{p_h}{p_a}.$$

In what follows we assume that the public raw water enterprise operates under the cost-covering (or zero-profit) constraint

$$(22) \quad G^q(b_q, e_q) = -p_b^n b_q - p_e e_q = 0.$$

This constraint turns out to be a necessary condition for production efficiency because with  $p_b^n = p_b - \eta p_e - \rho p_r$  equation (20) equals (22). For predetermined prices  $p_b$ ,  $p_e$  and  $p_r$  the rule (22) determines the factor intensity  $e_q/b_q$  and hence raw water quality – but not the scale of inputs in the production function of raw water quality. The efficient activity level of the raw water enterprise is implicitly given by (21). To see this observe first that the marginal factor productivities  $W_a$ ,  $W_q$ ,  $Y_a$ ,  $Y_h$ , and  $H_e$  are all fixed for predetermined prices since profit maximisation in these sectors determines the input ratios and hence marginal productivities. Therefore, the raw water enterprise must choose  $Q_e$  such that  $Q_e$

$$= \frac{Y_h W_a H_e}{Y_a W_q}. \text{ From } Q_e = \frac{\delta}{b_q} \left[ \frac{e_q}{b_q} \right]^{\delta-1} = \frac{\delta}{b_q} \left[ \eta + \frac{\rho p_r}{p_e} - \frac{p_b}{p_e} \right] \text{ it is obvious that equation}$$

(20) serves to determine  $b_q$  and hence  $e_q = (\eta + \rho p_r/p_e - p_b/p_e) b_q$ . To summarise, the raw water enterprise can achieve production efficiency by the strategy of cost covering (22) combined with a choice of its input levels guided by (21).

To obtain specific results we simplify the production functions  $H$ ,  $Q$ ,  $W$ , and  $Y$  again, similar as in the preceding subsection. The functions  $Y$ ,  $W$ , and  $H$  are assumed Cobb–Douglas and function  $Q$  is determined as  $Q(b_q, e_q) = (e_q/b_q)^\delta$ . The demand side is given by (19).



Under these restrictions Table 2b summarises the comparative statics for the case of a zero homogeneous technology. It contains a few ambiguities in sign, but to a large extent it is similar to Table 2a. In particular, it follows that if the economy has an initial competitive equilibrium characterised by excess supply of raw water at zero price, then

*raw water becomes less abundant – and possibly eventually scarce – with increasing demand parameter  $\gamma$ , with increasing labor endowment  $a_o$ , and with decreasing land endowment  $b_o$ .*

Table 2b also shows that the real price of drinking water and raw water quality increases with diminishing raw water abundance. Moreover, it can be demonstrated exactly as in the preceding section that raw water becomes scarce in a growing economy (increasing  $a_o$ ). A different approach to establish scarcity of raw water is the following: Suppose assumption B holds and the (public) raw water enterprise observes the equations (21) and (22). Then the price of raw water is positive ( $p_r > 0$ ) in competitive equilibrium, if and only if

$$(23) \quad (1-\alpha)\rho b_o < W[a_o, Q(\eta)]$$

and the equilibrium value of drinking water satisfies  $w \in ((1-\alpha)\rho b_o, W[a_o, Q(\eta)])$ . To see this observe first that the conditions

$$(24) \quad \frac{\alpha}{1-\alpha} \frac{e_h}{b_h} = \frac{p_b}{p_e} \quad \text{and} \quad -\frac{e_q}{b_q} = \frac{p_b}{p_e} - \eta - \frac{\rho p_r}{p_e}$$

are implied by maximising profits in the H-sector and by cost-covering production in the Q-sector, respectively. The combination of these equations gives

$$(25) \quad b_q = \frac{p_e}{p_b - p_r \rho} e_h = \frac{p_b(1-\alpha)}{(p_b - p_r \rho)\alpha} b_h.$$

With the help of  $b_o = b_q + b_h$  one obtains

$$(26) \quad b_q = \frac{(1-\alpha)p_b}{p_b - \alpha p_r \rho} b_o.$$

To prove sufficiency assume that (23) holds but suppose contrary to the assertion that  $p_r = 0$  for all  $w \in [(1-\alpha)\rho b_o, W[a_o, Q(\eta)]]$ . Then  $b_q = (1-\alpha)b_o$  by (25) and  $r = (1-\alpha)\rho b_o$  by (2).  $p_r = 0$  implies  $r = (1-\alpha)\rho b_o \geq w$ , hence  $w \notin [(1-\alpha)\rho b_o, W[a_o, Q(\eta)]]$ . Contradiction.

The necessary condition holds for the following reason: suppose that  $p_r > 0$  and  $(1-\alpha)\rho b_o \geq W[a_o, Q(\eta)]$ . Then we know from (26) that  $b_q > (1-\alpha)b_o$  and therefore  $r = \rho b_q > W[a_o, Q(\eta)]$ . Since  $W[a_o, Q(\eta)]$  is the maximum attainable amount of drinking water, feasibility requires  $w \in [0, W(q_o, Q(\eta))]$  in equilibrium. Hence  $r > w$  for all  $w \in [0, W(q_o, Q(\eta))]$ . Contradiction. Suppose now  $p_r > 0$  and (23) holds, but  $w \in [0, (1-\alpha)\rho b_o]$ . Again it is true that  $r = \rho b_q > (1-\alpha)\rho b_o$  which yields immediately  $r > w$  and therefore  $p_r = 0$ . Contradiction.

It is fairly surprising that the markedly different technological hypotheses A and B yield similar results on raw water abundance. But it should also be pointed out that the analysis is sensitive with respect to the technological assumptions (Cobb–Douglas). If for example the function  $H$  is not Cobb–Douglas but rather a general linear homogeneous function with unrestricted elasticity of substitution, then it is no longer true that  $\hat{r} = \hat{b} = 0$  for any exogenous shock of  $a_o$  or  $\gamma$ . One rather obtains in that case

$$(27) \quad \hat{r} = \hat{b}_q = \frac{b_h e_q (1-\sigma_h)}{b_o \delta e_h} \pi_{eb}$$

so that the sign of  $\hat{r}/\hat{a}_o$  and  $\hat{r}/\hat{\gamma}$  is non-zero for  $\sigma_h \neq 1$ , since the relative factor price  $\pi_{eb}$  does respond to exogenous shocks of  $a_o$  or  $\gamma$ . Depending on the sign of  $\hat{\pi}_{eb}$  and the size of  $\sigma_h$  the above conclusions about changes of excess supply of raw water may be reversed<sup>7</sup>.

<sup>7</sup> The comparative statics with production function  $H$  specified as a CES-function become very complex and do not yield unambiguous signs under conditions with sensible economic interpretations.

## 5. Concluding Remarks

The principal message of the preceding analysis is that efficient pricing of drinking water must not ignore that raw water quality and possibly raw water itself are scarce goods. As a policy implication our analysis demands the water works to set consumer prices for drinking water well above their average operating costs. Such a mark-up on average operating costs is required for the sake of efficiency to reflect the scarcity of both raw water quality and (possibly) quantity.

Another message of the preceding analysis is that the appropriate organisation of water management depends heavily on the nature of the ecological technology regarding assimilative services, the size of water protection areas and both the quantity and quality of raw water. Since this empirical relationship is not yet well understood, the paper offered two alternative hypotheses both of which may claim some a priori plausibility and investigated the pertaining implications. It turned out that in case of non-convex technology the water sector should be managed as a public enterprise or regulated private monopoly. Under both technological hypotheses it is shown that raw water may be a scarce natural resource. A positive rent for raw water is the more likely, the stronger is the demand for drinking water, the greater is the economy's labor endowment, and the smaller is the country in terms of its overall land endowment.

Recall that water as such is assumed to be abundant but that drinking water can only be generated from raw water out of water protection areas. The land reserved for water protection has opportunity costs which are sufficient to render raw water scarce if water pollution would not spill over into the water protection area. However, the production costs of drinking water increase with diminishing quality of the raw water. Therefore it may pay to enlarge the water protection area for the purpose of raising raw water quality. As a by-pro-

duct of the extension of the water protection area the quantitative supply of raw water may increase so strongly that not all of it is needed to satisfy the demand for drinking water. In this case raw water does not carry an economic rent.

Our analysis also points to the relevance of additional empirical investigations. For one thing, it is of course necessary to get more information about the ecological technology regarding raw water and raw water quality. It is important to know, in addition, whether the scarcity of raw water is not only a theoretical possibility but also a real phenomenon. Without doubt, this would have important implications for the design of efficiency-oriented real-world water management and pricing.

It is interesting, for example, to assess the merits of the "Wasserpennig" (water penny) as introduced in the German state of Baden-Württemberg in the light of the preceding analysis. The "water penny" is a charge on the withdrawal of raw water the allocative impact of which depends crucially on how drinking water was priced prior to its introduction. Suppose that drinking water prices were designed to cover the (average) operating costs of the water works before the implementation of the water penny. Then the water penny enhances allocative efficiency as long as its rate is not too high. This conclusion must be considered with the proviso, however, that real-world water management (in the absence of water charges) is likely to involve additional distortions other than the neglect of the scarcity of raw water quality and raw water. In such scenarios the impact on welfare of economic policy interventions like the water penny is difficult to assess.

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## Appendix

### Appendix I: The dynamics of changes in raw water quality

Recall that the fraction  $e_y = \mu m_y$  of the industrial or agricultural waste emission flow is assumed to leak into the water protection area. This waste spillover affects the raw water resource in the water protection area according to the dynamic ecological function

$$(A1) \quad \frac{dq}{dt} = E(b_q, q) - e_y,$$

where  $E(b_q, q)$  is the assimilation function of the raw water resource. The ecological evidence reviewed by Fiedler (1989) suggests, that the function  $E$  has a unique maximum at some  $q_0 < q_m$  and there is  $q_q(b_q) < q_0$  such that  $E[b_q, q_q(b_q)] = 0$ .

It would be an adequate approach to incorporate (A1) explicitly into the model substituting the representative consumer's preferences (13) by

$$(A2) \quad \int_0^{\infty} e^{-\delta t} U(w_t, y_t) dt.$$

Under the assumption that a complete set of future markets exists one would then obtain a time path of instantaneous competitive equilibria for the entire economy which tends towards an ecological steady state under certain qualifications.

Rather than undertaking such a complex investigation we proceed by ignoring the path of adjustment to the ecological state. It is assumed, instead, that consumers consider only their satisfaction in ecological steady states characterised by

$$(A3) \quad \frac{dq}{dt} = 0 \Leftrightarrow e_y = E(b_q, q).$$

To simplify even further the steady states are assumed to be exclusively attained in the range  $[q_0, q_m]$  which can be secured in the intertemporal approach when the externality  $q$  in the production function of drinking water (equation (11)) is sufficiently strong.

## Appendix II: The "production" of raw water quality

### Assumption A:

We wish to show that the assumptions made in Section 2 about the assimilation function  $E$  together with equation (7) are compatible with a function  $Q$  on  $[q_0, q_m]$  which is concave and has the properties  $Q_b, Q_e > 0$ . From (7) we get the following derivatives using the implicit function theorem:

$$\frac{\partial q}{\partial b_q} := Q_b = \frac{\eta - \overset{+}{E}_b}{\underset{-}{E}_q} > 0 \Leftrightarrow \eta := E_b|_{q=q_0} < E_b|_{q>q_0} \quad \text{and}$$

$$\frac{\partial^2 q}{\partial b_q^2} := Q_{bb} = - \frac{\overset{+}{E}_{bb} \overset{-}{E}_q + (\eta - \overset{-}{E}_b) \overset{+}{E}_{qb}}{\overset{+}{E}_{bq} \overset{-}{E}_q + (\eta - \overset{-}{E}_b) \overset{+}{E}_{qq}} < 0.$$

(Observe that  $E_{bq} = E_{qb}$  and that  $E_{bq} > 0$  is implied by  $\eta < E_b$ ).

$$\frac{\partial q}{\partial e_q} := Q_e = - \frac{\underset{-}{1}}{\underset{-}{E}_q} = - \overset{-}{E}_e^{-1} > 0 \quad \text{and} \quad \frac{\partial^2 q}{\partial e_q^2} := Q_{ee} = - \overset{+}{E}_{ee}^{-1} = - \overset{+}{E}_{qq}^{-1} < 0$$

Observe finally that  $\frac{\partial^2 q}{\partial e_q \partial b_q} := Q_{eb} = - \frac{\overset{+}{E}_{qb}}{(\overset{-}{E}_q)^2} > 0$  and that the condition  $\overset{-}{Q}_{ee} \overset{-}{Q}_{bb} > (\overset{+}{Q}_{eb})^2$  is sufficient for  $Q$  to be concave.

### Assumption B:

We wish to demonstrate that the function

$$(10) \quad Q(b_q, e_q) = F^{-1}\left(\eta - \frac{e_q}{b_q}\right)$$

is increasing in  $e_q$ , decreasing in  $b_q$  and non-concave, when  $F$  satisfies  $F(q_\ell) = F(q_m) = 0$  and is strictly concave on  $[q_\ell, q_m]$ . Define, for convenience,  $k := e_q/b_q$  and use the properties of  $E$  from (9) to turn (7) into  $e_q = b_q(\eta - F(q))$  or  $F(q) = \eta - k$ . Since  $F(q)$  is

strictly concave and decreasing on  $[q_o, q_m]$  its inverse exists on that interval and reads  $q = F^{-1}(\eta - k)$  which proves equation (10). The derivatives of  $Q$  are

- (i)  $Q_k = -1/F_q > 0$  and  $Q_{kk} = 1/F_{qq} < 0$ ,
- (ii)  $Q_e = Q_k/b_q > 0$  and  $Q_{ee} = Q_{kk}/b_q^2 < 0$ ,
- (iii)  $Q_b = -e_q Q_k/b_q^2 < 0$  and  $Q_{be} = -e_q Q_{kk}/b_q^3 - Q_k/b_q^2$
- (iv)  $Q_{bb} = Q_{kk}e_q^2/b_q^4 + 2e_q Q_k/b_q^3 =$   
 $= kQ_k(kQ_{kk}/Q_k + 2)/b_q^2 \gtrless 0 \Leftrightarrow kQ_{kk}/Q_k \gtrless 2.$

There are parametric specifications of  $F$  satisfying  $Q_{bb} > 0$  on the entire domain as, for example, the quadratic function  $F(q) = h_o q - h_1 q^2$  ( $h_o, h_1 > 0$ ). On the other hand, consider the parametric function  $Q(k) = c(1 - e^{-bk})$  with  $c, b \in \mathbb{R}_+$ . The domain of this function is  $[0, c)$  and the interval on which  $Q$  satisfies  $Q_{bb} < 0$  is  $[2/b, c)$ . Hence for  $b \rightarrow \infty$  the interval  $[2/b, c)$  tends to  $[0, c)$ . A parametric specification of (10) could not be found which satisfies  $Q_{bb} < 0$  on the entire domain. But even if such a function should exist observe that concavity also requires  $Q_{ee}Q_{bb} - (Q_{eb})^2 > 0$ . In view of (i) – (iv) above one obtains

$$Q_{ee}Q_{bb} - (Q_{eb})^2 = -Q_k^2/b_q^4 < 0.$$

It follows that the function  $Q$  from (10) is not concave.

### Appendix III: Comparative statics for the case of abundant raw water ( $p_r = 0$ ), when the technology of raw water quality is convex

Consider the production functions

$$(A4) \quad h = b_h^\alpha \cdot e_h^{1-\alpha} \quad \text{or} \quad h = b_h \cdot k_h^{1-\alpha} \quad \text{with} \quad k_h := e_h/b_h$$

$$(A5) \quad q = b_q^\delta \cdot e_q^{1-\delta} \quad \text{or} \quad q = b_q \cdot k_q^{1-\delta} \quad \text{with} \quad k_q := e_q/b_q$$

$$(A6) \quad y = a_y^\beta \cdot h^{1-\beta} \quad \text{or} \quad y = h \cdot k_y^\beta \quad \text{with} \quad k_y := a_y/h,$$

$$(A7) \quad w = a_w^\xi \cdot q^{1-\xi} \quad \text{or} \quad w = q \cdot k_w^\xi \quad \text{with} \quad k_w := a_w/q.$$

and the resource constraints

$$(A8) \quad b_o = b_q + b_h, \quad \eta b_q = e_h + e_q, \quad \text{and} \quad a_o = a_w + a_y.$$

The demand side is described by (19) or

$$(A9) \quad \gamma = \frac{w}{y}.$$

For given relative prices  $\pi_{be} := p_b/p_e$ ,  $\pi_{hq} := p_h/p_q$ ,  $\pi_{ha} := p_h/p_a$ ,  $\pi_{qa} := p_q/p_a$ ,  $\pi_{yw} := p_y/p_w$  profit maximisation implies

$$(A10) \quad \pi_{ha} = \frac{1-\beta}{\beta} k_y,$$

$$(A11) \quad \pi_{qa} = \frac{1-\xi}{\xi} k_w,$$

$$(A12) \quad \pi_{yw} = \frac{\xi}{\beta} (k_y^{1-\beta} / k_w^{1-\xi}),$$

$$(A13) \quad \pi_{be} = \frac{\alpha}{1-\alpha} k_h = \frac{\delta}{1-\delta} k_q + \eta \quad \text{and}$$

$$(A14) \quad \pi_{hq} = \frac{1-\delta}{1-\alpha} (k_h^\alpha / k_q^\delta).$$

With the help of (A10) and (A11) equation (A12) reads

$$(A12') \quad \pi_{yw} = \frac{\xi^\xi (1-\xi)^{1-\xi}}{\beta^\beta (1-\beta)^{1-\beta}} \frac{\pi_{ha}^{1-\beta}}{\pi_{qa}^{1-\xi}}.$$

Similarly, equation (A13) turns (A14) into

$$(A14') \quad \pi_{hq} = \frac{\delta^\delta (1-\delta)^{1-\delta}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \cdot \frac{\pi_{be}^\alpha}{(\pi_{be} - \eta)^\delta}.$$

Total differentiation of (A12 and (A14')) gives us

$$(A12'') \quad \hat{\pi}_{yw} = (1-\beta)\hat{\pi}_{ha} - (1-\xi)\hat{\pi}_{qa},$$

$$(A14') \quad \hat{\pi}_{be} = K_b \hat{\pi}_{hq},$$

where  $K_b := \frac{\delta K}{\alpha(1-\alpha)(1-\delta)\eta}$  and  $K := \frac{(1-\alpha)(1-\delta)\eta}{\delta[1-\alpha]k_q - (1-\delta)k_h}$ . Observe that  $K_b, K \lesssim 0$  if and

only if  $k_q \lesssim \frac{1-\delta}{1-\alpha} k_h$ . Empirical observation suggests that  $b_h$  is much greater than  $b_q$  while  $e_q$  is probably greater than  $e_h$  pointing to assume  $K > 0$ . Note, however, that

(A13) implies  $k_q = \frac{(1-\delta)^\alpha}{(1-\alpha)\delta} k_h - \frac{(1-\delta)\eta}{\delta}$ , so that  $\delta \geq \alpha$  is sufficient for  $k_q < k_h$ . Most of the subsequent comparative-statics results do not depend on the sign of  $K$ .

The equations (A8) and (A13) can be transformed to

$$(A15) \quad b_q = \frac{\delta(1-\alpha)b_o}{(\alpha\eta/\pi_{be}) + \delta - \alpha}.$$

In view of (A14'') and (A15) the response of  $b_q$  to changes in  $b_o$  and  $\pi_{hq}$  is

$$(A16) \quad \hat{b}_q = \hat{b}_o - K K_b \hat{\pi}_{hq}$$

with  $K$  as defined above. From  $b_h = b_o - b_q$  or  $\hat{b}_h = (b_o/b_h)\hat{b}_q$  one has in view of (A16)

$$(A17) \quad \hat{b}_h = \hat{b}_o + \frac{b_q}{b_h} K K_b \hat{\pi}_{hq}.$$

Moreover, (A13) implies  $e_q = ((1-\delta)/\delta(\pi_{be}-\eta))b_q$  which yields, when combined with (A16)

$$(A18) \quad \hat{e}_q = \hat{b}_o + \left( \frac{\pi_{be}}{\pi_{be} - \eta} - K \right) K_b \hat{\pi}_{hq}.$$

Since  $e_h = \eta b_q - e_q$  by (A8), we also have

$$(A19) \quad \hat{e}_h = \hat{b}_o - (K + \frac{\alpha(1-\delta)b_q}{\delta(1-\alpha)b_h})K_b \hat{\pi}_{hq}$$

(A4), (A13) and (A17) yield

$$(A20) \quad \hat{h} = \hat{b}_o + A_h \hat{\pi}_{hq} \text{ with } A_h := (\frac{b_q}{b_h} K + 1 - \alpha)K_b.$$

Similarly, (A4), (A13) and (A16) give us

$$(A21) \quad \hat{q} = \hat{b}_o - A_q \hat{\pi}_{hq} \text{ with } A_q := K_b K_b [K - (1-\delta)(1 + \frac{(1-\delta)\eta}{\delta k_q})].$$

In general, the sign of the terms  $A_h$  and  $A_q$  is ambiguous. But since all production functions are assumed to be concave, the transformation curve are assumed to be concave, the transformation curve of the goods  $Q$  and  $H$  must be concave and downward sloping. As an implication, with increasing  $\pi_{hq}$  the quantity produced of good  $h$  must rise and that of good  $q$  must fall. In view of (A20) and (A21) this requires  $A_h > 0$  and  $A_q > 0$ . This must be true irrespective of the sign of  $K$  and  $K_b$ .

Observe that  $\hat{\pi}_{ha} = \hat{k}_y$  and  $\hat{\pi}_{qa} = \hat{k}_w$  is implied by (A10) and (A11) and that  $\hat{y} = \hat{h} + \beta \hat{k}_y$  and  $\hat{w} = \hat{q} + \xi \hat{k}_w$  in view of (A6) and (A7). this information implies

$$(A22) \quad \hat{w} - \hat{y} = \hat{q} - \hat{h} + \xi \hat{\pi}_{qa} - \beta \hat{\pi}_{ha}$$

Moreover, it follows from (A20), (A21) and (A14'') that

$$(A23) \quad \hat{q} - \hat{h} = -(A_q + A_h)(\hat{\pi}_{ha} - \hat{\pi}_{qa}).$$

Combine (A22) and (A23) with  $\hat{\gamma} = \hat{w} - \hat{y}$  from (A9) to obtain

$$(A24) \quad \hat{\gamma} = -B_h \hat{\pi}_{ha} + B_q \hat{\pi}_{qa}$$

with  $B_h := A_q + \beta$  and  $B_q := A_h + \xi$ . An additional equation in  $\hat{\pi}_{ha}$  and  $\hat{\pi}_{qa}$  can be constructed by considering  $\hat{a}_o = \lambda_{aw} \hat{a}_w + \lambda_{ay} \hat{a}_y = \lambda_{aw} \hat{k}_w + \lambda_{ay} \hat{k}_y + \lambda_{aw} \hat{q} + \lambda_{ay} \hat{h}$

from (A8) together with (A14''), (A20) and (A21):

$$(A25) \quad \hat{a}_o = C_h \hat{\pi}_{ha} - C_q \hat{\pi}_{qa} + \hat{b}_o,$$

where  $C_h := \lambda_{ay}A_h - \lambda_{aw}A_q + \lambda_{ay}$  and  $C_q := (\lambda_{ay}A_h - \lambda_{aw}A_q) - \lambda_{aw}$ . Solving (A24) and (A25) for  $\hat{\pi}_{ha}$  and  $\hat{\pi}_{qa}$  yields

$$(A26) \quad \hat{\pi}_{ha} = \overset{+}{B}_q \overset{-}{D}(\hat{b}_o - \hat{a}_o) - C_q D \hat{\gamma},$$

$$(A27) \quad \hat{\pi}_{qa} = \overset{+}{B}_h \overset{-}{D}(\hat{b}_o - \hat{a}_o) - C_h D \hat{\gamma}, \text{ and}$$

$$(A28) \quad \hat{\pi}_{hq} = \hat{\pi}_{ha} - \hat{\pi}_{qa} = (B_q - B_h)D(\hat{b}_o - \hat{a}_o) - (C_q - C_h)D\hat{\gamma},$$

where  $D := (B_h C_q - B_q C_h)^{-1}$ . It is demonstrated above that  $A_h > 0$  and  $A_q > 0$  must hold. Therefore the terms  $B_h$  and  $B_q$  are positive. Moreover, by definition we also have  $C_q < C_h$ . That leaves indeterminate the sign of  $C_q$ ,  $C_h$  and  $D$ . We wish to show that  $D < 0$  if  $\beta$  and  $\xi$  are not very different. For simplicity assume  $\beta = \xi = \theta$  in what follows and rewrite (A22) combined with (A23) as

$$(A29) \quad \hat{w} - \hat{y} = -(A_h + A_q + \theta)\hat{\pi}_{hq}.$$

From (A9) we know that  $\hat{\gamma} > 0$  implies  $\hat{w} > \hat{y}$ . Suppose  $D > 0$ . Then (A28) tells us that  $\hat{\pi}_{hq} > 0$  for  $\hat{\gamma} > 0$  (and  $\hat{a}_o = \hat{b}_o = 0$ ). But it follows from (A29) that  $\hat{\pi}_{hq} > 0$  means  $\hat{w} < \hat{y}$  contradicting the demand-side change  $\hat{w} > \hat{y}$ . Hence  $D < 0$ .

Consideration of (A28) and (A29) also shows that  $B_q = B_h$  must hold. Suppose not. Then for  $\hat{\gamma} = 0$  and  $\hat{b}_o - \hat{a}_o \neq 0$  (A28) yields  $\hat{\pi}_{hq} \neq 0$  and hence  $\hat{w} \neq \hat{y}$  by (A29). But (A9) implies  $\hat{w} = \hat{y}$  for  $\hat{\gamma} = 0$ . Hence  $B_q = B_h$ . (Note that  $B_q = B_h$  and  $C_q < C_h$  imply  $D < 0$ ). By definition of  $B_q$  and  $B_h$ , with  $B_q = B_h$  the terms  $A_q$  and  $A_h$  are of similar size. As can be empirically observed,  $a_y$  is substantially larger than  $a_w$ . Therefore  $C_q$  and  $C_h$  are assumed positive. With these presuppositions it is possible to develop the complete comparative statics.

**Appendix IV: Comparative statics for  $p_r = 0$ , when the technology of raw water quality is homogeneous of degree zero**

**The first stage of production**

Consider first the production function H as specified in (A4) and define

$$(A30) \quad Q(b_q, e_q) = (e_q/b_q)^\delta.$$

Differentiation gives

$$(A31) \quad \hat{h} = \alpha \hat{b}_h + (1-\alpha) \hat{e}_h,$$

$$(A32) \quad -\hat{q} = \delta(\hat{e}_q - \hat{b}_q).$$

Differentiation of the resource constraints  $b_o = b_h + b_q$  and  $e = e_h + e_q$ ,  $e = \eta b_q$  ( $\eta > 0$ ) from (12) and the raw water supply function  $r = \rho b_q$  ( $\rho > 0$ ) from (2) yields

$$(A33) \quad \hat{b}_o = \lambda_{bh} \hat{b}_h + \lambda_{bq} \hat{b}_q,$$

$$(A34) \quad \hat{e} = \lambda_{eh} \hat{e}_h + \lambda_{eq} \hat{e}_q,$$

$$(A35) \quad \hat{e} = \hat{b}_q \quad (\text{for } \hat{\eta} = 0),$$

$$(A36) \quad \hat{r} = \hat{b}_q \quad (\text{for } \hat{\rho} = 0).$$

In equilibrium firm H is characterised by zero profit and the raw water enterprise covers its costs according to assumption (17). Hence the price equations are given by  $p_h = c_{bh}p_b + c_{eh}p_e$  and  $p_b^n b_q = -p_e e_q$  with  $p_b^n := p_b - \eta p_e$ . Using the definition of  $p_b^n$  and the resource constraint  $\eta b_q = e_h + e_q$ , the cost covering condition is transformed into  $b_q = \pi_{eb} e_h$ . With the help of the hat calculus we obtain, therefore,

$$(A37) \quad \pi_{hb} = \Theta_{eh} \pi_{eb},$$

$$(A38) \quad \pi_{eb} = \hat{b}_q - \hat{e}_h.$$



Cost minimising production of the intermediate good H satisfies the marginal condition

$$\pi_{eb} = \frac{H_e(b_h, e_h)}{H_b(b_h, e_h)} = \frac{1-\alpha}{\alpha} \frac{b_h}{e_h} \text{ and hence}$$

$$(A39) \quad \hat{\pi}_{eb} = \hat{b}_h - \hat{e}_h.$$

Observe that our equation system consists of the 9 equations (A31) – (A39) but contains the 10 variables  $\hat{h}$ ,  $\hat{q}$ ,  $\hat{b}_h$ ,  $\hat{b}_q$ ,  $\hat{e}_h$ ,  $\hat{e}_q$ ,  $\hat{e}$ ,  $\hat{r}$ ,  $\hat{\pi}_{eb}$ ,  $\hat{\pi}_{hb}$ , implying a degree of freedom of one. (A38) and (A39) to substitute out  $\pi_{eb}$  and obtain

$$(A40) \quad \hat{b}_h = \hat{b}_q.$$

Moreover, substitute (A40) into (A33) and consider (A34) and (A36) to get

$$(A41) \quad \hat{b}_q = \hat{b}_h = \hat{e} = \hat{r} = \hat{b}_o$$

Employing this equation in (A32) yields

$$(A42) \quad \hat{e}_q = \hat{b}_o + \frac{1}{\delta} \hat{q}.$$

Plugging (A41) and (A42) into (A34) gives

$$(A43) \quad \hat{e}_h = \hat{b}_o - \frac{e_q}{\delta e_h} \hat{q}.$$

Combining the equations (A31), (A41) and (A43) leads to

$$(A44) \quad \hat{h} = \hat{b}_o - \frac{(1-\alpha)e_q}{e_h \delta} \hat{q}.$$

Next we consider (A37), (A41) and (A43) in the cost covering constraint (A38) and obtain as a result

$$(A45) \quad \hat{\pi}_{eb} = \frac{e_q}{\delta e_h} \hat{q},$$

$$(A46) \quad \hat{\pi}_{hb} = \Theta_{eh} \frac{e_q}{\partial e_h} \hat{q}; \quad \Theta_{eh} = 1 - \alpha.$$

The second stage of production:

Specify the production functions Y and W as in (A6) and (A7). Differentiation gives us

$$(A47) \quad \hat{y} = \beta \hat{a}_y + (1 - \beta) \hat{h},$$

$$(A48) \quad \hat{w} = \xi \hat{a}_w + (1 - \xi) \hat{q}.$$

Applying the hat calculus to the labor resource constraint  $a_o = a_w + a_y$  yields

$$(A49) \quad \hat{a}_o = \lambda_{aw} \hat{a}_w + \lambda_{ay} \hat{a}_y.$$

The price equations at the second stage of production are  $p_y = c_{ay} p_a + c_{hy} p_h$  and  $p_w = c_{aw} p_a + c_{qw} p_q$ , where  $g_w = p_q q$  is the profit in the water work. Note that  $p_q$  is a fictitious price. Its introduction is convenient for analytical purposes because the profit accruing to the water work and then being transferred to the consumers is equal to the share of the (unpaid) factor "water quality". The relative changes of the price equations are

$$(A50) \quad \hat{\pi}_{ya} = \Theta_{hy} \hat{\pi}_{ha} \quad \text{with } \Theta_{ay} = \beta \text{ and } \Theta_{hy} = 1 - \beta;$$

$$(A51) \quad \hat{\pi}_{wa} = \Theta_{qw} \hat{\pi}_{qa} \quad \text{with } \Theta_{aw} = \xi \text{ and } \Theta_{qw} = 1 - \xi;$$

$$(A52) \quad \hat{g}_w - \hat{p}_a = \hat{\pi}_{qa} + \hat{q}.$$

Cost minimisation in sector Y and profit maximisation in Sector W implies the marginal conditions

$$\pi_{ha} = \frac{Y_h(a_y, h)}{Y_a(a_y, h)} = \frac{(1-\beta)}{\beta} \frac{a_y}{h} \quad \text{and} \quad \pi_{qa} = \frac{W_q(a_w, q)}{W_a(a_w, q)} = \frac{(1-\xi)}{\xi} \frac{a_w}{q}.$$

We differentiate these terms to obtain

$$(A53) \quad \hat{\pi}_{ha} = \hat{a}_y - \hat{h} \text{ and}$$

$$(A54) \quad \hat{\pi}_{qa} = \hat{a}_w - \hat{q}.$$

With the demand side as given by (19), our model of the second stage of production contains the 11 variables  $\hat{y}$ ,  $\hat{a}_y$ ,  $\hat{h}$ ,  $\hat{w}$ ,  $\hat{a}_w$ ,  $\hat{q}$ ,  $\hat{\pi}_{ya}$ ,  $\hat{\pi}_{ha}$ ,  $\hat{\pi}_{wa}$ ,  $\hat{\pi}_{qa}$  and  $(\hat{g}_w - \hat{p}_a)$  but only the 10 equations (19), (A44), (A47), (A48), (A49), (A50), (A51), (A52), (A53) and (A54). We need an additional equation to determinate all variables. This problem arises because the intermediate goods Q and H – and hence their prices  $\pi_{qa}$  and  $\pi_{ha}$  – are not identical as in the standard two-sector two-factor model. One could use that degree of freedom by an ad hoc determination of the variable q or by determining its efficient level. In what follows we take the second route by using the marginal condition (21) from the Lagrangean (14). With the help of the parametric production functions and after some rearrangements of terms the equation (21) is turned into

$$(A55) \quad \frac{1 - \xi}{\xi} \frac{a_w}{e_q} \delta e_h = \frac{1 - \beta}{\beta} a_y.$$

Differentiation gives us our missing equation

$$(A56) \quad \hat{a}_y = \hat{a}_w + \hat{e}_h - \hat{e}_q$$

The next step is to solve the system of equations by performing a number of simple but tedious substitution steps. We only indicate briefly which equations are used.

(A42), (A43) and (A49) into (A56):

$$(A57) \quad \hat{a}_y = \hat{a}_o - \frac{ea_w}{\delta e_h a_o} \hat{q}.$$

(A57) into (A49):

$$(A58) \quad \hat{a}_w = \hat{a}_o + \frac{ea_y}{\delta e_h a_o} \hat{q}.$$

(A44) and (A57) into (A47):

$$(A59) \quad \hat{y} = \beta \hat{a}_0 + (1-\beta) \hat{b}_0 - \frac{\beta e a_w + (1-\beta)(1-\alpha) a_0 e_q}{a_0 \delta e_h} \hat{q}.$$

(A58) into (A48):

$$(A60) \quad \hat{w} = \xi \hat{a}_0 + \frac{\xi e a_y + (1-\xi) a_0 \delta e_h}{a_0 \delta e_h} \hat{q}.$$

(19), (A59) and (A60):

$$(A61) \quad \hat{q} = \frac{\beta - \xi}{A} \hat{a}_0 + \frac{1 - \beta}{A} \hat{b}_0 + \frac{1}{A} \hat{\gamma},$$

where A is defined as  $A := \frac{\xi e a_y + (1-\xi) a_0 \delta e_h + \beta e a_w + (1-\beta)(1-\alpha) a_0 e_q}{a_0 \delta e_h} > 0$ .

(A61) into (A45) and (A46):

$$(A62) \quad \pi_{eb} = \frac{e_q (\beta - \xi)}{A \delta e_h} \hat{a}_0 + \frac{e_q (1 - \beta)}{A \delta e_h} \hat{b}_0 + \frac{1}{A \delta e_h} \hat{\gamma}$$

$$(A63) \quad \pi_{hb} = \frac{\Theta_{eh} e_q (\beta - \xi)}{A \delta e_h} \hat{a}_0 + \frac{\Theta_{eh} e_q (1 - \beta)}{A \delta e_h} \hat{b}_0 + \frac{\Theta_{eh}}{A \delta e_h} \hat{\gamma}.$$

The central equation (A61) is now successively plugged into the equations (A42), (A43), (A44), (A57), (A58) (A59), (A60):

$$(A64) \quad \hat{e}_q = \frac{\beta - \xi}{\delta A} \hat{a}_0 + \frac{\delta A + (1 - \beta)}{\delta A} \hat{b}_0 + \frac{1}{\delta A} \hat{\gamma}$$

$$(A65) \quad \hat{e}_h = -\frac{e_q (\beta - \xi)}{A \delta e_h} \hat{a}_0 + \frac{a_y \xi e + a_0 \delta e_h (1 - \xi) + \beta e a_w - a_0 \alpha (1 - \beta) e_q}{a_0 A \delta e_h} \hat{b}_0 - \frac{e_q}{A \delta e_h} \hat{\gamma}$$

$$(A66) \quad \hat{h} = -\frac{(1 - \alpha) e_q (\beta - \xi)}{A \delta e_h} \hat{a}_0 + \frac{a_y \xi e / a_0 + \delta e_h (1 - \xi) + \beta e a_w / a_0}{A \delta e_h} \hat{b}_0 - \frac{(1 - \alpha) e_q}{A \delta e_h} \hat{\gamma}$$

$$(A67) \quad \hat{a}_y = \frac{\delta \xi e + \delta e_h (1 - \xi) + \delta (1 - \alpha) (1 - \beta) e_h}{A \delta e_h} \hat{a}_0 - \frac{a_w e (1 - \beta)}{a_0 A \delta e_h} \hat{b}_0 - \frac{e a_w}{a_0 A \delta e_h} \hat{\gamma}$$

$$(A68) \quad \hat{a}_w = \frac{\delta(1-\xi)e_h + \delta\beta e + \delta(1-\alpha)(1-\beta)e_q}{A\delta e_h} \hat{a}_o + \frac{ea_y(1-\beta)}{a_o A\delta e_h} \hat{b}_o + \frac{ea_y}{a_o A\delta e_h} \hat{\gamma}$$

$$(A69) \quad \hat{y} = \frac{(1-\beta)\xi ea_y + \beta(1-\xi)a_o e_h + \beta\xi ea_w + \xi(1-\alpha)e_q a_o}{a_o A\delta e_h} \hat{a}_o +$$

$$+ \frac{(1-\beta)\xi ea_y + (1-\beta)(1-\xi)a_o e_h + (1-\beta)(1-\alpha)\beta a_o e_q}{a_o A\delta e_h} \hat{b}_o -$$

$$- \frac{\beta ea_w + (1-\alpha)e_q a_o}{a_o A\delta e_h} \hat{\gamma}$$

$$(A70) \quad \hat{w} = \frac{\beta\xi ea_o + a_o e_q \xi(1-\alpha) + e_h a_o(1-\xi)\beta}{a_o A\delta e_h} \hat{a}_o +$$

$$+ \frac{ea_y \xi + e_h a_o(1-\xi)}{a_o A\delta e_h} (1-\beta) \hat{b}_o + \frac{ea_y \xi + e_h a_o(1-\xi)}{a_o A\delta e_h} \hat{\gamma}$$

Using (A37) and (A70) the excess supply of raw water changes according to

$$(A71) \quad \hat{r} - \hat{w} = - \frac{\beta\xi e + e_q \xi(1-\alpha) + e_h(1-\xi)\beta}{A\delta e_h} \hat{a}_o - \frac{ea_y \xi + e_h a_o(1-\xi)}{a_o A\delta e_h} \hat{\gamma} +$$

$$+ \frac{\beta ea_w + a_o(1-\alpha)e_q + ea_y \xi\beta + e_h a_o(1-\xi)\beta}{a_o A\delta e_h} \hat{b}_o$$

Moreover, subtract (A53) from (A54):

$$(A72) \quad \hat{\pi}_{qh} = \frac{\alpha e_q}{e_h A} (\beta - \xi) \hat{a}_o + \frac{e_h A + \alpha e_q(1-\beta)}{e_h A} \hat{b}_o + \frac{\alpha e_q}{A} \hat{\gamma}$$

Put (A66), (A67) into (A53) and (A61), (A68) into (A54):

$$(A73) \quad \hat{\pi}_{ha} = \frac{\delta\xi e_q + \delta e_h + \delta(1-\alpha)(1-\beta)e_h + (1-\alpha)(\beta-\xi)e_q}{A\delta e_h} \hat{a}_o -$$

$$- \frac{a_w e(1-\beta) + a_y \xi e + a_o \delta e_h(1-\xi) + \beta ea_w}{a_o A\delta e_h} \hat{b}_o + \frac{a_o(1-\alpha)e_q - a_w e}{a_o A\delta e_h} \hat{\gamma}$$

$$(A74) \quad \hat{\pi}_{qa} = \frac{\delta e_h + \delta \beta e_q + \delta(1-\alpha)(1-\beta)e_q}{A \delta e_h} \hat{a}_o + \frac{(1-\beta)(e a_y - a_o \delta e_h)}{a_o A \delta e_h} \hat{b}_o \\ + \frac{e a_y - a_o \delta e_h}{a_o A \delta e_h} \hat{\gamma}$$

Thus we get the following information by using (A50) and (A51)

$$(A75) \quad \hat{\pi}_{ya} = \Theta_{hy} \frac{\delta \xi e_q + \delta e_h + \delta(1-\alpha)(1-\beta)e_h + (1-\alpha)(\beta-\xi)e_q}{A \delta e_h} \hat{a}_o - \\ - \Theta_{hy} \frac{a_w e(1-\beta) + a_y \xi e + a_o \delta e_h(1-\xi) + \beta e a_w}{a_o A \delta e_h} \hat{b}_o + \Theta_{hy} \frac{a_o(1-\alpha)e_q - a_w e}{a_o A \delta e_h} \hat{\gamma}$$

$$(A76) \quad \hat{\pi}_{wa} = \Theta_{qw} \frac{\delta e_h + \delta \beta e_q + \delta(1-\alpha)(1-\beta)e_q}{A \delta e_h} \hat{a}_o + \Theta_{qw} \frac{(1-\beta)(e a_y - a_o \delta e_h)}{a_o A \delta e_h} \hat{b}_o \\ + \Theta_{qw} \frac{e a_y - a_o \delta e_h}{a_o A \delta e_h} \hat{\gamma}$$

Finally combine (A52), (A54), (A68) to obtain:

$$(A77) \quad \hat{g}_w - \hat{p}_a = \hat{a}_w = \frac{\delta(1-\xi)e_h + \delta \beta e + \delta(1-\alpha)(1-\beta)e_q}{A \delta e_h} \hat{a}_o + \frac{e a_y(1-\beta)}{a_o A \delta e_h} \hat{b}_o + \\ + \frac{e a_y}{a_o A \delta e_h} \hat{\gamma}$$

Table 2b in Section 3 summarises the results of this Appendix.

**Seit 1989 erschienene Diskussionsbeiträge:  
Discussion papers released as of 1989/1990:**

- 1-89 **Klaus Schöler**, Zollwirkungen in einem räumlichen Oligopol
- 2-89 **Rüdiger Pethig**, Trinkwasser und Gewässergüte. Ein Plädoyer für das Nutzerprinzip in der Wasserwirtschaft
- 3-89 **Rüdiger Pethig**, Calculus of Consent: A Game-theoretic Perspective. Comment
- 4-89 **Rüdiger Pethig**, Problems of Irreversibility in the Control of Persistent Pollutants
- 5-90 **Klaus Schöler**, On Credit Supply of PLS-Banks
- 6-90 **Rüdiger Pethig**, Optimal Pollution Control, Irreversibilities, and the Value of Future Information
- 7-90 **Klaus Schöler**, A Note on "Price Variation in Spatial Markets: The Case of Perfectly Inelastic Demand"
- 8-90 **Jürgen Eichberger and Rüdiger Pethig**, Constitutional Choice of Rules
- 9-90 **Axel A. Weber**, European Economic and Monetary Union and Asymmetries and Adjustment Problems in the European Monetary System: Some Empirical Evidence
- 10-90 **Axel A. Weber**, The Credibility of Monetary Target Announcement: An Empirical Evaluation
- 11-90 **Axel A. Weber**, Credibility, Reputation and the Conduct of Economic Policies Within the European Monetary System
- 12-90 **Rüdiger Ostermann**, Deviations from an Unidimensional Scale in the Unfolding Model
- 13-90 **Reiner Wolff**, Efficient Stationary Capital Accumulation Structures of a Biconvex Production Technology
- 14-90 **Gerhard Brinkmann**, Finanzierung und Lenkung des Hochschulsystems – Ein Vergleich zwischen Kanada und Deutschland
- 15-90 **Werner Güth and Rüdiger Pethig**, Illegal Pollution and Monitoring of Unknown Quality – A Signaling Game Approach
- 16-90 **Klaus Schöler**, Konsistente konjekturale Reaktionen in einem zweidimensionalen räumlichen Wettbewerbsmarkt
- 17-90 **Rüdiger Pethig**, International Environmental Policy and Enforcement Deficits

